Arizona State University
POLYTECHNIC CAMPUS

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MAGNETIC CATALYSIS IN QCD IN A SUPERSTRONG MAGNETIC FIELD

XIV International Workshop on Hadron Physics

Lecture #2

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TOY MODEL

- Let us consider a Nambu-Jona-Lasino model ($m = 0$) with four-fermion contact interaction

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu} D_{\mu}\right)\psi + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma^5 \psi)^2]$$

- After the Hubbard–Stratonovich transformation, this is equivalent to

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu} D_{\mu} - \sigma - i\gamma^5 \pi\right)\psi - \frac{\sigma^2 + \pi^2}{2G}$$

where the following composite fields were introduced

$$\sigma = -G\,\bar{\psi}\psi \quad \text{and} \quad \pi = -\mathrm{G}\,\bar{\psi} i\gamma^5 \psi$$

- The effective action for the composite fields reads

$$\Gamma(\sigma, \pi) = -\frac{1}{2G}\int d^4x(\sigma^2 + \pi^2) - i\,\mathrm{Tr}\ln\left[i\gamma^{\mu} D_{\mu} - \sigma - i\gamma^5 \pi\right]$$

Symmetry of the Model

- $U_L(1)$ symmetry transformations, $\psi \to e^{i\alpha_L(1-\gamma^5)/2}\psi$

$$\bar{\psi}\psi \to \cos\alpha_L\,\bar{\psi}\psi - \sin\alpha_L\,\bar{\psi}i\gamma^5\psi$$
$$\bar{\psi}i\gamma^5\psi \to \sin\alpha_L\,\bar{\psi}\psi + \cos\alpha_L\,\bar{\psi}i\gamma^5\psi$$

- $U_R(1)$ symmetry transformations, $\psi \to e^{i\alpha_R(1+\gamma^5)/2}\psi$

$$\bar{\psi}\psi \to \cos\alpha_R\,\bar{\psi}\psi + \sin\alpha_R\,\bar{\psi}i\gamma^5\psi$$
$$\bar{\psi}i\gamma^5\psi \to -\sin\alpha_R\,\bar{\psi}\psi + \cos\alpha_R\,\bar{\psi}i\gamma^5\psi$$

- In terms of the composite fields, $U_L(1)$ / $U_R(1)$ transformations:

$$\sigma \to \cos\alpha_L\,\sigma - \sin\alpha_L\,\pi$$
$$\pi \to \sin\alpha_L\,\pi + \cos\alpha_L\,\sigma$$

(Note that $\rho^2 = \sigma^2 + \pi^2$ remains an invariant.)

- Just like the original action $\int \mathcal{L}\, d^4x$, the effective action $\Gamma(\sigma,\pi)$ should be invariant under the symmetry transformations, i.e.,

$$\Gamma(\sigma,\pi) = \Gamma(\rho) + \frac{1}{2} f_1^{\mu\nu}\left(\partial_\mu\sigma\partial_\nu\sigma + \partial_\mu\pi\partial_\nu\pi\right) + \cdots$$

EFFECTIVE POTENTIAL: DERIVATION

- Let us consider a homogeneous ground state with a *uniform* σ

$$\sigma = -G\langle\bar{\psi}\psi\rangle \neq 0$$

(Because of the chiral symmetry, we can always set $\pi = 0$.)

- In this case, $\Gamma(\sigma) = -\int V(\sigma) d^4x$, where the effective action is

$$V(\sigma) = \frac{\sigma^2}{2G} - \frac{i}{2}\int_0^\infty \frac{ds}{s}\,\mathrm{tr}\left\langle x\middle| e^{-is\left(D^\mu D_\mu - i\gamma^1\gamma^2 eB + \sigma^2\right)}\middle| x\right\rangle - (\infty)$$

- By using the Schwinger result [Phys. Rev. 82, 664 (1951)]

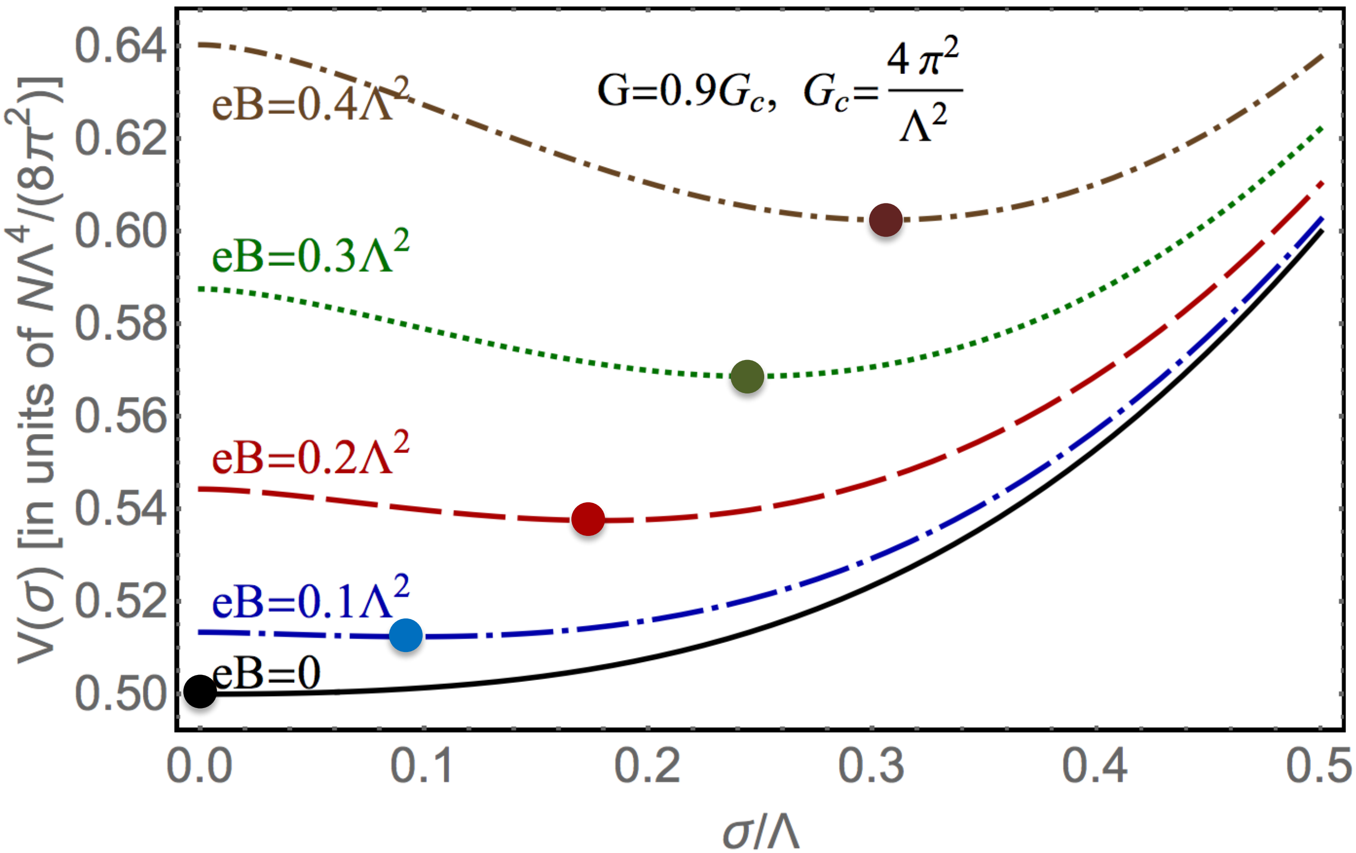
$$\left\langle x\middle| e^{-is\left(D^\mu D_\mu - i\gamma^1\gamma^2 eB + \sigma^2\right)}\middle| x\right\rangle = \frac{e^{-is\sigma^2 - i\pi/4}}{8(\pi s)^{3/2}} eBs[\cot eBs + \gamma^1\gamma^2]$$

