Exploring the Thermodynamics of Confining Models*

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Establishing a description for confinement is not something simple. In order to try to understand a little about this phenomenon, we will explore the thermodynamics of models that try to describe it in terms of propagators with violation of positivity. In this work, "confinement" is always understood in the sense of positivity violation of the propagator of the elementary fields. For simplicity, we will define a model for scalar fields with a momentum dependent and nonlocal mass term. One of our objectives is to verify the thermodynamic properties of this Lagrangian in order to analyze possible inconsistencies. For this we use the functional formalism of Quantum Field Theory at finite temperature, from which we obtain the partition function and, consequently, the thermodynamic variables such as pressure, energy density, entropy density, etc. Then, we obtain the two-point function at finite temperature of the scalar field, in order to study whether or not there is a restoration of positivity (hence, deconfinement, in our language).

Keywords: : Quantum Field Theory, Quantum Field Theory at finite Temperature, Model of Confining Fields, Gribov-Zwanziger Theory

I. INTRODUCTION

Although the Higgs mechanism is considered to be the main responsible for the mass of leptons (such as the electron, for example) practically the whole mass of the visible matter of the Universe (not counting Dark Matter and Dark Energy) is concentrated in the atomic nuclei. The atomic nuclei are formed by protons and neutrons, which in turn are formed by three quarks each. At high energies, when the strong nuclear interaction is less relevant, we can infer what is the portion of the mass of the quarks that is due only to the electroweak interaction. The value obtained for the sum of the individual masses of the three quarks that form a proton corresponds to less than 2% of the total mass of the proton. Thus, about 98% of the mass of a proton or a neutron (and hence visible matter) originates from strong nuclear interaction.

The role played by the strong interaction is evident, but its detailed mechanism is little understood. The fundamental theory of strong interactions, Quantum Chromodynamics (QCD), can be solved by perturbative methods reliably only in very high energies. QCD is a gauge theory in which quarks (which are fermions) interact with each other through gauge bosons, called gluons. Because it is a non-abelian gauge theory, the gluons interact with each other, bringing enormous richness and complexity to the theory.

One of the fundamental properties of strongly interacting matter is confinement, that is, there are no asymptotic states of a single quark, nor of a single gluon. At great distances (low energies), the interaction between quarks and gluons becomes so intense that it is not possible to isolate one of these particles from the others. Another way of understanding confinement is that strong interaction causes any state of a particle with non-zero color charge to be a non-physical state (for example, in violation of positivity). This second approach has been used as a confinement criterion for many years in analyses of the dynamic mass generation of quarks and gluons [1–6], for example. Such a violation (which is associated with a function, called the spectral function, be negative) can be roughly understood as a negative probability of propagation, and thus the absence of the quark state of the physical spectrum. Propagators that exhibit this violation, in momentum space, may have complex poles. This is how the model describes the confinement of an isolated quark. The nonlocal interaction that causes this effect can be understood as a result of the action of a background of gluons, on which the quarks move.

It is expected that, at high energies, quarks and gluons deconfinement occurs due to the phenomenon of asymptotic freedom. Confinement is an eminently nonperturbative phenomenon. Therefore, non-perturbative approaches, such as Functional Renormalization Group [7, 8], Dyson-Schwinger Equations [9], Monte Carlo simulations in the lattice [10], holographic methods [11], and finally effective models [12–19], are valuable tools in the study of strong interactions.

From the theoretical point of view, the exploration of physical models in a medium can also reveal important properties of the physical systems, as well as possible inconsistencies of the model that could not be very easily perceived in vacuum analyzes. Thus, this work fits as part of the great theoretical effort of High Energy Theoretical Physics in search of a better understanding of the strong nuclear interactions in extreme regimes. Such an

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undertaking requires methods that go beyond perturbation theory and have the capacity to predict experimental results, as well as the ability to compare with other theoretical methods. This is the case of the effective models method, which we will employ in our investigation.

Finally, using the effective models method, we will propose a Lagrangian for a scalar field with a momentum dependent and nonlocal mass term. First, we will make an approach to the free scalar field, obtaining its propagator and the free partition function. Having the partition function in hand, we will then explore the thermodynamics of the theory, analyzing the behavior of some thermodynamic variables, such as pressure and entropy density.

II. MODEL

From the point of view of confinement as a violation of positivity of the two-point function, it is possible to define a scalar field model with a nonlocal mass term such that fundamental degrees of freedom are confined.

