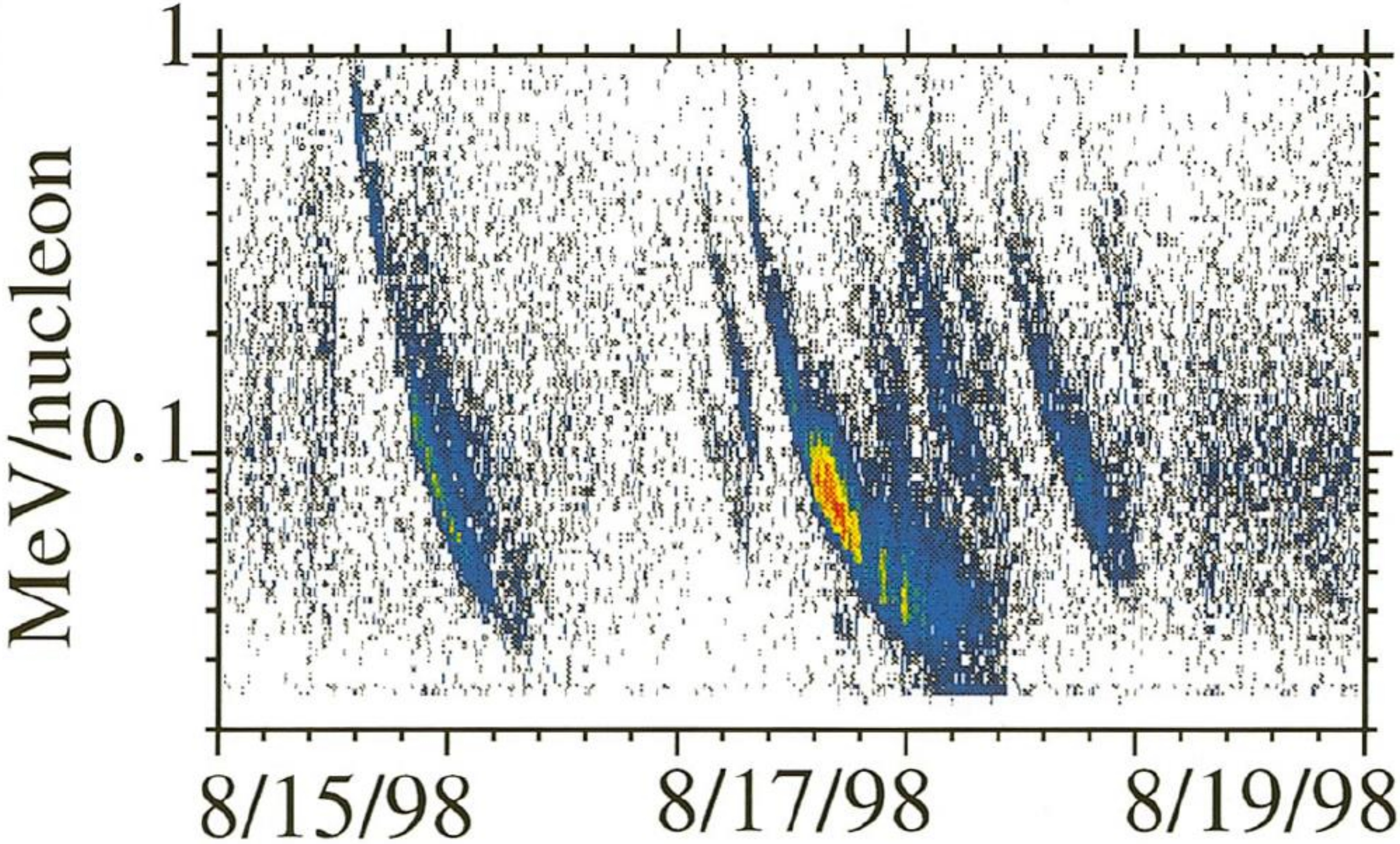
The Theory of SEP Acceleration and Transport

Marty Lee

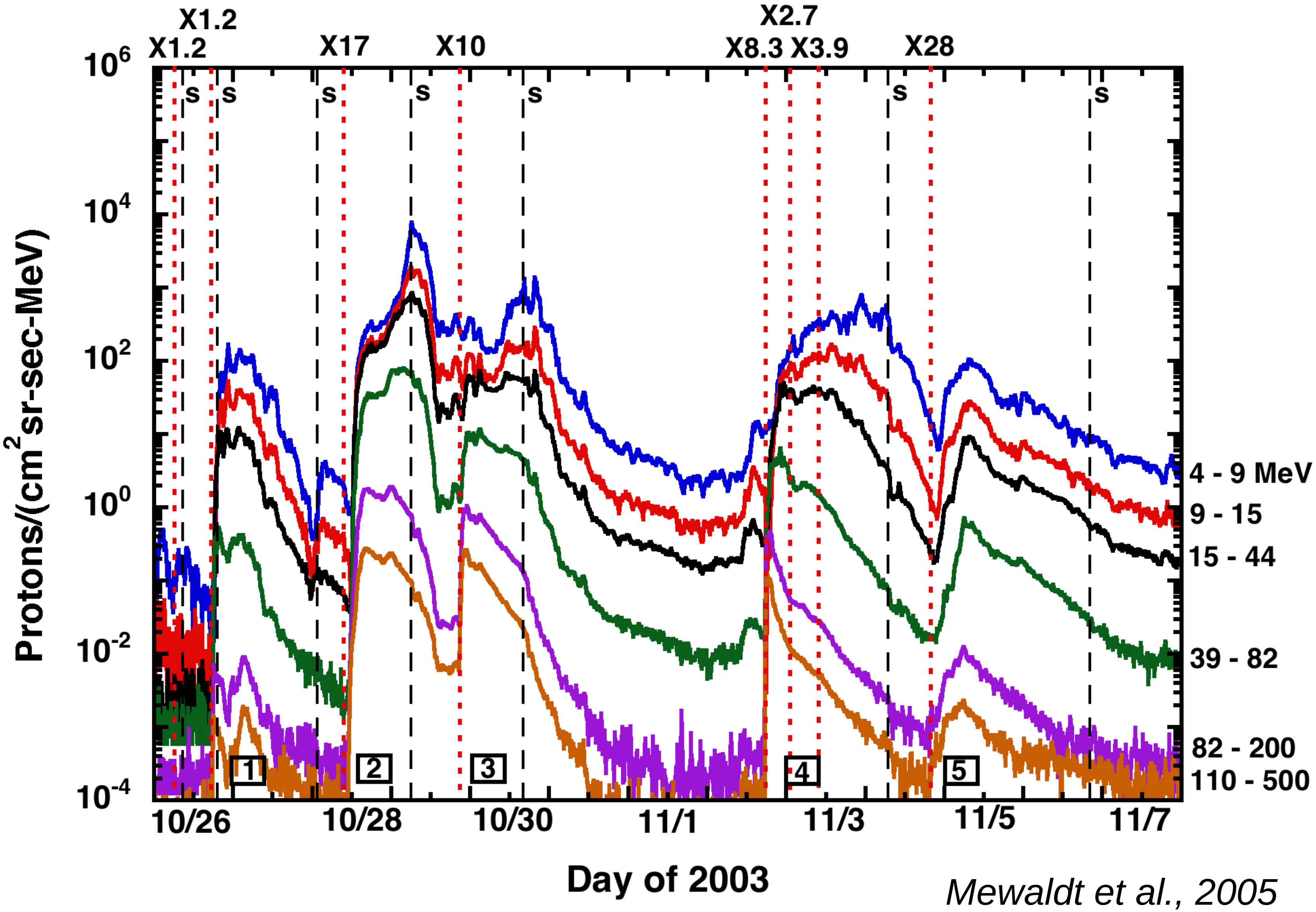
University of New Hampshire
USA

Impulsive Events



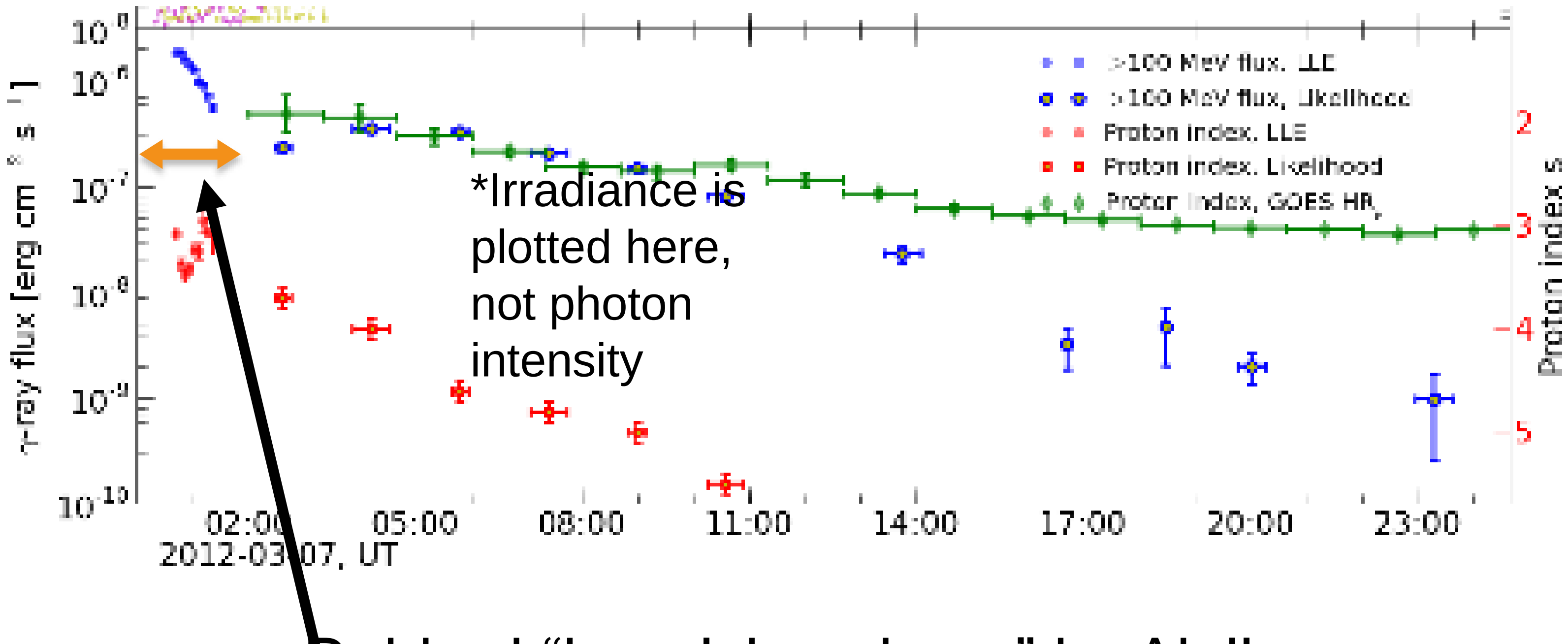
Mason et al., 1999

2003 Haloween Events



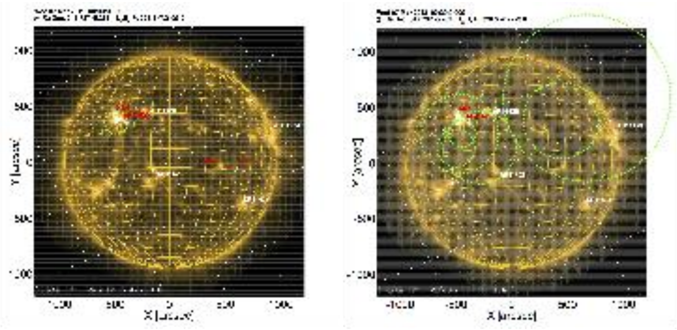
Mewaldt et al., 2005

2012 March 7 events(s)



*Irradiance is plotted here, not photon intensity

Dubbed "impulsive phase" by Ajello et al., but extended for >1 hour



Significant and large scale motion of *centroid* over hours—basically first vs. second flare

Focused Transport Equation

$$\frac{\partial f}{\partial t} + (U + v\mu)\frac{\partial f}{\partial r} - \frac{(1-\mu^2)}{r}Uv\frac{\partial f}{\partial v}$$

$$+\frac{(1-\mu^2)}{r}(v+\mu U)\frac{\partial f}{\partial \mu} = \frac{\partial}{\partial \mu}\left[(1-\mu^2)D_{\mu\mu}\frac{\partial f}{\partial \mu}\right]$$

$$D_{\mu\mu} = \frac{\pi\Omega^2}{2B_0^2\,|\,\mu\,|\,v} I\left(\frac{\Omega}{\mu v}\right)$$

Parker (1964)

$$\underset{\sim}{B} = B_0 \left[\frac{dF}{dz}\,\hat{e}_x + \frac{dG}{dz}\,\hat{e}_y + \hat{e}_z \right]$$

field lines: $x = F(z) + x_0$

$y = G(z) + y_0$

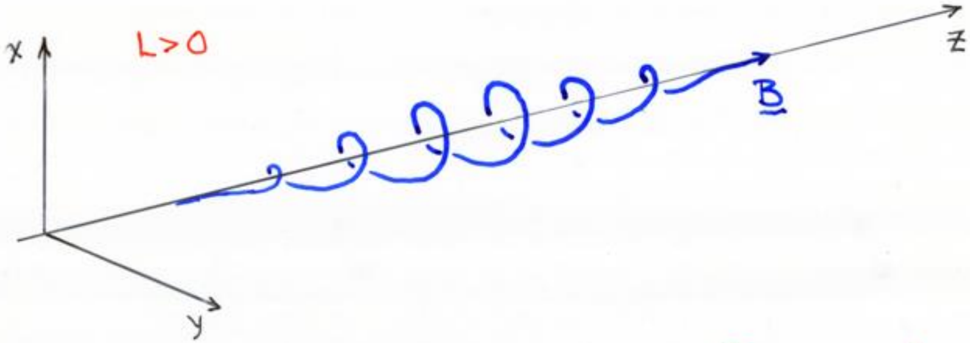
if $F, G \rightarrow F_{\pm}, G_{\pm}$ as $z \rightarrow \pm\infty$

particle remains on field line

Jokipii
Kota

take: $F(z) = \varepsilon \sin\left(2\pi \frac{z}{L}\right) e^{-z^2/\ell^2}$

$G(z) = \varepsilon \cos\left(2\pi \frac{z}{L}\right) e^{-z^2/\ell^2}$ $\quad \varepsilon \ll 1$



$$\Delta v_z = \varepsilon \sin\Phi\, \pi^{1/2} \ell\, v_{\perp} (\Omega^2/v_z^2)\, e^{-\frac{1}{4}\left(\frac{\Omega\ell}{v_z}\right)^2 \left(\frac{2\pi v_z}{\Omega L} - 1\right)^2}$$

resonance: $\tau_{\xi} = \frac{L}{v_z}$

Parker’s Energetic Particle Transport Equation

$$\frac{\partial f}{\partial t} + (\mathbf{V} + \mathbf{V}_D) \cdot \nabla f - \nabla \cdot \mathbf{K} \cdot \nabla f - \frac{1}{3} \nabla \cdot \mathbf{V} v \frac{\partial f}{\partial v} = Q$$

$$v \gg V \qquad\qquad S \ll vf$$

$$\mathbf{K}_{ij} = K_{\perp} \delta_{ij} + (K_{\mathrm{P}} - K_{\perp}) b_i b_j$$

