Scale independence in an asymptotically free theory at finite temperatures

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Abstract

A recently developed variational resummation technique incorporating renormalization group properties has been shown to solve the scale dependence problem that plagues the evaluation of thermodynamical quantities, e.g., within the framework of approximations such as in the hard-thermal-loop resummed perturbation theory. This method is used in the present work to evaluate thermodynamical quantities within the two-dimensional nonlinear sigma model, which shares some common features with Yang-Mills theories, like asymptotic freedom, trace anomaly and the nonperturbative generation of a mass gap. Besides the fact that nonperturbative results can be readily generated solely by considering the lowest-order contribution to the thermodynamic effective potential, we also show that its next-to-leading correction indicates convergence to the sought-after scale invariance.

INTRODUCTION

The theoretical description of the quark-gluon plasma phase transition requires the use of nonperturbative methods, since the use of perturbation theory near the transition is unreliable. LQCD has been very successful at finite temperatures and near vanishing baryonic densities, however, currently, the complete description of compressed baryonic matter cannot be achieved due to the so-called sign problem. In this case, an alternative is to use approximate but more analytical nonperturbative approaches. One of these is the Optimized Perturbation Theory (OPT), which reorganizes the series using a variational approximation, where the result of a related solvable case is rewritten in terms of a variational parameter that allows for nonperturbative results to be obtained. On the other hand, the results of the Hard Thermal Loop Perturbation Theory (HTLpt), done in a gauge-invariant framework, exhibit a strong sensitivity to the arbitrary renormalization scale M used in the regularization procedure [1]. A solution to this problem has been recently proposed, by generalizing to thermal theories a related variational resummation approach, Renormalization Group Optimized Perturbation Theory (RGOPT). In this work we apply the RGOPT to the nonlinear sigma model (NLSM) in 1+1 dimensions at finite temperatures in order to pave the way for future applications concerning other asymptotically free theories, such as thermal QCD. As we will illustrate, the scale invariant results obtained in the present application give further support to the method as a robust analytical nonperturbative approach to thermal theories.

THE MODEL

The two-dimensional NLSM partition function [2]

$$Z=\int\prod_{i=1}^{N}\mathcal{D}\Phi_i(x)\exp\left[\frac{1}{2g_0}\int d^2x(\partial\Phi_i)^2\right]\delta(\sum_{i=1}^{N}\Phi_i\Phi_i-1), \quad (1)$$

where g_0 is a (dimensionless) coupling and the scalar field is parametrized as $\Phi_i=(\sigma,\pi_1,...,\pi_{N-1})$.
The bare Lagrangian density expanded to order-g_0 is

$$\mathcal{L}_0=\frac{1}{2}\left[(\partial\pi_i)^2+m_0^2\pi_i^2\right]+\frac{g_0 m_0^2}{8}(\pi_i^2)^2+\frac{g_0}{2}(\pi_i\partial\pi_i)^2-\mathcal{E}_0, \quad (2)$$

RG IMPROVED OPTIMIZED PERTURBATION THEORY

Performing the correct replacements on the renormalized pressure and re-expanding it to the first order in δ, and finally taking $\delta\to 1$, one gets the two-loop PT result:

$$\begin{aligned}P_{2L}^{\rm RGOPT} &= -\frac{(N-1)}{2}I_0^{\rm r}(m,T)+(N-1)\left(\frac{\gamma_0}{b_0}\right)m^2 I_1^{\rm r}(m,T)\\ &- g(N-1)\frac{(N-3)}{8}m^2\left[I_1^{\rm r}(m,T)\right]^2\\ &+\frac{(N-1)}{4\pi}\frac{m^2}{g\,b_0}\left(1-\frac{\gamma_0}{b_0}\right). \end{aligned} \quad (3)$$

with the correct two-loop running coupling.

