Supersymmetry

António P. Morais[1,2]

[1]Theoretical High Energy Physics (THEP)
Lund University, Lund, Sweden
amorais@thep.lu.se

[2]Center for Research and Development in Mathematics and Applications (CIDMA)
Aveiro University, Aveiro, Portugal

July 23, 2015

HASCO Summer School 2015
Goettingen University

Suggested Literature

1. S. P. Martin, ***A Supersymmetry primer***, Adv.Ser.Direct.High Energy Phys. **21** (2010) 1-153, [hep-ph/9709356]
2. D. Bailin and A. Love, ***Supersymmetric gauge field theory and string theory***, Bristol, UK: IOP (1994) 322 p. (Graduate student series in physics)
3. I. J. R. Aitchison, ***Supersymmetry in Particle Physics. An Elementary Introduction***, Cambridge, UK: Univ. Pr. (2007) 222 p
4. K. A. Intriligator and N. Seiberg, ***Lectures on Supersymmetry Breaking*** Class. Quant. Grav. **24** (2007) S741 [hep-ph/0702069]
5. I. J. R. Aitchison, ***Supersymmetry and the MSSM: An Elementary introduction***, [hep-ph/0505105]

Outline

Outline

Introduction

Motivations for physics beyond the Standard Model (BSM)

- For many years the SM proved to be the most accurate description of Particle Physics, however theoretical and experimental disagreements:

Introduction

Motivations for physics beyond the Standard Model (BSM)

- For many years the SM proved to be the most accurate description of Particle Physics, however theoretical and experimental disagreements:
  - The SM does not provide a **dark matter (DM)** particle
  - No explanation for the origin of electric and color charges (gauge structure of the SM)
  - No explanation for fermion masses and mixings and flavour structure
  - Observation of neutrino oscillations requires mass eigenstates $\rightarrow$ not predicted in the SM
  - Anomalous magnetic moment of the muon
  - The SM suffers from the **Hierarchy Problem**
  - Hard to reconcile with the theory of General Relativity

Features of supersymmetry (SUSY)

- A possible cold dark matter particle
- Unification of the gauge couplings (SUSY GUT theories)
- Possible solution for the anomalous magnetic moment of the muon
- Connection to gravity in the limit of Local SUSY a.k.a supergravity (SUGRA)
- Mathematical beauty

Features of supersymmetry (SUSY)

- A possible cold dark matter particle
- Unification of the gauge couplings (SUSY GUT theories)
- Possible solution for the anomalous magnetic moment of the muon
- Connection to gravity in the limit of Local SUSY a.k.a supergravity (SUGRA)
- Mathematical beauty

However the one really good feature in favour os supersymmetry is

The Hierarchy Problem

Outline

The Hierarchy Problem

A QED analogy — hep-ph/0002232

Consider an electron as in classical electrostatics

1. Model the electron as solid sphere with radius R and uniform charge density

$$\Delta E_{\text{Coulomb}} = \frac{1}{4\pi\varepsilon_0}\frac{3}{5}\frac{e^2}{R} \quad (1)$$

The Hierarchy Problem

A QED analogy — hep-ph/0002232

Consider an electron as in classical electrostatics

1. Model the electron as solid sphere with radius R and uniform charge density

$$\Delta E_{\text{Coulomb}} = \frac{1}{4\pi\varepsilon_0}\frac{3}{5}\frac{e^2}{R} \tag{1}$$

2. This implies a correction to the electron mass $\delta m_e = \Delta E_{\text{Coulomb}}/c^2 \propto \frac{e^2}{4\pi}\Lambda$
3. The physical/observable mass is

$$m_{e,obs} = m_{e,bare} + \left(\frac{0.86\times 10^{-15}\ \text{meters}}{R}\right)\left(\frac{\text{MeV}}{c^2}\right) \tag{2}$$

The Hierarchy Problem

A QED analogy — hep-ph/0002232

Consider an electron as in classical electrostatics

1. Model the electron as solid sphere with radius R and uniform charge density

$$\Delta E_{\text{Coulomb}} = \frac{1}{4\pi\varepsilon_0}\frac{3}{5}\frac{e^2}{R} \tag{1}$$

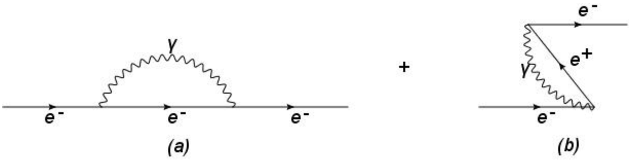
2. This implies a correction to the electron mass $\delta m_e = \Delta E_{\text{Coulomb}}/c^2 \propto \frac{e^2}{4\pi}\Lambda$
3. The physical/observable mass is

$$m_{e,obs} = m_{e,bare} + \left(\frac{0.86\times 10^{-15}\ \text{meters}}{R}\right)\left(\frac{\text{MeV}}{c^2}\right) \tag{2}$$

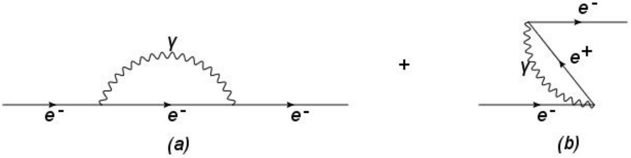
4. Experimentally $R \lesssim 10^{-19}$m, which implies that $\Delta E_{\text{Coulomb}} \gtrsim 8.6$ GeV
5. Requires an **unnatural** fine cancellation to obtain the observed mass

$$0.511\ \text{MeV/c}^2 = -8599.489\ \text{MeV/c}^2 + 8600.000\ \text{MeV/c}^2 \tag{3}$$

Quantum effects
electron self-energy



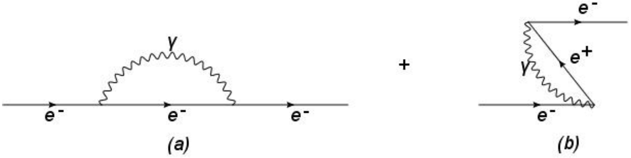
Quantum effects
electron self-energy



1. Naive calculation of the first diagram seems to vary linearly with a cutoff scale Λ

$$\delta m_e^{(a)} \propto \frac{e^2}{4\pi} \int^{\Lambda} \frac{d^4k}{\not{k} k^2} \sim \frac{e^2}{4\pi}\Lambda \quad (4)$$

Quantum effects
electron self-energy



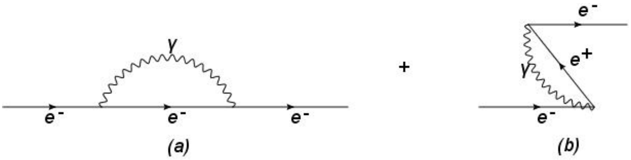
1. Naive calculation of the first diagram seems to vary linearly with a cutoff scale Λ

$$\delta m_e^{(a)} \propto \frac{e^2}{4\pi} \int^{\Lambda} \frac{d^4k}{\not{k} k^2} \sim \frac{e^2}{4\pi}\Lambda \tag{4}$$

2. The presence of a positron in the second diagram (flows backwards in time) provides the opposite contribution

$$\delta m_e^{(b)} \propto \frac{e^2}{4\pi} \int^{\Lambda} \frac{d^4k}{(-\not{k})\, k^2} \sim -\frac{e^2}{4\pi}\Lambda \tag{5}$$

Quantum effects
electron self-energy



1. Naive calculation of the first diagram seems to vary linearly with a cutoff scale Λ

$$\delta m_e^{(a)} \propto \frac{e^2}{4\pi} \int^{\Lambda} \frac{d^4 k}{\not{k} k^2} \sim \frac{e^2}{4\pi} \Lambda \quad (4)$$

2. The presence of a positron in the second diagram (flows backwards in time) provides the opposite contribution

$$\delta m_e^{(b)} \propto \frac{e^2}{4\pi} \int^{\Lambda} \frac{d^4 k}{(-\not{k})\, k^2} \sim -\frac{e^2}{4\pi} \Lambda \quad (5)$$

The leading linear divergence is canceled in QED due to the presence of a partner of the electron, the positron!

1. When the electron self-energy is calculated in detail in QED one finds

$$m_{e,obs} = m_{e,bare}\left(1 + \underbrace{\frac{3e^2}{8\pi^2}}_{3\alpha/2\pi}\log\left(\frac{\Lambda}{m_{e,bare}}\right) + O\left(e^4\right)\right) \quad (6)$$

2. Even if $\Lambda = M_{Planck} \sim 10^{19}$ GeV the correction is of order $O(1)$

1. When the electron self-energy is calculated in detail in QED one finds

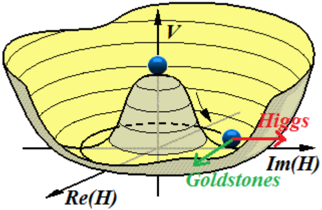
$$m_{e,obs} = m_{e,bare}\left(1 + \underbrace{\frac{3e^2}{8\pi^2}}_{3\alpha/2\pi}\log\left(\frac{\Lambda}{m_{e,bare}}\right) + O\left(e^4\right)\right) \quad (6)$$

2. Even if $\Lambda = M_{Planck} \sim 10^{19}$ GeV the correction is of order $O(1) \rightarrow$ **stabilized hierarchy**
3. This is due to a symmetry of the SM Lagrangian as the fermion masses go to zero called **chiral symmetry**
4. Chiral symmetry guarantees that radiative corrections to m vanish as $m \rightarrow 0$
5. In the same way **gauge symmetry** protects gauge bosons from acquiring **radiatively generated** masses for unbroken gauge theories

The Hierarchy Problem in the Standard Model

Revisiting the Higgs Mechanism — classical theory (see SM and Higgs Physics lectures)

$$\mathcal{L}_{SM} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\overline{\psi}\not{D}\psi + h.c. - \overline{\psi}_i\,(y_f)_{ij}\,\psi_j H + h.c. + (D^\mu H)^\dagger\,(D_\mu H) - V\left(H^\dagger H\right)$$

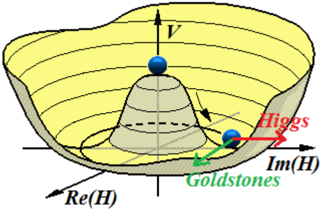


- $V\left(H^\dagger H\right) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2$
- Minimization $\longrightarrow \langle H \rangle = \frac{\mu}{\sqrt{2\lambda}} \equiv \frac{v}{\sqrt{2}}$
- $H = \frac{1}{\sqrt{2}}\begin{pmatrix} G^+ \\ v + h + iG^0 \end{pmatrix}$, $v = 246$ GeV

The Hierarchy Problem in the Standard Model

Revisiting the Higgs Mechanism — classical theory (see SM and Higgs Physics lectures)

$$\mathcal{L}_{SM} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\overline{\psi}\not{D}\psi + h.c. - \overline{\psi}_i\,(y_f)_{ij}\,\psi_j H + h.c. + (D^\mu H)^\dagger\,(D_\mu H) - V\left(H^\dagger H\right)$$



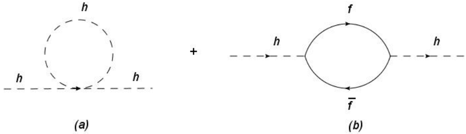
- $V\left(H^\dagger H\right) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2$
- Minimization $\longrightarrow \langle H \rangle = \frac{\mu}{\sqrt{2\lambda}} \equiv \frac{v}{\sqrt{2}}$
- $H = \frac{1}{\sqrt{2}}\begin{pmatrix} G^+ \\ v + h + iG^0 \end{pmatrix}$, $v = 246$ GeV

- Radial oscillations around vacuum generate a bare Higgs mass term

$$\mathcal{L}_{mass,h} = \frac{1}{2}\underbrace{\left(2\mu^2\right)}_{m_h^2} h^2 \tag{7}$$

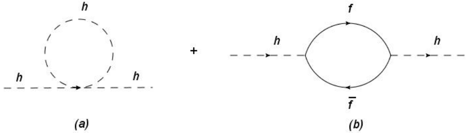
Quantum effects

Consider radiative corrections to the Higgs mass in the SM



Quantum effects

Consider radiative corrections to the Higgs mass in the SM



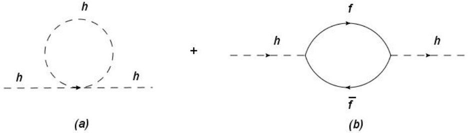
- Higgs self energy diagrams are **quadratically divergent**

$$\delta m_h^2 = \delta m_h^{2(a)} + \delta m_h^{2(b)} = \frac{1}{8\pi^2}\left[\left(\lambda - y_f^2\right)\Lambda^2 + \log\ terms\right] \quad (8)$$

- Correction to the Higgs mass: $m_{h,obs}^2 = m_h^2 + \underbrace{\delta m_h^2}_{\sim\Lambda^2}$

