Charged pion masses under strong magnetic fields in the NJL model

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The behavior of charged pion masses in the presence of a static uniform magnetic field is studied in the framework of the two-flavor NJL model, using a magnetic field-independent (MFIR) regularization scheme. Analytical calculations are carried out employing the Ritus eigenfunction method, which allows us to properly take into account the presence of Schwinger phases in the quark propagators. Numerical results are obtained for definite model parameters, comparing the predictions of the model with present lattice QCD (LQCD) results.

SU(2)$_f$ NJL model with B – Schwinger Form

The Euclidean Lagrangian density in an external static and uniform magnetic field along the z-axis, choosing the Landau gauge, is given by

$$\mathcal{L} = \bar\psi\,(-i\not{D} + m_0)\,\psi - G\left[(\bar\psi\psi)^2 + (\bar\psi\, i\gamma_5\vec\tau\,\psi)\right], \quad D_\mu = \partial_\mu - i\hat{Q}Bx_1\delta_{\mu 2}$$

After bosonization and expansion of the meson field around the Mean Field (MF) values, the Euclidean action reads

$$S_{\rm bos} = S^{\rm MF}_{\rm bos} + S^{\rm quad}_{\rm bos} + \dots$$

The quadratic contribution is given by

$$S^{\rm quad}_{\rm bos} = \frac{1}{2}\sum_{M=\sigma,\pi^0,\pi^\pm}\int_{x,x'} \delta M(x)^*\left[\frac{1}{2G}\,\delta^{(4)}(x-x') - J_M(x,x')\right]\delta M(x')$$

$$J_{\pi^0}(x,x') = N_c\sum_f {\rm tr}\left[\mathcal{S}^{{\rm MF},f}_{x,x'}\,\gamma_5\,\mathcal{S}^{{\rm MF},f}_{x',x}\,\gamma_5\right], \quad J_{\pi^\pm}(x,x') = 2N_c\,{\rm tr}\left[\mathcal{S}^{{\rm MF},u}_{x,x'}\,\gamma_5\,\mathcal{S}^{{\rm MF},d}_{x',x}\,\gamma_5\right]$$

where $\mathcal{S}^{{\rm MF},f}_{x,x'} = e^{i\Phi_f(x,x')}\int_p e^{ip(x-x')}\,\tilde{S}^f_p$ is the MF quark propagator.

Here $\Phi_f(x,x') = \exp\left[iq_f B(x_1+x_1')(x_2-x_2')/2\right]$ is the Schwinger phase and

$$\tilde{S}^f_p = \int_0^\infty d\tau\,\exp\left[-\tau\left(M^2+p_\parallel^2+p_\perp^2\frac{\tanh\tau B_f}{\tau B_f}\right)\right]\left[(M - p_\parallel\cdot\gamma_\parallel)\,(1+is_f\,\gamma_1\gamma_2\tanh\tau B_f) - \frac{p_\perp\cdot\gamma_\perp}{\cosh^2\tau B_f}\right]$$

where $B_f = |q_f B|,\ s_f = sign(q_f B),\ p_\perp = (p_1,p_2),\ p_\parallel = (p_3,p_4)$.