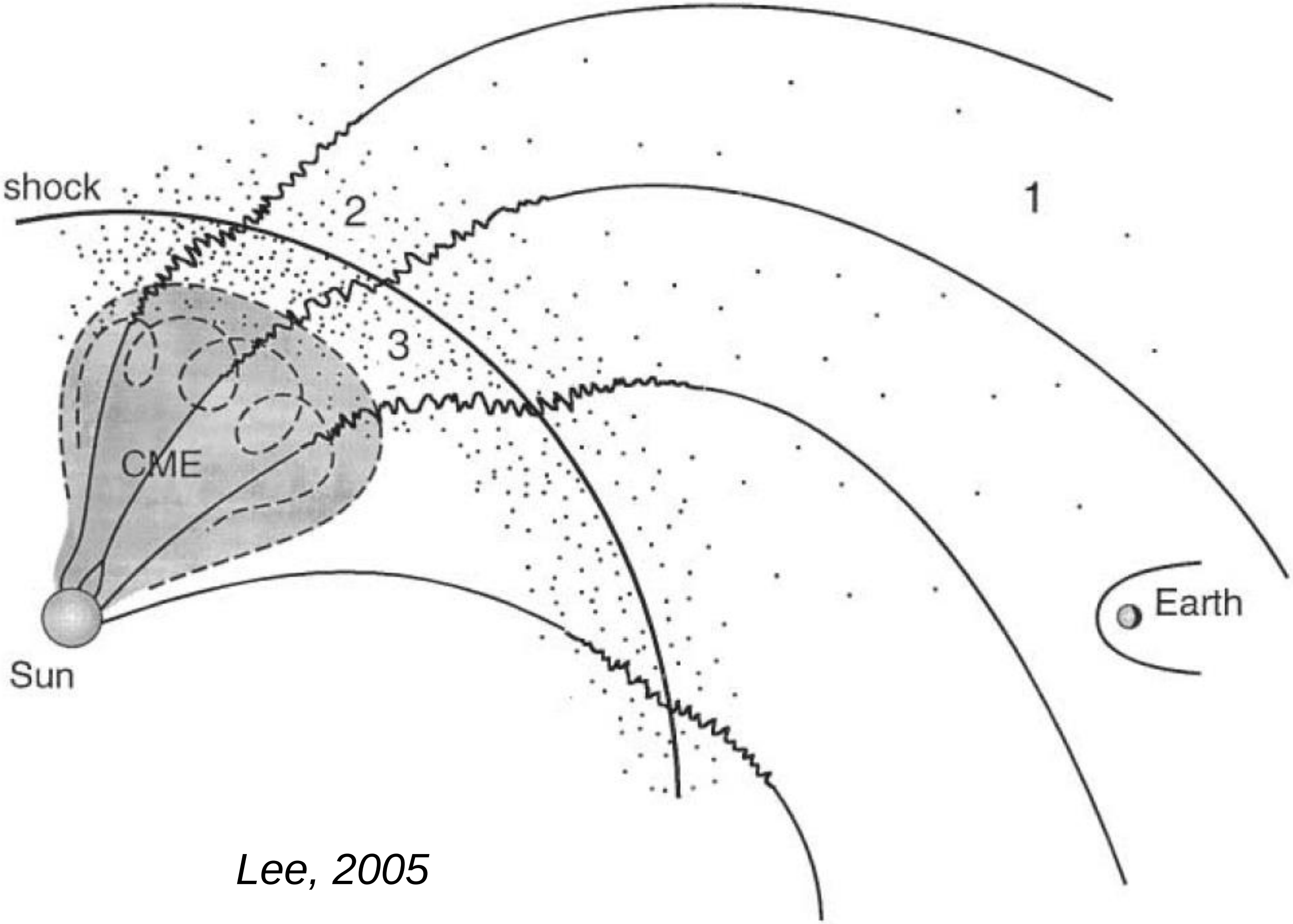
$$K_{\mathrm{P}} = \frac{v^2}{8} \int_{-1}^{1} d\mu \frac{(1-\mu^2)}{D_{\mu\mu}(v, \mu, \mathbf{r})}$$

Constraint on Acceleration

$$\mathbf{E} = -c^{-1}\mathbf{V} \times \mathbf{B}$$

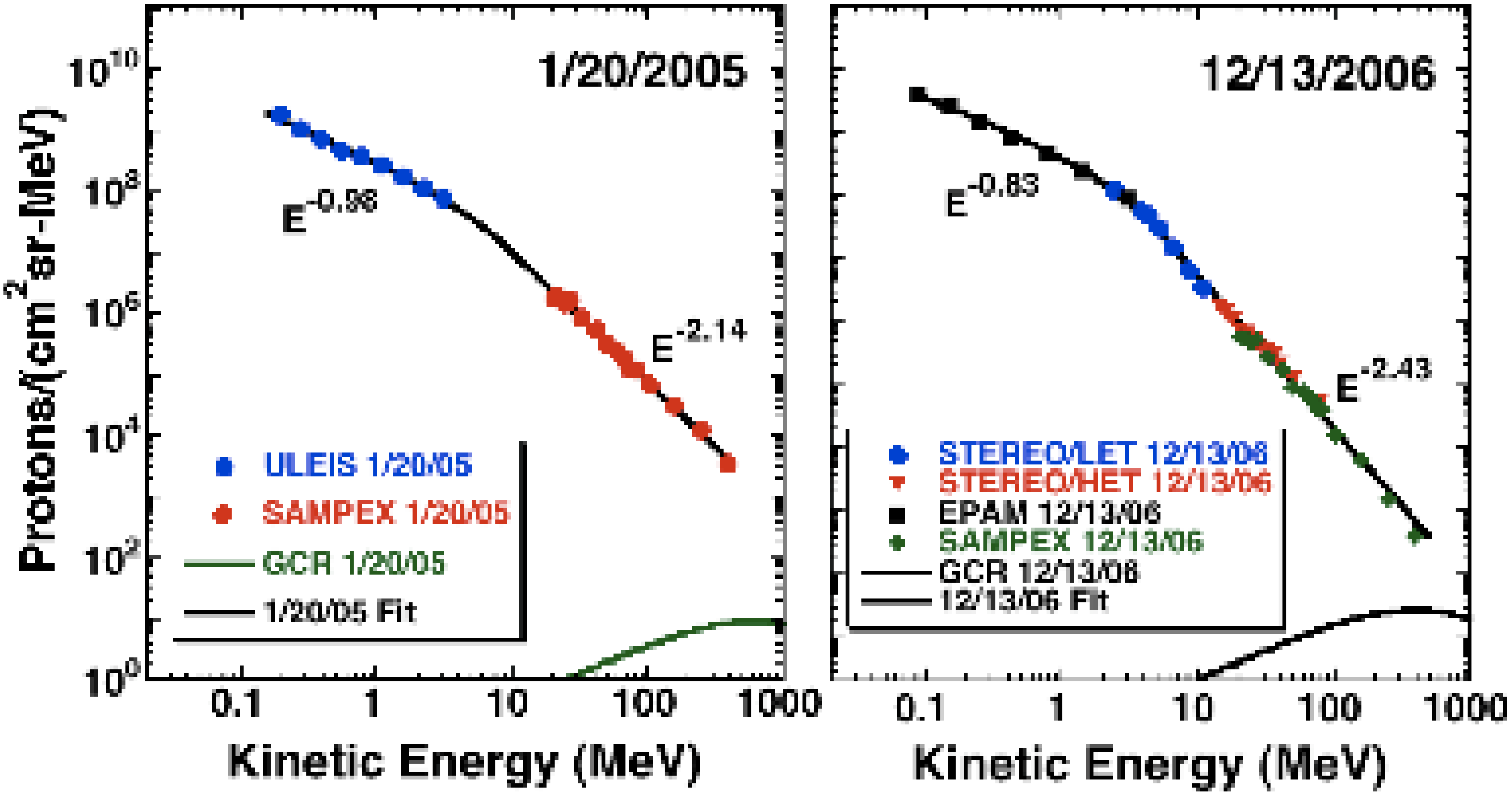
Gradual Events

Acceleration at a CME-Driven Shock

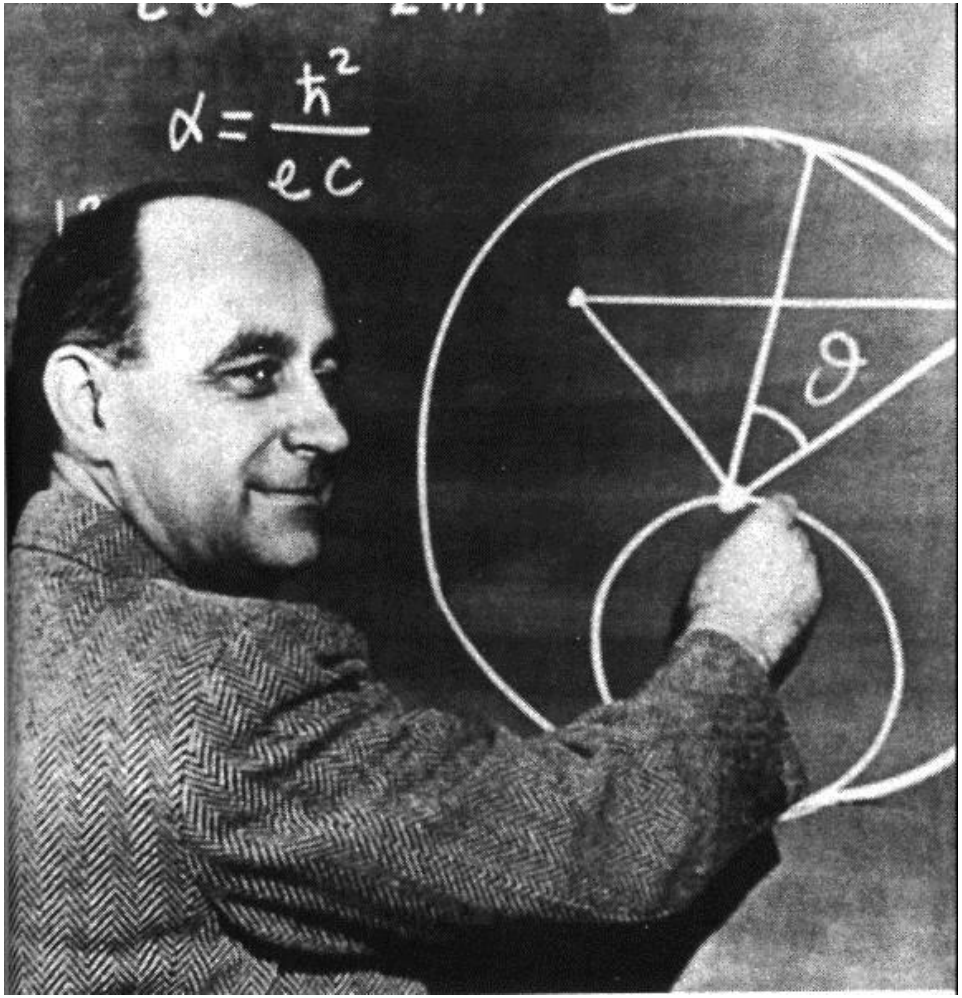


Lee, 2005

GLE Events (Mewaldt et al., 2012)

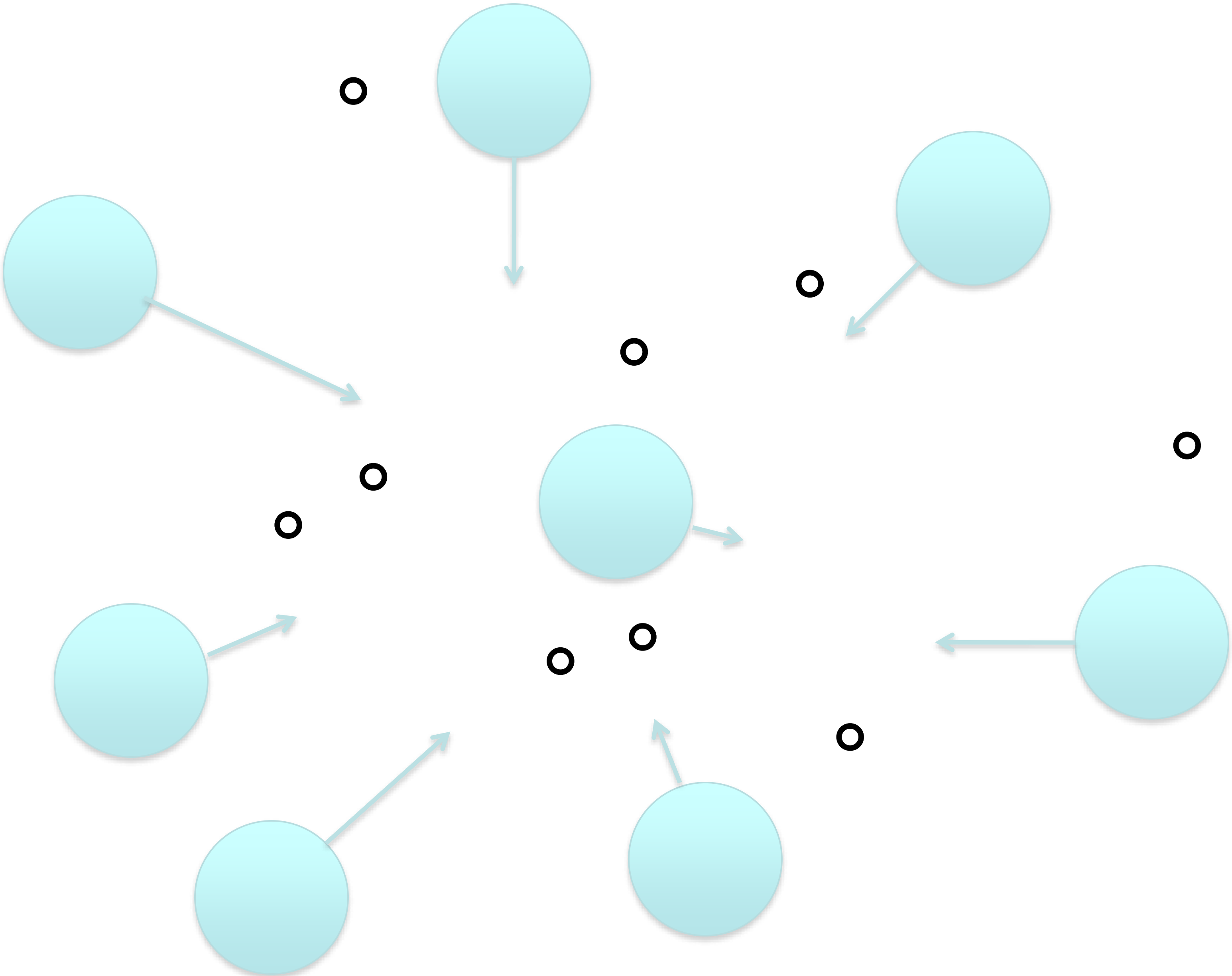


(See Li and Lee, ApJ, 2015)

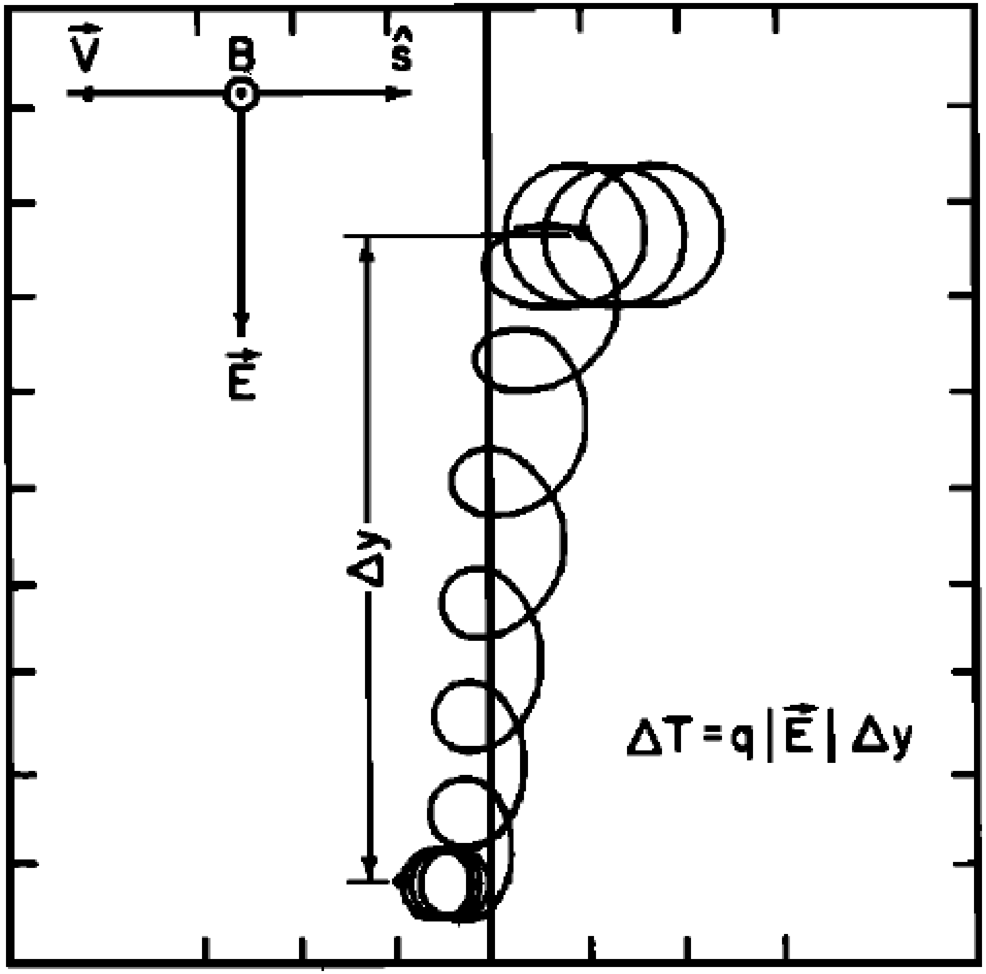


Enrico Fermi
(1949, 1954):