- We derive the effective potential (after $s \to -is$):

$$V(\sigma) = \frac{\sigma^2}{2G} + \frac{eB}{8\pi^2}\int_{1/\Lambda^2}^\infty \frac{ds}{s^2} e^{-s\sigma^2} \coth eBs - (\infty)$$

Effective potential: Results

Lowest energy ground state is defined by: $\frac{dV(\sigma)}{d\sigma} = 0$ (gap equation)

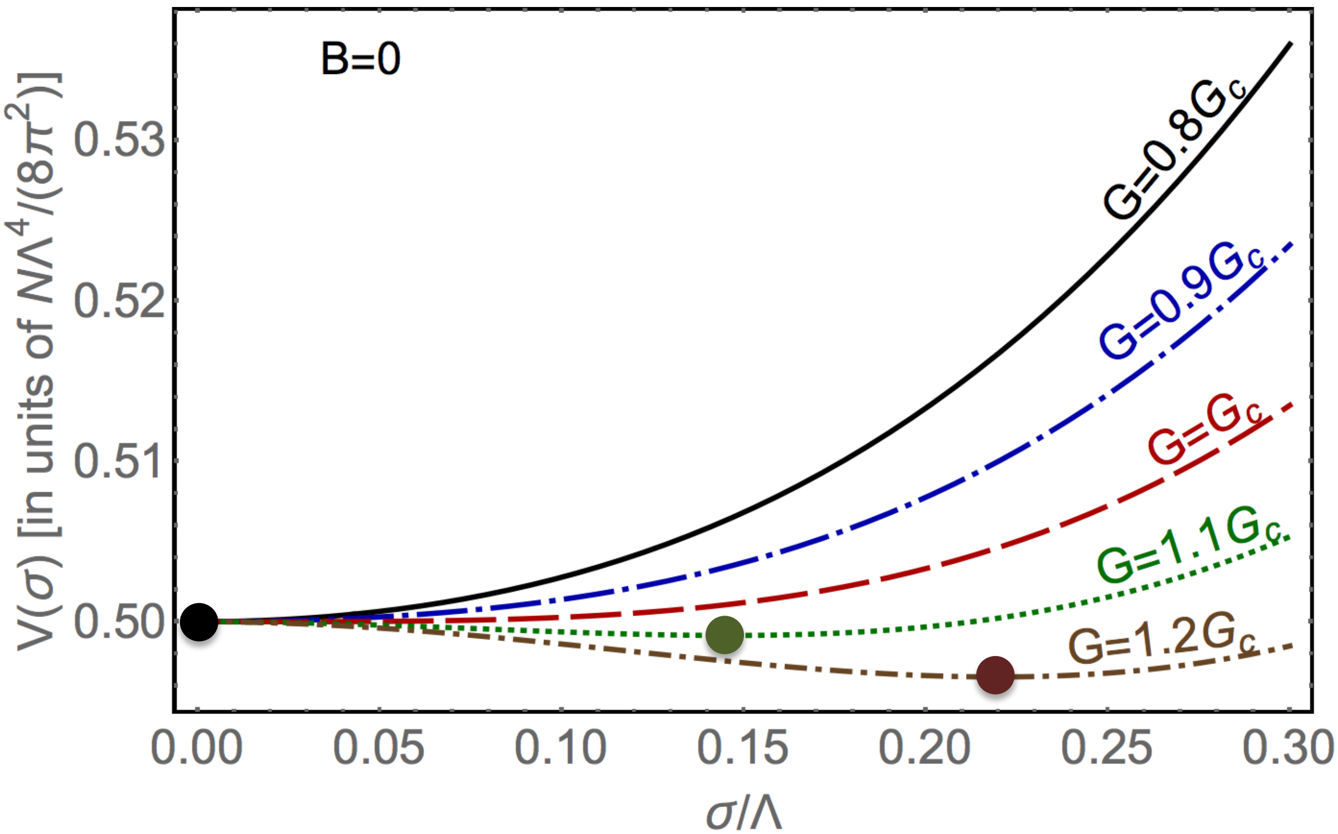


At weak coupling ($G \to 0$), the analytical solution for the minimum

$$\sigma_{\min} \simeq \frac{eB}{\pi} \exp\left(\frac{\Lambda^2}{|eB|}\right) \exp\left(-\frac{4\pi^2}{|eB|G}\right)$$

