Generalized solution for RS metric and LHC phenomenology

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Plan of the talk

- Randall-Sundrum scenario with two branes. Original RS solution
- Generalization of RS solution
- Solution with explicit orbifold symmetries
- Role of constant term. Different physical schemes
- Dilepton production at LHC in RS-like scenario
- Conclusions

Randall-Sundrum scenario

Background metric (y is extra coordinate)

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

warp factor

Periodicity: $(x, y \pm 2\pi r_c) = (x, y)$
Z_2-symmetry: $(x, y) = (x, -y)$

→ **orbifold S^1/Z_2** $0 \le y \le \pi r_c$

Two fixed points: $y=0$ and $y=\pi r_c$

→ **two (1+3)-dimensional branes**

5-dimensional action $S = S_g + S_1 + S_2$

$$S_g = \int d^4x \int dy \sqrt{G}\left(2M_5^3 R^{(5)} - \Lambda\right)$$ **(gravity term)**

$$S_{1(2)} = \int d^4x \sqrt{g_{1(2)}}\left(L_{1(2)} - \Lambda_{1(2)}\right)$$ **(brane terms)**

Einstein-Hilbert's equations:

$$\sigma'^2(y) = -\frac{\Lambda}{24M_5^3}$$

$$\sigma''(y) = \frac{1}{12M_5^3}[\Lambda_1\delta(y) + \Lambda_2\delta(\pi r_c - y)]$$

Original Randall-Sundrum solution

(Randall & Sundrum, 1999)

$$\sigma_{RS}(y) = \kappa\,|\,y\,|$$

$$\Lambda_{RS} = -24M_5^3\kappa^2, \quad (\Lambda_1)_{RS} = -(\Lambda_2)_{RS} = 24M_5^3\kappa$$

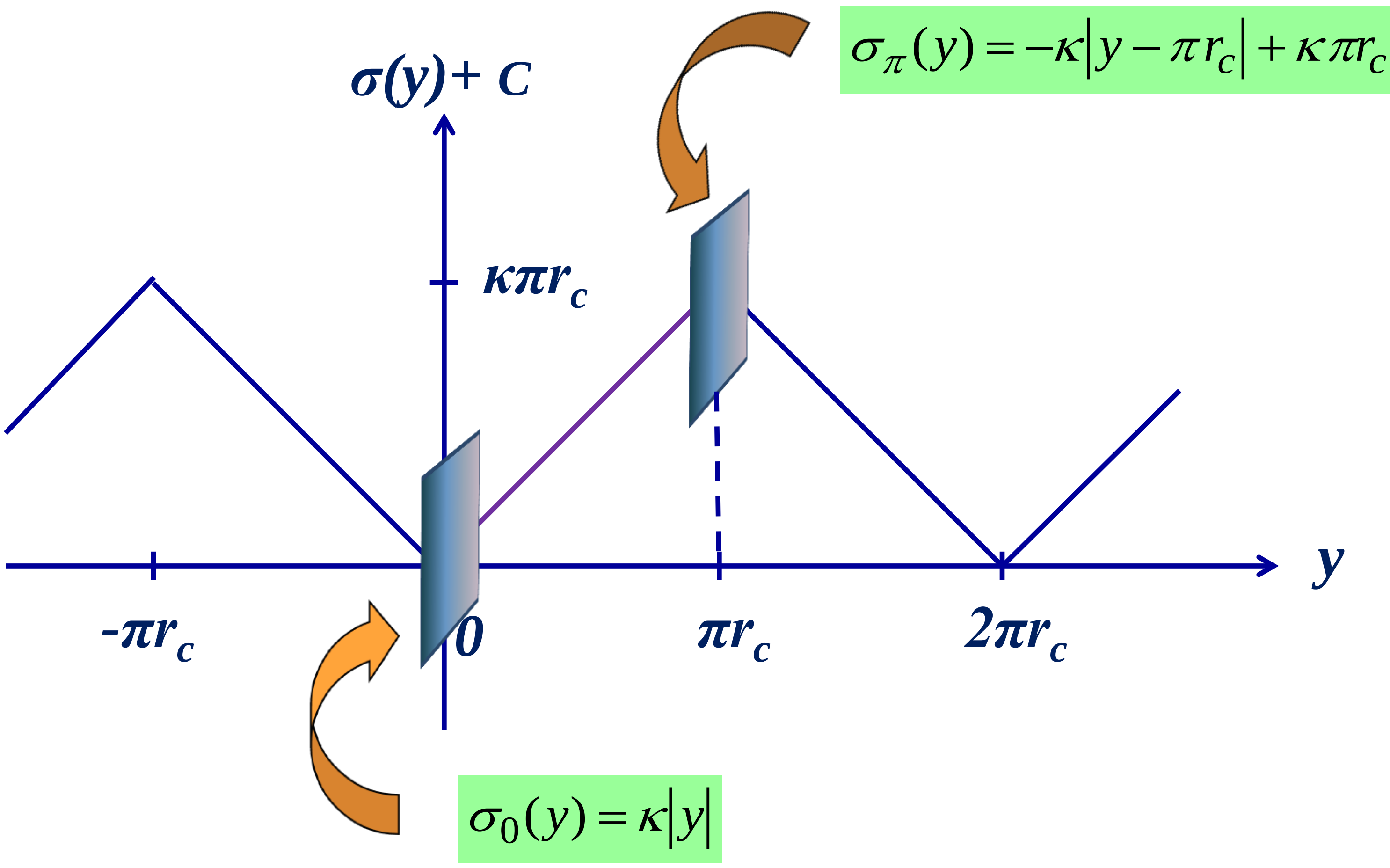
$$\longrightarrow \quad \sigma'_{RS}(y) = \kappa\varepsilon(y) \qquad \sigma''_{RS}(y) = 2\kappa\delta(y)$$