First-Order and Second-Order Fermi Acceleration



“Shock Drift” Acceleration



Pesses, 1981

Planar Stationary DSA - I

$$V_z \frac{\partial f_i}{\partial z} - \frac{\partial}{\partial z}\left(K_{i,zz}\frac{\partial f_i}{\partial z}\right) - \frac{1}{3}\frac{dV_z}{dz} v \frac{\partial f_i}{\partial v} = Q_i$$

$$\zeta(z) = \int_0^z [V/K_{zz}(z')]dz'$$

$$f(z>0,v) = f_0 - (f_0 - f_\infty)(1-e^{-\zeta})(1-e^{-\zeta_\infty})^{-1}$$

$$f_0 \equiv \beta\int_0^v \frac{dv'}{v'} f_\infty(v')(1-e^{-\zeta'_\infty})^{-1}\left(\frac{v}{v'}\right)^{-\beta}\exp\left[-\beta\int_{v'}^{v}\frac{dv''}{v''}e^{-\zeta''_\infty}(1-e^{-\zeta''_\infty})^{-1}\right]$$

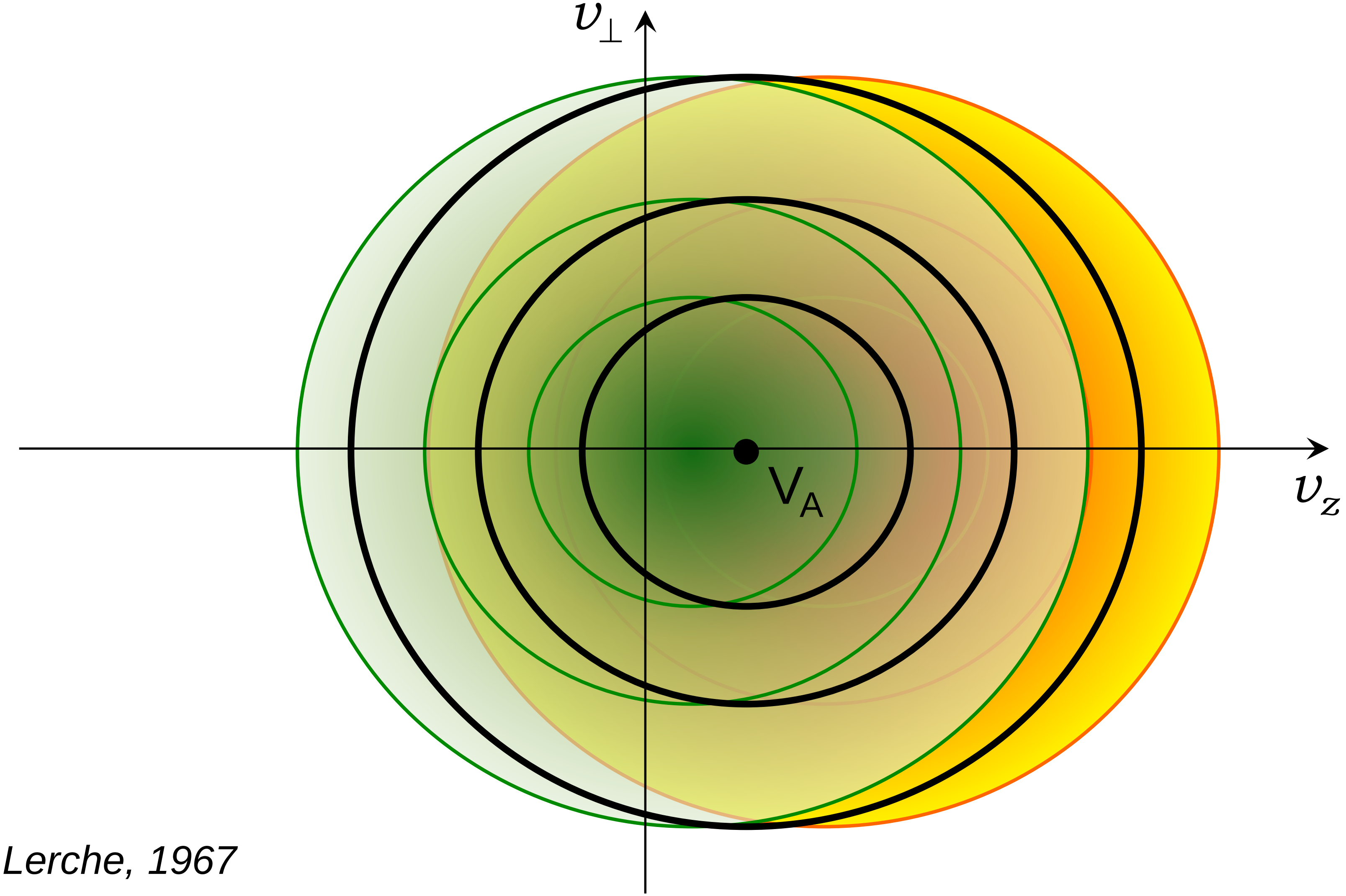
$$\beta = 3X/(X-1)$$

Planar Stationary DSA - II

$$-K_{zz}\frac{\partial f}{\partial z}\bigg|_{z\to\infty} = V(f_0 - f_\infty)e^{-\zeta_\infty}(1-e^{-\zeta_\infty})^{-1}$$

$$f_{p,\infty} = \bar{n}_p(4\pi v_{p,0}^2)^{-1}\delta(v - v_{p,0}) + \bar{C}v^{-\gamma}S(v - \bar{v}_{p,0})$$

Instability Mechanism



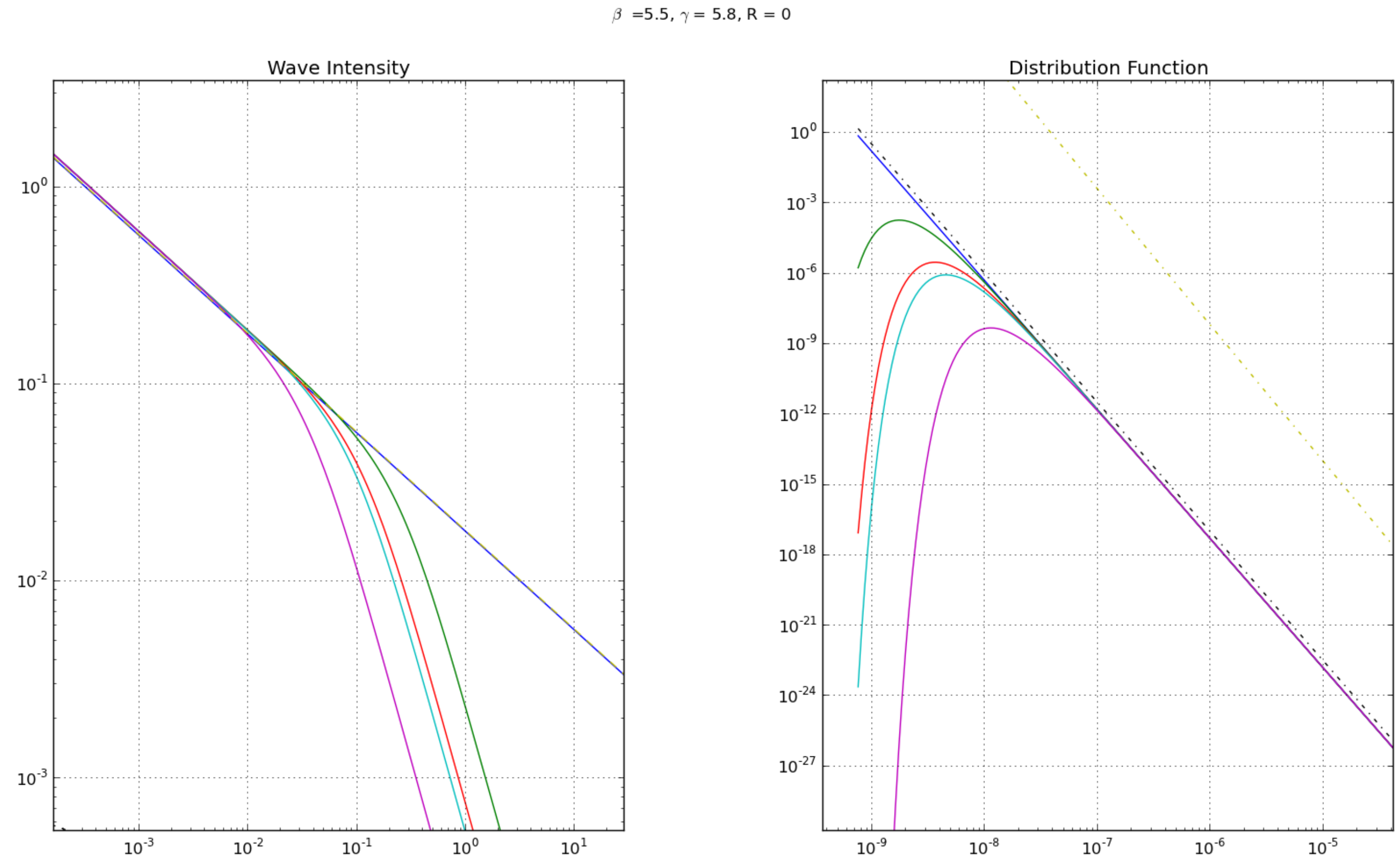
Lerche, 1967

Wave Excitation - I

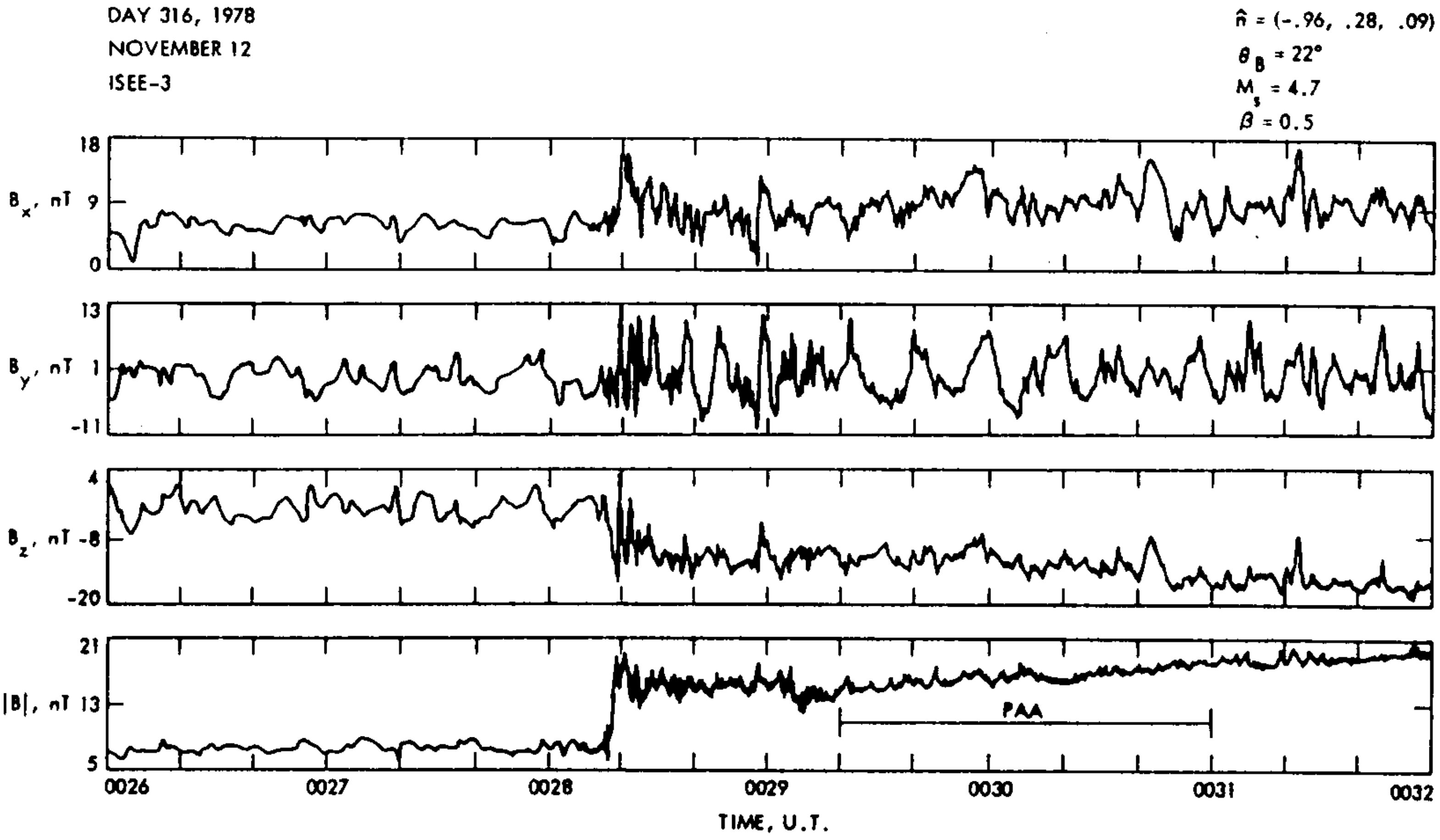
$$-V\partial I_{\pm} / \partial z = 2\gamma_{\pm} I_{\pm}$$

$$I \cong I_{+} = I^{\circ}_{+}(k) + \frac{4\pi^2}{k^2}\frac{V_A}{V}|\Omega_p| m_p \cos\psi \int_{|\Omega_p / k|}^{\infty} dv v^3 \left(1 - \frac{\Omega_p^2}{k^2 v^2}\right)(f_p - f_{p,\infty})$$

$$f_{p,\infty} = \bar{n}_p (4\pi v_{p,0}^2)^{-1} \delta(v - v_{p,0}) + \bar{C} v^{-\gamma} S(v - \bar{v}_{p,0})$$


β =5.5, γ = 5.8, R = 0
Wave Intensity
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
Distribution Function
10^{0}
10^{-3}
10^{-6}
10^{-9}
10^{-12}
10^{-15}
10^{-18}
10^{-21}
10^{-24}
10^{-27}
10^{-9}
10^{-8}
10^{-7}
10^{-6}
10^{-5}

