Theory perspective on $B_{d,s} \to \bar{\ell}\ell$ and $B \to K^{(*)}\bar{\ell}\ell$

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St. Petersburg

Outline

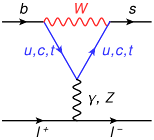
- **Motivation / Introduction**
- **Predictions / Uncertainties**

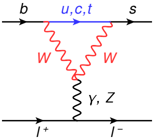
  $B_{d,s} \to \bar{\ell}\ell$

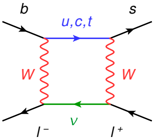
  $B \to K^{(*)}\bar{\ell}\ell$

Motivation to study $b \to q\bar{\ell}\ell$ ($q = d, s$ and $\ell = e, \mu, \tau$)

- test of the SM CKM-picture at loop-level (FCNC decay) $\Rightarrow$ loop-suppressed







- angular distributions $\Rightarrow$ plenty of CP-averaged and CP-asymmetric observables

  $B \to K^*(\to K\pi)\bar{\ell}\ell$: (12 + 12), $B \to K\bar{\ell}\ell$: (3 + 3), $B_s \to \bar{\mu}\mu$: 2

  also $B_s \to \phi(\to KK)\bar{\ell}\ell$ and $b \to d\bar{\ell}\ell$ decays

- constrain semi-leptonic new physics couplings $\propto [\bar{q}\,\Gamma_{qb}\,b][\bar{\ell}\,\Gamma_{\ell\ell}\,\ell]$
  - $\Gamma_{qb} \otimes \Gamma_{\ell\ell} = \gamma_\mu(\gamma_5) \otimes \gamma^\mu(\gamma_5)$ right-handed currents
  - $\Gamma_{qb} \otimes \Gamma_{\ell\ell} = 1(\gamma_5) \otimes 1(\gamma_5)$ (pseudo-) scalar currents
  - $\Gamma_{qb} \otimes \Gamma_{\ell\ell} = \sigma_{\mu\nu} \otimes \sigma^{\mu\nu}(\gamma_5)$ tensor currents

- test of non-standard CP violation in $b \to s$ $\Rightarrow$ since SM contribution $\propto V_{ub}$: $A_{CP}^{\rm SM} \lesssim 0.1\%$

- test of lepton flavour universality (LFU) among $\ell = e, \mu, \tau$

- excellent experimental prospects (for $\ell = e, \mu$) @ LHCb and Belle II, some cross checks from CMS & ATLAS possible

Global fits and "anomalies"

[LHCb 3/fb LHCb-CONF-015-002]

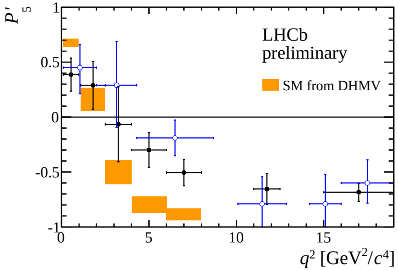
Global fits of $b \to s + (\gamma, \bar{\ell}\ell)$ prefer non-SM values

of $C_9 = C_9^{SM} + C_9^{NP}$ with $C_9^{SM} \approx +4$ and $C_9^{NP} \approx -1$

driven in part by

- angular obs's in $B \to K^* \bar{\ell}\ell$ at low q^2
- measured branching ratios $B \to K\bar{\mu}\mu$ and $B_s \to \phi\bar{\mu}\mu$ smaller than SM at low q^2

[Descotes-Genon et al. 1307.5683, Altmannshofer/Straub 1308.1501 + 1411.3161, Beaujean/CB/van Dyk 1310.2478, Mahmoudi et al. 1312.5267 + 1410.4545]



largest deviation from SM of 3.7σ in $P'_5(B \to K^* \bar{\ell}\ell)$ in $q^2 \in [4,6]\&[6,8]$ GeV^2

[DHMV = Descotes-Genon et al. 1407.8526]

$\Rightarrow$ Q: underestimated hadronic effects mimic new physics (NP) in C_9 ???

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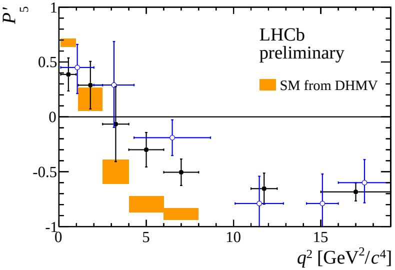

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Non-LFU in $B \to K \bar{\ell}\ell$?

$$R_M^{[q^2_{\rm min},\, q^2_{\rm max}]} \equiv \frac{\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \, \frac{d\Gamma[B \to M \bar{\mu}\mu]}{dq^2}}{\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \, \frac{d\Gamma[B \to M \bar{e}e]}{dq^2}}$$

for $M = K,\ K^*,\ X_s$

$\Rightarrow$ hadronic uncertainties cancel for LFU

Recent measurement of

$R_K^{[1,6]} = 0.745^{+0.090}_{-0.074} \pm 0.036$ [LHCb 3/fb 1406.6482]

deviates by 2.6σ from SM (@ LO in QED)

$R_{K,\rm SM}^{[1,6]} = 1.0008 \pm 0.0004$ [Bouchard et al. 1306.0434]

[Krüger/Hiller hep-ph/0310219, CB/Hiller/Piranishvili 0709.4174]

B-Hadron decays are a Multi-scale problem . . .

. . . with hierarchical interaction scales

electroweak IA	$\gg$	ext. mom'a in B restframe	$\gg$	QCD-bound state effects
$m_W \approx 80$ GeV $m_Z \approx 91$ GeV		$m_B \approx 5$ GeV		$\Lambda_{\mathrm{QCD}} \approx 0.5$ GeV

B-Hadron decays are a Multi-scale problem . . .

