

$b \rightarrow sll$ at (very) low q^2

Sebastian Jäger

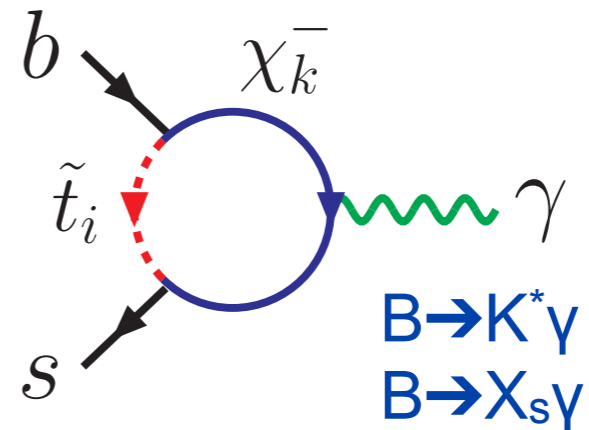
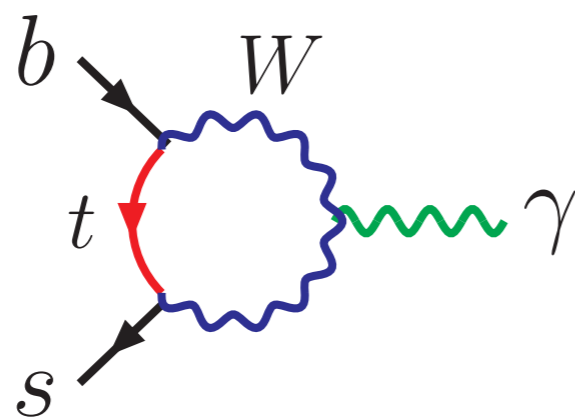


Workshop “Rare B decays in 2015: experiment and theory”
Edinburgh, Tuesday 12 May 2015

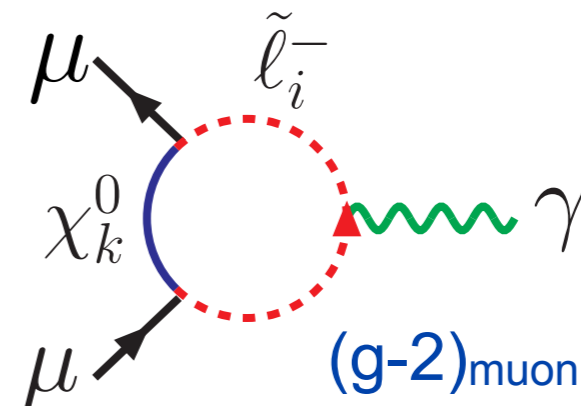
Why rare decays

New particles' couplings tend to mediate flavour changes (they do in all the “natural” proposals for TeV physics)