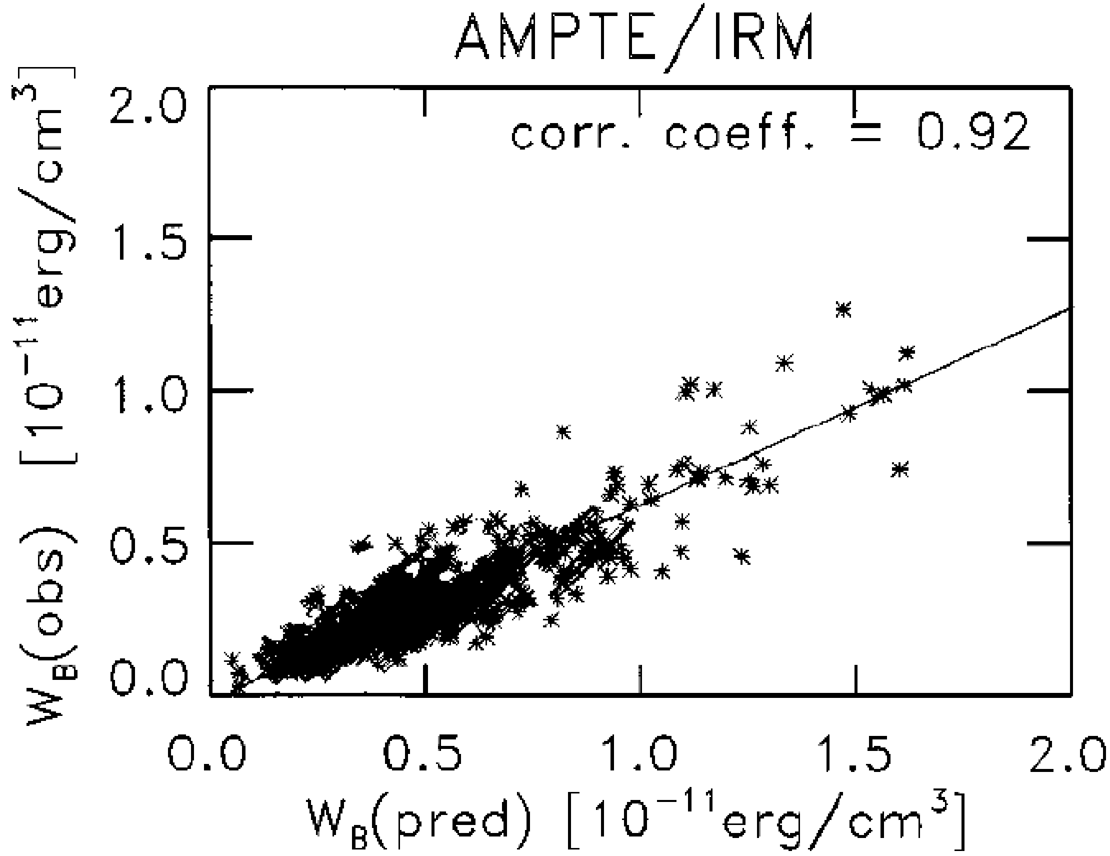
Collisionless Shock on 11/12/78: ISEE-3



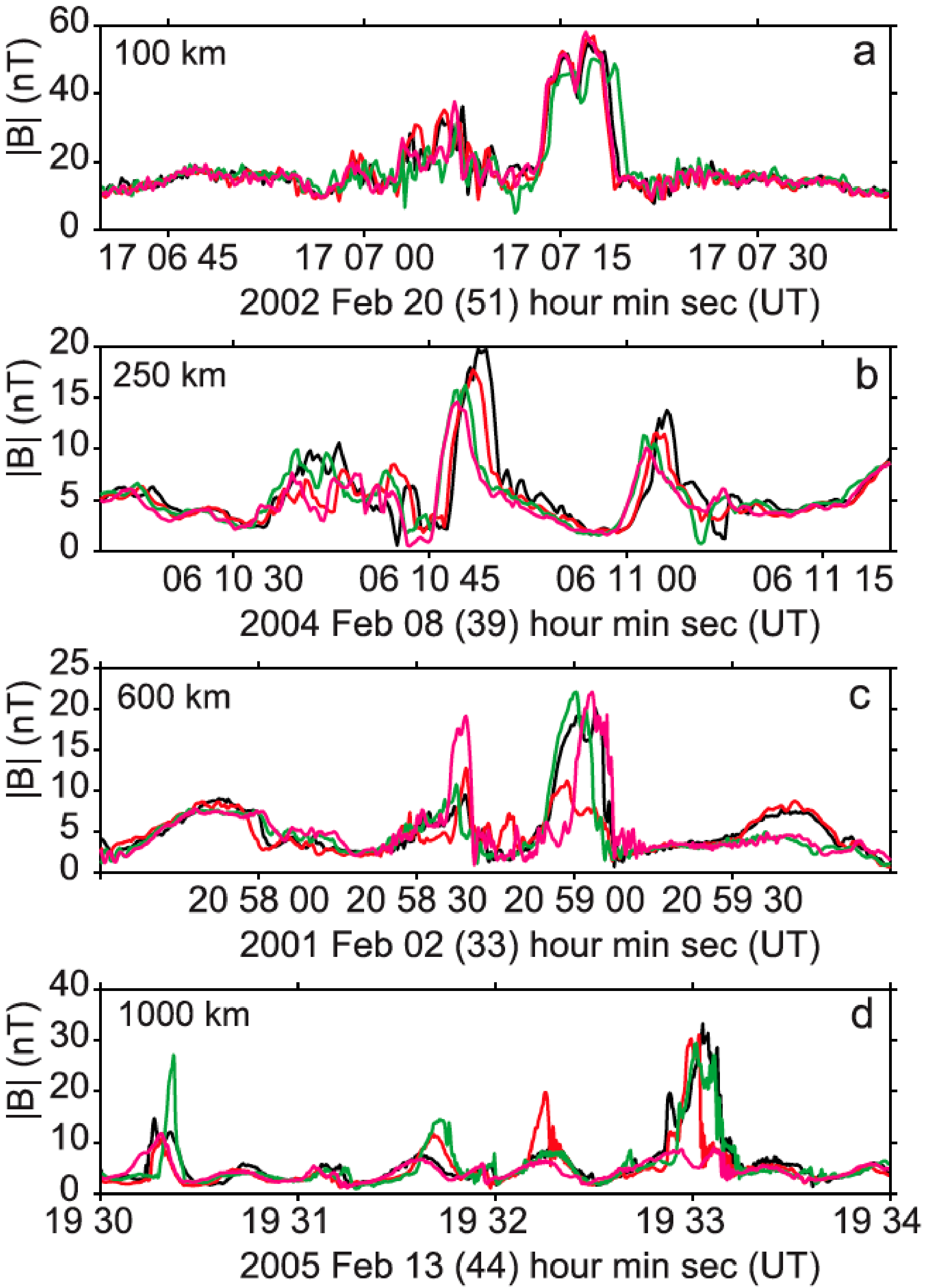
Tsurutani et al., 1983

Waves Upstream of Earth's Bow Shock

$$W_B = \frac{1}{3}\,\frac{V_A(\hat{e}_b \cdot \hat{e}_g)}{V_{sw}(\hat{e}_z \cdot \hat{e}_g) - V_A(\hat{e}_b \cdot \hat{e}_g)}\,W_p$$



Gordon et al., 1999



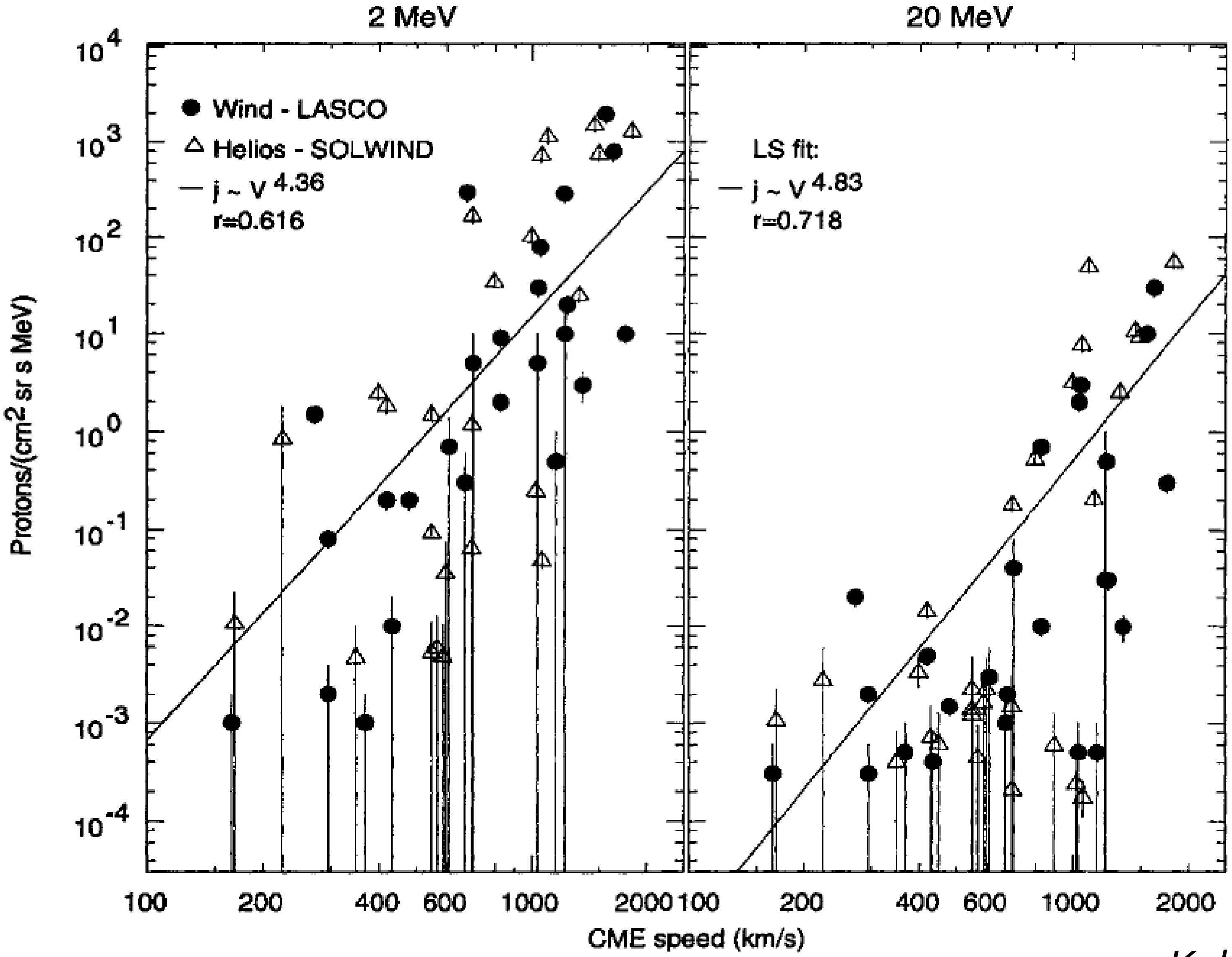
SLAMS

Lucek et al., 2008

Puzzles and Challenges

- Extreme Variability
- Power-law Index β
- Drift & Geometry
- Injection
- Magnetic Obliquity
- Time Limitation & Escape
- Conditions at the Sun

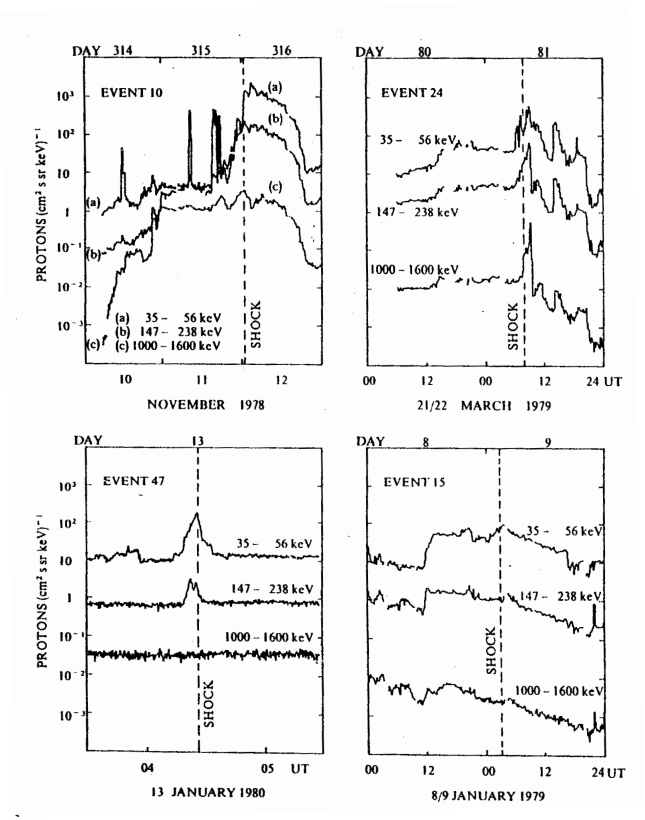
SEP Intensity versus CME Speed

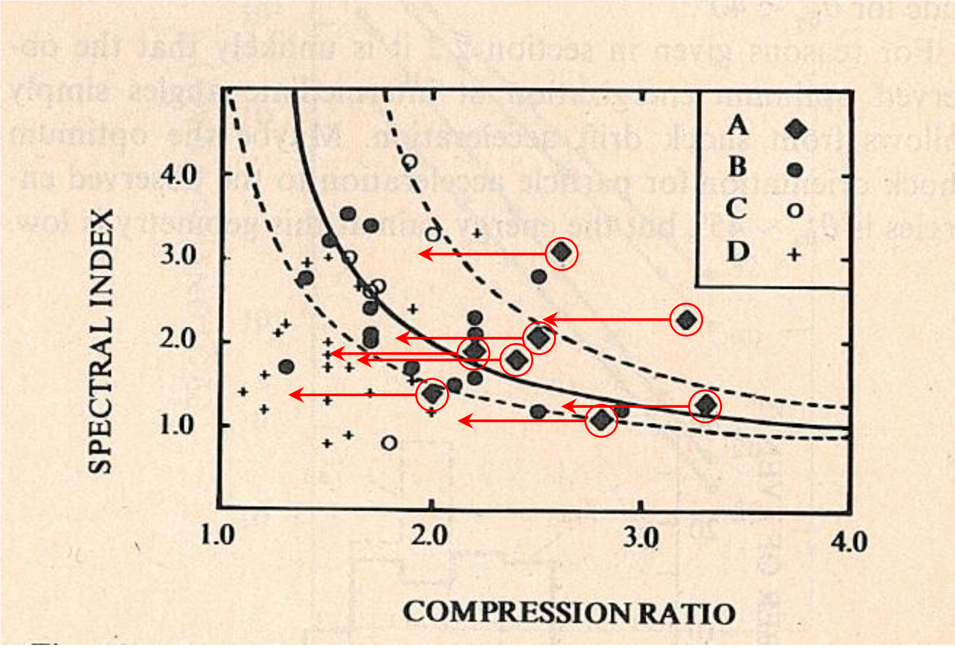


Kahler, 2001

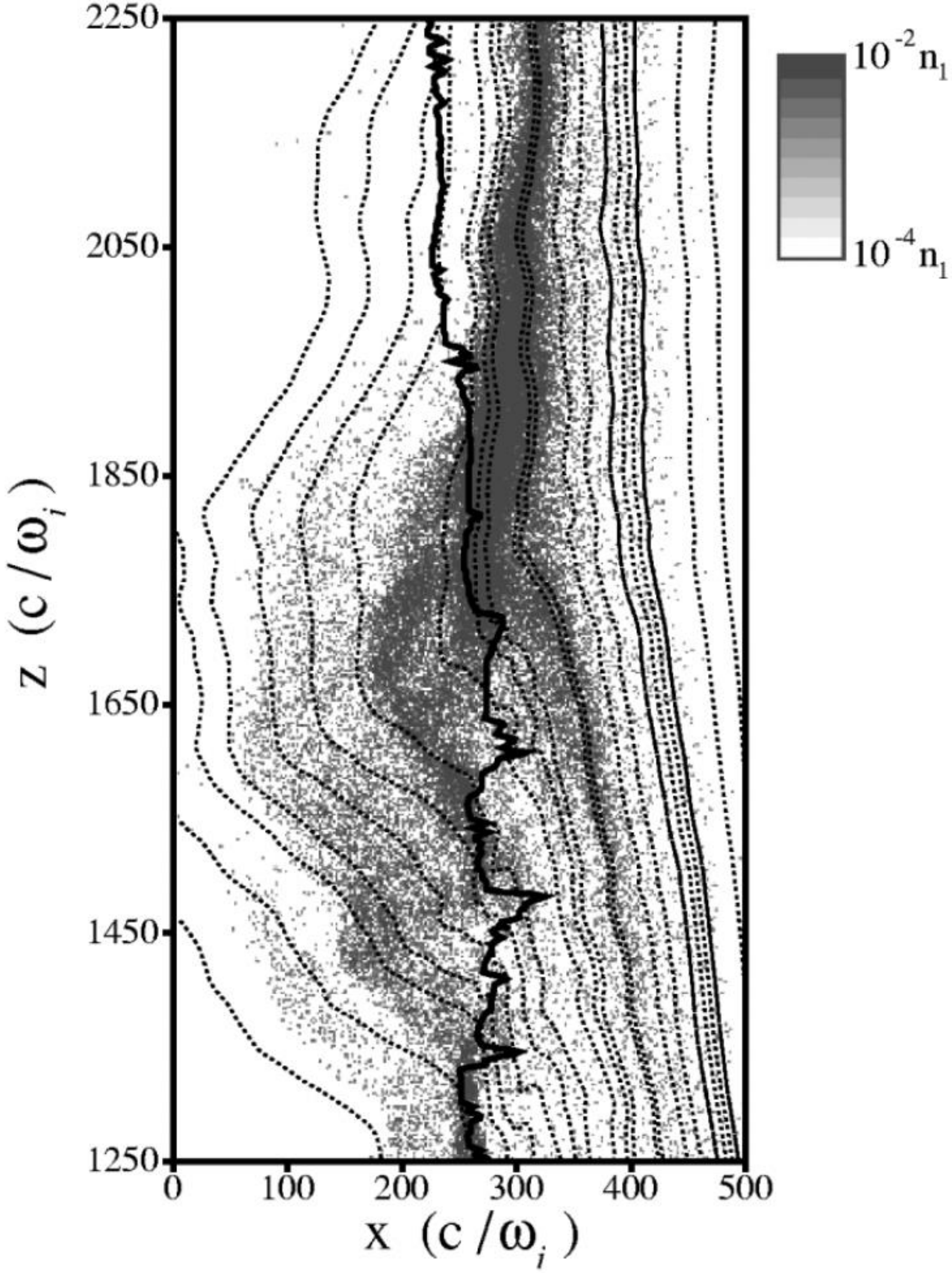
Testing the Predicted Shock Index

Wave-frame compression ratio:





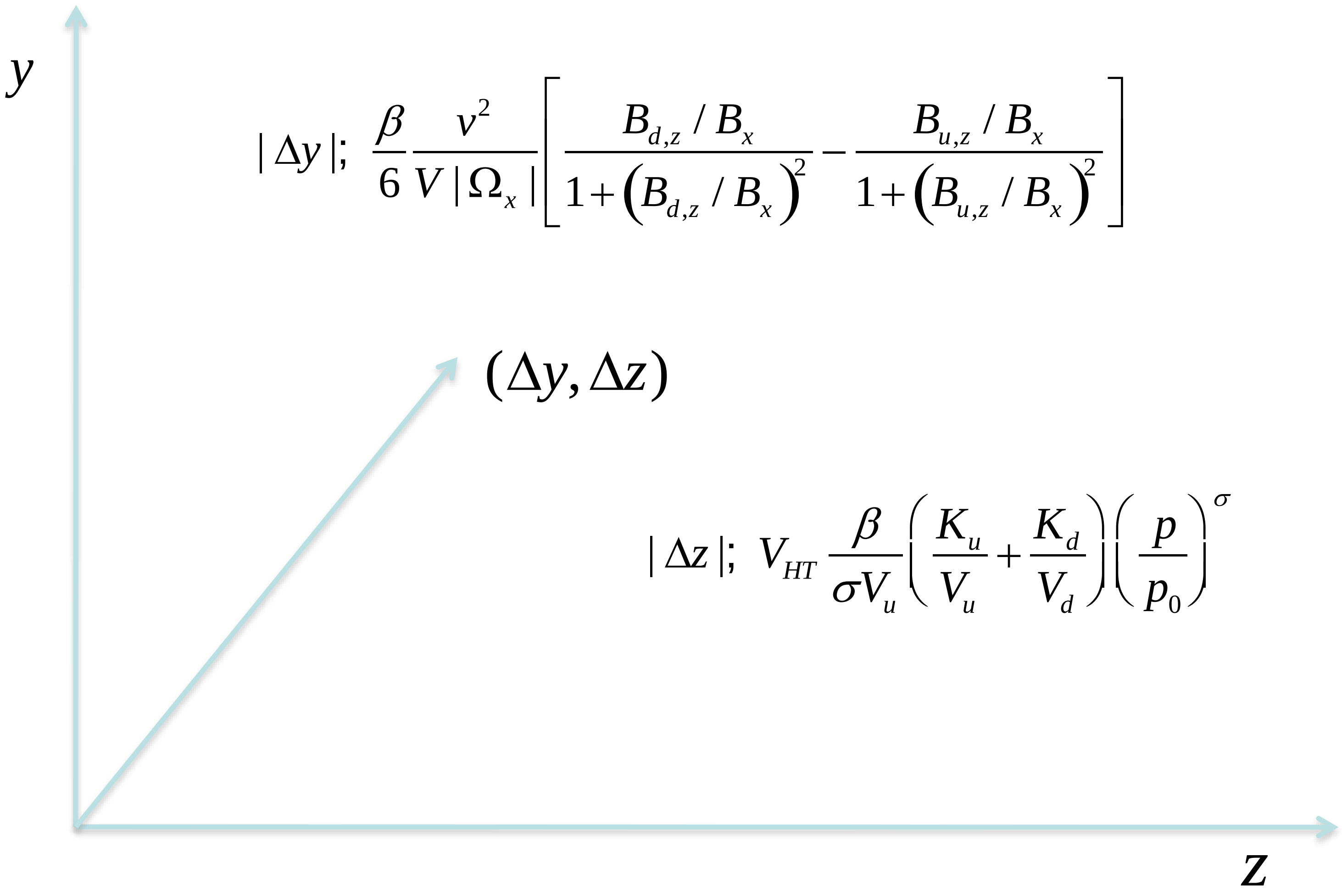
Van Nes et al., 1984



Quasi-Perpendicular Shock Simulation: Be Careful!

Giacalone, 1999

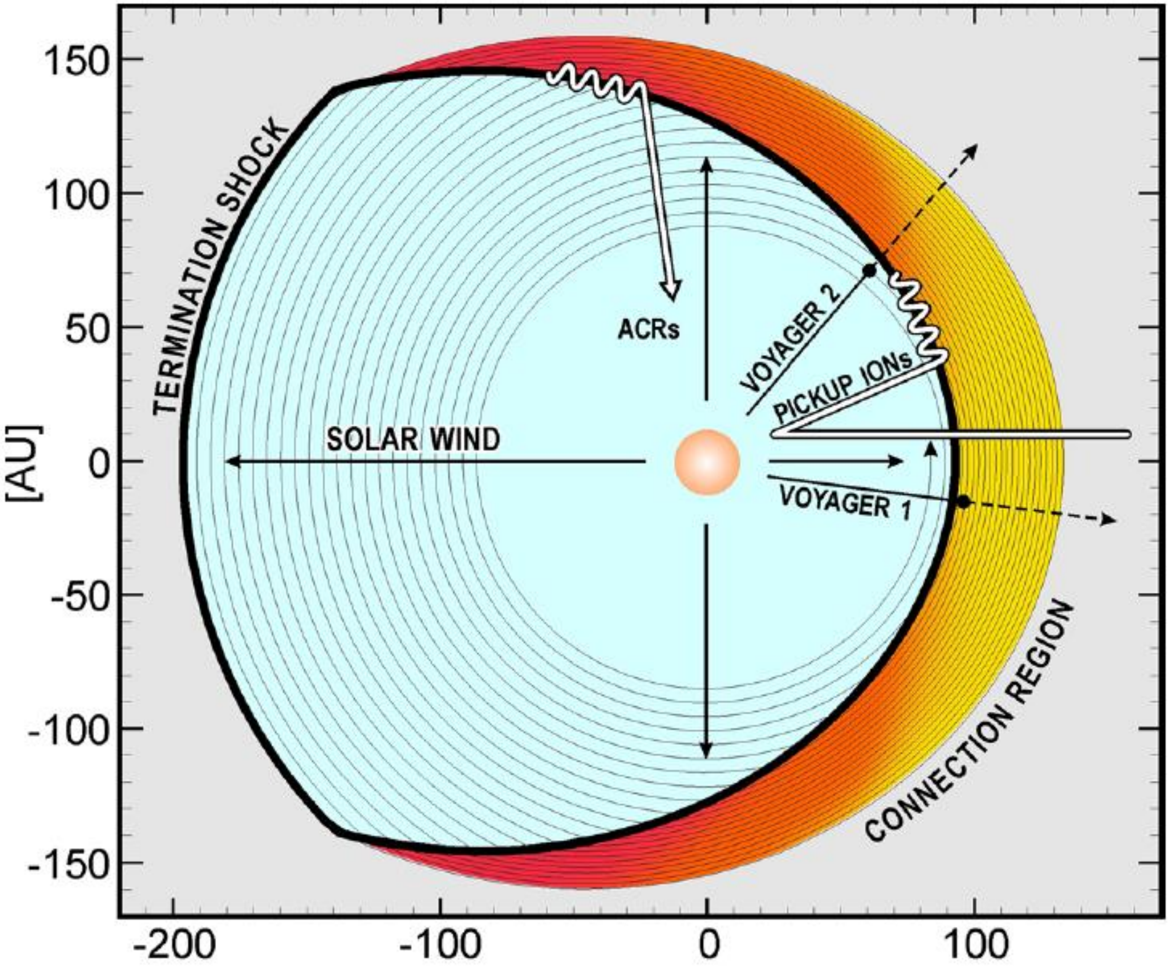
Particle Drift Along Shock



$$|\Delta y|;\ \frac{\beta}{6}\frac{v^2}{V\,|\Omega_x|}\left[\frac{B_{d,z}/B_x}{1+\left(B_{d,z}/B_x\right)^2}-\frac{B_{u,z}/B_x}{1+\left(B_{u,z}/B_x\right)^2}\right]$$

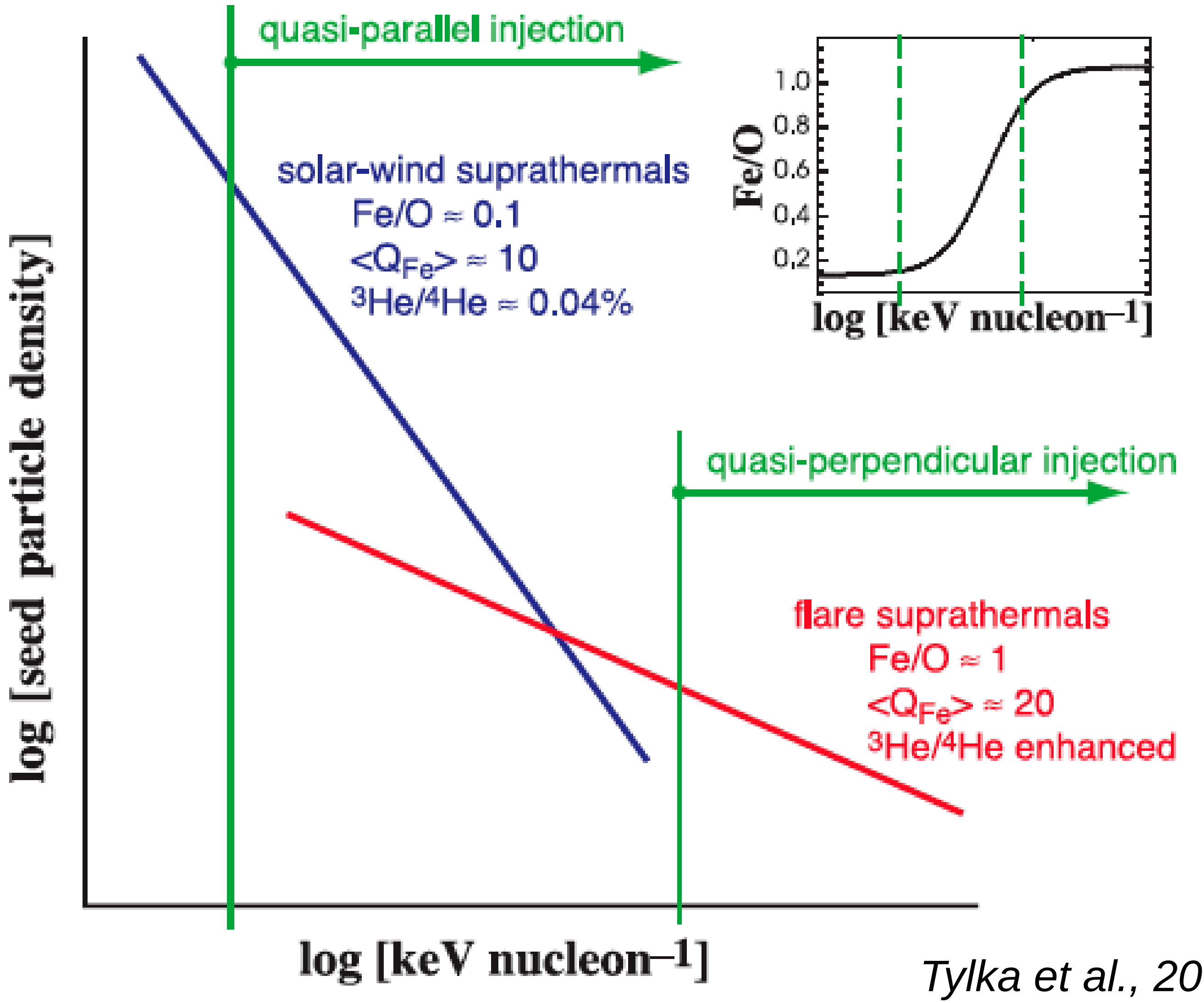
$$|\Delta z|;\ V_{HT}\,\frac{\beta}{\sigma V_u}\left(\frac{K_u}{V_u}+\frac{K_d}{V_d}\right)\left(\frac{p}{p_0}\right)^{\sigma}$$

