QED and Electroweak effects in $b \to s\, l^+ l^-$ transitions

``Rare B Decays 2015 – Experiment and Theory“
Edinburgh, 12 May 15

Based on works with Christoph Bobeth, Emanuel Stamou [PRD 89, 034023 (2014)]
Christoph Bobeth, Thomas Hermann, Mikolaj Misiak, Emanuel Stamou and Matthias Steinhauser [PRL 112, 101801 (2014)]
Ulrich Haisch [unpublished]

Martin Gorbahn
University of Liverpool

Content

Rare B decays

QED effects for $B_s \to \mu^+ \mu^-$ and R_K

Electroweak corrections

Conclusions

Rare B Decays

FCNCs which are dominated by top-quark loops:

$b \to s:$ $|V_{tb}^* V_{ts}| \propto \lambda^2$

$b \to d:$ $|V_{tb}^* V_{td}| \propto \lambda^3$

$s \to d:$ $|V_{ts}^* V_{td}| \propto \lambda^5$

B decays do not show the CKM suppression of K decays

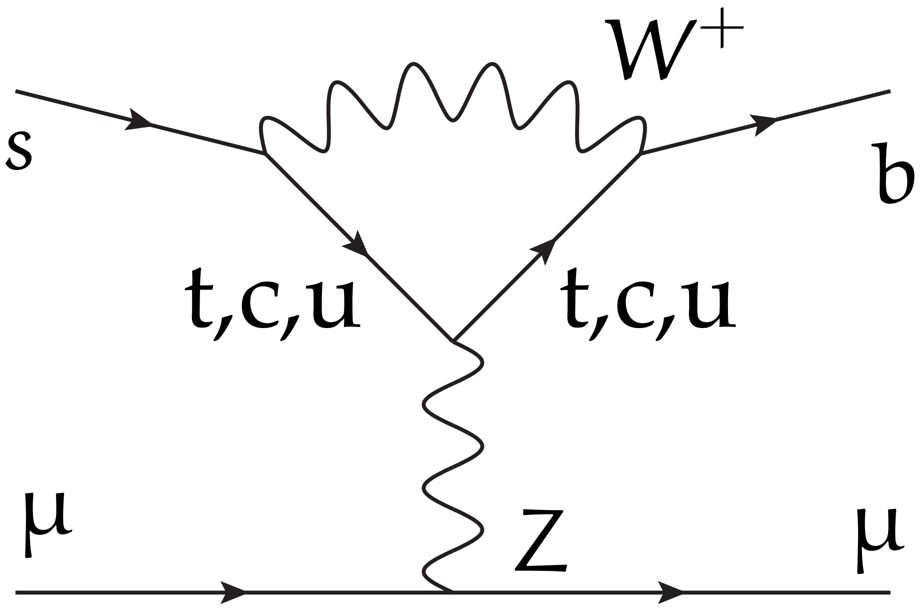
2 photon pollution is much smaller in $b \to s\, l^+ l^-$ decays

We can test helicity suppressed modes and more operators

$$Q_7 = (\bar{b}_L \sigma_{\mu\nu} s_L) F^{\mu\nu},\ Q_V = (\bar{b}_L \gamma_\mu s_L)(\bar{l}\gamma_\mu l),\ Q_A = (\bar{b}_L \gamma_\mu s_L)(\bar{l}\gamma_\mu \gamma_5 l)$$

E.g. $B_{(s)} \to l^+ l^-$, $B \to K^{(*)} l^+ l^-$, $B \to X_s \gamma$, ...

$B_s \to \mu^+ \mu^-$ in the Standard Model



+ Box diagrams

B_s is (pseudo)scalar – no photon penguin

$$Q_A = (\bar{b}_L \gamma_\mu s_L)(\bar{l}\gamma_\mu \gamma_5 l)$$

Dominant operator in the SM

helicity suppression $\left(\propto \frac{m_l^2}{M_B^2}\right)$

$$\propto |V_{tb}^* V_{ts}| \simeq \left|1 - \lambda^2 \left(\tfrac{1}{2} - i\eta - \rho\right)\right| V_{cb}$$

Effective Lagrangian in the SM:

$$\mathcal{L}_{\text{eff}} = G_F^2 M_W^2\, V_{tb}^* V_{ts}\, (C_A Q_A + C_S Q_S + C_P Q_P) + h.c.$$

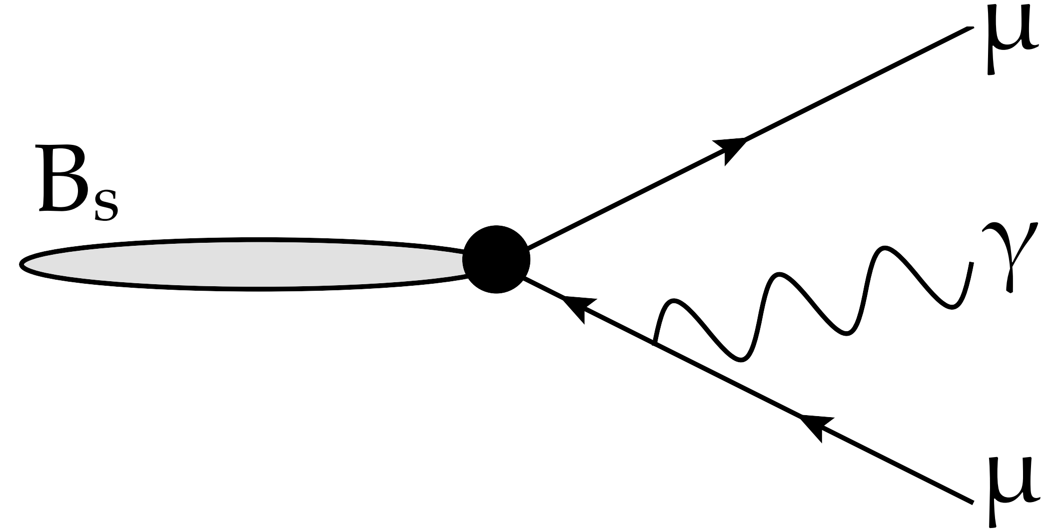
Scalar operators: $Q_S = (\bar{b}_R q_L)(\bar{l}l)$ $\quad Q_P = (\bar{b}_R q_L)(\bar{l}\gamma_5 l)$

Standard Model: C_S & C_P are highly suppressed

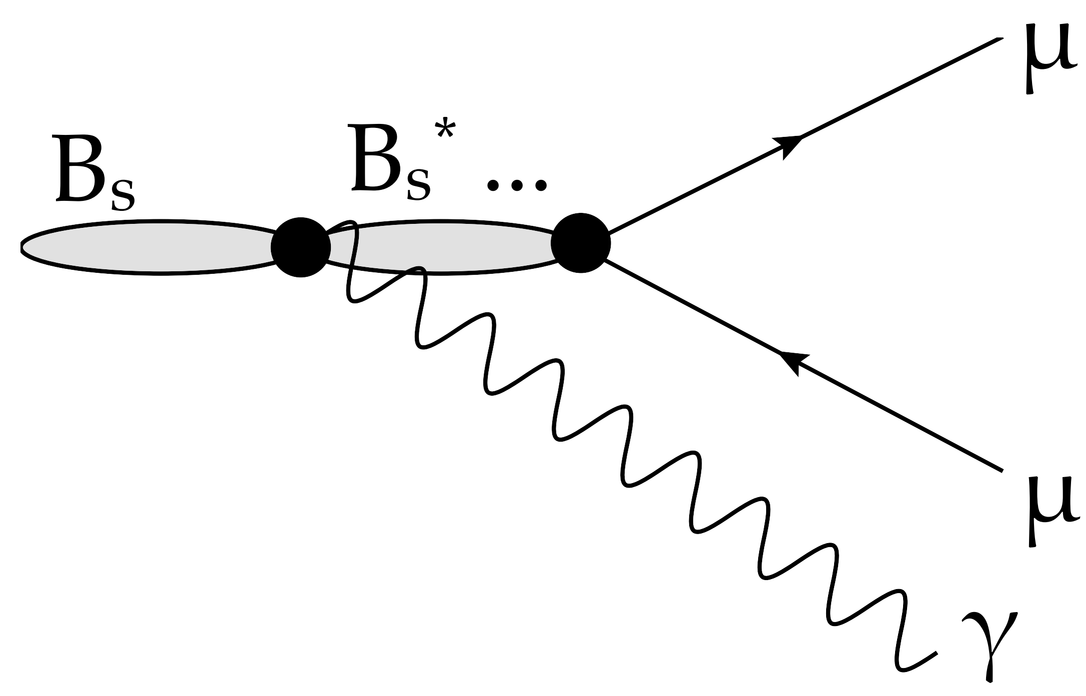
QED corrections I

B_s decay into a 2 lepton final state always helicity suppressed

Soft photon radiation from muons: Theoretical branching ratio is fully inclusive of bremsstrahlung. There would be sizeable corrections otherwise [Buras, Girrbach, Guadagnoli, Isidori] arXiv:1208.0934.