Compare with B=0 case

- Effective potentials for different coupling constants

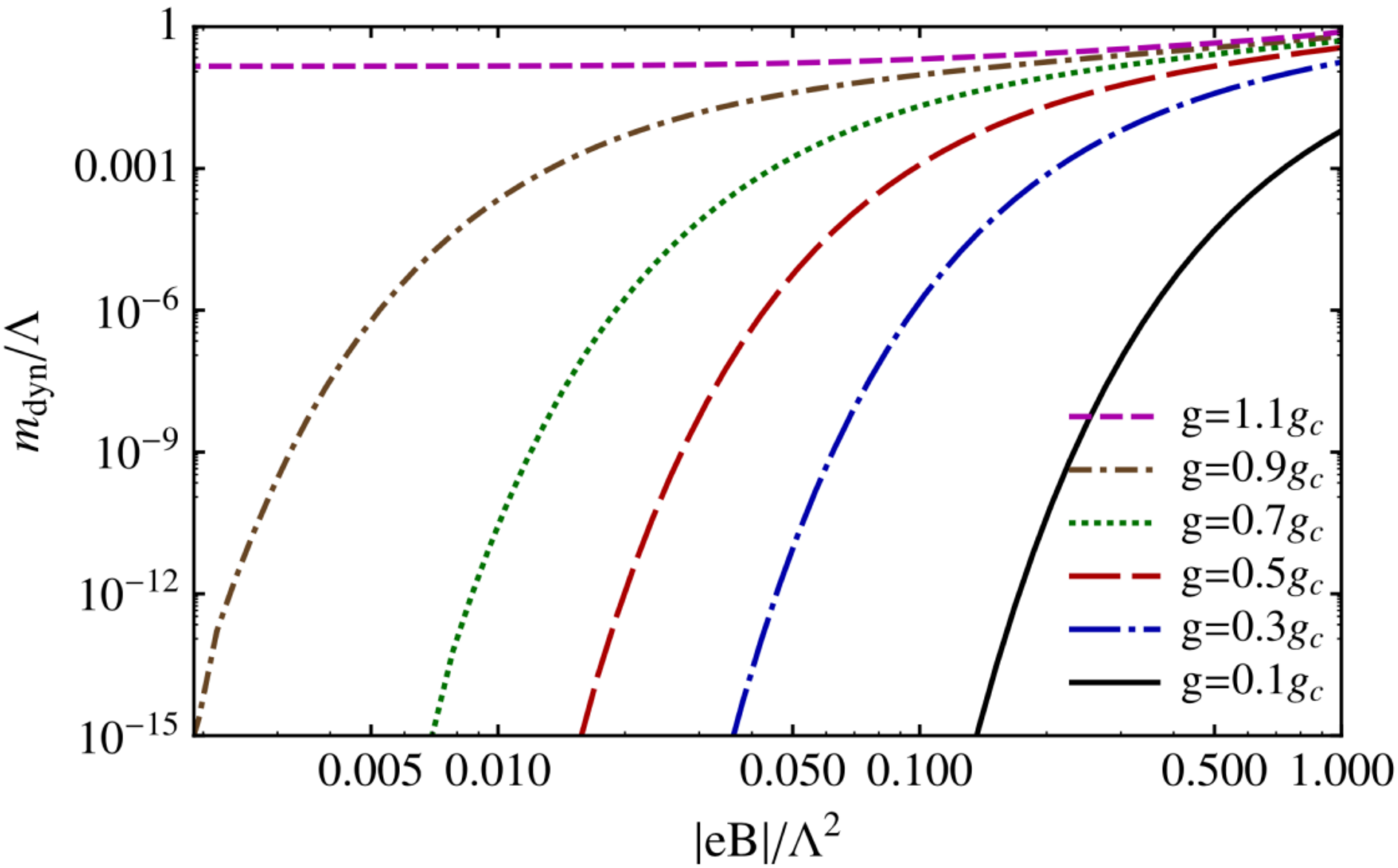


In fact, the gap equation at B=0 reads $\frac{G\Lambda^2-4\pi^2}{G}=\sigma^2\ln\frac{\Lambda^2}{\sigma^2}$

It has a nontrivial solution $\sigma_{\min}\neq 0$ only when the coupling strength is sufficiently strong, i.e., $G>G_c=4\pi^2/\Lambda^2$

Dynamical mass

- Recall: $\mathcal{L} = \bar{\psi}\left(i\gamma^\mu D_\mu - \sigma - i\gamma^5\pi\right)\psi - \frac{\sigma^2+\pi^2}{2G}$
- The ground state expectation value $\langle\sigma\rangle = \sigma_{\min}$ determines the dynamical mass of fermions m_{dyn} in the new Dirac vacuum



- Also, the chiral symmetry is broken in a state with $\langle\sigma\rangle \neq 0$

Nambu-Goldstone bosons

- When a continuous global symmetry breaks down, massless Nambu-Goldstone bosons appear in the particle spectrum

$$(D_\pi)^{-1} = \text{------} + \text{---}\bigcirc\text{---} = \frac{\delta^4(x)}{G} + i\,\mathrm{tr}[G(x,0)i\gamma^5 G(0,x)i\gamma^5]$$

- The dispersion relation of NG bosons at $\vec{p} \to 0$

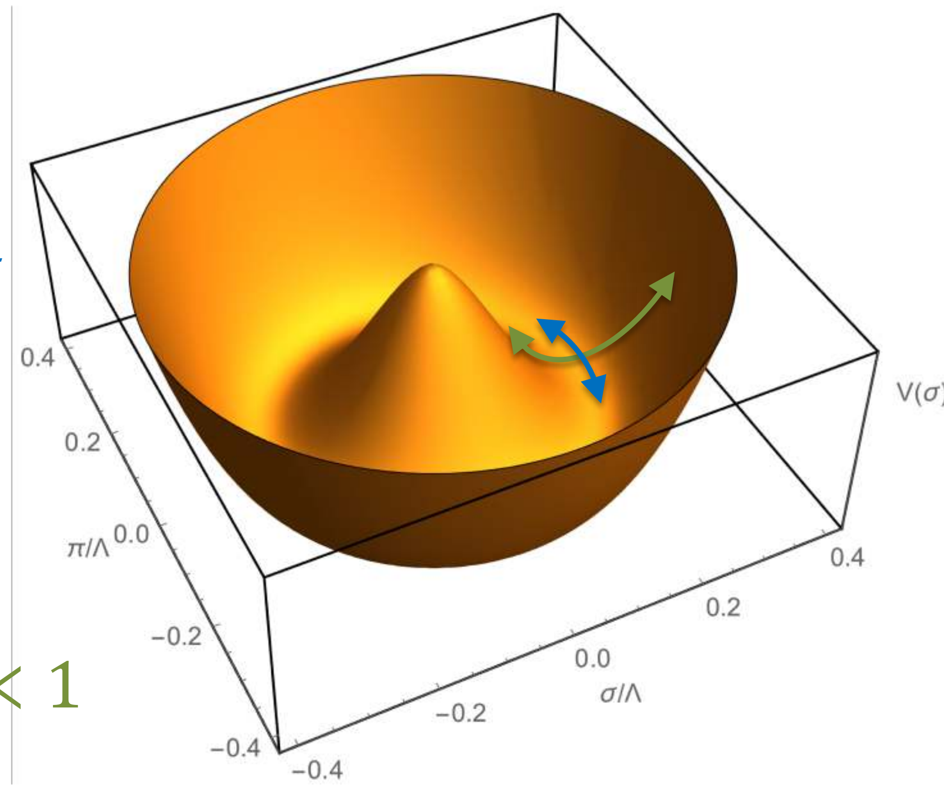
$$E_\pi = \sqrt{v_{\pi,\perp}^2 \vec{p}_\perp^{\,2} + p_z^2}$$

where $v_{\pi,\perp} \ll 1$ at *weak coupling*

- The relation for the σ-boson

$$E_\sigma = \sqrt{M_\sigma^2 + v_{\sigma,\perp}^2 \vec{p}_\perp^{\,2} + p_z^2}$$

where $M_\sigma = 2\sqrt{3} m_{dyn}$ & $v_{\sigma,\perp} \ll 1$



NONZERO TEMPERATURE

- Partition function:

$$Z_{T,\mu} = \mathrm{Tr}\left[\exp\left(-\frac{H-\mu N}{T}\right)\right]$$

$$= \int [d\psi d\bar{\psi} d\sigma d\pi] \exp\left(i \int_0^{-i/T} dt \int d^3x \left[\bar{\psi}\left(i\gamma^\mu D_\mu - \sigma - i\gamma^5\pi\right)\psi - \frac{\sigma^2+\pi^2}{2G}\right]\right)$$

where the fermion/boson fields satisfy (anti)periodic boundary conditions in imaginary time, e.g., $\psi(0) = -\psi(-i/T)$

