Magnetized color superconducting quark matter under compact star conditions: Phase structure within the SU(2)$_f$ NJL model

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The properties of magnetized color superconducting cold dense quark matter under compact star conditions are investigated using an SU(2)f Nambu Jona-Lasinio (NJL)-type model in which the divergences are treated using a magnetic field independent regularization (MFIR) scheme in order to avoid unphysical oscillations. We study the phase diagram for several model parametrizations. We analyze the behavior of the chiral and superconducting condensates together with the different particle densities for increasing chemical potential or magnetic field. With respect to previous studies, we show how the phases are modified in the presence of β-equilibrium as well as color and electric charge neutrality conditions.

The model: SU(2)f NJL + 2SC + B + μ + β-equilibrium + color and charge neutrality + T=0 + MFIR scheme

When diquark interaction channels and a magnetic field are introduced in the NJL model, we must work in a rotated base where modified charges appear, which are a combination of electric and color charges.

Quark flavor-color	ub	ur	ug	db	dr	dg
$\tilde{Q} = Q_f \otimes 1_c - 1_f \otimes \frac{T_c^8}{\sqrt{3}}$	+1	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$

Due to the B field, the 1-2 plane is quantized in Landau levels (LL) of index k.

$$\Omega_{\rm MFA} = \frac{(M-m_c)^2}{4G} + \frac{\Delta^2}{4H} - \sum_{|\tilde q|=0,\frac{1}{2},1} P_{|\tilde q|} - P_{lep}$$

$$\begin{cases} P_{|\tilde q|=0} = \int \frac{d^3p}{(2\pi)^3}\left(E^+_{db} + |E^-_{db}|\right), \quad P_{|\tilde q|=1} = \frac{\tilde e B}{8\pi^2}\sum_{k=0}^\infty \alpha_k \int_{-\infty}^\infty dp_z \left(E^+_{ub} + |E^-_{ub}|\right) \\ P_{|\tilde q|=1/2} = \frac{\tilde e B}{8\pi^2}\sum_{k=0}^\infty \alpha_k \int_{-\infty}^\infty dp_z \sum_{\lambda,s=\pm} |E^\lambda_{\Delta^s}|, \quad P_{lep} = \sum_{l=e,\mu} P_{|\tilde q|=1}\Big|_{\substack{M=m_l\\ \mu_{ub}=\mu_l}} \end{cases}$$

$$\begin{cases} E^\pm_{db} = \sqrt{p^2+M^2} \pm \mu_{db}, \quad E^\pm_{u_b} = \sqrt{p_z^2 + 2k\tilde e B + M^2} \pm \mu_{ub} \\ E^\pm_{\Delta^\pm} = \sqrt{\left(\sqrt{p_z^2 + k\tilde e B + M^2} \pm \bar\mu\right)^2 + \Delta^2} \pm \delta\mu \end{cases}$$

$$\begin{cases} \hat\mu = \mu + Q\mu_Q + T^8\mu_8, \ \text{where} \ \mu_3 = 0 \Rightarrow \mu_{ur} = \mu_{ug}, \ \mu_{dr} = \mu_{dg} \\ \beta\text{-equilibrium and neutrality:} \ \mu_Q = -\mu_e, \ \mu_\mu = \mu_e \\ \text{We define:} \ \bar\mu = \frac{\mu_{dg}+\mu_{ur}}{2}, \ \delta\mu = \frac{\mu_{dg}-\mu_{ur}}{2} = \frac{\mu_e}{2} \end{cases}$$

We regularize using the MFIR scheme. Gap equations: $\frac{\partial \Omega^{reg}_{\rm MFA}}{\partial \xi} = 0, \ \xi = M, \Delta, \mu_e, \mu_8$

We will consider H/G=0.75 and 1, and the following parameterization sets

	M_0	m_c	$G\Lambda^2$	Λ	$-<u\bar u>^{1/3}$
Set 1	340 MeV	5.59 MeV	2.21	621 MeV	244 MeV
Set 2	400 MeV	5.83 MeV	2.44	588 MeV	241 MeV