The RS solution:

- **does not explicitly reproduce the jump of derivative on TeV brane (at y=πr_c)**
- **is not symmetric with respect to both branes (located at y=0 and y=πr_c)**
- **does not include a constant term**

Generalization of RS solution

Two equivalent solutions related to different branes



Generalized solution $(0 \le C \le \kappa\pi r_c)$

$$\sigma(y) = \frac{1}{2}\left[\sigma_0(y) + \sigma_\pi(y)\right] - C = \frac{\kappa}{2}\left(|y| - |y - \pi r_c|\right) + \frac{\kappa\pi r_c}{2} - C$$

with fine tuning

$$\Lambda = -24 M_5^3 \kappa^2, \quad \Lambda_1 = -\Lambda_2 = 12 M_5^3 \kappa$$

factor of 2 different than that of RS

1-st derivative of σ(y)

$$\sigma'(y) = \frac{\kappa}{2}\left[\varepsilon(y) - \varepsilon(y - \pi r_c)\right]$$

2-nd derivative of σ(y) $\sigma''(y) = \kappa\left[\delta(y) - \delta(y - \pi r_c)\right]$

Explicit account of periodicity and Z_2-symmetry

Solution for the warp function in variable $x = y / r_c$ *(A.K., 2015)*

$$\sigma(y) = \frac{\kappa r_c}{2}\left[\left|\mathrm{Arccos}(\cos x)\right| - \left|\mathrm{Arccos}(\cos x) - \pi\right|\right] + \frac{\pi \kappa r_c}{2} - C$$

Arccos(z) is principal value of inverse cosine

$$0 \le \mathrm{Arccos}(z) \le \pi, \quad -1 \le z \le 1$$

$$\mathrm{Arccos}(\cos x) = \begin{cases} x - 2n\pi, & 2n\pi \le x \le (2n+1)\pi \\ -x + 2(n+1)\pi, & (2n+1)\pi \le x \le 2(n+1)\pi \end{cases}$$

(see, for instance, Gradshteyn &Ryzhik)

In particular, $\sigma(y) = \kappa y$ for $0 \le y \le \pi r_c$

Orbifold symmetries:

$$\sigma(y+2\pi r_c)=\sigma(y)$$ **(periodicity)**

$$\sigma(-y)=\sigma(y)$$ **(Z_2 symmetry)**

1-st derivative of σ(y): $(y \neq \pi n r_c\,,\ n = 0, \pm 1, \pm 2, \ldots)$

$$\sigma'(y)=\kappa\,\mathrm{sign}\big(\sin(\mathrm{x})\big)$$

$$\sigma'(-y)=-\sigma'(y)$$

2-nd derivative of σ(y):

$$\sigma''(y)=\frac{\kappa}{r_c}\sum_{n=-\infty}^{\infty}\left[\delta\big(x+2\pi n\big)-\delta\big(x-\pi+2\pi n\big)\right]$$

