

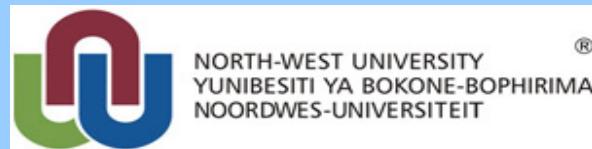
# **Solar Modulation of Cosmic Rays in the Heliosphere**

## **A global view**

**Marius Potgieter  
&**

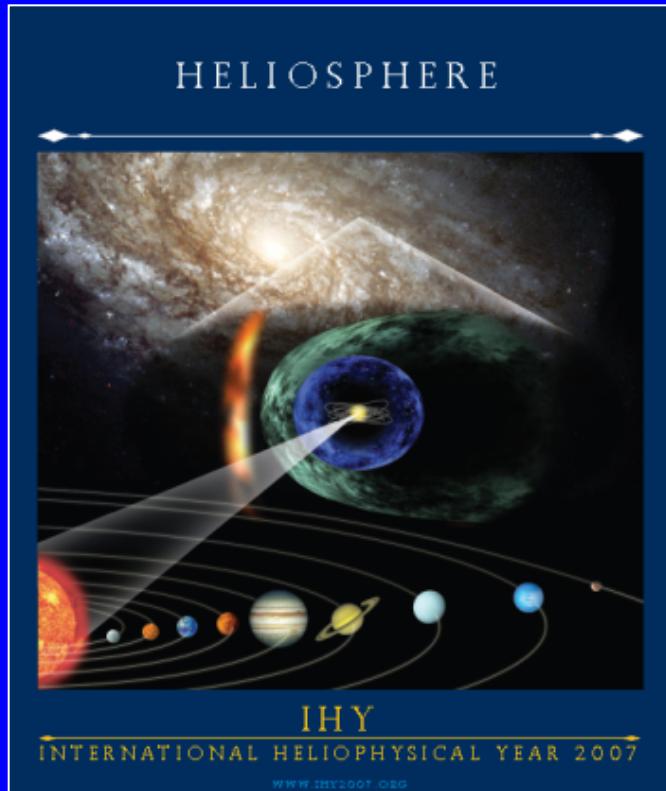
**Rendani Nndanganeni, Sibusiso Nkosi  
Lourens Prinsloo, Jan-Louis Raath & Etienne Vos**

**Centre for Space Research, North-West University,  
Potchefstroom, South Africa**



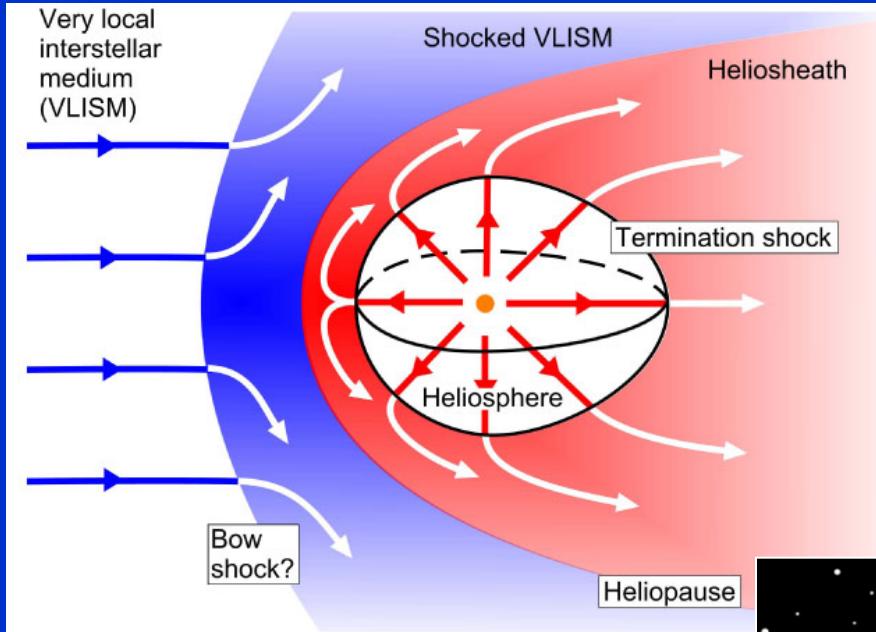
Honolulu, 19 October 2015

# Outline of presentation

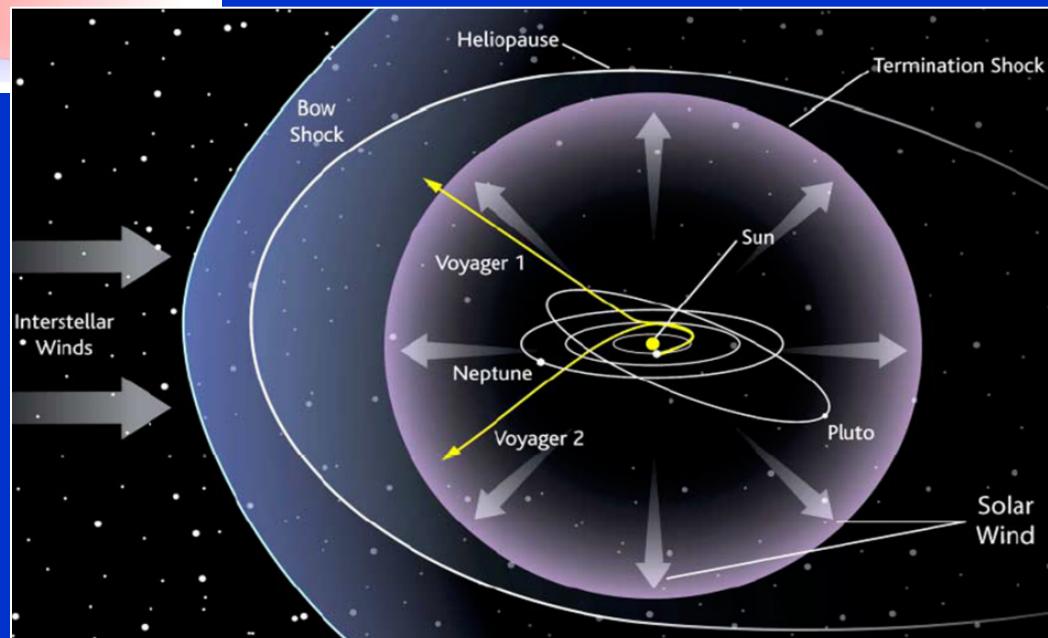


- **The heliosphere; modulation boundaries**
- **New heliopause spectra (HPS)**
- **Solar modulation observations**
- **Transport and modulation theory**
- **Numerical models**
- **Modeling results and implications**

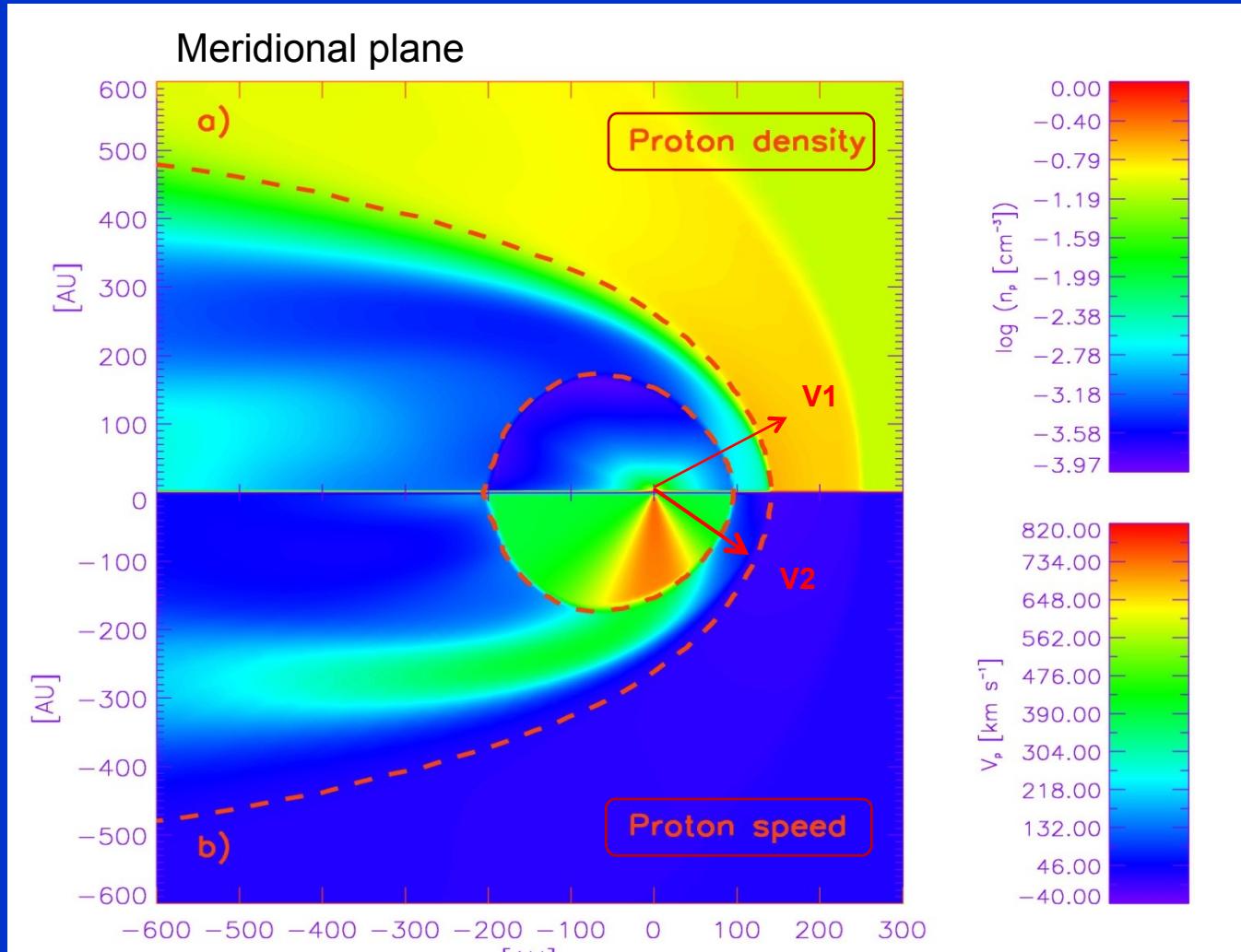
# The schematic heliosphere



Owens and Forsyth. Living Rev. 2013



# Major heliospheric structures (HD simulated)



## Fact sheet:

**Heliosphere is highly asymmetric**

TS was observed by V1 at 94 AU, by V2 at 84 AU...

V1 crossed the HP at 121.5 AU

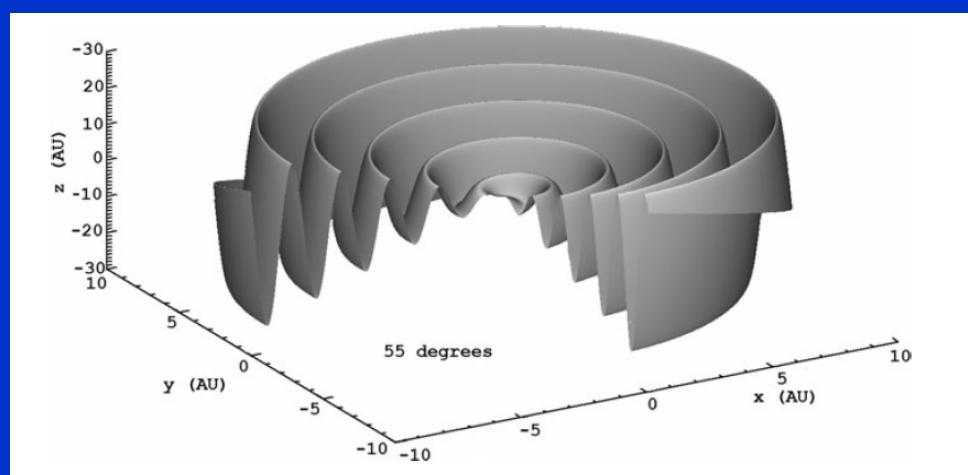
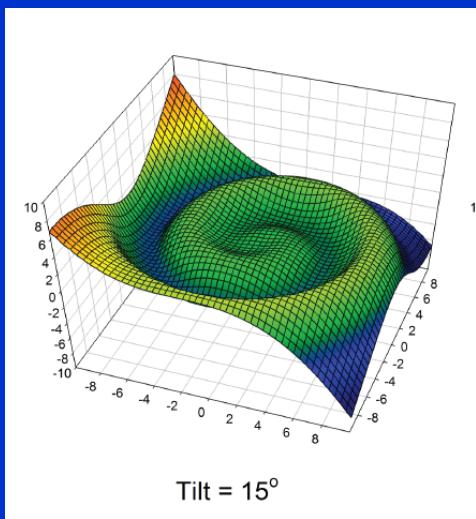
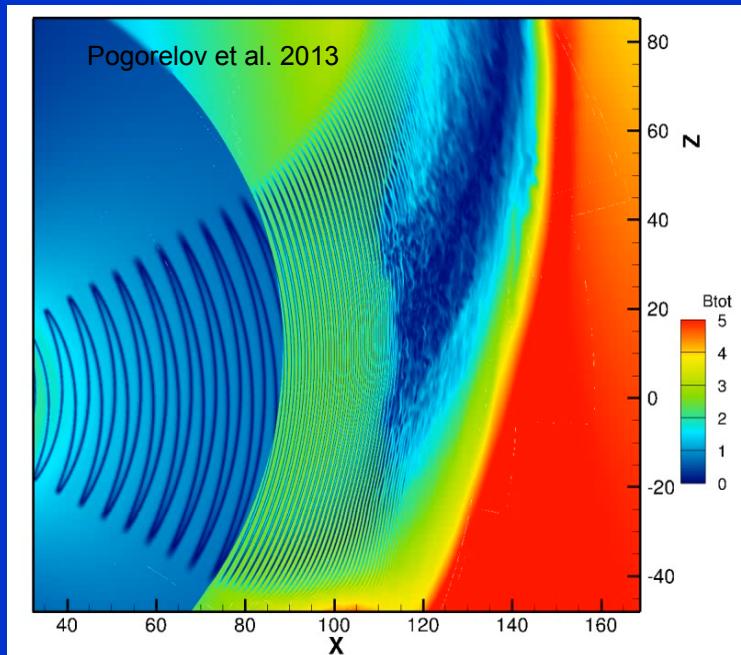
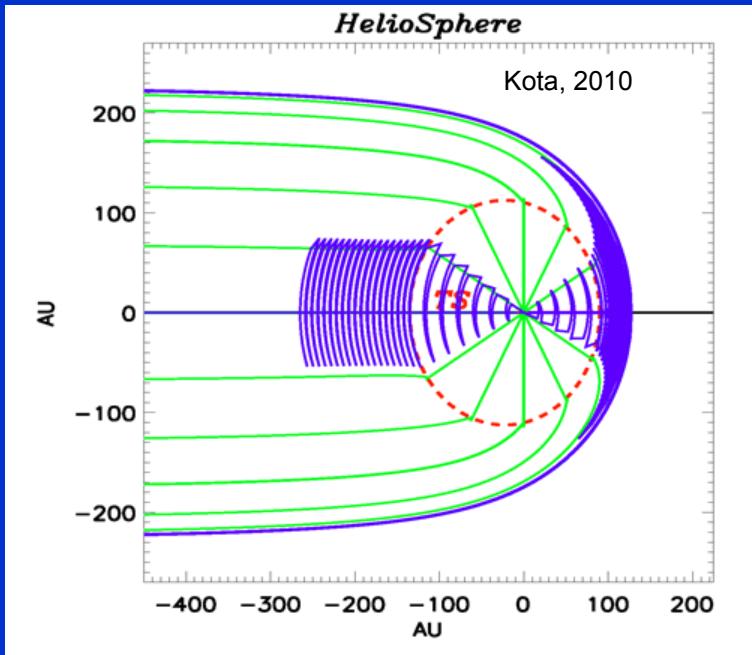
V1 is now at 131.9 AU from the Sun

V2 is still inside the heliosheath at 108.4 AU

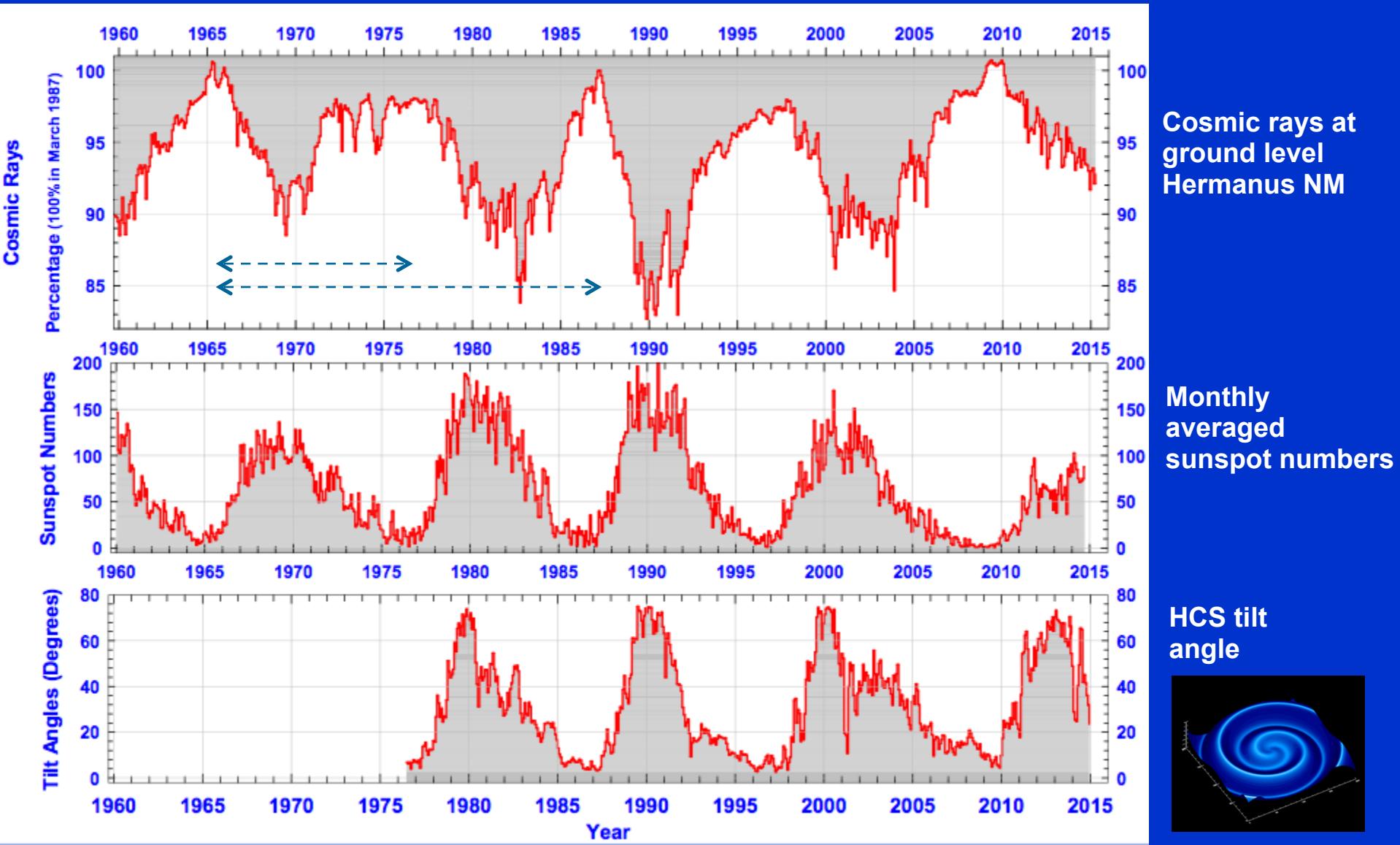
Roundtrip time for light: 29:49:03

<http://voyager.jpl.nasa.gov/>

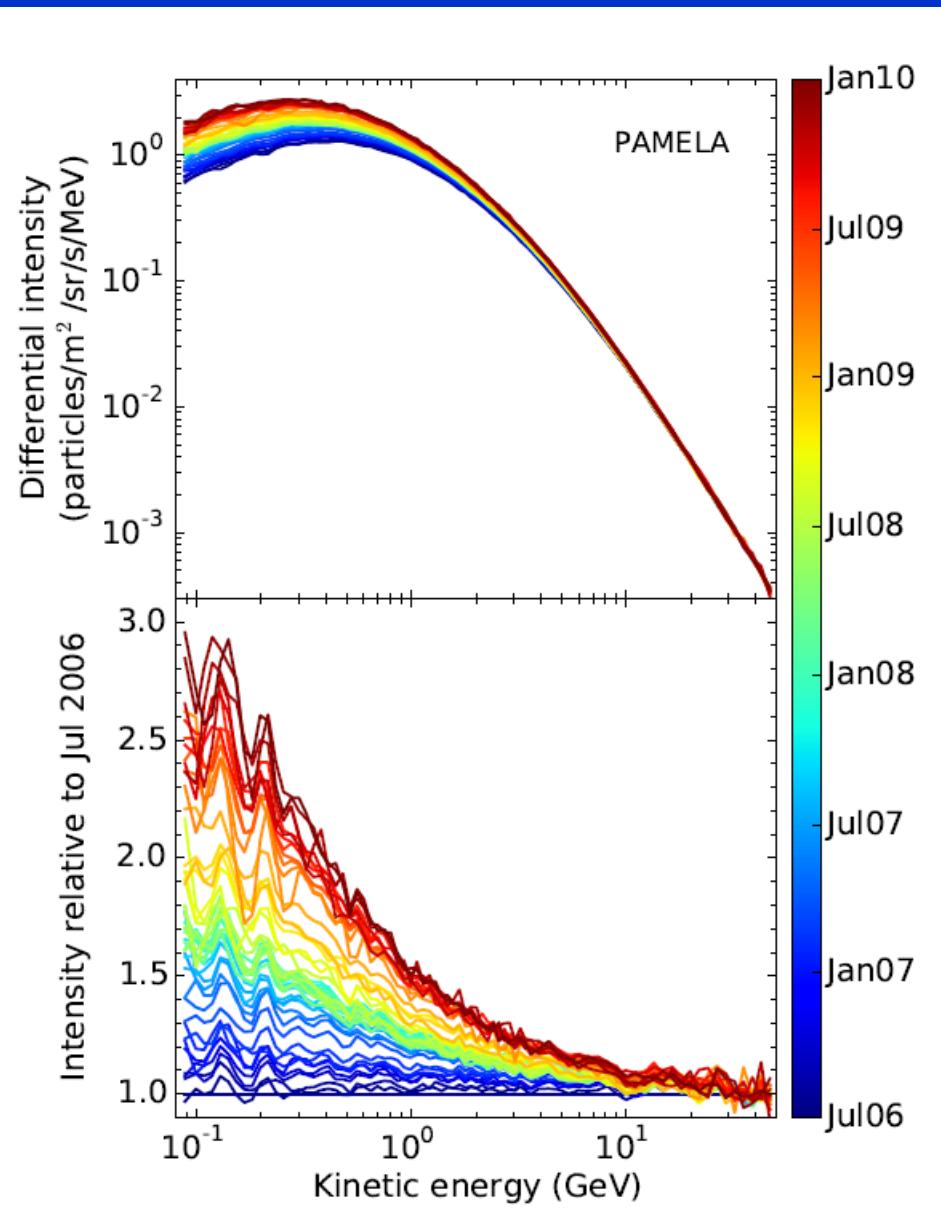
# The conceptual HCS (wavy current sheet)