The conventions for the different phases can be summarized as

Phase	Characteristics
B-*Vacuum Phase*	χ-symmetry broken, M=M(B,$\mu=0$), MC, low μ, $\mu < M$, $\Delta = \mu_8 = \mu_e = n = 0$
A-*CSC Phase*	χ-symmetry almost restored, IMC, big μ, $(\Delta, \mu_8, \mu_e, n) \neq 0$, vA-dH transitions
D-*Mixed Phase*	$M_B > M_D > M_A$, IMC, $(\Delta,\mu_e,n)_A > (\Delta,\mu_e,n)_D > (\Delta,\mu_e,n)_B$
2SC	Present in D and A phases, $\Delta > \delta\mu$, four gapped, $n_{dr} = n_{ur}$
g2SC-*Gapless*	$\Delta_\pm = \Delta \pm \delta\mu$ doubly-degenerate modes; $\delta\mu > \Delta$, two gapless, $n_{dr} \neq n_{ur}$

Fixed ẽB – two values of H/G and three of ẽB, Set 1

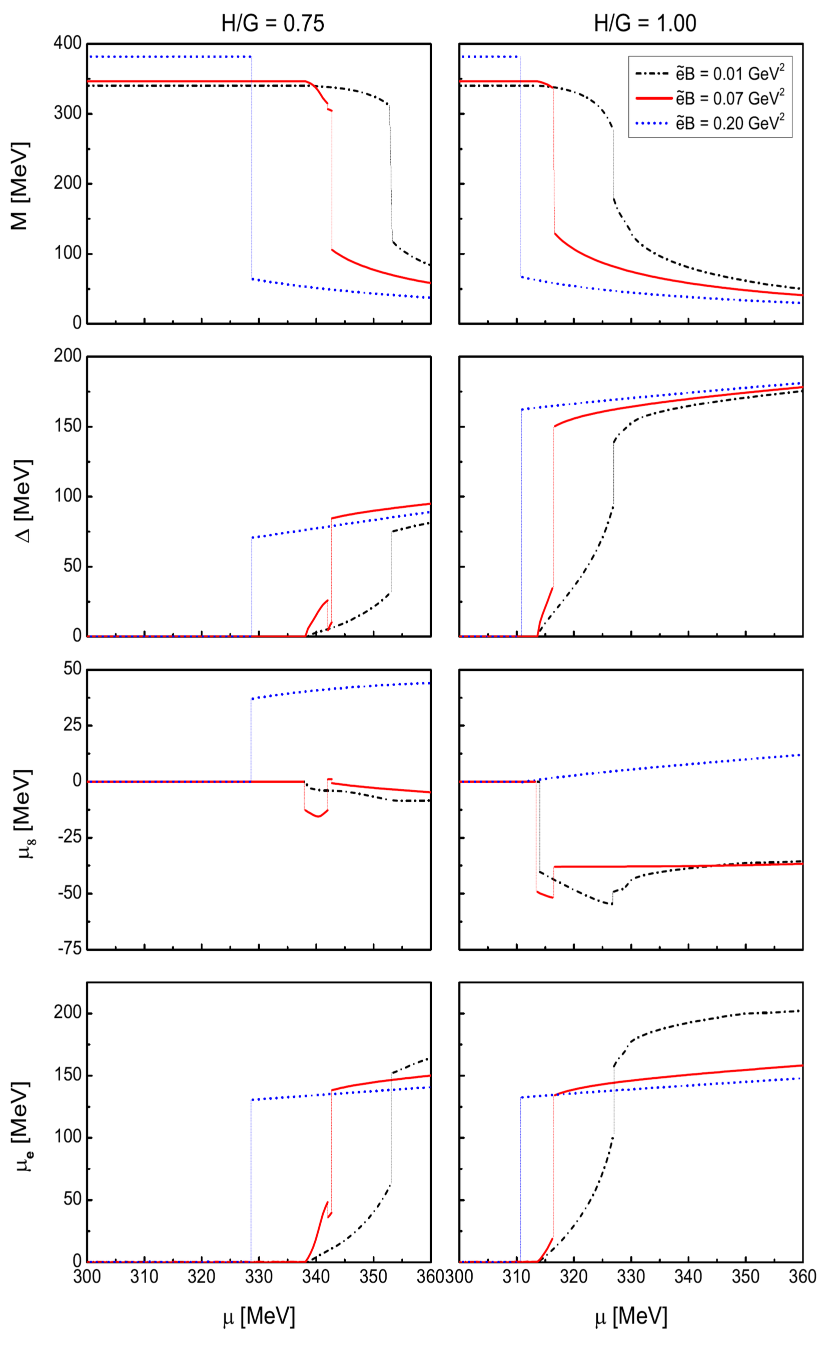

Figure 1. M, Δ, μ_8 and μ_e vs μ.