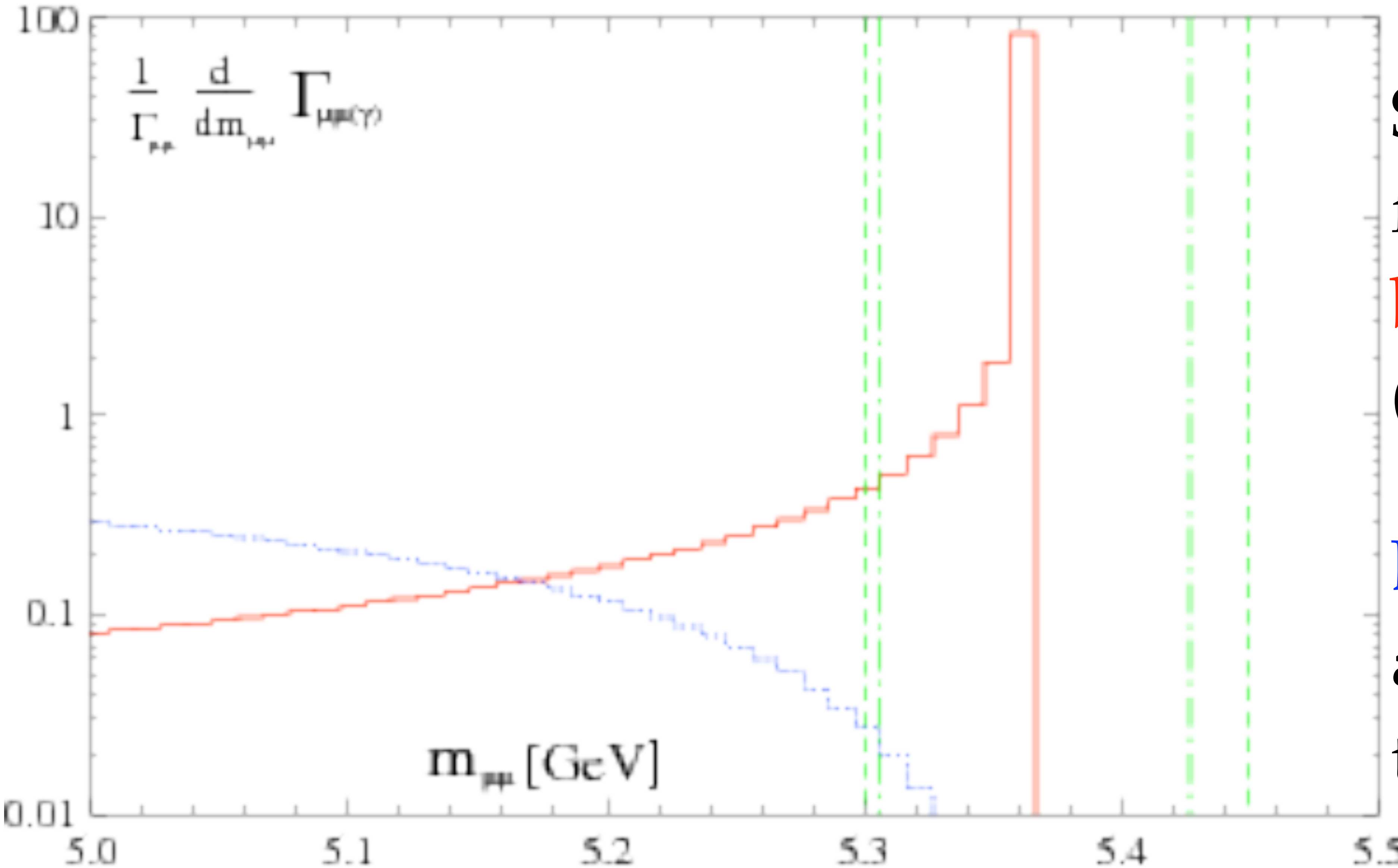
Direct emission is IR safe (B_s is neutral) and phase space suppressed for invariant mass $m_{\mu\mu}$ close to M_{Bs}.
[Aditya, Healey, Petrov] arXiv: 1212.4166



Next correction would be $O(\alpha^3)$

Illustration

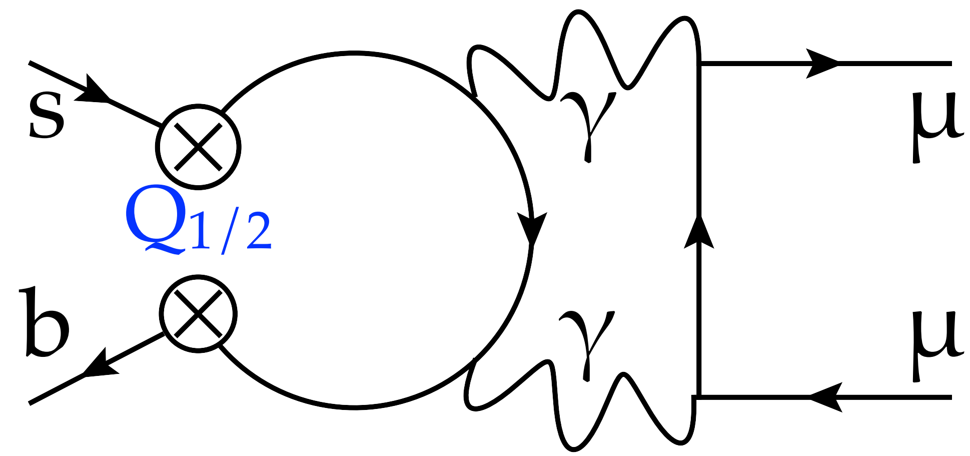
Consider an experimental signal window for the invariant mass of the muon pair $m_{\mu\mu}$



Simulate signal fully inclusive of bremsstrahlung (PHOTOS)

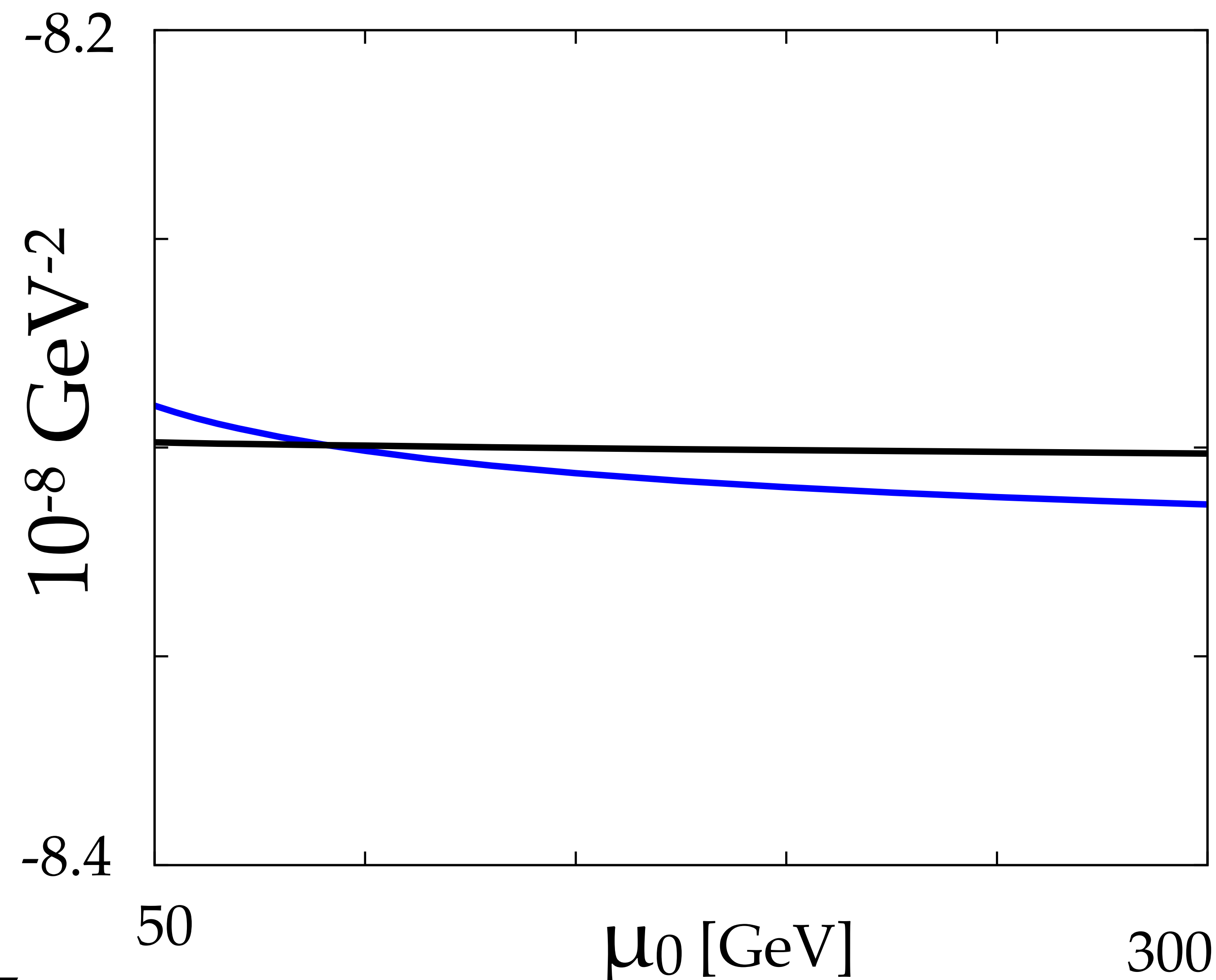
Direct emission is a background in the signal window