Hierarchy relation

$$M_{\mathrm{Pl}}^2 = \frac{M_5^3}{\kappa}\exp(2C)$$

Interaction Lagrangian
(*massive gravitons only*)

$$L(x) = -\frac{1}{\Lambda_\pi}\sum_{n=1}^{\infty} h_{\mu\nu}^{(n)}(x)T_{\alpha\beta}(x)\,\eta^{\mu\alpha}\eta^{\nu\beta}$$

Masses of KK gravitons
(x_n are zeros of $J_1(x)$)

$$m_n = x_n \frac{M_{\mathrm{Pl}}}{\sqrt{\exp(2\pi\kappa r_c)-1}}\left(\frac{\kappa}{M_5}\right)^{3/2}$$

Masses of KK gravitons m_n and coupling Λ_π depend on C via M_5 and κ

→ **Different values of C result in quite diverse physical models**

Different physical scenarios

I. C = 0 $\sigma(0)=0, \quad \sigma(\pi r_c)=\kappa\pi r_c$

$M_{\rm Pl}^2 \cong \dfrac{M_5^3}{\kappa}$ **that requires** $M_5 \sim \kappa \sim M_{\rm Pl}$

Masses of KK resonances $m_n \cong x_n \kappa \exp(-\kappa\pi r_c)$

RS1 model ***(Randall & Sundrum, 1999)***

Graviton spectrum - heavy resonances, with the lightest one above 1 TeV

II. C = κπr$_c$

$$\sigma(0) = -\kappa\pi r_c, \quad \sigma(\pi r) = 0$$

$$\longrightarrow \quad M_{\rm Pl}^2 \cong \frac{M_5^3}{\kappa} \exp(2\pi\kappa r_c)$$

$\kappa \ll M_5$ $\kappa r_c \approx 9.5$ for $M_5 = 1\,\mathrm{TeV}, \kappa = 100\,\mathrm{MeV}$

Masses of KK resonances $m_n \cong x_n \kappa$

→ **RSSC model: scenario with small curvature of 5-dimensional space-time**

For small κ, graviton spectrum is similar to that of the ADD model

(Giudice, 2005 Petrov & A.K., 2005)

III. C = $\kappa\pi r_c/2$

$$\sigma(0) = -\sigma(\pi r_c) = \kappa\pi r_c/2$$

"symmetric" scheme

$$\rightarrow \quad M_{\mathrm{Pl}}^2 \cong \frac{2M_5^3}{\kappa}\sinh(2\pi\kappa r_c)$$

Masses of gravitons $m_n \cong x_n \kappa \exp(-\kappa\pi r_c/2)$

Let $M_5 = 2\cdot 10^9\,\mathrm{GeV},\ \kappa = 10^4\ \mathrm{GeV}$

$$\rightarrow \quad m_n \cong 3.7\, x_n\,(\mathrm{MeV})$$

(A.K., 2015)

Almost continuous spectrum of KK gravitons

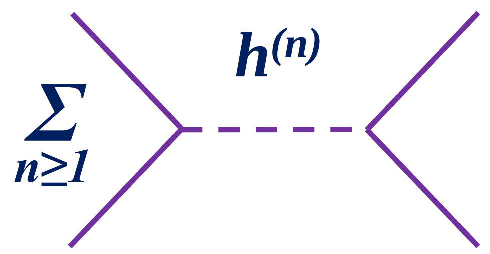
Virtual Gravitons at the LHC

pp-collisions at LHC mediated by KK graviton exchange in *s*-channel

Processes: $pp \to l^+ l^- (\gamma\gamma, 2\,jets) + X$

Sub-processes: $q\bar{q},\ gg \to h^{(n)} \to f\bar{f}, \gamma\gamma$

KK gravitons



Matrix element of sub-process

$$\mathrm{M} = \mathrm{A} \times \mathrm{S}$$

Tensor part of graviton propagator

where $\mathrm{A} = \mathrm{T}^{\mathrm{in}}_{\mu\nu}\, \mathrm{P}^{\mu\nu\alpha\beta}\, \mathrm{T}^{\mathrm{f}}_{\alpha\beta}$

Energy-momentum tensors

and $$\mathrm{S}(s) = \frac{1}{\Lambda_\pi^2} \sum_{n=1}^{\infty} \frac{1}{s - m_n^2 + i m_n \Gamma_n}$$ ***(process independent)***

Graviton widths $$\Gamma_n = \eta\, m_n^3 / \Lambda_\pi^2, \;\; \eta \cong 0.1$$

Dilepton production at LHC in scenario with small curvature

Scenario II ($C = \kappa\pi r_c$)

