Parton distribution functions and absorptive corrections

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Abstract

The gluon distribution function becomes negative at the region of small-x $\left(\lesssim 10^{-3}\right)$ and $Q^2 \lesssim 2$ GeV^2 in the HERAPDF2.0 PDF set at NLO and NNLO, but this is not the expected physical behavior. In this kinematic region, non-perturbative corrections must be include, such as power corrections in the coeffecent functions, due to the exclusion of soft contributions, and absorptive corrections, which are taken into account in the Balitsky-Kovchegov (BK) equation, but have not been implemented in a global analysis via DGLAP. It was observed that the power corrections do not alter the gluon distribution or the χ^2 adjustment. While the absorptive one is not fully implemented, a simpler modification to DGLAP show an enhancement of $xg(x, Q^2)$.

Parton Distribution Functions

Deep Inelastic Scattering (DIS) denominates the scattering of a lepton ($l = e^{\pm}, \nu_e$) by a proton P mediated by an electroweak boson ($\gamma, Z^0, W^{\pm}$) of high-virtuality Q^2. The high-virtuality implies a small wavelenght, allowing the boson to probe the constituents of the proton, denoted partons. Therefore, the boson is exchanged not with the proton, but with a parton inside it [1].

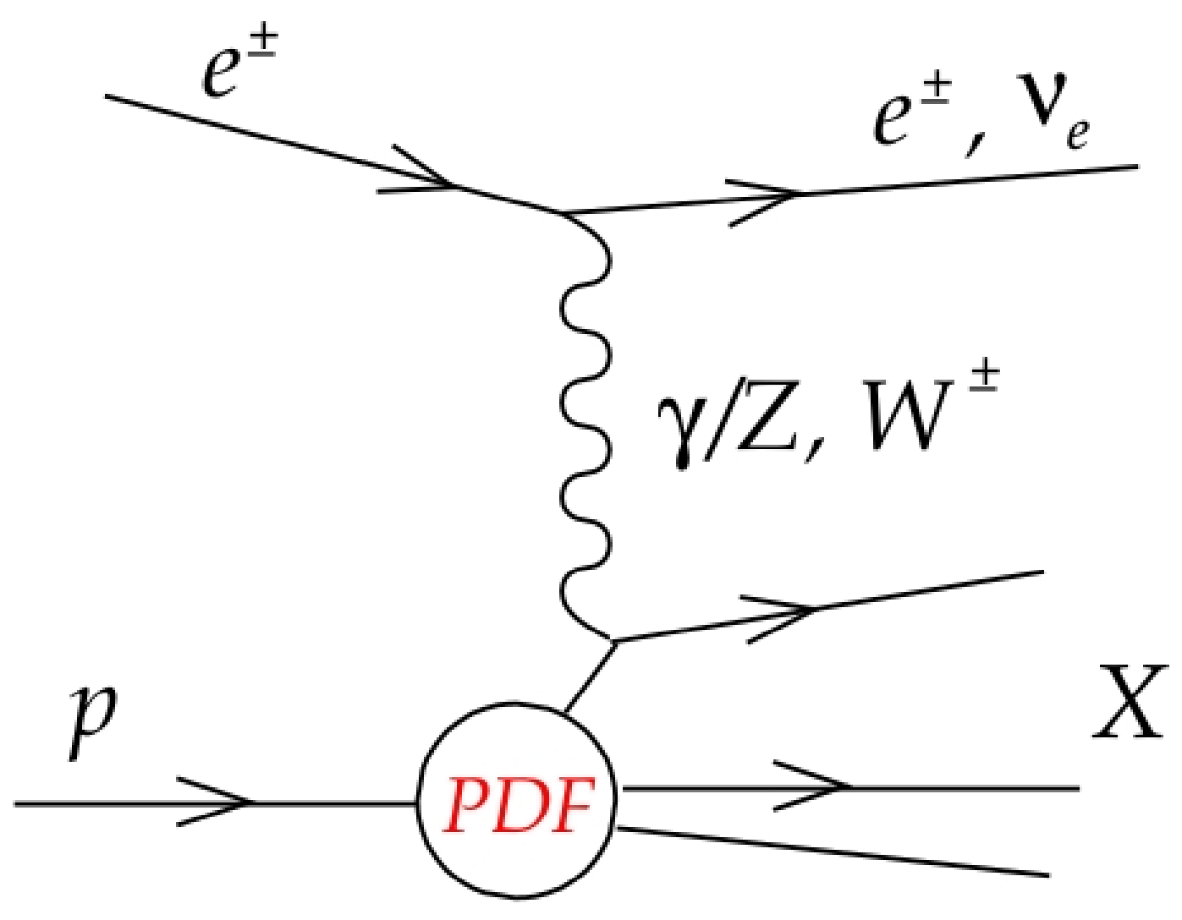

Figure 1: Deep inelastic scattering at leading order [2]