The Blunt Termination Shock



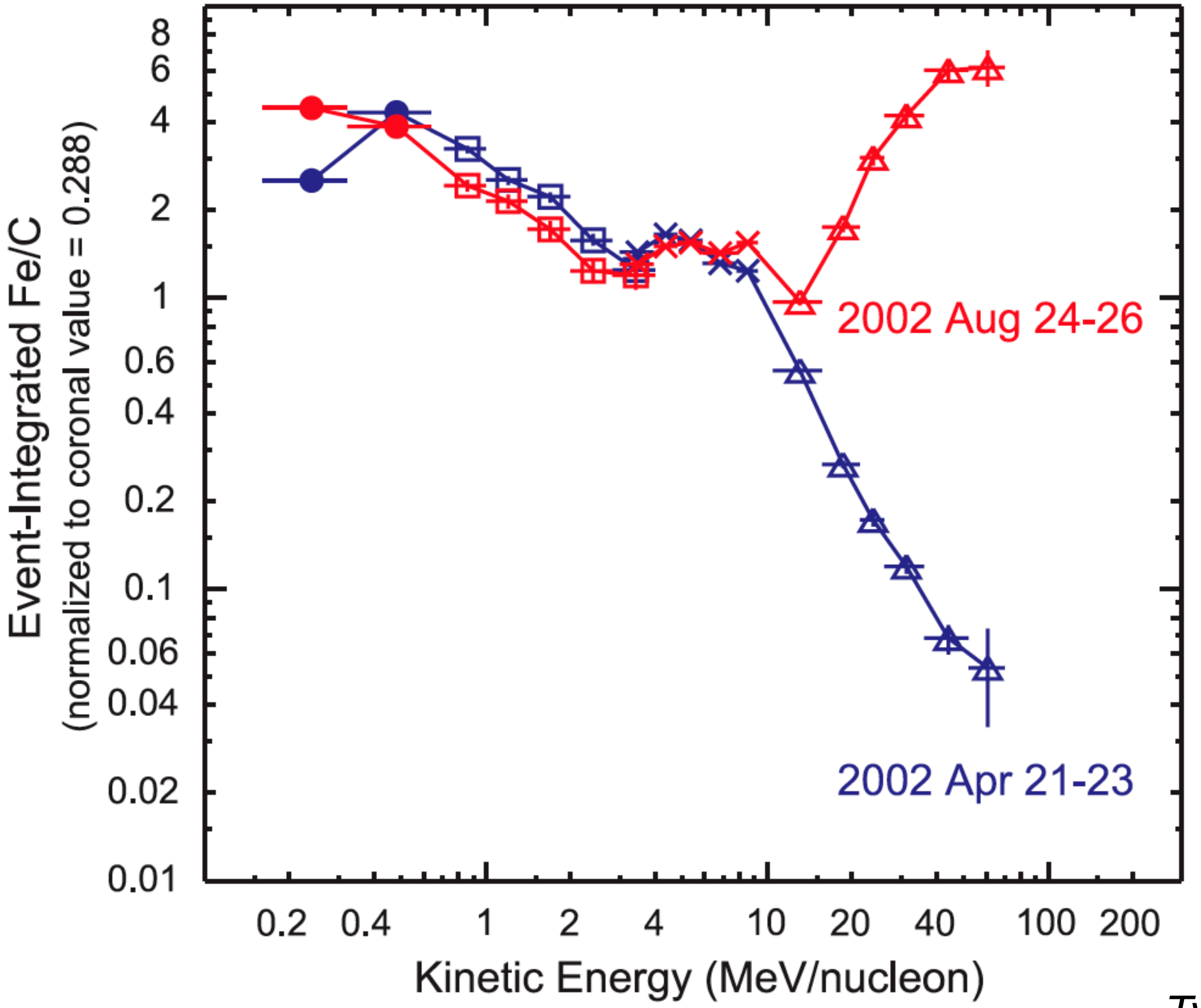
McComas and Schwadron, 2006

Injection and Magnetic Obliquity



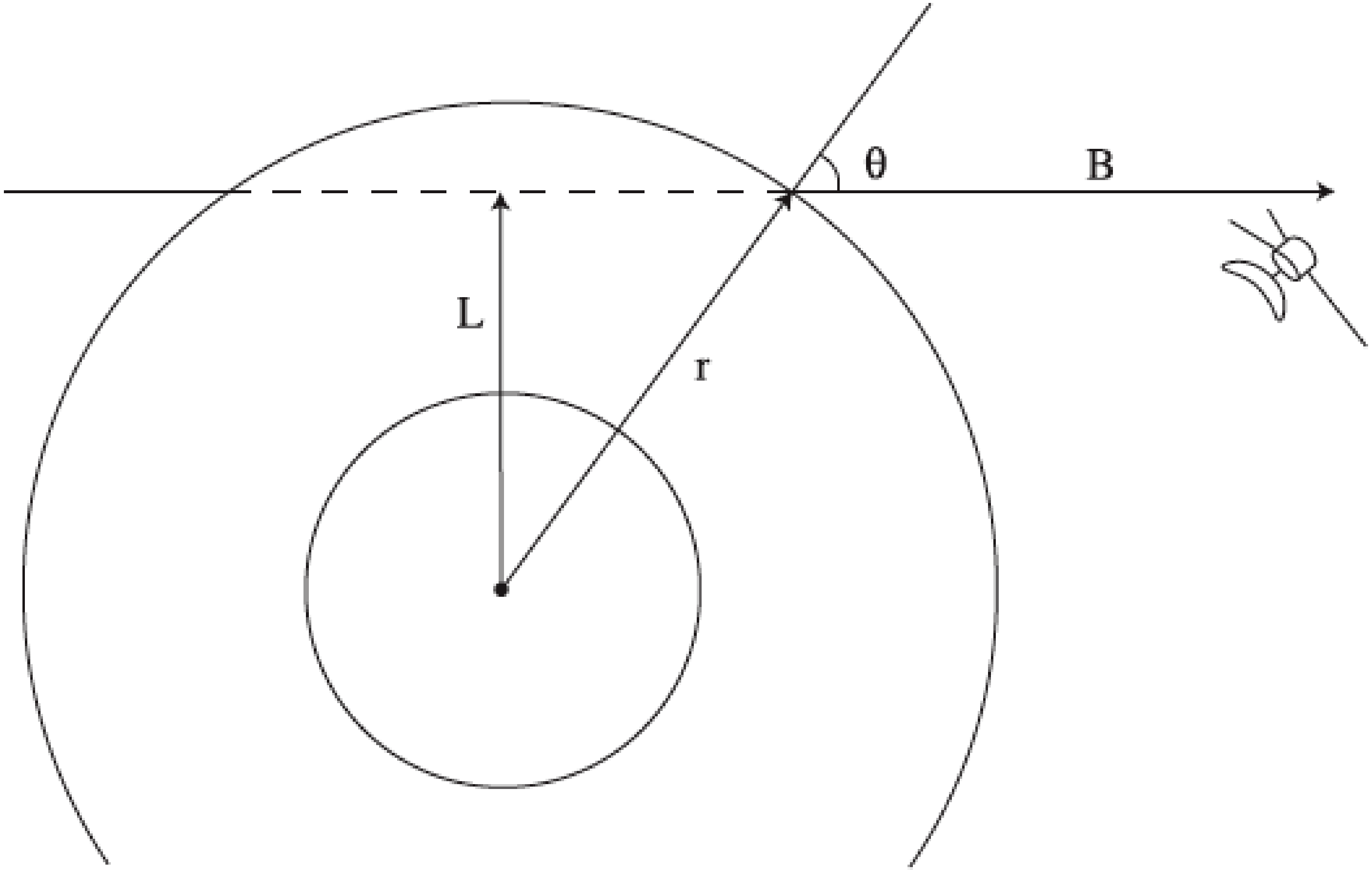
Tylka et al., 2005

Fe/O Versus Energy



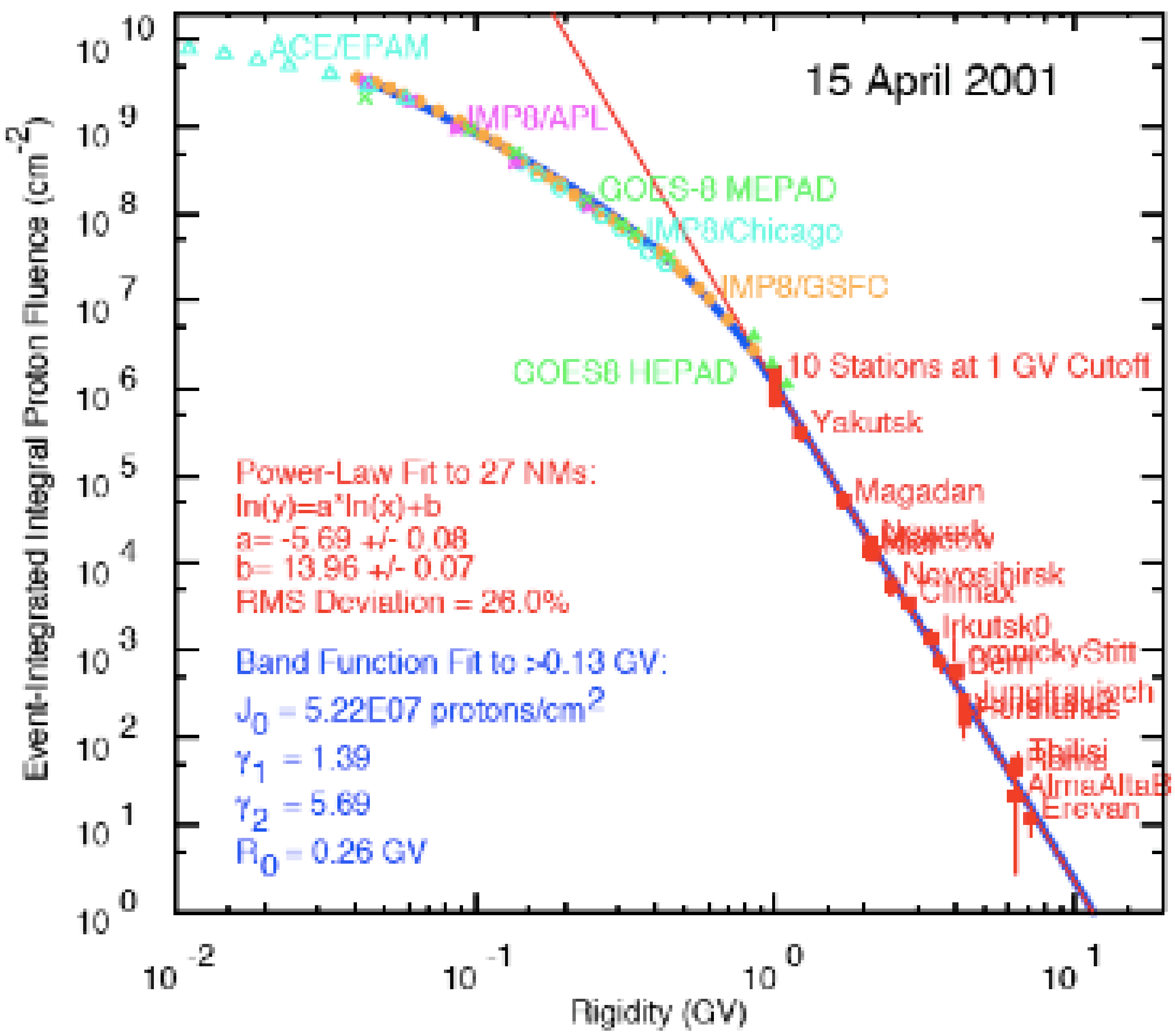
Tylka et al., 2005

Time Evolution



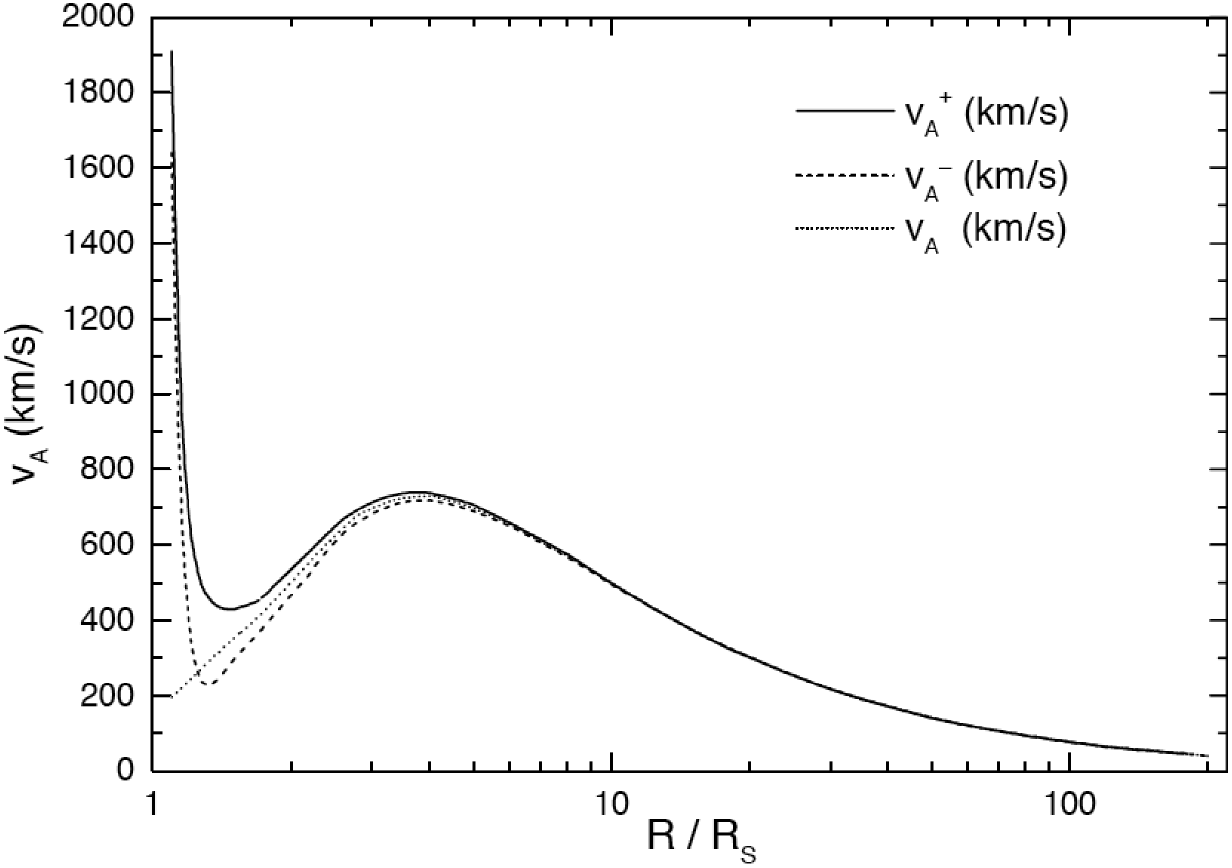
$$v_t \approx V \sec\theta$$

GLE Event of 15 April 2001



Tylka and Dietrich, 2009

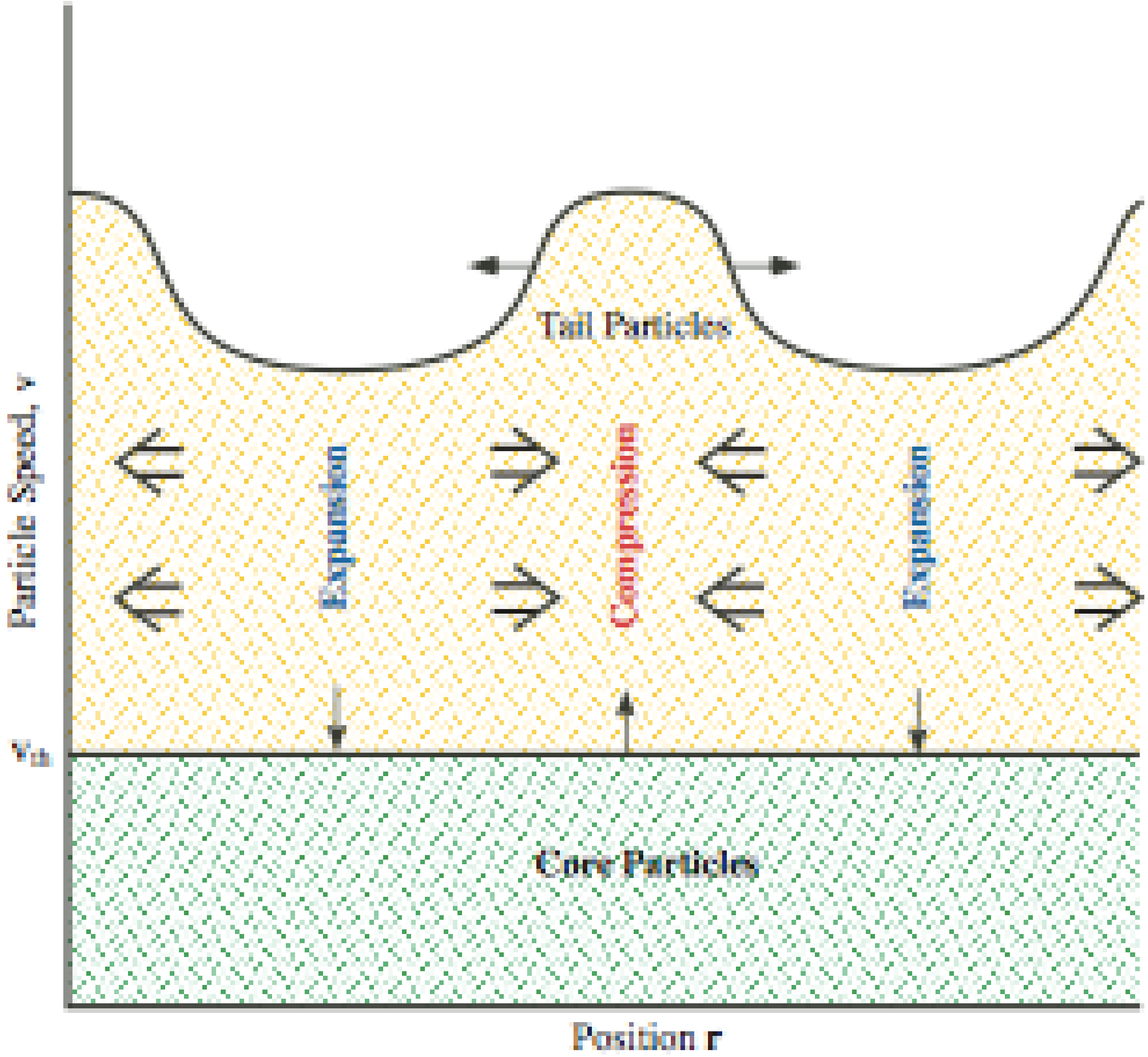
Alfven Speed Profile



Mann et al., 2003

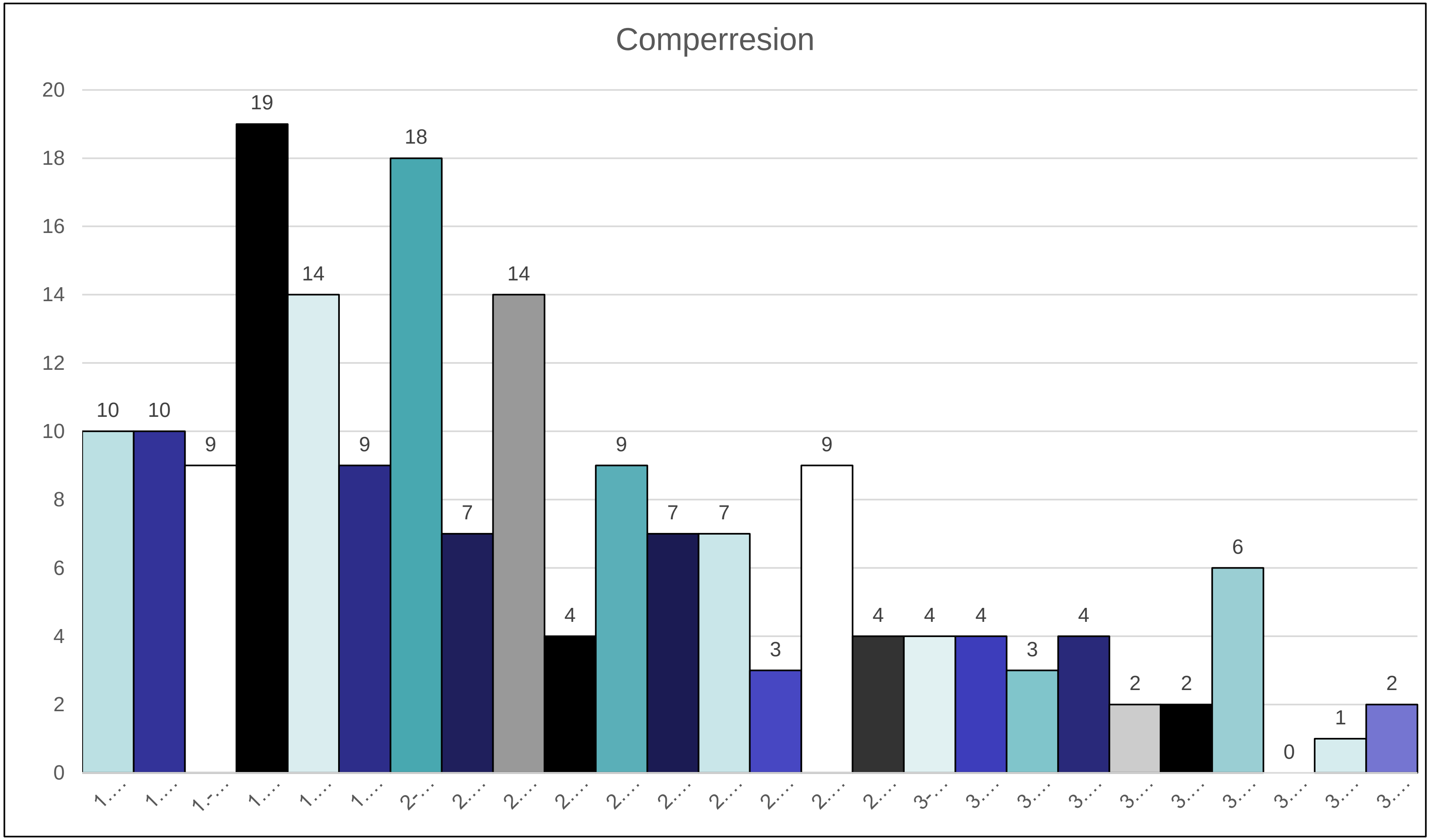
See poster by Li and Lee

Pump Mechanism of Fisk & Gloeckler



Fisk et al., 2010

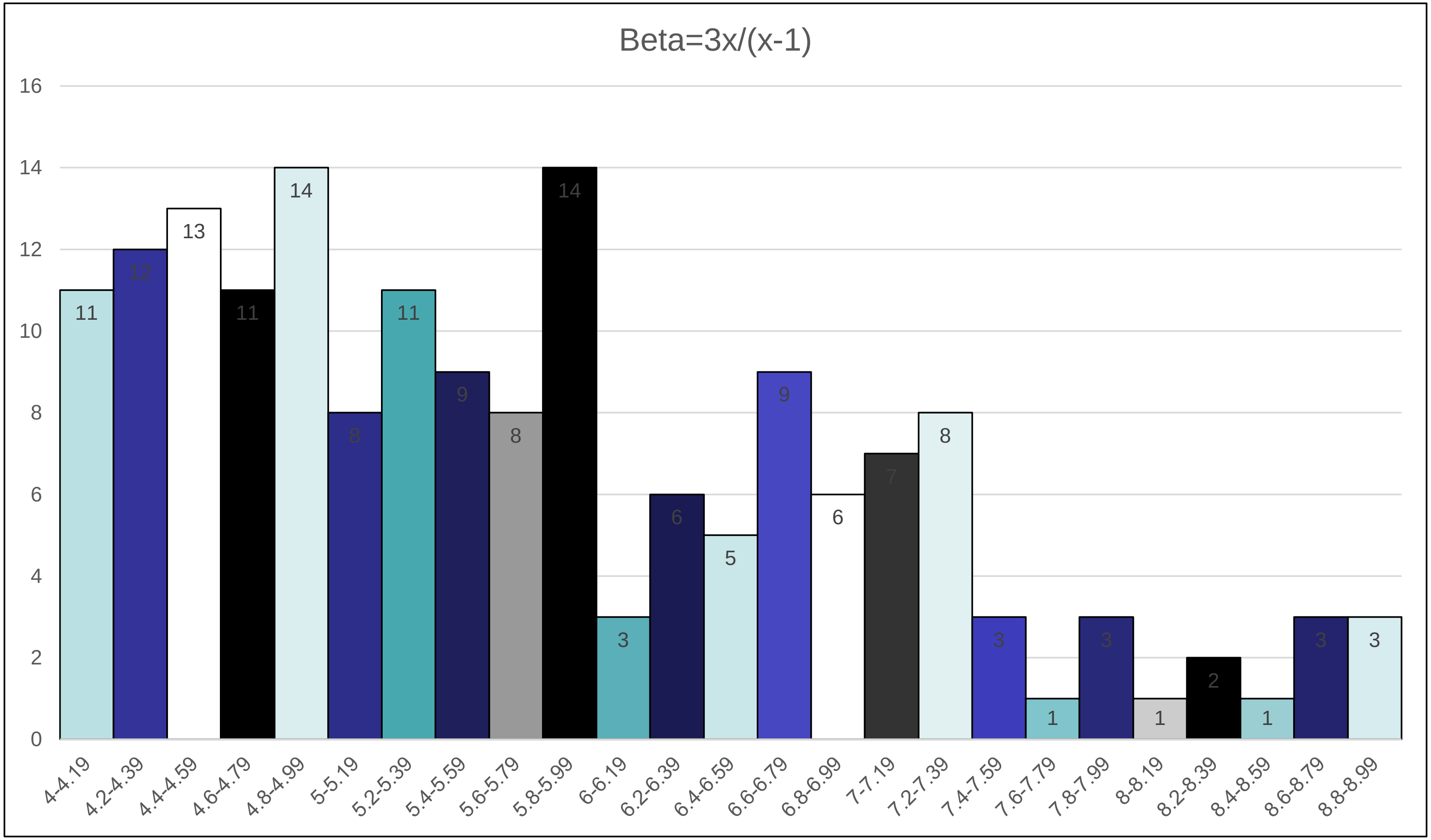
Compression



Reference: CfA Interplanetary Shock Database
https://www.cfa.harvard.edu/shocks/

$$\text{Beta}=\frac{3x}{x-1}$$

bin=0.2

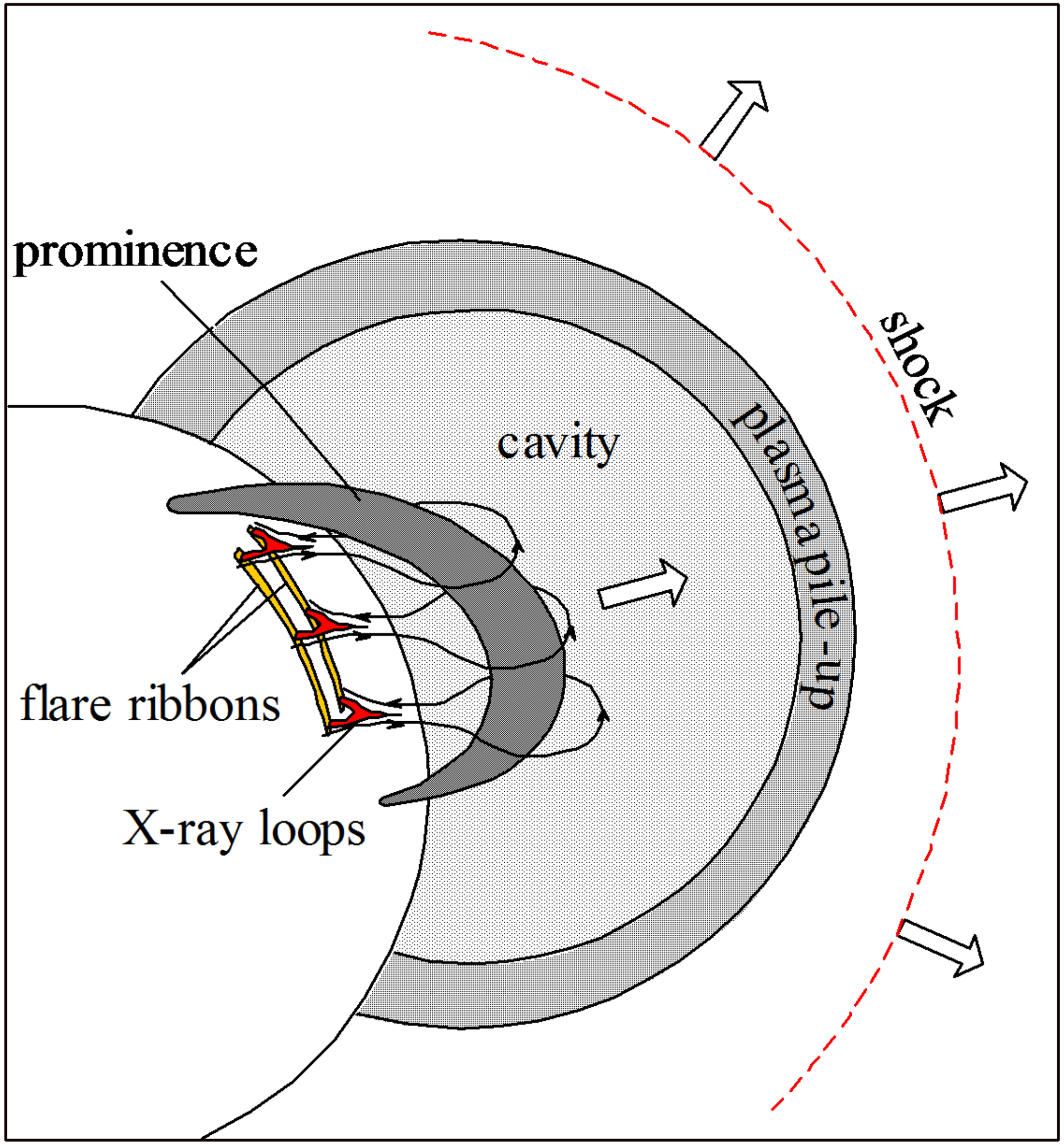


Reference: CfA Interplanetary Shock Database