$$\mathcal{L}_E = \frac{1}{2}\left[\phi(x)\left(-\partial^2 + m^2 + \frac{\Lambda^4}{-\partial^2 + M^2}\right)\phi(x)\right], \quad (1)$$

where m, M and Λ are mass parameters, (roughly in the 0.1 GeV scale), so we say that the scalar field ϕ has a momentum dependent mass, given by

$$\mathcal{M}^2(p) = m^2 + \frac{\Lambda^4}{p^2 + M^2}. \quad (2)$$

Note that the Lagrangian (1) can be seen as a (very) simplified version of refined Gribov-Zwanziger (RGZ) [20] Lagrangian, where the A_μ gauge field in the adjoint representation of an $SU(N_c)$ color group has been replaced by a simple scalar field without internal color structure. Another important detail is that in this effective theory for low energy QCD, the nonlocal term in (1) (which gives rise to the nontrivial mass function (2)) corresponds to the horizon function of Gribov (or Gribov's horizon) [21, 22] in its quadratic approximation in the gauge fields. We can easily visualize this by making a simple comparison with the Zwanziger horizon function

$$H(A) = \gamma^2 \int d^4x\, g f^{bal} A^a_\mu (\mathcal{M}^{-1})^{lm} g f^{bkm} A^k_\mu, \quad (3)$$

where $\mathcal{M}^{-1}$ is the nonlocal term, called the Faddeev-Popov operator, which is given in the Landau gauge by

$$\mathcal{M}^{ab} = -\partial^2\delta^{ab} + g f^{abc} A^c_\mu \partial_\mu, \quad (4)$$

and γ is the Gribov parameter.

In the next section, we will look at the propagator of this theory and compare it with the RGZ propagator.

III. FREE PROPAGATOR AT FINITE TEMPERATURE

First we are interested in obtaining the free propagator for such a model, so we can see if it is comparable with the RGZ propagator. Thus, the free propagator is given by

$$D_F(\omega_n, \vec{p}^2) = \frac{p^2 + M^2}{(p^2 + M^2)(p^2 + m^2) + \Lambda^4} = \frac{\omega_n^2 + \vec{p}^2 + M^2}{(\omega_n^2 + \vec{p}^2 - \Lambda_1)(\omega_n^2 + \vec{p}^2 - \Lambda_2)}, \quad (5)$$

where Λ_1 and Λ_2 are given by

$$\Lambda_1 = \frac{-(m^2 + M^2) + \sqrt{(m^2 - M^2)^2 - 4\Lambda^4}}{2} \quad (6)$$

and

$$\Lambda_2 = \frac{-(m^2 + M^2) - \sqrt{(m^2 - M^2)^2 - 4\Lambda^4}}{2}, \quad (7)$$

and $\omega_n = 2\pi n T$ are the Matsubara frequencies at temperature T, with $n \in \mathbb{Z}$. Figure 1 shows the behavior of the free propagator with respect to the momentum squared, for the zeroth and for the first Matsubara mode at different temperatures.

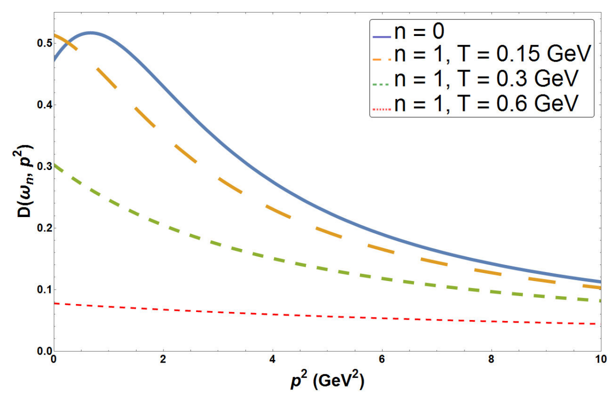

FIG. 1. Free propagator of a scalar particle. The solid line corresponds to the first Matsubara mode (n = 0) which corresponds to T = 0. The dashed lines correspond to the second Matsubara mode (n = 1) for three different temperatures.

We can see that for low momenta, the second derivative of the free propagator with respect to p^2 is negative, i.e., it is in a confining regime. However, by introducing temperature (Matsubara modes, in this case the first mode), as it increases, we see that such a derivative tends to zero. Therefore, of course, we may think that from a certain temperature value the free propagator will no longer have the positivity violation characteristic. As, in this work, such violation implies confinement, from this temperature value the particles would then be "deconfined". The values used, to have a notion of how is the behavior of the free propagator, for the mass parameters were [23]: $M^2 = 2.51\,\text{GeV}^2$, $m^2 = -1.91\,\text{GeV}^2$ e

$\Lambda^4 = 5.31\,\text{GeV}^4$. We also see that for the Matsubara zero mode, the graph shows a curve very similar to that of the RGZ gluon form factor at T = 0, showing that this model, although simple, really resembles the RGZ model which in turn presents a propagator to the gluon qualitatively similar to the gluon propagator in the lattice, as we can see in the figure 2.

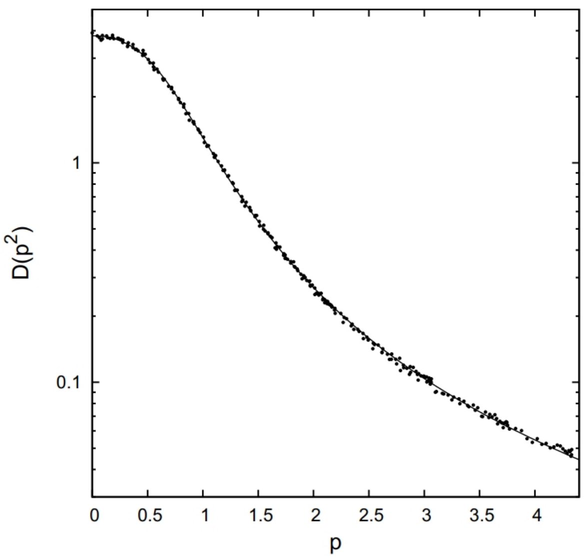

FIG. 2. SU(2) gluon form factor for a lattice volume $V = 128^4$ [23].