- **Note #1:** $Z_{T,\mu}$ is similar to the generating functional at T=0
- **Note #2:** Hubbard–Stratonovich trick ⟺ Gaussian integral
- The effective potential is similar to that at T=0, but with the energy integration replaced by the Matsubara sum:

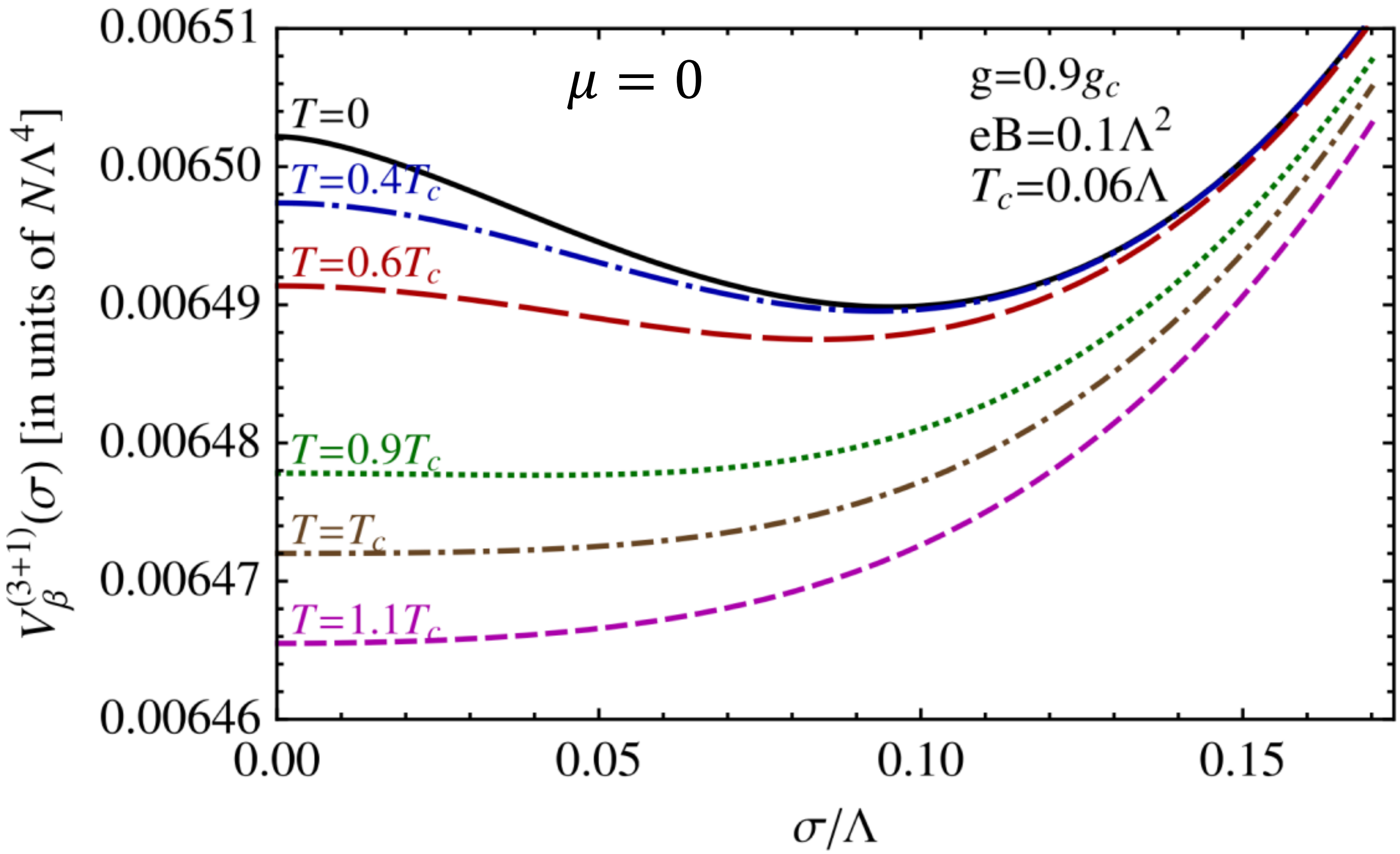
$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{is\omega^2}(\ldots) \to iT \sum_{n=-\infty}^{\infty} e^{is(i\omega_n)^2}(\ldots)$$