Number of events with $p_t > p_t^{cut}$

$$N_S = \int_{p_\perp^{cut}} \mathrm{dp}_\perp \frac{d\sigma(\mathrm{grav})}{\mathrm{dp}_\perp}, \quad N_B = \int_{p_\perp^{cut}} \mathrm{dp}_\perp \frac{d\sigma(\mathrm{SM})}{\mathrm{dp}_\perp}$$

Interference (SM-gravity) contribution is negligible

Statistical significance $$S = \frac{N_S}{\sqrt{N_S + N_B}}$$

No deviations from the CM were seen at the LHC

LHC search limits on M_5 ($p_t^{cut} = 200$ GeV)

$M_5 > 6.4$ TeV
for 7 + 8 TeV,
$L = 5\ \mathrm{fb}^{-1} + 20\ \mathrm{fb}^{-1}$

$M_5 > 9.0$ TeV
for 13 TeV,
$L = 30\ \mathrm{fb}^{-1}$

Week dependence of limits on parameter κ

Scenario III ($C = \kappa\pi r_c/2$)

$$S(s) = \frac{1}{2\Lambda_\pi^3\sqrt{s}}\left(\frac{M_5}{\kappa}\right)^{3/2}\frac{J_2(z)}{J_1(z)}$$ **with** $$z \cong \frac{\sqrt{s}}{\Lambda_\pi}\left(\frac{M_5}{\kappa}\right)^{3/2}$$

(***relation of*** m_n ***with*** x_n ***was used***)

For chosen values of parameters

$$|S(s)| = \frac{\mathrm{O}(1)}{(1\,\mathrm{TeV})^3\sqrt{s}}$$

→ **TeV physics at LHC energies**

Conclusions

- **Generalized solution for metric in RS-like scenario is derived**
- **This solution σ(y):**

  - is symmetric with respect to the branes

  - has the jumps of its derivative on both branes

  - is consistent with the orbifold symmetries

  - depends on constant C ($0 \leq C \leq \kappa\pi r_c$)
- **Different values of C result in quite diverse physical scenarios: RS1, RSSC, "symmetric" models**

Conclusions (continued)

- All these schemes lead to TeV physics at LHC, but with different experimental signatures
- LHC limits on gravity scale M_5 are obtained in RS-like scheme with small curvature

Thank you for your attention

Back-up slides

Extra dimensions: LHC limits

ADD model :

M_D > 4 TeV (for $n_{ED} = 4$) **real graviton production**

(CMS, EPJC 75 (2015) 235)

M_S > 7 TeV (for $n_{ED} = 4$) **virtual graviton exchange**

(HLZ convention)

(CMS, PLB 746 (2015) 79)

RS model :

m_{G*} > 2.7 TeV (for $\kappa/M_{Pl} = 0.1$)

(ATLAS , PRD 90 (2014) 052005)

Dilepton production

Pseudorapidity cuts:

$|\eta| \leq 2.4$ (muons)

$|\eta| \leq 1.44$, $1.57 \leq |\eta| \leq 2.4$ (electrons)

Efficiency: 85 %

K-factor:

1.5 for SM background

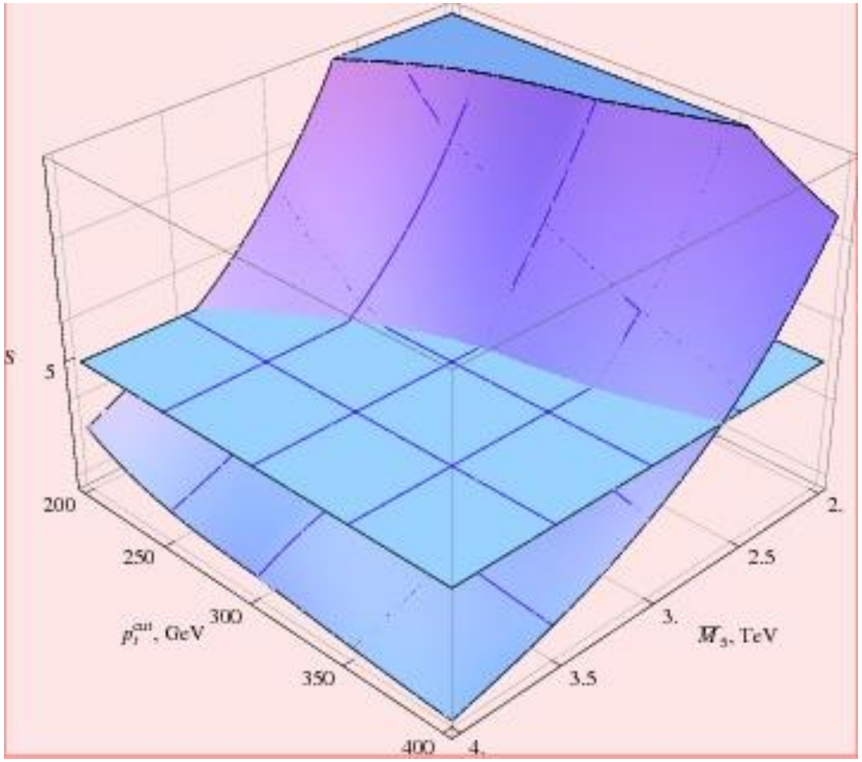
1.0 for signal

Uncertainties in search limit
($L = 30\ \mathrm{fb}^{-1}$)

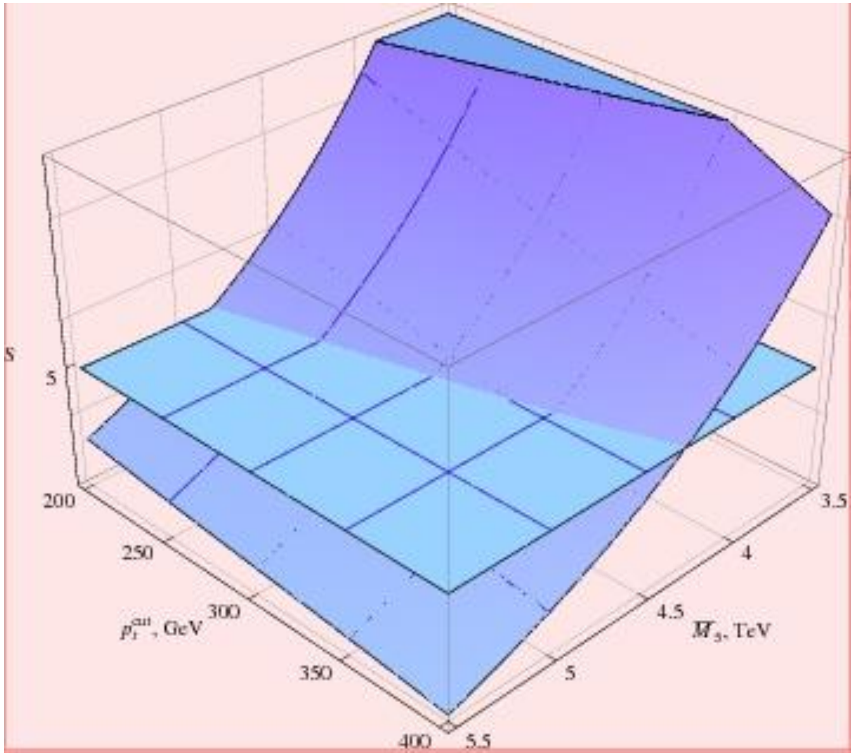
$\mu/2 \rightarrow \mu \rightarrow 2\mu$ ➡ $|\Delta M_5| = 17\ \mathrm{GeV}$