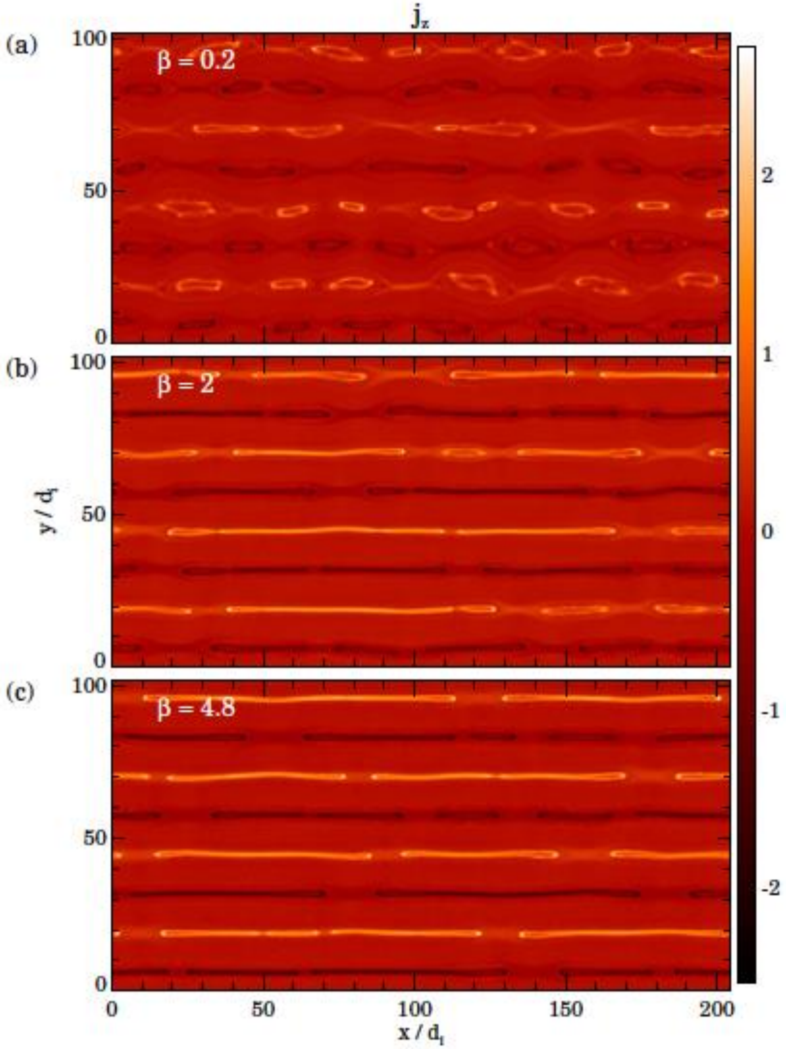
https://www.cfa.harvard.edu/shocks/

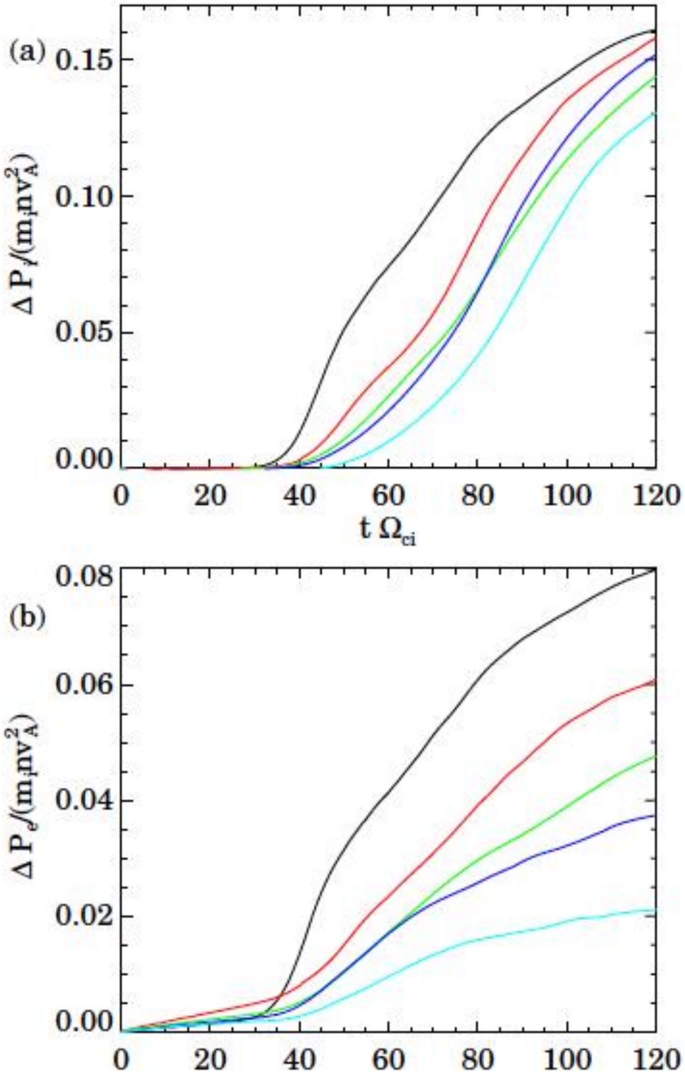
Impulsive Events

Large Solar Eruptions



Reconnection Acceleration





Schoeffler et al., 2013

Extra Slides

Wave Excitation - II

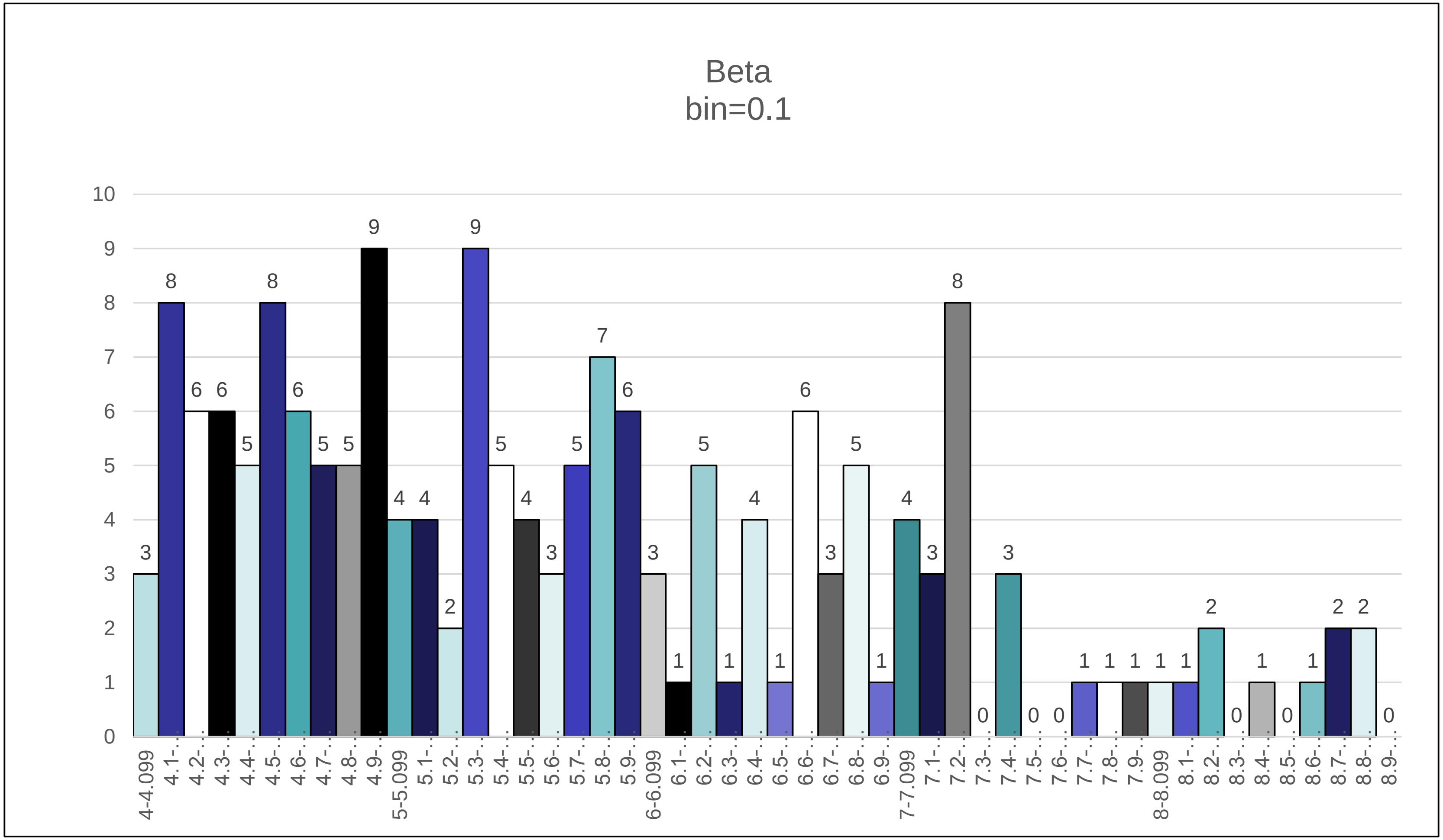
$$I = I_{+}^{\circ} + \frac{4\pi^2}{k^2}\frac{V_A}{V}|\Omega_p| m_p \cos\psi \int_{|\Omega_p/k|}^{\infty} dv v^3 \left(1 - \frac{\Omega_p^2}{k^2 v^2}\right) \cdot$$

$$\cdot \left\{ \frac{\beta \bar{n}}{4\pi v_{0,p}^3}\left(\frac{v}{v_{0,p}}\right)^{-\beta} S(v - v_{0,p}) - \frac{\bar{n}}{4\pi v_{0,p}^2}\delta(v - v_{0,p}) \right.$$

$$\left. + \frac{\bar{C}\bar{v}_{0,p}^{-\gamma}}{\beta - \gamma}\left[\gamma\left(\frac{v}{\bar{v}_{0,p}}\right)^{-\gamma} - \beta\left(\frac{v}{\bar{v}_{0,p}}\right)^{-\beta}\right] S(v - \bar{v}_{0,p}) \right\} \cdot$$

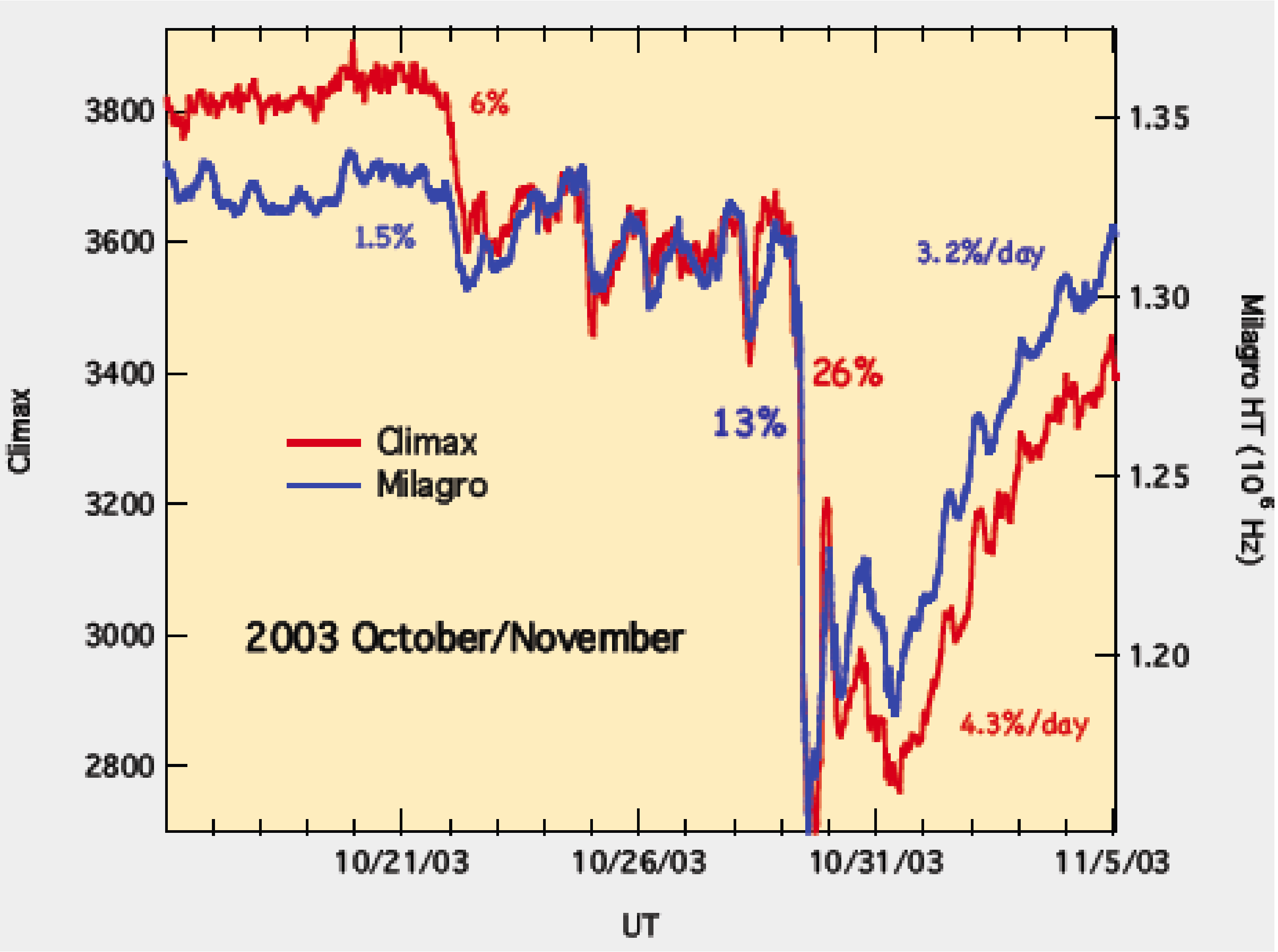