where $\omega \to i\omega_n = i\pi T(2n+1)$

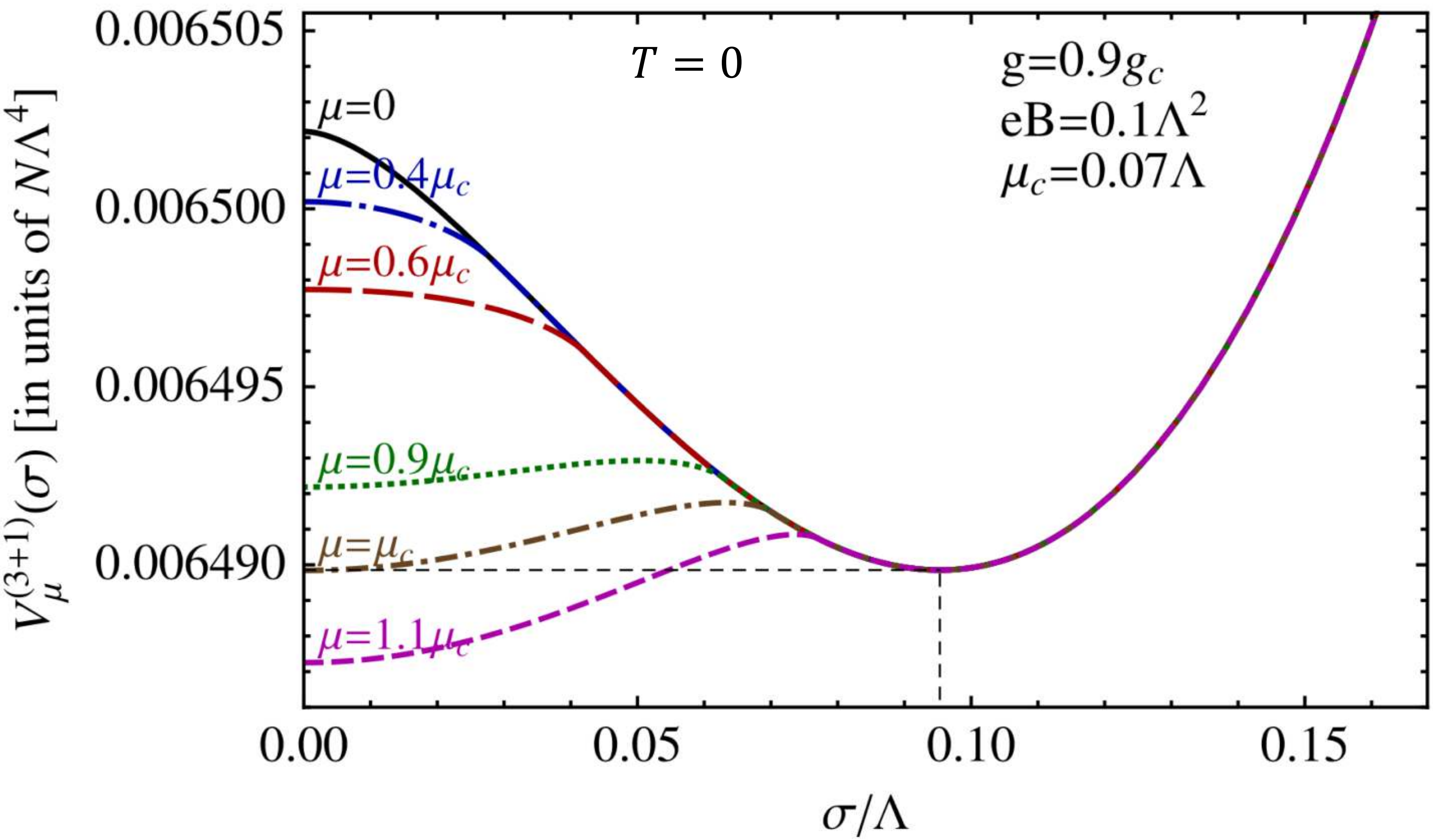
Effects of nonzero temperature

$$V_{\beta,\mu}(\rho) = V(\rho) - \frac{N}{2\beta\pi^2 l^2}\int_0^\infty dk_3 \left\{ \ln\left[1+e^{-\beta\left(\sqrt{\rho^2+k_3^2}-\mu\right)}\right] \right.$$

$$\left. + \; 2\sum_{n=1}^{\infty}\ln\left[1+e^{-\beta\left(\sqrt{\rho^2+k_3^2+2n/l^2}-\mu\right)}\right] + (\mu\to-\mu)\right\}$$



EFFECTS OF NONZERO CHEMICAL POTENTIAL



Notice that at $T = 0$ the chemical potential μ has no effect on the effective potential when $\sigma > \mu$ (This is not true at $T \neq 0$)

Symmetry breaking: Methods used

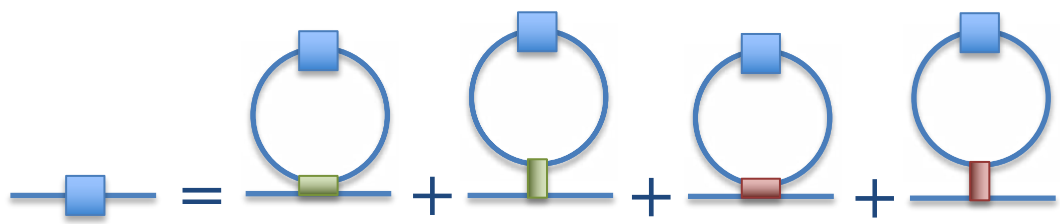
- Effective potential for the composite field, e.g., $\sigma = -G\,\bar{\psi}\psi$

$$\frac{dV(\sigma)}{d\sigma} = 0 \quad \text{(gap equation)}$$

- In NJL, e.g., $V_{NJL}(\sigma) = \frac{\sigma^2}{2G} + i\,\mathrm{tr}\ln\left[i\gamma^\mu D_\mu - \sigma\right]$, giving

$$\frac{\sigma}{G} - i\,\mathrm{tr}\left[\frac{1}{i\gamma^\mu D_\mu - \sigma}\right] = 0 \qquad \Longrightarrow \qquad \sigma = G\,\mathrm{tr}[G(x,x)]$$

- The same gap equation can be obtained from the Schwinger-Dyson equation for the fermion self-energy/propagator

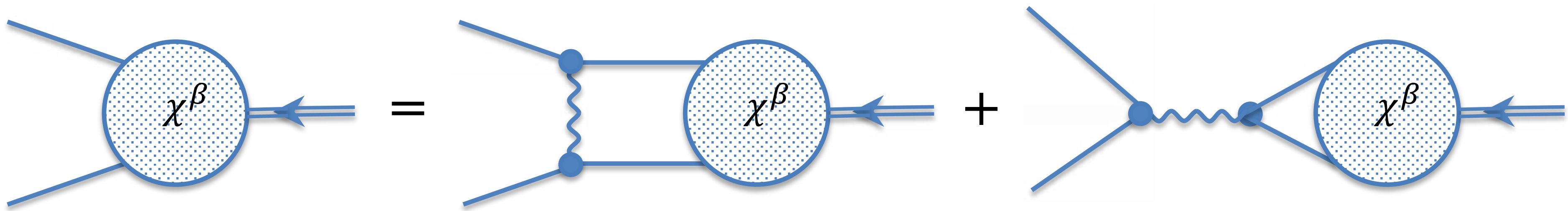


$$G^{-1}(x,x') - G_0^{-1}(x,x') = -iG\sum_i \Gamma_i[G(x,x)\Gamma_i - \mathrm{tr}\{G(x,x)\Gamma_i\}]\,\Gamma_i\delta^4(x-x')$$

where ansatz $G^{-1}(x,x') = -i\,\left(i\gamma^\mu D_\mu - m_{dyn}\right)\delta^4(x-x')$ is used

ANOTHER WAY: PION AS A BOUND STATE

- Homogeneous Bethe-Salpeter equation for a *massless* bound state with quantum numbers of the NG boson