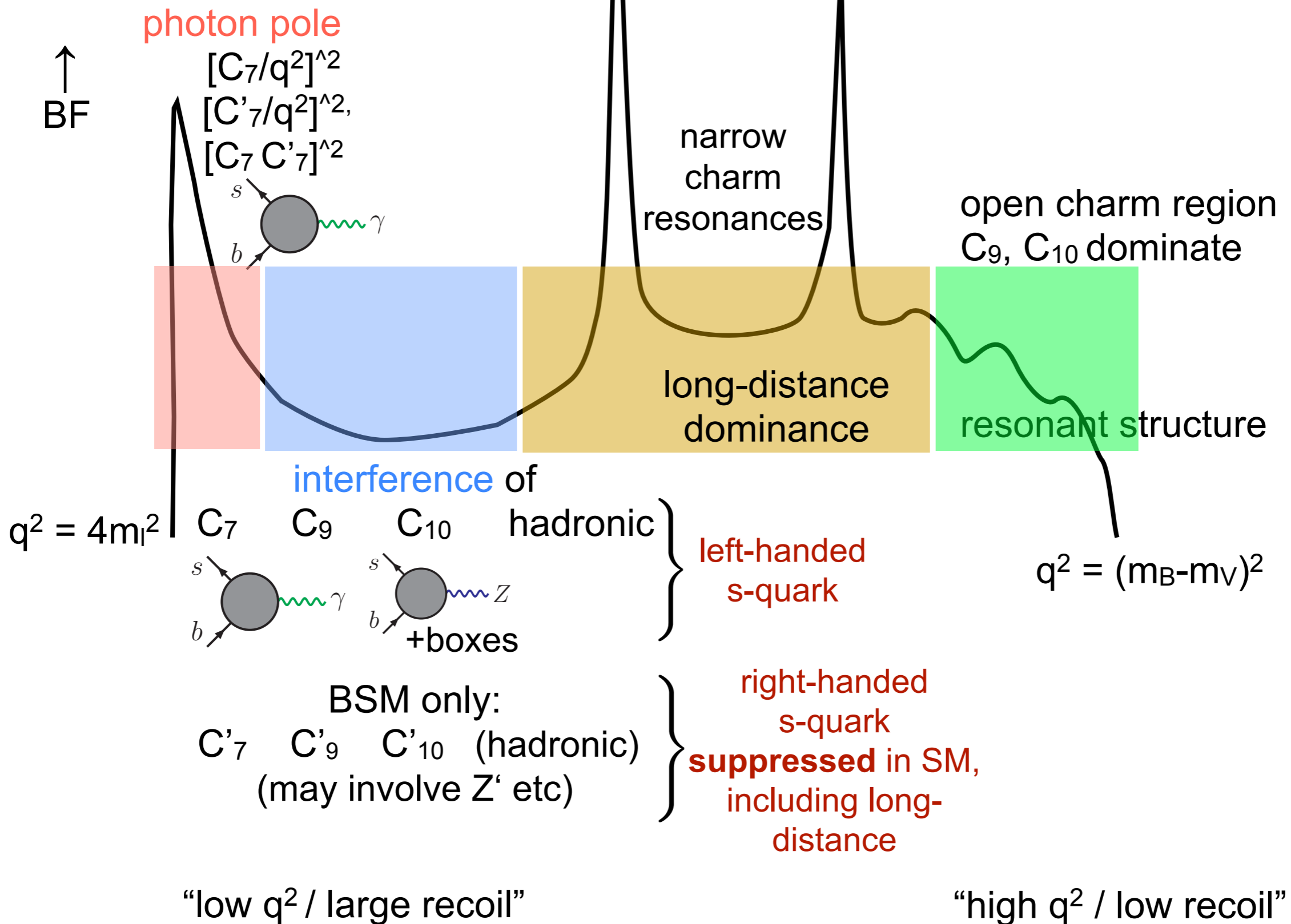
At least they will have CKM-like flavour violations (minimal flavour violation), so will always affect rare decays.



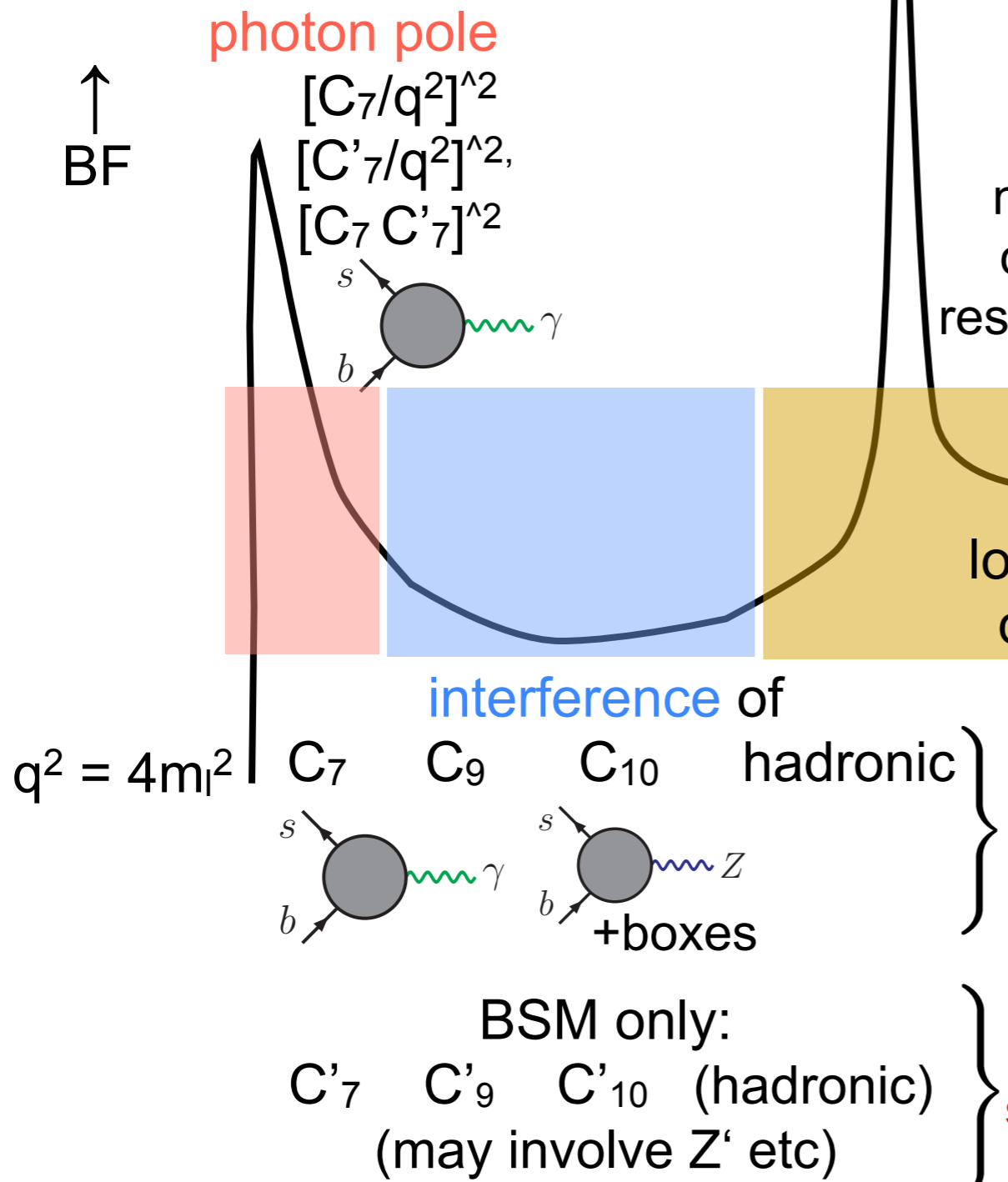
Of course BSM particles will mediate flavour-*conserving* processes, too.



