Theory of Heavy Quarkonium Production

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Production and Decay Rates of Heavy Quarkonia

Heavy Quarkonia: Bound states of heavy quark and antiquark.

The classic approach: Color-singlet model

- Calculate cross section for heavy quark pair in physical **color singlet** (=color neutral) state. In case of J/ψ: $c\bar{c}[{}^3S_1^{[1]}]$
- Multiply by quarkonium wave function at origin
- Leftover IR singularities in case of P wave quarkonia
- Mid 90's: Strong disagreement with Tevatron data apparent

Nonrelativistic QCD (NRQCD):

- Rigorous effective field theory: Bodwin, Braaten, Lepage (1995)
- Based on **factorization of soft and hard scales**
  (Scale hierarchy: $Mv^2 << Mv \approx \Lambda_{QCD} << M$)
- Large part of talk: **Is NRQCD factorization compatible with data?**

Further approaches: k_T factorization, Color Evaporation Model

Quarkonium Production with NRQCD (e.g. J/ψ)

Factorization theorem: $\sigma_{J/\psi} = \sum_n \sigma_{c\bar{c}[n]} \cdot \langle O^{J/\psi}[n] \rangle$

- ***n***: Every possible Fock state, including **color-octet** (CO) states.
- $\boldsymbol{\sigma_{c\bar{c}[n]}}$: Production rate of $c\bar{c}[n]$, calculated in perturbative QCD.
- $\boldsymbol{\langle O^{J/\psi}[n] \rangle}$: Long distance matrix elements (LDMEs): describe $c\bar{c}[n] \rightarrow J/\psi$, supposedly universal, nonperturbative.

Scaling rules: LDMEs scale with definite power of v ($v^2 \approx 0.2$):

scaling	v^3	v^7 ("CO states")	v^{11}
n	${}^3S_1^{[1]}$	${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, ${}^3P_J^{[8]}$	...

- **Double expansion** in v and α_s
- Leading term in v ($n = {}^3S_1^{[1]}$) equals **color-singlet model**.

Test NRQCD factorization at NLO

What we have:

- **Short distance coefficients $\sigma_{Q\bar{Q}[n]}$:**
  Three different groups/codes for inclusive NLO $Q\bar{Q}[n]$ production via Color Singlet + Color Octet states (Summary of publications, mostly since 2009):
  - **M.B., He, Mihaila, Klasen, Kniehl, Steinhauser:**
    J/ψ, $\psi(2S)$, η_c, h_c in $\gamma\gamma$, e^+e^-, γp, pp, including relativistic corrections.
  - **Chao, Han, Ma, Meng, Shao, K. Wang, Y.-J. Zhang:**
    J/ψ, $\psi(2S)$, χ_{cJ}, η_c, h_c, $\Upsilon(nS)$, χ_{bJ} in e^+e^-, pp.
  - **Li, Gong, Sang, Sun, Wan, J.-X. Wang, H.-F. Zhang:**
    J/ψ, $\psi(2S)$, χ_{cJ}, η_c, h_c, $\Upsilon(nS)$, χ_{bJ} in pp.
- **Color Singlet (CS) production LDMEs:**
  Related to decay CS LDMEs ➡ Extracted from decays like $J/\psi \to \mu^+\mu^-$ (or from potential model calculation).

What we have to fit:

- **Color Octet (CO) LDMEs** (no lattice calculation yet).

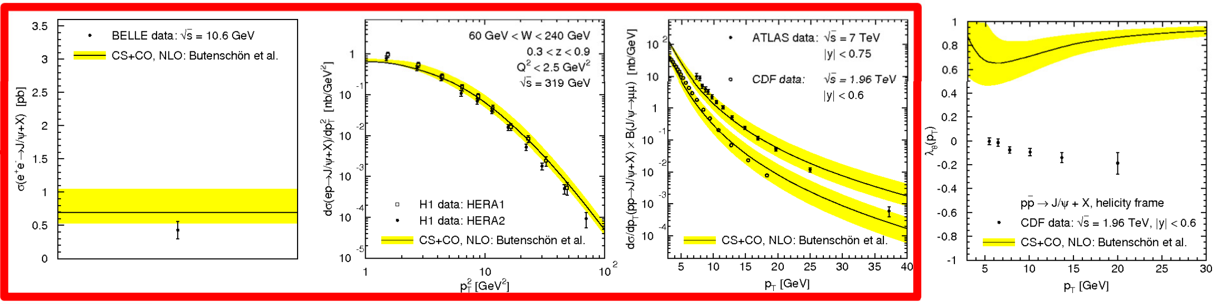
J/ψ Production Fits until 2013:

e+e- yield: **γp yield:** **pp yield:** **pp polarization:**

Butenschön, Kniehl:

$\langle O_8^{J/\psi}(^1S_0)\rangle = 0.0497\ \mathrm{GeV}^3$
$\langle O_8^{J/\psi}(^3S_1)\rangle = 0.0022\ \mathrm{GeV}^3$
$\langle O_8^{J/\psi}(^3P_0)\rangle = -0.0161\ \mathrm{GeV}^5$

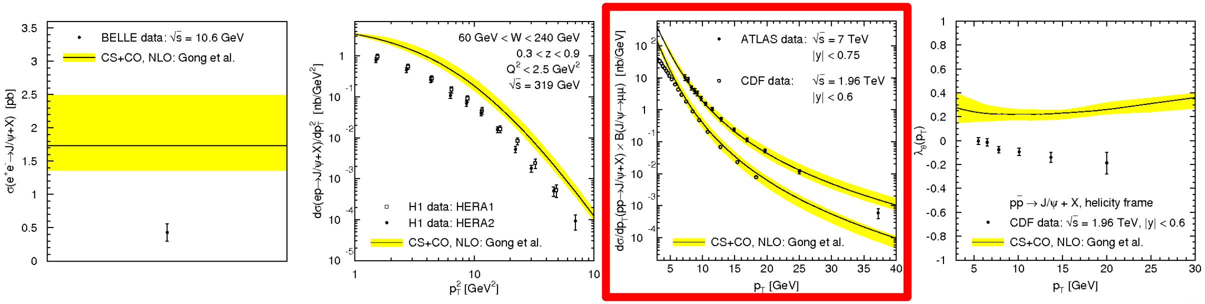
Data fitted



Gong, Wan, J.-X. Wang, H.-F. Zhang:

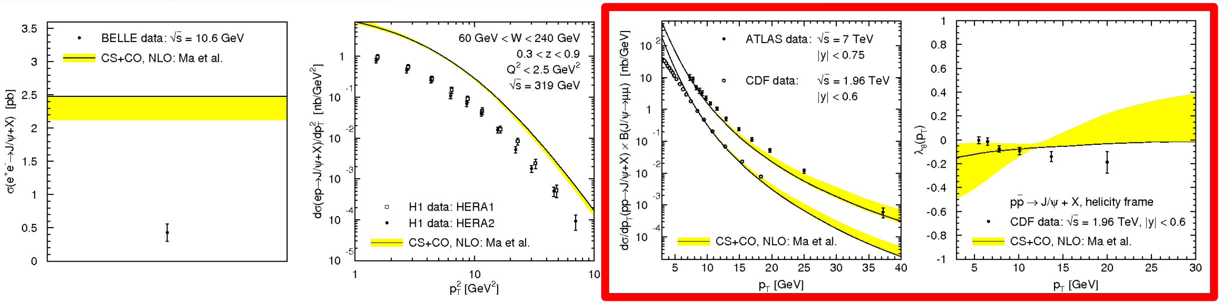
$\langle O_8^{J/\psi}(^1S_0)\rangle = 0.097\ \mathrm{GeV}^3$ $\langle O_8^{\psi'}(^1S_0)\rangle = -0.0001\ \mathrm{GeV}^3$
$\langle O_8^{J/\psi}(^3S_1)\rangle = -0.0046\ \mathrm{GeV}^3$ $\langle O_8^{\psi'}(^3S_1)\rangle = 0.0034\ \mathrm{GeV}^3$
$\langle O_8^{J/\psi}(^3P_0)\rangle = -0.0214\ \mathrm{GeV}^5$ $\langle O_8^{\psi'}(^3P_0)\rangle = 0.0095\ \mathrm{GeV}^5$

$\langle O_8^{\chi_0}(^3S_1)\rangle = 0.0022\ \mathrm{GeV}^3$



Chao, Ma, Shao, K. Wang, Y.-J. Zhang:

$\langle O_8^{J/\psi}(^1S_0)\rangle = 0.089\ \mathrm{GeV}^3$
$\langle O_8^{J/\psi}(^3S_1)\rangle = 0.003\ \mathrm{GeV}^3$
$\langle O_8^{J/\psi}(^3P_0)\rangle = 0.0126\ \mathrm{GeV}^5$



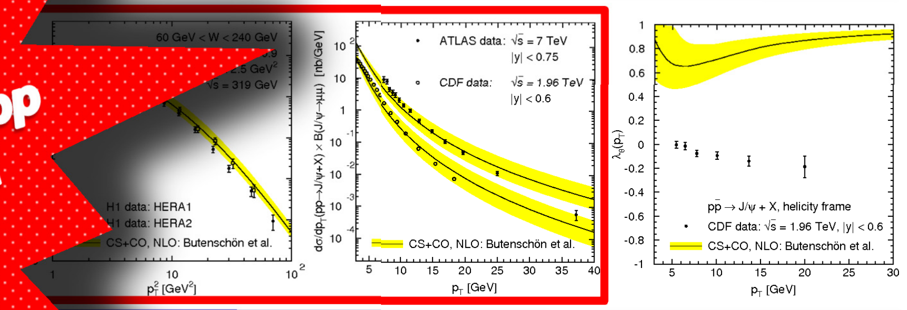
J/ψ Production Fits until 2013:

AGREEMENT: Can NOT describe e^+e^-, γp, pp yield and pp polarization with same LDMEs.

e^+e^- yield: **γp yield:** **pp yield:** **pp polarization:**

Butenschön, K

$\langle O_8^{J/\psi}(^1S$
$\langle O_8^{J/\psi}(^3S_1)\rangle$
$\langle O_8^{J/\psi}(^3P_0)\rangle = -$