Quantum effects

Consider radiative corrections to the Higgs mass in the SM



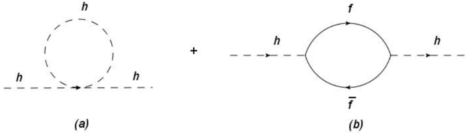
- Higgs self energy diagrams are **quadratically divergent**

$$\delta m_h^2 = \delta m_h^{2(a)} + \delta m_h^{2(b)} = \frac{1}{8\pi^2}\left[\left(\lambda - y_f^2\right)\Lambda^2 + \log\ terms\right] \quad (8)$$

- Correction to the Higgs mass: $m_{h,obs}^2 = m_h^2 + \underbrace{\delta m_h^2}_{\sim\Lambda^2}$
- $m_{h,obs}$ of the order of $\Lambda_{EW} \sim 100$ GeV
  - If new physics only at $\Lambda_{Plankk} \longrightarrow$ **Remarkable cancellation needed! UNNATURAL**

Quantum effects

Consider radiative corrections to the Higgs mass in the SM



- Higgs self energy diagrams are **quadratically divergent**

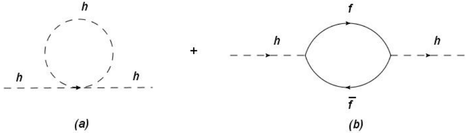
$$\delta m_h^2 = \delta m_h^{2(a)} + \delta m_h^{2(b)} = \frac{1}{8\pi^2}\left[\left(\lambda - y_f^2\right)\Lambda^2 + \log\ terms\right] \tag{8}$$

- Correction to the Higgs mass: $m_{h,obs}^2 = m_h^2 + \underbrace{\delta m_h^2}_{\sim\Lambda^2}$
- $m_{h,obs}$ of the order of $\Lambda_{EW} \sim 100$ GeV
  - If new physics only at $\Lambda_{Plankk} \longrightarrow$ **Remarkable cancellation needed! UNNATURAL**

Unless there is some symmetry to relate λ to y_f such that $\lambda = y_f^2$

Quantum effects

Consider radiative corrections to the Higgs mass in the SM



- Higgs self energy diagrams are **quadratically divergent**

$$\delta m_h^2 = \delta m_h^{2(a)} + \delta m_h^{2(b)} = \frac{1}{8\pi^2}\left[\left(\lambda - y_f^2\right)\Lambda^2 + \log\ terms\right] \tag{8}$$

- Correction to the Higgs mass: $m_{h,obs}^2 = m_h^2 + \underbrace{\delta m_h^2}_{\sim\Lambda^2}$
- $m_{h,obs}$ of the order of $\Lambda_{EW} \sim 100$ GeV
  - If new physics only at $\Lambda_{Plankk} \longrightarrow$ **Remarkable cancellation needed! UNNATURAL**

Unless there is some symmetry to relate λ to y_f such that $\lambda = y_f^2$

SUPERSYMMETRY

Back

Outline

Supersymmetric Algebra

An extension to the *Coleman and Mandula's Theorem*

Supersymmetry originally introduced as an extension to the *Coleman and Mandula's Theorem*

Coleman and Mandula's Theorem (1967) implies that...

... there is no way of relating fields with distinct space-time properties **such as spin**

Supersymmetric Algebra

An extension to the *Coleman and Mandula's Theorem*

Supersymmetry originally introduced as an extension to the *Coleman and Mandula's Theorem*

Coleman and Mandula's Theorem (1967) implies that...

... there is no way of relating fields with distinct space-time properties **such as spin**

R. Haag, J. Lopuszanski and M. Sohnius (1975)

Proved that it is possible to go around Coleman-Mandula's theorem by allowing anti-commuting generators

Supersymmetric Algebra

An extension to the *Coleman and Mandula's Theorem*

Supersymmetry originally introduced as an extension to the *Coleman and Mandula's Theorem*

Coleman and Mandula's Theorem (1967) implies that...

... there is no way of relating fields with distinct space-time properties **such as spin**

R. Haag, J. Lopuszanski and M. Sohnius (1975)

Proved that it is possible to go around Coleman-Mandula's theorem by allowing anti-commuting generators

- The concept of Graded Lie Algebra emerged, $\mathcal{V}_b \oplus \mathcal{V}_f$
  - $\mathcal{V}_b \to$ bosonic elements related by commutation relations
  - $\mathcal{V}_f \to$ fermionic elements related by anti-commutation relations

- In supersymmetry the bosonic vector space $\mathcal{V}_b$ is the Poincaré group with algebra

$$
\begin{aligned}
[P_\mu, P_\nu] &= 0 && (9)\\
[M_{\mu\nu}, P_\lambda] &= i\,(g_{\nu\lambda}P_\mu - g_{\mu\lambda}P_\nu) && (10)\\
[M_{\mu\nu}, M_{\rho\sigma}] &= i\,(g_{\nu\rho}M_{\mu\sigma} + g_{\mu\sigma}M_{\nu\rho} - g_{\mu\rho}M_{\nu\sigma} - g_{\nu\sigma}M_{\mu\rho}) && (11)
\end{aligned}
$$

$P_\mu \rightarrow$ translations generator, $M_{\mu\nu} \rightarrow$ angular momentum generator

- In supersymmetry the bosonic vector space $\mathcal{V}_b$ is the Poincaré group with algebra

$$[P_\mu, P_\nu] = 0 \quad (9)$$
$$[M_{\mu\nu}, P_\lambda] = i\,(g_{\nu\lambda}P_\mu - g_{\mu\lambda}P_\nu) \quad (10)$$
$$[M_{\mu\nu}, M_{\rho\sigma}] = i\,(g_{\nu\rho}M_{\mu\sigma} + g_{\mu\sigma}M_{\nu\rho} - g_{\mu\rho}M_{\nu\sigma} - g_{\nu\sigma}M_{\mu\rho}) \quad (11)$$

$P_\mu \rightarrow$ translations generator, $M_{\mu\nu} \rightarrow$ angular momentum generator

- To close the SUSY algebra a two-component Weyl spinor generator Q_α is introduced

$$Q|Fermion\rangle = |Boson\rangle \quad (12)$$
$$Q|Boson\rangle = |Fermion\rangle \quad (13)$$

- In supersymmetry the bosonic vector space $\mathcal{V}_b$ is the Poincaré group with algebra

$$\begin{aligned} [P_\mu, P_\nu] &= 0 & (9)\\ [M_{\mu\nu}, P_\lambda] &= i\left(g_{\nu\lambda}P_\mu - g_{\mu\lambda}P_\nu\right) & (10)\\ [M_{\mu\nu}, M_{\rho\sigma}] &= i\left(g_{\nu\rho}M_{\mu\sigma} + g_{\mu\sigma}M_{\nu\rho} - g_{\mu\rho}M_{\nu\sigma} - g_{\nu\sigma}M_{\mu\rho}\right) & (11)\end{aligned}$$

$P_\mu \to$ translations generator, $M_{\mu\nu} \to$ angular momentum generator

- To close the SUSY algebra a two-component Weyl spinor generator Q_α is introduced

$$Q|Fermion\rangle = |Boson\rangle \quad (12)$$

$$Q|Boson\rangle = |Fermion\rangle \quad (13)$$

- Supersymmetric algebra given by eqs.(9)-(11) and (14)-(19)

$$\begin{aligned} [P_\mu, Q_\alpha] &= \left[P_\mu, \overline{Q}^{\dot\alpha}\right] = 0 & (14)\\ [M_{\mu\nu}, Q_\alpha] &= -i\left(\sigma_{\mu\nu}\right)_\alpha{}^\beta Q_\beta & (15)\\ \left[M_{\mu\nu}, \overline{Q}^{\dot\beta}\right] &= -i\left(\overline{\sigma}_{\mu\nu}\right)^{\dot\beta}{}_{\dot\alpha}\,\overline{Q}^{\dot\alpha} & (16)\\ \{Q_\alpha, Q_\beta\} &= \left\{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\right\} = 0 & (17)\\ \left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\} &= 2\sigma^\mu_{\alpha\dot\beta}P_\mu & (18)\\ [T^a, Q_\alpha] &= \left[T^a, \overline{Q}_{\dot\alpha}\right] = 0 & (19)\end{aligned}$$

$$\begin{aligned} \sigma^\mu &\equiv (\mathbb{1}, \sigma^i),\\ \overline{\sigma}^\mu &\equiv (\mathbb{1}, -\sigma^i),\\ \sigma^{\mu\nu} &\equiv \frac{1}{4}(\sigma^\mu\overline{\sigma}^\nu - \sigma^\nu\overline{\sigma}^\mu),\\ \overline{\sigma}^{\mu\nu} &\equiv \frac{1}{4}(\overline{\sigma}^\mu\sigma^\nu - \overline{\sigma}^\nu\sigma^\mu).\end{aligned}$$

LUND UNIVERSITY

Consequences of the supersymmetric algebra

(1) Take the anti-commutation relation $\left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$ and the identity $tr\left(\sigma^\mu \overline{\sigma}^\nu\right) = 2g^{\mu\nu}$, and apply $\left(\overline{\sigma}^\nu\right)^{\dot\beta\alpha}$ to the anti-commutator...

Consequences of the supersymmetric algebra

(1) Take the anti-commutation relation $\left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$ and the identity $tr\left(\sigma^\mu \overline{\sigma}^\nu\right) = 2g^{\mu\nu}$, and apply $\left(\overline{\sigma}^\nu\right)^{\dot\beta\alpha}$ to the anti-commutator...

$$\left(\overline{\sigma}^\nu\right)^{\dot\beta\alpha}\left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\} = 4P^\nu \ , \qquad (20)$$

Consequences of the supersymmetric algebra

(1) Take the anti-commutation relation $\left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$ and the identity $tr\left(\sigma^\mu\overline{\sigma}^\nu\right) = 2g^{\mu\nu}$, and apply $\left(\overline{\sigma}^\nu\right)^{\dot\beta\alpha}$ to the anti-commutator...

$$\left(\overline{\sigma}^\nu\right)^{\dot\beta\alpha}\left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\} = 4P^\nu \ , \tag{20}$$

If we take the matrix element of the zeroth component $4P^0$ we get

$$\langle\psi|Q_\alpha(Q_\alpha)^* + (Q_\alpha)^* Q_\alpha|\psi\rangle \geqslant 0 \tag{21}$$

where the identity among spinor components $\overline{\xi}_{\dot\alpha} \equiv (\xi_\alpha)^*$ was used.

Consequences of the supersymmetric algebra

(1) Take the anti-commutation relation $\left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$ and the identity $tr\left(\sigma^\mu \overline{\sigma}^\nu\right) = 2g^{\mu\nu}$, and apply $\left(\overline{\sigma}^\nu\right)^{\dot\beta\alpha}$ to the anti-commutator...

$$\left(\overline{\sigma}^\nu\right)^{\dot\beta\alpha}\left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\} = 4P^\nu \ , \tag{20}$$

If we take the matrix element of the zeroth component $4P^0$ we get

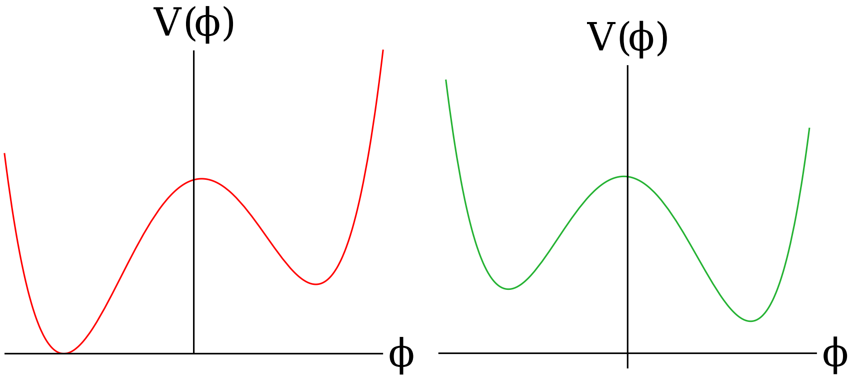
$$\langle\psi|Q_\alpha (Q_\alpha)^* + (Q_\alpha)^* Q_\alpha|\psi\rangle \geqslant 0 \tag{21}$$

where the identity among spinor components $\overline{\xi}_{\dot\alpha} \equiv (\xi_\alpha)^*$ was used.