Rate: q^2 dependence (qualitative)



Rate: q^2 dependence (qualitative)



This talk is about picking up the photon pole

Specific sensitivity to C_7 (constrained from $b \rightarrow s$ gamma) and C'_7 (well-motivated BSM effect)

Related to $B \rightarrow K^* \gamma$ (completely model-independently)

Unlike other observables, form factor ratios play almost not role.

Main issue is to rule out (or control) sizable effects from the nonleptonic hamiltonian (charm loops etc). Good complementarity of QCDF + LCSR

“low q^2 / large recoil”

“high q^2 / low recoil”

Optimised angular observables

=functions of the angular coefficients for which form factors drop out in the heavy quark limit if perturbative QCD corrections neglected.

E.g.

neglecting strong phase differences
[tiny; take into account in numerics]

Krueger, Matias 2005; Egede et al 2008
Becirevic, Schneider 2011
Matias, Mescia, Ramon, Virto 2012
Descotes-Genon et al 2012

$$P_1 \equiv \frac{I_3 + \bar{I}_3}{2(I_{2s} + \bar{I}_{2s})} = \frac{-2 \operatorname{Re}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2}$$

$$P_3^{CP} \equiv -\frac{I_9 - \bar{I}_9}{4(I_{2s} + \bar{I}_{2s})} = -\frac{\operatorname{Im}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2}$$

$$P_5' = \frac{\operatorname{Re}[(H_V^- - H_V^+) H_A^{0*} + (H_A^- - H_A^+) H_V^{0*}]}{\sqrt{(|H_V^0|^2 + |H_A^0|^2)(|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2)}}$$

= 0. } (Melikhov 1998)
Krueger, Matias 2002
Lunghi, Matias 2006
Becirevic, Schneider 2011
Becirevic, Kou, et al 2012

$$= \frac{C_{10} (C_{9,\perp} + C_{9,\parallel})}{\sqrt{(C_{9,\parallel}^2 + C_{10}^2)(C_{9,\perp}^2 + C_{10}^2)}}$$

where

$$C_{9,\perp} = C_9^{\text{eff}}(q^2) + \frac{2 m_b m_B}{q^2} C_7^{\text{eff}}$$

$$C_{9,\parallel} = C_9^{\text{eff}}(q^2) + \frac{2 m_b E}{q^2} C_7^{\text{eff}}$$

in SM, neglecting power corrections and pert. QCD corrections

C₇ and C₉ opposite sign

destructive interference enhances vulnerability to anything that violates the large-energy form factor relations (or more generally underestimated errors on form factors)

much less of an issue in than to P₁ or P₃^{CP} than eg in P₅' (and others)

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Two approximate null tests of the SM
What are the leading corrections?

where

$$C_{9,\perp} = C_9^{\text{eff}}(q^2) + \frac{2 m_b m_B}{q^2} C_7^{\text{eff}}$$

$$C_{9,\parallel} = C_9^{\text{eff}}(q^2) + \frac{2 m_b E}{q^2} C_7^{\text{eff}}$$