- As we'll see, in NJL model in the strong-field limit, the pion's wave function in momentum space should have the structure:

$$\chi(p; P \to 0) = A(p_\parallel)e^{-p_\perp^2 l^2} \frac{\omega\gamma^0 - p_z\gamma^3 - m}{\omega^2 - p_z^2 - m^2}\gamma^5\mathcal{P}_+ \frac{\omega\gamma^0 - p_z\gamma^3 - m}{\omega^2 - p_z^2 - m^2}$$

where $A(p_\parallel)$ with $p_\parallel = (\omega, p_z)$ satisfies a simple integral equation

$$A(p_{\parallel,E}) = \frac{G|eB|}{4\pi^3}\int \frac{A(k_{\parallel,E})d^2k_{\parallel,E}}{k_{\parallel,E}^2 + m^2}$$

(here mass parameter *m* is treated as a variational parameter)

AUXILIARY SCHRODINGER PROBLEM

- It is instructive to recast the problem in terms of

$$\Psi(r_\parallel) = \int \frac{d^2k_\parallel}{(2\pi)^2} \frac{e^{-ir_\parallel\cdot k_\parallel}}{k_\parallel^2 + m^2} A(k_\parallel)$$

- Function $\Psi(r_\parallel)$ satisfies the following 2D Schrodinger equation:

$$\left[-\nabla_{r_\parallel}^2 + m^2 + V(r_\parallel)\right] \Psi(r_\parallel) = 0$$

where $-m^2$ plays the role of energy ϵ, and $V(r_\parallel)$ is a model-dependent potential (as we will see later)

- In the NJL model, $V(r_\parallel)$ is proportional to a δ-function

$$V(r_\parallel) = -\frac{G|eB|}{\pi}\delta_\Lambda^2(r_\parallel) = -\frac{G|eB|}{\pi}\int_0^\Lambda \frac{d^2k_\parallel}{(2\pi)^2} e^{-ir_\parallel\cdot k_\parallel}$$

- There exists a bound state solution ($\epsilon_b < 0$) in this Schrodinger problem and, thus, also a real solution for m, i.e.,

$$m^2 = -\epsilon_b \simeq \Lambda^2 \exp\left(-\frac{4\pi^2}{|eB|G}\right)$$ (LLL & weak coupling) ✓

Magnetic catalysis in QED

- Lagrangian density invariant under $\mathrm{SU_L}(N_f)\times\mathrm{SU_R}(N_f)\times\mathrm{U}(1)$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}_f\left(i\gamma^\mu D_\mu\right)\psi_f$$

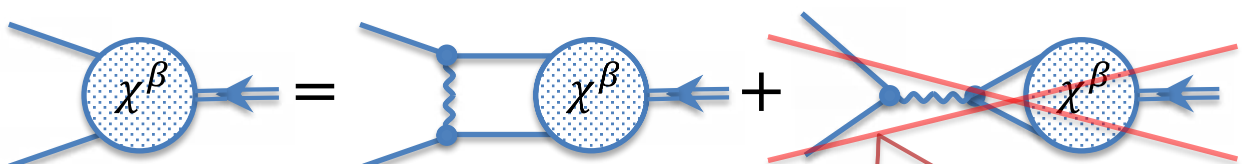
where $D_\mu = \partial_\mu + ie\left(A_\mu + a_\mu\right)$ and $F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$

- The Bethe–Salpeter equation for NG states $(\beta = 1, \ldots, N_f^2 - 1)$:

$$\chi^\beta_{AB}(u, u'; P) = -i\int d^4u_1 d^4u_1' d^4u_2 d^4u_2' G_{AA_1}(u, u_1) K_{A_1B_1;A_2B_2}(u_1u_1', u_2u_2')\chi^\beta_{A_2B_2}(u_2, u_2'; P) G_{B_1B}(u_2', u')$$

where the wave function is defined by $\chi^\beta_{AB} = \langle 0|T\psi_A(u)\bar{\psi}_B(u')|P;\beta\rangle$

Diagrammatically



where the kernel (in the ladder approximation) is

Hartree term plays no role for NG bound states

$$K_{A_1B_1;A_2B_2}(u_1u_1', u_2, u_2') = -4\pi i\alpha\delta_{a_1a_2}\delta_{b_2b_1}\gamma^\mu_{n_1n_2}\gamma^\nu_{m_2m_1}\mathcal{D}_{\mu\nu}(u_2' - u_2)\delta(u_1 - u_2)\delta(u_1' - u_2')$$
$$+\cancel{4\pi i\alpha\delta_{a_1b_1}\delta_{b_2a_2}\gamma^\mu_{n_1m_1}\gamma^\nu_{m_2n_2}\mathcal{D}_{\mu\nu}(u_1 - u_2)\delta(u_1 - u_1')\delta(u_2 - u_2')}$$

SOLUTION IN STRONG FIELD LIMIT

- Structure of the NG-boson wave function ($r_\mu = u_\mu - u'_\mu$):

$$\chi^\beta_{AB}(u,u';P) = \lambda^\beta_{ab} e^{-iPR} \exp\left[-ier^\mu A^{\text{ext}}_\mu(R)\right] \tilde{\chi}_{nm}(R,r;P)$$

- In the LLL approximation, the equation reduces to

$$\varphi(p_\parallel) = \frac{\pi\alpha}{(2\pi)^4}\int d^2k_\parallel \left(1 - i\gamma^1\gamma^2\right)\gamma^\mu \frac{\hat{k}_\parallel + m_{\text{dyn}}}{k_\parallel^2 - m_{\text{dyn}}^2}\varphi(k_\parallel)\frac{\hat{k}_\parallel + m_{\text{dyn}}}{k_\parallel^2 - m_{\text{dyn}}^2}\gamma^\nu \left(1 - i\gamma^1\gamma^2\right)\boxed{D^\parallel_{\mu\nu}(k_\parallel - p_\parallel)}$$

where we introduced $(\hat{p}_\parallel - m_{\text{dyn}})\tilde{\chi}(p)(\hat{p}_\parallel - m_{\text{dyn}}) = \exp(-l^2\mathbf{p}_\perp^2)\varphi(p_\parallel)$

and
$$\boxed{D^\parallel_{\mu\nu}(k_\parallel - p_\parallel) = i\pi\delta_{\mu\nu}\int_0^\infty \frac{dx\exp(-l^2x/2)}{(k_\parallel - p_\parallel)^2 + x}}$$

- The solution should have the following Dirac structure

$$\varphi(p_\parallel) = A\gamma_5\left(1 - i\gamma_1\gamma_2\right)$$

Compare with the NJL model

- Finally, the equation for $A(p_\parallel)$ reads

$$A(p_\parallel) = \frac{\alpha}{2\pi^2}\int \frac{A(k_\parallel)d^2k_\parallel}{k_\parallel^2 + m_{dyn}^2}\boxed{\int_0^\infty dx\frac{e^{-xl^2/2}}{x + (k_\parallel - p_\parallel)^2}}$$