PDF set (2.7%) ➡ $|\Delta M_5| = 23\ \mathrm{GeV}$

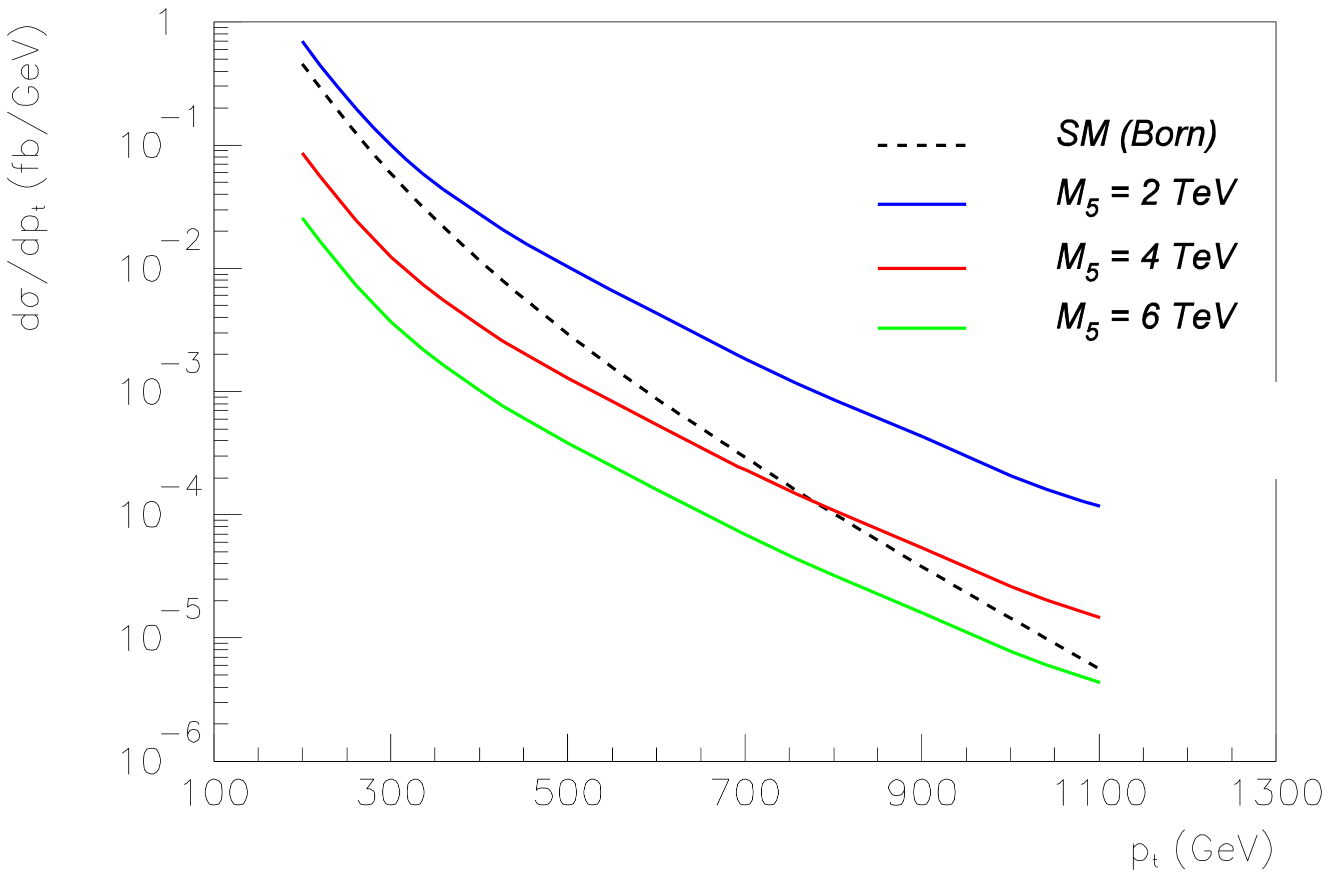
ΔL (3%) ➡ $|\Delta M_5| = 26\ \mathrm{GeV}$