In the next section we will obtain the free partition function, in order to investigate the behavior of the thermodynamic variables.

IV. FREE PARTITION FUNCTION

One of our objectives is to analyze the thermodynamic variables of this model. For this, we will use the free partition function, which is given by

$$
\begin{aligned}
Z_F(\beta) &= \int \mathcal{D}\phi \exp^{-S_E(\beta)} \\
&= \int \mathcal{D}\phi \exp\left\{-\frac{1}{2}\int d^4x \right. \\
&\left. \times\, \phi(x)\left(-\partial^2 + m^2 + \frac{\Lambda^4}{-\partial^2 + M^2}\right)\phi(x)\right\} .
\end{aligned} \tag{8}
$$

From it we can then obtain the free energy, which is given by

$$
\Omega = -\frac{1}{\beta}\ln Z_F(\beta) , \tag{9}
$$

and consequently, obtain some thermodynamic variables for this simple model of a free gas, as for example the pressure. For a better analysis of this variable, figure 3 shows a comparison between models. Recalling that our model resembles RGZ, we then compared with a similar model to that of Gribov-Zwanziger (GZ), which corresponds to $M = 0$ and $m = 0$ in (1), and also with a model free massive gas with $m_{free} = m$ ($\Lambda = 0$). We

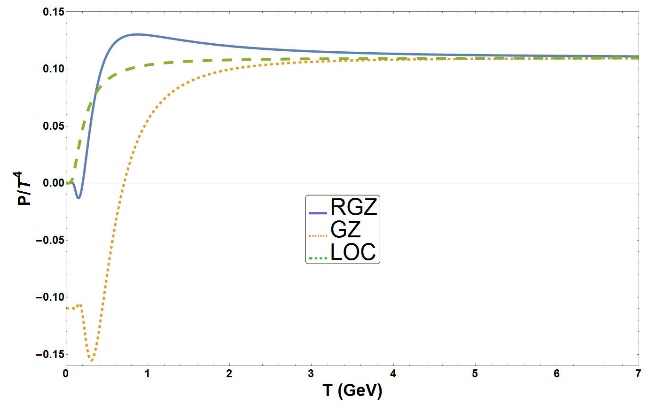

FIG. 3. Behavior of pressure for a free gas. The solid line corresponds to the RGZ model, while the dashed lines corresponds to the similar GZ model and free massive gas model (LOC) (green and orange) respectively.

can see that for high temperatures (energies), the models converge to the Stefan-Boltzmann limit. However for low temperatures, certain oscillations occur, implying a negative pressure. We see that, for our model, in the region where the pressure is negative, i.e. in the temperature range between (approximately) 0.07 and 0.2 GeV, the positivity violation still appears as we see from figure 1. Another important variable to be studied is the entropy density. The graph figure 4, analogous to pressure, makes a comparison with other models as well (RGZ, GZ and model with a local Lagrangian i.e., a free massive gas).

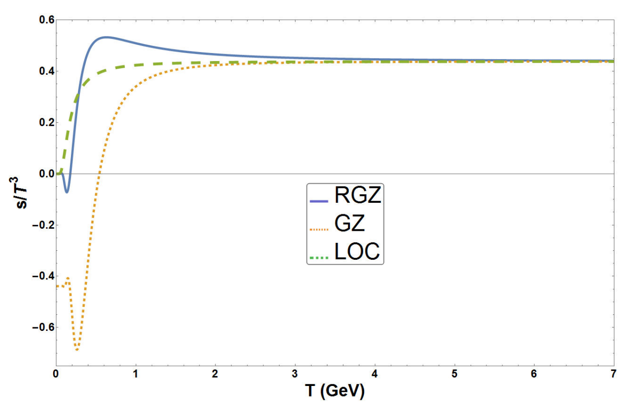

FIG. 4. Behavior of the entropy density for a free gas. The solid line corresponds to the RGZ model, while the dashed lines corresponds to the similar GZ model and the free massive gas model (LOC) (green and orange) respectively.

We also perceive an agreement of the models for high energies, but for low energies, there are again certain oscillations and negative entropy.

So far we have seen that the model suggested for free scalar field is comparable with RGZ, even though it is a simple model (toy model). Our future goal is then to take into account the interaction and analyze its influence on the thermodynamic variables.

V. CONCLUSIONS AND PERSPECTIVES

The RGZ propagator has great agreement with the calculation of the lattice, but when one introduces the temperature in the theory some inconsistencies appear that are the same presented here for this simple model of scalar fields. Since QCD is a theory that describes the strong interactions, considering the interaction terms is obviously something important in this model, so one of the future calculations will be to obtain the first order correction to the propagator, and consequently for the partition function and for the thermodynamic variables in order to, perhaps, see some improvement in the results.

VI. ACKNOWLEDGMENTS

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