# Modulation of Galactic Cosmic Rays observed at the Earth & solar activity proxies



# Global modulation of cosmic ray protons: mid-2006 to end of 2009



## PAMELA proton observations at the Earth

PAMELA-SA bilateral cooperation:  
Mirko Boezio & colleagues

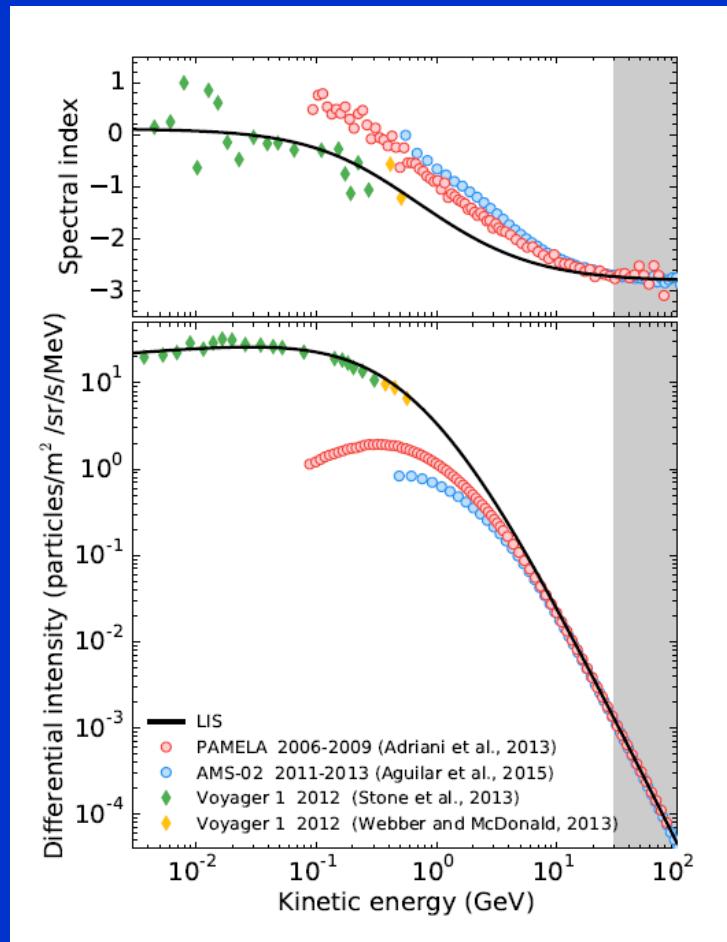
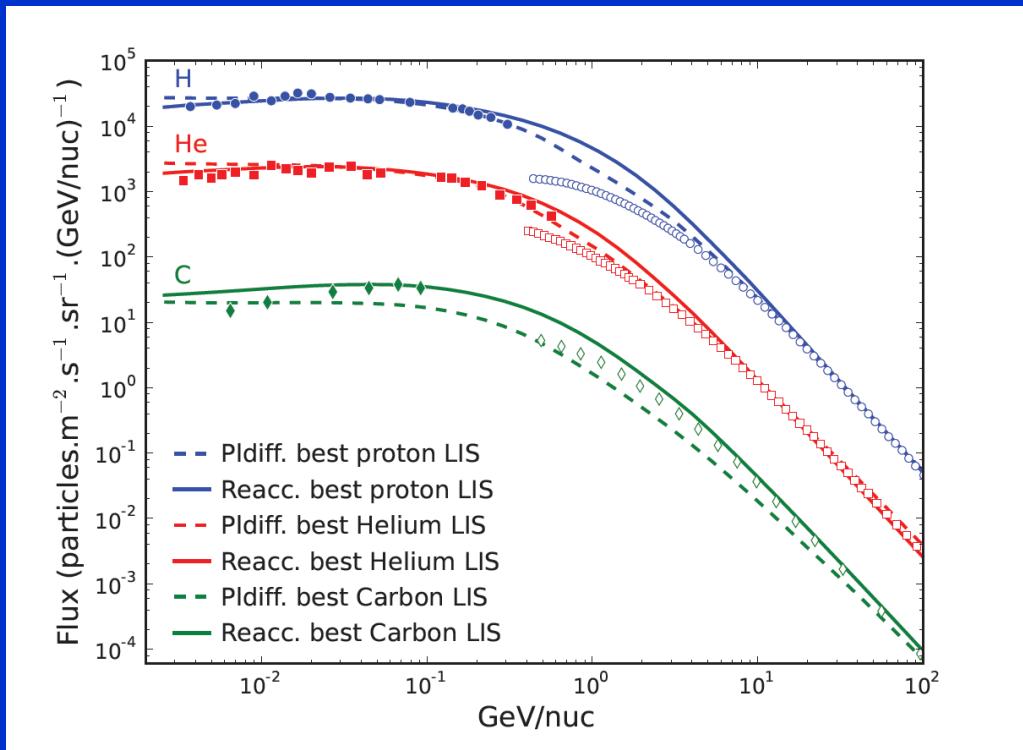
Proton spectra published  
Adriani et al. ApJ 2013  
Potgieter et al. Solar Phys 2014

PhD's: Valeria Di Felice, Nico De Simone,  
Valerio Formato

MSc's: Riccardo Munini, Etienne Vos

# The Heliopause Spectra: Voyager 1, PAMELA and AMS2 Observations and GALPROP computations

GALPROP; Plain Diffusion and with re-acceleration



# Transport equation for the transport, modulation and acceleration of cosmic rays in the heliosphere

$$\frac{\partial f}{\partial t} = \nabla \cdot [\mathbf{K} \cdot \nabla f] - \mathbf{V} \cdot \nabla f - \langle \mathbf{v}_D \rangle \cdot \nabla f + \frac{1}{3} (\nabla \cdot \mathbf{V}) \frac{\partial f}{\partial \ln p} + Q(r, p, t)$$

Time-dependent, pitch-angle-averaged distribution function

Diffusion

Convection with solar wind

Particle Drifts

Adiabatic energy changes

Any local source

$$\dots = \dots + \frac{1}{p^2} \frac{\partial}{\partial p} \left( p^2 D_{pp} \frac{\partial f}{\partial p} \right)$$

Second order Fermi acceleration

# TPE in spherical coordinates; diffusion tensor based on a simple HMF geometry

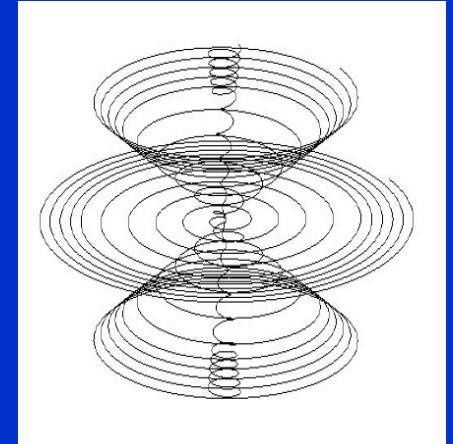
$$\begin{aligned}
\frac{\partial f}{\partial t} = & \left[ \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 K_{rr}) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (K_{\theta r} \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial K_{\phi r}}{\partial \phi} - V \right] \frac{\partial f}{\partial r} \\
& + \left[ \frac{1}{r^2} \frac{\partial}{\partial r} (r K_{r\theta}) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (K_{\theta\theta} \sin \theta) + \frac{1}{r^2 \sin \theta} \frac{\partial K_{\phi\theta}}{\partial \phi} \right] \frac{\partial f}{\partial \theta} \\
& + \left[ \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial r} (r K_{r\phi}) + \frac{1}{r^2 \sin \theta} \frac{\partial K_{\theta\phi}}{\partial \theta} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial K_{\phi\phi}}{\partial \phi} \right] \frac{\partial f}{\partial \phi} \\
& + K_{rr} \frac{\partial^2 f}{\partial r^2} + \frac{K_{\theta\theta}}{r^2} \frac{\partial^2 f}{\partial \theta^2} + \frac{K_{\phi\phi}}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2} + \frac{2 K_{r\phi}}{r \sin \theta} \frac{\partial^2 f}{\partial r \partial \phi} \\
& + \frac{1}{3r^2} \frac{\partial}{\partial r} (r^2 V) \frac{\partial f}{\partial \ln p} + Q_{source}(r, \theta, \phi, p, t), \quad (4)
\end{aligned}$$

The diffusion tensor can then be written as:  $(r, \theta, \phi)$  is:

$$\begin{bmatrix} K_{rr} & K_{r\theta} & K_{r\phi} \\ K_{\theta r} & K_{\theta\theta} & K_{\theta\phi} \\ K_{\phi r} & K_{\phi\theta} & K_{\phi\phi} \end{bmatrix} \begin{bmatrix} \kappa_{||} & 0 & 0 \\ 0 & \kappa_{\perp\theta} & \kappa_A \\ 0 & -\kappa_A & \kappa_{\perp r} \end{bmatrix} \begin{bmatrix} \cos \psi & 0 & -\sin \psi \\ 0 & 1 & 0 \\ \sin \psi & 0 & \cos \psi \end{bmatrix} = \\
\begin{bmatrix} \kappa_{||} \cos^2 \psi + \kappa_{\perp r} \sin^2 \psi & -\kappa_A \sin \psi & (\kappa_{\perp r} - \kappa_{||}) \cos \psi \sin \psi \\ \kappa_A \sin \psi & \kappa_{\perp\theta} & \kappa_A \cos \psi \\ (\kappa_{\perp r} - \kappa_{||}) \cos \psi \sin \psi & -\kappa_A \cos \psi & \kappa_{||} \sin^2 \psi + \kappa_{\perp r} \cos^2 \psi \end{bmatrix}, \quad (5)$$

with  $\psi$  the spiral angle of the magnetic field with respect to the radial direction. The components of the gradient and curvature drift velocity are:

$$\begin{aligned}
\langle v_d \rangle_r &= -\frac{A}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta K_{\theta r}), \\
\langle v_d \rangle_\theta &= -\frac{A}{r} \left[ \frac{1}{\sin \theta} \frac{\partial}{\partial \phi} (K_{\phi\theta}) + \frac{\partial}{\partial r} (r K_{r\theta}) \right], \\
\langle v_d \rangle_\phi &= -\frac{A}{r} \frac{\partial}{\partial \theta} (K_{\theta\phi}), \quad (6)
\end{aligned}$$



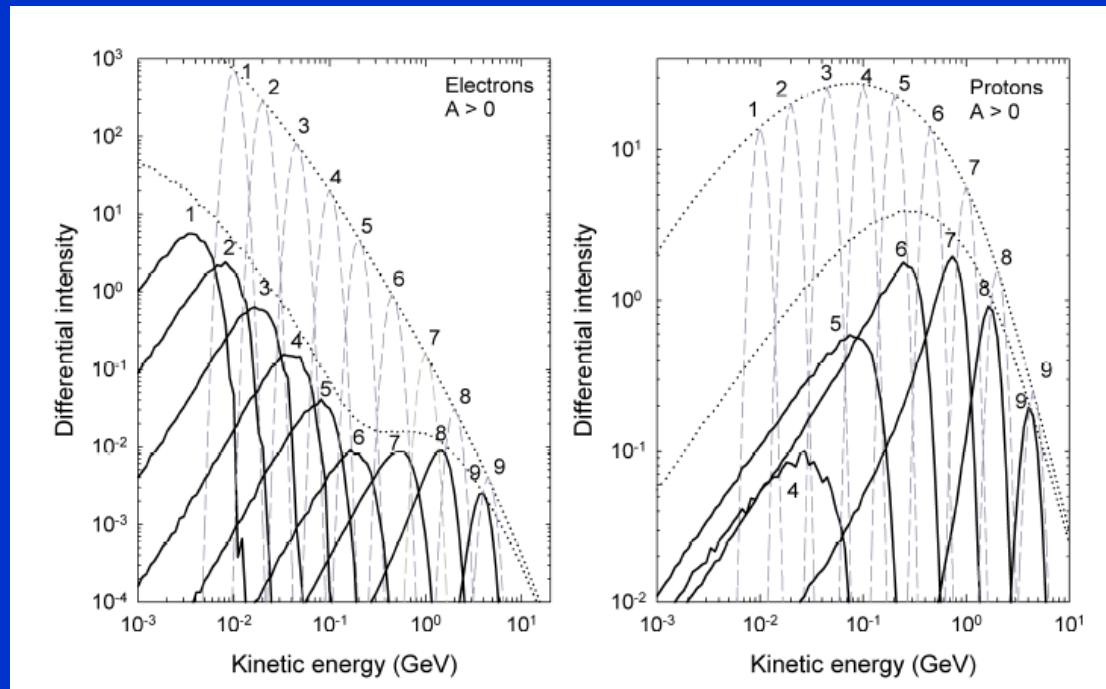
$$B = B_0 \left[ \frac{r_0}{r} \right]^2 \sqrt{1 + \left[ \frac{\Omega(r - r_\odot) \sin \theta}{V_{sw}} \right]^2 + \left[ \frac{r \delta(\theta)}{r_\odot} \right]^2},$$

**Then comes the generalized tensor...**

$$\begin{aligned}
K_{rr} &= \cos^2 \zeta (\kappa_{||} \cos^2 \psi + \kappa_{\perp,3} \sin^2 \psi) + \kappa_{\perp,2} \sin^2 \zeta, \\
K_{r\theta} &= \sin \zeta \cos \zeta (\kappa_{||} \cos^2 \psi + \kappa_{\perp,3} \sin^2 \psi - \kappa_{\perp,2}) - \kappa_A \sin \psi, \\
K_{r\phi} &= \sin \psi \cos \psi \cos \zeta (\kappa_{\perp,3} - \kappa_{||}) - \kappa_A \cos \psi \sin \zeta, \\
K_{\theta r} &= \sin \zeta \cos \zeta (\kappa_{||} \cos^2 \psi + \kappa_{\perp,3} \sin^2 \psi - \kappa_{\perp,2}) + \kappa_A \sin \psi, \\
K_{\theta\theta} &= \sin^2 \zeta (\kappa_{||} \cos^2 \psi + \kappa_{\perp,3} \sin^2 \psi) + \kappa_{\perp,2} \cos^2 \zeta, \\
K_{\theta\phi} &= \sin \psi \cos \psi \sin \zeta (\kappa_{\perp,3} - \kappa_{||}) + \kappa_A \cos \psi \cos \zeta, \\
K_{\phi r} &= \sin \psi \cos \psi \cos \zeta (\kappa_{\perp,3} - \kappa_{||}) + \kappa_A \cos \psi \sin \zeta, \\
K_{\phi\theta} &= \sin \psi \cos \psi \sin \zeta (\kappa_{\perp,3} - \kappa_{||}) - \kappa_A \cos \psi \cos \zeta, \\
K_{\phi\phi} &= \kappa_{||} \sin^2 \psi + \kappa_{\perp,3} \cos^2 \psi.
\end{aligned}$$

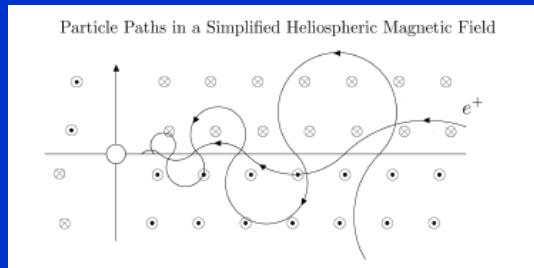
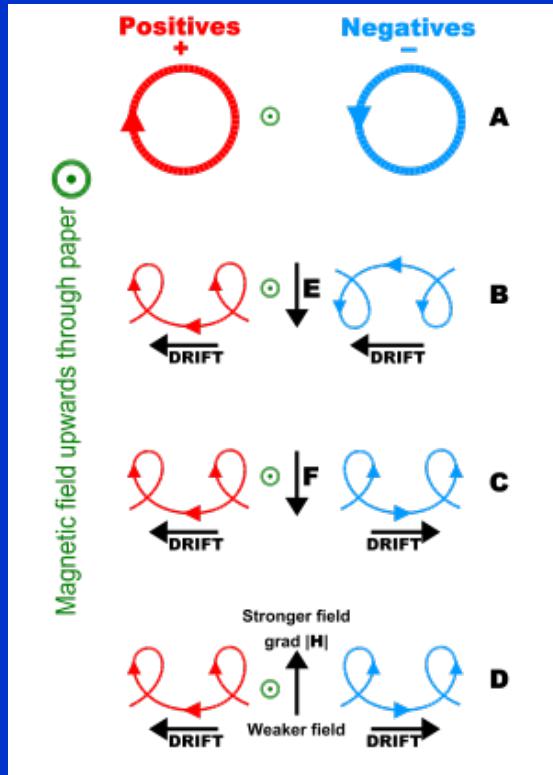
# Adiabatic energy loss mechanism

$$\frac{\partial f}{\partial t} + \mathbf{V} \cdot \nabla f - \nabla \cdot (\mathbf{K} \cdot \nabla f) \left[ -\frac{1}{3} (\nabla \cdot \mathbf{V}) \frac{\partial f}{\partial \ln p} \right] = 0$$



e.g. Strauss et al (2011)

# Gradient, curvature and current sheet drifts: Basic Theory



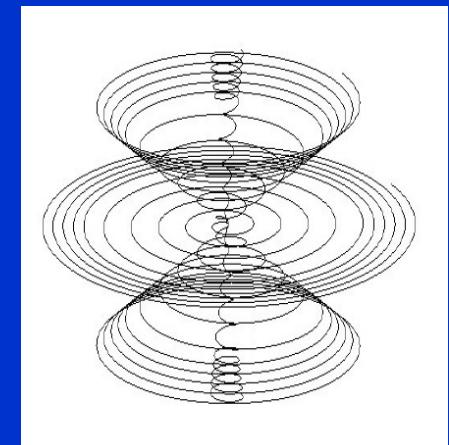
$$\langle \vec{v}_D \rangle = \frac{pv}{3Q} \frac{(\omega\tau_d)^2}{1 + (\omega\tau_d)^2} \nabla \times \frac{\vec{B}}{B^2}$$

$$(\omega\tau_d)^{ws} \gg 1$$

$$\begin{aligned} \langle \vec{v}_D \rangle^{ws} &= \nabla \times \frac{v}{3} r_L \mathbf{e}_B \\ &= \nabla \times \kappa_D \mathbf{e}_B, \end{aligned}$$

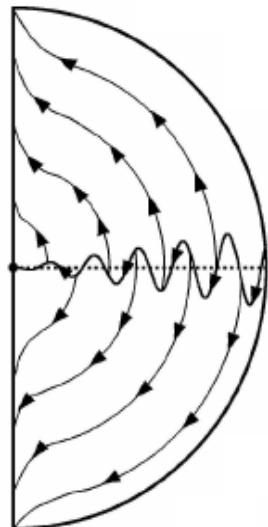
$$\mathbf{K}_S \equiv \begin{bmatrix} \kappa_{||} & 0 & 0 \\ 0 & \kappa_{\perp\theta} & 0 \\ 0 & 0 & \kappa_{\perp r} \end{bmatrix}$$

$$\mathbf{K}_D \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \kappa_D \\ 0 & -\kappa_D & 0 \end{bmatrix},$$

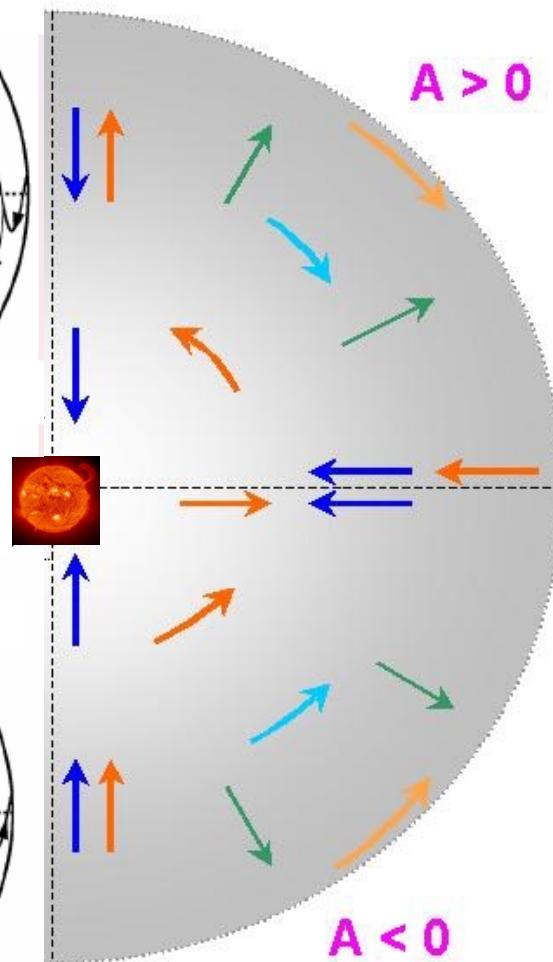
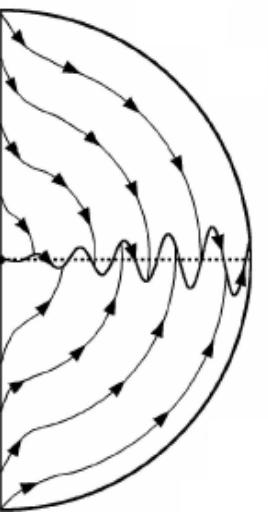