- The Hamiltonian $H = P^0$ is positive semi-definite

(A) **The vacuum of a supersymmetric therory has zero energy**
(B) **If SUSY is spontaneously broken the vacuum has positive energy**

LUND UNIVERSITY



Case (A)
Unbroken SUSY

Case (B)
Spontaneously broken SUSY

(2) Q_α and $\overline{Q}_{\dot\alpha}$ change the fermion number by one unit

Introduce the fermion number operator $\mathcal{N}_f$ such that $(-1)^{\mathcal{N}_f}|F\rangle = -|F\rangle$ and $(-1)^{\mathcal{N}_f}|B\rangle = |B\rangle$ in order to obtain

$$(-1)^{\mathcal{N}_f} Q_\alpha = -Q_\alpha (-1)^{\mathcal{N}_f} \ . \tag{22}$$

(2) Q_α and $\overline{Q}_{\dot\alpha}$ change the fermion number by one unit

Introduce the fermion number operator $\mathcal{N}_f$ such that $(-1)^{\mathcal{N}_f}|F\rangle = -|F\rangle$ and $(-1)^{\mathcal{N}_f}|B\rangle = |B\rangle$ in order to obtain

$$(-1)^{\mathcal{N}_f} Q_\alpha = -Q_\alpha (-1)^{\mathcal{N}_f} \; . \tag{22}$$

Use the cyclic property of the trace to show that

$$\begin{aligned} tr\left[(-1)^{\mathcal{N}_f}\left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\}\right] &= tr\left[(-1)^{\mathcal{N}_f} Q_\alpha \overline{Q}_{\dot\beta} + \underbrace{(-1)^{\mathcal{N}_f}\overline{Q}_{\dot\beta} Q_\alpha}_{\text{cyclic property}}\right] \\ &= tr\left[-Q_\alpha(-1)^{\mathcal{N}_f}\overline{Q}_{\dot\beta} + Q_\alpha(-1)^{\mathcal{N}_f}\overline{Q}_{\dot\beta}\right] = 0 \end{aligned}$$

(2) Q_α and $\overline{Q}_{\dot\alpha}$ change the fermion number by one unit

Introduce the fermion number operator $\mathcal{N}_f$ such that $(-1)^{\mathcal{N}_f}|F\rangle = -|F\rangle$ and $(-1)^{\mathcal{N}_f}|B\rangle = |B\rangle$ in order to obtain

$$(-1)^{\mathcal{N}_f} Q_\alpha = -Q_\alpha (-1)^{\mathcal{N}_f} \ . \tag{22}$$

Use the cyclic property of the trace to show that

$$\begin{aligned} tr\left[(-1)^{\mathcal{N}_f}\left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\}\right] &= tr\left[(-1)^{\mathcal{N}_f} Q_\alpha \overline{Q}_{\dot\beta} + \underbrace{(-1)^{\mathcal{N}_f}\overline{Q}_{\dot\beta} Q_\alpha}_{\text{cyclic property}}\right] \\ &= tr\left[-Q_\alpha(-1)^{\mathcal{N}_f}\overline{Q}_{\dot\beta} + Q_\alpha(-1)^{\mathcal{N}_f}\overline{Q}_{\dot\beta}\right] = 0 \end{aligned}$$

For non-zero P^μ we deduce that

$$tr\left[(-1)^{\mathcal{N}_f} 2\sigma^\mu_{\alpha\dot\beta} P_\mu\right] = 0 \Rightarrow tr\left[(-1)^{\mathcal{N}_f}\right] = 0 \ . \tag{23}$$

- Consider a SUSY representation $\mathbf{R}$ with $n_F(\mathbf{R})$ fermions and $n_B(\mathbf{R})$ bosons

$$\mathbf{R} = |F_1, \dots, F_{n_F}; B_1, \dots, B_{n_B}\rangle \ ,$$

then

$$tr\left[(-1)^{\mathcal{N}_f}\right] = n_B(\mathbf{R}) - n_F(\mathbf{R}) = 0 \ , \tag{24}$$

- Consider a SUSY representation $\mathbf{R}$ with $n_F(\mathbf{R})$ fermions and $n_B(\mathbf{R})$ bosons

$$\mathbf{R} = |F_1, \ldots, F_{n_F}; B_1, \ldots, B_{n_B}\rangle \ ,$$

then

$$tr\left[(-1)^{\mathcal{N}_f}\right] = n_B(\mathbf{R}) - n_F(\mathbf{R}) = 0 \ , \tag{24}$$

The number of fermions and bosons in a representation of the supersymmetric algebra or SUPERMULTIPLET is the same

- Consider a SUSY representation $\mathbf{R}$ with $n_F(\mathbf{R})$ fermions and $n_B(\mathbf{R})$ bosons

$$\mathbf{R} = |F_1, \ldots, F_{n_F}; B_1, \ldots, B_{n_B}\rangle \ ,$$

then

$$tr\left[(-1)^{\mathcal{N}_f}\right] = n_B(\mathbf{R}) - n_F(\mathbf{R}) = 0 \ , \tag{24}$$

The number of fermions and bosons in a representation of the supersymmetric algebra or SUPERMULTIPLET is the same

(3) As P^2 and T^a commute with Q_α and $\overline{Q}_{\dot{\alpha}}$, all members of a given **supermultiplet** must have the same mass and gauge quantum numbers

Types of supermultiplets

(4) It is possible to show (D. Bailin & A. Love chap 1) that in a given supermultiplet there are only states with helicities λ and $\lambda - \frac{1}{2}$

Types of supermultiplets

(4) It is possible to show (D. Bailin & A. Love chap 1) that in a given supermultiplet there are only states with helicities λ and $\lambda - \frac{1}{2}$

- **Chiral Supermultiplet:** $\lambda = \frac{1}{2}$ (fermion) and $\lambda - \frac{1}{2} = 0$ (sfermion)
  - (i) 1 two-component Weyl fermion $(n_F = 2)$
  - (ii) 2 real scalars = 1 complex scalar $(n_B = 2)$

    Standard Model quarks, leptons and Higgs bosons fit here
- **Gauge Supermultiplet:** $\lambda = 1$ (gauge boson) and $\lambda - \frac{1}{2} = \frac{1}{2}$ (gaugino)
  - (i) 1 two-component Weyl gugino fermion $(n_F = 2)$
  - (ii) 1 real massless gauge vector boson (2 transverse polarizations) $(n_B = 2)$

    Standard Model gauge bosons fit here
- **Gravity Supermultiplet:** $\lambda = 2$ (graviton) and $\lambda - \frac{1}{2} = \frac{3}{2}$ (gravitino)
  - (i) 1 two-component Weyl gravitino fermion $(n_F = 2)$
  - (ii) 1 real massless graviton $(n_B = 2)$

Outline

Constructing supersymmetric Lagrangians

Superfields (D. Bailin & A. Love chap 2 and 3)

- The easiest way to construct SUSY Lagrangians is by introducing superfields $\mathcal{S}\left(x^{\mu}, \theta^{\alpha}, \overline{\theta}_{\dot{\alpha}}\right)$
  - θ and $\overline{\theta}$ are anti-commuting variables or Grassman variables
  - together with space-time coordinates they span the **superspace** with well defined differentiation and integration

Constructing supersymmetric Lagrangians

Superfields (D. Bailin & A. Love chap 2 and 3)

- The easiest way to construct SUSY Lagrangians is by introducing superfields $\mathcal{S}\left(x^{\mu}, \theta^{\alpha}, \overline{\theta}_{\dot{\alpha}}\right)$
  - θ and $\overline{\theta}$ are anti-commuting variables or Grassman variables
  - together with space-time coordinates they span the **superspace** with well defined differentiation and integration

(1) **Chiral superfields expansion**

$$\begin{aligned} \mathbf{\Phi} &= \phi\left(x\right) + \sqrt{2}\theta\psi\left(x\right) + \theta\theta F\left(x\right) + i\partial_{\mu}\phi\left(x\right)\theta\sigma^{\mu}\bar{\theta} \\ &- \frac{i}{\sqrt{2}}\theta\theta\partial_{\mu}\psi\left(x\right)\sigma^{\mu}\bar{\theta} - \frac{1}{4}\partial_{\mu}\partial^{\mu}\phi\left(x\right)\theta\theta\bar{\theta}\bar{\theta}\,, \end{aligned} \tag{25}$$

$$\begin{aligned} \mathbf{\Phi}^{\dagger} &= \phi^{\dagger}\left(x\right) + \sqrt{2}\bar{\theta}\bar{\psi}\left(x\right) + \bar{\theta}\bar{\theta}F^{\dagger}\left(x\right) - i\partial_{\mu}\phi^{\dagger}\left(x\right)\theta\sigma^{\mu}\bar{\theta} \\ &+ \frac{i}{\sqrt{2}}\bar{\theta}\bar{\theta}\theta\sigma^{\mu}\partial_{\mu}\bar{\psi}\left(x\right) - \frac{1}{4}\partial_{\mu}\partial^{\mu}\phi^{\dagger}\left(x\right)\theta\theta\bar{\theta}\bar{\theta}\,. \end{aligned} \tag{26}$$

(2) Vector/gauge superfields (real) expansion

$$\mathcal{V}^a = \theta\sigma^\mu\bar{\theta}A^a_\mu(x) + i\theta\theta\bar{\theta}\bar{\lambda}^a(x) - i\bar{\theta}\bar{\theta}\theta\lambda^a(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\mathcal{D}^a(x)\ . \tag{27}$$

(2) Vector/gauge superfields (real) expansion

$$\mathcal{V}^a = \theta\sigma^\mu\bar{\theta}A^a_\mu(x) + i\theta\theta\bar{\theta}\bar{\lambda}^a(x) - i\bar{\theta}\bar{\theta}\theta\lambda^a(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\mathcal{D}^a(x) \,. \tag{27}$$

- **Chiral:** $\psi \to$ fermions, $\phi \to$ scalars, $F \to$ auxiliary fields
- **Gauge:** $A^a_\mu \to$ gauge bosons, $\lambda^a \to$ gauginos, $\mathcal{D}^a \to$ auxiliary fields

Lund University

(2) Vector/gauge superfields (real) expansion

$$\mathcal{V}^a = \theta\sigma^\mu\bar{\theta}A^a_\mu(x) + i\theta\theta\bar{\theta}\bar{\lambda}^a(x) - i\bar{\theta}\bar{\theta}\theta\lambda^a(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\mathcal{D}^a(x)\ . \tag{27}$$

- **Chiral:** $\psi \to$ fermions, $\phi \to$ scalars, $F \to$ auxiliary fields
- **Gauge:** $A^a_\mu \to$ gauge bosons, $\lambda^a \to$ gauginos, $\mathcal{D}^a \to$ auxiliary fields

F and $\mathcal{D}^a$ fields are non-propagating degrees of freedom (do not affect dynamics) that guarantee the same degrees of freedom on-shell and off-shell $\Longrightarrow$ SUSY Lagrangians invariant both on-shell and off-shell

$$\frac{\partial\mathcal{L}}{\partial F_i} = 0\ , \quad \frac{\partial\mathcal{L}}{\partial F^{*i}} = 0\ , \quad \frac{\partial\mathcal{L}}{\partial\mathcal{D}^a} = 0\ , \quad \cancel{\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\right)} - \frac{\partial\mathcal{L}}{\partial\phi_i} = 0 \tag{28}$$

LUND UNIVERSITY

The Kähler potential and the superpotential

All the information needed to construct a SUSY Lagrangian is encoded in three objects, the Kähler potential, the superpotential and the field strength superfield

- The Kähler potential encodes kinetic terms and gauge-scalar interactions
- The superpotential encodes chiral field interactions (like Yukawa interactions)

The Kähler potential and the superpotential

All the information needed to construct a SUSY Lagrangian is encoded in three objects, the Kähler potential, the superpotential and the field strength superfield

- The Kähler potential encodes kinetic terms and gauge-scalar interactions
- The superpotential encodes chiral field interactions (like Yukawa interactions)

(1) **Construction of chiral-free (non interacting) Lagrangian**

Define the Kähler potential as $K\left(\Phi^\dagger,\Phi\right)=\Phi^{\dagger i}\Phi_i$ (vector superfield) and extract D-terms ($\theta\theta\bar{\theta}\bar{\theta}$ coefficient)

$$\mathcal{L}_{chiral-free}=\int d^2\theta d^2\bar{\theta}\,K\left(\Phi^\dagger,\Phi\right)=-\partial^\mu\phi^{*i}\partial_\mu\phi_i+i\psi^{\dagger i}\overline{\sigma}^\mu\partial_\mu\psi_i+F^{*i}F_i+\cdots\,. \quad (29)$$

(2) Construction of chiral-interacting Lagrangian

Construct **the most generic** holomorphic function of the superfields that respects **gauge invariance** and ensures **renormalizability** of $\mathcal{L}$, the **SUPERPOTENTIAL**

$$W = L^i \Phi_i + \frac{1}{2}\mu^{ij}\Phi_i\Phi_j + \frac{1}{6}y^{ijk}\Phi_i\Phi_j\Phi_k \ , \tag{30}$$