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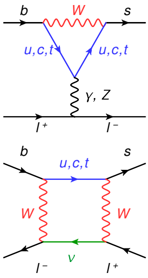
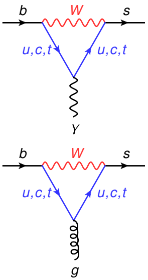
electroweak IA	$\gg$	ext. mom'a in *B* restframe	$\Rightarrow$	decoupling heavy particles
$m_W \approx 80$ GeV $m_Z \approx 91$ GeV		$m_B \approx 5$ GeV		*W*, *Z*-boson, top-quark

$$\mathcal{L}_{\rm eff} \sim G_F\, V_{\rm CKM} \times \Big[\sum_{9,10} C_i\, \mathcal{O}_i + \sum_{7\gamma,\, 8g} C_i\, \mathcal{O}_i + \text{CC} + (\text{QCD \& QED-peng}) \Big]$$

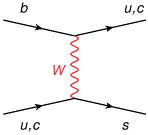
semi-leptonic



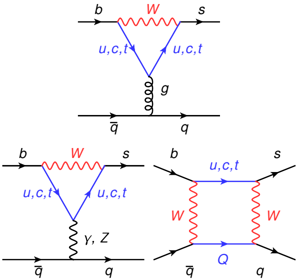
electro- & chromo-mgn



charged current



QCD & QED -penguin



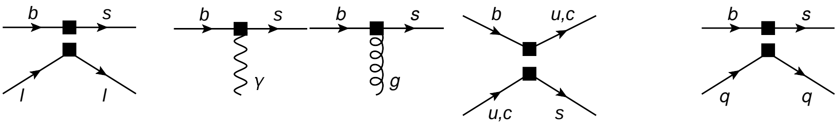
B-Hadron decays are a Multi-scale problem ...

... with hierarchical interaction scales

electroweak IA	$\gg$	ext. mom'a in B restframe	$\Rightarrow$	effective theory
$m_W \approx 80$ GeV $m_Z \approx 91$ GeV		$m_B \approx 5$ GeV		at scales below m_B

$$\mathcal{L}_{\rm eff} \sim G_F\, V_{\rm CKM} \times \left[\sum_{9,10} C_i\, \mathcal{O}_i + \sum_{7\gamma, 8g} C_i\, \mathcal{O}_i + \text{CC} + (\text{QCD \& QED-peng})\right]$$

semi-leptonic | electro- & chromo-mgn | charged current | QCD & QED -penguin



C_i = Wilson coefficients: contains short-dist. pmr's (heavy masses $M_t, \ldots$ – CKM factored out) and leading logarithmic QCD-corrections to all orders in α_s

$\Rightarrow$ in SM known up to NNLO QCD and NLO EW/QED

$\mathcal{O}_i$ = higher-dim. operators: flavour-changing coupling of light quarks

$B_{d,s} \to \bar{\ell}\ell$

experimental prospects for $\overline{\mathcal{B}}(B_s \to \bar{\mu}\mu)$

@ LHCb after Run 2 : $\sim 0.50 \times 10^{-9} \simeq$ 18% error of current measurement (only stat. err)
with 50 fb^{-1} : $\sim 0.15 \times 10^{-9} \simeq$ 5% [LHCb arXiv:1208.3355]

@ CMS after Run 2 (100 fb^{-1}) : 15% relative error
with 300 fb^{-1} : 12% relative error [CMS-PAS-FTR-13-022]

Hadronic matrix elements for $B_q \to \bar{\ell}\ell$

Semileptonic op's

$$\mathcal{O}_{10(10')} = [\bar{s}\,\gamma^\mu P_{L(R)}\, b][\bar{\ell}\,\gamma_\mu\gamma_5\,\ell]$$

Factorisation into well-defined hadronic objects (@ LO QED)

$\Rightarrow$ No conceptual problems !!!

Hadronic amplitude $B_q \to \bar{\ell}\ell$

$$\mathcal{A} \propto C_{10}\,\langle\bar{\ell}\ell|[\bar{s}\,\gamma^\mu P_L\, b][\bar{\ell}\,\gamma_\mu\gamma_5\,\ell]|B(p)\rangle$$

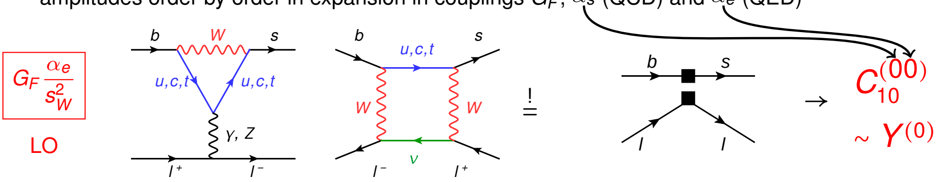
$$\overset{\text{LO QED}}{\propto} C_{10}\,\underbrace{\langle\bar{\ell}\ell|\bar{\ell}\,\gamma_\mu\gamma_5\,\ell|0\rangle}_{L_\mu} \times \underbrace{\langle 0|\bar{s}\,\gamma^\mu\gamma_5\, b]|B(p)\rangle}_{i\, f_B\, p^\mu}$$

$$\propto C_{10} \quad L_\mu \quad \times \quad i\, f_B\, p^\mu$$

- only $\mathcal{O}_{10}$ contributes, since $p^\mu(\bar{\ell}\,\gamma_\mu\,\ell) = 0$ for on-shell leptons
- other $\mathcal{O}_i$ can contribute only at NLO in QED
- $\mathcal{O}_{10}$ is a conserved current under QCD ($\to$ no "Running")
- (pseudo-) scalar $\mathcal{O}_{S(P)} \propto [\bar{s}\gamma_5 b][\bar{\ell}1(\gamma_5)\ell]$ from Higgs-penguins are down by $m_{B_q}^2/m_W^2$
- hadronic uncertainty from B-meson decay constant f_B
  $\Rightarrow$ from lattice $\lesssim 2\%$ accuracy [FLAG 1310.8555]
- branching ratio @ LO QED: $\overline{\mathcal{B}} \propto |C_{10}|^2 \times (f_{B_q})^2$
  $\Rightarrow \lesssim 4\%$ hadronic uncertainty

C_{10} at LO EW

⇒ C_i are determined by requiring equality of full theory (=SM) and effective theory (=EFT) amplitudes order by order in expansion in couplings G_F, α_s (QCD) and α_e (QED)

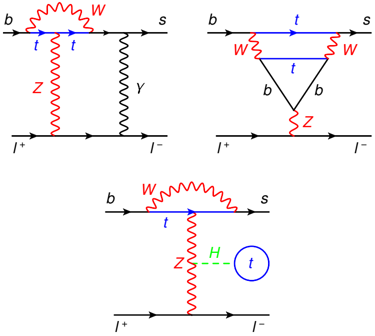


[Inami/Lim Prog.Theor.Phys. 65 (1981) 297]

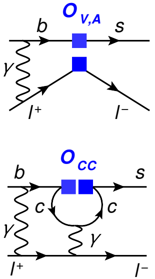
Adding NLO EW corrections . . .