# Charge-sign dependent modulation

Drift direction of electrons in  $A > 0$  cycle



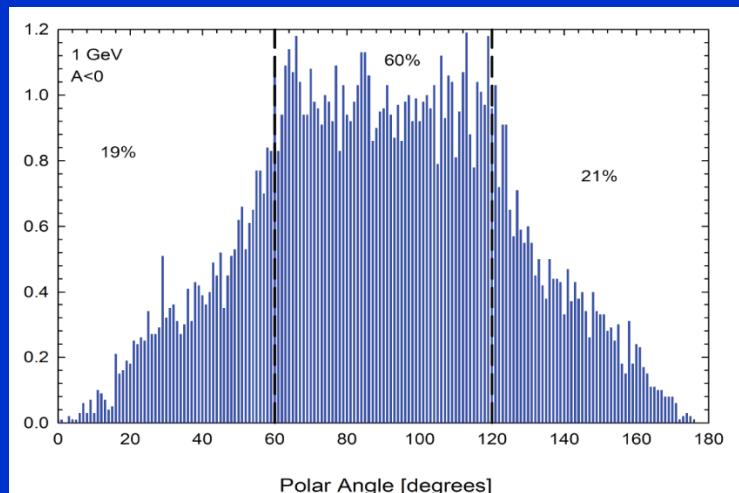
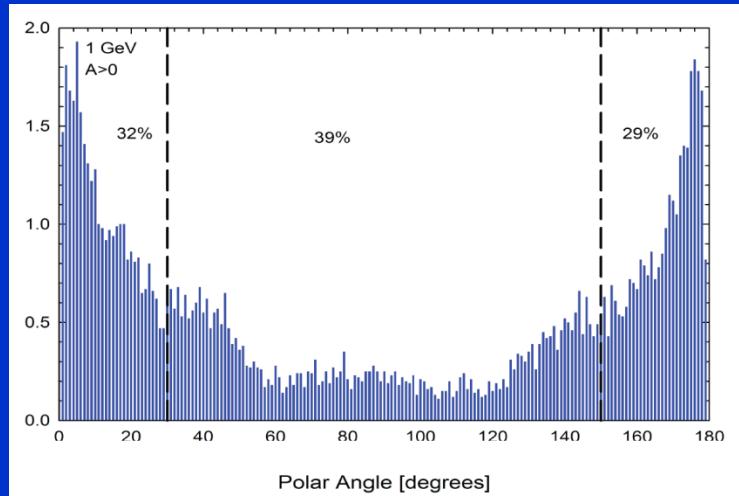
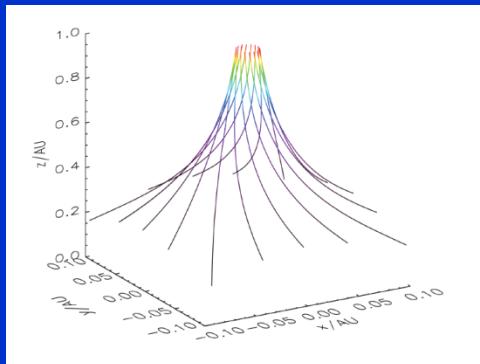
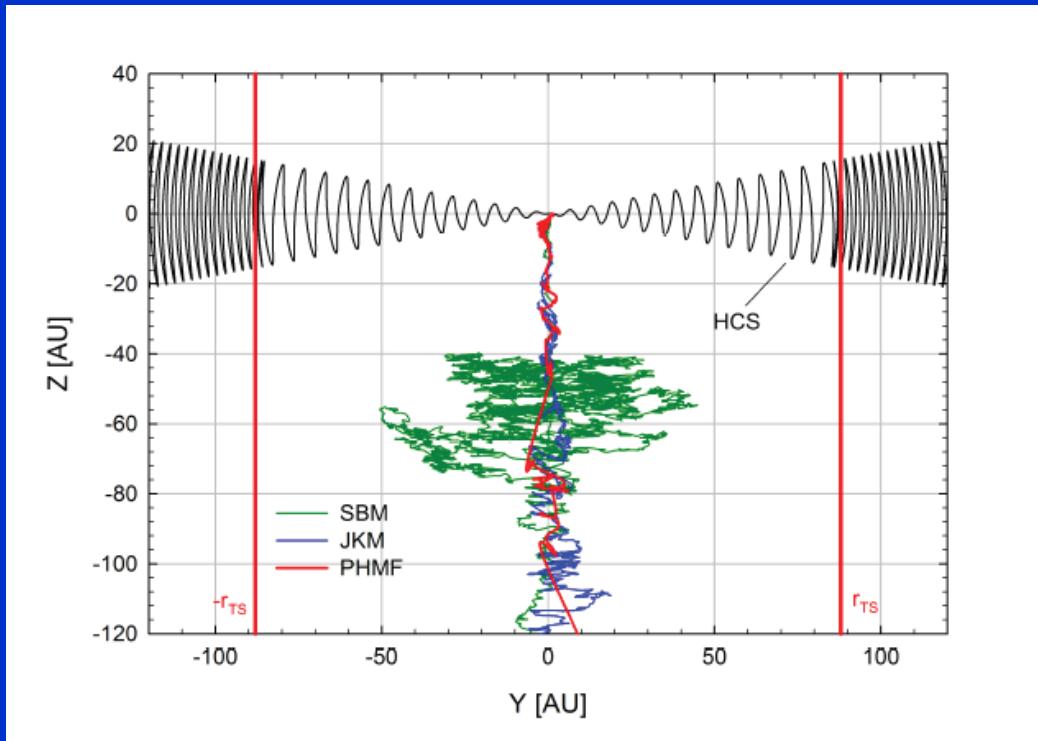
Drift direction of electrons in  $A < 0$  cycle



Modulation mechanisms

- Convection
- Diffusion
- Perpendicular diffusion
- G,C & NS Drifts
- Shock-drift

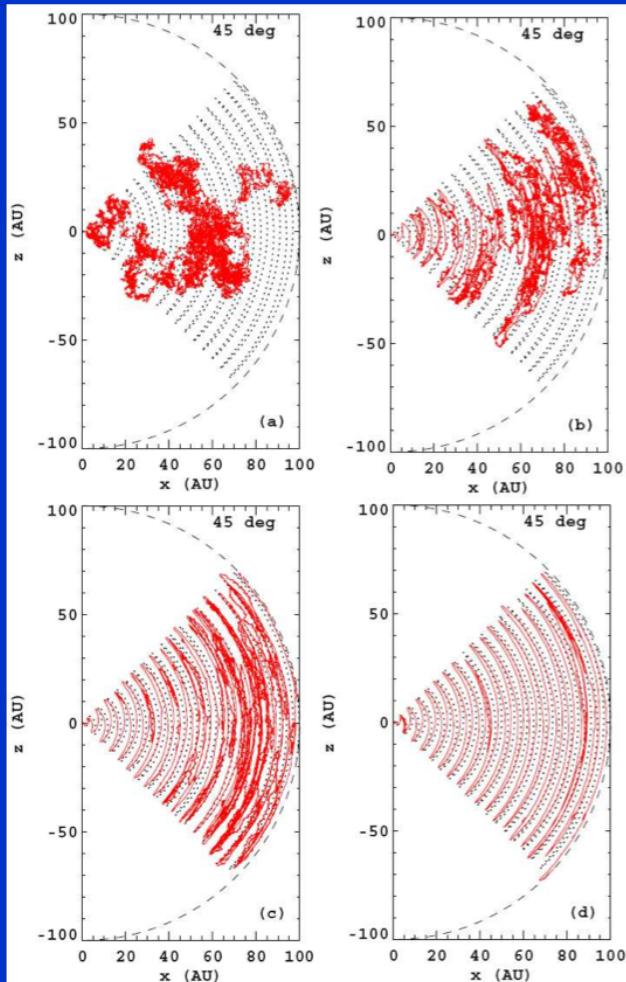
# Particle drifts in the heliospheric polar regions; SDE approach



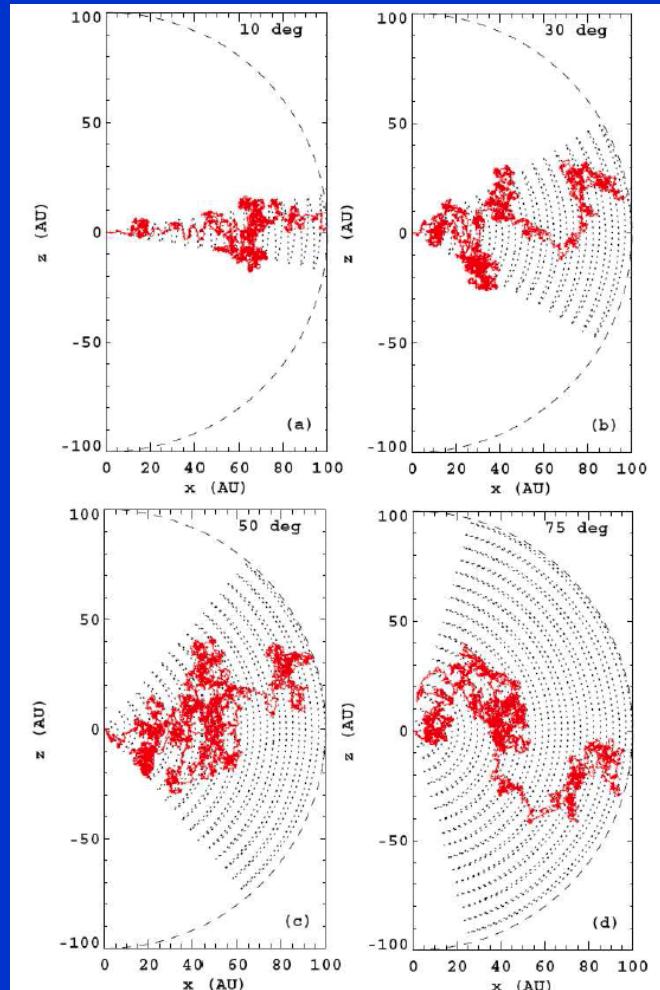
# Proton trajectories in the heliosphere along the HCS

## Impact of SDE models

Decreasing diffusion causes increasing drift effects

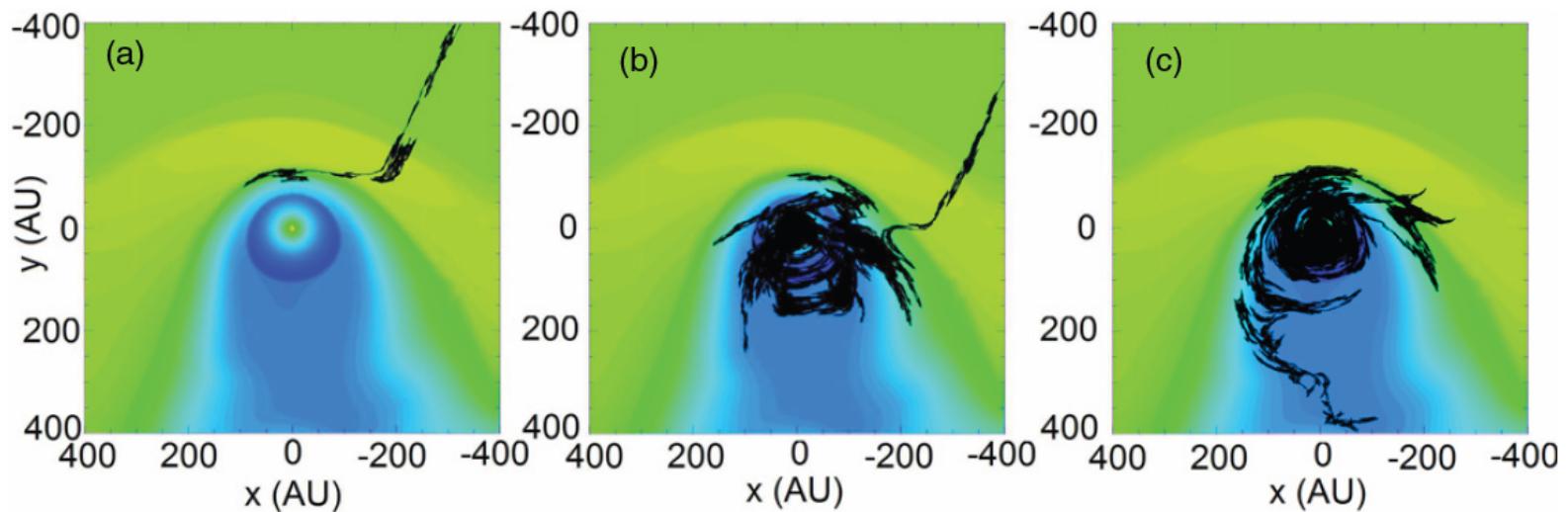


Decreasing HCS tilt angle

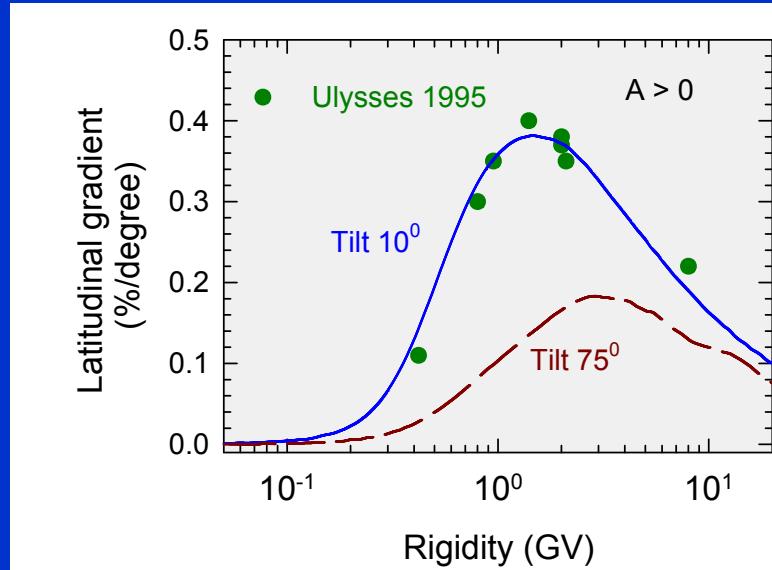
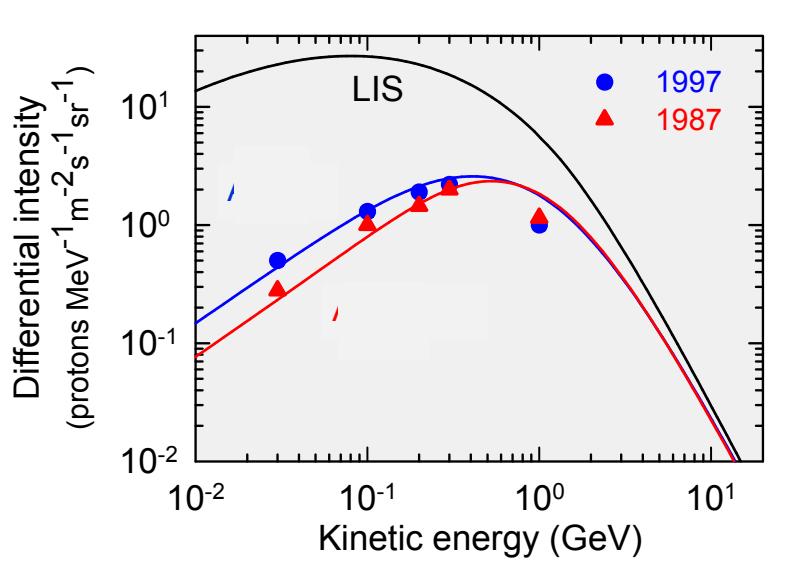


# 100 MeV Proton Trajectories to reach Voyager 1

SDE approach to numerical modeling



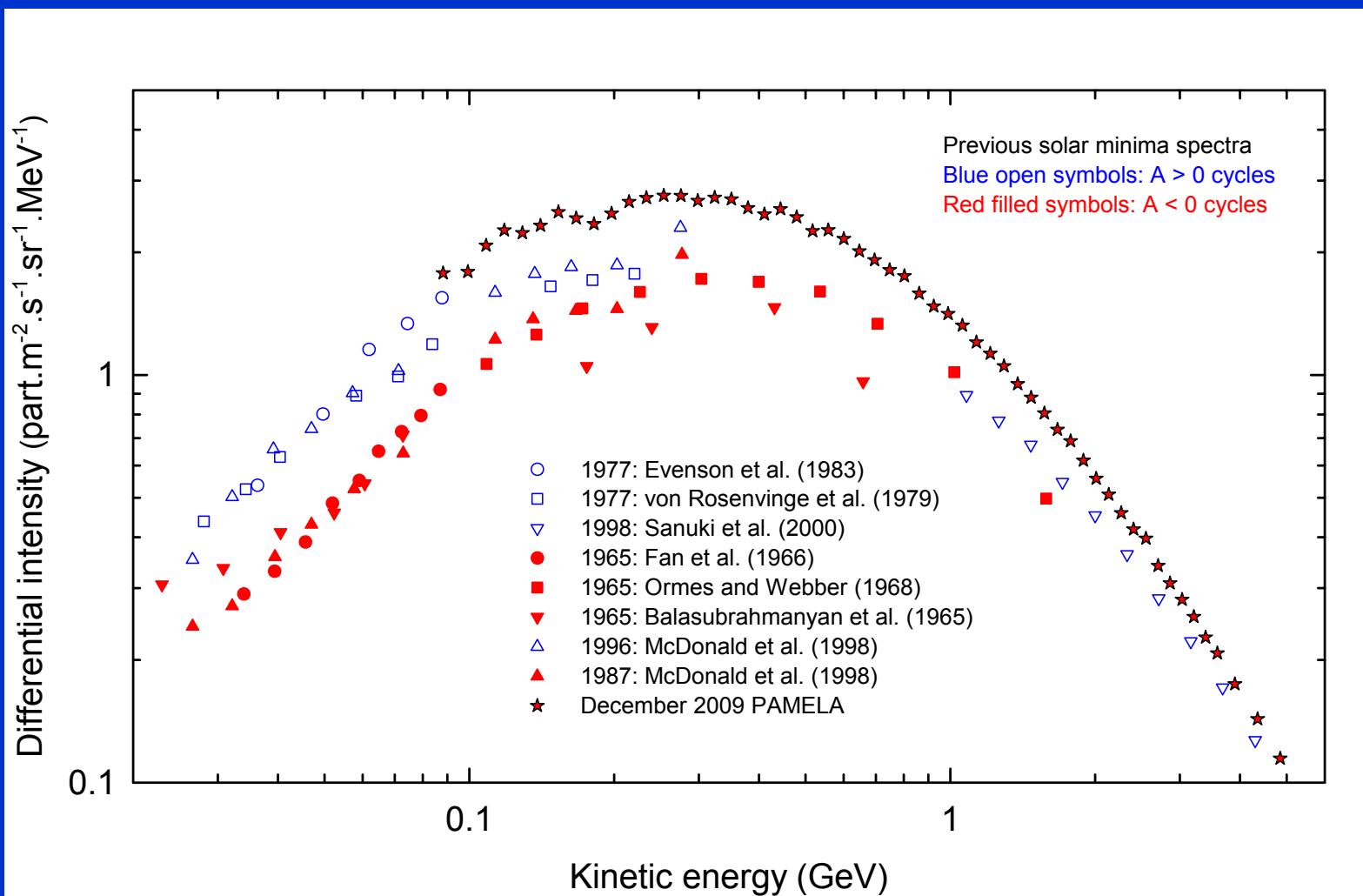
# Major features of observed modulated cosmic rays near Earth: Required as validation for ALL numerical models



Observed and computed spectra crossings  
at Earth, for  $A > 0$  and  $A < 0$  solar minima  
polarity cycles...

Computed latitudinal gradients compared to  
Ulysses-KET observations, for  $A > 0$ , solar  
minimum to maximum...

# Highest every recorded cosmic ray protons in 2009



# Diffusion theory: Updated - Difference between electrons and protons

A. Teufel and R. Schlickeiser: Cosmic ray parallel mean free path. I.

711

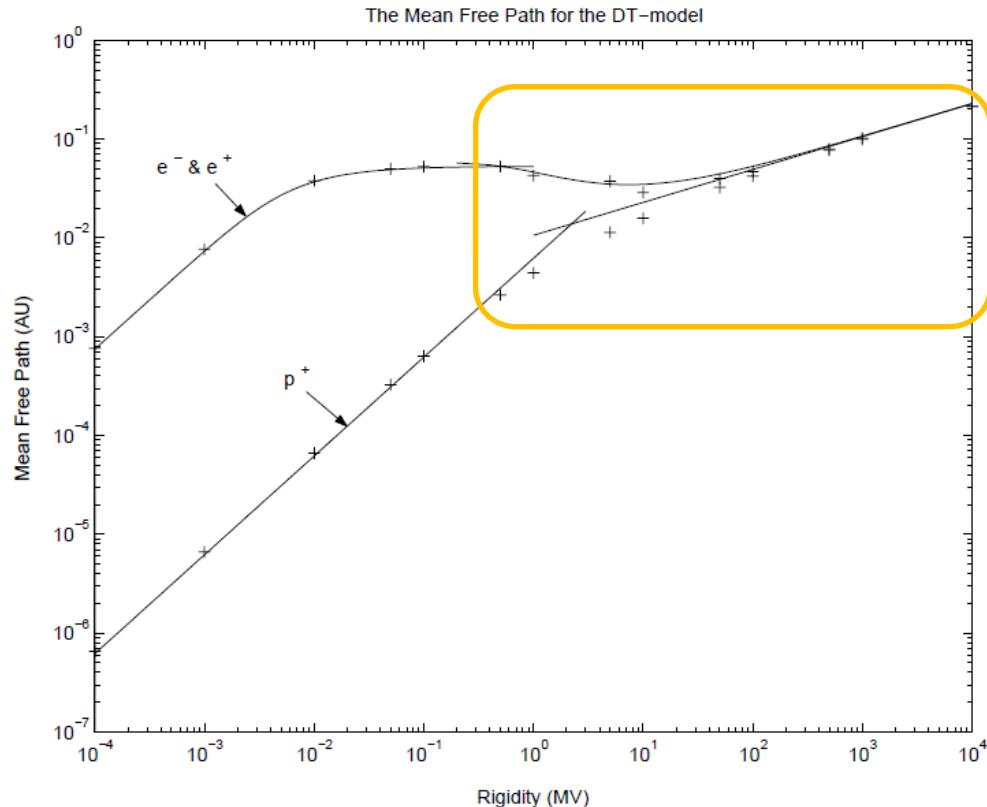


Fig. 2. Parallel mean free paths for electrons, positrons and protons in the damping model of dynamical turbulence. The results, the lines are our approximations.