$$\cdot \exp\left\{ -V \int_0^z dz' \left[\cos^2\psi \frac{v^3}{4\pi}\frac{B_0^2}{\Omega_p^2} \int_{-1}^{1} d\mu \frac{|\mu|(1-\mu^2)}{I(\Omega_p \mu^{-1} v^{-1})} + \sin^2\psi K_{\perp} \right]^{-1} \right\}$$

Beta (bin=0.1)

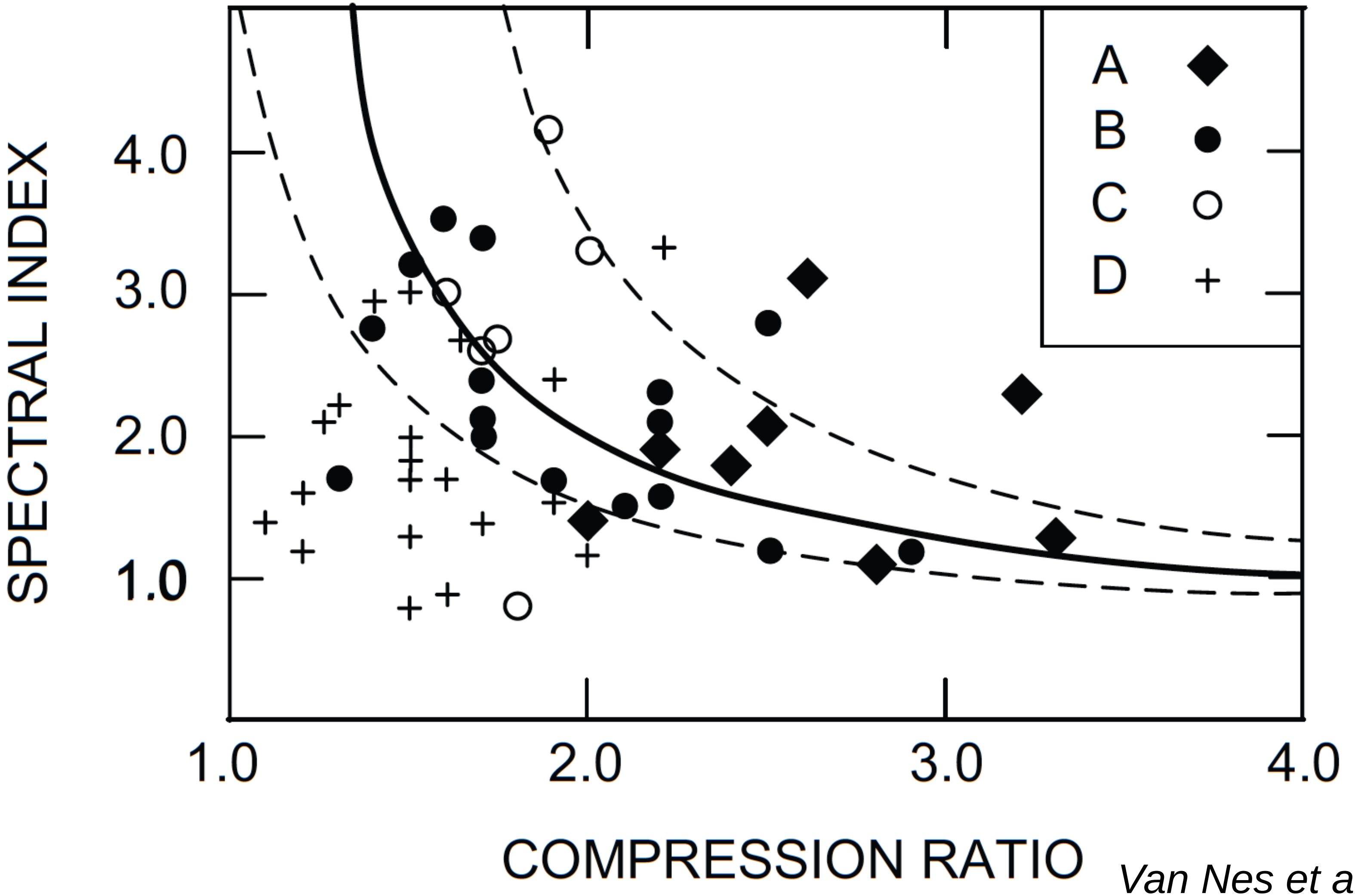


Reference: CfA Interplanetary Shock
Database
https://www.cfa.harvard.edu/shocks/

Halloween Forbush Decreases

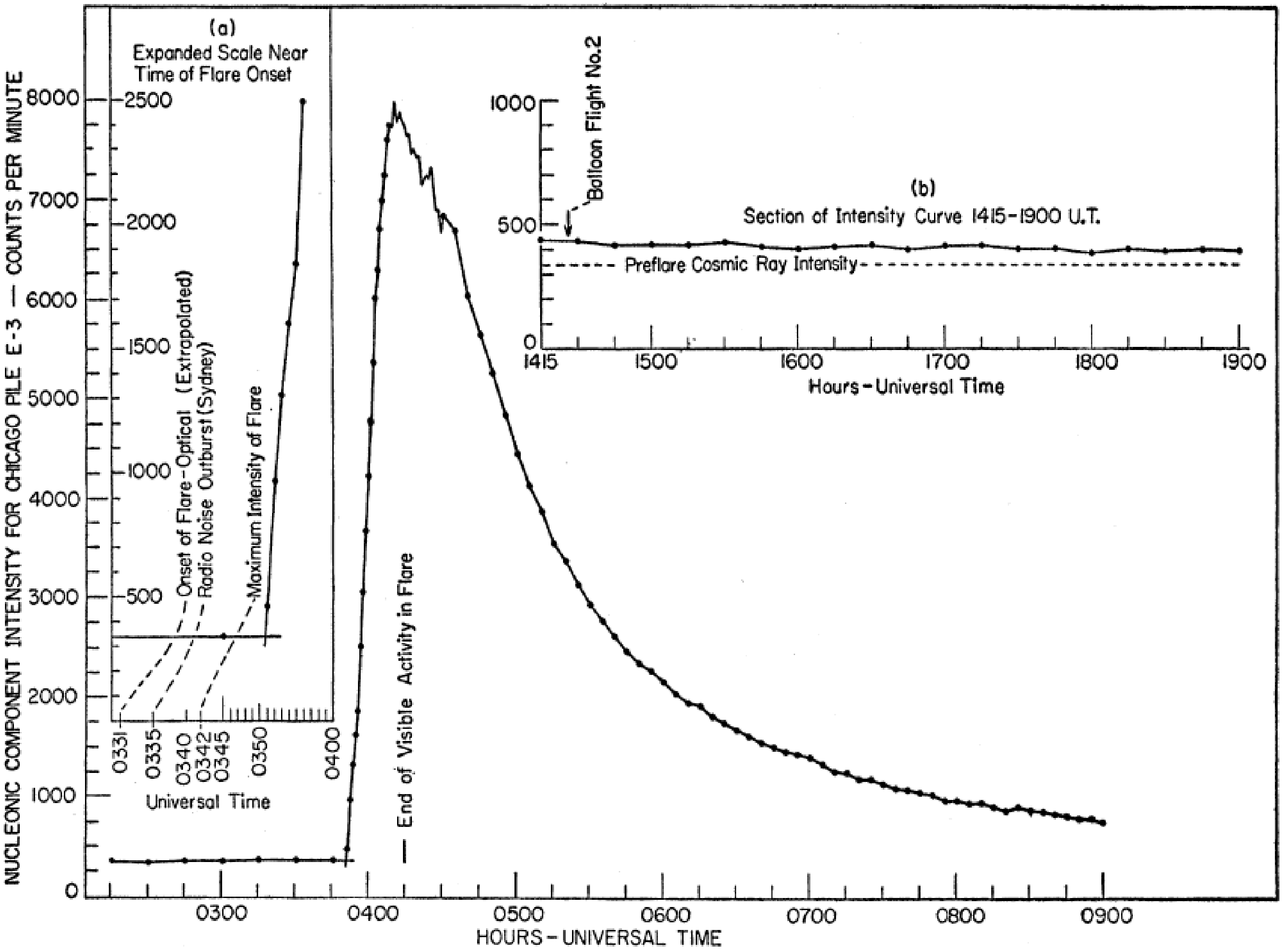


$$\beta = \frac{3X}{X-1}?$$



Van Nes et al., 1984

23 February 1956 GLE Event



Meyer, Parker and Simpson, 1956

Shock Modification

$$\partial/\partial t = \partial/\partial y = \partial/\partial z = \mathbf{V}_D = Q = 0$$

$$V\frac{dP_c}{dx} - \frac{d}{dx}\left(\bar{K}\frac{dP_c}{dx}\right) + \gamma_c\frac{dV}{dx}P_c \cong 0$$

$$\frac{d}{dx}\left(\rho V\right) = 0$$

$$\rho V\frac{dV}{dx} = -\frac{d}{dx}\left(P_g + P_c\right)$$

$$V\frac{dP_g}{dx} + \gamma_g\frac{dV}{dx}P_g = 0$$