Parameterization – NJL vs LQCD condensate

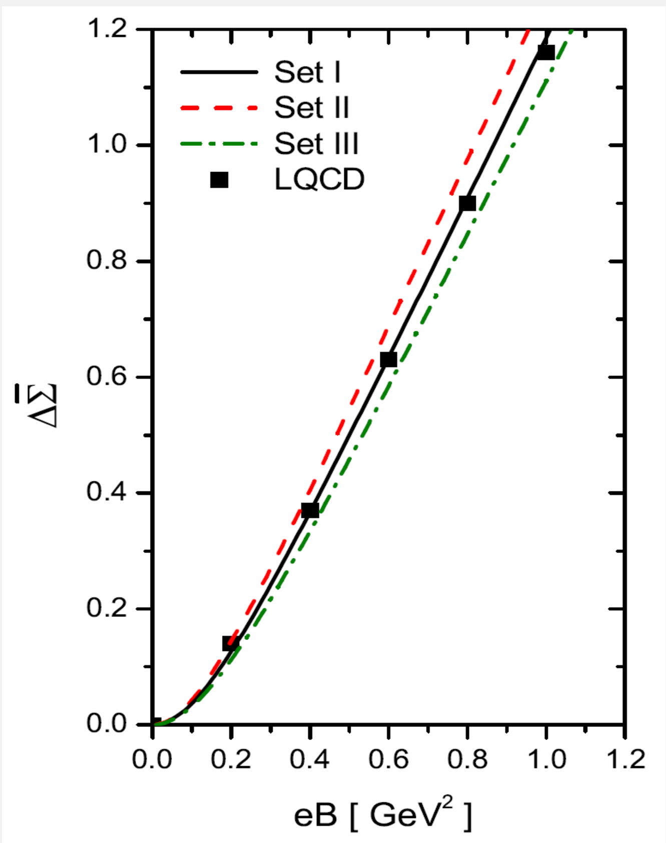
At the MF level, the condensates are given by

$$\langle\bar{f}f\rangle_B = -N_c\,{\rm tr}\int d^4x\,\mathcal{S}^{{\rm MF},f}_{x,x} = -\frac{N_c M}{4\pi^2}\int_0^\infty \frac{d\tau}{\tau^2}\,e^{-\tau M^2}\,\tau B_f\coth(\tau B_f)$$

The integral can be regulated in the MFIR scheme through a 3D cutoff. To compare with LQCD results we introduce the quantities

$$\Delta\bar\Sigma(B) = \frac{\Delta\Sigma_u(B)+\Delta\Sigma_d(B)}{2}, \quad \Delta\Sigma_f(B) = -2\,m_0\frac{\langle\bar{f}f\rangle_B - \langle\bar{f}f\rangle_0}{(135\times 86)^2}\ {\rm MeV}^{-4}$$

We consider the following parameterizations which reproduce the empirical vacuum values $m_\pi(B{=}0){=}138$ MeV and $f_\pi(B{=}0){=}92.4$ MeV.



Set	M_0 (MeV)	m_0 (MeV)	$G\Lambda^2$	Λ (MeV)
I	350.00	5.66	2.25	613.39
II	320.00	5.42	2.14	639.49
III	380.00	5.79	2.36	596.11

Table 1. Parameters m_0, G and Λ for Sets I, II and III.

Figure 1. Condensate average vs eB for Sets I,II,III and LQCD **[5]** (squares). Similar agreement is found for the condensate difference.

Neutral Pion

π^0 properties have been previously studied in the MFIR scheme expanding the propagator in Landau levels (LL) **[1,2]**. We use the Schwinger form instead. Since the Schwinger phases cancel out, J_{π^0} depends only on x-x' (i.e. it is translational invariant). Momentum is conserved and J_{π^0} is diagonalizable in the Fourier basis. The final expression coincides with the one found in **[1,2]**.

Charged Pions

For charged pions, Schwinger phases do not cancel due to their different quark flavors, and therefore J_{π^+} is not translational invariant. However, it is still diagonal in the Ritus basis. The pion field is expanded as

$$\pi^+(x) = \sum_{\bar q}\mathbb{F}^+_{\bar q}(x)\,\pi^+_{\bar q}, \quad \mathbb{F}^+_{\bar q}(x) = N_k\,e^{i(q_2x_2+q_3x_3+q_4x_4)}\,D_k(\rho_+)$$

$D_k(x)$ are the cylindrical parabolic functions

$$N_k = (4\pi|q_{\pi^+}B|)^{1/4}/\sqrt{k!}, \quad \rho_+ = \sqrt{2|q_{\pi^+}B|}\,\left[x_1 - q_2/(q_{\pi^+}B)\right], \quad \bar q \equiv (k,q_2,q_3,q_4)$$