A&A 393, 703–715 (2002)  
DOI: 10.1051/0004-6361:20021046  
© ESO 2002

Astronomy  
&  
Astrophysics

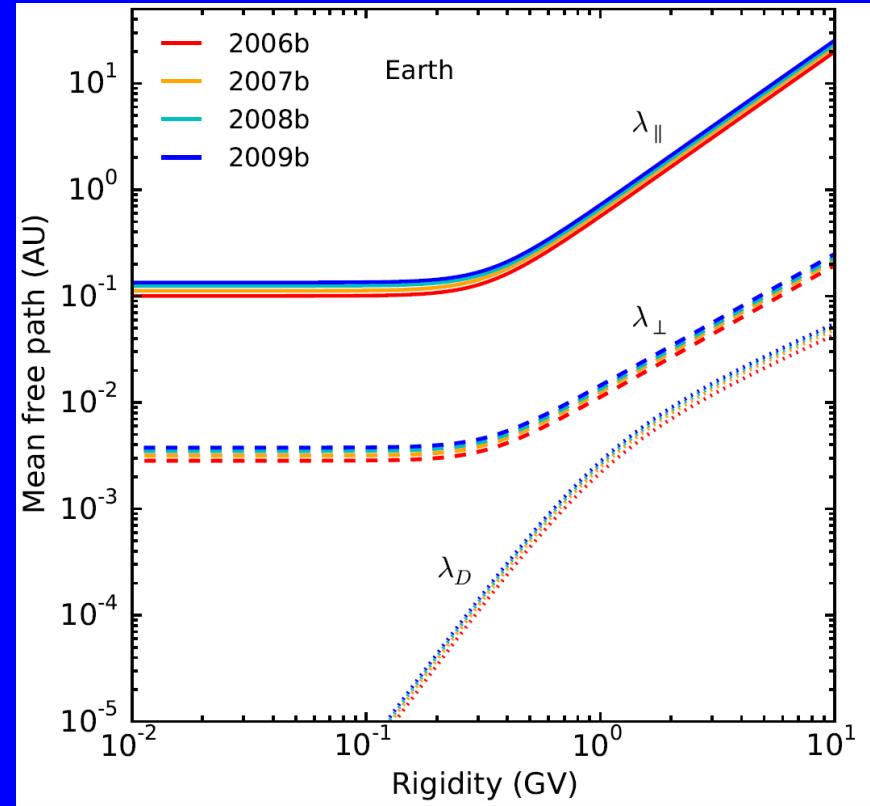
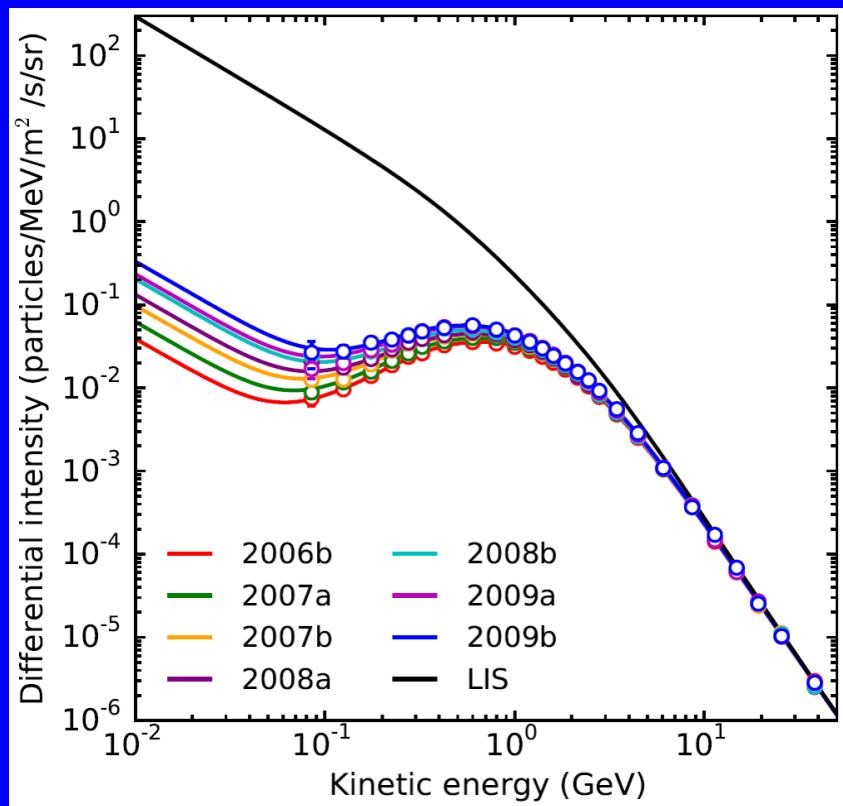
Analytic calculation of the parallel mean free path of heliospheric cosmic rays

I. Dynamical magnetic slab turbulence and random sweeping slab turbulence

A. Teufel and R. Schlickeiser

# PAMELA Electron Observations and Modeling

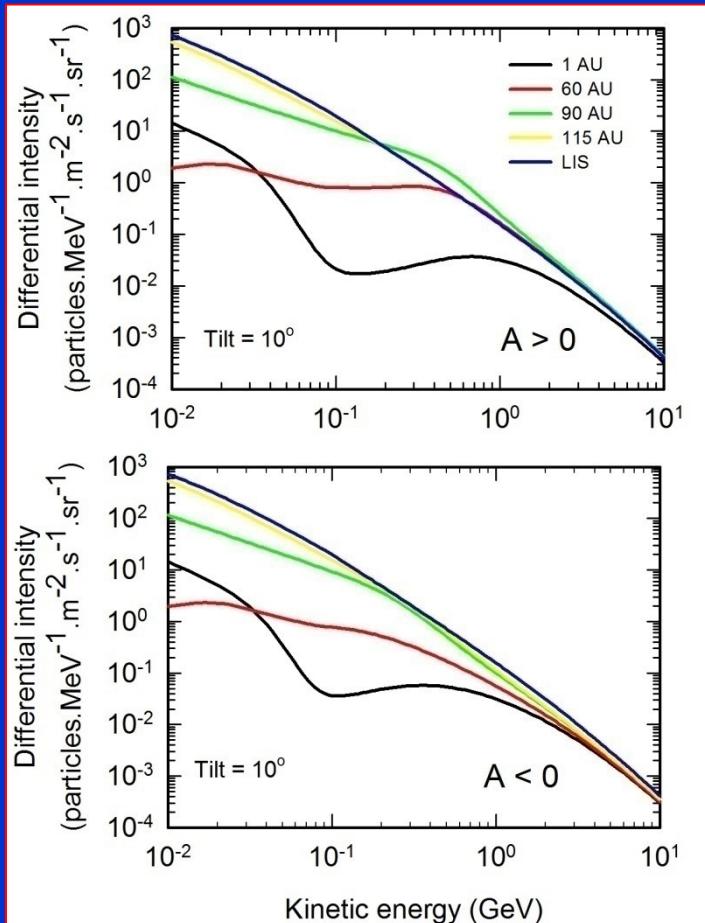
Consequences for electron diffusion theory



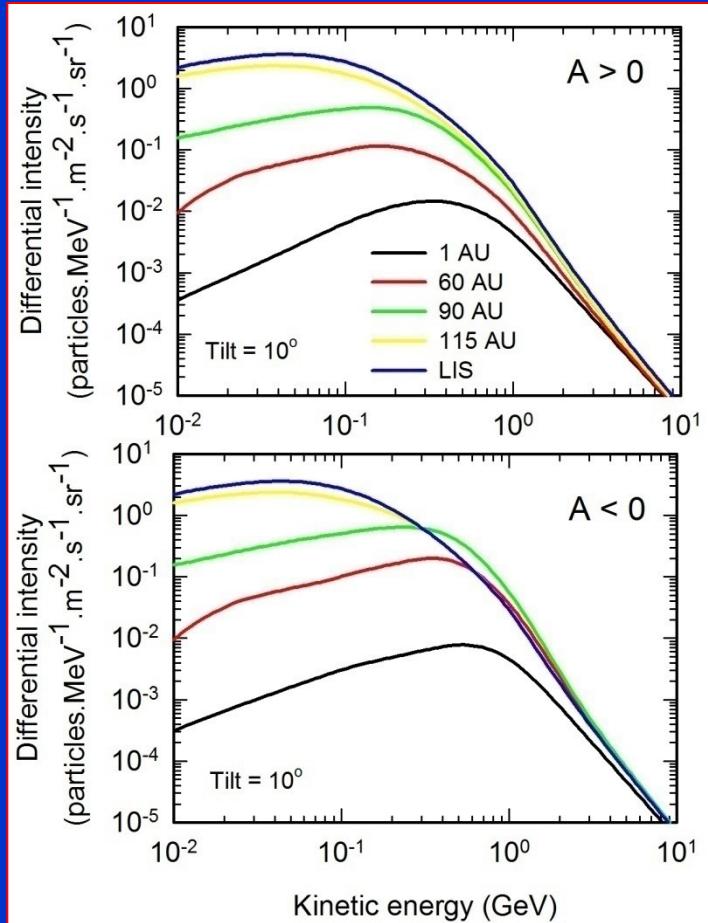
Potgieter, Vos, Boezio et al. 2015

# Computed modulation of galactic electrons and positrons at solar minimum for two polarity cycles

## Electrons

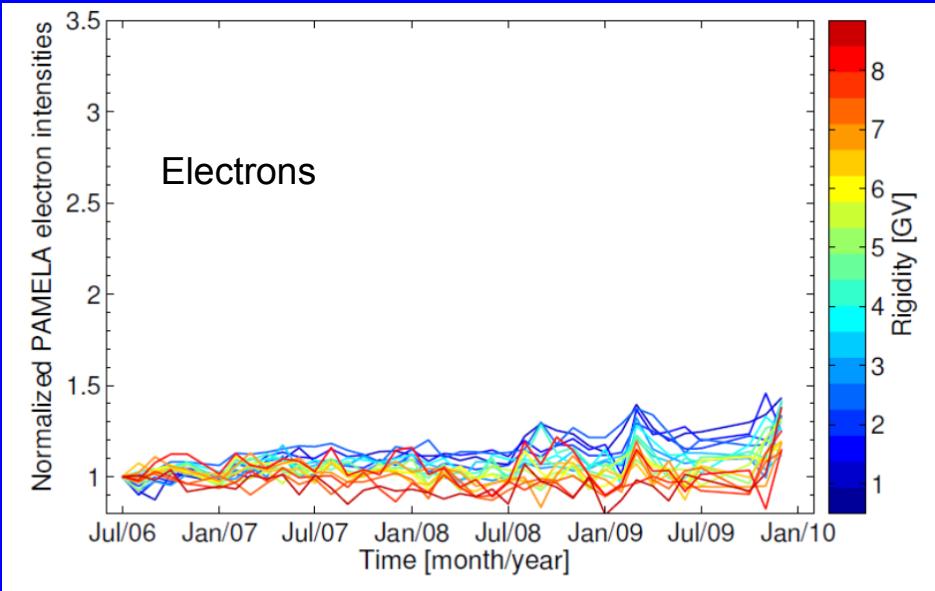
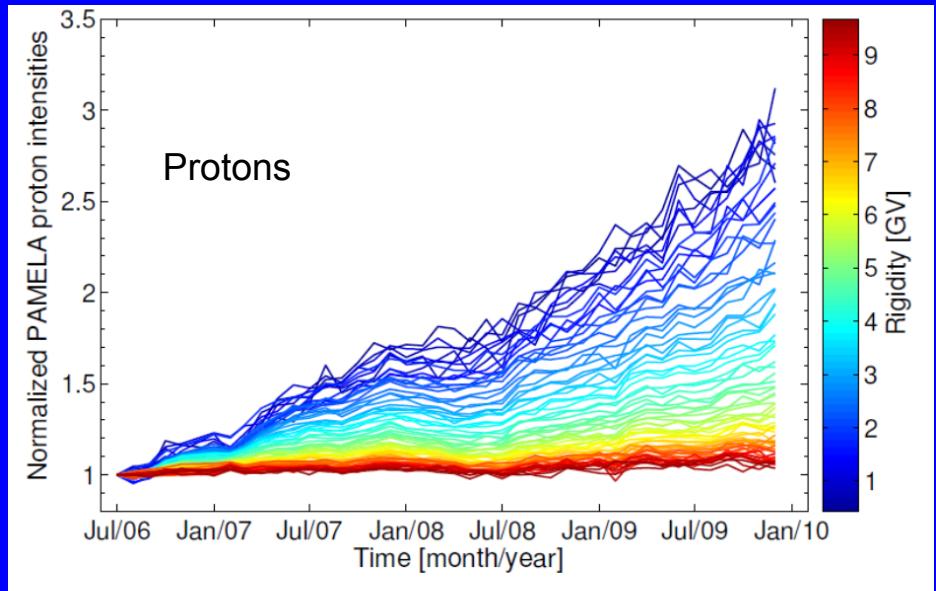


## Positrons



Langner & Potgieter, Solar wind termination shock and heliosheath effects on charge-sign dependent modulation for protons and anti-protons, JGR, 109, 2004; Potgieter & Langner, Heliospheric modulation of cosmic ray positrons and electrons: Effects of the heliosheath and solar wind termination shock, ApJ, 602, 2004.

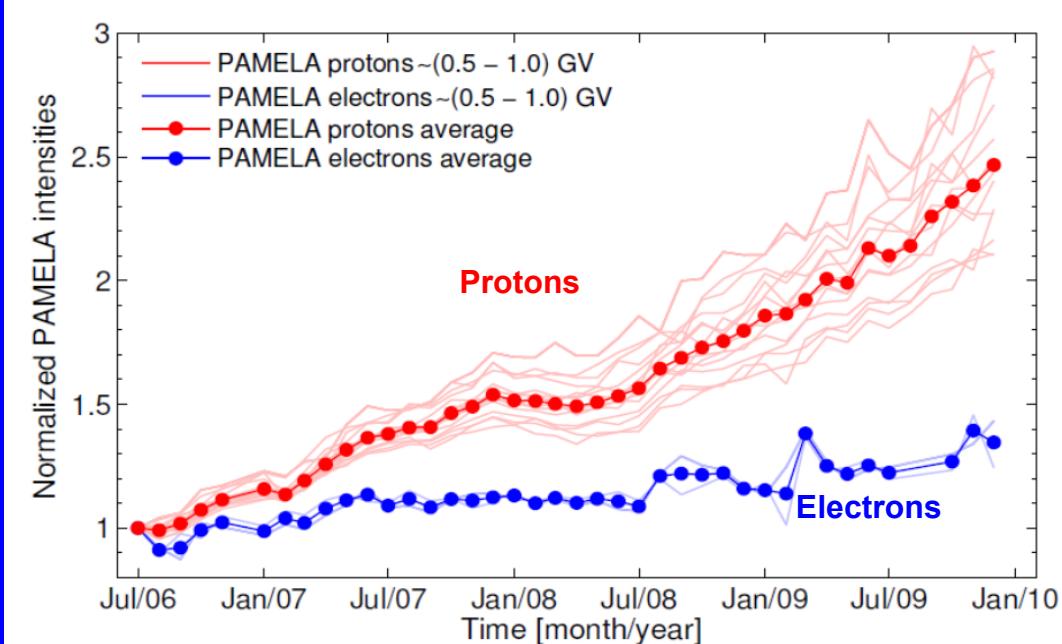
# PAMELA Observations and Charge-sign Dependence



New evidence of charge-sign-dependent (drift) modulation for 2006-2009 from PAMELA...!

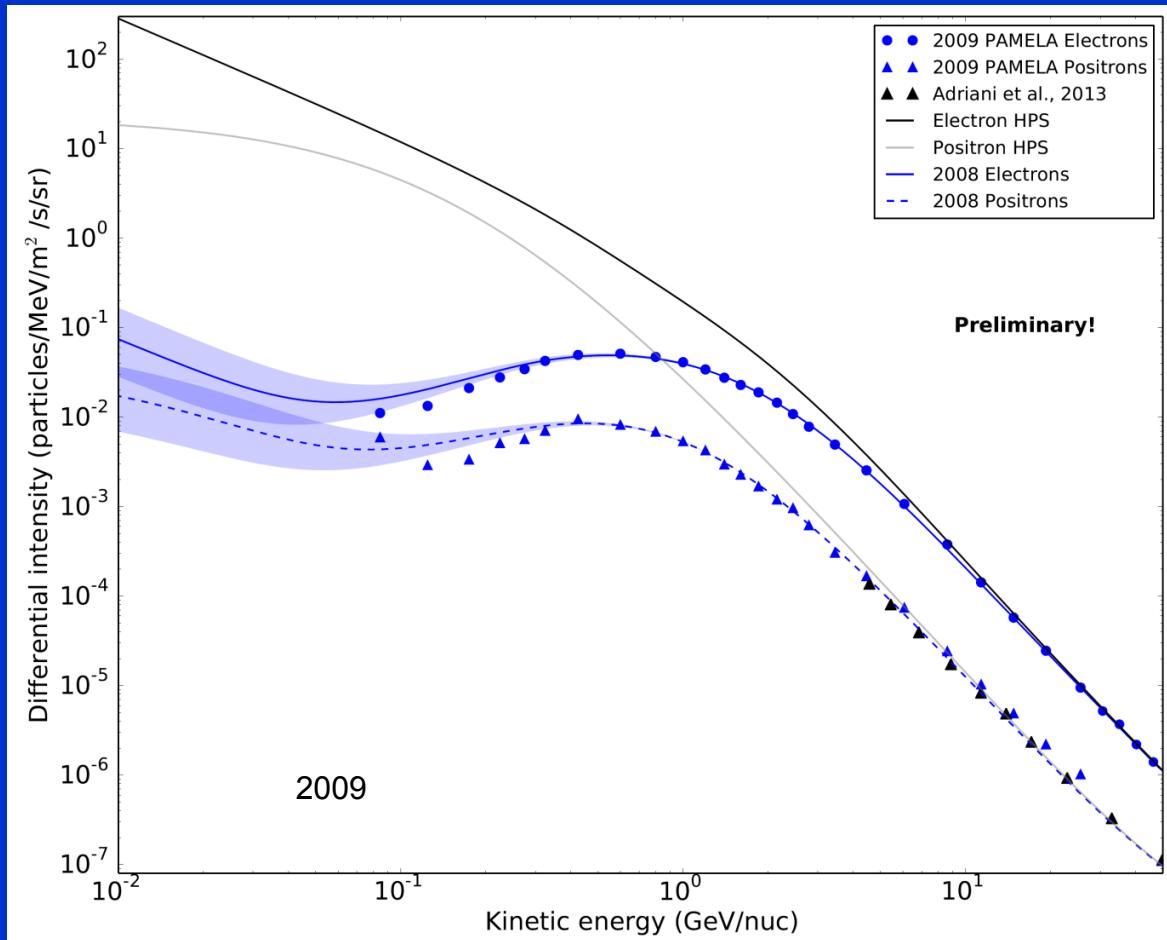
But, much smaller than anticipated from modelling of previous solar minimum activity periods....!

The 2009 minimum was different...



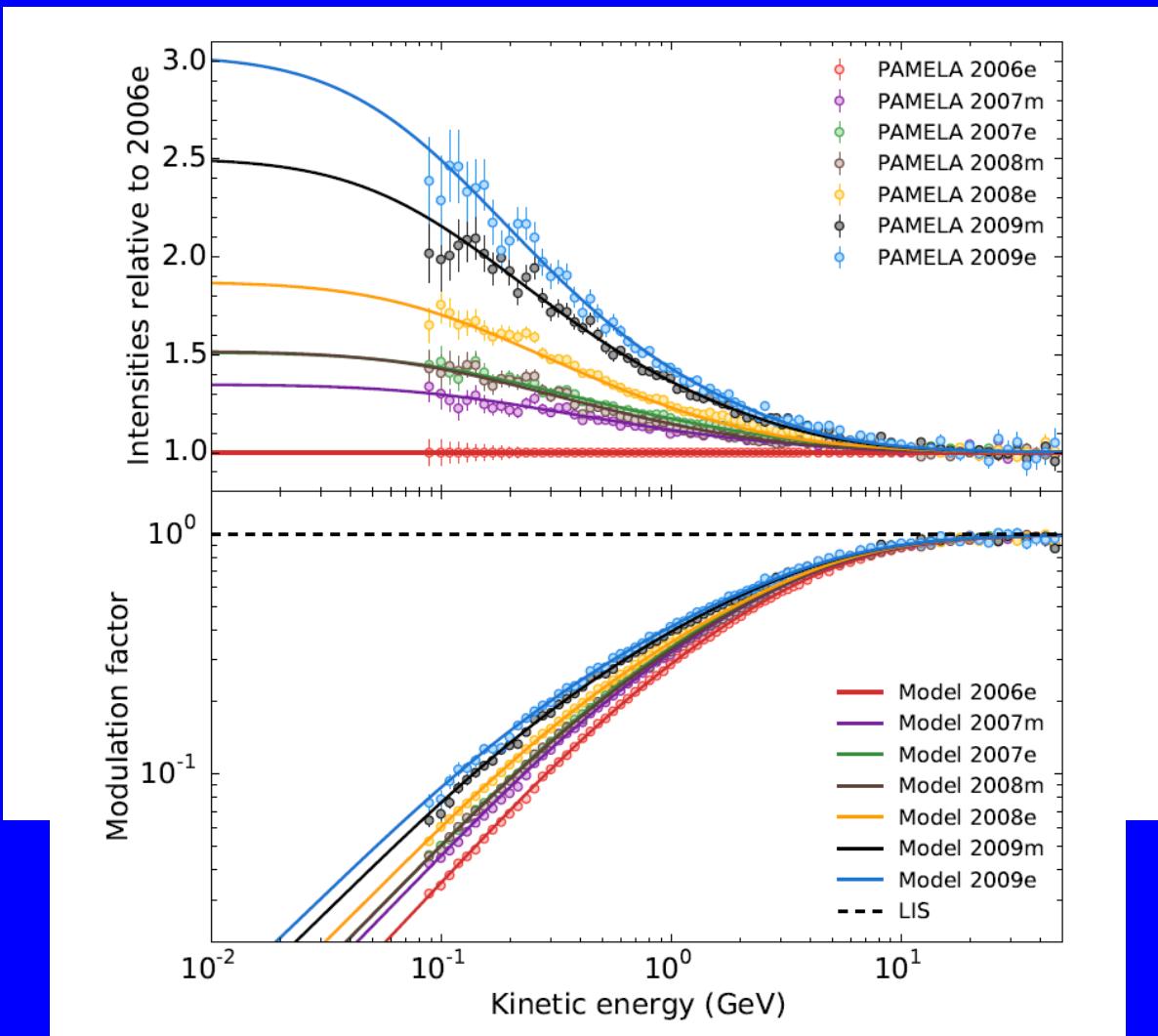
# PAMELA Electrons and Positrons for 2009