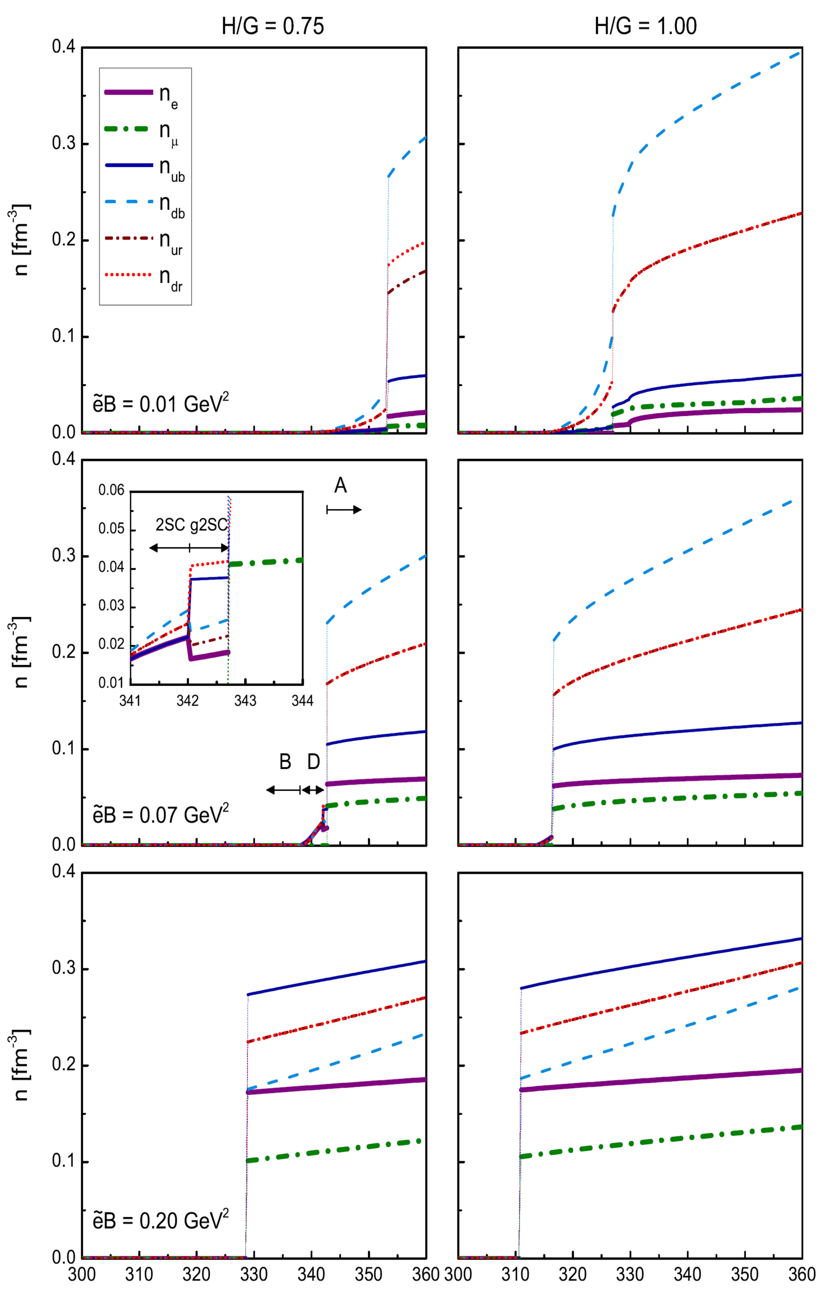

Figure 2. Quarks and leptons densities vs μ. Since μ_3=0, n_{fr}=n_{fg}.