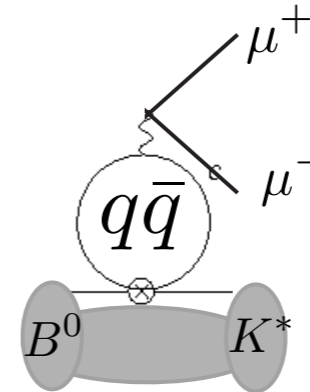
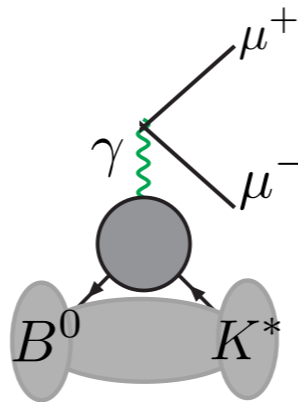
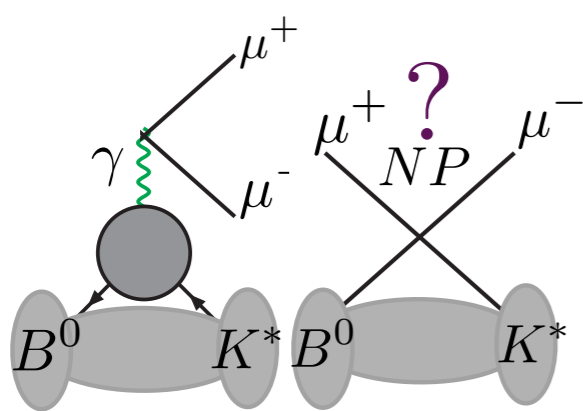
C_7 and C_9 opposite sign

destructive interference enhances vulnerability to anything that violates the large-energy form factor relations (or more generally underestimated errors on form factors)

much less of an issue in than to P_1 or P_3^{CP} than eg in P_5' (and others)

B->Vll vector amplitudes

Only helicity +1 and -1 contribute to P_1 and P_3^{CP}



$$H_V(\lambda) \propto \underbrace{\tilde{V}_\lambda(q^2) C_9 - V_{-\lambda}(q^2) C'_9}_{\text{no photon pole: vanishing relative contribution as } q^2 \rightarrow 0} - \underbrace{\frac{2 m_b m_B}{q^2} \left(\tilde{T}_\lambda(q^2) C_7 - \tilde{T}_{-\lambda}(q^2) C'_7 \right)}_{\text{photon pole at } q^2=0} - \underbrace{\frac{16 \pi^2 m_B^2}{q^2} h_\lambda(q^2)}_{\text{photon pole at } q^2=0}$$

Only one form factor, drops out up to interference
complicated nonlocal correction

Helicity +1 power suppressed in the heavy-quark limit

Burdman, Hiller 2000

form factor T_+ doubly suppressed (further q^2/m_B^2 factor)

nonlocal term known to be singly suppressed (Λ/m_b)

Beneke, Feldmann, Seidel 2001

could be the dominant uncertainty for null tests

Grinstein et al 2004
Khodjamirian et al 2010
(Ball, Jones, Zwicky 2006)

however, extra suppression $\sim \Lambda/m_b$

SJ, Martin Camalich 2012

Heavy-quark limit and corrections

At most 1-2%
over entire 0..6
GeV² range ->
ignore

$$F(q^2) = F^\infty(q^2) + a_F + b_F q^2/m_B^2 + \mathcal{O}([q^2/m_B^2]^2)$$

heavy quark limit

Power corrections - parameterise

SJ, Martin Camalich 2012

$$F^\infty(q^2) = F^\infty(0)/(1 - q^2/m_B^2)^p + \Delta_F(\alpha_s; q^2)$$

(Charles et al)

(Beneke, Feldmann)

- q^2 dependence in heavy-quark limit not known (model by a power p, and/or a pole model)

Corrections are calculable in terms of perturbation theory, decay constants, light cone distribution amplitudes

$$\begin{aligned} V_+^\infty(0) = 0 & \quad T_+^\infty(0) = 0 & \text{from heavy-quark/} \\ V_-^\infty(0) = T_-^\infty(0) & & \text{large energy} \\ V_0^\infty(0) = T_0^\infty(0) & & \text{symmetry} \end{aligned}$$