$G_F \frac{\alpha_e}{s_W^2} \alpha_e$

NLO EW



$\stackrel{!}{=}$



$\rightarrow \quad C_{10}^{(01)}$

[CB/Gorbahn/Stamou arXiv:1311.1348]

⇒ Technical complications arise:

1) electroweak parameters have to be renormalised

→ choose a set of input $\{G_F, \alpha_e, m_Z, m_t, m_H\}$

Only product $G_F \frac{\alpha_e}{s_W^2} C_{10}\left(\frac{m_t}{m_W}\right)$ invariant under change of EW renormalization scheme

!!! m_W and $s_W \equiv \sin(\theta_W)$ are dependant (1-loop relations must be used)

2) $\mathcal{O}_{10}$ not conserved current under QED = mixing occurs

→ need to resum large $(\alpha_e/\alpha_s)^n \log^n(\mu_b/\mu_0)$ with RGE $\qquad \mu_b \sim m_b,\ \mu_0 \sim m_W$

Impact of NLO EW at μ_0 . . . neglecting operator mixing

- α_e always $\overline{\text{MS}}$ and m_t always $\overline{\text{MS}}$ w.r.t QCD
- 3 different EW renormalization schemes:
  - OS) masses and ($s_W^2 = 1 - m_W^2/m_Z^2$) on-shell renormalized
  - $\overline{\text{MS}}$) masses and s_W^2 in minimal subtraction
  - HY) masses on-shell but s_W^2 in minimal subtraction
- 2 normalizations of $\mathcal{L}_{\text{eff}}$ [Misiak arXiv:1112.5978]

1) $\frac{4G_F}{\sqrt{2}}\frac{\alpha_e}{s_W^2}\left[C_{10}^{(00)} + \frac{\alpha_e(\mu_0)}{4\pi}C_{10}^{(01)}(\mu_0)\right]$ 2) $\frac{G_F^2 m_W^2}{\pi^2}\left[\widetilde{C}_{10}^{(00)} + \frac{\alpha_e(\mu_0)}{4\pi}\widetilde{C}_{10}^{(01)}(\mu_0)\right]$

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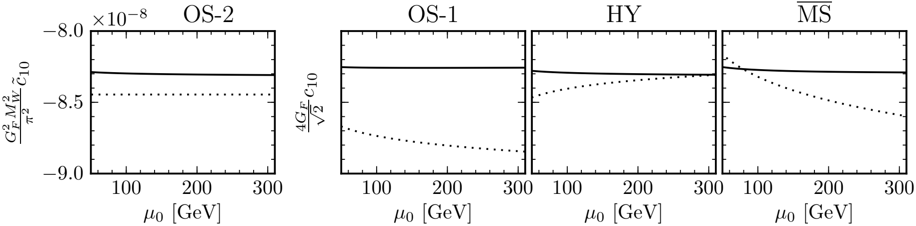

scheme dependence @ LO: $[-8.9\ , -8.2\ ] \to \pm 8\%$ at $\overline{\mathcal{B}}$

reduces @ NLO: $[-8.31, -8.25] \to \pm 0.8\%$ at $\overline{\mathcal{B}}$

[CB/Gorbahn/Stamou arXiv:1311.1348]

LO = dotted

NLO = solid

Branching ratios in the SM

(in OS-2 renorm. scheme)

- NNLO QCD crrs. reduce μ_0-dep. from 1.8 % at NLO → 0.2 % at NNLO at $\overline{\mathcal{B}}$

[Hermann/Misiak/Steinhauser arXiv:1311.1347]

SM predictions @ NNLO QCD & NLO EW

$$\overline{\mathcal{B}}(B_s \to \bar{\mu}\mu)_{\rm SM} = (3.65 \pm 0.23) \times 10^{-9}$$

$$\overline{\mathcal{B}}(B_d \to \bar{\mu}\mu)_{\rm SM} = (1.06 \pm 0.09) \times 10^{-10}$$

[CB/Gorbahn/Hermann/Misiak/Stamou/Steinhauser 1311.0903]

Measurement

$$\overline{\mathcal{B}}(B_s \to \bar{\mu}\mu)_{\rm Exp} = (2.8^{+0.7}_{-0.6}) \times 10^{-9} \qquad (6.2\sigma)$$

$$\overline{\mathcal{B}}(B_d \to \bar{\mu}\mu)_{\rm Exp} = (3.9^{+1.6}_{-1.4}) \times 10^{-10} \qquad (3.2\sigma)$$

[LHCb + CMS 1411.4413]

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[LHCb + CMS 1411.4413]

Error budget	f_{B_q}	CKM	τ_H^q	m_t	α_s	other param.	non-param.	$\sum$
$\overline{\mathcal{B}}(B_s \to \bar{\mu}\mu)$	4.0%	4.3%	1.3%	1.6%	0.1%	< 0.1%	1.5%	6.4%
$\overline{\mathcal{B}}(B_d \to \bar{\mu}\mu)$	4.5%	6.9%	0.5%	1.6%	0.1%	< 0.1%	1.5%	8.5%

!!! used $|V_{cb}|_{\rm incl} = (42.4 \pm 0.9) \cdot 10^{-3}$ $\Rightarrow$ rescale $\overline{\mathcal{B}} \propto \left(|V_{cb}|_{\rm your\ favorite} / |V_{cb}|_{\rm incl}\right)^2$

comparable to indirect determinations (CKM-fits)

UTfit: $|V_{cb}|_{\rm UTfit} = (42.05 \pm 0.65) \cdot 10^{-3}$

CKMfitter: $|V_{cb}|_{\rm CKMfitter} = (41.4^{+2.4}_{-1.4}) \cdot 10^{-3}$

Non-parametric uncertainties:

- 0.3% from $\mathcal{O}(\alpha_e)$ corrections from $\mu_b \in [m_b/2,\, 2m_b]$
- $2 \times 0.2\%$ from $\mathcal{O}(\alpha_s^3, \alpha_e^2, \alpha_s\alpha_e)$ matching corrections from $\mu_0 \in [m_t/2,\, 2m_t]$
- 0.3% from top-mass conversion from on-shell to $\overline{\rm MS}$ scheme
- 0.5% further uncertainties (power corrections $\mathcal{O}(m_b^2/m_W^2), \ldots$)

$\boldsymbol{B \to K^{(*)} \bar{\ell}\ell}$

Hadronic matrix elements for $B \to K^{(*)}\bar{\ell}\ell$ – Part 1

Radiative & Semileptonic op's

$$\mathcal{O}_{7\gamma(7\gamma')} = m_b[\bar{s}\,\sigma^{\mu\nu} P_{R(L)}\, b]F_{\mu\nu}$$

$$\mathcal{O}_{9(9')} = [\bar{s}\,\gamma^{\mu} P_{L(R)}\, b][\bar{\ell}\,\gamma_{\mu}\,\ell]$$

$$\mathcal{O}_{10(10')} = [\bar{s}\,\gamma^{\mu} P_{L(R)}\, b][\bar{\ell}\,\gamma_{\mu}\gamma_5\,\ell]$$

Factorisation into well-defined hadronic objects (@ LO QED)
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Hadronic amplitude $B \to K^{(*)}\bar{\ell}\ell$ (@ LO in QED)

$$\mathcal{A}_7 \propto C_7 L_\mu \frac{q_\nu}{q^2}\langle K_\lambda^{(*)}|[\bar{s}\,\sigma^{\mu\nu} P_R\, b]|B(p)\rangle \qquad \propto C_7\, T_\lambda(q^2)$$

$$\mathcal{A}_9 \propto C_9 L_\mu \langle K_\lambda^{(*)}|[\bar{s}\,\gamma^{\mu} P_L\, b]|B(p)\rangle \qquad \propto C_9\, V_\lambda(q^2)$$

- $q = p_B - p_K$ dilepton invariant mass
- $\lambda = K^{(*)}$ polarization
- V_λ and T_λ: $B \to K^{(*)}$ vector and tensor form factors (FF)

@ low q^2: FF's from LCSR	$B \to K$	[Ball/Zwicky hep-ph/0406232, Khodjamirian et al. 1006.4945]
$(10 - 15)\%$ accuracy	$B \to K^*$	[Khodjamirian et al. 1006.4945, Bharucha/Straub/Zwicky 1503.05534]
@ high q^2: FF's from lattice	$B \to K$	[Bouchard et al. 1306.2384]
$(6 - 9)\%$ accuracy	$B \to K^*$	[Horgan/Liu/Meinel/Wingate 1310.3722, 1501.00367]

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$$\mathcal{A}_9 \propto C_9 L_\mu \langle K_\lambda^{(*)}|[\bar{s}\,\gamma^{\mu} P_L\, b]|B(p)\rangle \qquad \propto C_9\, V_\lambda(q^2)$$

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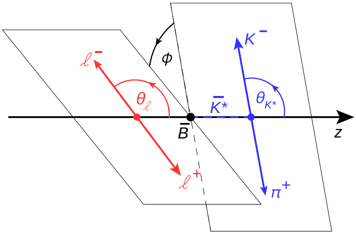
FF relations at low & high q^2

- allow to relate FF's $\Rightarrow$ reduce their number
- valid up to corrections of $\Lambda_{\mathrm{QCD}}/m_b \simeq 0.5/4 \approx 13\%$

$\Rightarrow$ "optimized observables" in $B \to K^*\bar{\ell}\ell$

Angular observables $J_i(q^2)$ in $B \to K^* [\to K\pi] + \bar{\ell}\ell$

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} \simeq J_{1s} \sin^2\theta_K + J_{1c} \cos^2\theta_K$$
$$+ (J_{2s} \sin^2\theta_K + J_{2c} \cos^2\theta_K) \cos 2\theta_\ell + J_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi$$
$$+ J_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi + J_5 \sin 2\theta_K \sin\theta_\ell \cos\phi$$
$$+ (J_{6s} \sin^2\theta_K + J_{6c} \cos^2\theta_K) \cos\theta_\ell + J_7 \sin 2\theta_K \sin\theta_\ell \sin\phi$$
$$+ J_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi + J_9 \sin^2\theta_K \sin^2\theta_\ell \sin 2\phi$$



Angular observables $J_i(q^2)$ in $B \to K^* [\to K\pi] + \bar{\ell}\ell$

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} \simeq J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K$$
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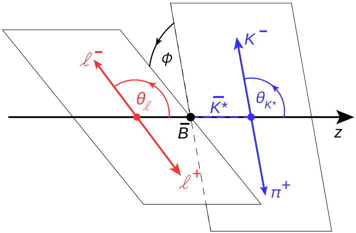

"Optimized observables" $\Rightarrow$ reduced FF sensitivity

- guided by large energy limit @ low-q^2 and Isgur-Wise @ high-q^2 FF-relations
- FF's cancel up to corrections $\sim \Lambda_{\rm QCD}/m_b$

@ low q^2

[Krüger/Matias hep-ph/0502060, Egede/Hurth/Matias/Ramon/Reece arXiv:0807.2589 + 1005.0571]
[Becirevic/Schneider arXiv:1106.3283]
[Matias/Mescia/Ramon/Virto arXiv:1202.4266]
[Descotes-Genon/Matias/Ramon/Virto arXiv:1207.2753]

$$A_T^{(2)} \equiv P_1 \equiv \frac{J_3}{2\,J_{2s}} \qquad A_T^{(\rm re)} \equiv 2\,P_2 \equiv \frac{J_{6s}}{4\,J_{2s}} \qquad A_T^{(\rm im)} \equiv -2\,P_3 \equiv \frac{J_9}{2\,J_{2s}}$$

$$P_4' \equiv \frac{J_4}{\sqrt{-J_{2c}J_{2s}}} \qquad P_5' \equiv \frac{J_5/2}{\sqrt{-J_{2c}J_{2s}}} \qquad P_6' \equiv \frac{-J_7/2}{\sqrt{-J_{2c}J_{2s}}} \qquad P_8' \equiv \frac{-J_8}{\sqrt{-J_{2c}J_{2s}}}$$