QED RGE for C_A



$G^2_F\, M^2_W\, C_A(M_Z)$

NLL running cancels matching scale dependence in Q_A



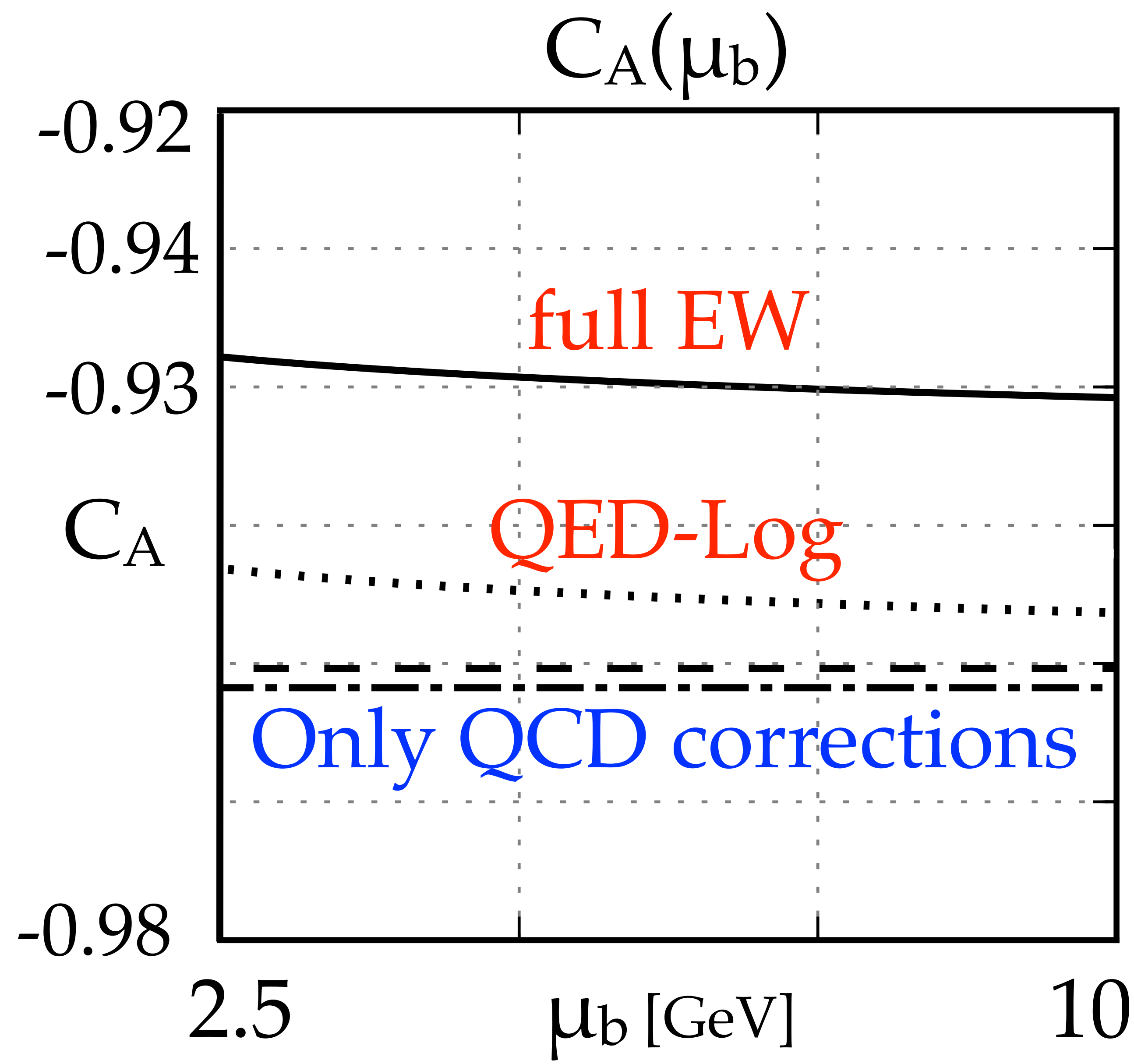
Study residual scale dependence for the G_F^2 M_W^2 normalised results

$G_F^2\, M_W^2\, C(\mu_0)$ is scale dependent, while $U(M_Z, \mu_0)\, G_F^2\, M_W^2\, C(\mu_0)$ is only residually scale dependent.

Wilson Coefficient at m_b

The log enhanced QED corrections further reduce the modulus of the Wilson coefficient further.

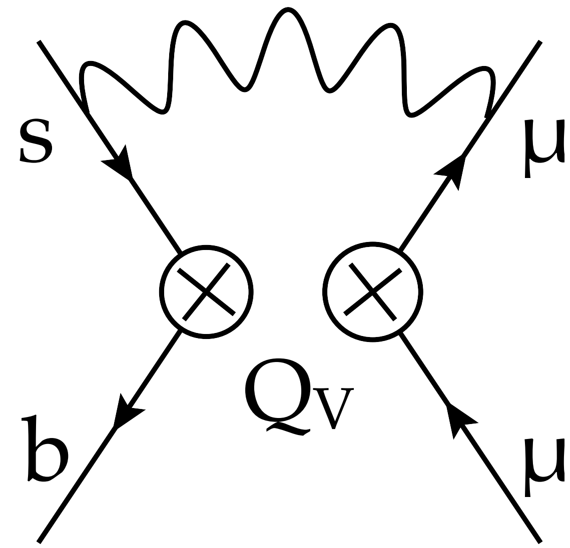
Varying μ_b in $U(\mu_b, m_t)\, G_F^2 M_W^2\, C(m_t)$ gives a measure of uncertainty regarding the contributions of virtual QED corrections at m_b.



The 0.3% scale dependence is not canceled at the scale μ_b

Remaining QED uncertainty

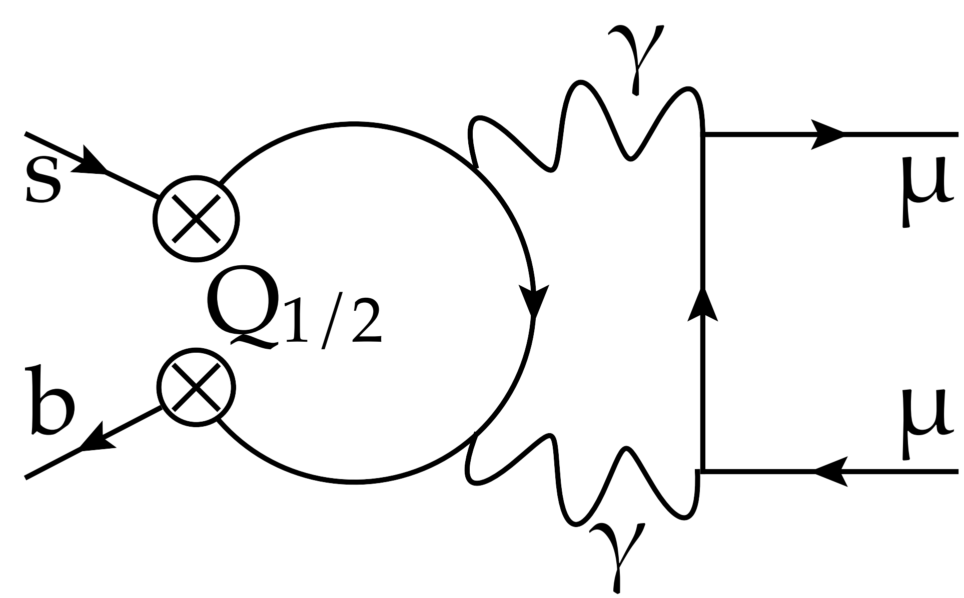
The remaining 0.3% μ_b scale dependence will only be removed after non-perturbative QED corrections are included.



I.e. QED⊗QCD Matrix elements of

$$Q_1 = (\bar{b}\gamma_\mu T^a q_L)(\bar{q}\gamma_\mu T^a s_L)$$
$$Q_2 = (\bar{b}\gamma_\mu q_L)(\bar{q}\gamma_\mu s_L)$$
$$Q_V = (\bar{b}\gamma_\mu s_L)(\bar{l}\gamma_\mu l)$$

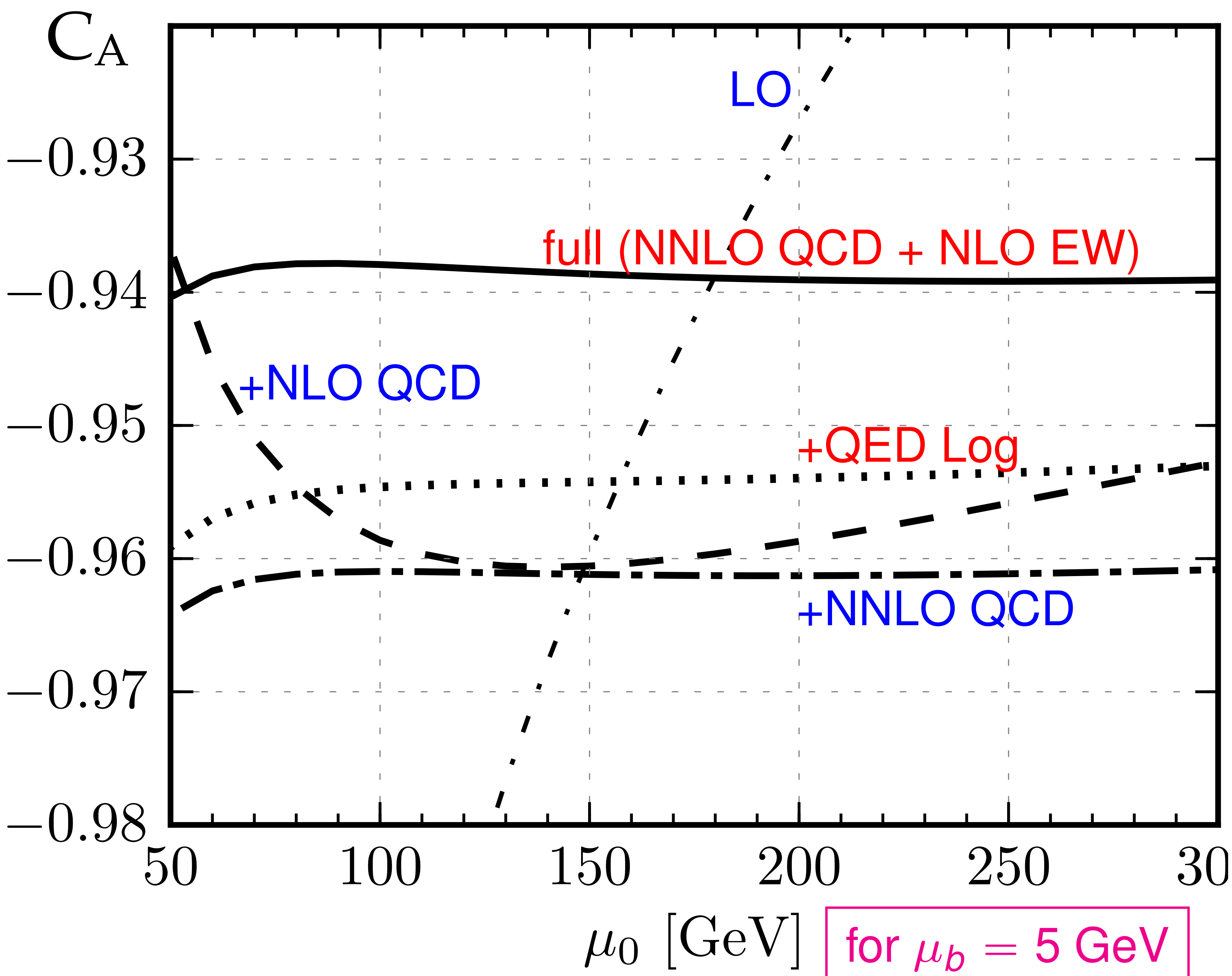
could be considered, but they are $O(\alpha/\pi) \lesssim 0.3\%$ – our error estimate