$$V_+^\infty(q^2) = 0 \quad T_+^\infty(q^2) = 0$$

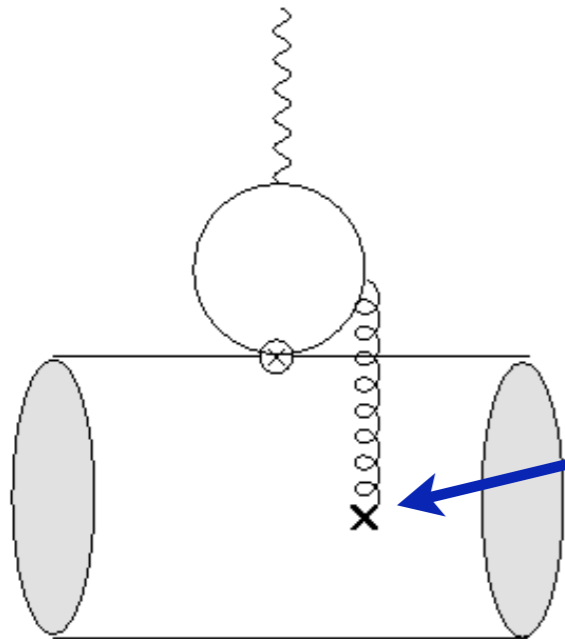
hence

$$\begin{aligned} T_+(q^2) &= \mathcal{O}(q^2) \times \mathcal{O}(\Lambda/m_b) \\ V_+(q^2) &= \mathcal{O}(\Lambda/m_b). \end{aligned}$$

- “naively factorizing” part of the helicity amplitudes $H_{V,A}^+$ strongly suppressed as a consequence of chiral SM weak interactions
- We see the suppression is particularly strong near low- q^2 endpoint
- Form factor relations imply reduced uncertainties in suitable observables

Burdman, Hiller 1999 (quark picture) confirmed in QCDF/SCET Beneke, Feldmann, ...

Charm loop estimate



$$h_\lambda|_{c\bar{c}} = \frac{1}{m_B^2} \frac{2}{3} \epsilon^{\mu*}(\lambda) \int d^4y e^{iq \cdot y} \langle M [T[(\bar{c}\gamma^\mu c)(y)(C_1^c Q_1^c + C_2^c Q_2^c)(0)]] | \bar{B} \rangle$$

consider soft gluon (in B rest frame)

From collinear factorisation viewpoint this represents the endpoint region, which is known to give a power-suppressed contribution

perform a “light-cone OPE”

(This is equivalent to expanding the charm loop, treating $\Lambda^2/(4 m_c^2) \sim \Lambda/m_b$) [Khodjamirian et al 2010](#)

obtain

$$h_\lambda|_{c\bar{c},LD} = \epsilon^{\mu*}(\lambda) \langle M(k, \lambda) | \tilde{\mathcal{O}}_\mu | \bar{B} \rangle$$

$$\tilde{\mathcal{O}}_\mu = \int d\omega I_{\mu\rho\alpha\beta}(q, \omega) \bar{s}_L \gamma^\rho \delta\left(\omega - \frac{in_+ \cdot D}{2}\right) \tilde{G}^{\alpha\beta} b_L$$

(a nonlocal, light-cone operator)

need estimate of $\langle M(k, \lambda) | \tilde{\mathcal{O}}_\mu | \bar{B} \rangle$ (which goes into H_V^λ)

light-cone SR based on [Khodjamirian et al 2010](#) for K^* helicity amplitudes [SJ, Martin Camalich 2012](#)

outcome: helicity hierarchy remains for the endpoint region

same conclusion for (anyway CKM-suppressed) light-quark LD effects at low q^2 (estimated via VMD)

RH current probes

Extending to BSM Wilson coefficients, have

$$\begin{aligned}
 P_1 &\equiv \frac{I_3 + \bar{I}_3}{2(I_{2s} + \bar{I}_{2s})} \stackrel{\substack{\text{neglecting strong phase differences} \\ \text{[tiny; take into account in numerics]}}}{\approx} \frac{-2 \operatorname{Re}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2} \stackrel{\substack{\text{close to } q^2 = 0 \text{ (photon} \\ \text{pole dominance)}}}{\approx} 2 \frac{\operatorname{Re}(C_7 C_7'^*)}{|C_7|^2 + |C_7'|^2} \\
 P_3^{CP} &\equiv -\frac{I_9 - \bar{I}_9}{4(I_{2s} + \bar{I}_{2s})} = -\frac{\operatorname{Im}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2} \approx \frac{\operatorname{Im}(C_7 C_7'^*)}{|C_7|^2 + |C_7'|^2}
 \end{aligned}$$

- **double** suppression $T_+(q^2) = \mathcal{O}(q^2/m_B^2) \times \mathcal{O}(\Lambda/m_b)$

- extra suppression of LD contribution to H_V^+ (model by effective helicity-dependent C_7 (or C_9) shift, within range established by power counting)

Helicity hierarchy survives power corrections
and is highly effective close to $q^2=0$