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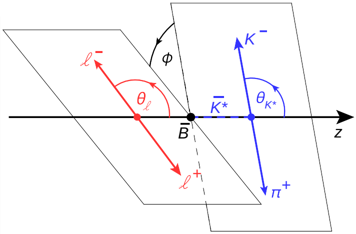

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- FF's cancel up to corrections $\sim \Lambda_{\mathrm{QCD}}/m_b$

@ high q^2

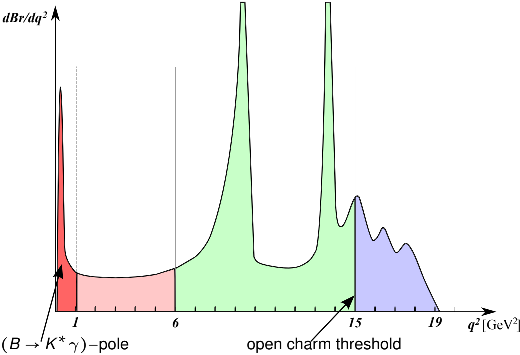
[CB/Hiller/van Dyk arXiv:1006.5013]
[Matias/Mescia/Ramon/Virto arXiv:1202.4266]
[CB/Hiller/van Dyk arXiv:1212.2321]

$$H_T^{(1)} \equiv P_4 \equiv \frac{\sqrt{2}J_4}{\sqrt{-J_{2c}(2J_{2s} - J_3)}}$$

$$H_T^{(2)} \equiv P_5 \equiv \frac{J_5/\sqrt{2}}{\sqrt{-J_{2c}(2J_{2s} + J_3)}} \qquad H_T^{(3)} \equiv \frac{J_{6s}/2}{\sqrt{(2J_{2s})^2 - (J_3)^2}}$$

$$H_T^{(4)} \equiv Q \equiv \frac{\sqrt{2}J_8}{\sqrt{-J_{2c}(2J_{2s} + J_3)}} \qquad H_T^{(5)} \equiv \frac{-J_9}{\sqrt{(2J_{2s})^2 - (J_3)^2}}$$

Hadronic matrix elements for $B \to K^{(*)}\bar{\ell}\ell$ – Part 2



Nonleptonic = "the bad guys"

$$\mathcal{O}_{(1)2} = [\bar{s}\gamma^{\mu} P_L (T^a)\, c][\bar{c}\gamma_{\mu} P_L (T^a)\, b]$$

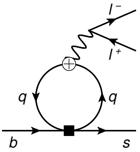
$$\mathcal{O}_{3,4,5,6} = [\bar{s}\,\Gamma_{sb} P_L (T^a)\, b] \sum_q [\bar{q}\,\Gamma_{qq} (T^a)\, q]$$

$$\mathcal{O}_{8g(8g')} = m_b [\bar{s}\,\sigma^{\mu\nu} P_{R(L)}\, T^a\, b] G^a_{\mu\nu}$$

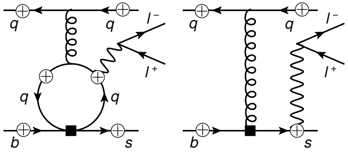
- at LO in QED $\Rightarrow$ solved with different approaches depending on q^2

$$\mathcal{A}_{\lambda,\mathrm{hadr}} = \frac{\alpha_e}{4\pi} \frac{L^{\mu}}{q^2} \int \mathrm{d}^4x\, e^{i\,q\cdot x} \left\langle K_{\lambda}^{(*)} \middle| \mathrm{T}\left\{ j_{\mu}^{\mathrm{em}}(x), \sum_i C_i \mathcal{O}_i(0) \right\} \middle| B(p) \right\rangle$$

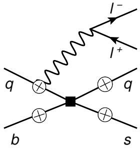
"resonant contributions"



"spectator scattering"



"weak annihilation"



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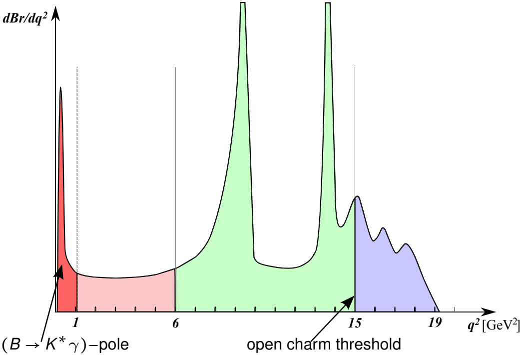

$$\mathcal{A}_{\lambda,\mathrm{hadr}} = \frac{\alpha_e}{4\pi}\frac{L^{\mu}}{q^2} \int \mathrm{d}^4 x\, e^{i\,q\cdot x} \left\langle K^{(*)}_{\lambda} \middle| \mathrm{T}\left\{ j^{\mathrm{em}}_{\mu}(x), \sum_i C_i \mathcal{O}_i(0) \right\} \middle| B(p) \right\rangle$$

Large Recoil (low-q^2)

- very low-q^2 ($\lesssim 1$ GeV^2) dominated by $\mathcal{O}_7$
  low-q^2 ($[1,6]$ GeV^2) dominated by $\mathcal{O}_{9,10}$
- 1) QCD factorization or SCET
  2) LCSR
  3) non-local OPE of $\bar{c}c$-tails

[Beneke/Feldmann/Seidel hep-ph/0106067 + 0412400;
Lyon/Zwicky et al. 1212.2242 +1305.4797; Khodjamirian et al. 1006.4945 + 1211.0234 + 1506.07760]

Low Recoil (high-q^2)

- dominated by $\mathcal{O}_{9,10}$
- local OPE (+ HQET) ⇒ theory only for sufficiently large q^2-integrated obs's
  [Grinstein/Pirjol hep-ph/0404250, Beylich/Buchalla/Feldmann 1101.5118]