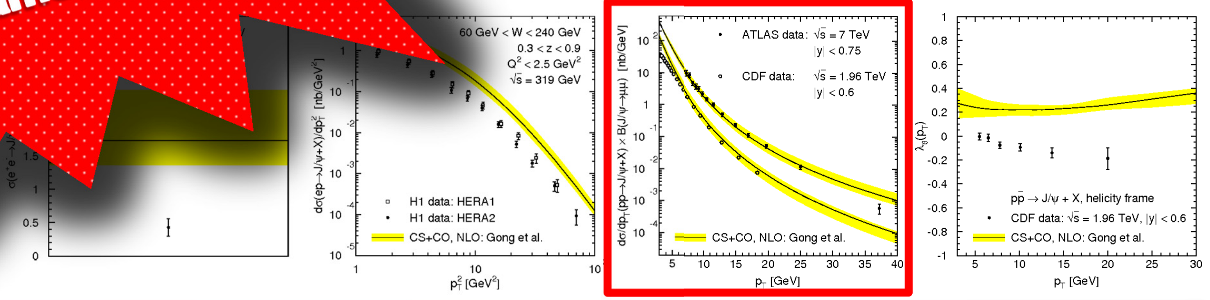


Gong, W

H.-F. Zha

$\langle O_8^{J/\psi}(^1S_0$
$\langle O_8^{J/\psi}(^3S_1)\rangle = -0.0046$ GeV
$\langle O_8^{J/\psi}(^3P_0)\rangle = -0.0214\ \mathrm{GeV}^5$ 0.0095

$\langle O_8^{\chi_0}(^3S_1)\rangle = 0.$ eV³



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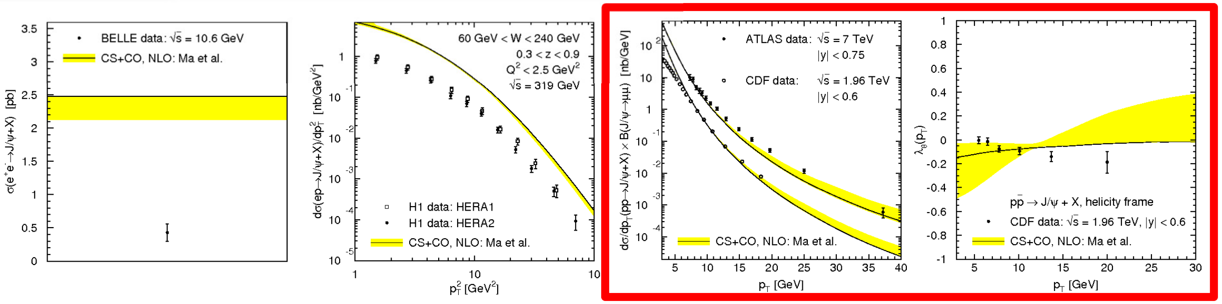


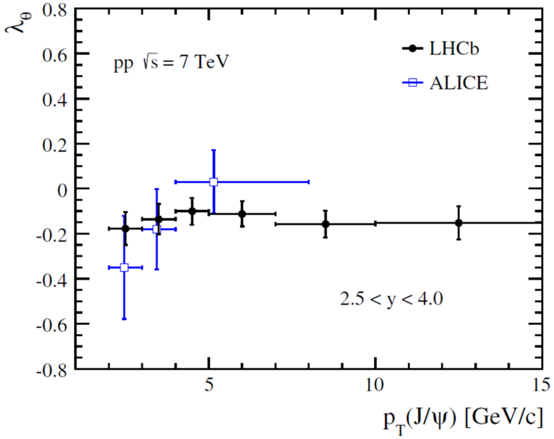
Ways Out (1): Is J/ψ really unpolarized?

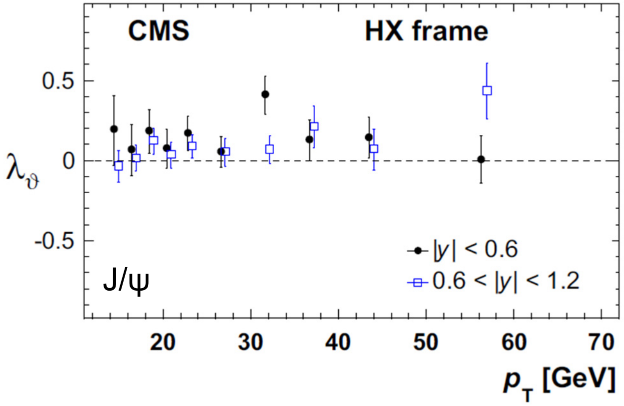
One option discussed:

- Maybe CDF data (unpolarized *J/ψ*: $\lambda_\theta \approx 0$) cannot be trusted. (Disagreement between Tevatron Run I and Run II data)
- **Strong transverse polarization** ($\lambda_\theta \approx +1$) would solve the problem!

BUT:

- In 2013: ALICE, LHCb and CMS have all succeeded in difficult polarization measurements and found **no significant transverse polarization** either:



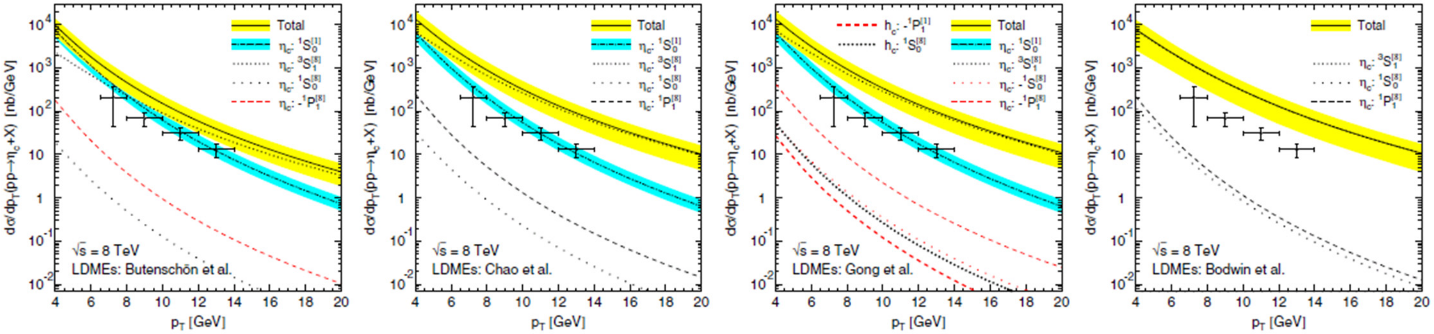


Ways Out (2): Is NRQCD valid only at high-p_T?

Another option: Maybe NRQCD holds only at $p_T > 9$ GeV.
(Idea: Factorization breaking terms in non-fragmentation-like contributions?)
At $p_T > 9$ GeV only *pp* data was available ➡ No disagreement with data.

BUT:

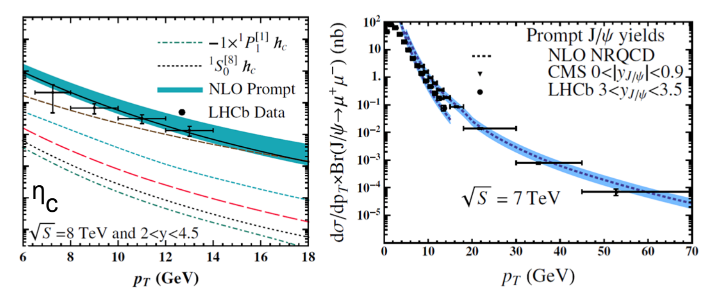
- In 2015 it was explicitly shown that there are **no factorization breaking terms** at any perturbative order. [Nayak (2015)]
- In 2014 LHCb measured ***η_c* production rate.** η_c and *J/ψ* LDMEs are related via Heavy quark spin symmetry of the NRQCD Lagrangian:

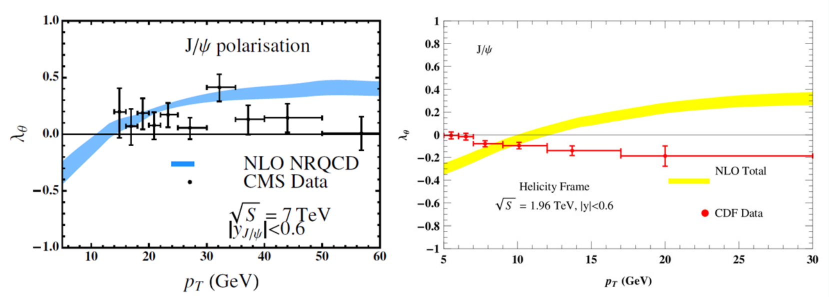


➡ **None of the *J/ψ* CO LDME sets** on the market describes η_c data, even though $p_T > 9$ GeV. [M.B., He, Kniehl (2014)]

Possible Other Ways Out

- **Carry on with only *pp* data anyway, but new fits including η_c:**
  - ☐ Han, Ma, Meng, Shao, Chao (2014):





  - ☐ Or: Accept lower CS LDME to get agreement [Zhang, Sun, Sang, Li (2014)]

➡ **Increasing tensions** even with high-p_T *pp* data only.