Predictions at very low q^2

SJ, Martin Camalich
1412.3183

Bin [GeV^2]	Br [10^{-8}]	P_1	P_2	P_3^{CP} [10^{-4}]
[0.1, 0.98]	$9.5^{+5.2}_{-3.5}$	$0.024^{+0.053}_{-0.055}$	$-0.16^{+0.05}_{-0.04}$	$0.1^{+0.7}_{-0.8}$
Electron	26^{+12}_{-9}	$0.030^{+0.047}_{-0.044}$	$-0.073^{+0.020}_{-0.016}$	$0.1^{+0.6}_{-0.6}$

[0.0004, 1.12 \pm 0.06]

- Very clean, very insensitive to form factor input
- Boost in BR: nearly 3x more electrons, most of the extra ones in the relevant q^2 region \rightarrow partly offsets lower efficiency in LHCb

	Result	QCDF	Fact. p.c.'s	Non-fact. p.c.'s
P_1	$0.030^{+0.047}_{-0.044}$	$+0.008$ -0.003	± 0.012	$+0.028$ -0.026
P_3^{CP} [10^{-4}]	$0.1^{+0.7}_{-0.6}$	± 0.3	± 0.2	± 0.3

$$A_T^{(2)} = -0.23 \pm 0.23 \pm 0.05$$

$$A_T^{\text{Im}} = +0.14 \pm 0.22 \pm 0.05$$

$$A_T^{\text{Re}} = +0.10 \pm 0.18 \pm 0.05$$

LHCb, 1501.03028, JHEP 1504 (2015) 064

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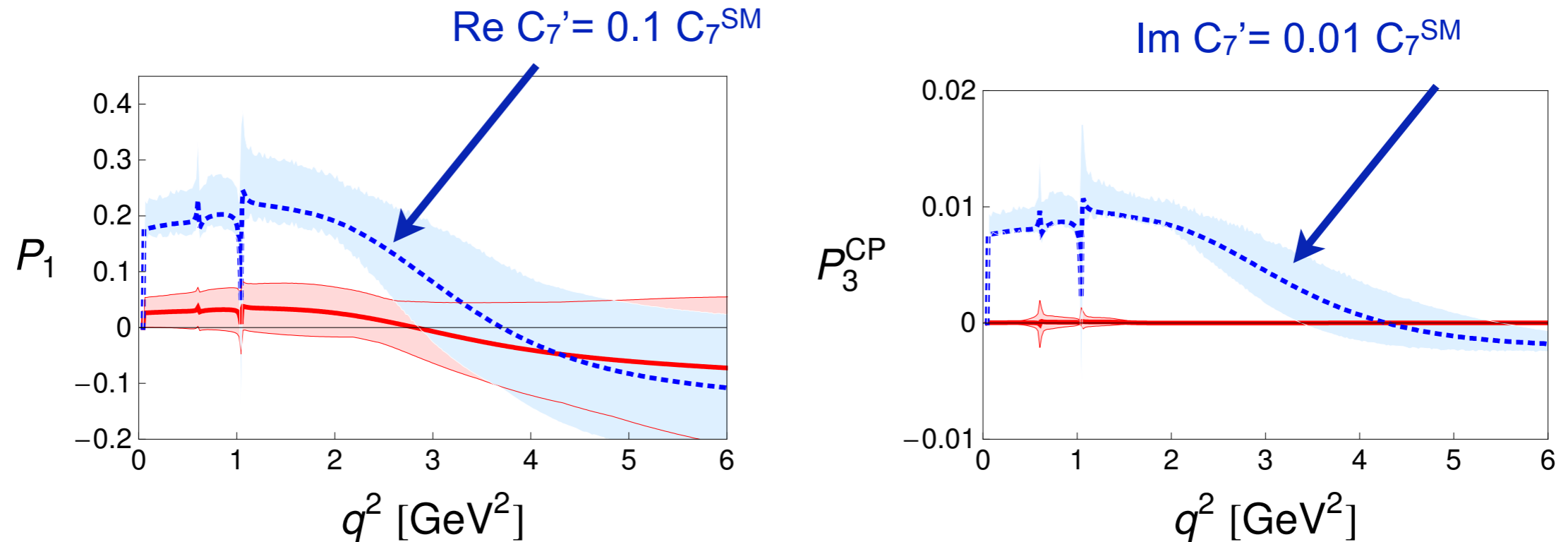
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Experiment (electrons) $A_T^{(2)} = -0.23 \pm 0.23 \pm 0.05$ LHCb, 1501.03028, JHEP 1504 (2015) 064

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Sensitivity to C_7' (muonic mode)



SJ, Martin Camalich 2012

- Two angular observables remain clean null tests of the SM in the presence of long-distance corrections
- (theoretical limit on) sensitivity to Re C_7' at $<10\%$ (C_7^{SM}) level, to Im C_7' at $<1\%$
- sensitivity stems from $q^2 < 2 \text{ GeV}^2$