No relevant lifting of Helicity suppression

Combine with NNLO QCD

Three loop QCD matching, i.e. NNLO, removes scale ambiguities – fixes top mass [Hermann, Misiak, Steinhauser `14]



$\mathrm{R_K}$

$$R_K = \frac{\int_{q^2_{\min}}^{q^2_{\max}} \frac{\mathrm{d}\Gamma[B^+ \to K^+\mu^+\mu^-]}{\mathrm{d}q^2}\mathrm{d}q^2}{\int_{q^2_{\min}}^{q^2_{\max}} \frac{\mathrm{d}\Gamma[B^+ \to K^+e^+e^-]}{\mathrm{d}q^2}\mathrm{d}q^2}$$

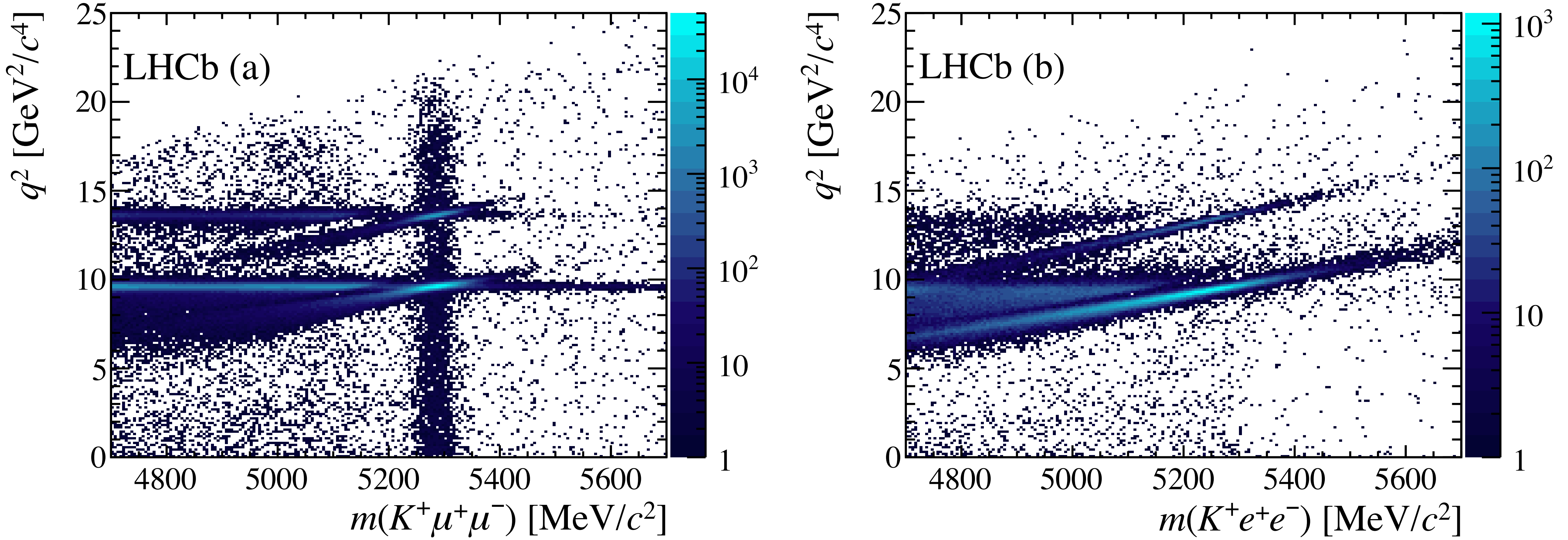
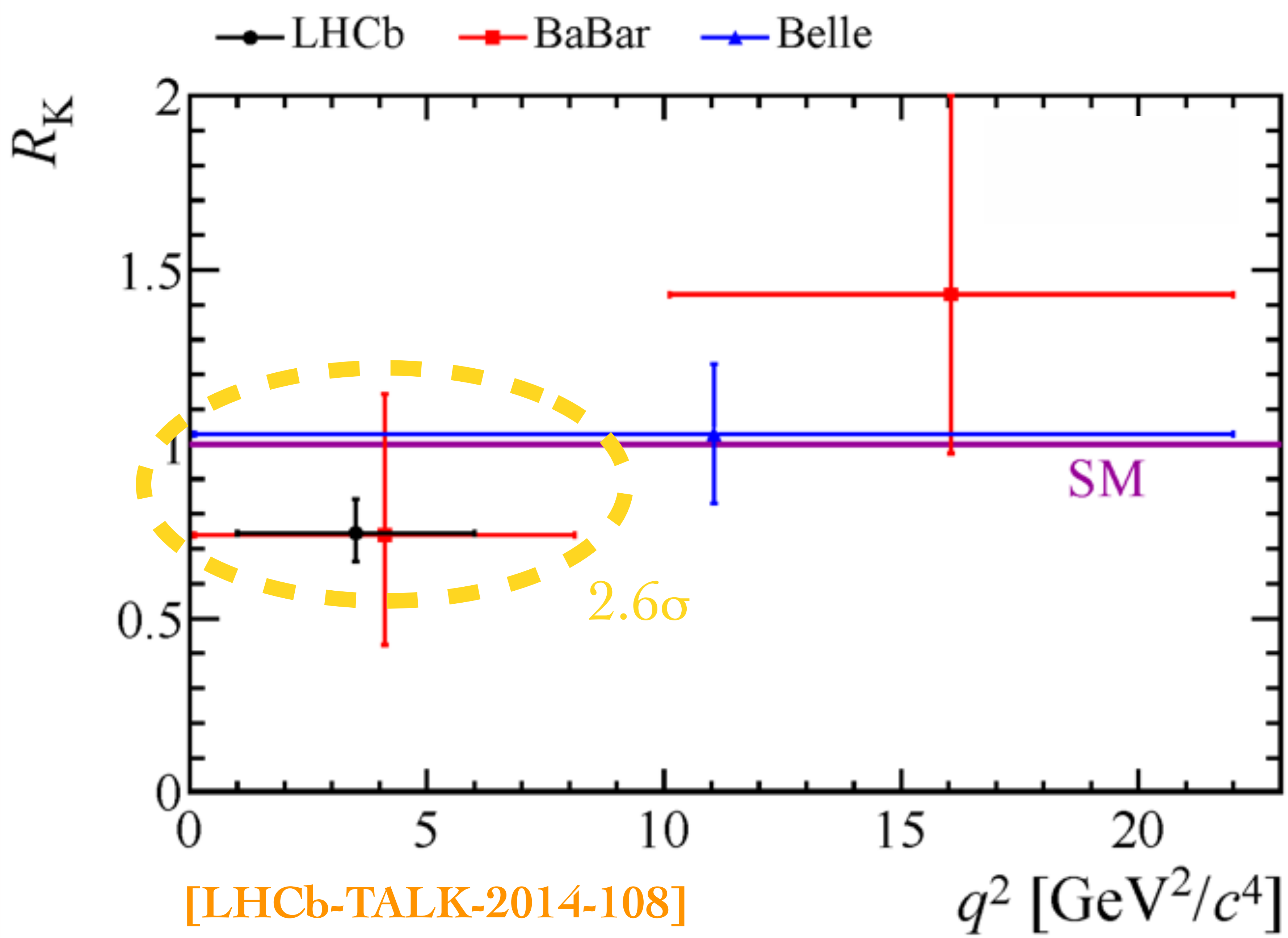


Figure 1: Dilepton invariant mass squared, q^2, as a function of the $K^+\ell^+\ell^-$ invariant mass, $m(K^+\ell^+\ell^-)$, for selected (a) $B^+ \to K^+\mu^+\mu^-$ and (b) $B^+ \to K^+e^+e^-$ candidates. The radiative tail of the J/ψ and $\psi(2S)$ mesons is most pronounced in the electron mode due to the larger bremsstrahlung and because the energy resolution of the ECAL is lower compared to the momentum resolution of the tracking system.

[U. Haisch]

R_K anomaly



[LHCb-TALK-2014-108]

Maybe R_K not alone

[U. Haisch]

$$R_{X_s} = \frac{\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} dq^2 \, \frac{d\Gamma\left(B \to X_s \mu^+ \mu^-\right)}{dq^2}}{\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} dq^2 \, \frac{d\Gamma\left(B \to X_s e^+ e^-\right)}{dq^2}} = 0.34 \pm 0.16$$

$$R_{X_s}^{\mathrm{SM}} = 1 - 4.3\%$$

3.9σ

[http://belle.kek.jp/belle/theses/doctor/2009/Nakayama.pdf]

$\mathrm{R_K}$: null test in SM?

