Naturalness, Renormalization Group and the Scale of New Physics

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Introduction

Two opposite points of view:

G: Higgs is unnatural, and "the multiverse offers the most plausible answer at our disposal." [1]

BRS: Standard Model is self-consistent up to the Plank scale. [2]

WHO IS RIGHT, G OR BRS?

Method

Resummation of leading logarithms in quantum corrections to the running scalar mass squared.

Naturalness criterion: A theory is unnatural if the rescaling factor, $r(m^2_{ph}/Q^2)$, needed to transform the relative error in the scalar physical mass squared to the relative error in the scalar running mass squared defined at momentum Q, vanishes as Q^2 goes to infinity.

I call the dimensionless function r the r-function. When the value $r(m^2(Q^2)/Q^2) = 10^{-3}$ is reached, $\sqrt{Q^2}$ is taken as the "scale of new physics" (approximately 1 Tev for some estimates of the Standard Model r-function).

Conclusion

Resumation of leading logs in the quantum corrections to scalar running mass may push up the scale of new physics. Its concrete value for the Standard Model is a subject of an unfinished computation. For ϕ^4, the resummation outcome supports the point of view BRS of the Introduction.

Neglecting leading logs in the quantum corrections to scalar running mass leads to expectations of new physics at 1 Tev, and to the point of view G of the Introduction.

Emploing minimal subtractions in computing the scalar running mass is unreliable, because minimal subtractions do not subtract the poles located away from the physical dimension and related to the quadratic divergences. [3]

References

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The case of ϕ^4

The running mass: For minimal subtractions the running mass is

$$m^2_{ms}(Q^2, m^2_{ph}) = Q^2\left(\frac{m^2_{ph}}{Q^2}\right)^{1-\gamma_\phi}$$

The r-function: For minimal subtractions the r-function is

$$r_{ms}\left(\frac{m^2_{ph}}{Q^2}\right) = 1 - \gamma_\phi$$

In these formulas $\gamma_\phi = \frac{g^2}{12(16\pi^2)^2}$ is the anomalous dimension of the scalar field.

There is no naturalness problem for ϕ^4 renormalized within minimal subtractions
(Supports point of view BRS of the Introduction)

The running mass: For momentum subtractions the running mass without resummation of the leading logarithms of Q^2 is

$$m^2_{MOM}(Q^2, m^2_{ph}) = m^2_{ph} + \gamma_\phi Q^2$$

The r-function: For momentum subtractions the r-function without resummation of the leading logarithms of Q^2 is

$$r_{MOM}\left(\frac{m^2_{ph}}{Q^2}\right) = \frac{m^2_{ph}/Q^2}{m^2_{ph}/Q^2 + \gamma_\phi} \approx \frac{m^2_{ph}/Q^2}{\gamma_\phi}$$

Without leading log resummation, there is a strong naturalness problem for ϕ^4 renormalized within momentum subtractions
(Supports point of view G of the Introduction)

Momentum subtractions without resummation and minimal subtractions lead to opposite conclusions about the naturalness problem. A possible source for this qualitative difference is that minimal subtractions do not subtract the poles at the dimensions [$4 - 2/$(number of loops)] related to the quadratic divergences. [3]

Scalar running mass with resummation

The leading logarithms of Q^2 in the scalar mass squared renormalized with momentum subtractions at momentum Q^2 have been resummed for ϕ^4 with solving an analog of the Gell-Mann–Low equation. [4]

The scalar running mass squared: After the resummation, at large Q^2, the scalar running mass squared is

$$m^2_{GML}(Q^2, m^2_{ph}) = \frac{\gamma_\phi Q^2}{1 + 4\gamma_\phi \log\frac{m^2_{ph}}{Q^2}}$$

The r-function: After the resummation, at large Q^2, the r-functon is

$$r_{GML}\left(\frac{m^2_{ph}}{Q^2}\right) = \frac{-4\gamma_\phi}{1 + 4\gamma_\phi \log\frac{m^2_{ph}}{Q^2}}$$

Assumption: Both the scalar running mass and the r-function of ϕ^4 obtain a Landau pole after the resummation. This means that they both grow with Q^2 until they become infinite at $Q^2 = m^2_{ph}\exp(1/\gamma_\phi)$. I assume that the Landau pole in the mass is a feature of ϕ^4, and that in the Standard model it is replaced with the asymptotic freedom behavior. This can be modeled by changing the sign by the scalar field anomalous dimension in the denominators:

$$m^2_{GML}(Q^2, m^2_{ph}) = \frac{\gamma_\phi Q^2}{1 - 4\gamma_\phi \log\frac{m^2_{ph}}{Q^2}}$$

$$r_{GML}\left(\frac{m^2_{ph}}{Q^2}\right) = \frac{-4\gamma_\phi}{1 - 4\gamma_\phi \log\left(\frac{m^2_{ph}}{Q^2}\right)} \approx \frac{1}{\log\left(\frac{m^2_{ph}}{Q^2}\right)}$$

A caveat: At the moment, no analog of the Gell-Mann–Low evolution equation for the Standard Model is available, and no resummation of the leading logarithms in the scalar running mass has been performed for the Standard Model. But the example of ϕ^4 demonstrates that the resummation of the leading logarithms may qualitatively change the naturalness analysis.

The outcome: If $r\left(\frac{m^2_{ph}}{Q^2}\right) = 1/\log\left(\frac{m^2_{ph}}{Q^2}\right)$, the Standard Model has a naturalness problem, but a weak one, because the magnitude of the r-function decreases very slowly with growing Q^2, and reaches dangerously small values beyond the Plank scale (at the Plank scale it is about 1/78). This supports the point of view BRS of the Introduction.