## Numerical modeling with particle drifts

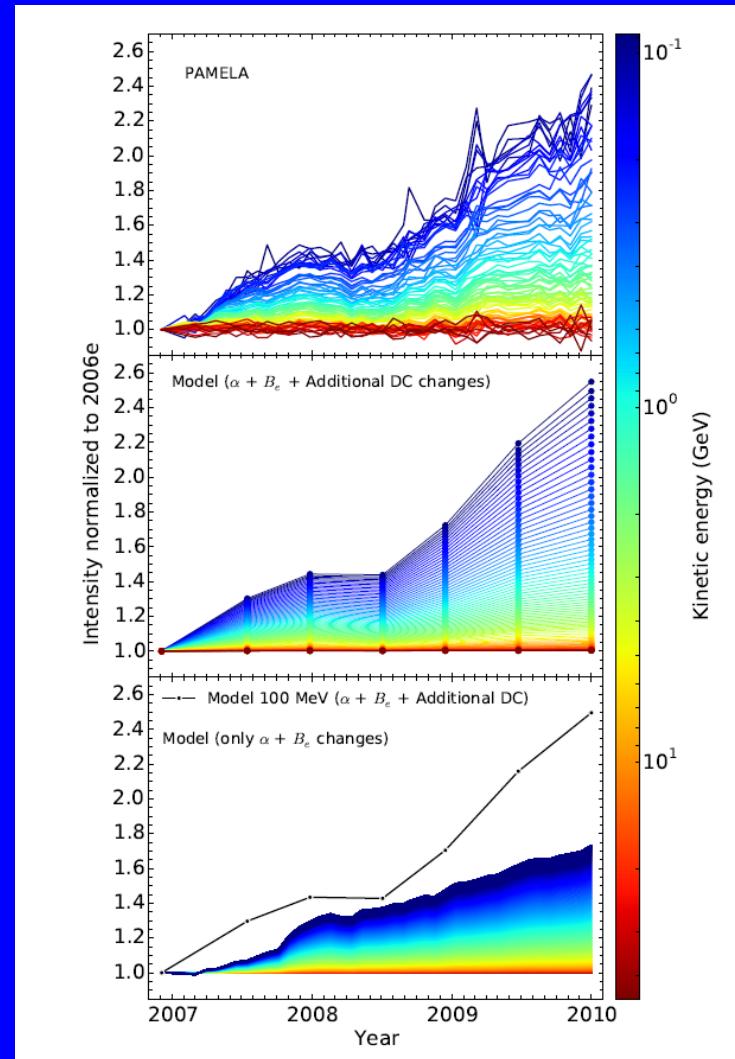
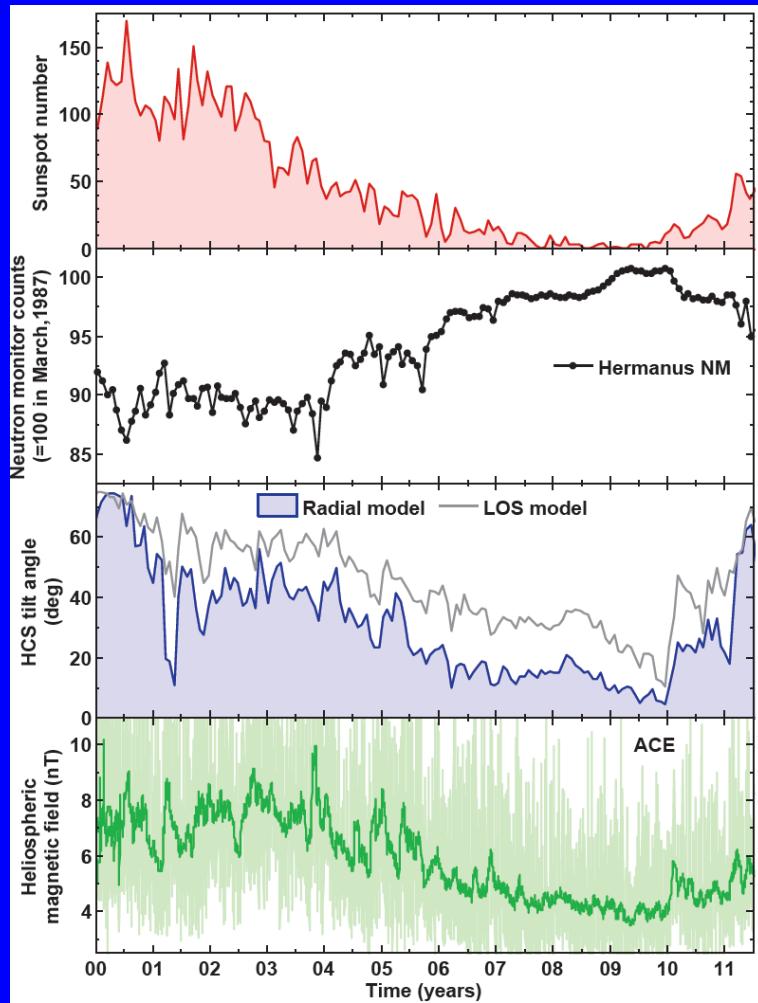


Preliminary; see PhDs of Riccardo Munini and Etienne Vos

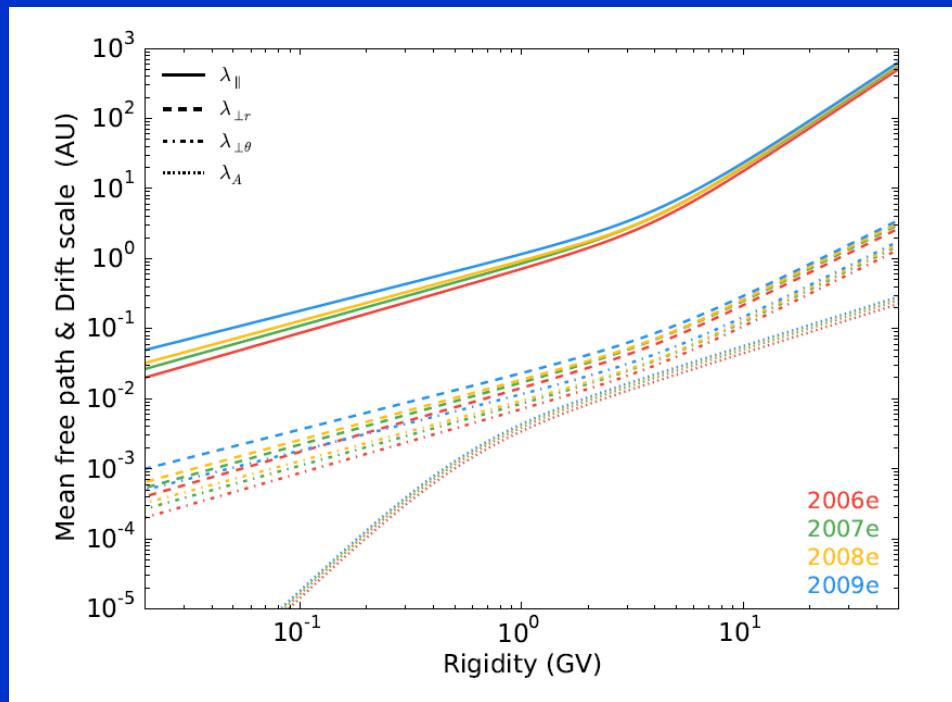
# Total Modulation of Protons: Observations and Modeling



# Proton modulation during the unusual 2009 minimum period



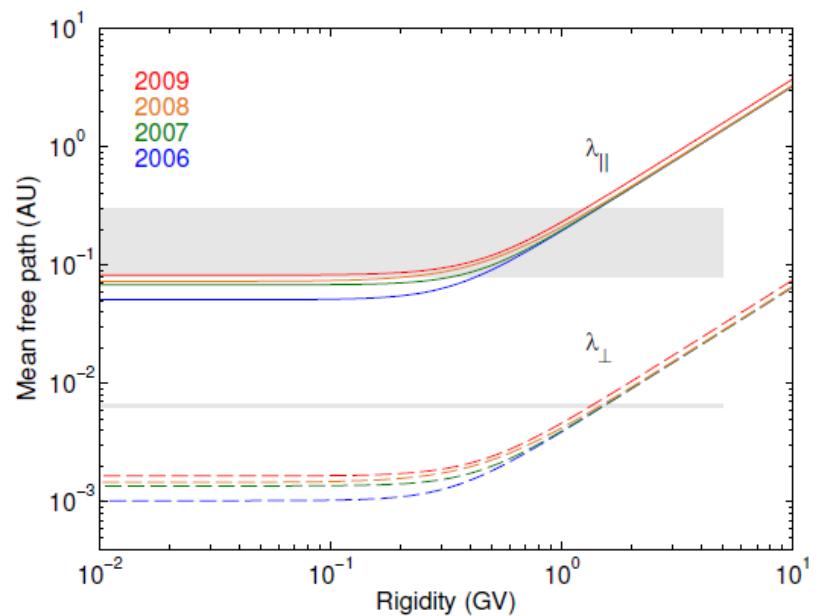
# Consequences for Diffusion & Drift Theory



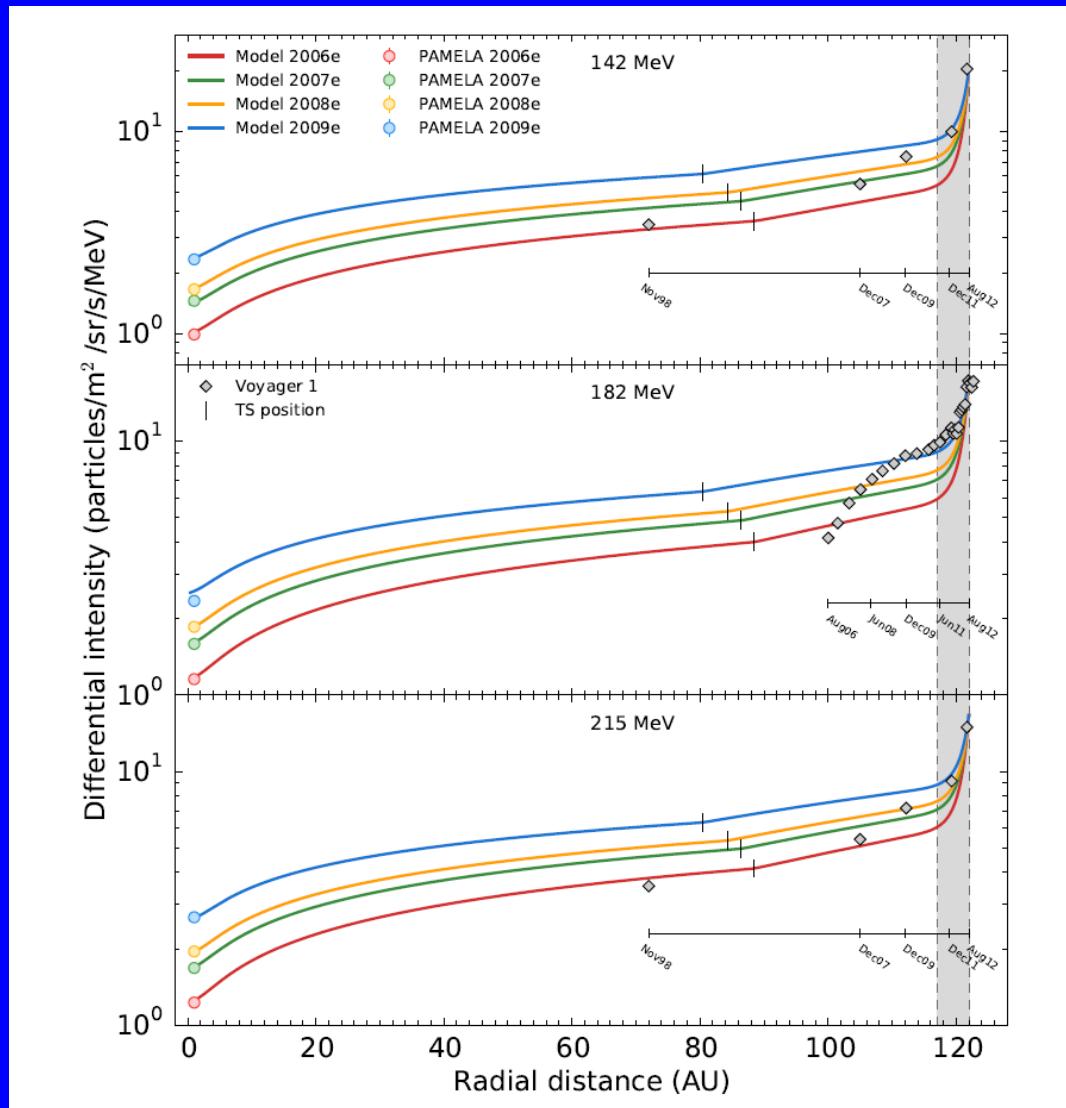
Protons

Electrons

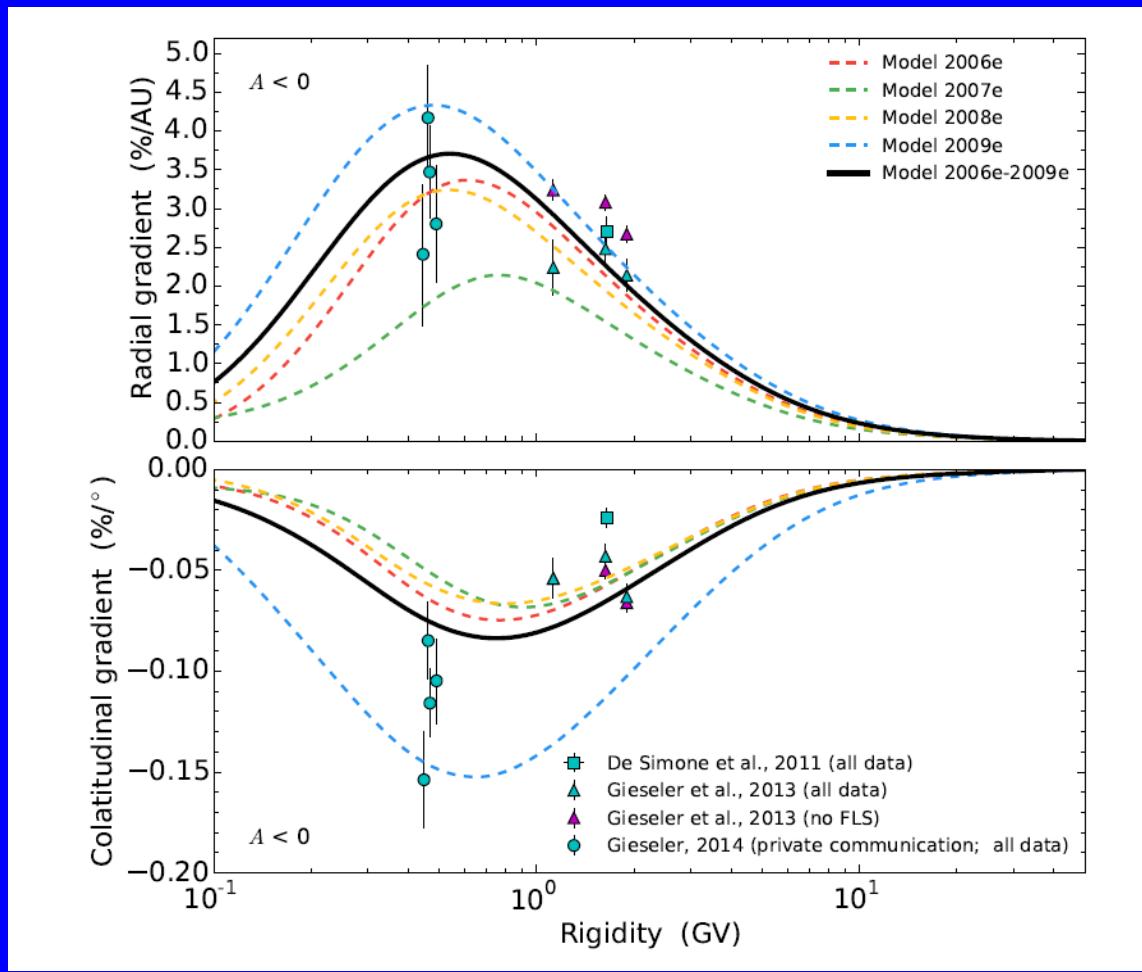
At Earth



# Proton Radial Profile Observations and Modeling

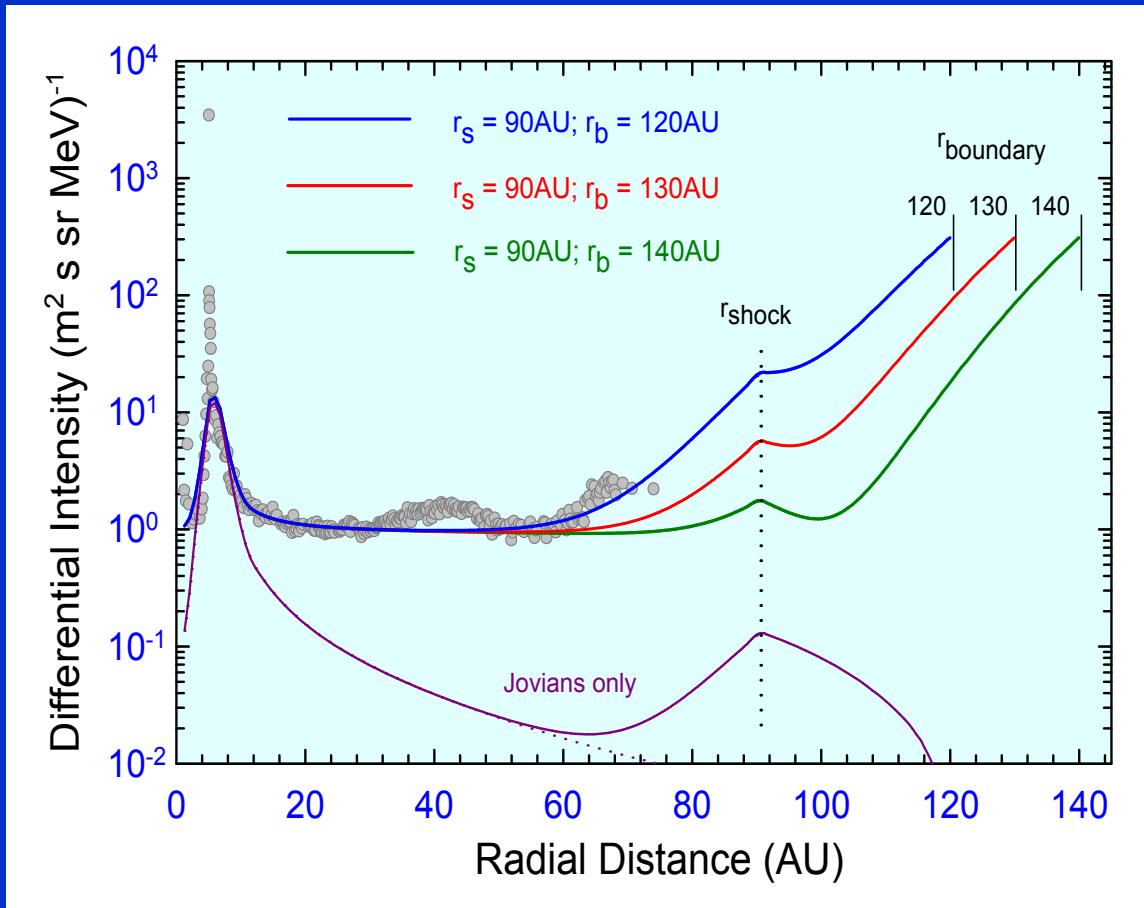


# Proton Radial and Latitudinal Modeling and Observations



# Predicted Galactic and Jovian electron modulation at 16 MeV

## HP at different positions; extending heliosheath



### Conclusions:

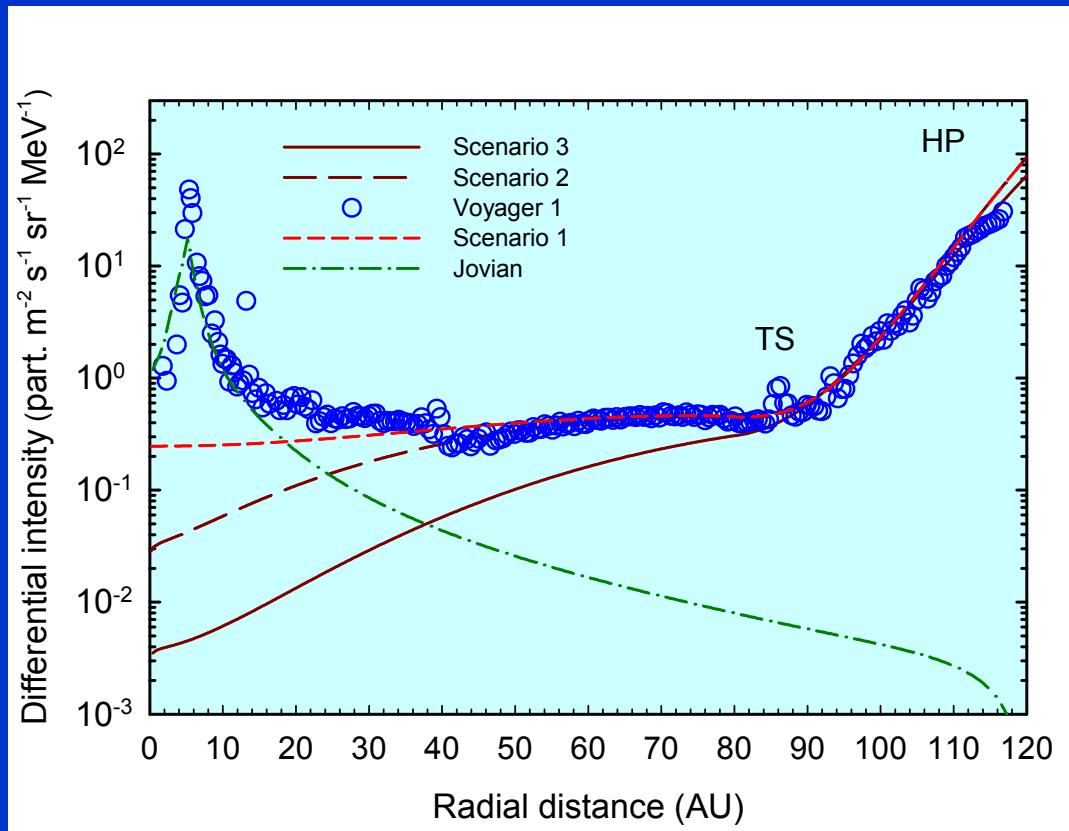
The heliosheath was predicted to act as a strong modulation 'barrier'...