Fixed μ – two values of H/G and three of μ, Set 1

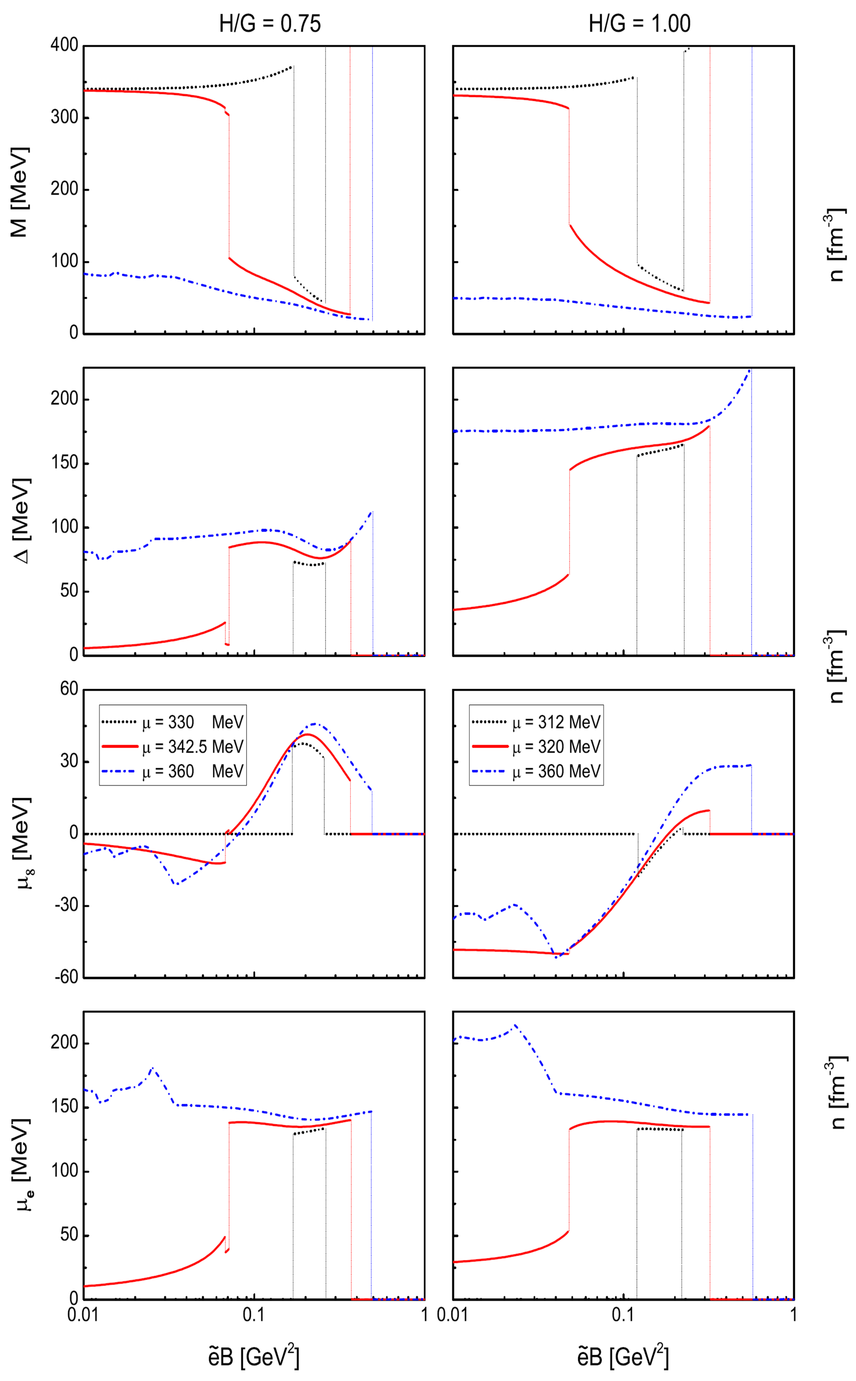

Figure 3. M, Δ, μ_8 and μ_e vs ẽB.