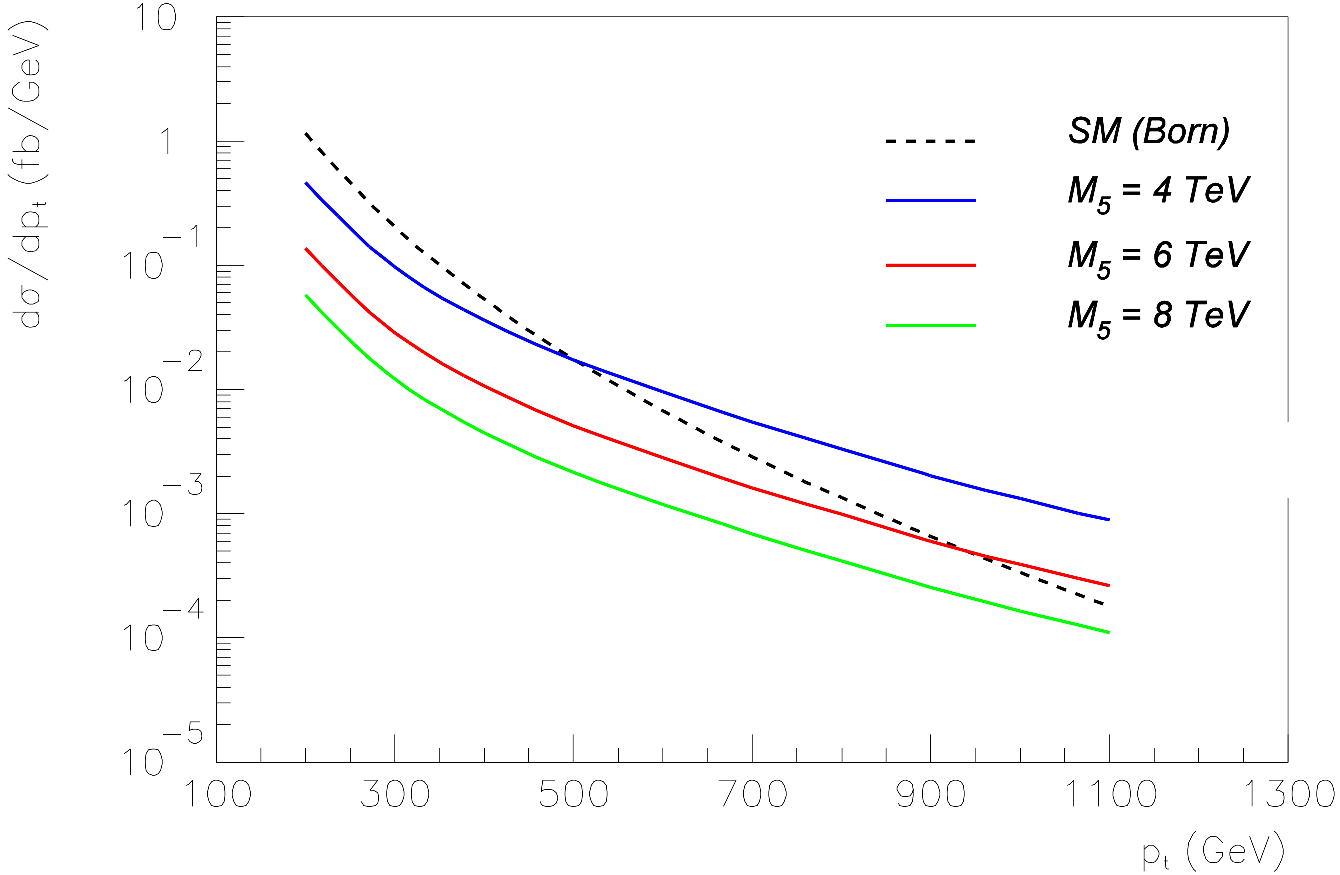
Statistical significance in RSSC for pp → e⁺e⁻ + X as a function of 5-dimensional reduced Planck scale M_5 and cut on electron transverse momentum p_t^{cut} for 7 TeV ($L=5$ fb^{-1}) and 8 TeV ($L=20$ fb^{-1})



Statistical significance for the process pp → e^+e^- + X as a function of 5-dimensional reduced Planck scale M_5 and cut on electron transverse momentum p_t^{cut} for 13 TeV (L=30 fb^{-1})



Graviton contributions in RSSC to the process
pp → μ+μ- + X (solid lines)
vs. SM contribution (dashed line) for 7 TeV



Graviton contributions in RSSC to the process pp → μ+μ- + X (solid lines) vs. SM contribution (dashed line) for 14 TeV

$$\frac{d\sigma(SM)}{dp_t} = \frac{1}{s^{3/2}} f(x_\perp, \ln s)$$

Weak logarithmic dependence on energy comes from PDFs

$$\frac{d\sigma(grav)}{dp_t} = \frac{1}{M_5^3} g(x_\perp, \ln s)$$

$$\Rightarrow \quad \frac{d\sigma(grav)/dp_t}{d\sigma(SM)/dp_t} \approx \left(\frac{\sqrt{s}}{M_5}\right)^3$$ **(for fixed $x_\perp$)**

dσ(grav): week dependence on curvature κ

$$d\sigma(pp \to h^{(n)} \to \gamma\gamma) = 2 \cdot d\sigma(pp \to h^{(n)} \to l^+ l^-)$$

universal for all subprocesses

$$q\bar{q} \to G^{(n)} \to l^+ l^- : 1 - 3\cos^2\theta + 4\cos^4\theta$$

$$gg \to G^{(n)} \to l^+ l^- : 1 - \cos^4\theta$$

while in SM: $1 + \cos^2\theta$

Effective gravity action

$$S_{\rm eff} = \frac{1}{4}\sum_{n=0}^{\infty}\int d^4x[\partial_\mu h^{(n)}_{\rho\sigma}(x)\partial_\nu h^{(n)}_{\delta\lambda}(x)\eta^{\mu\nu} - m_n^2 h^{(n)}_{\rho\sigma}(x)h^{(n)}_{\delta\lambda}(x)]\eta^{\rho\delta}\eta^{\sigma\lambda}$$