(2) Construction of chiral-interacting Lagrangian

Construct **the most generic** holomorphic function of the superfields that respects **gauge invariance** and ensures **renormalizability** of $\mathcal{L}$, the **SUPERPOTENTIAL**

$$W = L^i \Phi_i + \frac{1}{2}\mu^{ij}\Phi_i\Phi_j + \frac{1}{6}y^{ijk}\Phi_i\Phi_j\Phi_k \ , \tag{30}$$

W is a chiral superfield on its own $\rightarrow$ Extract F-terms ($\theta\theta$ coefficients)

$$\mathcal{L}_{chiral-int} = \int d^2\theta W\Big|_{\bar{\theta}=0} + h.c. = \left(-\frac{1}{2} \underbrace{\frac{\partial W}{\partial\Phi_i\partial\Phi_j}}_{W^{ij}} \psi_i\psi_j + \underbrace{\frac{\partial W}{\partial\Phi_i}}_{W^i=-F^{*i}} F_i \right) + h.c. \ , \tag{31}$$

(2) Construction of chiral-interacting Lagrangian

Construct **the most generic** holomorphic function of the superfields that respects **gauge invariance** and ensures **renormalizability** of $\mathcal{L}$, the **SUPERPOTENTIAL**

$$W = L^i \boldsymbol{\Phi}_i + \frac{1}{2}\mu^{ij}\boldsymbol{\Phi}_i\boldsymbol{\Phi}_j + \frac{1}{6}y^{ijk}\boldsymbol{\Phi}_i\boldsymbol{\Phi}_j\boldsymbol{\Phi}_k \ , \tag{30}$$

W is a chiral superfield on its own → Extract F-terms ($\theta\theta$ coefficients)

$$\mathcal{L}_{chiral-int} = \int d^2\theta W\Big|_{\bar\theta=0} + h.c. = \left(-\frac{1}{2}\underbrace{\frac{\partial W}{\partial\Phi_i\partial\Phi_j}}_{W^{ij}}\psi_i\psi_j + \underbrace{\frac{\partial W}{\partial\Phi_i}}_{W^i=-F^{*i}} F_i \right) + h.c. \ , \tag{31}$$

(3) Add both contributions

$$\mathcal{L}_{chiral} = -\partial^\mu\phi^{*i}\partial_\mu\phi_i + i\psi^{\dagger i}\bar\sigma^\mu\partial_\mu\psi_i - \frac{1}{2}\left(W^{ij}\psi_i\psi_j + h.c.\right) - W^i W^*_i \ , \tag{32}$$

Back

(4) Adding gauge contributions

Redefine the Kähler potential such that it is explicitly gauge invariant

$$K\left(\mathbf{\Phi}^{\dagger}, e^{2g_a T^a \mathcal{V}^a}\mathbf{\Phi}\right) = \mathbf{\Phi}^{\dagger i} e^{2g_a T^a \mathcal{V}^a}\mathbf{\Phi}_{i} \tag{33}$$

(4) **Adding gauge contributions**

Redefine the Kähler potential such that it is explicitly gauge invariant

$$K\left(\Phi^{\dagger}, e^{2g_a T^a \mathcal{V}^a}\Phi\right) = \Phi^{\dagger i} e^{2g_a T^a \mathcal{V}^a}\Phi_i \quad (33)$$

Extract D-terms ($\theta\theta\bar{\theta}\bar{\theta}$ coefficient of (33))

$$\begin{aligned}\mathcal{L}_{gauge-chiral} &= \int d^2\theta d^2\bar{\theta}\, K\left(\Phi^{\dagger}, e^{2g_a T^a \mathcal{V}^a}\Phi\right) \\ &= -D^{\mu}\phi^{*i}D_{\mu}\phi_i + i\psi^{\dagger i}\bar{\sigma}^{\mu}D_{\mu}\psi_i + F^{*i}F_i - \sqrt{2}g\left(\phi^{*i}T^a\psi_i\right)\lambda^a \\ &- \sqrt{2}g\lambda^{a\dagger}\left(\psi^{\dagger i}T^a\phi_i\right) + g\left(\phi^{*i}T^a\phi_i\right)\mathcal{D}^a\,. \end{aligned} \quad (34)$$

$D_{\mu}\phi_i = \partial_{\mu}\phi_i + i g_a T^a A^a_{\mu}\phi_i$

(4) Adding gauge contributions

Redefine the Kähler potential such that it is explicitly gauge invariant

$$K\left(\Phi^\dagger, e^{2g_a T^a \mathcal{V}^a}\Phi\right) = \Phi^{\dagger i} e^{2g_a T^a \mathcal{V}^a}\Phi_i \tag{33}$$

Extract D-terms ($\theta\theta\bar{\theta}\bar{\theta}$ coefficient of (33))

$$\begin{aligned}\mathcal{L}_{gauge-chiral} &= \int d^2\theta d^2\bar{\theta}\, K\left(\Phi^\dagger, e^{2g_a T^a \mathcal{V}^a}\Phi\right) \\ &= -D^\mu \phi^{*i} D_\mu \phi_i + i\psi^{\dagger i}\bar{\sigma}^\mu D_\mu \psi_i + F^{*i}F_i - \sqrt{2}g\left(\phi^{*i}T^a\psi_i\right)\lambda^a \\ &- \sqrt{2}g\lambda^{a\dagger}\left(\psi^{\dagger i}T^a\phi_i\right) + g\left(\phi^{*i}T^a\phi_i\right)\mathcal{D}^a\,. \end{aligned} \tag{34}$$

$D_\mu\phi_i = \partial_\mu\phi_i + ig_a T^a A^a_\mu \phi_i$

- $\mathcal{L}_{gauge-chiral}$ provides scalar-gauge, fermion gauge, 4-scalar and scalar-fermion-gaugino interactions

(5) Gauge kinetic terms

Construct the **field strength superfield** for generic gauge theory (supersymmetrized version of $F^a_{\mu\nu}$)

$$\mathcal{W}_\alpha = -\frac{1}{4}\bar{D}^2 e^{-2g_a T^a \mathcal{V}^a} D_\alpha e^{2g_a T^a \mathcal{V}^a} \tag{35}$$

D_α and $D^{\dot{\alpha}}$ are covariant derivatives for Grassmann variables.

(5) Gauge kinetic terms

Construct the **field strength superfield** for generic gauge theory (supersymmetrized version of $F^a_{\mu\nu}$)

$$\mathcal{W}_\alpha = -\frac{1}{4}\bar{D}^2 e^{-2g_a T^a \mathcal{V}^a} D_\alpha e^{2g_a T^a \mathcal{V}^a} \tag{35}$$

D_α and $D^{\dot{\alpha}}$ are covariant derivatives for Grassmann variables.

Since $\mathcal{W}_\alpha$ is a chiral superfield on its own, extract F-terms ($\theta\theta$ coefficients)

$$\begin{aligned} \mathcal{L}_{gauge} &= \int d^2\theta \frac{1}{4g_a^2} Tr\left[\mathcal{W}^\alpha \mathcal{W}_\alpha\right]\Big|_{\bar{\theta}=0} + h.c. \\ &= -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\lambda^{a\dagger}\overline{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2}\mathcal{D}^a\mathcal{D}^a , \end{aligned} \tag{36}$$

- Using the EOM for the auxiliary fields, the last term in (36) combined with the $g\left(\phi^{*i}T^a\phi_i\right)\mathcal{D}^a$ terms provides an algebraic expression for $\mathcal{D}^a$

$$\frac{\partial \mathcal{L}_{tot}}{\partial \mathcal{D}^a} = 0 \Rightarrow \mathcal{D}^a = -g\left(\phi^{*i}T^a\phi_i\right) \tag{37}$$

Total SUSY Lagrangian

- The total supersymmetric Lagrangian is given by $\mathcal{L}_{SUSY} = \mathcal{L}_{chiral-int} + \mathcal{L}_{gauge-chiral} + \mathcal{L}_{gauge}$

$$\begin{aligned}\mathcal{L}_{SUSY} = & - D^{\mu}\phi^{*i}D_{\mu}\phi_i + i\psi^{\dagger i}\overline{\sigma}^{\mu}D_{\mu}\psi_i - \frac{1}{2}\left(W^{ij}\psi_i\psi_j + h.c.\right) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} \\ & + i\lambda^{a\dagger}\overline{\sigma}^{\mu}D_{\mu}\lambda^a - \sqrt{2}g\left(\phi^{*i}T^a\psi_i\right)\lambda^a - \sqrt{2}g\lambda^{a\dagger}\left(\psi^{\dagger i}T^a\phi_i\right) \\ & - V(\phi^{*i}, \phi_i)\ . \end{aligned} \tag{38}$$

The scalar potential $V\left(\phi^{*i}, \phi_i\right)$ is entirely derived from the *F* and *D-terms*

$$V\left(\phi^{*i}, \phi_i\right) = F^{i*}F_i + \frac{1}{2}\mathcal{D}^a\mathcal{D}^a\ , \tag{39}$$

The Hierarchy Problem revisited

- We have seen in eq. (8) that corrections to the Higgs mass are quadratically divergent $\delta m_h^2 \sim \left(\lambda - y_f^2\right) \Lambda^2 \phi^* \phi$
- If the theory has more scalars they **all** suffer from this "*pathology*"
- **Unless the model is supersymmetric!**

The Hierarchy Problem revisited

- We have seen in eq. (8) that corrections to the Higgs mass are quadratically divergent $\delta m_h^2 \sim \left(\lambda - y_f^2\right) \Lambda^2 \phi^* \phi$
- If the theory has more scalars they **all** suffer from this "*pathology*"
- **Unless the model is supersymmetric!**

Take the superpotential of eq. (30) and plug it in the Lagrangian for chiral interactions $\mathcal{L}_{int} = -\frac{1}{2}\left(W^{ij}\psi_i\psi_j + h.c.\right) - W^i W_i^*$

Dropping indices for ease of notation we get the chiral interactions:

$$\mathcal{L}_{int} = -\frac{1}{2}\left[(\mu + y\phi)\psi \cdot \psi\right] - \left|\mu\phi + \frac{1}{2}y\phi^2\right|^2 + \cdots \quad . \tag{40}$$

The 4-scalar and Yuakwa-type interactions of SUSY models are related through the Yukawa coupling

$$-\underbrace{\frac{1}{2}y}_{y_f}\,\phi\psi \cdot \psi \quad \text{vs} \quad -\underbrace{\frac{1}{4}|y|^2}_{\lambda}\,\phi^2\phi^{*2} \tag{41}$$

The Hierarchy Problem revisited

- We have seen in eq. (8) that corrections to the Higgs mass are quadratically divergent $\delta m_h^2 \sim \left(\lambda - y_f^2\right) \Lambda^2 \phi^* \phi$
- If the theory has more scalars they **all** suffer from this "*pathology*"
- **Unless the model is supersymmetric!**

Take the superpotential of eq. (30) and plug it in the Lagrangian for chiral interactions $\mathcal{L}_{int} = -\frac{1}{2}\left(W^{ij}\psi_i\psi_j + h.c.\right) - W^i W_i^*$

Dropping indices for ease of notation we get the chiral interactions:

$$\mathcal{L}_{int} = -\frac{1}{2}\left[(\mu + y\phi)\psi \cdot \psi\right] - \left|\mu\phi + \frac{1}{2}y\phi^2\right|^2 + \cdots \quad . \tag{40}$$

The 4-scalar and Yuakwa-type interactions of SUSY models are related through the Yukawa coupling

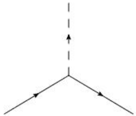
$$-\underbrace{\frac{1}{2}y}_{y_f}\,\phi\psi\cdot\psi \quad \text{vs} \quad -\underbrace{\frac{1}{4}|y|^2}_{\lambda}\,\phi^2\phi^{*2} \tag{41}$$

SUSY stabilizes the hierarchy!