[U. Haisch]

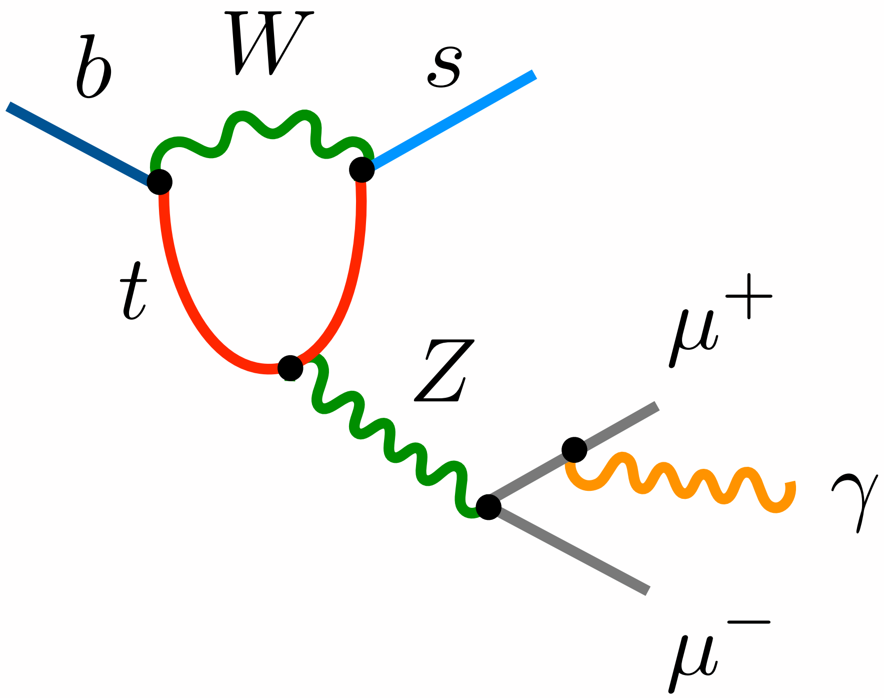
$$R_K^{\mathrm{SM}} = 1 + \mathcal{O}\left(\frac{m_\mu^2}{m_b^2}\right) = 1.0003 \pm 0.0001$$

[Bobeth et al., arXiv:0709.4174]

[U. Haisch]

R_K: null test in SM?

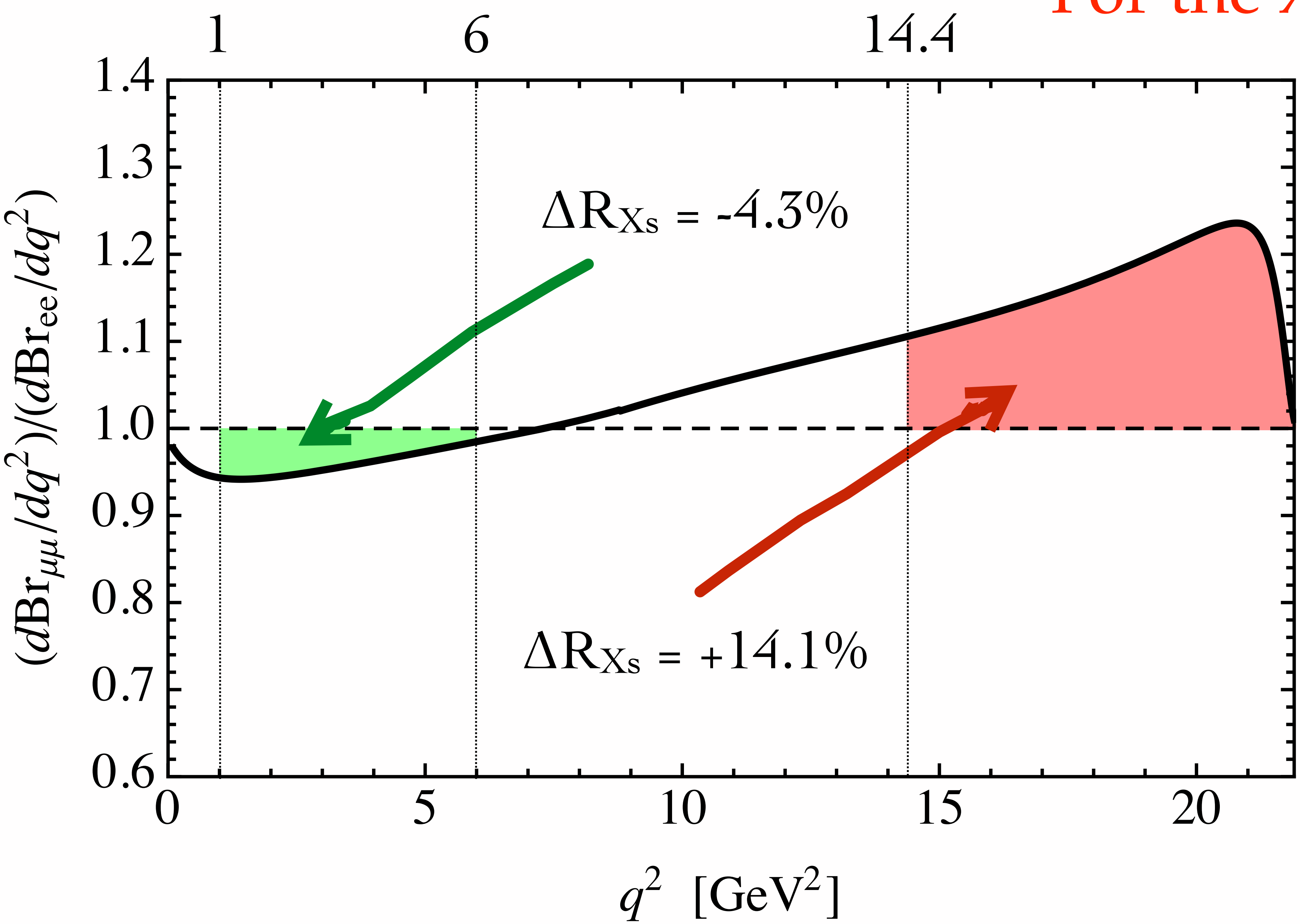
$$R_K^{\mathrm{SM}} = 1 + \mathcal{O}\left(\frac{m_\mu^2}{m_b^2}\right) + \mathcal{O}\left(\alpha \ln \frac{m_\mu^2}{m_b^2}\right) = 1 + \mathcal{O}(0.01)$$



How big is O(0.01)?

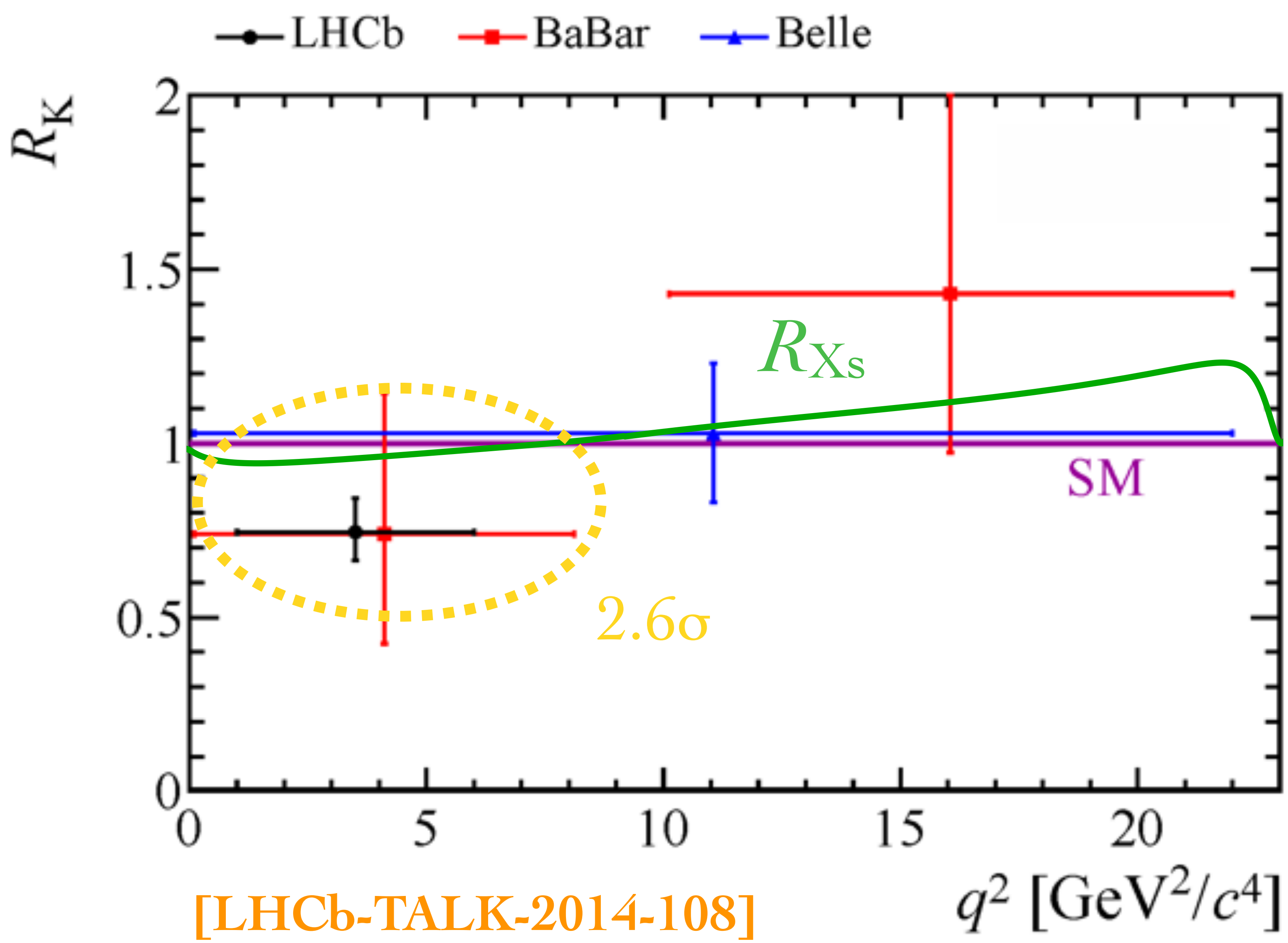
[U. Haisch]

For the X_s decay



[UH based on Huber et al., hep-ph/0510266]

How big is O(0.01)? [U. Haisch]



- Naive inclusion of collinear QED logarithms (from R_{Xs}) fails to explain anomaly, but corrections seem to improve tension in R_K