Figure 1: *Feynman diagrams contributing to the perturbative pressure at $\mathcal{O}(g)$.*

NUMERICAL RESULTS

Due to coupling arbitrarieness in the literature, we find it sensible to compare the results for a given finite N_{phys} input by taking

$$g_{LN}(M_0)\simeq(N_{phys}-2)g(M_0) \quad (4)$$

We set the arbitrary $\overline{ms}$ scale as $M=\alpha M_0=2\pi T\alpha$ and consider $0.5\le\alpha\le 2$, choosing moderately nonperturbative coupling choice, $g(M_0)\simeq 1$.

The $T\neq 0$ results

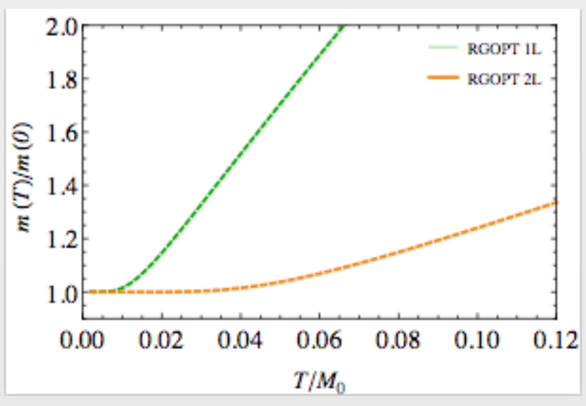

Figure 2: *The normalized thermal optimized masses for $N=4$, $g(M_0)=1=g_{LN}(M_0)/2$, and at the central scale choice $\alpha=1$, in the RGOPT at one- and two-loop cases.*

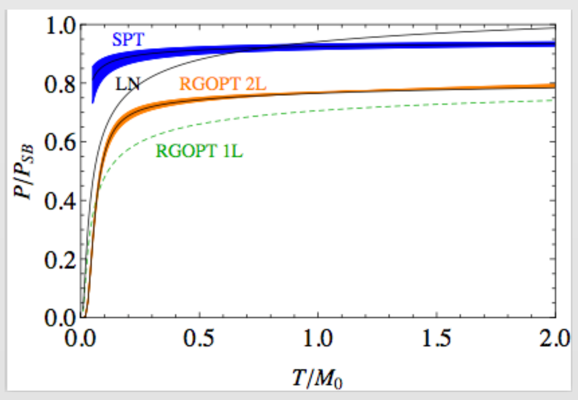

Figure 3: *$P/P_{\rm SB}$ as function of the temperature T for $N=4$ and $g(M_0)=1=g_{LN}(M_0)/2$, with $0.5\le\alpha\le 2$.*

Comparison with next-to-leading $1/N$ expansion

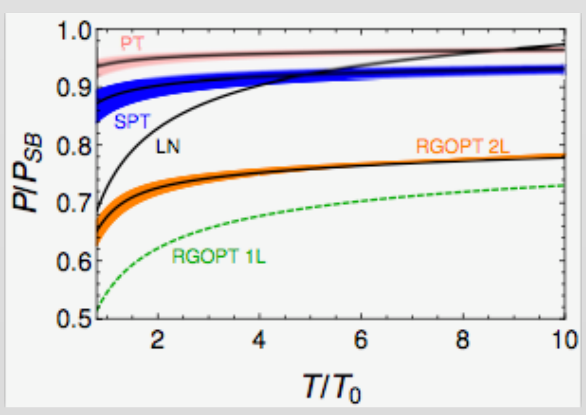

Figure 4: *$P/P_{\rm SB}(T/M_0)$ in high-T approximation for $N=4$, $g(M_0)=1=g_{LN}(M_0)/2$ and scale variation $1/2<\alpha<2$.*

Further improving residual scale dependence

Despite the explicit perturbatively RG invariant RGOPT construction, there is a residual scale dependence within the two-loop results. By defining an *effective* temperature-dependent coupling $g(T)$ and truncating the thermal mass to its purely perturbative first order high-T expression, we can minimize such dependence.

$$b_0\,g_{eff}(m,M)=\left[\ln\frac{M}{m}-2J_1(\tfrac{m}{T})\right]^{-1} \quad (5)$$

$$\simeq\left\{\ln\left[\frac{2M}{(N-2)gT}\right]-2J_1(\tfrac{(N-2)g}{2})\right\}^{-1}. \quad (6)$$