$\left(\frac{1}{2}y\right)^2 = \frac{1}{4}|y|^2 \Rightarrow \boldsymbol{y_f^2 = \lambda}$, **scalars** and fermions with equal masses

SUSY interactions Feynman diagrams

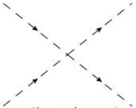
(1) $W^{ij}\psi_i\psi_j$



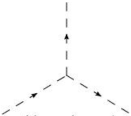
Yukawa-type vertex

fermion mass term

(2) $V(\phi^{*i},\phi_i)$



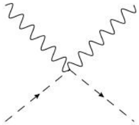
quartic scalar vertex



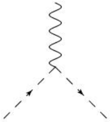
cubic scalar vertex

scalar mass term

(3) $D^\mu\phi^{*i}D_\mu\phi_i$

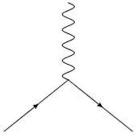


2 scalar - 2 gauge vertex



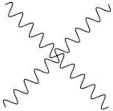
scalar-scalar-gauge vertex

(4) $i\psi^{\dagger i}\overline{\sigma}^\mu D_\mu\psi_i$

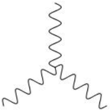


fermion-fermion-gauge vertex

(5) $F^a_{\mu\nu}F^{a\mu\nu}$

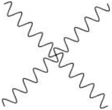


quartic gauge vertex

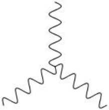


cubic gauge vertex

(5) $F^a_{\mu\nu}F^{a\mu\nu}$

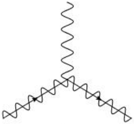


quartic gauge vertex



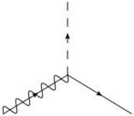
cubic gauge vertex

(6) $i\lambda^{a\dagger}\overline{\sigma}^{\mu}D_{\mu}\lambda^a$



gaugino-gaugino-gauge vertex

(7) $-\sqrt{2}g\left(\phi^{*i}T^a\psi_i\right)\lambda^a$



gaugino-fermion-scalar vertex

Outline

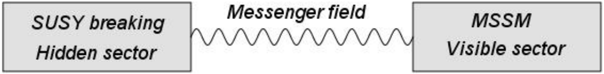
Soft Supersymmetry Breaking

- If SUSY is unbroken at low energies the masses of the matter fields are equal for both scalar and fermions
- **Not what we observe in experiments** (no SUSY so far)
- **SUSY has to be broken!**

Soft Supersymmetry Breaking

- If SUSY is unbroken at low energies the masses of the matter fields are equal for both scalar and fermions
- **Not what we observe in experiments** (no SUSY so far)
- **SUSY has to be broken!**

1. SUSY spontaneously broken (SSB) in a hidden sector
2. suppressed interactions (gravity, gauge, ...) communicate with visible sector
   - carry information about the breaking mechanism
   - dictate high-scale structure of observable couplings



Gravity mediation

SUSY breaking is transmitted to the visible sector by gravitational interactions that enter near the Planck mass scale M_P

Gravity mediation

SUSY breaking is transmitted to the visible sector by gravitational interactions that enter near the Planck mass scale M_P

- One possibility for spontaneous SUSY breaking in the hidden sector, is to let an F-field acquiring a VEV $\langle F \rangle$ $(\mathrm{mass}^2$ dimensions)
- In the visible sector the SUSY breaking terms should be of the order

$$m_{vis} \approx \frac{\langle F \rangle}{M_P}$$

Gravity mediation

SUSY breaking is transmitted to the visible sector by gravitational interactions that enter near the Planck mass scale M_P

- One possibility for spontaneous SUSY breaking in the hidden sector, is to let an F-field acquiring a VEV $\langle F \rangle$ $(\mathrm{mass}^2$ dimensions)
- In the visible sector the SUSY breaking terms should be of the order

$$m_{vis} \approx \frac{\langle F \rangle}{M_P}$$

- Since we know that $m_{vis} \sim 10^3 \text{ to } 10^5 \text{ GeV}$, given $M_P \sim 10^{19} \text{ GeV}$ then

$$\sqrt{\langle F \rangle} \sim 10^{11} \text{ to } 10^{12} \text{ GeV}$$

Effective non-renormalizable Lagrangian that couples F to the visible sector

$$\mathcal{L}_{grav-mediation} = - \frac{\kappa^i{}_j}{M_P^2}|F|^2\tilde{\phi}_i\tilde{\phi}^{*j} - \left(\frac{1}{2}\frac{\beta^{ij}}{M_P^2}|F|^2\tilde{\phi}_i\tilde{\phi}_j + \frac{1}{6}\frac{\eta^{ijk}}{M_P}F\tilde{\phi}_i\tilde{\phi}_j\tilde{\phi}_k + h.c.\right) - \left(\frac{f_a}{2M_P}F\lambda^a\lambda^a + h.c.\right)$$

- This is part of a fully supersymmetric Lagrangian that arises in supergravity

Effective non-renormalizable Lagrangian that couples F to the visible sector

$$\mathcal{L}_{grav-mediation} = - \frac{\kappa^i{}_j}{M_P^2}|F|^2\tilde{\phi}_i\tilde{\phi}^{*j} - \left(\frac{1}{2}\frac{\beta^{ij}}{M_P^2}|F|^2\tilde{\phi}_i\tilde{\phi}_j + \frac{1}{6}\frac{\eta^{ijk}}{M_P}F\tilde{\phi}_i\tilde{\phi}_j\tilde{\phi}_k + h.c.\right) - \left(\frac{f_a}{2M_P}F\lambda^a\lambda^a + h.c.\right)$$

- This is part of a fully supersymmetric Lagrangian that arises in supergravity
- When the F-fields acquire a VEV we get what we call **soft** SUSY-breaking terms
  - Gaugino masses: $M_a = \frac{f_a\langle F\rangle}{M_P}$
  - Scalar cubic couplings: $a^{ijk} \equiv \frac{\eta^{ijk}}{M_P}\langle F\rangle$
  - Scalar masses: $\left(m^2_{\tilde{\phi}}\right)^i{}_j \equiv \frac{\kappa^i{}_j}{M_P^2}|\langle F\rangle|^2$ and $\mathrm{b_{ij}} \equiv \frac{\cdot^{\mathrm{ij}}}{\mathrm{M_P^2}}|\langle \mathrm{F}\rangle|^2$

- Many breaking scenarios proposed
- Parametrize the unknown realistic scenario of SSB
  - Introduce terms that explicitly break supersymmetry $\longrightarrow$ **SOFT TERMS**
  - Should be of positive mass dimensions $\longrightarrow$ **renormalizable theory**
  - Should not generate new couplings upon renormalization

Generic soft SUSY Lagrangian

$$\mathcal{L}_{soft} = -\left(\frac{1}{2}M_a\lambda^a\lambda^a + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i\right) + h.c. - \left(m^2\right)^i{}_j\,\phi^{j*}\phi_i$$

Back to MSSM

LUND UNIVERSITY

- Many breaking scenarios proposed
- Parametrize the unknown realistic scenario of SSB
  - Introduce terms that explicitly break supersymmetry $\longrightarrow$ **SOFT TERMS**
  - Should be of positive mass dimensions $\longrightarrow$ **renormalizable theory**
  - Should not generate new couplings upon renormalization

Generic soft SUSY Lagrangian

$$\mathcal{L}_{soft} = -\left(\frac{1}{2}M_a\lambda^a\lambda^a + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i\right) + h.c. - \left(m^2\right)^i_{\ j}\phi^{j*}\phi_i$$

Back to MSSM

The total Lagrangian is given by the susy preserving and susy-breaking parts:

$$\mathcal{L}_{tot} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft}$$

Outline

LUND UNIVERSITY

Building a SUSY model

(1) Choose a gauge symmetry group

(2) Choose a superpotential invariant under the gauge symmetry
   - **All possible terms allowed by the symmetries should be included**

(3) Choose a soft-SUSY breaking Lagrangian (very popular) or else specify the breaking mechanism (non-trivial)

Building a SUSY model

(1) Choose a gauge symmetry group

(2) Choose a superpotential invariant under the gauge symmetry

- **All possible terms allowed by the symmetries should be included**

(3) Choose a soft-SUSY breaking Lagrangian (very popular) or else specify the breaking mechanism (non-trivial)

Let us follow this steps for the MSSM and explore the consequences

The Superpotential and Soft Lagrangian

- The MSSM has the same gauge structure as the SM for the strong and electroweak interactions

$$SU(3)_c \times SU(2)_L \times U(1)_y$$

The Superpotential and Soft Lagrangian

- The MSSM has the same gauge structure as the SM for the strong and electroweak interactions

$$SU(3)_c \times SU(2)_L \times U(1)_y$$

- **Superpotential**

$$W_{MSSM} = \varepsilon_{\alpha\beta}\left[(y_u)_{ij}\,\hat{\bar{u}}_{Rix}\hat{Q}^{\alpha x}_{Lj}\hat{H}^{\beta}_{u} - (y_d)_{ij}\,\hat{\bar{d}}_{Rix}\hat{Q}^{\alpha x}_{Lj}\hat{H}^{\beta}_{d} - (y_e)_{ij}\,\hat{\bar{e}}_{Ri}\hat{L}^{\alpha}_{Lj}\hat{H}^{\beta}_{d} + \mu\hat{H}^{\alpha}_{u}\hat{H}^{\beta}_{d}\right] \quad (42)$$

LUND UNIVERSITY

The Superpotential and Soft Lagrangian

- The MSSM has the same gauge structure as the SM for the strong and electroweak interactions

$$SU(3)_c \times SU(2)_L \times U(1)_y$$

- **Superpotential**

$$W_{MSSM} = \epsilon_{\alpha\beta}\left[(y_u)_{ij}\,\hat{\bar{u}}_{Rix}\hat{Q}^{\alpha x}_{Lj}\hat{H}^{\beta}_{u} - (y_d)_{ij}\,\hat{\bar{d}}_{Rix}\hat{Q}^{\alpha x}_{Lj}\hat{H}^{\beta}_{d} - (y_e)_{ij}\,\hat{\bar{e}}_{Ri}\hat{L}^{\alpha}_{Lj}\hat{H}^{\beta}_{d} + \mu\hat{H}^{\alpha}_{u}\hat{H}^{\beta}_{d}\right] \quad (42)$$

- Soft SUSY-breaking Lagrangian ▸ $\mathcal{L}_{soft}$

$$\begin{aligned} -\mathcal{L}_{soft} \;=\;& m^2_{H_d}\,|H_d|^2 + m^2_{H_u}\,|H_u|^2 + \tilde{Q}^{\alpha x}_{Li}\left(m^2_{\tilde{Q}_L}\right)^i_{\ j}\tilde{Q}^{*\,j}_{L\alpha x} + \tilde{L}^{\alpha}_{Li}\left(m^2_{\tilde{L}_L}\right)^i_{\ j}\tilde{L}^{*\,j}_{L\alpha} \\ &+ \tilde{u}^{*\,x}_{Ri}\left(m^2_{\tilde{u}_R}\right)^i_{\ j}\tilde{u}^{\,j}_{Rx} + \tilde{d}^{*\,x}_{Ri}\left(m^2_{\tilde{d}_R}\right)^i_{\ j}\tilde{d}^{\,j}_{Rx} + \tilde{e}^{*}_{Ri}\left(m^2_{\tilde{e}_R}\right)^i_{\ j}\tilde{e}^{\,j}_{R} \\ &+ \epsilon_{\alpha\beta}\left[a_{uij}H^{\alpha}_{u}\tilde{u}_{Rix}\tilde{Q}^{\beta x}_{Lj} - a_{dij}H^{\alpha}_{d}\tilde{d}_{Rix}\tilde{Q}^{\beta x}_{Lj} - a_{eij}H^{\alpha}_{d}\tilde{e}_{Ri}\tilde{L}^{\beta}_{Lj} + bH^{\alpha}_{d}H^{\beta}_{u} + h.c.\right] \\ &+ \frac{1}{2}\left[M_1\tilde{B}\cdot\tilde{B} + M_2\tilde{W}^a\cdot\tilde{W}^a + M_3\tilde{g}^a\cdot\tilde{g}^a + h.c.\right], \end{aligned} \quad (43)$$

R-parity

- **R-parity** for a particle with spin S is defined by

$$P_R = (-1)^{3(B-L)+2S}$$

  - **Particles within the same supermultiplet do not carry the same R-parity**
  - All Standard Model particles and Higgs bosons carry $P_R = +1$
  - All squarks, sleptons, higgsinos and gauginos carry $P_R = -1$

Consequences of R-parity

- Each interaction vertex must contain an even number of odd ($P_R = -1$) particles (sparticles)

Consequences of R-parity

- Each interaction vertex must contain an even number of odd $(P_R = -1)$ particles (sparticles)

1. SUSY particles are pair produced
2. Each sparticle must decay into a state that contains an **odd** number of sparticles

Consequences of R-parity

- Each interaction vertex must contain an even number of odd ($P_R = -1$) particles (sparticles)

1. SUSY particles are pair produced
2. Each sparticle must decay into a state that contains an **odd** number of sparticles
3. The Lightest Supersymmetric Particle (LSP) is **stable** as R-parity forbids decay to standard particles
4. If stable and neutral, the LSP interacts weakly with ordinary matter and can make a good candidate for **cold dark matter** (CDM)
5. **This parity eliminates lepton and barion number violating interactions** $\longrightarrow$ forbids proton decay

Consequences of R-parity

- Each interaction vertex must contain an even number of odd ($P_R = -1$) particles (sparticles)

1. SUSY particles are pair produced
2. Each sparticle must decay into a state that contains an **odd** number of sparticles
3. The Lightest Supersymmetric Particle (LSP) is **stable** as R-parity forbids decay to standard particles
4. If stable and neutral, the LSP interacts weakly with ordinary matter and can make a good candidate for **cold dark matter** (CDM)
5. **This parity eliminates lepton and barion number violating interactions** $\longrightarrow$ forbids proton decay