The cross-section must be written in terms of structure functions F_i, which contain information on the structure of the proton. There are two ramifications of DIS, neutral current (NC), $e^{\pm}P \to e^{\pm}X$ and charged current (CC), $e^{\pm}P \to \nu P$. For a neutral current process (mediated by γ and Z bosons):

$$\frac{d^2\sigma_{NC}^{e^{\pm}P}}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[Y_+ F_2(x,Q^2) \mp Y_- xF_3(x,Q^2) - y^2 F_L(x,Q^2)\right], \tag{1}$$

where x is the Bjorken variable, interpreted as the fraction of longitudinal momentum the strucked parton carries, α is the QED coupling constant and $Y_{\pm} = 1 \pm (1-y)^2$ with y being the inelasticity.

The factorization theorem states that the structure functions can be written as a convolution of perturbative and non-perturbative terms [3].

- Coefficient functions C_i: Perturbative; calculated from Feynman diagrams associated with the process.
- Parton Distribution Functions f_i: Non-perturbative; cannot be calculated from first principles in pQCD, requiring them to be adjusted from data.

The PDFs obey the DGLAP evolution equations, with these, given a PDF at an initial scale Q_0^2, it is possible to evolve it to another scale Q^2:

$$\frac{\partial}{\partial \ln Q^2} f_i(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\sum_j \int_x^1 \frac{dz}{z} P_{ij}\left(\frac{x}{z}, Q^2\right) f_j\left(z, Q^2\right). \tag{2}$$

Where P_{ij} are the splitting functions, that give the probability that a parton j emits a parton i, and are expanded in the strong coupling constant α_s.

There are various groups currently working in their determination, such as CT, MMHT, NNPD and H1 & ZEUS. The last of these uses purely DIS data from the HERA I+II run in their analisys, which is made in the xFitter/QCDNUM framework and their latest result is the HERAPDF2.0 set [4]. Our work will use this as a standard, mainly because we expect to include the corrections in their framework.

The HERAPDF parametrization is

$$xf_i(x) = A_i x^{B_i}(1-x)^{C_i} - \delta_{ig} A_i' x^{B_i'}(1-x)^{C_i'}. \tag{3}$$

where the second term is restricted to the gluon distribution in an alternative parametrization, in order to make it more malleable, but at the region of $Q^2 \lesssim 5$ GeV^2 and $x \lesssim 10^{-3}$:

- The gluon distribution becomes negative at NLO and NNLO.
- Non-perturbative corrections are known to play a considerable role.
- There are not many constraints from data.

We expect that our corrections to impose positivity to the gluon density in the region mentioned and to diminish the log-likehoold variable χ^2 that measures the agreement between the experimental measurements and theoretical predictions [5].

Power Corrections

When the DGLAP equation is used to evolve the PDFs, it never enters the low-virtuality domain $\left(|Q^2| < Q_0^2\right)$, which is all absorved into the input parton distribution $f_i(x, Q_0^2)$. But the whole kinematic space is taken into account when calculating the coefficient functions. To be consistent, we must exclude the soft region of the coefficients (*). To exemplify we explicity the simplest ones, from F_L:

$$C_{Lg} = 4T_R z \cdot (1-z) \cdot \left(1 - zQ_0^2/Q^2\right) \quad \text{and} \quad C_{Lq} = C_F 2z \cdot \left(1 - \left(zQ_0^2/Q^2\right)^2\right). \tag{4}$$

It can be seen that in the limit $Q_0 \to 0$ these are reduced to the usual expressions.

We found that the implementation of these corrections has produced no impact in the global analisys of the PDFs.

Absorptive Corrections

At low-x it is expected that the partons inside the proton begin to recombine due to large number density [6]. These are included in the BK equation, but the PDF global analysis uses the DGLAP formalism.