The quadratic action in the Ritus basis reads

$$S^{\rm quad}_{\pi^+} = \frac{1}{2}\sum_{\bar q}(\delta\pi^+_{\bar q})^*\left[\frac{1}{2G} - J_{\pi^+}(k,\Pi^2)\right]\delta\pi^+_{\bar q}, \quad \Pi^2 = (2k+1)\,B_{\pi^+} + q_\parallel^2$$

Again, we regularize the polarization function using the MFIR scheme

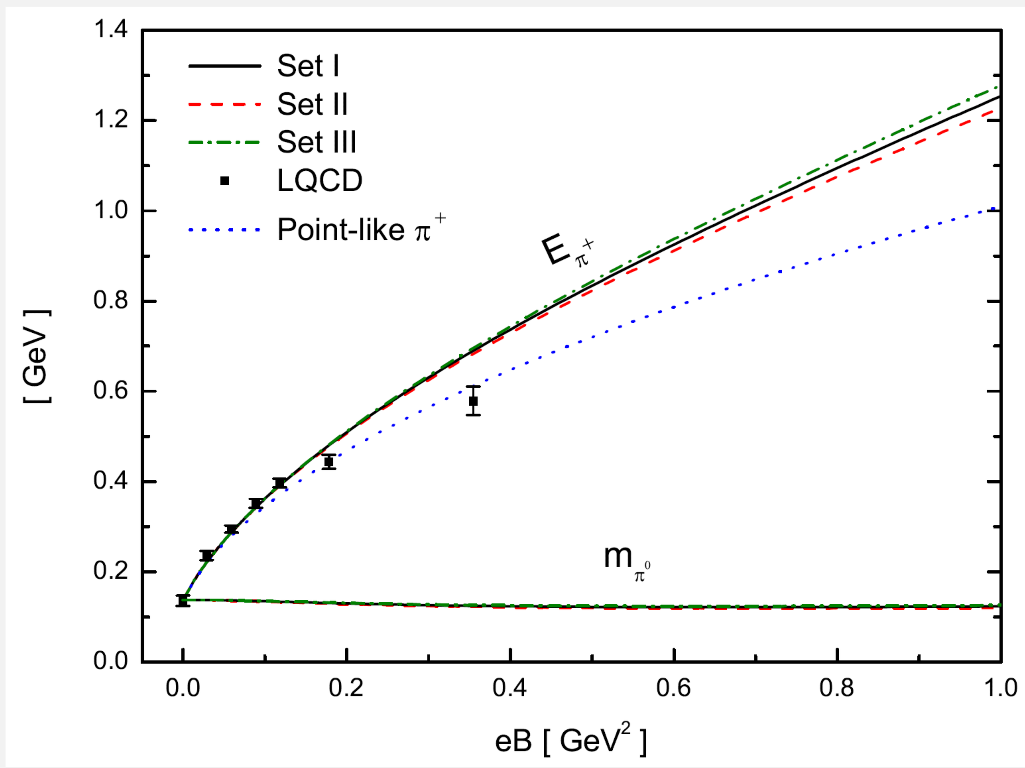
$$J^{\rm (reg)}_{\pi^+}(k,\Pi^2) = J^{\rm (reg)}_{\pi,B=0}(\Pi^2) + J^{\rm (mag)}_{\pi^+}(k,\Pi^2) \;\Rightarrow\; \frac{1}{2G} - J^{\rm (reg)}_{\pi^+}\left(k,-m^2_{\pi^+}(eB)\right) = 0$$

It is customary to define the B-dependent charged pion "mass" as

$$E_{\pi^+}(eB) = \sqrt{m^2_{\pi^+}(eB) + (2k+1)\,eB + q_3^2}\,\Big|_{q_3=0,\,k=0} = \sqrt{m^2_{\pi^+}(eB) + eB}$$