But the photons are not hard in the exclusive measurement

Comparison with EVTGEN+PHOTOS

The LHCb analysis setup agrees with the analytic calculation

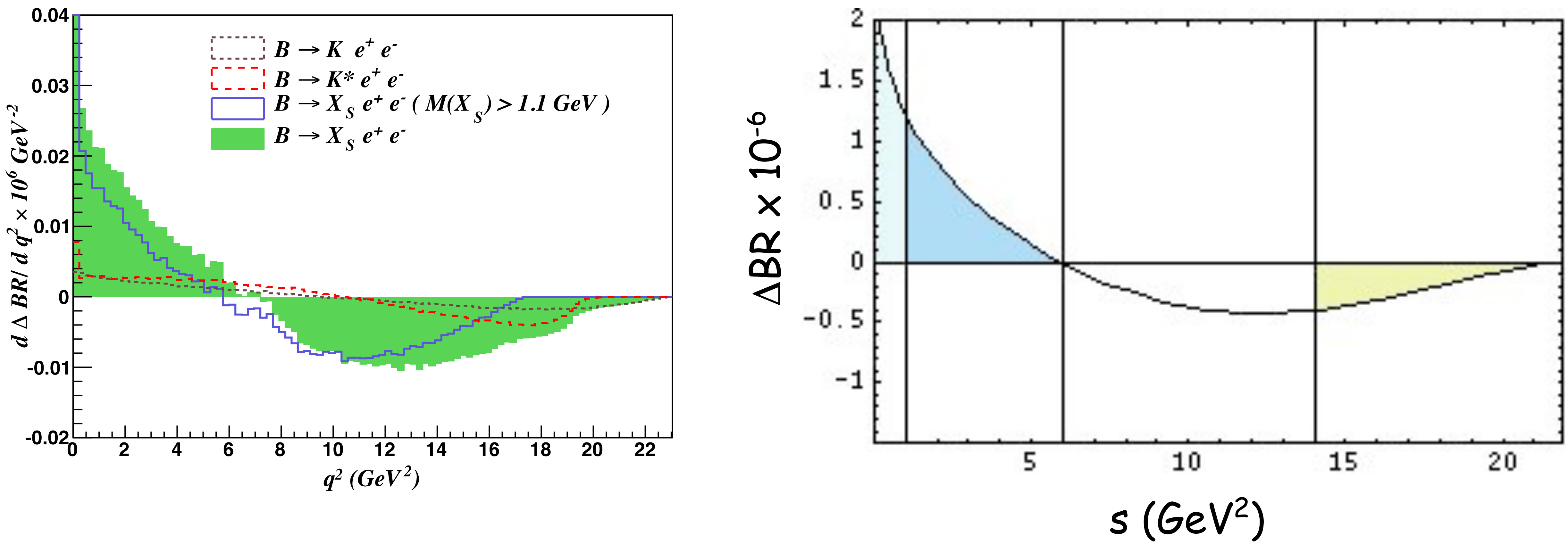


Figure 11. Effect of the inclusion of electromagnetic radiation calculated using EVTGEN + PHOTOS (left) and using analytical methods (right).

[Huber, Hurth, Lunghi 1503.0489]

Invariant Mass Cuts on $B^+ \to K^+ l^+ l^-$

Using soft universal photon corrections (Weinberg)

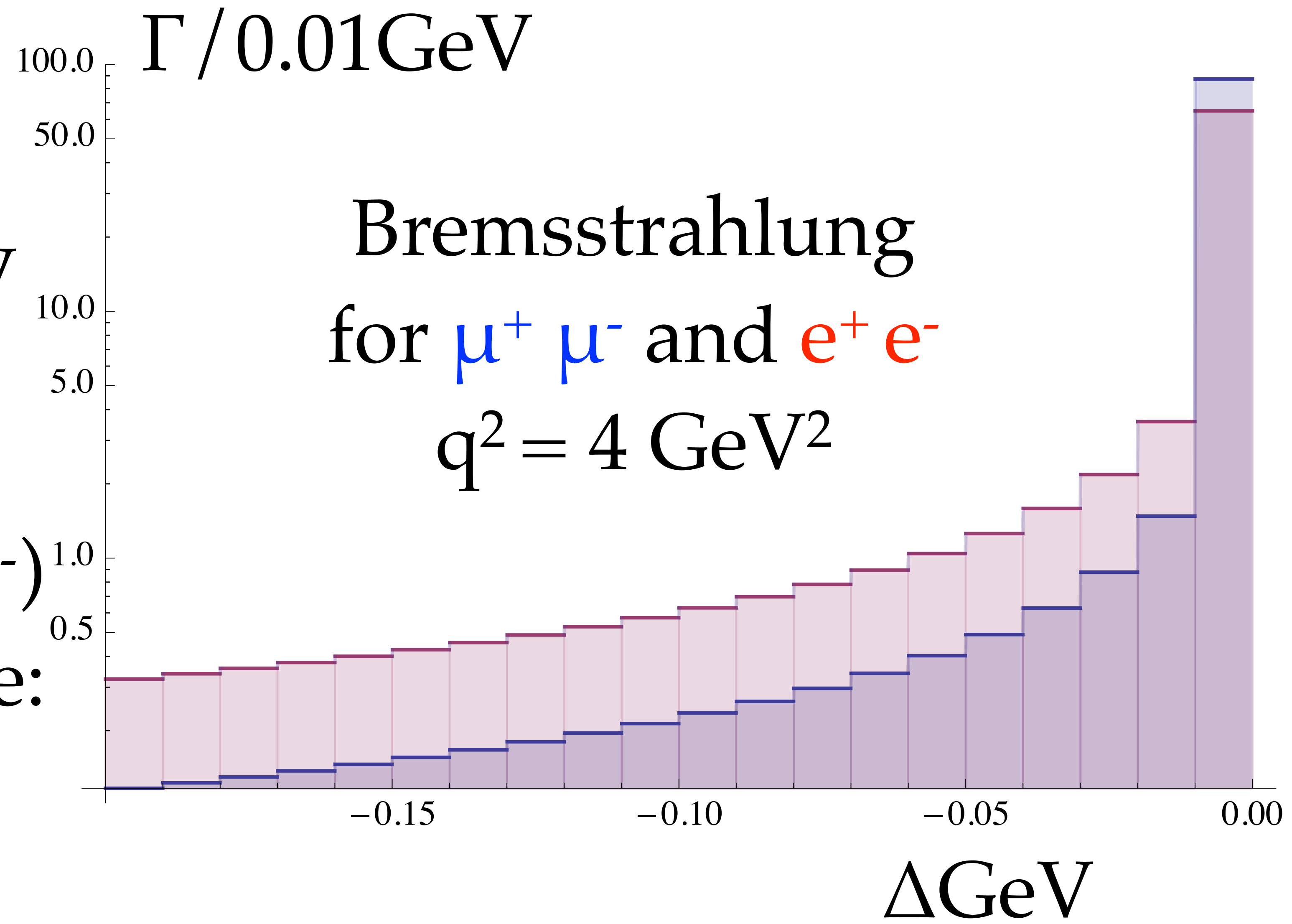
$m(K^+ e^+ e^-) > 4.880$ GeV

$m(K^+ \mu^+ \mu^-) > 5.175$ GeV

If we naively translate this into a cut on $m(e^+ e^-)$ and $m(\mu^+ \mu^-)$ we include:

92% of muons
86% of electrons

rough calculation



$R_K^{(exp)}$

Taking similar cuts for $m(e^+ e^-)$ and $m(\mu^+ \mu^-)$ in the J/Ψ normalisation should cancel this contribution.

The q^2 dependence from these soft photons should be negligible in the normalisation

Since photos accurately describes these effects, everything should be included in $R_K^{(exp)}$

One could check the cancellation by switching photos on and off in the analysis.

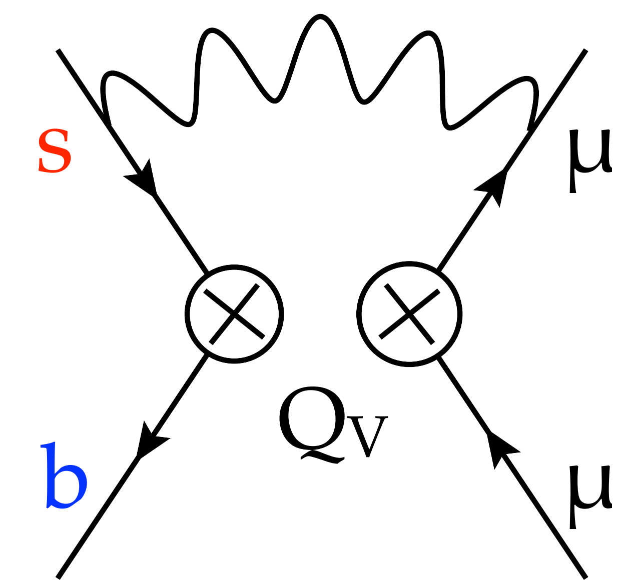
Other QED Corrections

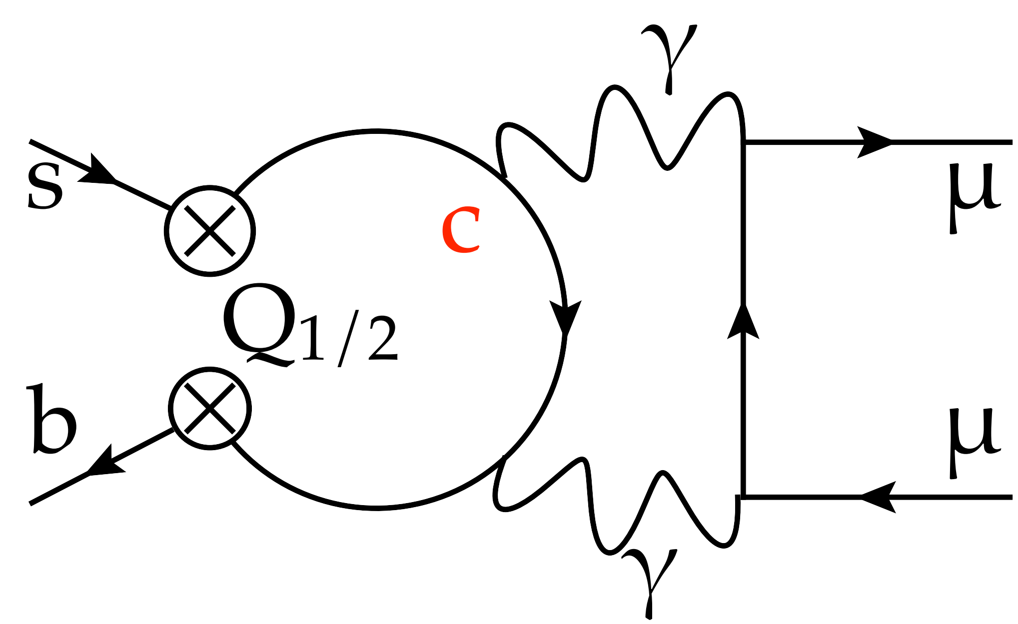
Purely leptonic corrections included in Photos