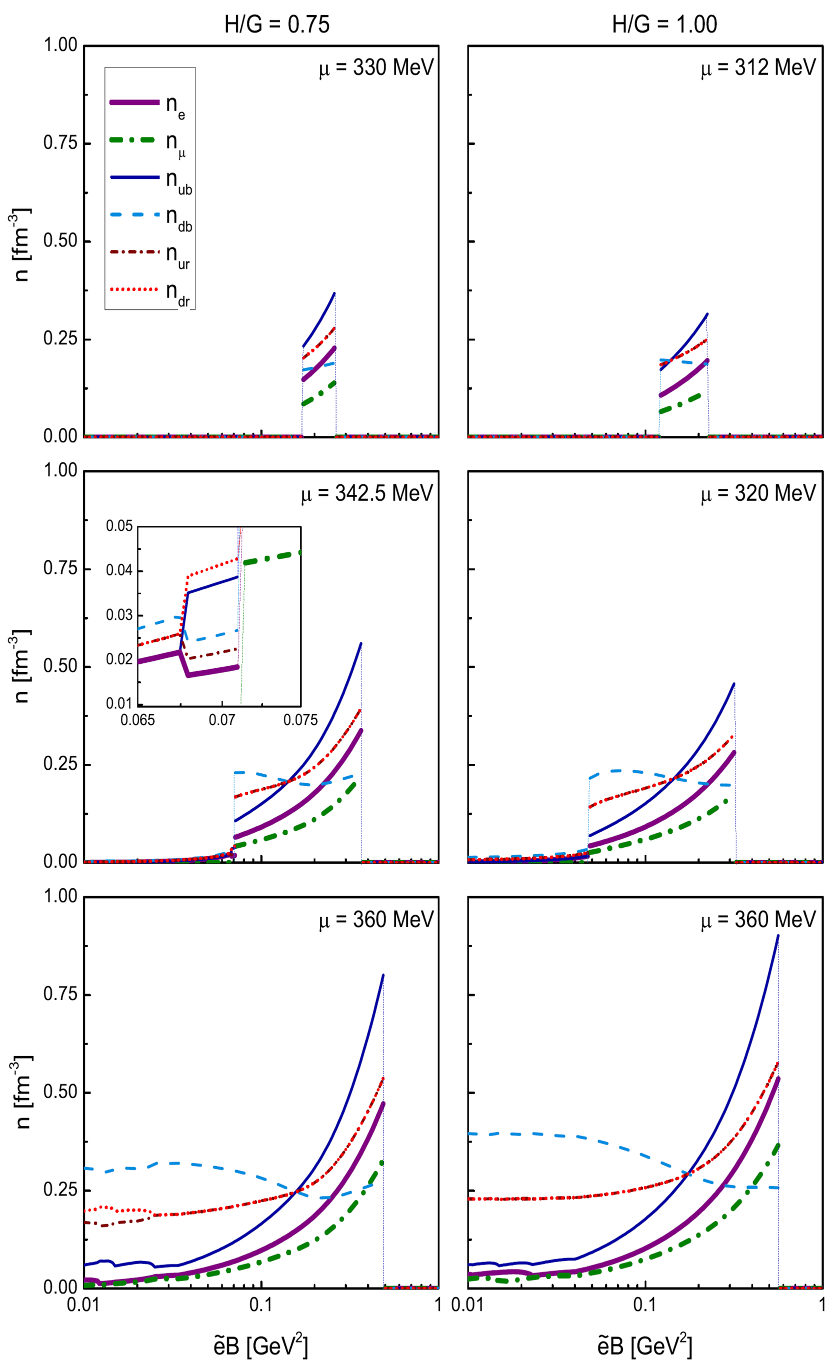

Figure 4. Quarks and leptons densities vs ẽB.