REDUCE TO A SCHRODINGER PROBLEM

- Rewrite the problem in terms of

$$\Psi(\mathbf{r}) = \int \frac{d^2 k_\parallel}{(2\pi)^2} \frac{e^{i\mathbf{r}\cdot k_\parallel}}{k_\parallel^2 + m_{dyn}^2} A(k_\parallel)$$

- Function $\Psi(\mathbf{r})$ satisfies the following 2D Schrodinger equation:

$$\left[-\nabla_{\mathbf{r}}^2 + m_{dyn}^2 + V(\mathbf{r})\right] \Psi(\mathbf{r}) = 0$$

where

$$V(\mathbf{r}) = -\frac{\alpha}{2\pi^2} \int d^2 p e^{i\mathbf{p}\cdot\mathbf{r}} \int_0^\infty \frac{dx \exp(-x/2)}{l^2 p^2 + x} = \frac{\alpha}{\pi l^2} \exp\left(\frac{r^2}{2l^2}\right) \mathrm{Ei}\left(-\frac{r^2}{2l^2}\right)$$

- The potential is long-ranged with the following asymptote

$$V(\mathbf{r}) \simeq -\frac{2\alpha}{\pi} \frac{1}{r^2}, \quad r \to \infty$$

- The lowest energy bound state gives

$$m_{dyn} \simeq C\sqrt{|eB|} \exp\left[-\frac{\pi}{2}\left(\frac{\pi}{2\alpha}\right)^{1/2}\right]$$

(LLL & weak coupling) ✓

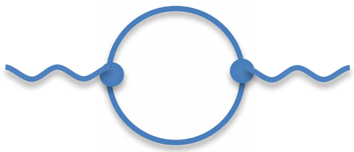
$\exp(-C/\sqrt{\alpha})$ is the result of a long-range interaction

No screening   not good

- Photon exchange interaction is screened in a strong B-field

$$\mathcal{D}^{-1}_{\mu\nu}(u,u') = D^{-1}_{\mu\nu}(u-u') + \Pi_{\mu\nu}(u,u')$$

strong-B limit

where $\Pi_{\mu\nu} \equiv$  $\simeq \left(q^{\parallel}_{\mu} q^{\parallel}_{\nu} - q^2_{\parallel} g^{\parallel}_{\mu\nu}\right) \boxed{e^{-q^2_{\perp} l^2}\Pi\left(q^2_{\parallel}\right)}$

- Then, the screened photon propagator reads

$$\mathcal{D}_{\mu\nu}(q) = -i\left[\frac{1}{q^2} g^{\perp}_{\mu\nu} + \frac{q^{\parallel}_{\mu} q^{\parallel}_{\nu}}{q^2 q^2_{\parallel}} + \frac{1}{q^2 + q^2_{\parallel}\boxed{\Pi(q^2_{\perp}, q^2_{\parallel})}}\left(g^{\parallel}_{\mu\nu} - \frac{q^{\parallel}_{\mu} q^{\parallel}_{\nu}}{q^2_{\parallel}}\right) - \frac{\lambda}{q^2}\frac{q_{\mu} q_{\nu}}{q^2}\right]$$

where the polarization function has the asymptotes

$$\Pi(q^2_{\parallel}) \simeq \frac{\bar{\alpha}}{3\pi}\frac{|eB|}{m^2_{\text{dyn}}}, \quad \text{as } |q^2_{\parallel}| \ll m^2_{\text{dyn}}$$ (extremely narrow range in $q^2_{\parallel}$)

$$\Pi(q^2_{\parallel}) \simeq -\frac{2\bar{\alpha}}{\pi}\frac{|eB|}{q^2_{\parallel}} \quad \text{as } |q^2_{\parallel}| \gg m^2_{\text{dyn}} \quad \Longrightarrow \quad \frac{1}{q^2 + q^2_{\parallel}\Pi(q^2_{\perp}, q^2_{\parallel})} \simeq \frac{1}{q^2 - M^2_{\gamma}}$$

where the effective photon screening mass is $M^2_{\gamma} = \dfrac{2\bar{\alpha}}{\pi}|eB|$

IMPROVED LADDER APPROXIMATION

- Let us re-analyze the problem with screening

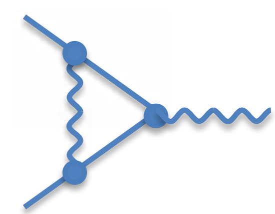
$$A(p_\parallel) = \frac{\alpha}{4\pi^2}\int \frac{A(k_\parallel)d^2k_\parallel}{k_\parallel^2+m^2}\int_0^\infty dx\left(\frac{e^{-xl^2/2}}{x+(k_\parallel-p_\parallel)^2}+\cancel{\frac{e^{-xl^2/2}}{x+(k_\parallel-p_\parallel)^2+M_\gamma^2}}\right)$$

- Improved vs. simple ladder approximations: $\alpha \to \alpha/2$
- Note, the dynamical mass is very sensitive to small α (or $\alpha/2$):

$$m_{dyn} \simeq C\sqrt{|eB|}\exp\left[-\frac{\pi}{2}\left(\frac{\pi}{2\alpha}\right)^{1/2}\right] \quad \text{(ladder approximation)}$$

and, thus, changes drastically with inclusion of screening

- The bigger problem is that the improved ladder approximation is *not* reliable either
  - The vertex corrections will change the result too
  - Singularities $\sim \ln\left(|eB|/m_{dyn}^2\right) \sim 1/\sqrt{\alpha}$ in higher-order diagrams



- Re-summation of infinitely many diagrams is needed (!)

TOWARD EXACT RESULT

- QED in a strong field looks almost like (1+1)D
- Lesson from exactly solvable (1+1)D Schwinger model: find a gauge in which all (singular) vertex corrections vanish!
- Such a (non-local) gauge exists

$$D_{\mu\nu}(q) = -i\frac{1}{q^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) - id(q_\perp^2, q_\parallel^2)\frac{q_\mu^\parallel q_\nu^\parallel}{q^2 q_\parallel^2}$$

where

$$d = -q_\parallel^2 \Pi / [q^2 + q_\parallel^2 \Pi] + q_\parallel^2 / q^2$$

- The corresponding full photon propagator reads

$$\mathcal{D}_{\mu\nu}(q) = \boxed{-i\frac{g_{\mu\nu}^\parallel}{q^2 + q_\parallel^2 \Pi(q_\perp^2, q_\parallel^2)}} - i\frac{g_{\mu\nu}^\perp}{q^2} + i\frac{q_\mu^\perp q_\nu^\perp + q_\mu^\perp q_\nu^\parallel + q_\mu^\parallel q_\nu^\perp}{(q^2)^2}$$