QED corrections involving only quarks should drop out in ratio

Non-universal QED corrections involving b coupling to leptons should be m_b suppressed

Suppression not obvious for coupling to s or K^+ or charm loop





Electroweak Corrections

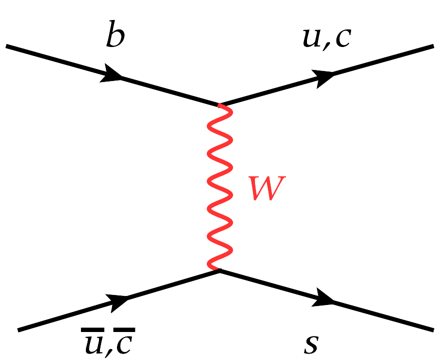
For $B_s \to \mu^+ \mu^-$:
Only electroweak corrections to $C_A(\mu_b)$ potentially large – enhanced by m_{top}/M_W, $1/s_W$, $\alpha_e \log^2(M_W/m_b)$.
NNLO is important to remove the scale uncertainty.

For R_K:
The effect drops out

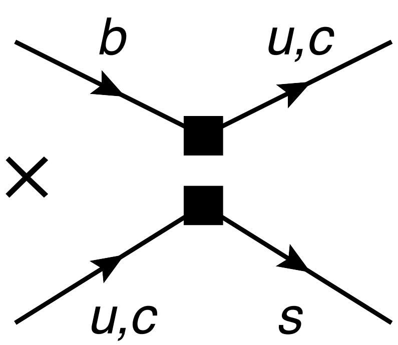
For $B \to K^{(*)} l^+ l^-$:
We should also have an assessment of higher order electroweak corrections to $C_V(\mu_b)$

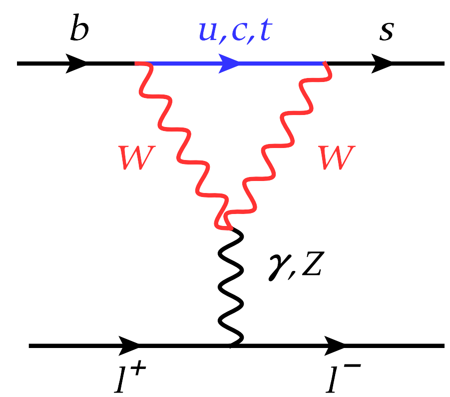
Status of $\mathcal{L}_{\text{eff}}$ for $b \to s\, l^+ l^-$

SM Wilson coefficients: Matching at $\mu \approx M_W$



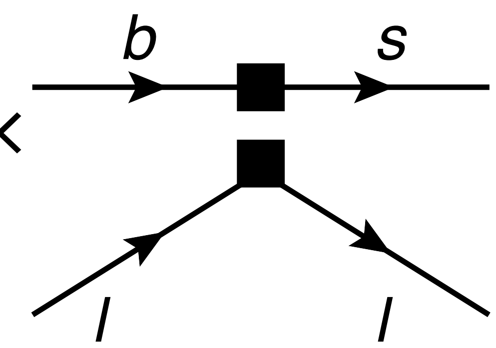
$\to C_2 \times$





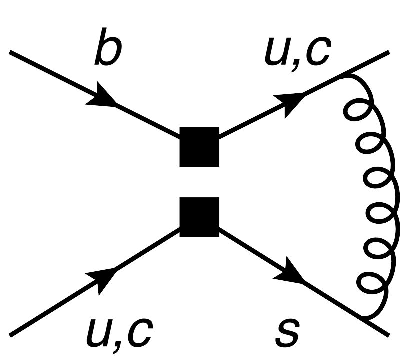
$\to C_{A,V} \times$

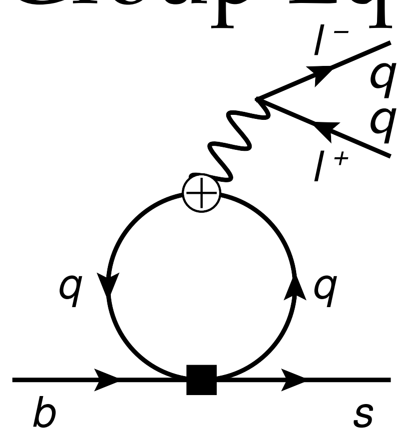
$(C_{10,9})$



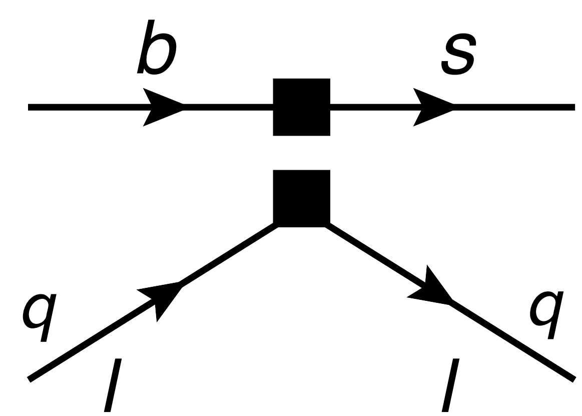
Known at two-loops in QCD for NNLL [Bobeth, Misiak, Urban, `99]

Renormalisation Group Equation $\to \mu \approx M_W$





$\to C_{1\ldots10} \times$



$\mathcal{L}_{\text{eff}}$ @ NNLL in QCD and NLL EW for all but C_9 & C_{10} EW matching [Gambino Haisch`01; Haisch `05, Bobeth, Gambino, MG, Haisch `04, MG, Haisch `05, Huber et. al. `05]

Electroweak Corrections for C_A

Consider $$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha \pi V_{tb}^* V_{ts}}{\sin^2 \theta_W} C_A Q_A + \text{h.c.}$$

$G_F \alpha / \sin^2\theta_W$ does not renormalise under QCD: can be factored out for QCD calculation

Only $G_F \alpha / \sin^2\theta_W \, C_A(m_t/M_W)$ invariant under electroweak scheme change

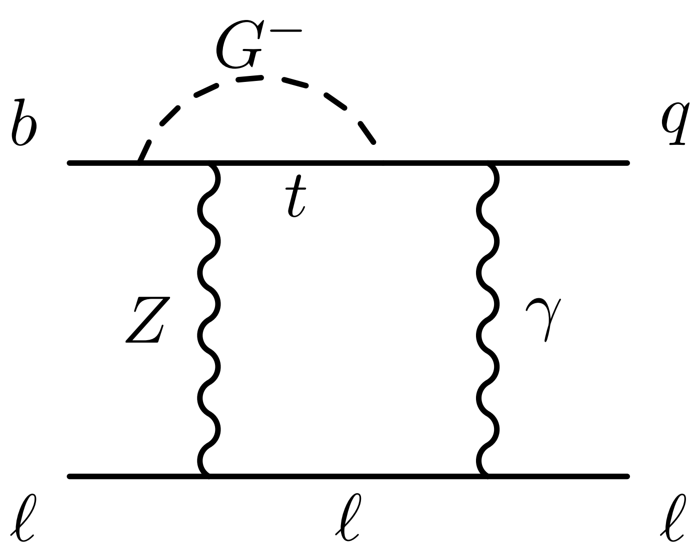
This combination should always give the same result if we use the same input (G_F, α, M_Z, M_t, M_H) up to higher order corrections

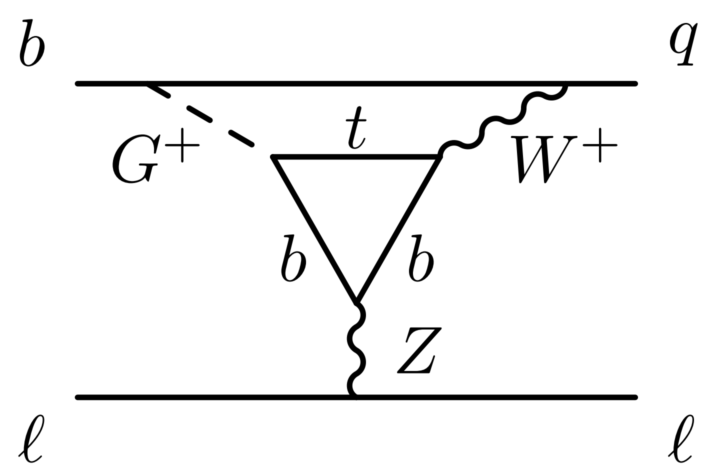
Electroweak Scheme Uncertainties

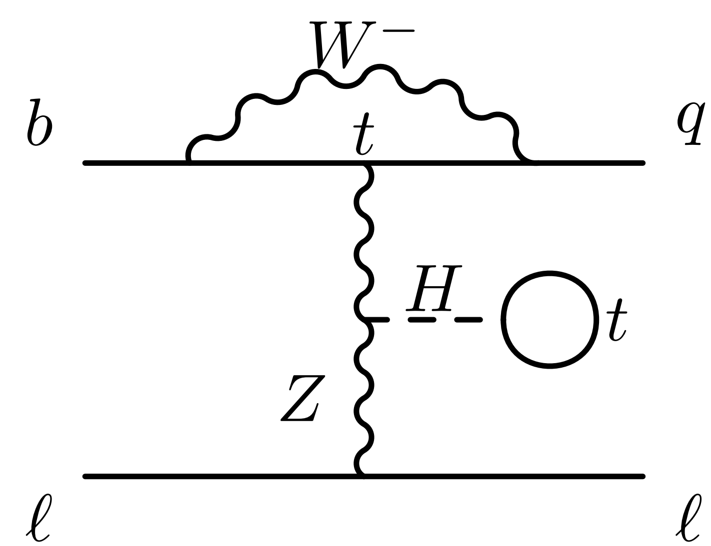
$$\mathcal{L}_{\rm eff} = \frac{G_F}{\sqrt{2}} \frac{\alpha \pi V_{tb}^* V_{ts}}{\sin^2 \theta_W} C_A\left(\frac{m_t}{M_W}\right) Q_A + \text{h.c.}$$

	MS-bar	OS	unct. $B_s\ \mu^+\ \mu^-$
$\sin \theta_W$	0,231	0,223	± 4 %
m_t(QCD-MS-bar)	163,5 GeV	164,8 GeV	± 1 %

These scheme uncertainties should be canceled by the 2-loop electroweak matching corrections!