Phase diagrams

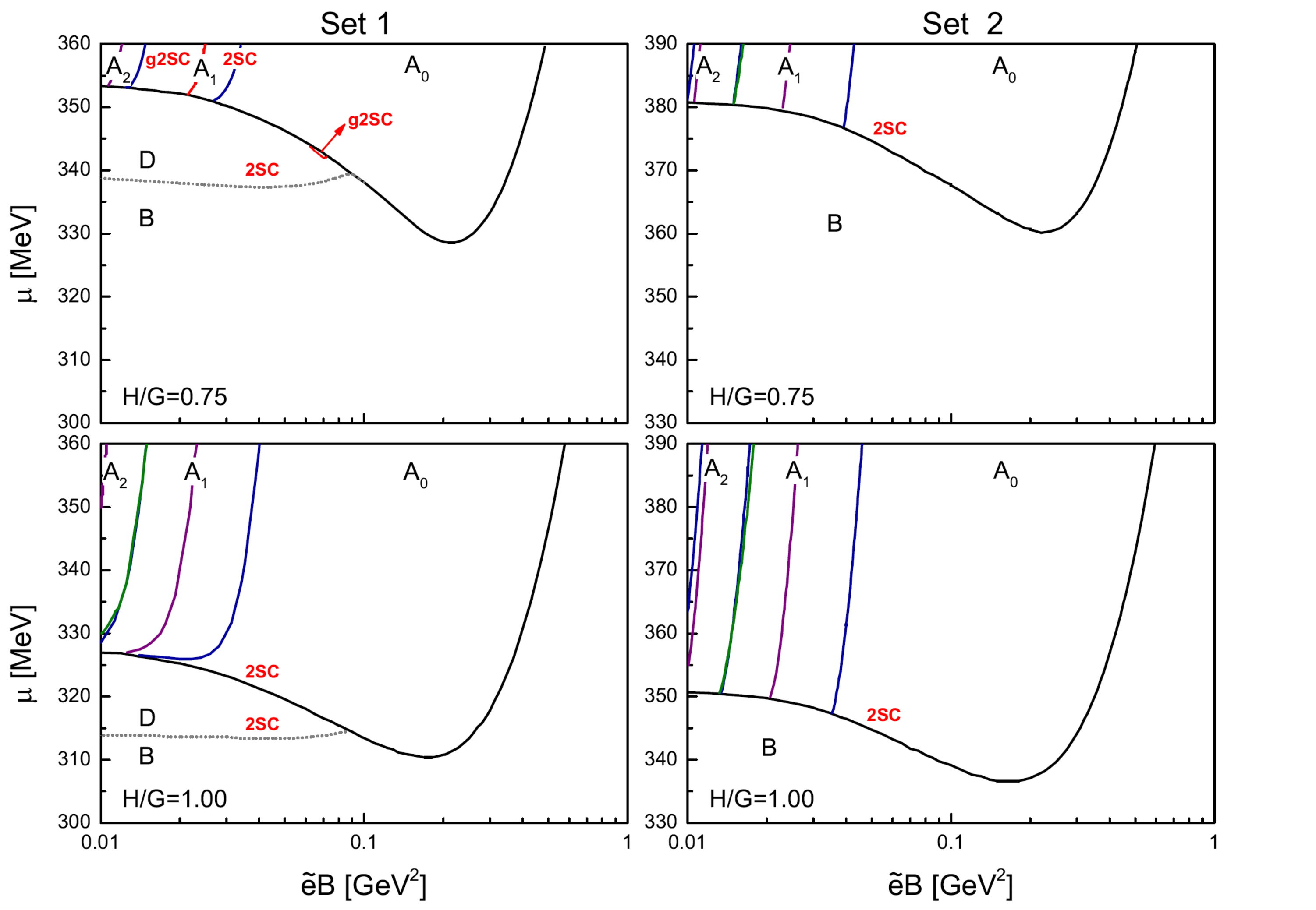

Figure 5. Phase diagrams. Full lines correspond to first order transitions and dotted lines to second order phase transitions. Black lines represent phase transitions. Van Alphen-de Haas (VA-dH) transitions for unit charged species are colored as: blue for ub quarks, violet for electrons and green for muons. Red lines indicate g2SC – 2SC transitions. In the A_i sub-phases, the ub quark populates up to i-th LL.

Conclusions

- Presence of chirally broken B, almost restored A and mixed D phases, the latter two composed of g2SC and/or 2SC regions.
- Based on a B=0 study, for H/G<0.75 the A and D phases are expected to become g2SC and eventually have Δ=0 for H/G<0.65, recovering the C phase.
- The introduction of compact star conditions induces the existence of g2SC and 2SC modes, diminishes the maximum LL reached in the vA-dH transitions and reduces the superconducting effect. The value of the critical μ is increased and the magnetic catalysis (MC) effect is attenuated, diminishing the depth of the IMC well and moving the phase diagram upwards.

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