Relation to B->K*γ

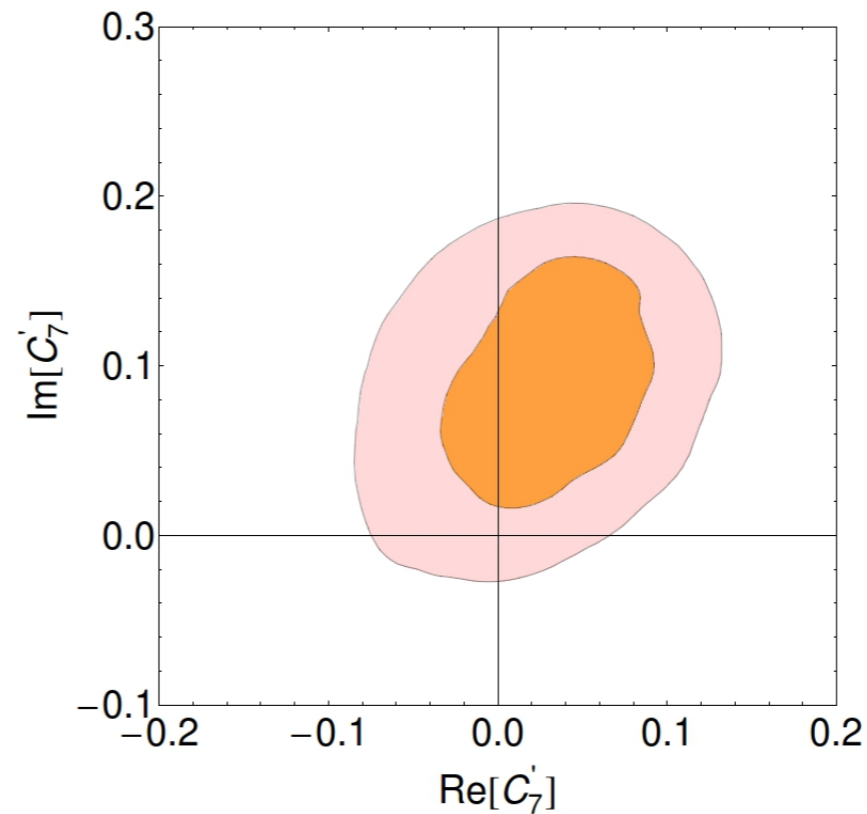
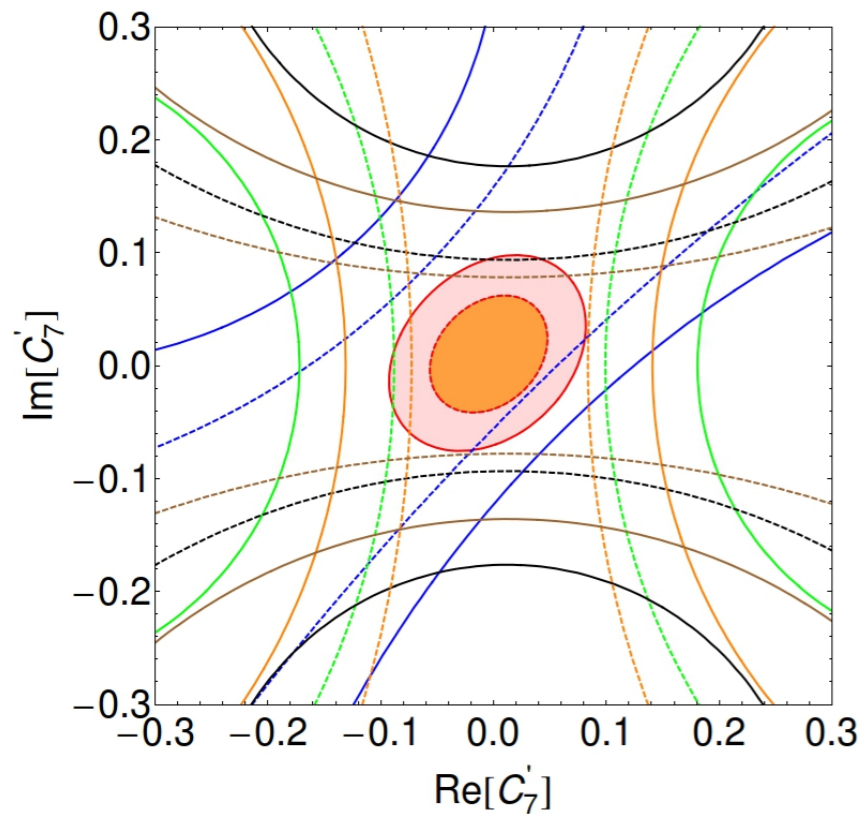
$$\begin{aligned}
 \mathcal{A}(\bar{B} \rightarrow V(\lambda)\gamma(\lambda)) &= \lim_{q^2 \rightarrow 0} \frac{q^2}{e} H_V(q^2 = 0; \lambda) && \text{exact (LSZ)} \\
 &= \frac{iNm_B^2}{e} \left[\frac{2\hat{m}_b}{m_B} (C_7 \tilde{T}_\lambda(0) - C'_7 \tilde{T}_{-\lambda}(0)) - 16\pi^2 h_\lambda(q^2 = 0) \right]
 \end{aligned}$$