Uncertainties @ high-q^2

Hard momentum transfer $(q^2 \sim m_B^2) \Leftrightarrow x \to 0$ allows for local OPE (at each value of q^2)

$$\int d^4x \, \frac{e^{i\,q\cdot x}}{q^2} \, \mathrm{T}\Big\{ j^{\mu}_{\rm em}(x), \sum_i C_i \mathcal{O}_i(0) \Big\} \stackrel{x\to 0}{=} \sum_a \mathcal{C}_{3a} \mathcal{Q}^{\mu}_{3a} + \text{no dim-4} + \sum_b \mathcal{C}_{5b} \mathcal{Q}^{\mu}_{5b} + \mathcal{O}(\text{dim} > 5)$$

[Buchalla/Isidori hep-ph/9801456, Grinstein/Pirjol hep-ph/0404250, Beylich/Buchalla/Feldmann 1101.5118]

$dim = 3$ $\propto B \to K^{(*)}$ FF's $\Rightarrow$ from lattice & also NLO-α_s corrections known

$dim = 5$ suppressed by $(\Lambda_{\rm QCD}/m_b)^2 \sim 2\%$, explicite estimate @ $q^2 = 15$ GeV^2: $< 1\%$

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Duality violating (DV) effects $\Rightarrow$ go beyond those neglected in $\Lambda_{\rm QCD}/m_b$ and α_s in the OPE

$$\mathcal{B} \simeq \underbrace{|A_{10}|^2 + |A_{9,\rm OPE}|^2}_{\text{similar in size}} + \underbrace{2\,\mathrm{Re}(A_{9,\rm OPE}\Delta^*_{9,\rm DV})}_{0 \gtrless} + \underbrace{|\Delta_{9,\rm DV}|^2}_{0 \le} + \ldots$$

- !!! no first principle methods to calculate $\Delta_{9,\rm DV}$
- $\Delta_{9,\rm DV}$ = oscillatory in q^2 $\Rightarrow$ hope to minimize DV effects by q^2 integration
- with exponential suppression for $q^2 \to q^2_{\rm max}$ $\Rightarrow$ or stay close to endpoint (not much data)
- using Shifman model for c-quark corr. $\Rightarrow$ $\Delta_{9,\rm DV}$ affects integrated rate ($q^2 > 15$ GeV^2) by $\pm 2\%$ [Beylich/Buchalla/Feldmann 1101.5118]
- OPE predicts relations: $H_T^{(1)} \simeq 1$ and $H_T^{(2)} \simeq H_T^{(3)}$
  large breaking from DV can be checked for experimentally [CB/Hiller/van Dyk 1006.5013, 1212.2321]
- allowing for large DV does NOT improve goodness of global fits [Altmannshofer/Straub 1411.3161]

Uncertainties @ low-q^2: $1/m_b$ corrections

- $\Lambda_{\rm QCD}/m_b \approx 13\%$ corrections to QCDF not known, only partially (contain endpoint divergences)
  [Kagan/Neubert hep-ph/0110078, Feldmann/Matias hep-ph/0212158, Beneke/Feldmann/Seidel hep-ph/0412400]

- $1/m_b$ corrections $\Rightarrow$ ruin "optimised observables" (some more, others less):
  A) due to use of FF-relations (factorisable) and B) due to QCDF to $B \to K^{(*)}\bar{\ell}\ell$ amplitude (non-factorisable)

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structure of vector ($\propto [\bar{\ell}\gamma_\mu \ell]$)-amplitudes (omitted helicity index $\lambda = 0, \pm 1$)

$$\mathcal{A} \propto \left(\xi_i + \Delta F_9^{\alpha_s} + \Delta F_9^{1/m_b}\right) C_9 + \left(\xi_i + \Delta F_7^{\alpha_s} + \Delta F_7^{1/m_b}\right) C_7 + \Delta^{\rm non-fac} + \Delta^{\bar{c}c}$$

- FF-relation breaking from α_s = known [Beneke/Feldmann hep-ph/0008255]
  $1/m_b$ = "unknown" (LCSR predictions of FF's account for some)
  ad-hoc parameterisation: $\Delta F_i^{1/m_b} = a_i + b_i \frac{q^2}{m_B^2} + c_i \frac{q^4}{m_B^4} + \ldots$ [Jäger/Martin-Camalich 1212.2263]

- $1/m_b$ corrections to non-factorisable parts (resonant, spectator scattering, WA)
  similarly $\Delta^{\rm non-fac} = \left(1 + A\, e^{i\phi_A} + B\, e^{i\phi_B}\frac{q^2}{m_B^2} + C\, e^{i\phi_C}\frac{q^4}{m_B^4}\right)\mathcal{A}^{\rm hadr}$ [Descotes-Genon et al. 1407.8526]
  with $A, B, C \in [0, 0.1]$, arbitrary $\phi_{A,B,C}$

- soft gluons to $\bar{c}c$ resonant-contributions [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945]

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How does it affect "optimised" observables ??? Example P_5'

ABSZ and DHMV =

simult. scan of groups of parameters (ABSZ incl. corr. of FF parameters)

$\Rightarrow$ error = linear or quadratic sum of spreads in observable

JMC (68%)=

gaussian priors for parameters

$\Rightarrow$ error = 68% of posterior predictive

JMC (max spread) =

simult. scan of all parameters

$\Rightarrow$ error = max spread in observable

Ref.	$q^2 \in [2.5, 4]$ GeV^2	$q^2 \in [4, 6]$ GeV^2
LHCb (3/fb)	$-0.07^{+0.34}_{-0.36}$	-0.30 ± 0.16
ABSZ (qua)	-0.50 ± 0.10	-0.77 ± 0.07
ABSZ (lin)	-0.50 ± 0.16	-0.77 ± 0.11
DHMV (qua)	$-0.49^{+0.14}_{-0.16}$	$-0.79^{+0.10}_{-0.12}$
DHMV (lin)	$-0.49^{+0.26}_{-0.30}$	$-0.79^{+0.16}_{-0.21}$
JMC (68%)	$-0.28^{+0.14}_{-0.13}$	$-0.71^{+0.11}_{-0.10}$
JMC (max spread)	$-0.28^{+0.54}_{-0.42}$	$-0.70^{+0.49}_{-0.31}$