Other ways out:

- Maybe ***v* expansion** converges too slowly (need more intermediate states).
- A wider **range of parameters** (scales and heavy quark mass) might help.
- **Resummation** of $m^4/p_T^4\ \log(p_T/m)$ terms via **Double Parton Fragmentation Functions** (FFs) could improve usual FF results (RGEs need to be solved). [Kang, Qiu, Sterman (2012); Fleming, Leibovich, Mehen, Rothstein (2012)]

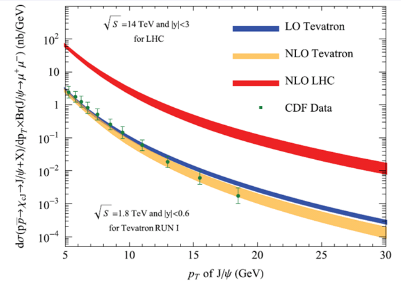
Ypsilon and $\chi_{c,J}$ Production with NRQCD

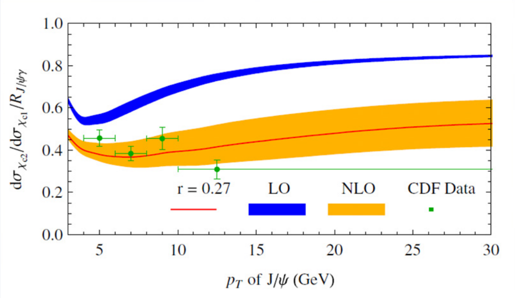
Bottomonia *Y(1S)*, *Y(2S)*, *Y(3S)* production:

- **Only *pp*** data: **More than enough** free parameters to easily describe production yield and polarization (Not yet a “test” on NRQCD factorization). [Gong, Wang, Wan, Zhang (2013)]

χ_{cJ} production:

- NRQCD Velocity Scaling: Leading LDMEs are $\langle O^{\chi_{cJ}}({}^3P_J^{[1]})\rangle$, $\langle O^{\chi_{cJ}}({}^3S_1^{[8]})\rangle$.
  ➡ Only **one free fit parameter** $\langle O^{\chi_{c0}}({}^3S_1^{[8]})\rangle$.
- **Nontrivial outcome:** Both χ_{cJ} yield and χ_{c2}/χ_{c1} ratio in *pp* collisions can simultaneously be described: [Ma, Wang, Chao (2010)]





k_T Factorization Approach

Apply k_T factorization to quarkonium production:

- **Idea:** Scales of quarkonium production much smaller than collision energy:

  $$p_T, m_c \ll \sqrt{s}$$

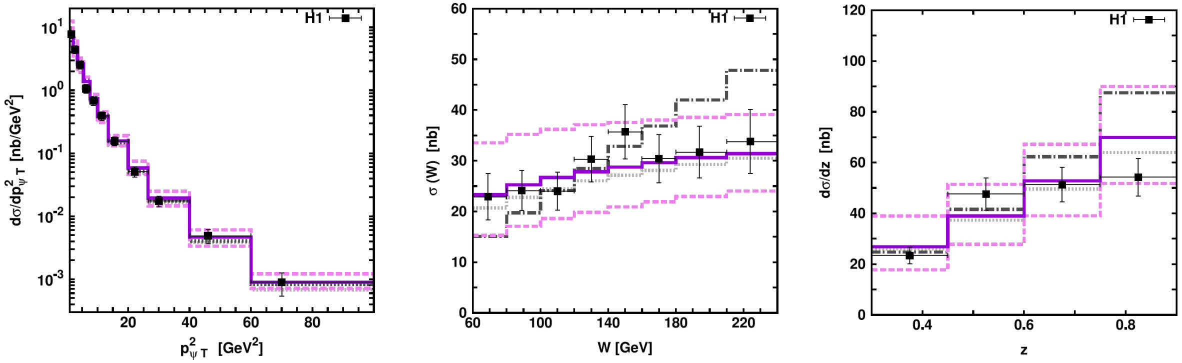
  ➡ Longitudinal parton momentum fractions x small,
  transverse parton momenta k_T should not be neglected.
- Use **off shell** matrix elements with k_T dependence entering via

  $$\varepsilon^\mu(k_T) = k_T^\mu / |\vec{k}_T| \, .$$
- Usually just LO matrix elements used.
- Fold with k_T dependent, **unintegrated PDFs**.
- **Various prescriptions** for deriving uPDFs from usual PDFs
  in DGLAP, BFKL or “CCFM” approach.
- Monte Carlo program **CASCADE** simulates initial state gluon radiation
  within k_T factorization framework [Jung, Salam (2001)].

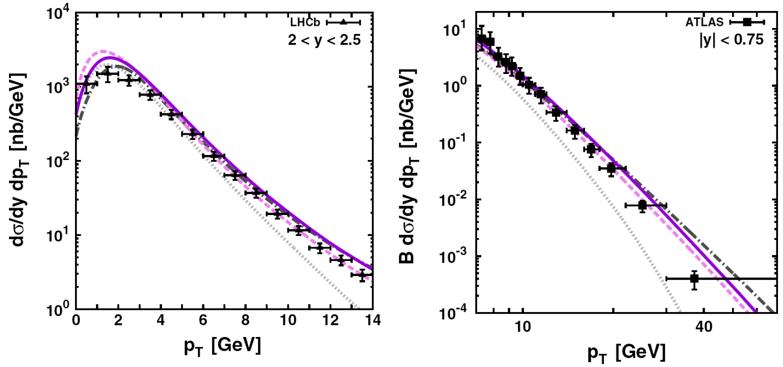
k_T Factorization Approach: Results (1)

- Baranov, Lipatov, Zotov (2011); Baranov, Lipatov, Zotov (2012): **Color Singlet Model** predictions for various uPDFs:

γp:



pp:



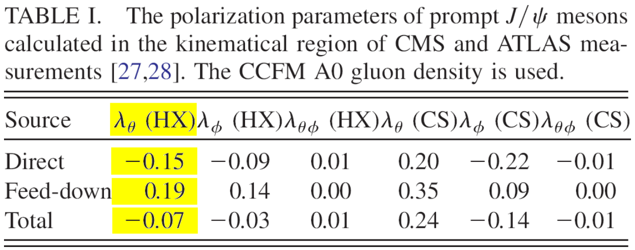


TABLE I. The polarization parameters of prompt J/ψ mesons calculated in the kinematical region of CMS and ATLAS measurements [27,28]. The CCFM A0 gluon density is used.

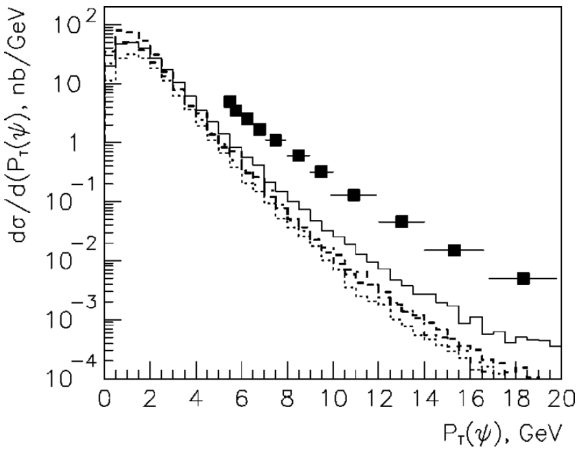
Source	λ_θ (HX)	λ_ϕ (HX)	$\lambda_{\theta\phi}$ (HX)	λ_θ (CS)	λ_ϕ (CS)	$\lambda_{\theta\phi}$ (CS)
Direct	-0.15	-0.09	0.01	0.20	-0.22	-0.01
Feed-down	0.19	0.14	0.00	0.35	0.09	0.00
Total	-0.07	-0.03	0.01	0.24	-0.14	-0.01

➡ **No room** and no need for **color octet** contributions.

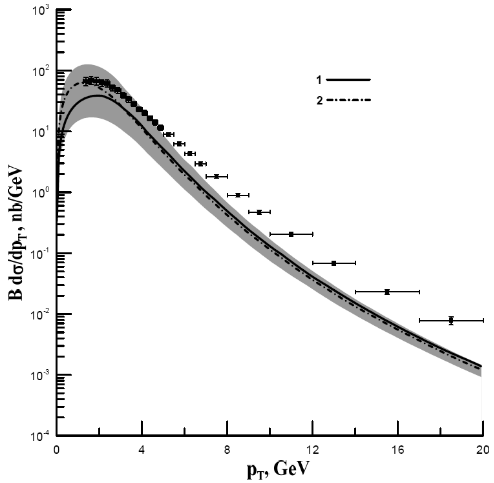
k_T Factorization Approach: Results (2)

But: Other calculations come to different conclusions:

(for hadroproduction)



[Baranov (2002)]



[Saleev, Nefedov, Shipilova (2012)]

- Effect of k_T **much smaller**, color singlet still **not enough.**
- In these works: Fits of **CO LDMEs** within k_T factorization framework.

  ➡ Maybe using k_T factorization **LDMEs** can be shown to be universal.

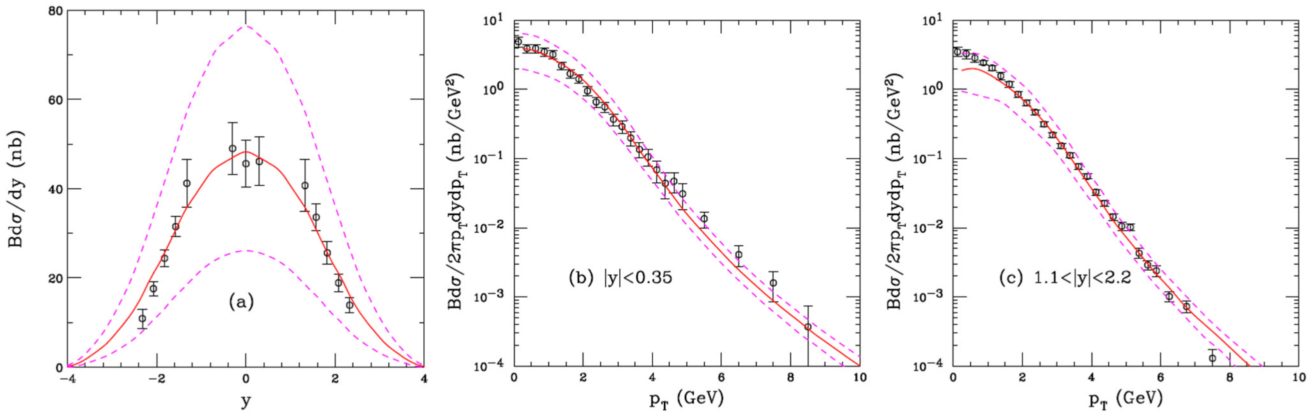
Color Evaporation Model

- Fritsch (1977); Halzen (1977); Glück, Owens, Reya (1978):

$$\sigma = F_H \int_{(2m_c)^2}^{(2m_D)^2} dm^2 \frac{d\sigma_{c\bar{c}}}{dm^2_{c\bar{c}}}$$

- Consider **open c+$\bar{c}$** production, regardless of c+$\bar{c}$ color, spin, momenta.
- Integrate over invariant c+$\bar{c}$ mass up to formation of next heavier meson pair.
- **F_H**: Number describing formation of quarkonium H by color “evaporation”.
- Qualitative picture rather than rigorous theory.