(only $\lambda = \pm 1$)

same amplitudes as in B->Kll including all long-distance details

$$S_{K^*\gamma} = 2 \frac{\text{Im}(e^{-i\phi_d} H_V^+(0) H_V^{-*}(0))}{|H_V^+(0)|^2 + |H_V^-(0)|^2} \approx 2 \frac{\text{Im}(e^{-i\phi_d} C_7 C_7'^*)}{|C_7|^2 + |C_7'|^2}$$

Status/prospects



SJ, Martin Camalich
1412.2183

awaiting update with
2015 electron and
muon data!

$$S \simeq \frac{2\text{Im}(e^{-2i\beta} C_7 C_7')}{|C_7|^2 + |C_7'|^2},$$

$$P_1 \simeq \frac{2\text{Re}(C_7 C_7')}{|C_7|^2 + |C_7'|^2},$$

$$P_3^{\text{CP}} \simeq \frac{2\text{Im}(C_7 C_7')}{|C_7|^2 + |C_7'|^2}$$

- Left: assuming $\sigma_{P_i} = 0.25$ for muons and electrons, no theory errors
- Right: Profile likelihood for 2014 data (1sigma and 95% CL)
- excellent sensitivity to right-handed currents remains with conservative treatment of QCD uncertainties

A note on LUV tests

SM predicts lepton universality to great accuracy. In particular, apart from lepton mass effects all helicity amplitudes coincide and hence, to our accuracy, the theory error on any LUV ratio or difference is zero.

Altmannshofer, Straub; Hiller, Schmaltz;...; SJ, Martin Camalich 1412.3183

Lepton-flavour-dependence of position of zero-crossings

$$\Delta_0^i \equiv (q_0^2)_{I_i}^{(\mu)} - (q_0^2)_{I_i}^{(e)}$$

have negligible uncertainty (ie zero in SM within our approximations)

SJ, Martin Camalich 1412.3183

Complementary to ratios, as around the zeroes ratios will have large uncertainties (due to low statistics)

Probably not for LHCb alone due to energy resolution issues, but could be good opportunity for LHCb-Belle2 interplay.

Conclusions

Very low q^2 provides excellent sensitivity to right-handed dipole transitions (Wilson coefficient C_7').

Reaching this conclusion involves combining heavy-quark expansions and LCSR methods to establish a double suppression of the “wrong-helicity” amplitude in the SM

Electrons are very useful: factor 3 higher rate partially offsets lower acceptance in LHCb.

Good complementarity of LHCb electron and muon data

Possible LHCb-Belle2 interplay

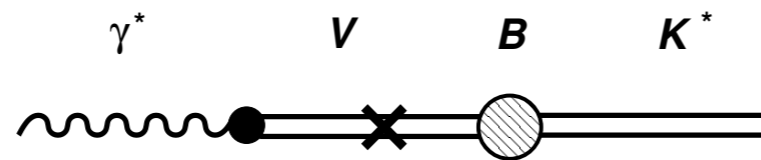
BACKUP

Light-quark contributions

Operators without charm have strong charm or CKM suppression; power corrections should be negligible.

However, they generate (mild) resonance structure even below the charm threshold, presumably “duality violation”

Presumably ρ, ω, ϕ most important; use vector meson dominance supplemented by heavy-quark limit $B \rightarrow VK^*$ amplitudes



$$\tilde{a}_\mu^{\text{had, lq}} = \int d^4x e^{-iq \cdot x} \sum_{P, P'} \langle 0 | j_\mu^{\text{em, lq}}(x) | P' \rangle \langle P'(x) | P(0) \rangle \langle K^* P | \mathcal{H}_{\text{eff}}^{\text{had}}(0) | B \rangle$$

estimate **uncertainty** from difference between VMD model and the subset of heavy-quark limit diagrams corresponding to intermediate V states.

Helicity hierarchies in **hadronic** B decays prevent large uncertainties in H_V^+ from this source, too.