NOTE: R-parity is just an *ad hoc* assumption without a solid theoretical justification for its origin

Outline

Chiral Supermultiplets

Chiral Supermultiplet Fields in the MSSM				
Names		Spin 0	Spin 1/2	$SU(3)_c \times SU(2)_L \times U(1)_y$
Squarks, Quarks (× 3)	$\hat{Q}_L$	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	**3**, **2**, 1/3
	$\hat{\bar{u}}_R$	$\tilde{\bar{u}}_L = \tilde{u}_R^*$	$\bar{u}_L = (u_R)^c$	$\bar{\mathbf{3}}$, **1**, -4/3
	$\hat{\bar{d}}_R$	$\tilde{\bar{d}}_L = \tilde{d}_R^*$	$\bar{d}_L = (d_R)^c$	$\bar{\mathbf{3}}$, **1**, 2/3
Sleptons, Leptons (× 3)	$\hat{L}_L$	$(\tilde{\nu}_{eL}, \tilde{e}_L)$	(ν_{eL}, e_L)	**1**, **2**, -1
	$\hat{\bar{e}}_R$	$\tilde{\bar{e}}_L = \tilde{e}_R^*$	$\bar{e}_L = (e_R)^c$	**1**, **1**, 2
Higgs, Higgsinos	$\hat{H}_u$	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$	**1**, **2**, 1
	$\hat{H}_d$	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	**1**, **2**, -1

Table : *Chiral supermultiplet fields in the MSSM. The leftmost column provides the usual designation for the fundamental particles, the two middle ones the spin and the rightmost the gauge charges.*

Gauge Supermultiplets

Gauge Supermultiplet Fields in the MSSM				
Names		Spin 1/2	Spin 1	$SU(3)_c \times SU(2)_L \times U(1)_y$
Gluinos, Gluons	$\hat{G}^a$	$\tilde{g}$	g	**8**, **1**, 0
Winos, W bosons	$\hat{W}^a$	$\tilde{W}^\pm,\ \tilde{W}^0$	$W^\pm,\ W^0$	**1**, **3**, 0
Bino, B Boson	$\hat{B}$	$\tilde{B}$	B	**1**, **1**, 0

Table : *Gauge supermultiplet fields in the MSSM. The left column provides the usual designation for the gauge fields, the middle one the spin and the right one the gauge charges.*

Why two Higgs doublets?

As noted in table 1 there are **two Higgs doublets** in the model. There are two strong reasons for that:

(A) Anomaly cancellation (physical)

(B) Analyticity of the superpotential (mathematical)

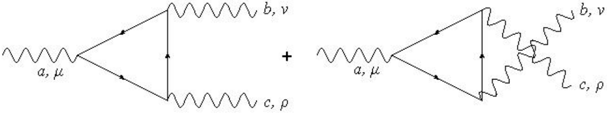
Anomaly cancellation

Anomalies are **quantum mechanical** effects that break the classical theory at loop level.

Anomaly cancellation

Anomalies are **quantum mechanical** effects that break the classical theory at loop level.

In quantum field theories anomalies that violate the gauge symmetry are called gauge anomalies and can be generated from triangle diagrams.

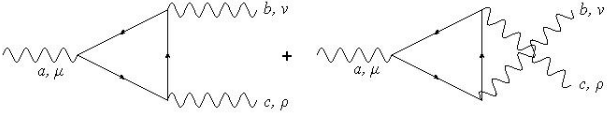


a, b and c are the adjoint representation indices of the A^a_μ, A^b_ν and A^c_ρ gauge bosons.

Anomaly cancellation

Anomalies are **quantum mechanical** effects that break the classical theory at loop level.

In quantum field theories anomalies that violate the gauge symmetry are called gauge anomalies and can be generated from triangle diagrams.



a, b and c are the adjoint representation indices of the A^a_μ, A^b_ν and A^c_ρ gauge bosons.

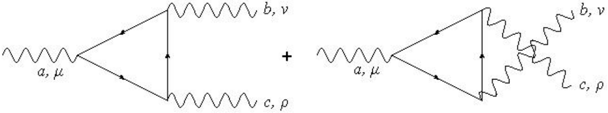
- The result of the loop-integration yields

$$\mathcal{A}^{abc} \propto Tr\left(T^a\left\{T^b, T^c\right\}\right). \tag{44}$$

Anomaly cancellation

Anomalies are **quantum mechanical** effects that break the classical theory at loop level.

In quantum field theories anomalies that violate the gauge symmetry are called gauge anomalies and can be generated from triangle diagrams.



a, b and c are the adjoint representation indices of the A^a_μ, A^b_ν and A^c_ρ gauge bosons.

- The result of the loop-integration yields

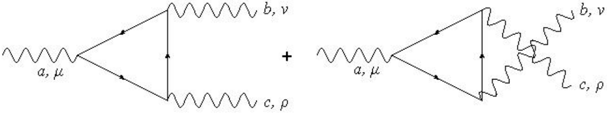
$$\mathcal{A}^{abc} \propto Tr\left(T^a\left\{T^b, T^c\right\}\right). \tag{44}$$

- In the SM there are no gauge anomalies due to complete generations of chiral fermions

Anomaly cancellation

Anomalies are **quantum mechanical** effects that break the classical theory at loop level.

In quantum field theories anomalies that violate the gauge symmetry are called gauge anomalies and can be generated from triangle diagrams.



a, b and c are the adjoint representation indices of the A^a_μ, A^b_ν and A^c_ρ gauge bosons.

- The result of the loop-integration yields

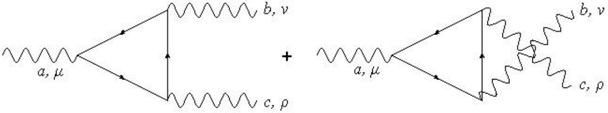
$$\mathcal{A}^{abc} \propto Tr\left(T^a\left\{T^b, T^c\right\}\right). \quad (44)$$

- In the SM there are no gauge anomalies due to complete generations of chiral fermions
- In the MSSM, if there was only one Higgs doublet, its fermion partner Higgsino doublet would provides non-zero contribution

Anomaly cancellation

Anomalies are **quantum mechanical** effects that break the classical theory at loop level.

In quantum field theories anomalies that violate the gauge symmetry are called gauge anomalies and can be generated from triangle diagrams.



a, b and c are the adjoint representation indices of the A^a_μ, A^b_ν and A^c_ρ gauge bosons.

- The result of the loop-integration yields

$$\mathcal{A}^{abc} \propto Tr\left(T^a\left\{T^b, T^c\right\}\right). \qquad (44)$$

- In the SM there are no gauge anomalies due to complete generations of chiral fermions
- In the MSSM, if there was only one Higgs doublet, its fermion partner Higgsino doublet would provides non-zero contribution

We need two Higgs/Higgsino doublets to guarantee that the model is anomaly-free

Analyticity of the Superpotential

Up and down-type quark masses in the Standard Model:

$$\mathcal{L}_d = -(y_d)^{ij}\,\overline{Q}_{Li} H d_{Rj} + h.c.\,, \quad \mathcal{L}_u = -(y_u)^{ij}\,\overline{Q}_{Li} \tilde{H} u_{Rj} + h.c. \qquad (45)$$

with $\tilde{H} = i\tau^2 H^*$

- One single doublet H does the job!

Analyticity of the Superpotential

Up and down-type quark masses in the Standard Model:

$$\mathcal{L}_d = -\left(y_d\right)^{ij} \overline{Q}_{Li} H d_{Rj} + h.c. \,, \quad \mathcal{L}_u = -\left(y_u\right)^{ij} \overline{Q}_{Li} \tilde{H} u_{Rj} + h.c. \tag{45}$$

with $\tilde{H} = i\tau^2 H^*$

- One single doublet H does the job!

Why don't we simply add a term like $\hat{\bar{u}}_R \hat{Q}_L \hat{H}_d^\dagger$ to the superpotential?

Analyticity of the Superpotential

Up and down-type quark masses in the Standard Model:

$$\mathcal{L}_d = -\left(y_d\right)^{ij} \overline{Q}_{Li} H d_{Rj} + h.c.\ , \quad \mathcal{L}_u = -\left(y_u\right)^{ij} \overline{Q}_{Li} \tilde{H} u_{Rj} + h.c. \tag{45}$$

with $\tilde{H} = i\tau^2 H^*$

- One single doublet H does the job!

Why don't we simply add a term like $\hat{\bar{u}}_R \hat{Q}_L \hat{H}_d^\dagger$ to the superpotential?

The superpotential is an analytic function of the fields ϕ_i. Therefore it can only depend on ϕ_i, $W(\phi_i)$, and not on ϕ_i^*, ~~$W(\phi_i, \phi_i^*)$~~

Analyticity of the Superpotential

Up and down-type quark masses in the Standard Model:

$$\mathcal{L}_d = -\left(y_d\right)^{ij} \overline{Q}_{Li} H d_{Rj} + h.c. \,, \quad \mathcal{L}_u = -\left(y_u\right)^{ij} \overline{Q}_{Li} \tilde{H} u_{Rj} + h.c. \tag{45}$$

with $\tilde{H} = i\tau^2 H^*$

- One single doublet H does the job!

Why don't we simply add a term like $\hat{\bar{u}}_R \hat{Q}_L \hat{H}_d^\dagger$ to the superpotential?

The superpotential is an analytic function of the fields ϕ_i. Therefore it can only depend on ϕ_i, $W(\phi_i)$, and not on ϕ_i^*, ~~$W(\phi_i, \phi_i^*)$~~

Need another doublet to couple to up-type quarks

Outline

Electroweak symmetry breaking

The scalar potential of the MSSM is derived from F-terms, D-terms and soft SUSY-breaking terms:

$$\begin{aligned} V_H &= \left(|\mu|^2 + m_{H_u}^2\right)\left(\left|H_u^0\right|^2 + |H_u^+|^2\right) + \left(|\mu|^2 + m_{H_d}^2\right)\left(\left|H_d^0\right|^2 + |H_d^-|^2\right) \\ &+ b\left[(H_u^+ H_d^- - H_u^0 H_d^0) + h.c.\right] + \frac{1}{2} g^2 \left|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}\right|^2 \\ &+ \frac{1}{8}(g^2 + g'^2)\left(\left|H_u^0\right|^2 + |H_u^+|^2 - \left|H_d^0\right|^2 - |H_d^-|^2\right)^2 , \end{aligned} \tag{46}$$

Electroweak symmetry breaking

The scalar potential of the MSSM is derived from F-terms, D-terms and soft SUSY-breaking terms:

$$
\begin{aligned}
V_H = & \left(|\mu|^2 + m_{H_u}^2\right)\left(\left|H_u^0\right|^2 + |H_u^+|^2\right) + \left(|\mu|^2 + m_{H_d}^2\right)\left(\left|H_d^0\right|^2 + |H_d^-|^2\right) \\
& + b\left[\left(H_u^+ H_d^- - H_u^0 H_d^0\right) + h.c.\right] + \frac{1}{2} g^2 \left|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}\right|^2 \\
& + \frac{1}{8}\left(g^2 + g'^2\right)\left(\left|H_u^0\right|^2 + |H_u^+|^2 - \left|H_d^0\right|^2 - |H_d^-|^2\right)^2 ,
\end{aligned} \tag{46}
$$

At the minimum of the potential, the Higgs VEVs can be parametrized by

$$
\langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix} \quad \text{and} \quad \langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix} \tag{47}
$$

and the minimization conditions yield

$$
\left(|\mu|^2 + m_{H_u}^2\right) v_u = b v_d + \frac{1}{4}\left(g^2 + g'^2\right)\left(v_d^2 - v_u^2\right) v_u \tag{48}
$$

$$
\left(|\mu|^2 + m_{H_d}^2\right) v_d = b v_u - \frac{1}{4}\left(g^2 + g'^2\right)\left(v_d^2 - v_u^2\right) v_d . \tag{49}
$$

g is the $SU(2)_L$ gauge coupling and g' that of the $U(1)_y$ gauge symmetry.

The gauge boson masses are obtained as in the SM and the result is

$$M_Z = \sqrt{\frac{1}{2}\left(g^2 + g'^2\right)\left(v_u^2 + v_d^2\right)}\ , \text{ and } M_W = \sqrt{\frac{1}{2}g^2\left(v_u^2 + v_d^2\right)} \tag{50}$$

with $v^2 = v_u^2 + v_d^2 = (246\ \text{GeV})^2$.

The gauge boson masses are obtained as in the SM and the result is

$$M_Z = \sqrt{\frac{1}{2}\left(g^2 + g'^2\right)\left(v_u^2 + v_d^2\right)}\ , \text{ and } M_W = \sqrt{\frac{1}{2}g^2\left(v_u^2 + v_d^2\right)} \tag{50}$$

with $v^2 = v_u^2 + v_d^2 = (246 \text{ GeV})^2$.