LHCb = LHCb-CONF-2015-002

ABSZ = 1411.3161 + 1503.05534, DHMV = 1407.8526 + 1503.03328, JMC = 1412.3183 + talk S. Jäger Portoroz '15

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How does it affect "optimised" observables ??? Example P_5'

- ABSZ (contrary to DHMV and JMC) uses full QCD FF's from LCSR

  $\Rightarrow$ do not need to consider $1/m_b$ corrections from FF relations,
  only due to $B \to K^*\bar{\ell}\ell$ amplitudes and $\bar{c}c$ tails

- DHMV try to implement error estimates as closely to JMC

  $\Rightarrow$ same parameterisation of FF-relation breaking corrections

- for linearly added errors: uncertainties of DHMV only half of JMC

- central values of P_5' between DHMV/ABSZ and JMC very different,
  due to choice of central values of FF-relation breaking corrections:
  JMC uses heavy quark limit $\Leftrightarrow$ DHMV/ABSZ use LCSR results

ABSZ = 1411.3161 + 1503.05534, DHMV = 1407.8526 + 1503.03328, JMC = 1412.3183 + talk S. Jäger Portoroz '15

Summary

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$B_q \to \bar{\ell}\ell$

- branching ratio

$$\overline{\mathcal{B}}(B_q \to \bar{\ell}\ell) \propto \underbrace{\overset{\text{hadronic}}{f_{B_q}^2/\Gamma_H^q}}_{4(7)\%\text{ for } B_s(B_d)} \times \underbrace{\overset{\text{short-distance}}{(G_F m_W)^4 \left|V_{tb}V_{tq}^* \, C_{10}\right|^2}}_{\text{in SM } 6(9)\% \text{ from } V_{ts}(V_{td})}$$

- short-distance C_{10} under control @ NLO EW + NNLO QCD

⇒ excellent probe of short-distance part, given experimental precision reaches $\lesssim 5\%$

($B_s \to \bar{\mu}\mu$ @ LHCb with 50 fb^{-1})

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$B \to K^{(*)}\bar{\ell}\ell$

- global fits favour -25% deviation in C_9 from SM prediction, but no individual measurement with significant ($> 5\sigma$) deviation from SM
- form factors uncertainties reducible in future by lattice (mainly @ high-q^2)
- need theory progress @ low q^2 concerning $1/m_b$ corrections: to go beyond simple parameterisations used in global fits
- show-stopper for ultimate precision @ high q^2: duality violation to local OPE (accounted for in global fits with some ad-hoc size), test duality violation @ high q^2 with data

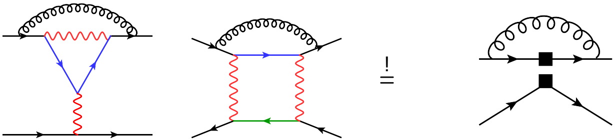
⇒ wait for final experimental results 3 fb^{-1} and results of LHC Run 2 after 2018 (about 8 fb^{-1})
continue global fits to tighten constraints on new physics

Backup Slides

Adding QCD corrections to $B_q \to \bar{\ell}\ell \ldots$

$$G_F \frac{\alpha_e}{s_W^2} \alpha_s$$

NLO QCD

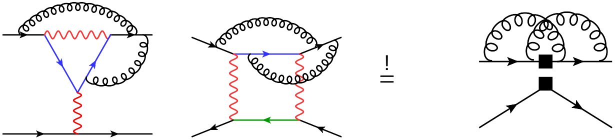


$\to C_A^{(10)}$

[Buchalla/Buras NPB 398 (1993) 285, NPB 400 (1993) 225, Misiak/Urban hep-ph/9901278]

$$G_F \frac{\alpha_e}{s_W^2} \alpha_s^2$$

NNLO QCD



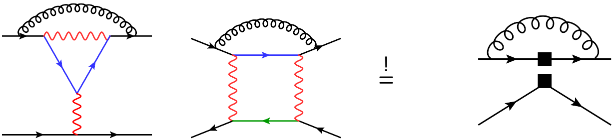
$\to C_A^{(20)}$

[Hermann/Misiak/Steinhauser arXiv:1311.1347]

Adding QCD corrections to $B_q \to \bar{\ell}\ell \ldots$

$$G_F \frac{\alpha_e}{s_W^2} \alpha_s$$

NLO QCD



$\to \; C_A^{(10)}$

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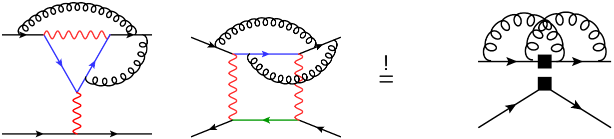


$\to \; C_A^{(20)}$

[Hermann/Misiak/Steinhauser arXiv:1311.1347]

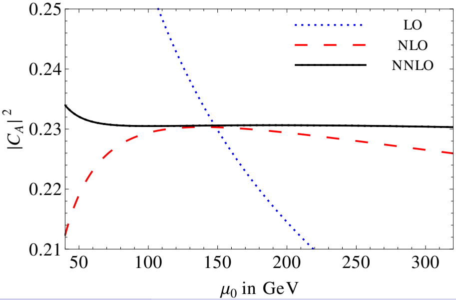
$$C_A = C_A^{(00)} + \frac{\alpha_s}{4\pi} C_A^{(10)} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_A^{(20)}$$

implicite – via $m_t(\mu_0)$ (= $\overline{\text{MS}}$ QCD) – and explicite μ_0 dependence

$m_t(m_t) = 163.5$ GeV
$m_t(\mu_0 = 50\,\text{GeV}) = 180.8$ GeV
$m_t(\mu_0 = 300\,\text{GeV}) = 156.2$ GeV