The closer the HP is to the TS, the less the effect of the TS is....

The 120 AU scenario seems to be the closest to recent observations.

Jovian electrons dominate over first ~25 AU.

# Computed radial profile of galactic and Jovian electrons at 12 MeV



## Conclusions:

Extraordinary type of modulation in heliosheath (HS) ...

The HS indeed acts as a strong modulation 'barrier' for these low energy electrons ...

With the HP position and LIS known, we can attempt to predict intensity of 12 MeV galactic electrons at the Earth.

V1: 4-16 MeV; Webber (private comm.)

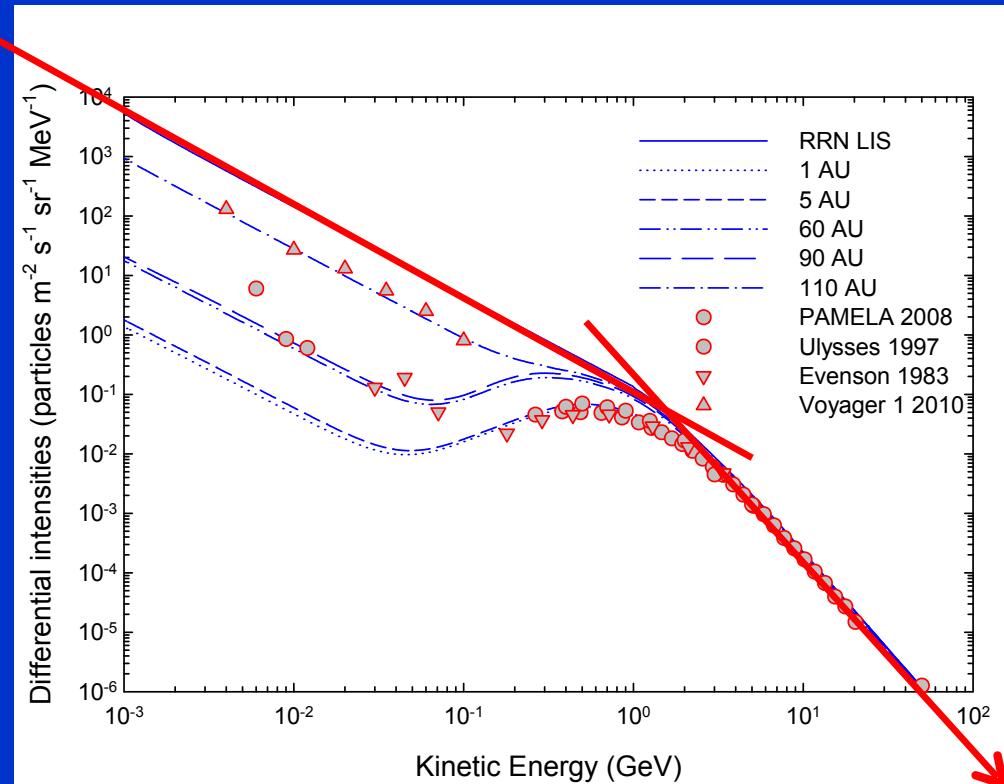
Potgieter & Nndanganeni, Astrophys. Space Sci. 2013

## Recap: The galactic electron HPS = VLIS (= LIS = GS)

Computed electron spectra at different radial distances

$$E^{-(1.5 \pm 0.1)}$$

Voyager 1



V1 2010 electron observations  
(Webber, May 2011)

$$E^{-(3.15 \pm 0.05)}$$

PAMELA

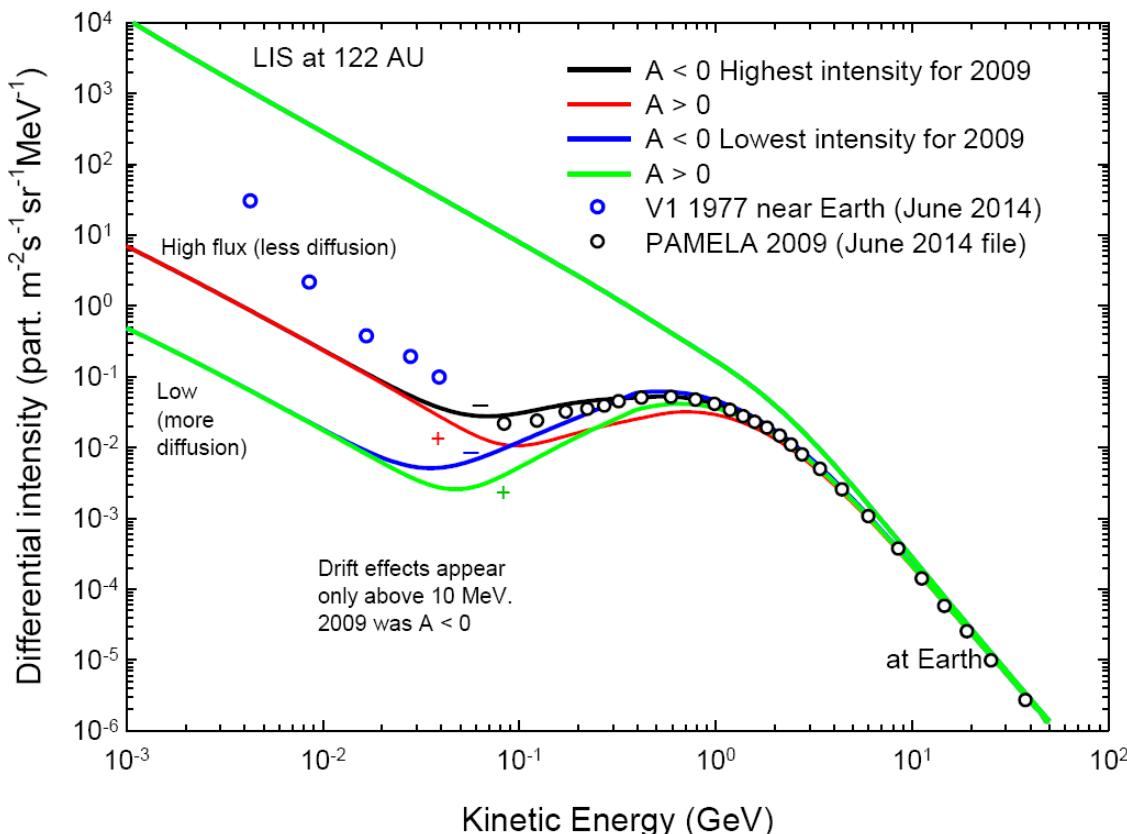
Galactic electron spectrum at the HP seems to consist of two power laws...!  
 $E < \sim 200 \text{ MeV}$  and  $E > \sim 3 \text{ GeV}$

# Main features of total Electron Modulation

## Modeling scenarios

### Galactic Electrons; two intensity scenarios at Earth

LIS spectral index is preserved at low energies because diffusion dominates there, not adiabatic cooling



HPS has a power-law 500 MeV.

At low energies this power-law is preserved up to Earth if....

Galactic electron intensity at Earth is not known because of ....

PAMELA data at lower energies reduce the predicted uncertainty ... below 200 MeV.

## Total modulation (Modulation Factor: LIS/Earth)

**Protons:** Modulation factor (MF) as the ratio of the very LIS intensity to the computed intensity at the Earth in terms of kinetic energy for the periods 2006 to 2009.

$E$ (GeV)	0.001	0.01	0.10	1.00	10.0	50.0
2006	2174	270	29	3.5	1.15	1.01
2007	1429	179	20	3.0	1.13	1.01
2008	1163	143	17	2.8	1.12	1.01
2009	714	89	12	2.4	1.11	1.01

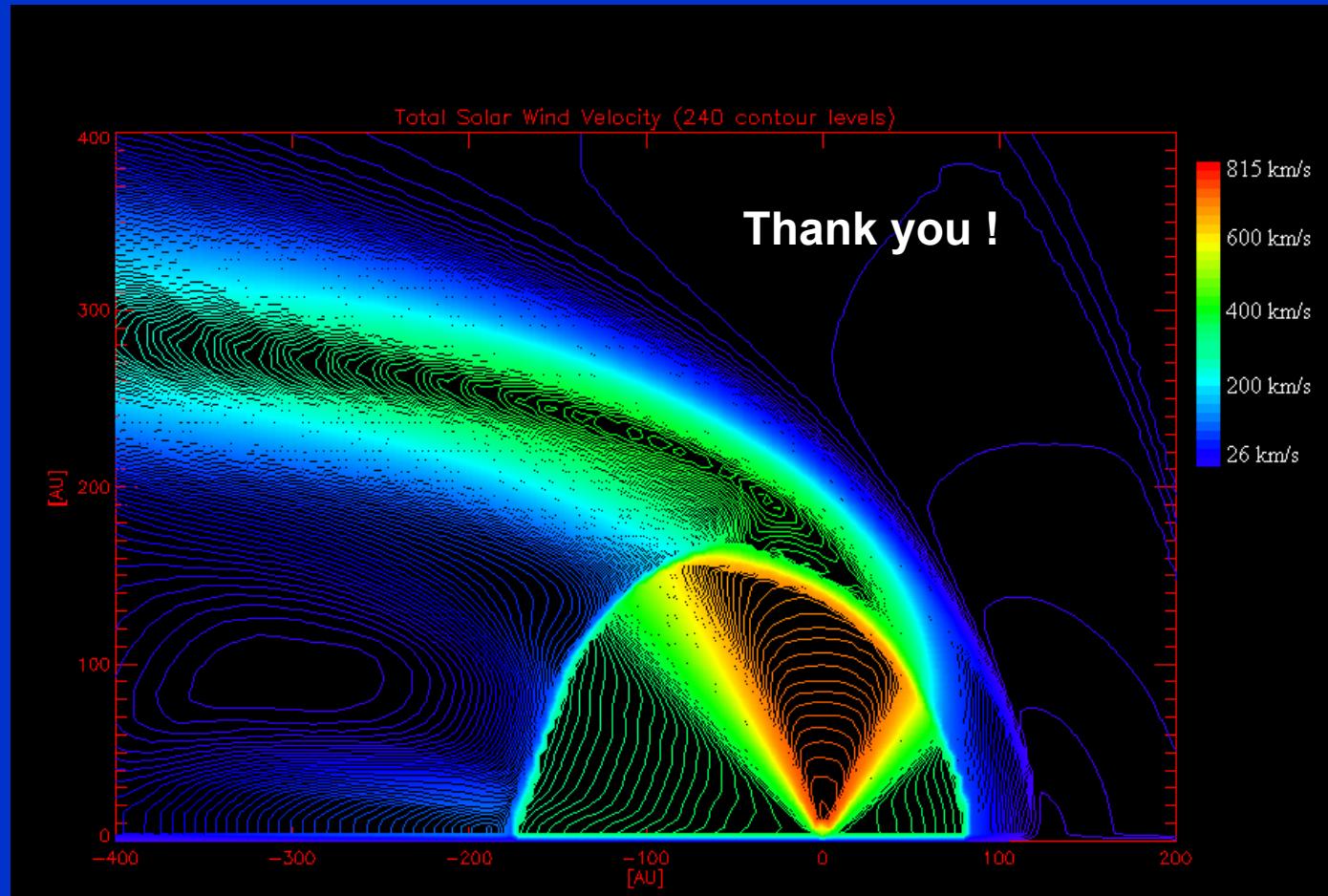
**Electrons:** Modulation factor (MF) as the ratio of the very LIS intensity to the computed intensity at the Earth in terms of kinetic energy  $E$  for 2009a and 2009b.

$E$ (GeV)	0.05	0.10	0.20	0.50	0.80	1.0	3.0	5.0	8.0	10.0
2009a	1030	530	140	17.9	7.30	5.14	1.76	1.38	1.21	1.16
2009b	759	438	128	17.3	7.21	5.08	1.75	1.38	1.21	1.16

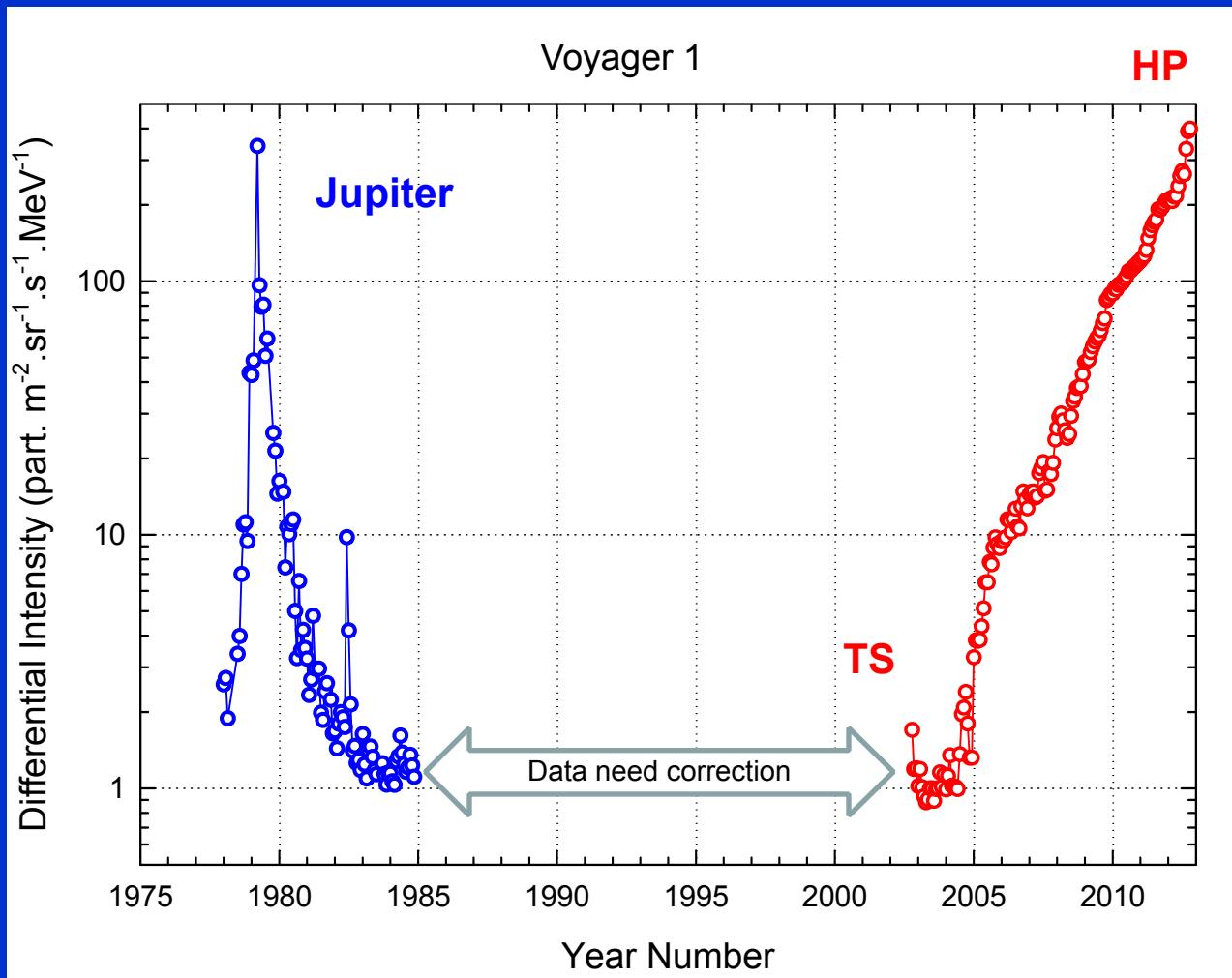
# Concluding Remarks

- Electron, proton, helium and carbon HPS (very LIS) are established...
- Finally, we can study and determine the total modulation of GCRs...
- Comprehensive modeling gives significantly useful insights...
- In particular concerning drift effects (cannot be done with FF-approach).
- Combined with observations we have made good progress,
- Towards a general diffusion and drift theory... but
- We need to address the complications introduced by the heliosheath.
- We need more good observational data...
- The AMS2 era...

# Solar wind velocity profiles in the heliosphere



## Electrons 5-12 MeV: First 25 AU compared to the last 30 AU



Extraordinary decrease in low energy electrons in the inner heliosheath

Webber et al. 2013

## Improvements for the HMF geometry - it can get ugly

$$B = B_0 \left[ \frac{r_0}{r} \right]^2 \sqrt{1 + \left[ \frac{\Omega(r - r_\odot) \sin \theta}{V_{sw}} \right]^2 + \left[ \frac{r \delta(\theta)}{r_\odot} \right]^2},$$

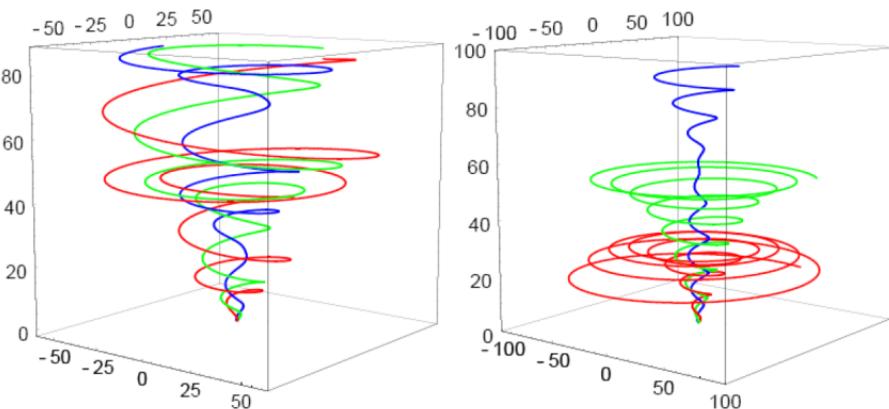
Modified Parker type HMF

$$\tan \psi = \frac{\Omega(r - b) \sin \theta}{V_{sw}(r, \theta)} - \frac{r}{b} \frac{V_{sw}(b, \theta)}{V_{sw}(r, \theta)} \left( \frac{B_T(b)}{B_R(b)} \right),$$

Smith & Bieber 1991

$$\begin{aligned} B_r &= B_0 \left[ \frac{r_0}{r} \right]^2 \\ B_\theta &= B_r \frac{(r - r_{ss})}{V_{sw}} \sin \beta \sin \left( \phi + \frac{\Omega(r - r_{ss})}{V_{sw}} \right) \\ B_\phi &= B_r \frac{(r - r_{ss})}{V_{sw}} \left[ \omega \sin \beta \cos \theta \cos \left( \phi + \frac{\Omega(r - r_{ss})}{V_{sw}} \right) + \sin \theta (\omega \cos \beta - \Omega) \right] \end{aligned}$$

Fisk (1996) type HMF

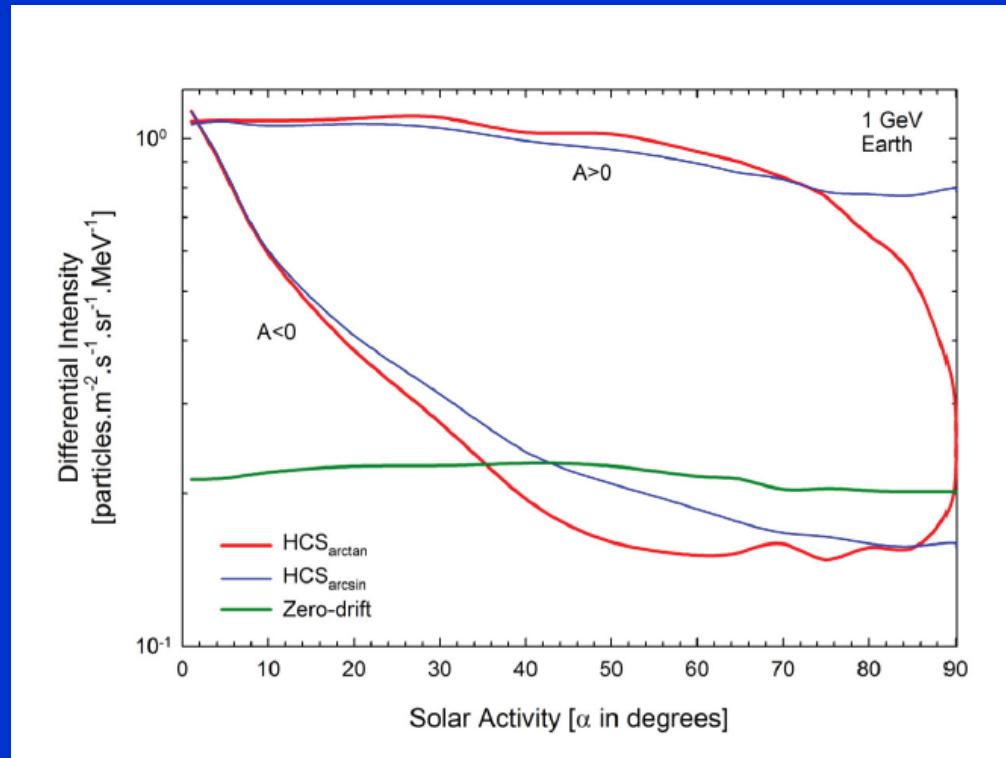


Is this reality...?