CEM predictions for RHIC data [Nelson, Voigt, Frawley (2013)]:



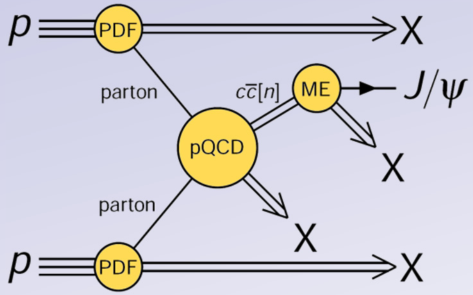
Summary

- **40 years after *J/ψ* discovery:**
  Mechanism behind heavy quarkonium production **still not clear.**
- **Traditional color singlet model:**
  - ☐ Can successfully describe **only e^+e^- data.**
  - ☐ Theoretically **incomplete** due to uncancelled IR divergences.
- **NRQCD factorization:**
  - ☐ **Rigorous theorem** based on a solid effective field theory.
  - ☐ But: Current analyses of experimental data cast doubt on the universality (process-independence) of the LDMEs.
- **Possible ways out today:**
  - ☐ Maybe v expansion does simply not converge well (at least for charmonia).
  - ☐ Maybe resummation of **large logarithms** p_T/m_c in region of small and/or large transverse momenta is necessary (e.g. NLP formula with double parton FFs).
  - ☐ Application of **k_T dependent** PDFs.

BACKUP SLIDES

Calculate Inclusive J/ψ Production within NRQCD

Factorization formulas (here hadroproduction):



- Convolute partonic cross section with **proton PDFs:**

$$\sigma_{\mathrm{hadr}} = \sum_{i,j} \int dx\, dy\, f_{i/p}(x)\, f_{j/p}(y) \cdot \sigma_{\mathrm{part,i,j}}$$

- **NRQCD factorization:**

$$\sigma_{\mathrm{part,i,j}} = \sum_{n} \sigma(ij \to c\bar{c}[n] + X) \cdot \langle O^{J/\psi}[n] \rangle$$

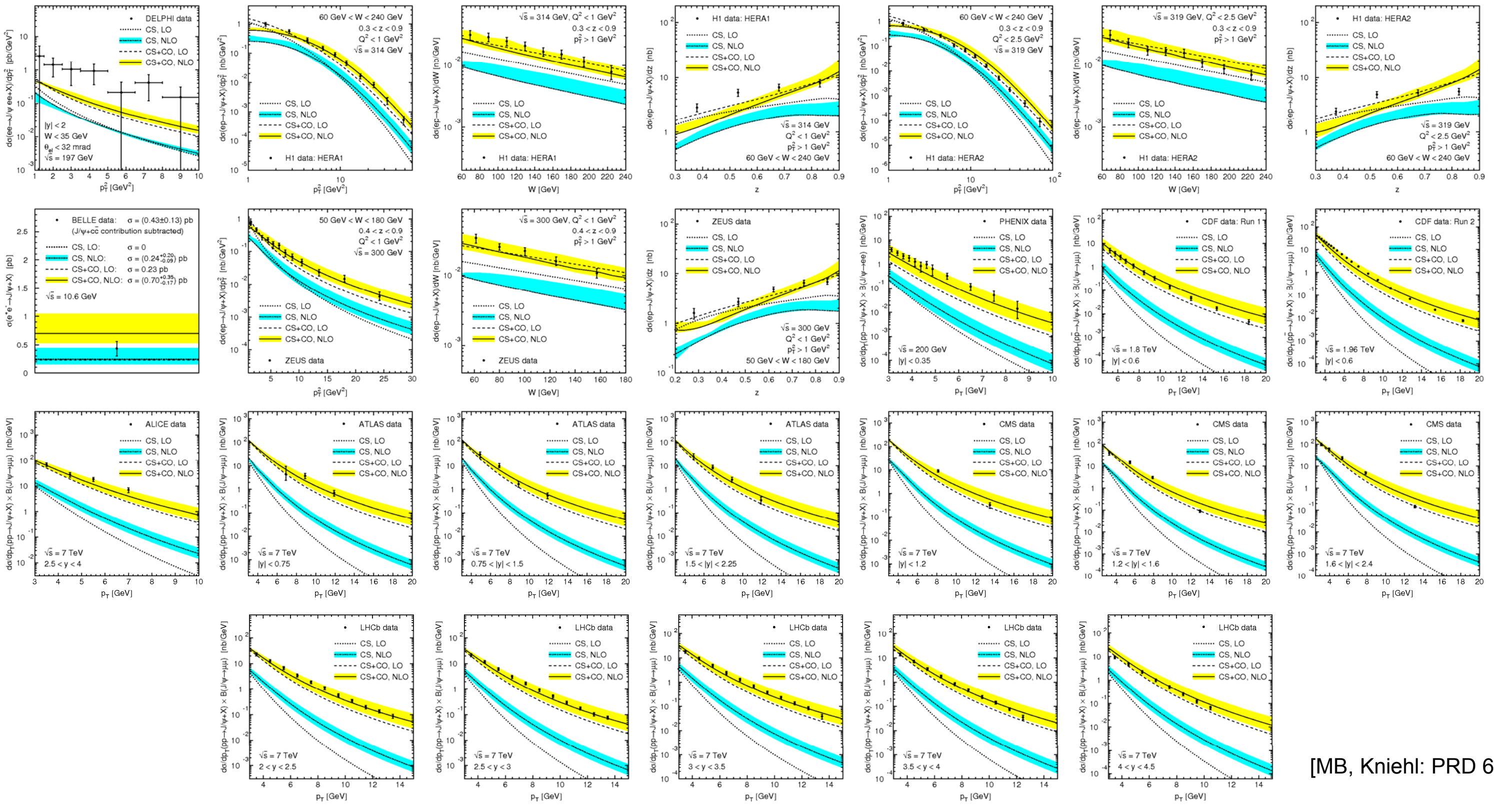
Amplitudes for $c\bar{c}[n]$ production by projector application, e.g.:

$$A_{c\bar{c}[^3S_1^{[1/8]}]} = \varepsilon_\alpha(m_S)\,\mathrm{Tr}\,[C\,\Pi^\alpha A_{c\bar{c}}]\,|_{q=0}$$

$$A_{c\bar{c}[^3P_J^{[8]}]} = \varepsilon_\alpha(m_S)\,\varepsilon_\beta(m_l)\,\frac{d}{dq_\beta}\,\mathrm{Tr}\,[C\,\Pi^\alpha A_{c\bar{c}}]\,|_{q=0}$$

- **$A_{c\bar{c}}$:** Amputated pQCD amplitude for open $c\bar{c}$ production.
- ***q*:** Relative momentum between *c* and *$\bar{c}$*. ***ε*:** Polarization vectors.

Global Fit to Unpolarized Data

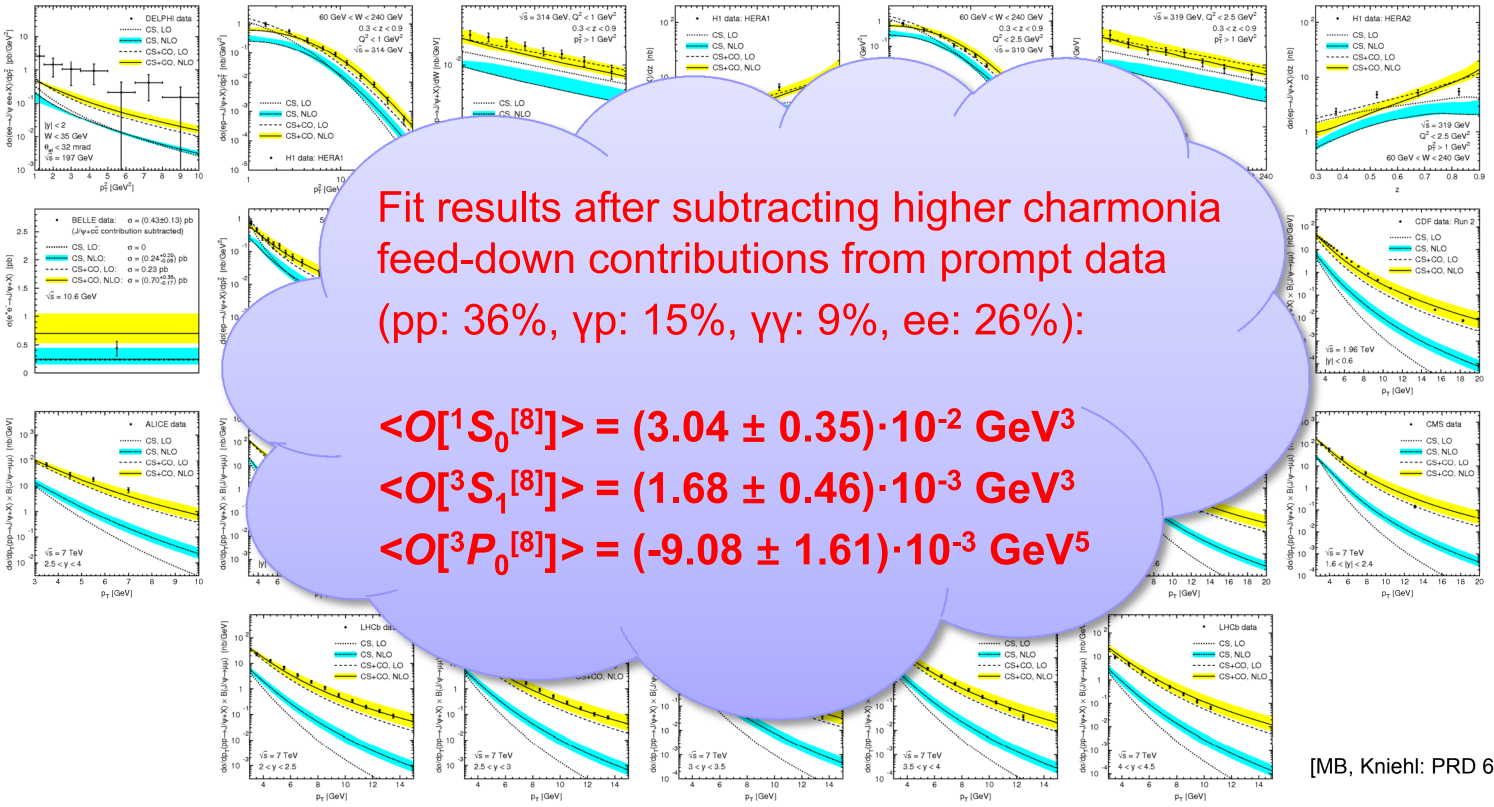


[MB, Kniehl: PRD 64, 051501R]