$\Rightarrow$ NNLO QCD crrs. reduce μ_0-dep. from

1.8% at NLO → 0.2% at NNLO on $Br \sim |C_A|^2$



Branching ratio $B_q \to \bar{\ell}\ell$

Up to now discussed (NNLO QCD + NLO EW) corrections to $C_A(\mu_b)$

$\Rightarrow$ need to calculate (matrix elements)^2 to obtain $\overline{\mathcal{B}}$

$$\overline{\mathcal{B}} \sim \left|\langle \ell\bar{\ell}|\mathcal{L}_{\text{eff}}|\bar{B}\rangle\right|^2 \sim \left|C_A(\mu_b)\right|^2 \langle 0|\bar{s}\gamma_\mu\gamma_5 b|\bar{B}\rangle^2 + \mathcal{O}(\alpha_e)$$

1) Hadronic matrix element (to all orders in QCD and LO in QED):

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}_q(p_B)\rangle \equiv i f_{B_q} p_B^\mu$$

$f_{B_d} = (190.5 \pm 4.2)$ MeV

$f_{B_s} = (227.7 \pm 4.5)$ MeV

[FLAG (lattice average) 2013]

2) Account for B_s-mixing: fully time-integrated CP-averaged *Br* [De Bruyn et al. arXiv:1204.1737]

Branching ratio $B_q \to \bar{\ell}\ell$

Up to now discussed (NNLO QCD + NLO EW) corrections to $C_A(\mu_b)$

$\Rightarrow$ need to calculate (matrix elements)^2 to obtain $\overline{\mathcal{B}}$

$$\overline{\mathcal{B}} \sim \left|\langle \ell\bar{\ell}|\mathcal{L}_{\rm eff}|\bar{B}\rangle\right|^2 \sim \left|C_A(\mu_b)\right|^2 \langle 0|\bar{s}\gamma_\mu\gamma_5 b|\bar{B}\rangle^2 + \mathcal{O}(\alpha_e)$$

1) Hadronic matrix element (to all orders in QCD and LO in QED):

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}_q(p_B)\rangle \equiv i f_{B_q} p_B^\mu$$

$$f_{B_d} = (190.5 \pm 4.2)\,\text{MeV}$$

$$f_{B_s} = (227.7 \pm 4.5)\,\text{MeV}$$

[FLAG (lattice average) 2013]

2) Account for B_s-mixing: fully time-integrated CP-averaged *Br* [De Bruyn et al. arXiv:1204.1737]

!!! for consistency should include $\mathcal{O}(\alpha_e)$ corrections:

A) bremsstrahlung

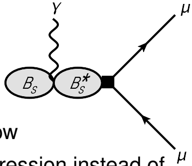
can be sizeable depending on photon energy cuts in experiment

B) virtual corrections

expected to be small, but non purely perturbative $\to$ no theoretical framework, perhaps Lattice calculations

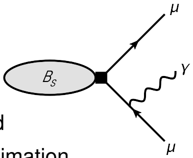
Soft photon bremsstrahlung: initial and final state

Initial state radiation



- tiny in signal window
- phase-space suppression instead of helicity suppression
- can be avoided with cuts

Final state radiation



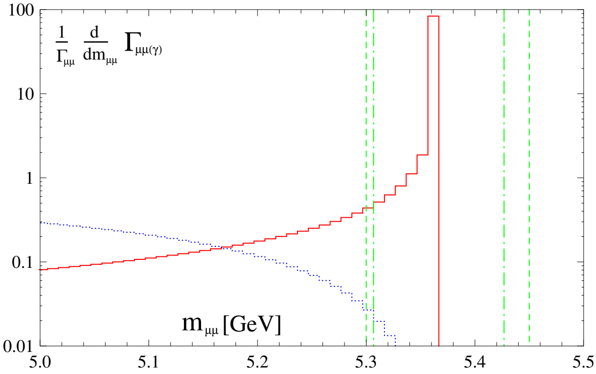
- helicity suppressed
- soft-photon approximation
- extrapolated from signal window over all $m^2_{\mu^+\mu^-}$ via PHOTOS by LHCb and CMS

ISR [Aditya/Healey/Petrov arXiv:1212.4166]

FSR [Buras et al. arXiv:1208.0934]

experimental signal windows (LHCb, CMS)

[LHCb arXiv:1307.5024, CMS arXiv:1307.5025]



What should theorists include in predictions of $\overline{\mathcal{B}}(B_q \to \bar{\ell}\ell)$?

- no need to include ISR → currently removed by cuts

  !!! it should be counted as background in MC

  ??? should not be included in PHOTOS in experimental analysis

  (especially when experimental accuracy increases)

- FSR should be included due to cuts in experimental analysis

  BUT already accounted for by LHCb and CMS using PHOTOS

  (extrapolation along red curve to zero on previous slide)

  ⇒ this corresponds to the limit where:

  photon-inclusive $\overline{\mathcal{B}}$ = non-radiative $\overline{\mathcal{B}}$

  [Buras/Girrbach/Guadagnoli/Isidori arXiv:1208.0934]

- virtual corrections → requires nonperturbative method

  ⇒ NOT included

  → here we assign 0.3% uncertainty from μ_b-variation

Another precise SM prediction @ (NLO EW + NNLO QCD)

Ratio of $\overline{\mathcal{B}}(B_q \to \bar{\ell}\ell)$ and B_q-mass difference Δm_{B_q}

$$\kappa_{q\ell} \equiv \frac{\overline{\mathcal{B}}(B_q \to \bar{\ell}\ell)}{\Delta m_{B_q}} \frac{\Gamma_H^q}{(G_F m_W m_\ell)^2 \beta_{q\ell}} \overset{(SM)}{=} \frac{3|C_A(\mu_b)|^2}{\pi^3 C_{LL}(\mu_b) B_{B_q}(\mu_b)}$$

NLO EW crr. to Δm_q [Gambino/Kwiatkowski/Pott hep-ph/9810400]

depends in addition only on

- $\Rightarrow C_{LL}(\mu_b)$ Wilson coefficient of $\Delta B = 2$ operator
- $\Rightarrow B_{B_q}(\mu_b)$ bag factor of $\Delta B = 2$ hadronic matrix element

SM prediction

$$\kappa_{s\ell} = 0.0126 \pm 0.0007$$

$$\kappa_{d\ell} = 0.0132 \pm 0.0012$$

with comparable theory uncertainty to $\overline{\mathcal{B}}(B_q \to \bar{\ell}\ell)$, dominated by lattice prediction of $B_{B_q}(\mu_b)$