Possible Evidence for a Fisk-type Heliospheric Magnetic Field I:  
Analysing Ulysses/KET Electron Observations

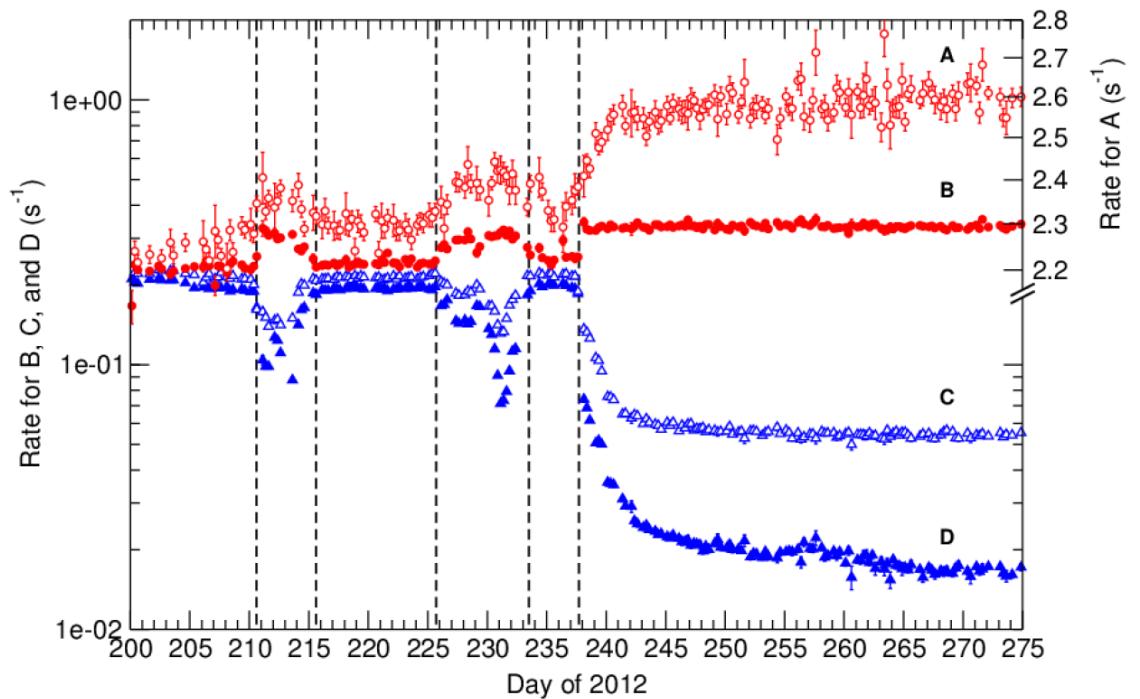
O. Sternal, N.E. Engelbrecht<sup>1</sup>, R.A. Burger<sup>1</sup>, S.E.S. Ferreira<sup>1</sup>, H. Fichtner<sup>2</sup>, B. Heber,  
A. Kopp, M.S. Potgieter<sup>1</sup> and K. Scherer<sup>2</sup>

# Effects of the wavy HCS on proton modulation



Raath, Potgieter, Strauss, ASS, 2015

# What happened in August 2012?



GCR protons K.E. > 70 MeV

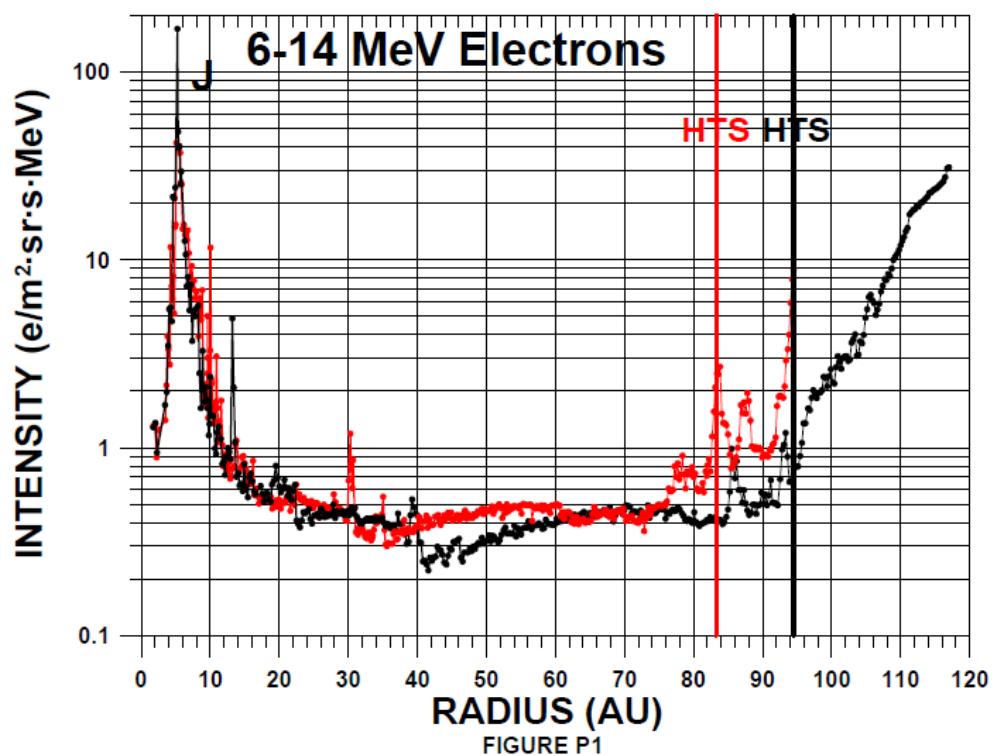
GCR electrons 7 to ~100 MeV

ACR protons 7 to 60 MeV

TSPs 0.5 to 30 MeV

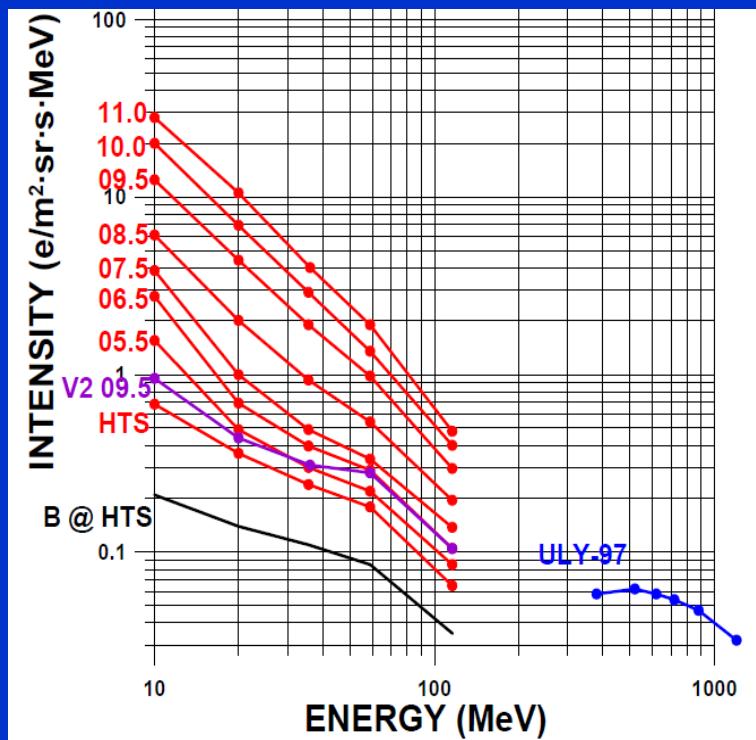
# Galactic electron observations into the heliosheath

Webber et al. 2012: Voyager 1 & 2 Observations



HTS: Heliospheric Termination Shock  
Red line: V2  
Black line: V1

Voyager 1 spectra



# Theory and dimensional complexity (1D to 2D to 3D)

## Analytical solution of Parker's basic TPE

- Convection-diffusion approach
- Force-Field approach

## 1D numerical approach to Parker's TPE

### Convection-diffusion approach

$$\frac{\partial f}{\partial t} + \nabla \cdot (\mathbf{V}f - \mathbf{K} \cdot \nabla f) - \frac{1}{3p^2} (\nabla \cdot \mathbf{V}) \frac{\partial}{\partial p} (p^3 f) = 0$$

$$\frac{\partial f}{\partial t} + \mathbf{V} \cdot \nabla f - \nabla \cdot (\mathbf{K} \cdot \nabla f) - \frac{1}{3} (\nabla \cdot \mathbf{V}) \frac{\partial f}{\partial \ln p} = 0$$

$$\mathbf{S} = \mathbf{V}f - \mathbf{K} \cdot \nabla f$$

$$= Vf - \kappa \frac{\partial f}{\partial r}$$

$$\approx 0$$

$$f_{Earth} = f_{LIS} \exp \left[ - \int_{r_e}^{r_{HP}} \frac{V dr}{\kappa} \right]$$

# Complexity & Dimension issues

## Convection-diffusion approach

$$f_{Earth} = f_{LIS} \exp \left[ - \int_{r_e}^{r_{HP}} \frac{V dr}{\kappa} \right]$$

$$M = \int_{r_{Earth}}^{r_{Boundary}} \frac{V}{\kappa} dr = \frac{V}{\kappa} [r_{boundary} - r_{Earth}]$$

Modulation parameter

# Complexity & Dimension issues

## Force-Field approach

$$\mathbf{S} = C \nabla f \cdot \mathbf{K} \cdot \nabla f = CVf - \kappa \frac{\partial f}{\partial r} = 0$$

$$f_{Earth} = f_{Boundary} \exp \left[ - \int_{r_E}^{r_B} \frac{CVdr}{\kappa} \right], \quad \text{with } C = -\frac{1}{3} \frac{\partial \ln f}{\partial \ln p}$$

Important: This gives an energy loss without considering the adiabatic process

## Approximated Force-Field approach

$$j(T) = j_{LIS}(T + \Phi) \frac{T(T + 2E_0)}{(T + \Phi)(T + \Phi + 2E_0)}$$

Valid if  $\kappa$  is separable :  $\kappa = \kappa_1(r)\kappa_2(P)$  with  $\kappa_2 = \beta P$  when  $\beta \approx 1$

# Complexity & Dimension issues

Force Field gives an indication of the modulation level (depth), nothing more, nothing about the physics responsible, always 'forced' approximated solutions in 1 D, so that your heliosphere looks like this:

---

**1 D spherically symmetric, steady-state, numerical approach**

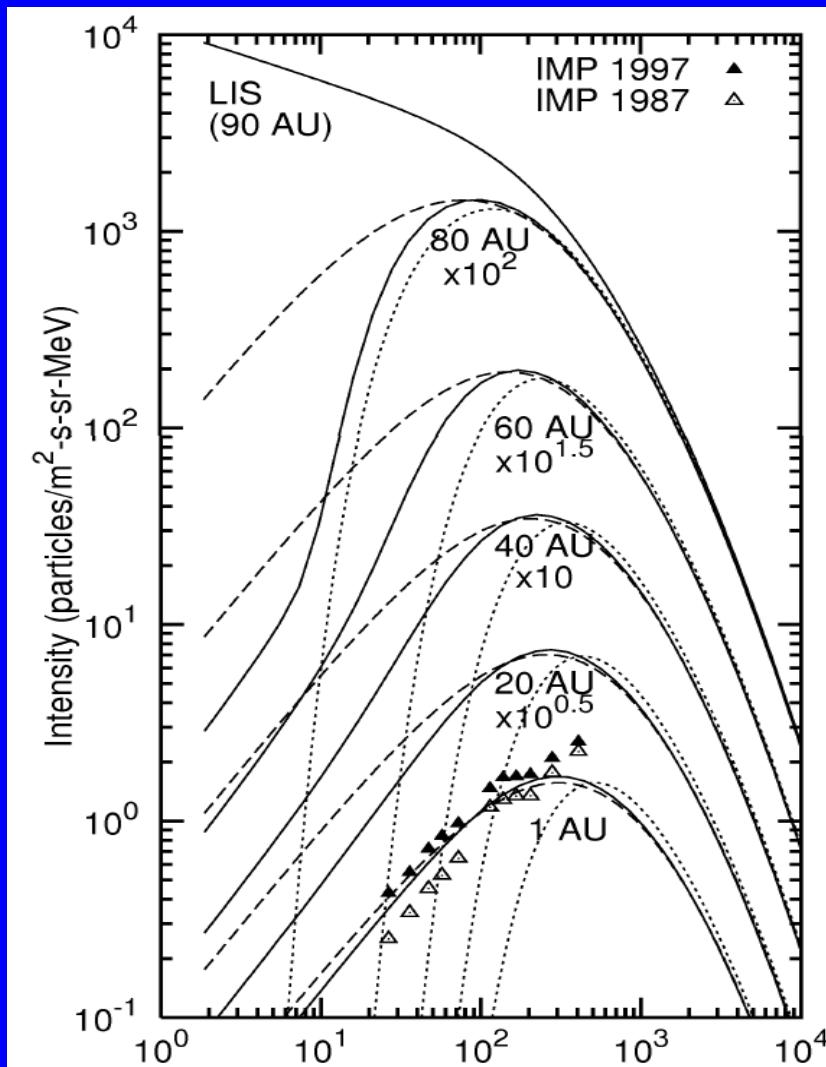
$$V \frac{\partial f}{\partial r} - \frac{1}{r^2} \left( r^2 \kappa \frac{\partial f}{\partial r} \right) - \frac{1}{3r^2} \frac{\partial}{\partial r} \left( r^2 V \right) \frac{\partial f}{\partial \ln p} = 0$$

Input: LIS,  $r_{Boundary}$ ,  $V(r)$  and  $\kappa(r, P)$

Ouput: Adiabatic energy loss now taken care of;

Approximation:  $\phi = \beta \kappa_2 \int_{r_{Anywhere}}^{r_B} \frac{V}{\kappa} dr$

# Comparison of the three 1 D modeling approaches



Modulation modeling dilemma:

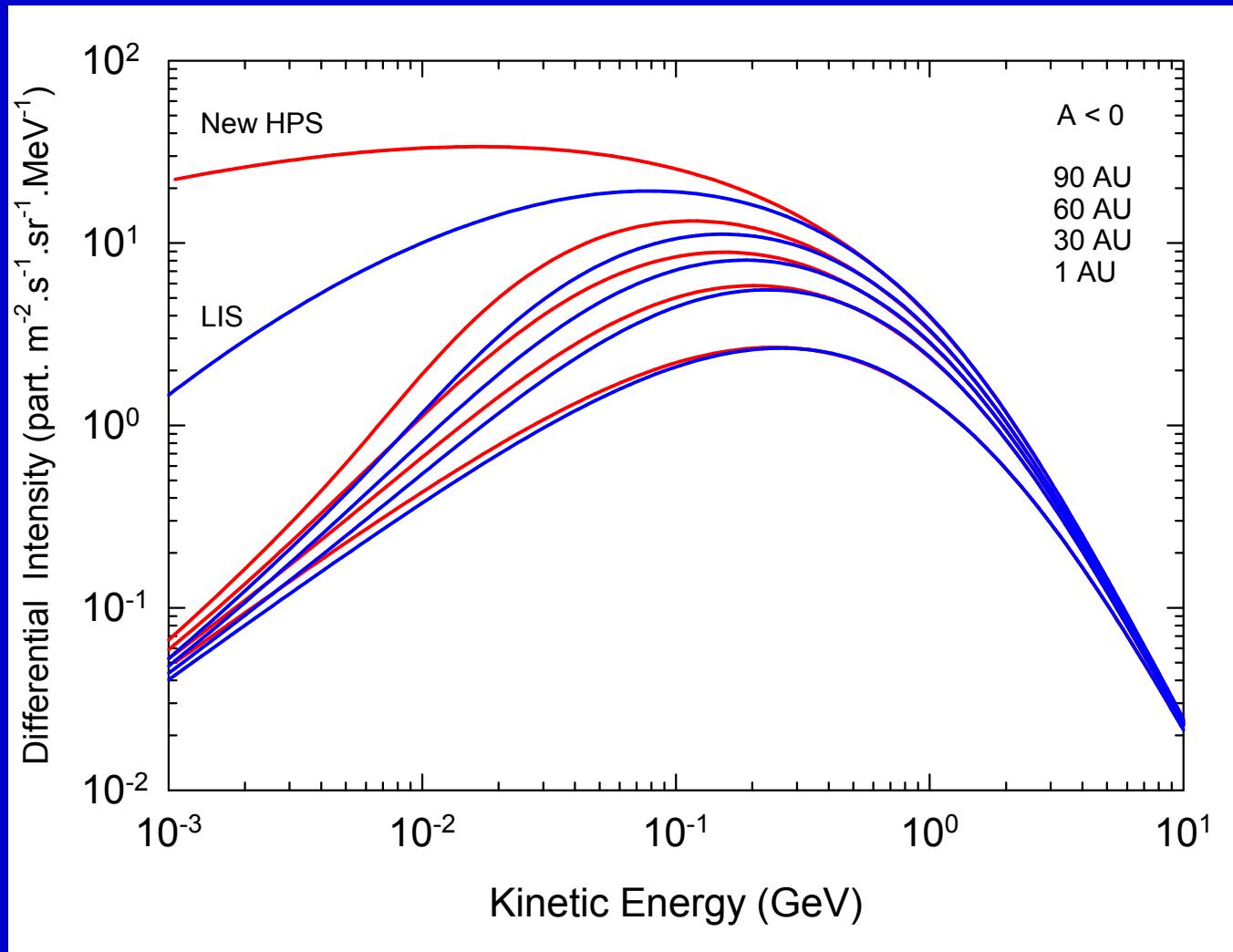
The diffusion tensor is not known well enough in terms of the spatial and rigidity dependence of its elements...

The VLIS is not known well enough below a few GeV.

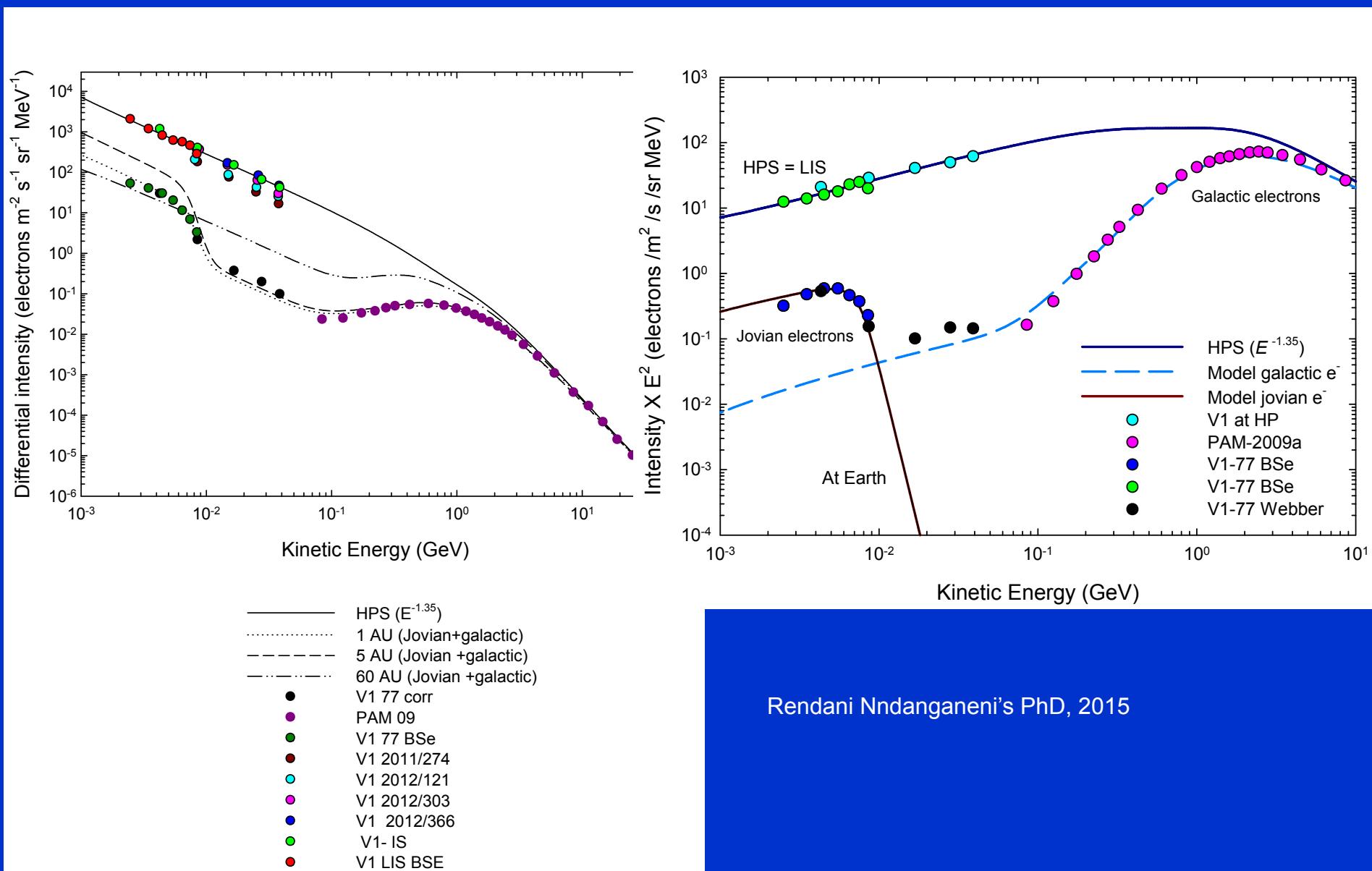
Only scenarios can thus be studied.

But, with good data at Earth and near the heliopause, progress can be made...