$\langle O[{}^1S_0^{[8]}]\rangle = (4.97 \pm 0.44)\cdot 10^{-2}\ \mathrm{GeV}^3$ $\langle O[{}^3S_1^{[8]}]\rangle = (2.24 \pm 0.59)\cdot 10^{-3}\ \mathrm{GeV}^3$

$\langle O[{}^3P_0^{[8]}]\rangle = (-1.61 \pm 0.20)\cdot 10^{-2}\ \mathrm{GeV}^5$

Global Fit to Unpolarized Data



Fit results after subtracting higher charmonia feed-down contributions from prompt data (pp: 36%, γp: 15%, γγ: 9%, ee: 26%):

$\langle O[^1S_0^{[8]}]\rangle = (3.04 \pm 0.35)\cdot 10^{-2}\ \mathrm{GeV}^3$

$\langle O[^3S_1^{[8]}]\rangle = (1.68 \pm 0.46)\cdot 10^{-3}\ \mathrm{GeV}^3$

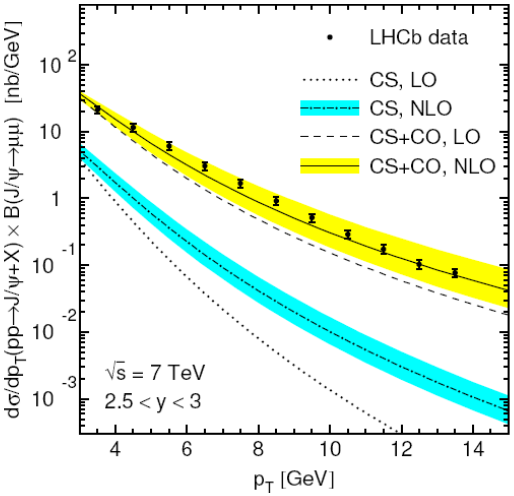
$\langle O[^3P_0^{[8]}]\rangle = (-9.08 \pm 1.61)\cdot 10^{-3}\ \mathrm{GeV}^5$

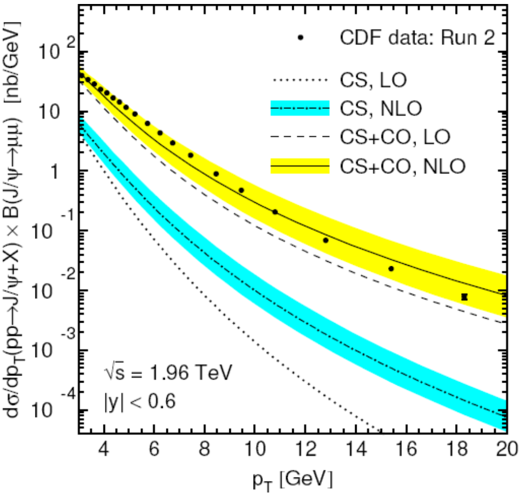
[MB, Kniehl: PRD 64, 051501R]

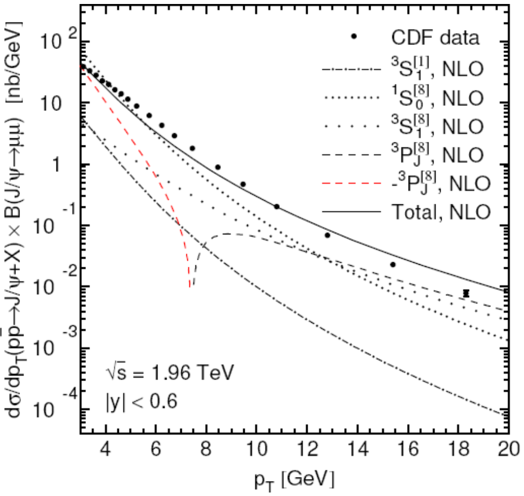
$\langle O[^1S_0^{[8]}]\rangle$ $= (4.97 \pm 0.44)\cdot 10^{-2}\ \mathrm{GeV}^3$ **$\langle O[^3S_1^{[8]}]\rangle$** $= (2.24 \pm 0.59)\cdot 10^{-3}\ \mathrm{GeV}^3$

$\langle O[^3P_0^{[8]}]\rangle$ $= (-1.61 \pm 0.20)\cdot 10^{-2}\ \mathrm{GeV}^5$

In Detail: Hadroproduction (LHC, Tevatron)



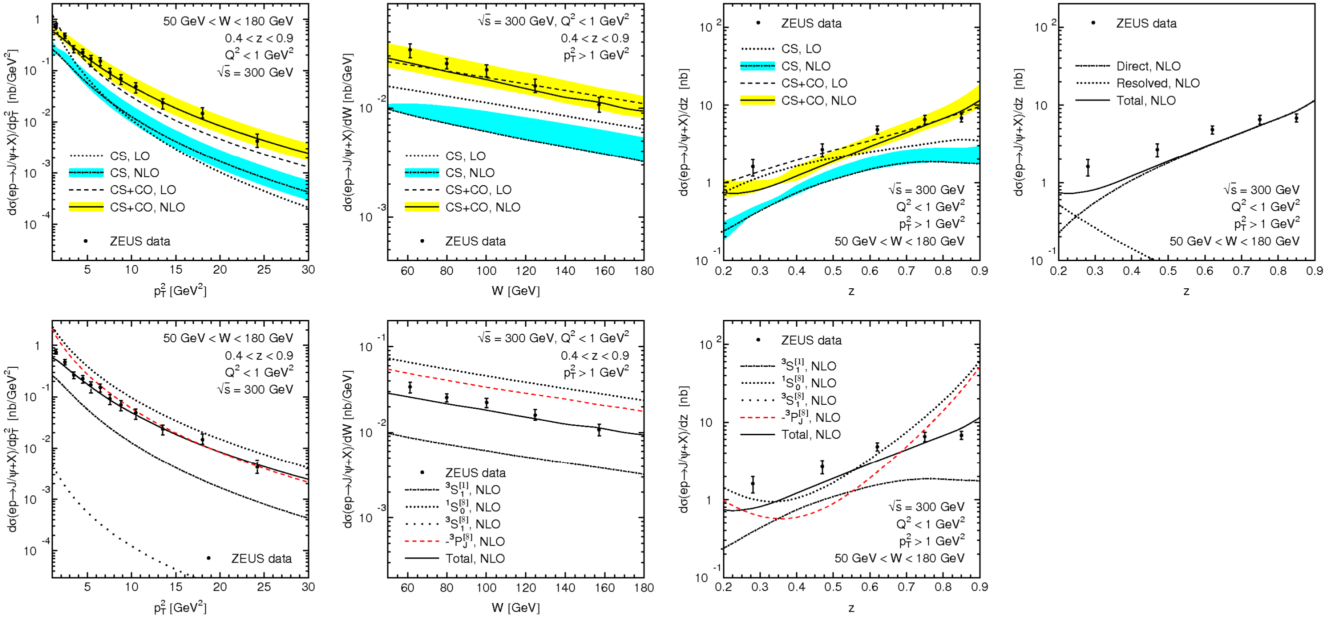




- Color singlet model **far below** data. **CS+CO** describes data **well**.
- $^3P_J^{[8]}$ short distance cross section **negative** at $p_T > 7$ GeV.
- But: Short distance cross sections and LDMEs **unphysical**

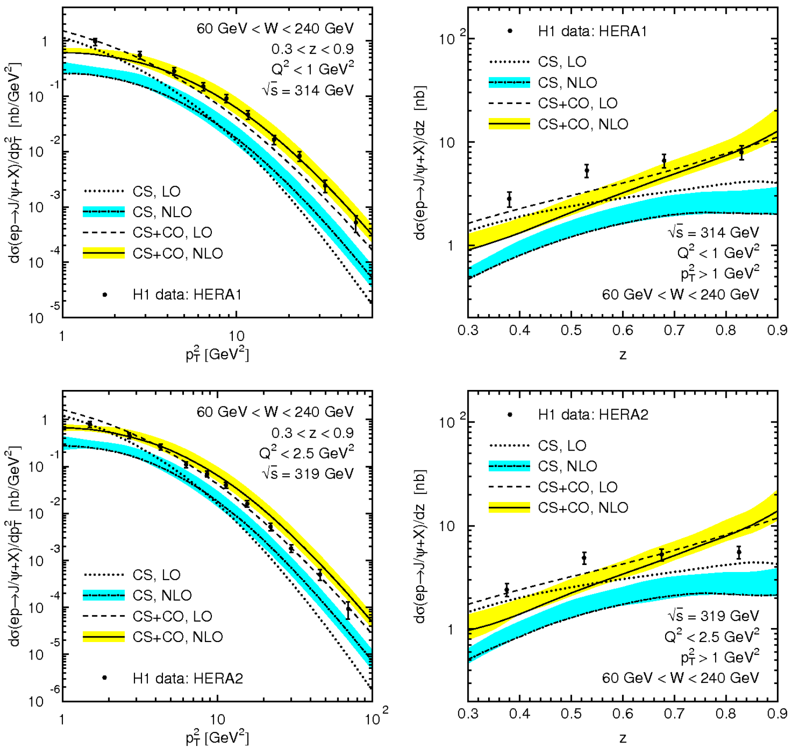
  ➡ No problem!
- Hadroproduction data below $p_T = 3$ GeV excluded from our fit.
- Observation: Change s or rapidity y just **rescaling** of cross sections: CO LDMEs describing RHIC or Tevatron must also describe LHC!