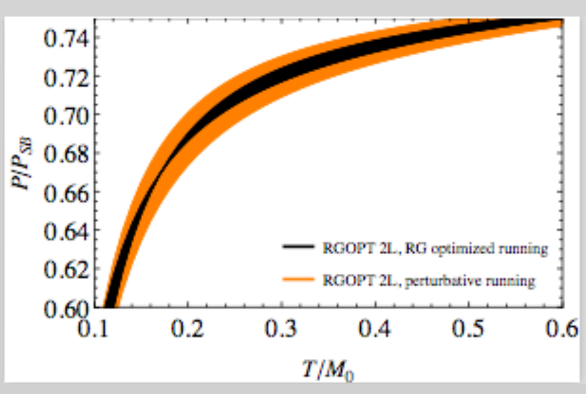

Figure 5: *Comparison of the regular pressure with that obtained by using the thermal coupling.*

Comparison with lattice simulations

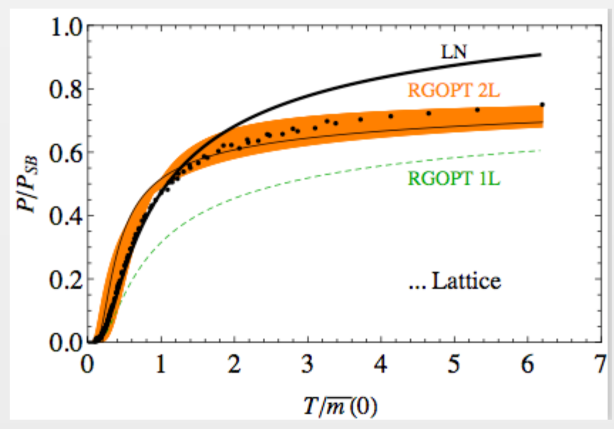

Figure 6: *$P/P_{\rm SB}$ as a function of the temperature T compared with lattice simulations (taken from [3]).*

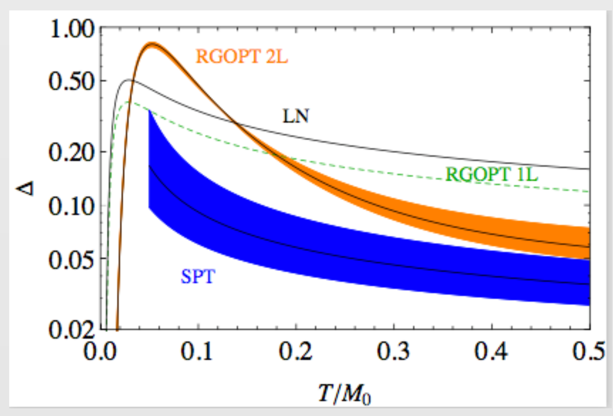

Figure 7: *The interaction measure Δ as a function of the temperature T (normalized by M_0) for $N=4$ and $g(M_0)=1=g_{LN}(M_0)/2$, with scale variation $0.5\le\alpha\le 2$.*

CONCLUSIONS

Our application shows how simple perturbative results can acquire a robust nonperturbative predictive power by combining renormalization group properties with a variational criterion used to fix the (arbitrary) "quasi-particle" RGOPT mass. A non-trivial scale invariant result was obtained by considering the lowest order contribution to the pressure and the NLO (two-loop) order RGOPT results display a very mild residual scale dependence when compared to the standard SPT/OPT results. We also obtain a reasonable agreement of the RGOPT pressure with known lattice results for $N=3$. The NLSM thermodynamical observables obtained from two-loop RGOPT display a physical behavior that is more in line with LQCD predictions for pure Yang-Mills four-dimensional theories, as compared with the two-loop order SPT. The one- and two-loop RGOPT interaction measure Δ exhibit some characteristic nonperturbative features somewhat similar to the QCD interaction measure. Finally it would be of much interest to compare our NLSM thermodynamical results with other lattice simulation results for other N values, but unfortunately to our knowledge no such simulations at finite temperature are available up to now for $N>3$.

References

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