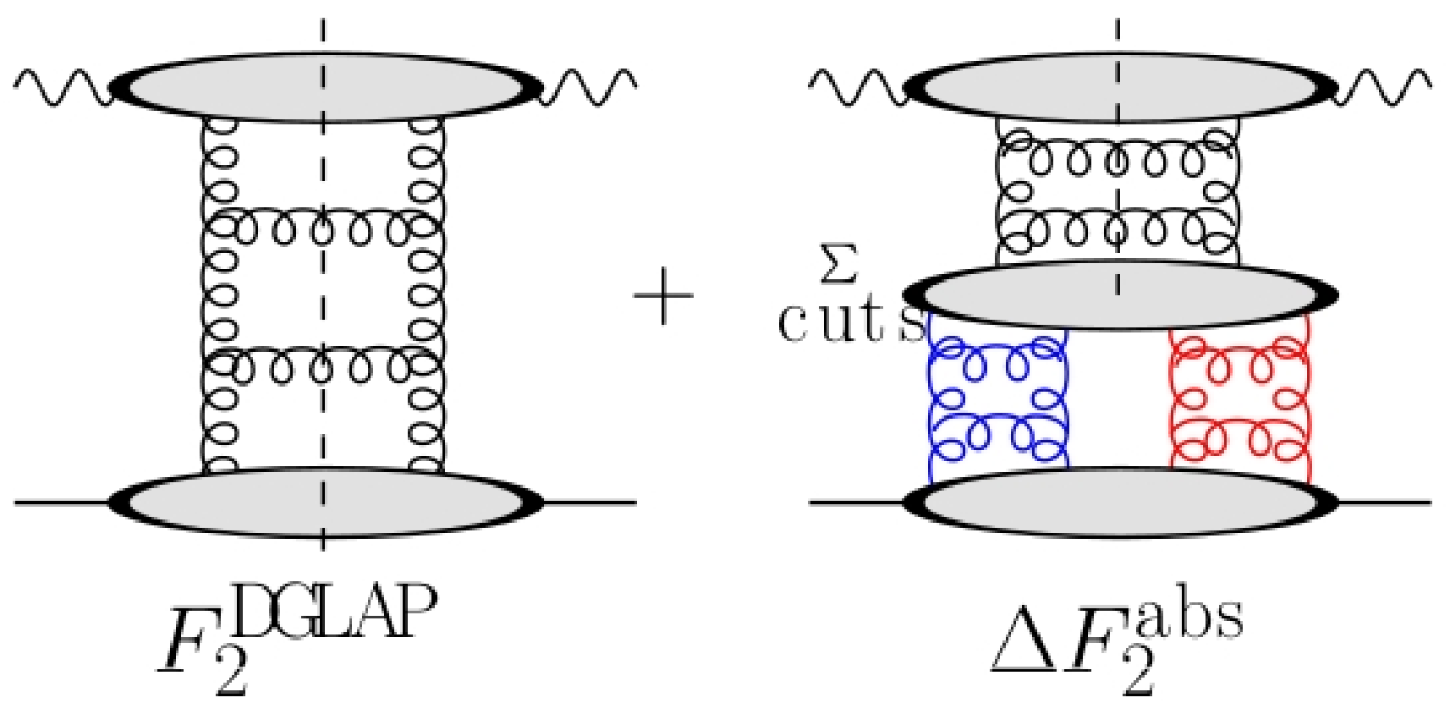

Figure 2: Absorptive corrections due to $2 \to 1$ Pomeron contribution. [6]

One way to include a large number of parton cascades in DGLAP is to replace the QCD coupling:

$$\alpha_s \to \alpha_s \exp\left(-\Omega(x,Q^2)\right), \tag{5}$$

where Ω may be interpreted as a proton opacity, and it is written as

$$\Omega(x,Q^2) = \frac{3\pi}{16}\frac{\alpha_s xg(x,Q^2)}{Q^2 a_0^2}. \tag{6}$$

Such a correction is very difficult is implement in the HERAPDF framework, mainly because the linearity of the DGLAP equations is used to speed-up the computation. A simpler form to test the absorptive corrections is to make the replacement

$$\alpha_s \to \frac{\alpha_s}{1 + a_0\,\alpha_s}. \tag{7}$$

where a_0 is a free parameter.