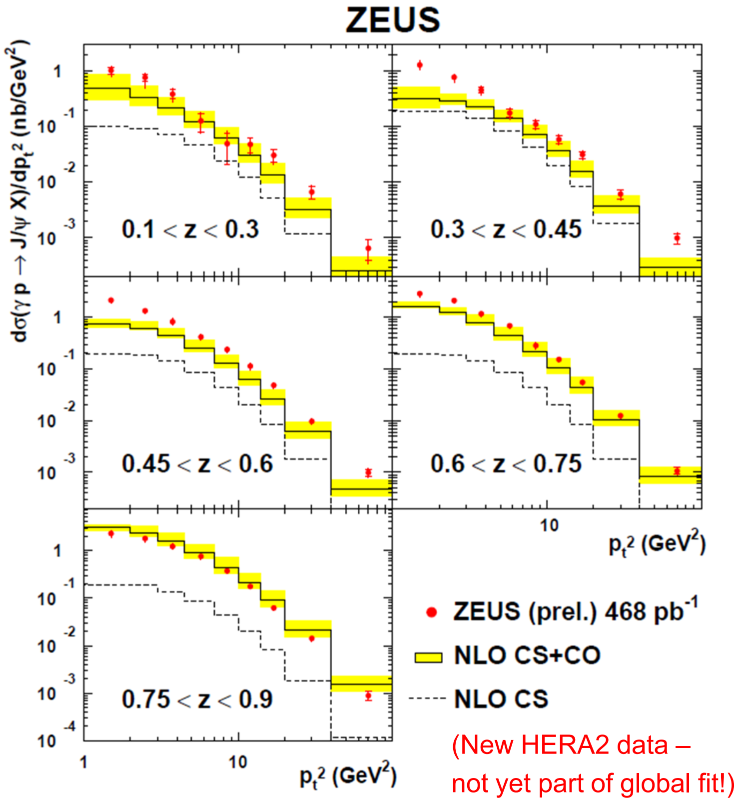
In Detail: Photoproduction at HERA



- **Distributions:** Transverse momentum (p_T), photon-proton c.m. energy (W), and z = Fraction of photon energy going to J/ψ.
- Again: Color singlet alone **below** the data, **CS+CO** describes data well.
- Calculation includes **resolved** photon contributions: Important at low z.
- **Good description at high z:** No increase like in older Born analyses!

In Detail: More Photoproduction



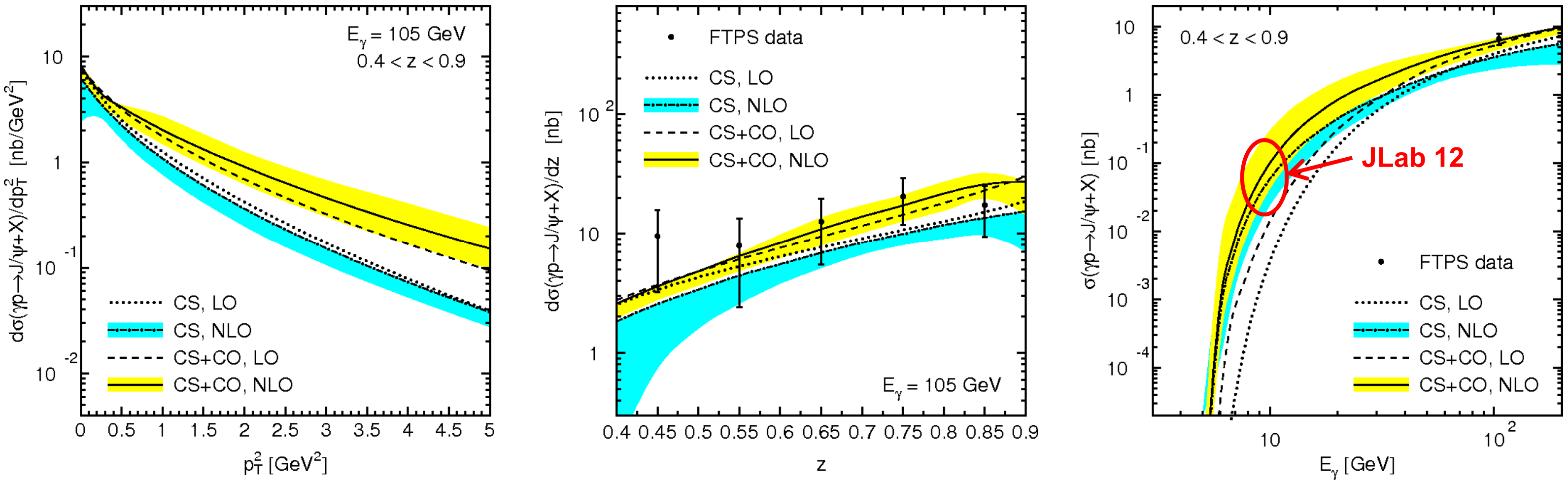


(New HERA2 data – not yet part of global fit!)

- Again: CS alone **below** data; **CS+CO** good description, especially at high *z*.
- H1 HERA2 data systematically below H1 HERA1 and ZEUS HERA1 + 2.

Low-energy inelastic J/ψ photoproduction

- **FTPS experiment** at Fermilab ('80s): 105 GeV photons on hydrogen target.
- Measured **inelastic** J/ψ production ($z = E_{J/\psi} / E_\gamma < 0.9$)
  NRQCD yields good description even at this low-energy range:

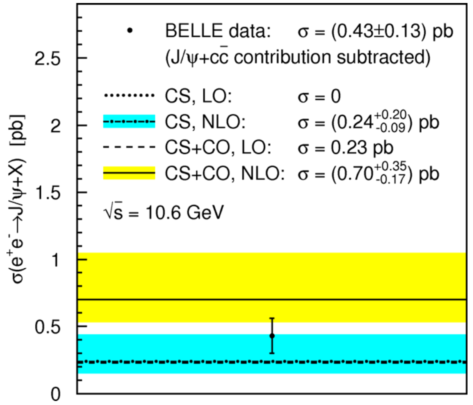


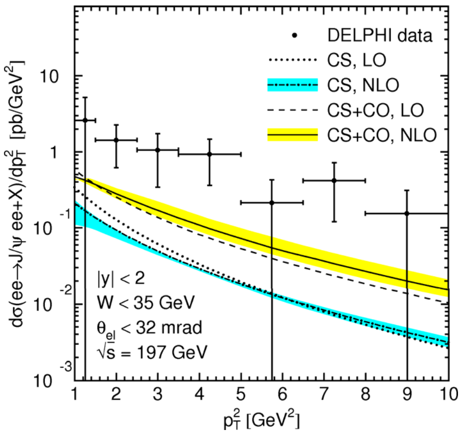
- Planned **JLab** near-threshold measurements: 12 GeV electrons on nuclei.
- **Total inelastic cross section**: ~ 10^{-2} nb. Measureable?
  (does of course increase with other nuclei than hydrogen)
- Close to threshold: Bad perturbative stability of parton model.

In Detail: e^+e^- and $\gamma\gamma$ Collisions

Electron-Positron Collisions at BELLE:

- **CS:** Large overlap with data, **CS+CO:** Small overlap.
- But: Only 4+ charged track events measured.
  ➡ Actual BELLE **data larger** by unknown factor.
- For e^+e^- **color singlet**, **NNLO** terms been calculated, increasing cross section. Not part of the global fit. [Ma, Zhang, Chao (2009); Gong, Wang (2009)]





Two Photon scattering at DELPHI (LEP):

- Includes direct, single and double resolved photons.
- CS below data, but also **CS+CO** curve **too low**. Possible explanations:
  - ☐ Uncertainties in the measurement (Just 16 events involved!)
  - ☐ Hint at problems with LDME universality.

Hadroproduction-only Fit

Global fit to hadroproduction data alone, vary low-p_T cut:

	$p_T > 1$ GeV	$p_T > 2$ GeV	$p_T > 3$ GeV	$p_T > 5$ GeV	$p_T > 7$ GeV
$\langle O[^1S_0^{[8]}]\rangle$ $[10^{-2}\ \mathrm{GeV}^3]$	8.54 ± 0.52	16.85 ± 1.23	11.02 ± 1.67	1.68 ± 2.20	2.18 ± 2.56
$\langle O[^3S_1^{[8]}]\rangle$ $[10^{-3}\ \mathrm{GeV}^3]$	-2.66 ± 0.69	-13.36 ± 1.60	-5.56 ± 2.19	8.75 ± 2.98	10.34 ± 3.55
$\langle O[^3P_0^{[8]}]\rangle$ $[10^{-2}\ \mathrm{GeV}^5]$	-3.63 ± 0.23	-7.70 ± 0.61	-4.46 ± 0.87	2.20 ± 1.23	3.50 ± 1.50
M_0 $[10^{-2}\ \mathrm{GeV}^3]$	**2.25 ± 0.12**	**3.51 ± 0.19**	**3.29 ± 0.20**	**5.50 ± 0.29**	**8.24 ± 0.58**
M_1 $[10^{-3}\ \mathrm{GeV}^3]$	**6.37 ± 0.19**	**5.80 ± 0.19**	**5.54 ± 0.20**	**3.27 ± 0.29**	**1.63 ± 0.43**

- Fit **underconstrained.** Therefore give two linear combinations of Ma *et al.*:

$$M_0 = \langle O(^1S_0^{[8]})\rangle + 3.9\,\langle O(^3P_0^{[8]})\rangle/m_c^2 \qquad M_1 = \langle O(^3S_1^{[8]})\rangle - 0.56\,\langle O(^3P_0^{[8]})\rangle/m_c^2$$

- Fit results **depend strongly** on low-p_T cut.