- All potentially dangerous infrared singularities vanish because

$$\mathcal{P}_+ \gamma_\mu \mathcal{P}_+ = \gamma_{\parallel,\mu} \quad \text{and} \quad \gamma_{\parallel,\alpha}\, \gamma_{\parallel,\mu_1}\, \gamma_{\parallel,\mu_2} \cdots \gamma_{\parallel,\mu_{2n+1}}\, \gamma_\parallel^\alpha = 0$$

RELIABLE STRONG-B LIMIT IN QED

- Let us use the method of Schwinger-Dyson equation this time:

$$\tilde{G}(x) = \tilde{G}_0(x) - 4\pi\alpha \int d^4y d^4z\, e^{-i\Phi(x,y)-i\Phi(y,z)} \tilde{G}_0(x-y)\gamma^\mu \tilde{G}(y-z)\gamma^\nu \tilde{G}(z)\mathcal{D}_{\mu\nu}(y-z)$$

where all Schwinger phases were carefully accounted for, and the nonlocal gauge is assumed in the photon propagator

$$\mathcal{D}^{-1}_{\mu\nu}(x-y) = D^{-1}_{\mu\nu}(x-y) - 4\pi\alpha\, \mathrm{tr}\left[\gamma_\mu \tilde{G}(x-y)\gamma_\nu \tilde{G}(y-x)\right]$$

- Perform Fourier transform and use LLL approximation,

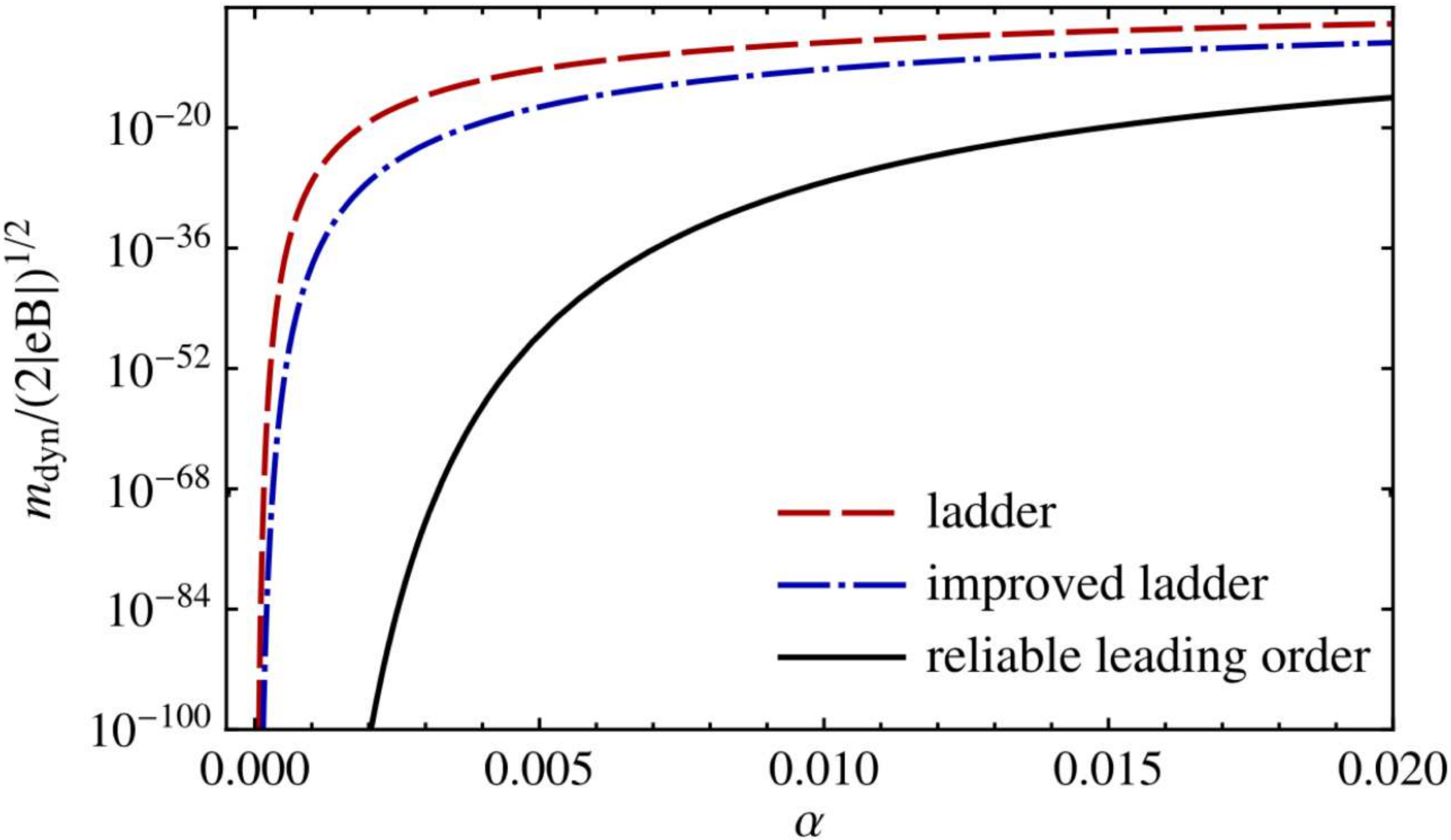
$$\tilde{G}_0(p_\parallel) = 2ie^{-\vec{p}_\perp^2 l^2} \frac{\hat{p}_\parallel}{p_\parallel^2}\mathcal{P}_+ \quad \text{and} \quad \tilde{G}(p_\parallel) = 2ie^{-\vec{p}_\perp^2 l^2} \frac{\hat{p}_\parallel + A(p_\parallel)}{p_\parallel^2 - A^2(p_\parallel)}\mathcal{P}_+$$

- Derive the following gap equation:

$$A(p_\parallel) = \frac{\alpha}{2\pi^2}\int \frac{d^2k_\parallel A(k_\parallel)}{k_\parallel^2 + A^2(p_\parallel)} \int_0^\infty dx \frac{e^{-xl^2/2}}{x + (k_\parallel - p_\parallel)^2 + M_\gamma^2 e^{-xl^2/2}}$$

- Compare with the gap equations in the (improved) ladder QED, obtained with Bethe-Salpeter method

DYNAMICAL MASS IN QED



- The numerical result is fitted well by

$$m_{dyn} \simeq \sqrt{2|eB|}\left(\alpha N_f\right)^{1/3} \boxed{\exp\left[-\frac{\pi}{\alpha \ln\frac{C_1}{\alpha N_f}}\right]}, \quad C_1 \approx 1.82 \pm 0.06$$