Results

- The implementation of absorptive corrections decreases χ^2 by around 0.75% with a cut at 3.5 GeV^2 in data, and 1.5% with the cut at 1.9 GeV^2.
- The gluon distribution has enhanced for $x \sim 10^{-1} - 10^{-4}$, becoming positive and closer to the leading order distribution.

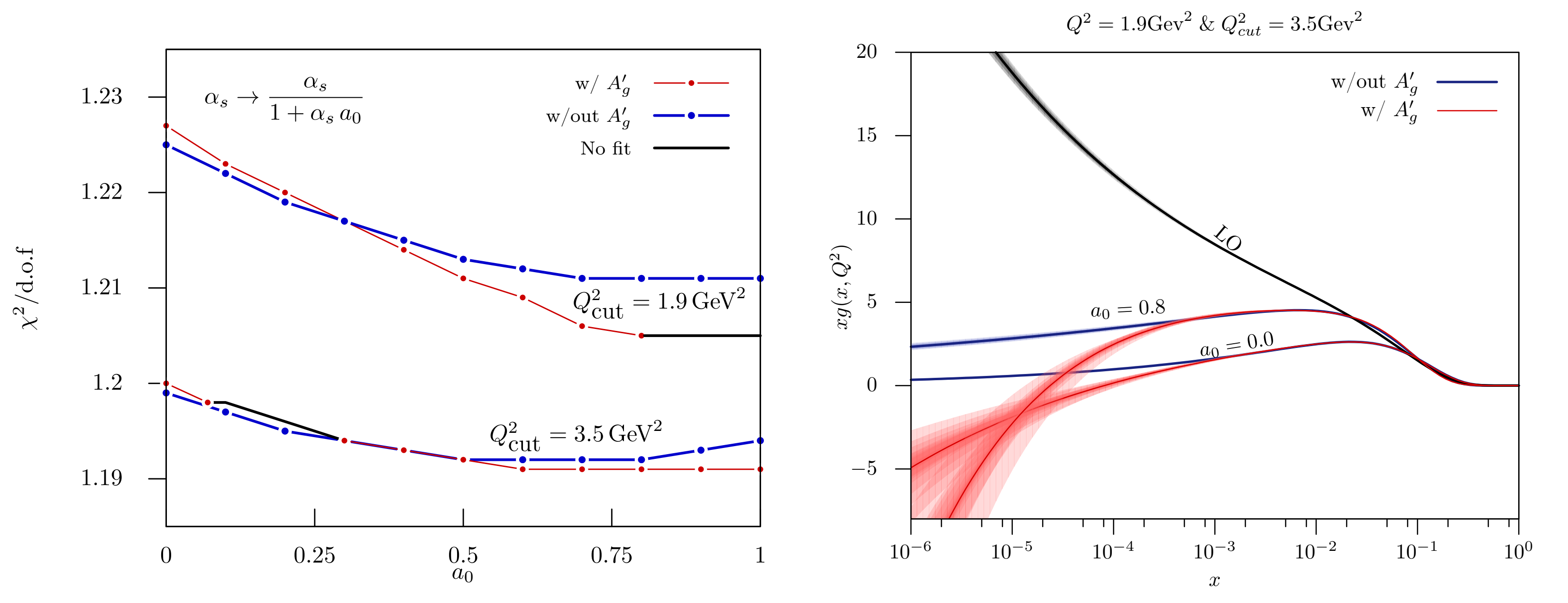

Figure 3: Left: Plot of the χ^2/d.o.f. varying with the free parameter a_0 from the correction. Fits were made for cuts in data at 1.9 and 3.5 GeV^2. Right: Comparison of the gluon distribution at $Q^2 = 1.9$ GeV^2 with and w/out corrections.

Conclusions

- The physically motivated power corrections have had no impact over the global fit in the xFitter framework.
- Absorptive corrections of the simplest form (Eq.7) enchance the gluon distribution in the region of $x \sim 10^{-4}$ at the input scale and reduce χ^2, giving hope that Eq.5 may produce even better results.

Forthcoming Research

- Implementation of the following at the HERAPDF framework:
  - Absorptive correction from Eq.5
  - Confinement corrections, as proposed at [7], that freeze the DGLAP at low-Q^2
  - Drell-Yan power corrections, with the same motivation as those of DIS.

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