Agreement with Ma *et al.*'s fit to Tevatron run II data with $p_T > 7$ GeV:

Default: Include feed-downs, directly fit M_0 and M_1:	**$M_0 = (7.4 \pm 1.9)\ 10^{-2}\ \mathrm{GeV}^3$**	**$M_1 = (0.5 \pm 0.2)\ 10^{-3}\ \mathrm{GeV}^3$**
Ignore feed-downs, directly fit M_0 and M_1:	**$M_0 = (8.92 \pm 0.39)\ 10^{-2}\ \mathrm{GeV}^3$**	**$M_1 = (1.26 \pm 0.23)\ 10^{-3}\ \mathrm{GeV}^3$**
Ignore feed-downs, M_0 and M_1 from 3-parameter fit:	**$M_0 = (8.54 \pm 1.02)\ 10^{-2}\ \mathrm{GeV}^3$**	**$M_1 = (1.67 \pm 1.05)\ 10^{-3}\ \mathrm{GeV}^3$**
[Ma, Wang, Chao: Table 1 of PRL 106, 042002 and Equation (18) of PRD 84, 114001		

Hadroproduction-only Fit

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$\langle O[{}^3S_1^{[8]}]\rangle$ [10^{-3} GeV^3]	-2.66 ± 0.69	-13.36 ± 1.60	-5.56 ± 2.19	8.75 ± 2.98	10.34 ± 3.55
$\langle O[{}^3P_0^{[8]}]\rangle$ [10^{-2} GeV^5]	-3.63 ± 0.23	-7.70 ± 0.61	-4.46 ± 0.87	2.20 ± 1.23	3.50 ± 1.50
M_0 [10^{-2} GeV^3]	**2.25 ± 0.12**	**3.51 ± 0.19**	**3.29 ± 0.20**	**5.50 ± 0.29**	**8.24 ± 0.58**
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[Ma, Wang, Chao: Table 1 of PRL 106, 042002 and Equation (18) of PRD 84, 114001

Global Fit: Dependence on Low-p_T Cuts (1)

Global fit: Vary low-p_T cut on hadroproduction data:

hadroproduction data left	$p_T > 1$ GeV 148 points	$p_T > 2$ GeV 134 points	$p_T > 3$ GeV 119 points	$p_T > 5$ GeV 86 points	$p_T > 7$ GeV 60 points
$\langle O[{}^1S_0^{[8]}]\rangle$ [10^{-2} GeV^3]	5.68 ± 0.37	4.25 ± 0.43	4.97 ± 0.44	4.92 ± 0.49	3.91 ± 0.51
$\langle O[{}^3S_1^{[8]}]\rangle$ [10^{-3} GeV^3]	0.90 ± 0.50	2.94 ± 0.58	2.24 ± 0.59	2.23 ± 0.62	2.96 ± 0.64
$\langle O[{}^3P_0^{[8]}]\rangle$ [10^{-2} GeV^5]	-2.23 ± 0.17	-1.38 ± 0.20	-1.61 ± 0.20	-1.59 ± 0.22	-1.16 ± 0.23
M_0 [10^{-2} GeV^3]	**1.81 ± 0.09**	**1.85 ± 0.09**	**2.18 ± 0.10**	**2.17 ± 0.12**	**1.89 ± 0.12**
M_1 [10^{-3} GeV^3]	**6.46 ± 0.17**	**6.37 ± 0.17**	**6.25 ± 0.17**	**6.18 ± 0.17**	**5.86 ± 0.18**

↑

Our default fit

- **Stabilizing** influence of **photoproduction** data.
- Fit **constrained** enough: Can now extract 3 CO LDMEs.
- Fit results now **almost independent** of low-p_T cut.
- Fit less stable with low-p_T cut below 2 GeV (nonperturbative effects).

Global Fit: Dependence on Low-p_T Cuts (2)

Global fit: Vary low-p_T cut on photoproduction (including γγ-scattering):

photoproduction data left	$p_T > 1$ GeV 74 points	$p_T > 2$ GeV 30 points	$p_T > 3$ GeV 15 points	$p_T > 5$ GeV 5 points	$p_T > 7$ GeV 1 point
$\langle O[^1S_0^{[8]}]\rangle$ [10^{-2} GeV^3]	4.97 ± 0.44	5.10 ± 0.92	4.05 ± 1.17	5.44 ± 1.27	9.56 ± 1.59
$\langle O[^3S_1^{[8]}]\rangle$ [10^{-3} GeV^3]	2.24 ± 0.59	2.11 ± 1.22	3.52 ± 1.56	1.73 ± 1.68	-3.66 ± 2.09
$\langle O[^3P_0^{[8]}]\rangle$ [10^{-2} GeV^5]	-1.61 ± 0.20	-1.58 ± 0.48	-0.97 ± 0.63	-1.63 ± 0.68	-3.73 ± 0.83
M_0 [10^{-2} GeV^3]	**2.18 ± 0.10**	**2.36 ± 0.12**	**2.37 ± 0.13**	**2.62 ± 0.15**	**3.10 ± 0.19**
M_1 [10^{-3} GeV^3]	**6.25 ± 0.17**	**6.05 ± 0.18**	**5.94 ± 0.19**	**5.78 ± 0.20**	**5.62 ± 0.20**

↑

Our default fit

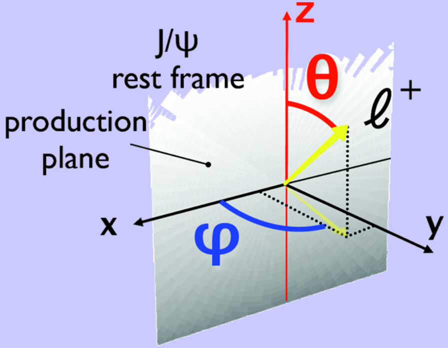
- **Fit stable** against varying low-p_T cut in region 1 GeV ~ 3 GeV.
- Just 5 or 1 photoproduction against 119 hadroproduction points not enough to stabilize the fit. ➡ **Not stable** with low-p_T cut much larger than 3 GeV. (Would need more high-p_T photoproduction data.)

J/ψ Polarization

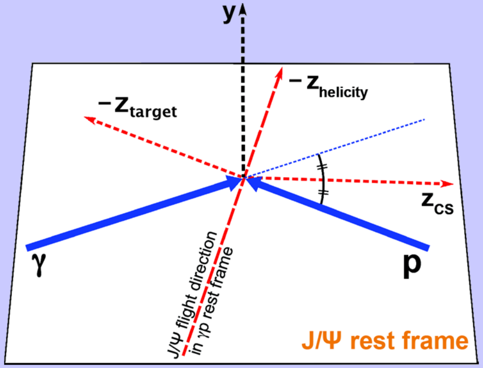
- **Angular distribution** of decay lepton l^+ in *J/ψ* rest frame

  ➡ Polarization observables ***λ***, ***μ***, ***ν*:**

$$\frac{d\Gamma(J/\psi \to l^+l^-)}{d\cos\theta\, d\phi} \propto 1 + \lambda\cos^2\theta + \mu\sin(2\theta)\cos\phi + \frac{\nu}{2}\sin^2\theta\cos(2\phi)$$



- Depends on choice of **coordinate system:**
  - ☐ Helicity frame: z axis $\parallel -(\vec{p}_\gamma + \vec{p}_p)$
  - ☐ Collins-Soper frame: z axis $\parallel \vec{p}_\gamma/|\vec{p}_\gamma| - \vec{p}_p/|\vec{p}_p|$
  - ☐ Target frame: z axis $\parallel -\vec{p}_p$

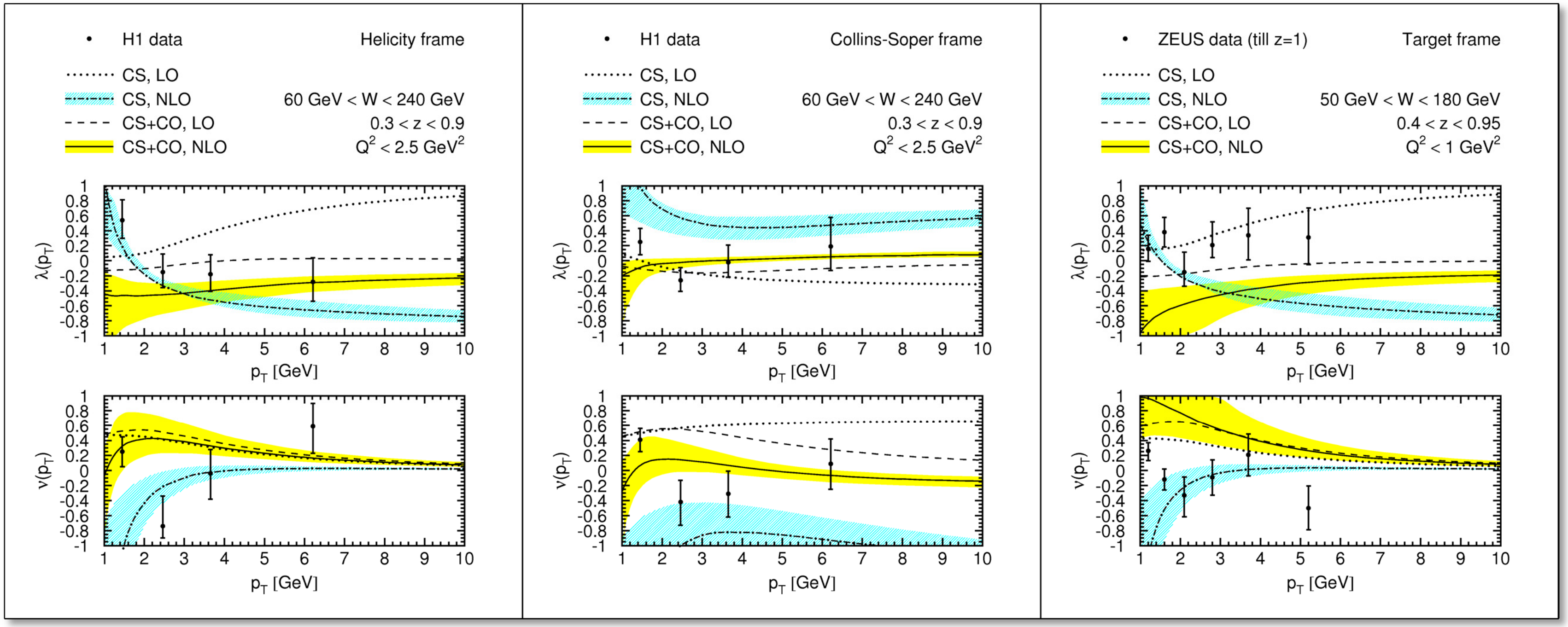


- **In Calculation:** Plug in explicit expressions for $c\bar{c}[n]$ spin polarization vectors according to

$$\lambda = \frac{d\sigma_{11} - d\sigma_{00}}{d\sigma_{11} + d\sigma_{00}}, \quad \mu = \frac{\sqrt{2}\mathrm{Re}\, d\sigma_{10}}{d\sigma_{11} + d\sigma_{00}}, \quad \nu = \frac{2d\sigma_{1,-1}}{d\sigma_{11} + d\sigma_{00}}$$

- We use the CO LDME set with feed-down contributions subtracted.