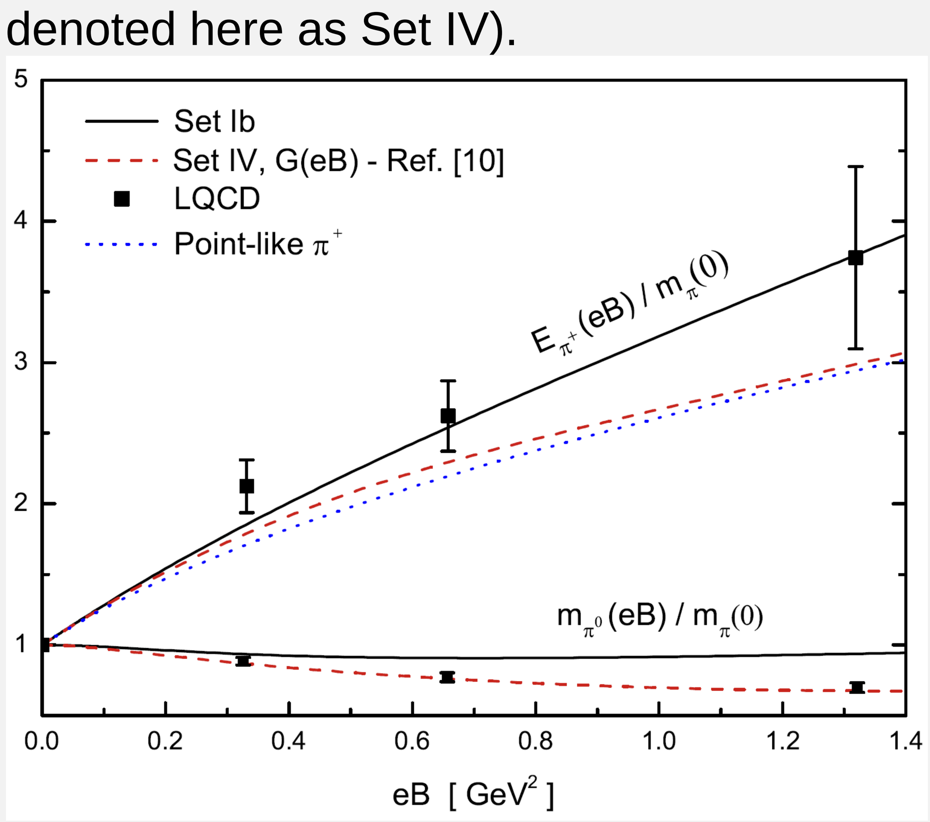
Numerical results for the masses

Figure 2. B-dependent neutral pion mass and charged pion "mass" as functions of eB for Sets I, II and III. Also shown are the charged pion "mass" for point-like case (dotted) and the results from a LQCD calculation with physical pion mass **[3]** (squares).



More recent LQCD simulations **[4]** consider larger values of eB but use a heavy pion mass of $m_\pi(B{=}0){=}415$ MeV. To compare we define the rescaled Set Ib, where m_0 is increased as in **[2]**. In that reference, the authors also consider a B-dependent coupling G(eB) to reproduce LQCD results for the π^0 mass (this is denoted here as Set IV).

Figure 3. Normalized neutral pion mass and magnetic field-dependent charged pion mass as functions of eB for Set Ib (solid), Set IV (dashed) of **[2]**, the charged point-like case (dotted) and LQCD **[4]** (squares).



Conclusions

- Within the NJL model, the charged pion two-point function was diagonalized in the Ritus basis, using the Schwinger form of the quark propagator and regularized in the MFIR scheme.
- The model parametrizations reproduce the quark condensates of LQCD.
- When eB is enhanced, the π^0 mass slightly decreases, while E_{π^+} steadily increases, remaining always larger than the one of a point-like pion.
- The results are rather independent of the parametrization.
- For a realistic charged pion mass, the results agree with LQCD **[3]** for low values of eB (eB$<$0.15 GeV^2). Higher values show some discrepancy.
- For a heavy pion mass, although rescaled parameters show consistency with LQCD **[4]** for E_{π^+}, the errors are large to be conclusive. For m_{π^0} the results disagree. The agreement is improved if G(eB) is introduced **[2]**.

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