We define $\tan\beta = \frac{v_u}{v_d}$ and rewrite the minimization conditions as

$$\left(|\mu|^2 + m_{H_u}^2\right) = b\cot\beta + \frac{M_Z^2}{4}\cos 2\beta \tag{51}$$

$$\left(|\mu|^2 + m_{H_d}^2\right) = b\tan\beta - \frac{M_Z^2}{4}\cos 2\beta \tag{52}$$

The gauge boson masses are obtained as in the SM and the result is

$$M_Z = \sqrt{\frac{1}{2}\left(g^2 + g'^2\right)\left(v_u^2 + v_d^2\right)}\ , \text{ and } \ M_W = \sqrt{\frac{1}{2} g^2 \left(v_u^2 + v_d^2\right)} \tag{50}$$

with $v^2 = v_u^2 + v_d^2 = (246\ \text{GeV})^2$.

We define $\tan\beta = \dfrac{v_u}{v_d}$ and rewrite the minimization conditions as

$$\left(|\mu|^2 + m_{H_u}^2\right) = b\cot\beta + \frac{M_Z^2}{4}\cos 2\beta \tag{51}$$

$$\left(|\mu|^2 + m_{H_d}^2\right) = b\tan\beta - \frac{M_Z^2}{4}\cos 2\beta \tag{52}$$

Eliminating b and expanding in powers of $1/\tan\beta$, the M_Z^2 mass in the MSSM becomes:

$$M_Z^2 = -2\left(m_{H_u}^2 + |\mu|^2\right) + \frac{2}{\tan^2\beta}\left(m_{H_d}^2 - m_{H_u}^2\right) + \mathcal{O}\left(1/\tan^4\beta\right). \tag{53}$$

typically $m_{H_u}^2 < 0$ for radiative EWSB

The gauge boson masses are obtained as in the SM and the result is

$$M_Z = \sqrt{\frac{1}{2}\left(g^2 + g'^2\right)\left(v_u^2 + v_d^2\right)}\ , \text{ and } \ M_W = \sqrt{\frac{1}{2} g^2 \left(v_u^2 + v_d^2\right)} \tag{50}$$

with $v^2 = v_u^2 + v_d^2 = (246 \text{ GeV})^2$.

We define $\tan\beta = \frac{v_u}{v_d}$ and rewrite the minimization conditions as

$$\left(|\mu|^2 + m_{H_u}^2\right) = b\cot\beta + \frac{M_Z^2}{4}\cos 2\beta \tag{51}$$

$$\left(|\mu|^2 + m_{H_d}^2\right) = b\tan\beta - \frac{M_Z^2}{4}\cos 2\beta \tag{52}$$

Eliminating b and expanding in powers of $1/\tan\beta$, the M_Z^2 mass in the MSSM becomes:

$$M_Z^2 = -2\left(m_{H_u}^2 + |\mu|^2\right) + \frac{2}{\tan^2\beta}\left(m_{H_d}^2 - m_{H_u}^2\right) + \mathcal{O}\left(1/\tan^4\beta\right). \tag{53}$$

typically $m_{H_u}^2 < 0$ for radiative EWSB

The lack of observation of SUSY **may** be pushing m_{H_u} to large values making the tuning between $m_{H_u}^2$ and μ^2 of the order of several percent (less natural) $\rightarrow$ MSSM under pressure?

In the MSSM there are 5 physical Higgs bosons, $h^0, H^0, A^0, H^{\pm}$, and 3 Goldstone bosons $G^0, G^{\pm}$ which are absorbed by the gauge bosons to give them mass.

In the MSSM there are 5 physical Higgs bosons, $h^0, H^0, A^0, H^\pm$, and 3 Goldstone bosons $G^0, G^\pm$ which are absorbed by the gauge bosons to give them mass.

This is a consequence of Goldstone's Theorem, which essentially implies that for a model with n_{real} real scalar degrees of freedom, if there is a SSB with m broken generators (the same number of Goldstone bosons), the number of **massive** physical degrees of freedom is $N_{phy} = n_{real} - m_{Goldstones}$.

$$m^2_{A^0} = \frac{2b}{\sin 2\beta}, \qquad m^2_{H^\pm} = M^2_W + m^2_{A^0}, \quad \text{with} \quad \tan\beta = \frac{v_u}{v_d}$$

$$m^2_{h^0,\ H^0} = \frac{1}{2}\left\{M^2_Z + m^2_{A^0} \mp \left[\left(M^2_Z + m^2_{A^0}\right)^2 - 4m^2_{A^0}M^2_Z\cos^2 2\beta\right]^{\frac{1}{2}}\right\}$$

Radiative Corrections

The tree-level Higgs mass a maximum value

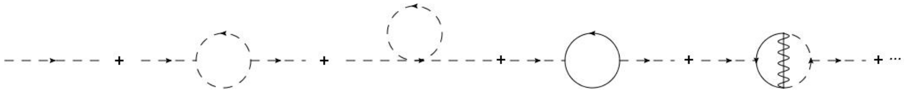
$$m_{h^0}^2 = M_Z^2 \cos^2(2\beta) < M_Z^2 \ll 126 \text{ GeV}$$

Radiative Corrections

The tree-level Higgs mass a maximum value

$$m_{h^0}^2 = M_Z^2 \cos^2(2\beta) < M_Z^2 \ll 126 \text{ GeV}$$

Radiative corrections are quite significant for the Higgs mass, where top quarks and stops provide the largest contribution (about 35 GeV)



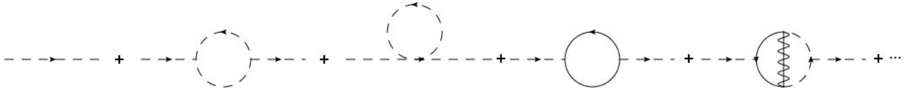
$$\Delta m_{h^0}^2 = \frac{3}{4\pi^2} \sin^2\beta \, y_t^2 m_t^2 \log \frac{m_{\tilde{t}}^2}{m_t^2} + \cdots$$

Radiative Corrections

The tree-level Higgs mass a maximum value

$$m_{h^0}^2 = M_Z^2 \cos^2(2\beta) < M_Z^2 \ll 126 \text{ GeV}$$

Radiative corrections are quite significant for the Higgs mass, where top quarks and stops provide the largest contribution (about 35 GeV)



$$\Delta m_{h^0}^2 = \frac{3}{4\pi^2} \sin^2\beta \, y_t^2 m_t^2 \log \frac{m_{\tilde{t}}^2}{m_t^2} + \cdots$$

Radiative corrections are of extreme importance for the calculation of the Higgs mass

Squark and Slepton masses

- Obtaining physical masses implies diagonalization of the mass matrices coming from the original Lagrangian

Squark and Slepton masses

- Obtaining physical masses implies diagonalization of the mass matrices coming from the original Lagrangian
- To treat the sfermions in complete generality we would have to consider arbitrary mixing and diagonalize

1. a 6×6 mass matrix for the up-type squarks $(\tilde{t}_L,\ \tilde{t}_R,\ \tilde{c}_L,\ \tilde{c}_R,\ \tilde{u}_L,\ \tilde{u}_R)$
2. a 6×6 mass matrix for the down-type squarks $(\tilde{b}_L,\ \tilde{b}_R,\ \tilde{s}_L,\ \tilde{s}_R,\ \tilde{d}_L,\ \tilde{d}_R)$
3. a 6×6 mass matrix for the charged sleptons $(\tilde{\tau}_L,\ \tilde{\tau}_R,\ \tilde{\mu}_L,\ \tilde{\mu}_R,\ \tilde{e}_L,\ \tilde{e}_R)$
4. a 3×3 mass matrix for sneutrinos $(\tilde{\nu}_\tau,\ \tilde{\nu}_\mu,\ \tilde{\nu}_e)$

LUND UNIVERSITY

Squark and Slepton masses

- Obtaining physical masses implies diagonalization of the mass matrices coming from the original Lagrangian
- To treat the sfermions in complete generality we would have to consider arbitrary mixing and diagonalize

1. a 6×6 mass matrix for the up-type squarks $(\tilde{t}_L,\ \tilde{t}_R,\ \tilde{c}_L,\ \tilde{c}_R,\ \tilde{u}_L,\ \tilde{u}_R)$
2. a 6×6 mass matrix for the down-type squarks $\left(\tilde{b}_L,\ \tilde{b}_R,\ \tilde{s}_L,\ \tilde{s}_R,\ \tilde{d}_L,\ \tilde{d}_R\right)$
3. a 6×6 mass matrix for the charged sleptons $(\tilde{\tau}_L,\ \tilde{\tau}_R,\ \tilde{\mu}_L,\ \tilde{\mu}_R,\ \tilde{e}_L,\ \tilde{e}_R)$
4. a 3×3 mass matrix for sneutrinos $(\tilde{\nu}_\tau,\ \tilde{\nu}_\mu,\ \tilde{\nu}_e)$

Fortunately most of mixing angles are small in viable models and the Yukawa couplings for the first and second generations are negligible. We end up with 7 unmixed pairs

$$(\tilde{c}_L,\ \tilde{c}_R)\,,\ (\tilde{u}_L,\ \tilde{u}_R)\,,\ (\tilde{s}_L,\ \tilde{s}_R)\,,\ \left(\tilde{d}_L,\ \tilde{d}_R\right)\,,\ (\tilde{\mu}_L,\ \tilde{\mu}_R)\,,\ (\tilde{e}_L,\ \tilde{e}_R)\,,\ (\tilde{\nu}_\mu,\ \tilde{\nu}_e)$$

and 3 **mixing** pairs (due to sizable Yukawa coupling)

$$(\tilde{t}_L,\ \tilde{t}_R)\,,\ \left(\tilde{b}_L,\ \tilde{b}_R\right)\,,\ (\tilde{\tau}_L,\ \tilde{\tau}_R)$$

LUND UNIVERSITY

For the stops we have

$$\mathcal{L}_{stop-mass} = -\begin{pmatrix} t_L^* & t_R^* \end{pmatrix} \begin{pmatrix} m_{\tilde{Q}_3}^2 + m_t^2 + \Delta_{\tilde{u}_L} & \frac{v}{\sqrt{2}} \sin\beta \,(a_t - y_t \mu \cot\beta) \\ \frac{v}{\sqrt{2}} \sin\beta \,(a_t - y_t \mu \cot\beta) & m_{\tilde{t}_R}^2 + m_t^2 + \Delta_{\tilde{u}_R} \end{pmatrix} \begin{pmatrix} t_L \\ t_R \end{pmatrix}.$$

where $\Delta_{\tilde{u}_{L,R}}$ is an $SU(2)_L \times U(1)_y$ D-term ($\Delta_{\Phi_{L,R}} = M_Z^2 (T_{3\Phi_{L,R}} - Q_{\Phi_{L,R}} \sin^2\theta_W) \cos 2\beta$)

For the stops we have

$$\mathcal{L}_{stop-mass} = -\begin{pmatrix} t_L^* & t_R^* \end{pmatrix} \begin{pmatrix} m_{\tilde{Q}_3}^2 + m_t^2 + \Delta_{\tilde{u}_L} & \frac{v}{\sqrt{2}} \sin\beta \, (a_t - y_t \mu \cot\beta) \\ \frac{v}{\sqrt{2}} \sin\beta \, (a_t - y_t \mu \cot\beta) & m_{\tilde{t}_R}^2 + m_t^2 + \Delta_{\tilde{u}_R} \end{pmatrix} \begin{pmatrix} t_L \\ t_R \end{pmatrix}.$$

where $\Delta_{\tilde{u}_{L,R}}$ is an $SU(2)_L \times U(1)_y$ D-term ($\Delta_{\Phi_{L,R}} = M_Z^2 (T_{3\Phi_{L,R}} - Q_{\Phi_{L,R}} \sin^2\theta_W) \cos 2\beta$)

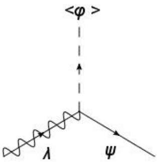
After diagonalization

$$\begin{aligned} m_{\tilde{t}_1,\,\tilde{t}_2}^2 &= \frac{1}{2}\Bigg[\left(m_{\tilde{Q}_3}^2 + m_{\tilde{t}_R}^2 + 2m_t^2 + \Delta_{u_L} + \Delta_{u_R}\right) \\ &\mp \sqrt{\left(m_{\tilde{Q}_3}^2 - m_{\tilde{t}_R}^2 + \Delta_{u_L} - \Delta_{u_R}\right)^2 + 2v^2 \sin^2\beta \, (a_t - y_t \mu \cot\beta)^2}\Bigg], \quad (54) \end{aligned}$$

where $\{\tilde{t}_1, \tilde{t}_2\}$ are the so called **mass eigenstates basis.**