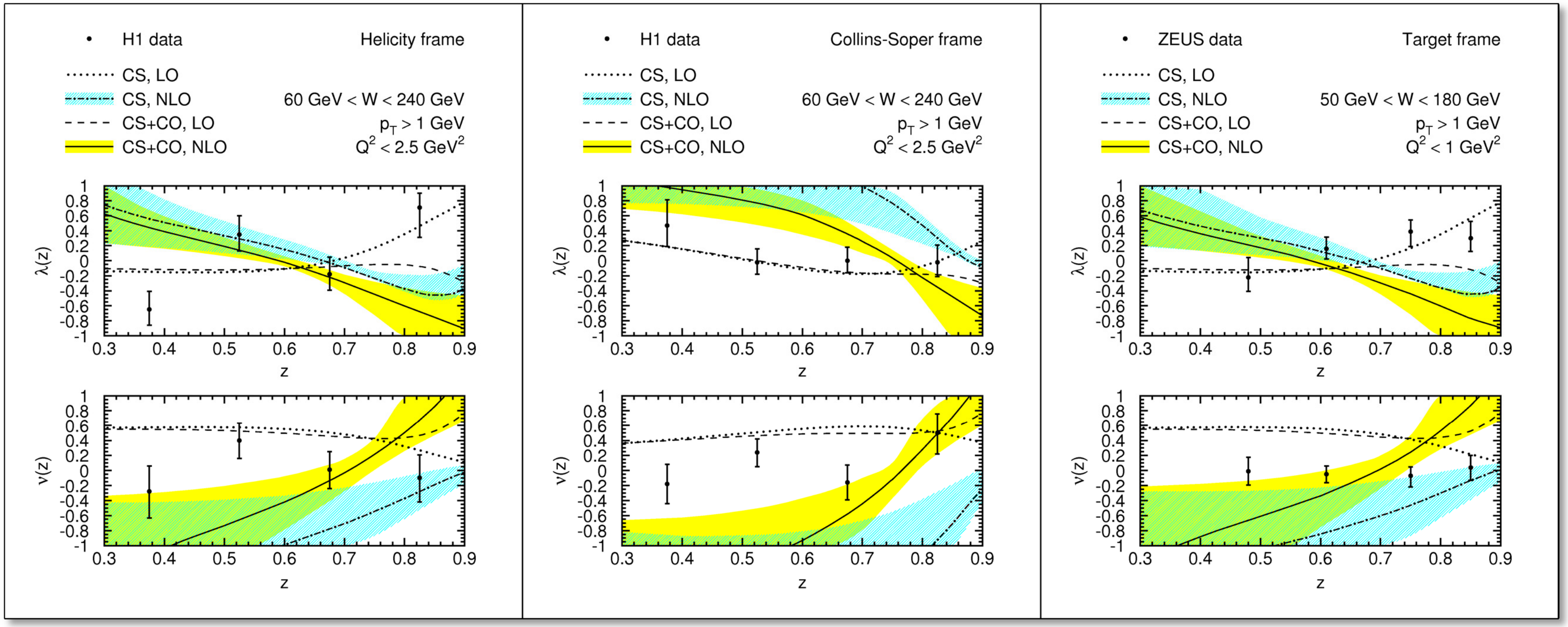
J/ψ Polarization in Photoproduction: p_T Distribution



[MB, Kniehl: PRL 107, 232001]

- Bands: Uncertainties due to scale variation and CO LDMEs.
- **CSM** predicts **longitudinal** *J/ψ* at high p_T.
- **CS+CO:** largely **unpolarized** *J/ψ* at high p_T. α_s expansion converges better.
- H1 and ZEUS **data not precise** enough to discriminate CSM / NRQCD.

J/ψ Polarization in Photoproduction: z Distribution

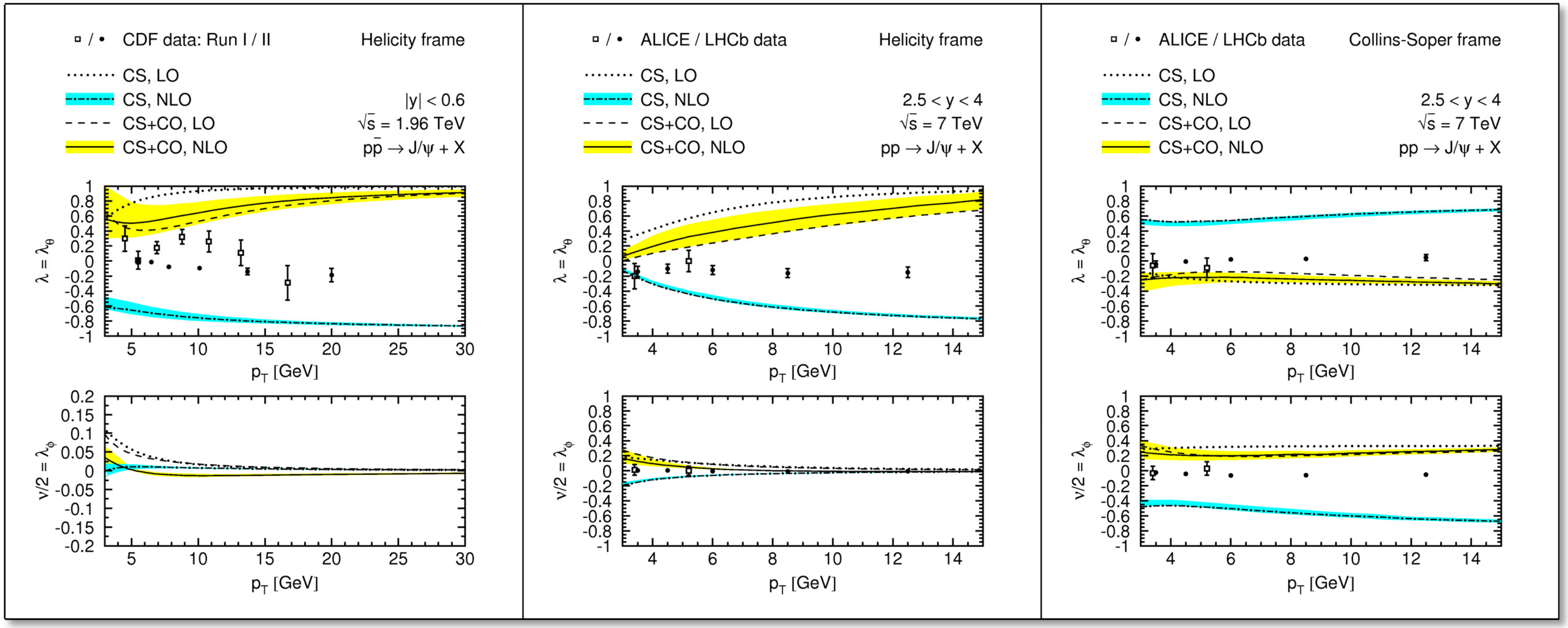


[MB, Kniehl: PRL 107, 232001]

- Bands: Uncertainties due to scale variation and CO LDMEs.
- **Scale** uncertainties very large.
- **Error bands** of CSM and NRQCD largely **overlap**.

➡ p_T distribution better suited to discriminate production mechanisms than z.

J/ψ Polarization in Hadroproduction



[MB, Kniehl: PRL 108, 172002]

- **Helicity frame**: NRQCD predicts strong **transverse** polarization at high p_T.
- **Collins-Soper frame**: NRQCD predicts slightly longitudinal *J/ψ*.
- **Disagreement** with CDF Run II data, and with new ALICE and LHCb data.
  ➡ **Challenge to LDME universality!**

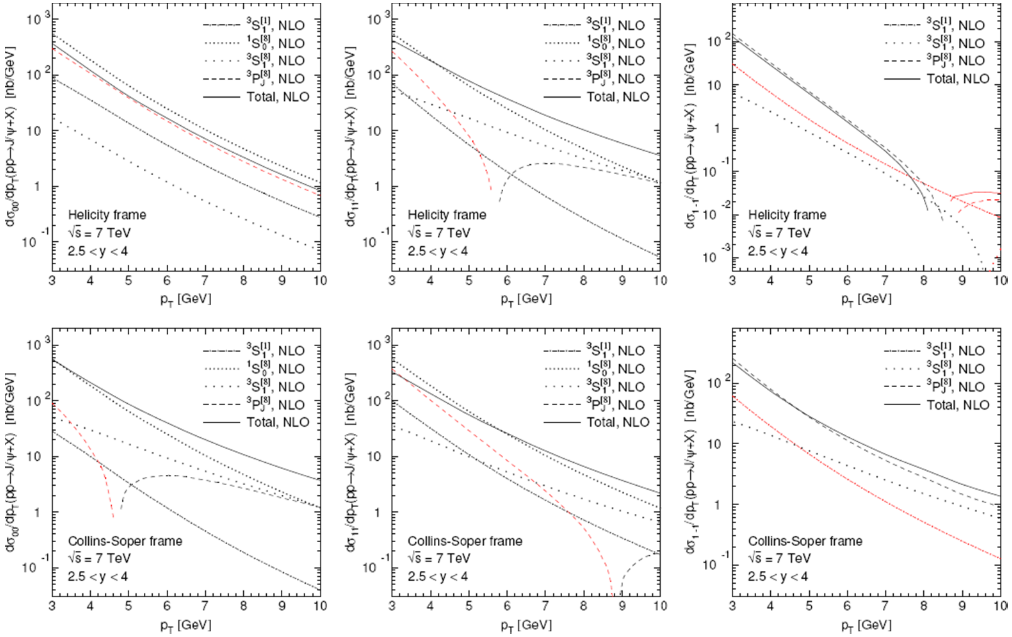
Polarization in Hadroproduction: Contributions

- **First:** Sum up contributions of intermediate states:

$d\sigma_{00}$ **$d\sigma_{11}$** **$d\sigma_{1\,-1}$**

Helicity frame:

Collins-Soper frame:



- **Then:** $\lambda_\theta = \dfrac{d\sigma_{11} - d\sigma_{00}}{d\sigma_{11} + d\sigma_{00}}, \quad \lambda_{\theta\phi} = \dfrac{\sqrt{2}\mathrm{Re}\, d\sigma_{10}}{d\sigma_{11} + d\sigma_{00}}, \quad \lambda_\phi = \dfrac{d\sigma_{1,-1}}{d\sigma_{11} + d\sigma_{00}}$