## Comparison of modulation with a LIS vs. HPS

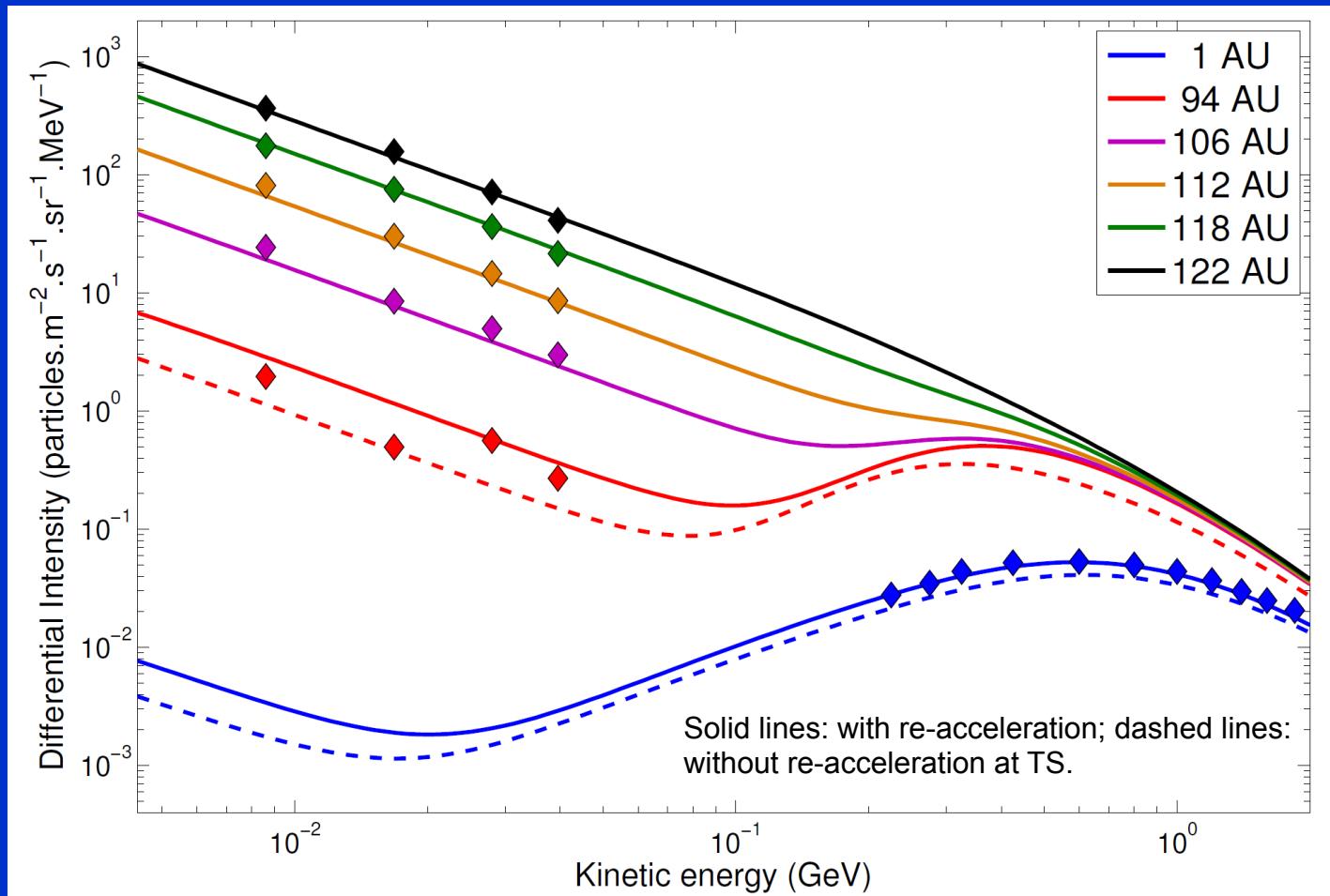


# Combining Voyager 1 and PAMELA observations with modeling

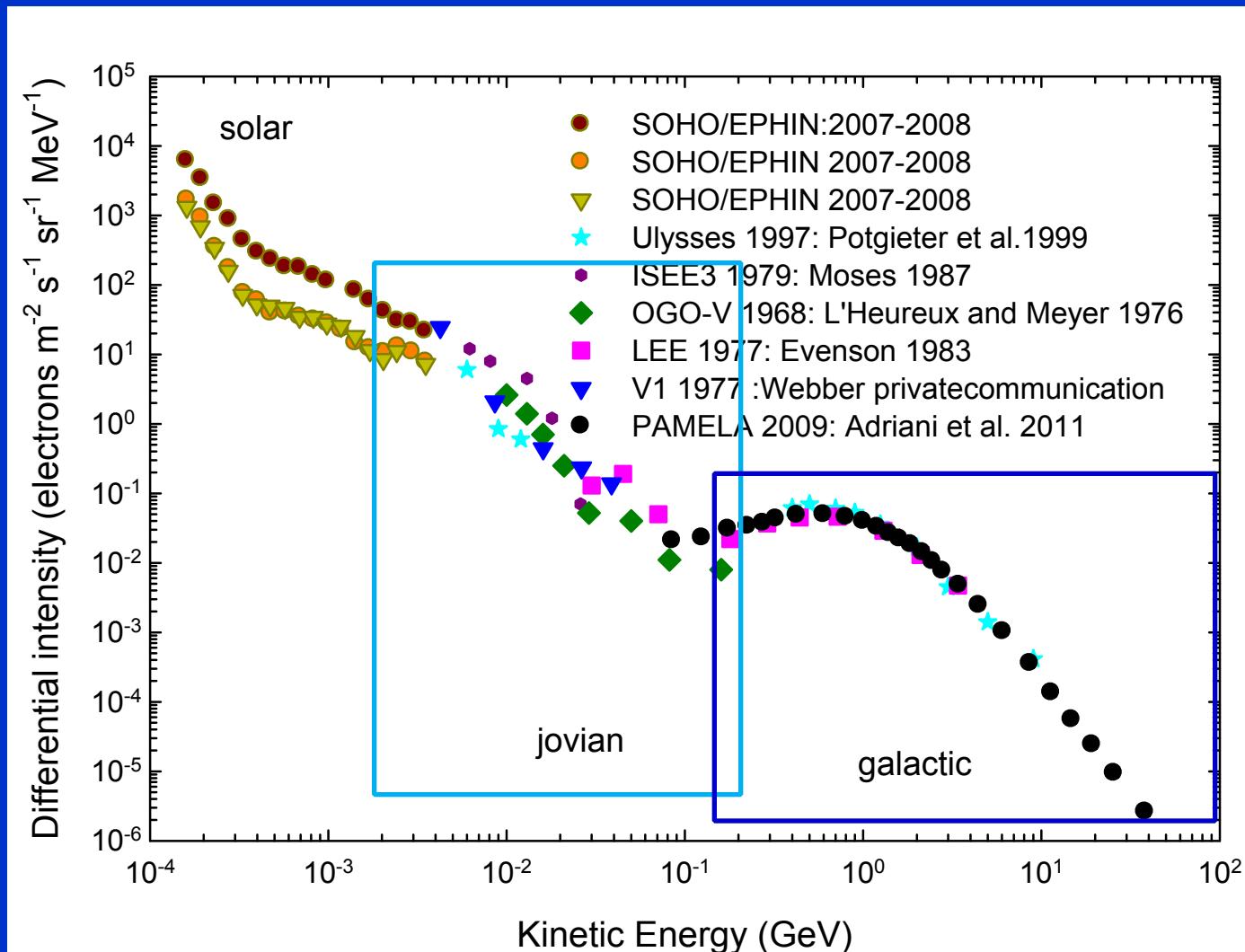


Rendani Nndanganeni's PhD, 2015

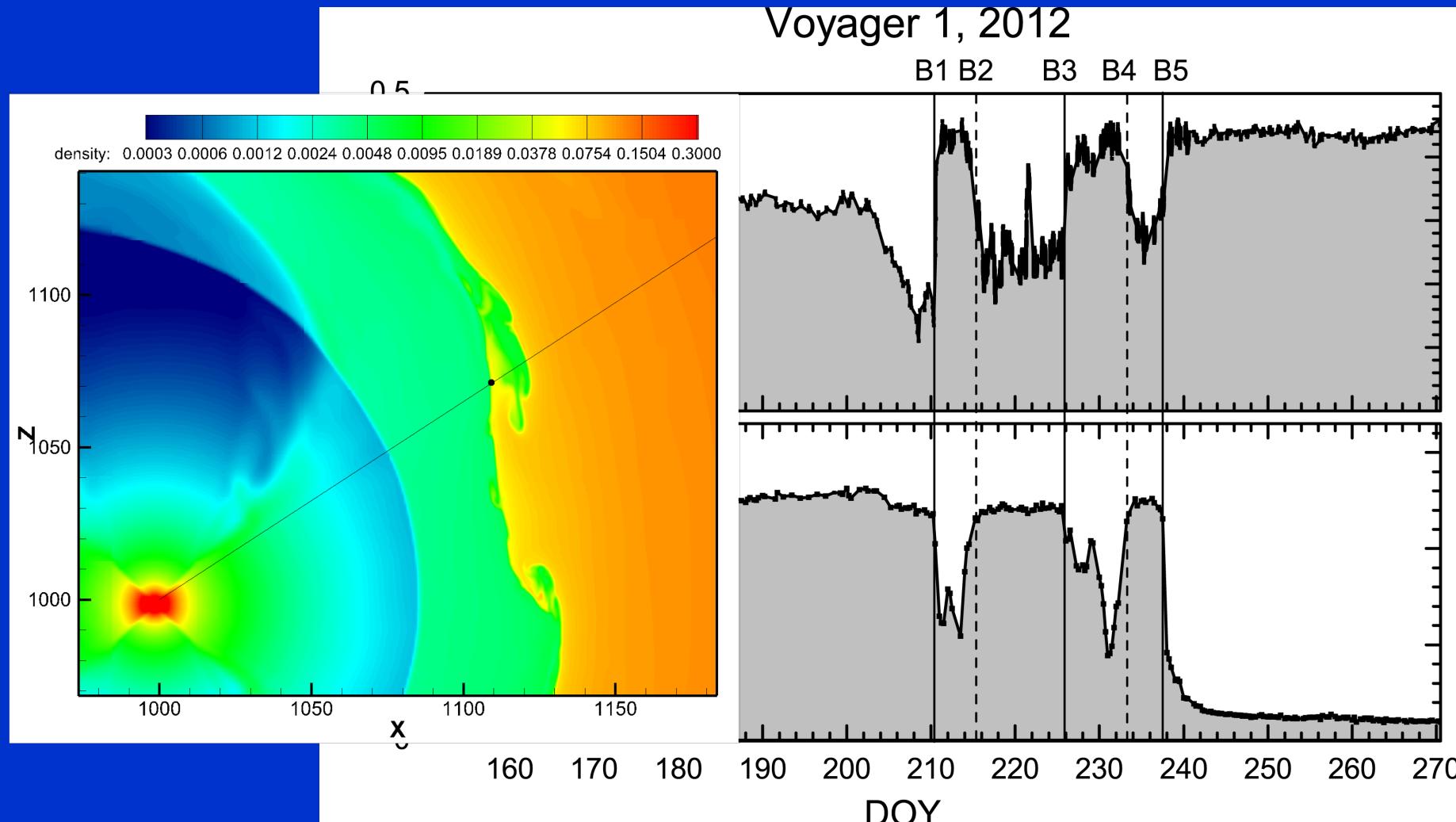
# What are the effects of re-acceleration at the TS ?



# Global Modulation of Galactic Electrons Observed at and close to Earth



# What happened inside the HP



# Very Local Interstellar Spectra

The LIS resulting from these parameters can be approximated (within 15%) over the energy range 3 MeV to 100 GeV by the following expressions.

The approximate proton LIS is given by:

$$J(E) = 3719 \frac{1}{\beta^2} E^{1.03} \left( \frac{E^{1.21} + 0.77^{1.21}}{1 + 0.77^{1.21}} \right)^{-3.18}, \quad (1)$$

the approximate Helium LIS is given by:

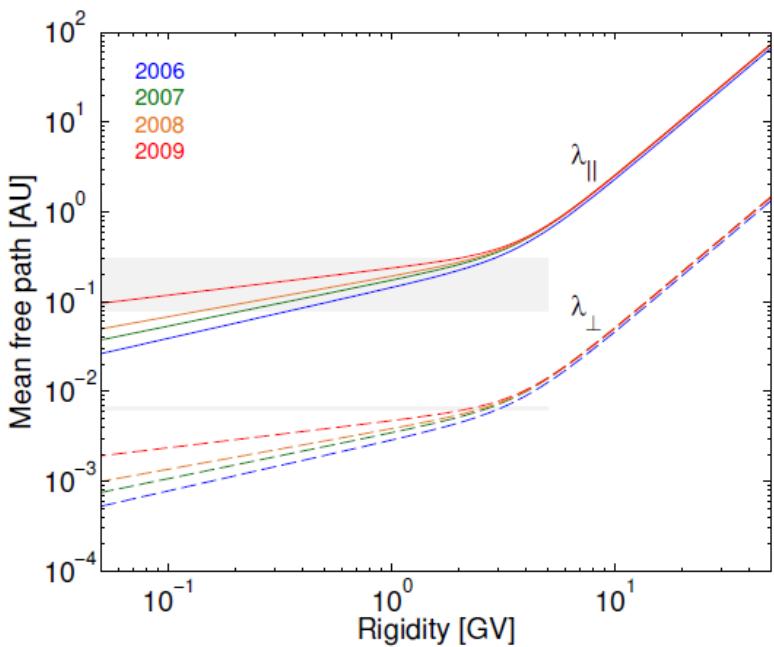
$$J(E) = 195.4 \frac{1}{\beta^2} E^{1.02} \left( \frac{E^{1.19} + 0.60^{1.19}}{1 + 0.60^{1.19}} \right)^{-3.15}, \quad (2)$$

and the approximate Carbon LIS is given by:

$$J(E) = 0.832 \frac{1}{\beta^2} E^{1.29} \left( \frac{E^{0.74} + 1.25^{0.74}}{1 + 1.25^{0.74}} \right)^{-5.62}, \quad (3)$$

where the CR intensity ( $J(E)$ ), given in  $\text{particles} \cdot \text{m}^{-2} \cdot \text{s}^{-1} \cdot \text{sr}^{-1} \cdot (\text{GeV/nuc})^{-1}$ ) is a function of the kinetic energy per nucleon ( $E$ , given in  $\text{GeV/nuc}$ ).

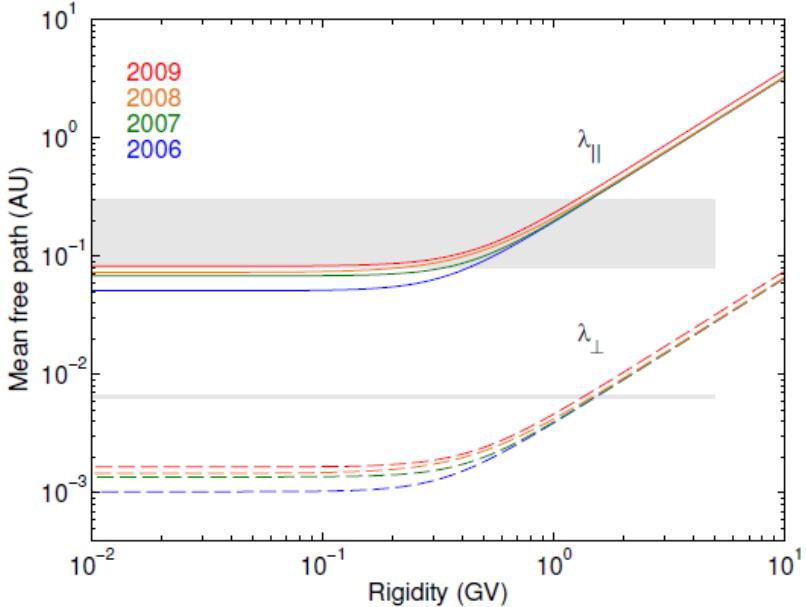
# Diffusion Theory: Difference between protons and electrons



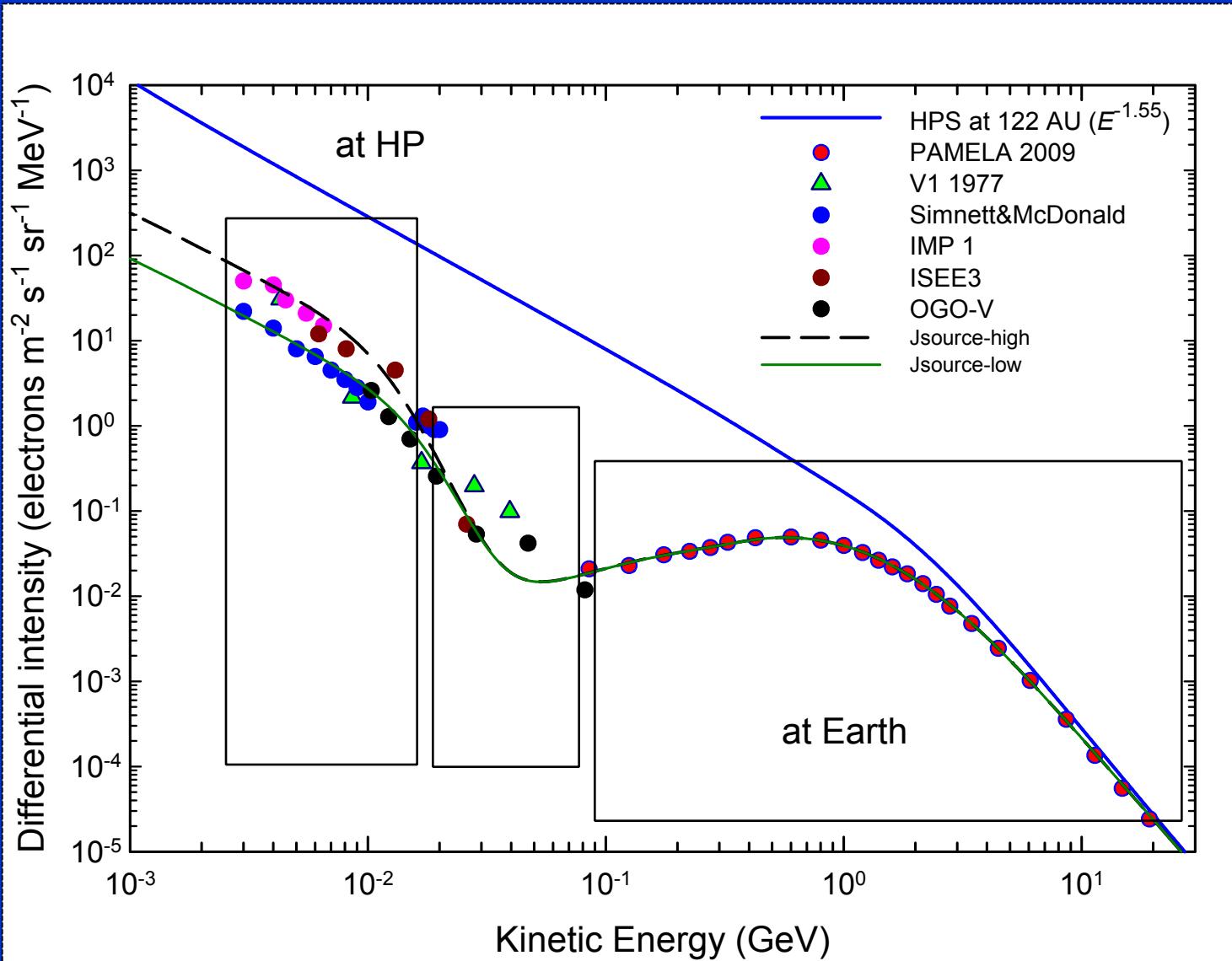
Protons

Electrons

At Earth

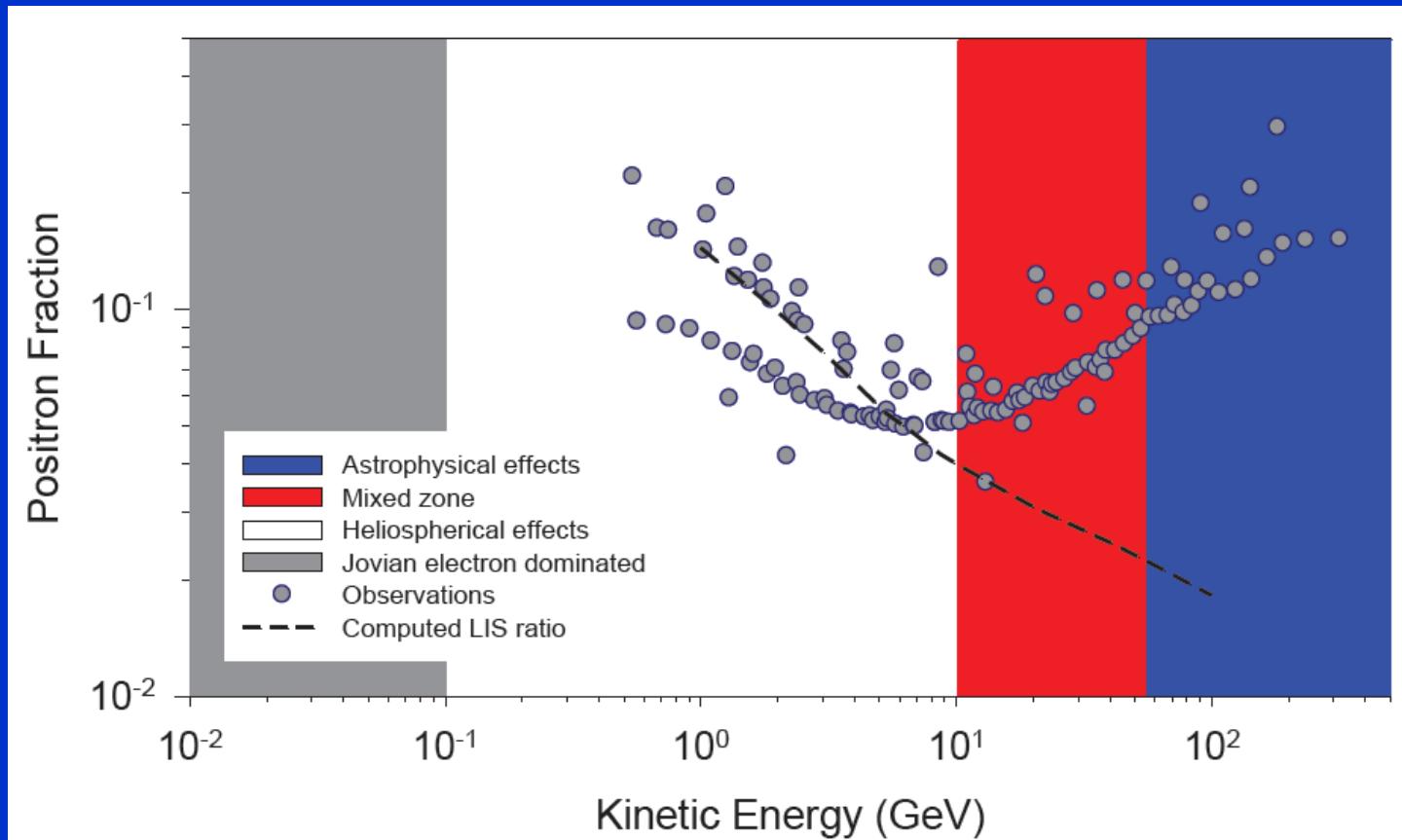


## Main features of electron modulation with a Jovian source

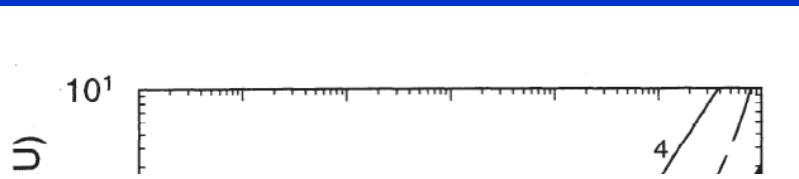


# Positron fraction at the Earth

## Effects of solar modulation with particle drifts



# Basics of Diffusion & Drift Theory



Energy (rigidity) dependence