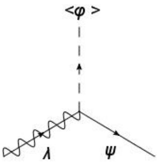
Gluinos, Charginos and Neutralinos

Gauginos mix with fermions due to the vertex



Gluinos, Charginos and Neutralinos

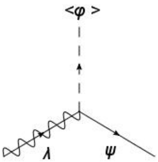
Gauginos mix with fermions due to the vertex



But gluinos are $SU(3)_C$ fermion octets carrying color charge $\rightarrow$

Gluinos, Charginos and Neutralinos

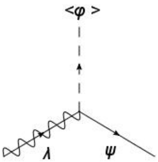
Gauginos mix with fermions due to the vertex



But gluinos are $SU(3)_C$ fermion octets carrying color charge $\rightarrow$ **mixing only possible if there was a colored Higgs which would also break QCD!!**

Gluinos, Charginos and Neutralinos

Gauginos mix with fermions due to the vertex



But gluinos are $SU(3)_C$ fermion octets carrying color charge $\rightarrow$ **mixing only possible if there was a colored Higgs which would also break QCD!!**

$$m_{\tilde{g}} = M_3 + \text{radiative corrections}$$

- After the breaking of the electroweak symmetry, $SU(2)_L \times U(1)_y \to U(1)_{Q_{em}}$, there is no way to prevent binos, winos and Higgsinos to mix
- However, the remnant symmetry, electromagnetism, forbids mixing between charged and neutral components

- After the breaking of the electroweak symmetry, $SU(2)_L \times U(1)_y \to U(1)_{Q_{em}}$, there is no way to prevent binos, winos and Higgsinos to mix
- However, the remnant symmetry, electromagnetism, forbids mixing between charged and neutral components

(i) Neutral components $\to$ Neutralinos

$$\mathcal{L}_{neutral} = -\frac{1}{2} \begin{pmatrix} \tilde{B} & \tilde{W}^0 & \tilde{H}_d^0 & \tilde{H}_u^0 \end{pmatrix} \underbrace{\begin{pmatrix} M_1 & 0 & -g' v_d/2 & g' v_u/2 \\ 0 & M_2 & g v_d/2 & -g v_u/2 \\ -g' v_d/2 & g v_d/2 & 0 & -\mu \\ g' v_u/2 & -g v_u/2 & -\mu & 0 \end{pmatrix}}_{M_\chi} \begin{pmatrix} \tilde{B} \\ \tilde{W}^0 \\ \tilde{H}_d^0 \\ \tilde{H}_u^0 \end{pmatrix} + h.c.\,,$$

- After the breaking of the electroweak symmetry, $SU(2)_L \times U(1)_y \to U(1)_{Q_{em}}$, there is no way to prevent binos, winos and Higgsinos to mix
- However, the remnant symmetry, electromagnetism, forbids mixing between charged and neutral components

(i) Neutral components $\to$ Neutralinos

$$\mathcal{L}_{neutral} = -\frac{1}{2}\left(\tilde{B} \quad \tilde{W}^0 \quad \tilde{H}_d^0 \quad \tilde{H}_u^0\right) \underbrace{\begin{pmatrix} M_1 & 0 & -g' v_d/2 & g' v_u/2 \\ 0 & M_2 & g v_d/2 & -g v_u/2 \\ -g' v_d/2 & g v_d/2 & 0 & -\mu \\ g' v_u/2 & -g v_u/2 & -\mu & 0 \end{pmatrix}}_{M_\chi} \begin{pmatrix} \tilde{B} \\ \tilde{W}^0 \\ \tilde{H}_d^0 \\ \tilde{H}_u^0 \end{pmatrix} + h.c.\,,$$

Mass eigenstates, the neutralinos, obtained after diagonalization

$$diag\left(m_{\tilde{\chi}_1^0}\,,\ m_{\tilde{\chi}_2^0}\,,\ m_{\tilde{\chi}_3^0}\,,\ m_{\tilde{\chi}_4^0}\right)\,,$$

where the lightest neutralino, $\tilde{\chi}_1^0$, is a good dark matter candidate with important cosmological implications.

(ii) Charged components → Charginos

$$\mathcal{L}_{charged} = -\frac{1}{2}\left[\begin{pmatrix}\tilde{W}^{+} & \tilde{H}_u^{+}\end{pmatrix}\mathbf{C}^{\mathbf{T}}\begin{pmatrix}\tilde{W}^{-} \\ \tilde{H}_d^{-}\end{pmatrix} + \begin{pmatrix}\tilde{W}^{-} & \tilde{H}_d^{-}\end{pmatrix}\mathbf{C}\begin{pmatrix}\tilde{W}^{+} \\ \tilde{H}_u^{+}\end{pmatrix}\right] + h.c. \quad (55)$$

with the mass matrix of the charged components given by

$$\mathbf{C} = \begin{pmatrix} M_2 & gv_u/2 \\ gv_d/2 & \mu \end{pmatrix}. \quad (56)$$

where the eigenstates are the charginos $\chi_1^{\pm}$ and $\chi_2^{\pm}$.

(ii) Charged components → Charginos

$$\mathcal{L}_{charged} = -\frac{1}{2}\left[\begin{pmatrix}\tilde{W}^{+} & \tilde{H}_u^{+}\end{pmatrix}\mathbf{C}^{\mathbf{T}}\begin{pmatrix}\tilde{W}^{-}\\ \tilde{H}_d^{-}\end{pmatrix} + \begin{pmatrix}\tilde{W}^{-} & \tilde{H}_d^{-}\end{pmatrix}\mathbf{C}\begin{pmatrix}\tilde{W}^{+}\\ \tilde{H}_u^{+}\end{pmatrix}\right] + h.c. \tag{55}$$

with the mass matrix of the charged components given by

$$\mathbf{C} = \begin{pmatrix} M_2 & gv_u/2 \\ gv_d/2 & \mu \end{pmatrix}. \tag{56}$$

where the eigenstates are the charginos $\chi_1^{\pm}$ and $\chi_2^{\pm}$.

In most popular scenarios, $M_1 < M_2 \ll \mu$. Here the lightest neutralino (LSP) is *bino-like*, the next-to-lightest is *wino-like* and the heavy ones are *Higgsino-like*.

(ii) Charged components $\to$ Charginos

$$\mathcal{L}_{charged} = -\frac{1}{2}\left[\begin{pmatrix}\tilde{W}^+ & \tilde{H}_u^+\end{pmatrix}\mathbf{C}^{\mathbf{T}}\begin{pmatrix}\tilde{W}^- \\ \tilde{H}_d^-\end{pmatrix} + \begin{pmatrix}\tilde{W}^- & \tilde{H}_d^-\end{pmatrix}\mathbf{C}\begin{pmatrix}\tilde{W}^+ \\ \tilde{H}_u^+\end{pmatrix}\right] + h.c. \quad (55)$$

with the mass matrix of the charged components given by

$$\mathbf{C} = \begin{pmatrix} M_2 & g v_u/2 \\ g v_d/2 & \mu \end{pmatrix}. \quad (56)$$

where the eigenstates are the charginos $\chi_1^\pm$ and $\chi_2^\pm$.

In most popular scenarios, $M_1 < M_2 \ll \mu$. Here the lightest neutralino (LSP) is *bino-like*, the next-to-lightest is *wino-like* and the heavy ones are *Higgsino-like*.

However there are UV completions of the MSSM where $M_2 < M_1$ yielding a *wino-like* LSP or $\mu \ll M_{1,2}$ where the dark matter candidate would be a Higgsino.

LUND UNIVERSITY

Summary of the Particle Spectrum

taken from S. Martin Primer, hep-ph/9709356

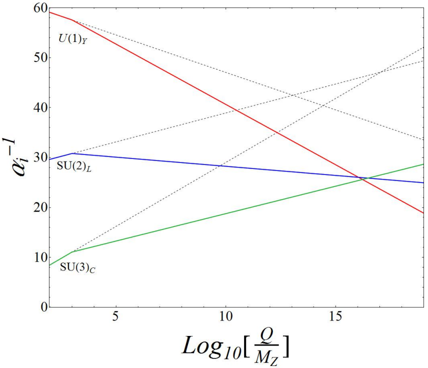
Names	Spin	P_R	Mass Eigenstates	Gauge Eigenstates
Higgs bosons	0	$+1$	$h^0\ H^0\ A^0\ H^\pm$	$H_u^0\ H_d^0\ H_u^+\ H_d^-$
squarks	0	-1	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{d}_L\ \widetilde{d}_R$	" "
			$\widetilde{s}_L\ \widetilde{s}_R\ \widetilde{c}_L\ \widetilde{c}_R$	" "
			$\widetilde{t}_1\ \widetilde{t}_2\ \widetilde{b}_1\ \widetilde{b}_2$	$\widetilde{t}_L\ \widetilde{t}_R\ \widetilde{b}_L\ \widetilde{b}_R$
sleptons	0	-1	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\nu}_e$	" "
			$\widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\nu}_\mu$	" "
			$\widetilde{\tau}_1\ \widetilde{\tau}_2\ \widetilde{\nu}_\tau$	$\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\nu}_\tau$
neutralinos	$1/2$	-1	$\widetilde{N}_1\ \widetilde{N}_2\ \widetilde{N}_3\ \widetilde{N}_4$	$\widetilde{B}^0\ \widetilde{W}^0\ \widetilde{H}_u^0\ \widetilde{H}_d^0$
charginos	$1/2$	-1	$\widetilde{C}_1^\pm\ \widetilde{C}_2^\pm$	$\widetilde{W}^\pm\ \widetilde{H}_u^+\ \widetilde{H}_d^-$
gluino	$1/2$	-1	$\widetilde{g}$	" "

The Idea of Grand Unification

- As a renormalizable quantum theory, **all couplings** of the model are scale dependent once we introduce quantum corrections (loops)

The Idea of Grand Unification

- As a renormalizable quantum theory, **all couplings** of the model are scale dependent once we introduce quantum corrections (loops)
  - Remarkable "*coincidence ?* " of the gauge couplings in the MSSM

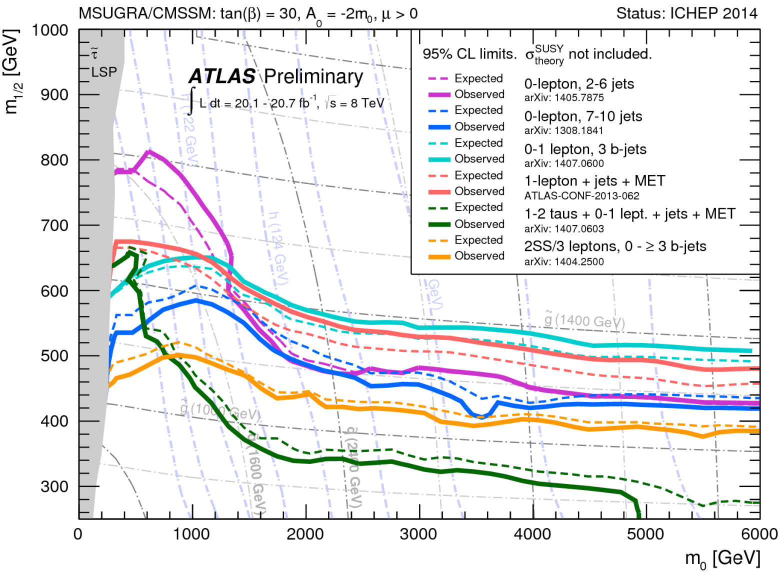


- Is this evidence of a Grand Unified Theory (GUT)?

Embed gauge symmetry into a larger simple group, eg., $SU(5)$, $SO(10)$, E_6, $\cdots$ or semi-simple as $SU(3)_C \times SU(3)_L \times SU(3)_R \ltimes \mathbb{Z}_3$

Outline

Latest ATLAS SUSY Searches



Concluding Remarks

However

(a) There is still **a lot** of parameter space to search

(b) 126 GeV Higgs requires rather heavy stops, therefore it is not surprising if the squarks are of the order of few TeV

(c) The CMSSM is the most restrictive model $(M_{1/2},\ m_0, a_0, \tan\beta, sign(\mu))$

(d) If we go beyond CMSSM, eg. GUTs, we find a huge search landscape

(e) Not yet much attention to the electroweak sector

Concluding Remarks

However

(a) There is still **a lot** of parameter space to search

(b) 126 GeV Higgs requires rather heavy stops, therefore it is not surprising if the squarks are of the order of few TeV

(c) The CMSSM is the most restrictive model $(M_{1/2},\ m_0, a_0, \tan\beta, sign(\mu))$

(d) If we go beyond CMSSM, eg. GUTs, we find a huge search landscape

(e) Not yet much attention to the electroweak sector

(f) We are still in the very beginning... stay tuned!!