Shift $\sigma \to \sigma - C$ **is equivalent to the change** $x^\mu \to x'^\mu = \mathrm{e}^{-C}x^\mu$

Invariance of the action → **rescaling of fields and masses:**

$$h^{(n)}_{\mu\nu} \to h'^{(n)}_{\mu\nu} = \mathrm{e}^C h^{(n)}_{\mu\nu}, \quad m_n \to m'_n = \mathrm{e}^C m_n$$

Massive theory is not scale-invariant

→ **From the point of view of 4- dimensional observer these two theories are not physically equivalent**

Randall-Sundrum solution $\sigma_{\mathrm{RS}}(y) = \kappa |y|$

Does it mean that $\sigma_{RS}(y)$ is a linear function for all y?

The answer is negative: one must use the periodicity condition first, and only then estimate absolute value $|y|$

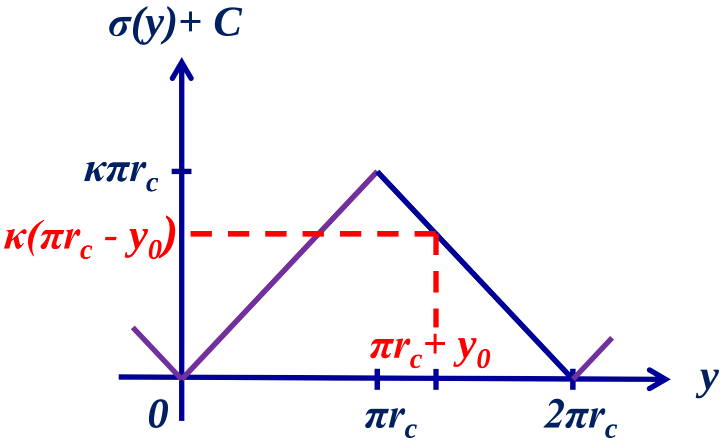
In other words, extra coordinate y must be reduced to the interval $-\pi r_c \leq y \leq \pi r_c$ (by using periodicity condition) before evaluating functions $|x|$, $\varepsilon(x)$, …

Examples: definition of $\sigma(y)$ outside interval $|y| \leq \pi r_c$

Let $y = \pi r_c + y_0$, where $0 < y_0 < \pi r_c$
(that is, $\pi r_c < y < 2\pi r_c$)

$$\sigma(\pi r_c + y_0) + C = \frac{\kappa}{2}(|\,\pi r_c + y_0\,| - |\,y_0\,|) + \frac{\pi r_c}{2}$$

$$= \frac{\kappa}{2}(|\,\pi r_c + y_0 - \mathbf{2\pi\, r_c}| - |\,y_0\,|) + \frac{\pi r_c}{2} = \kappa(\pi r_c - y_0)$$



RSSC model vs. ADD model

RSSC model is not equivalent to the ADD model with one ED of size R=$(\pi\kappa)^{-1}$ up to $\kappa \approx 10^{-20}$ eV

Hierarchy relation for small κ

$$M_{\mathrm{Pl}}^2 \cong \frac{M_5^3}{\kappa}\left[\exp(2\pi\kappa r_c)-1\right] \xrightarrow{2\pi\kappa r_c \ll 1} M_5^3(2\pi r_c)$$

But the inequality $2\pi\kappa r_c \ll 1$ **means that**

$$\kappa \ll \frac{M_5^3}{M_{\mathrm{Pl}}^2} \approx 0.17\cdot 10^{-18}\left(\frac{M_5}{1\mathrm{TeV}}\right)^3 \mathrm{eV}$$

$$\sum_{n=1}^{\infty} \frac{1}{z_{n,\nu}^2 - z^2} = \frac{1}{2z} \frac{J_{\nu+1}(z)}{J_\nu(z)}, \quad J_\nu(z_{n,\nu}) = 0$$

$$S(s) \approx -\frac{1}{4\bar{M}_5^3 \sqrt{s}} \frac{\sin 2A + i \sinh 2\varepsilon}{\cos^2 A + \sinh^2 \varepsilon}$$

(A.K, 2006)

where $$A = \frac{\sqrt{s}}{\kappa}, \quad \varepsilon = \frac{\eta}{2} \left(\frac{\sqrt{s}}{\bar{M}_5} \right)^3$$

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