Renormalisation Schemes

1. On-shell scheme: Determine M_W including loop corrections from input: results in $\sin\theta_W$, m_t and M_W counterterms to $C_A^{(EW)}$.

2. MS-bar scheme: Fit g_1, g_2, v, λ, m_t from data i.e. from G_F, α, M_Z, M_t, M_H

3. Hybrid scheme: Masses on-shell couplings MS-bar

4. OS2: Use $G_F^2 M_W^2$ normalisation and on-shell scheme

Note: QCD is MS-bar renormalised for all schemes i.e. we use a QCD MS-bar top mass at a fixed scale

Matching Correction for C_A

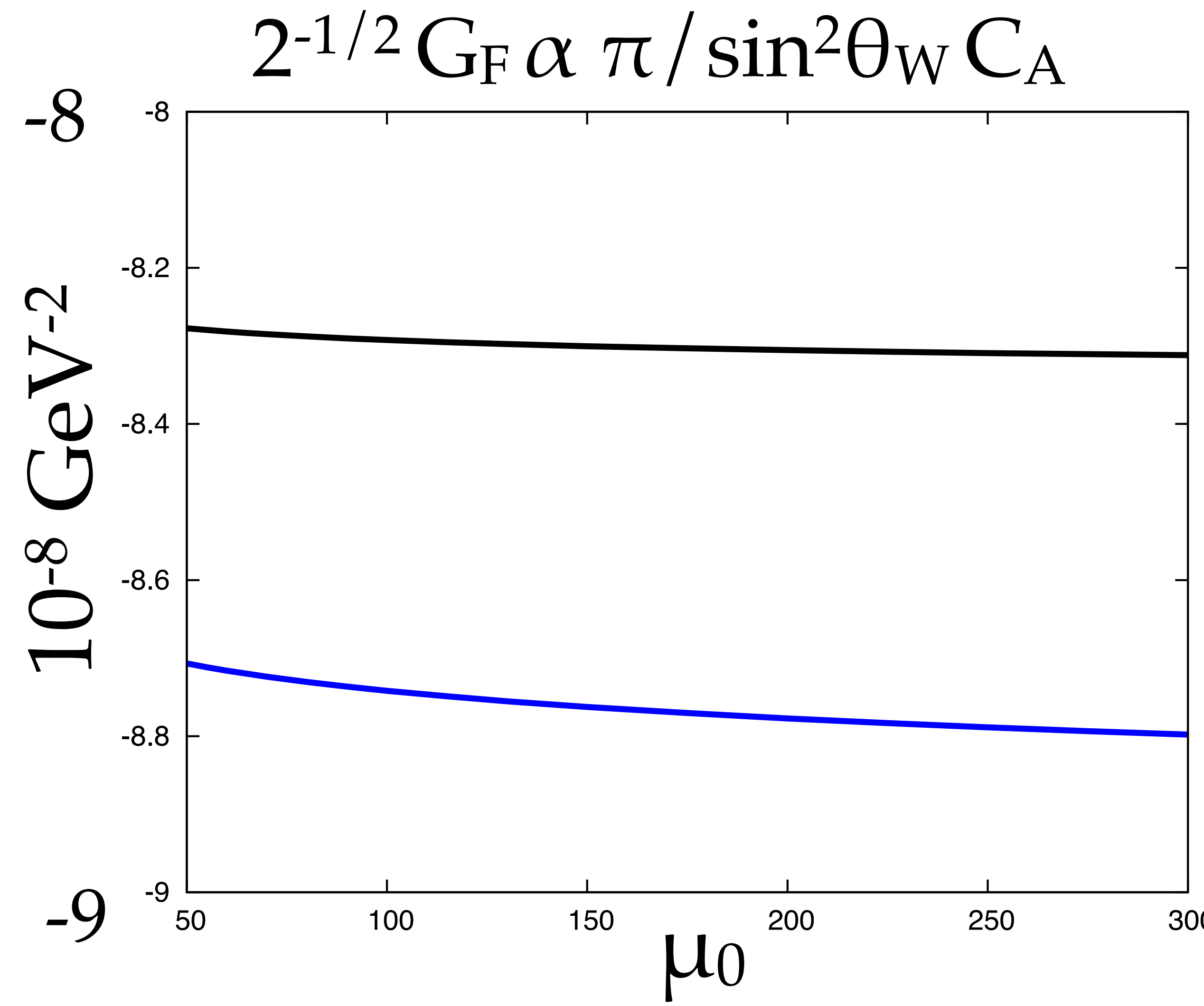
There are sizeable shifts and reduction of scale dependence if we go from 1-loop to 2-loop

Note: α(nq=6) used for plot

Matching Correction for C_A

There are sizeable shifts and reduction of scale dependence if we go from 1-loop to 2-loop

1. We find largest shift in the on-shell scheme,



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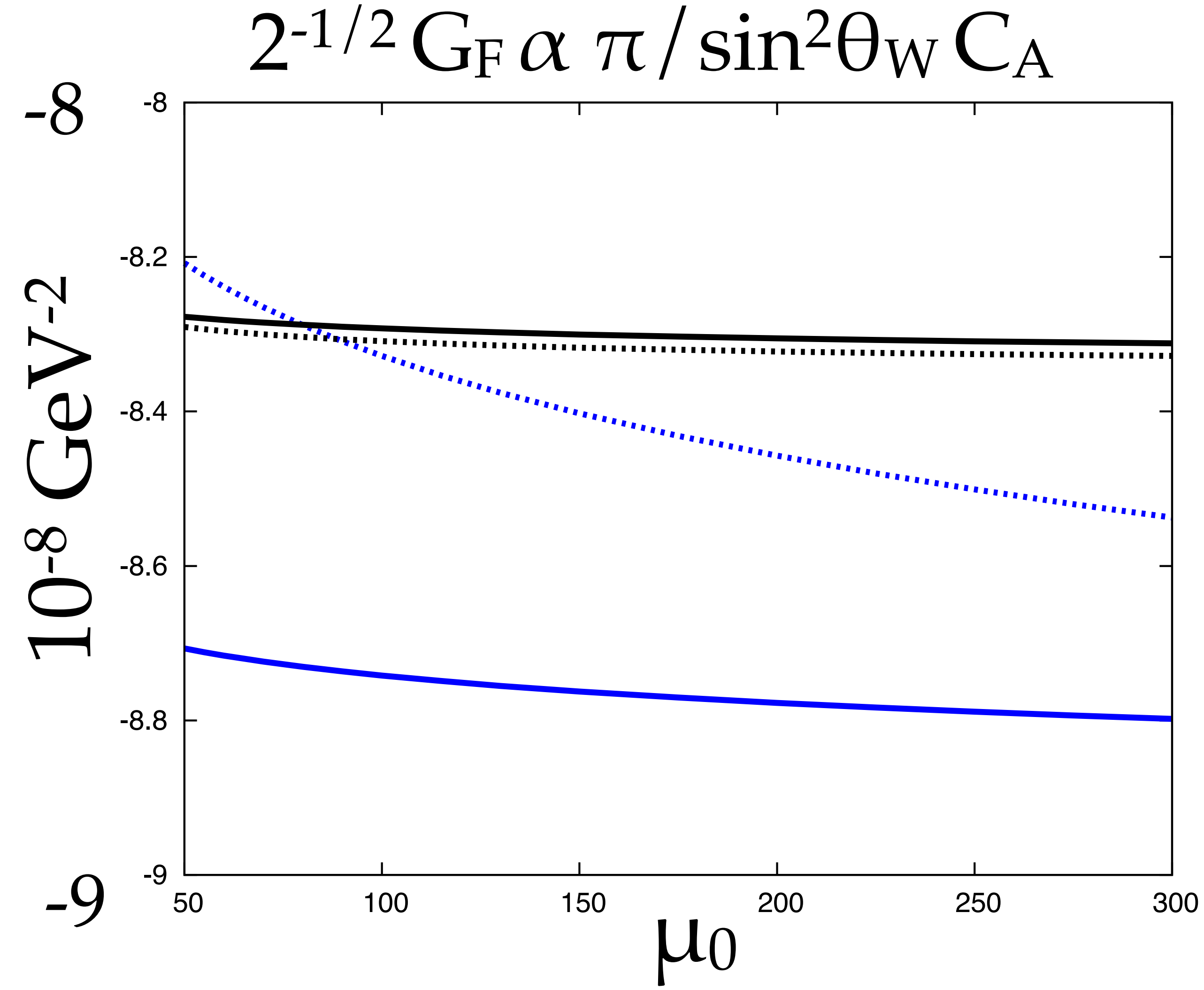
Matching Correction for C_A

There are sizeable shifts and reduction of scale dependence if we go from 1-loop to 2-loop

1. We find largest shift in the on-shell scheme,

2. large scale dependence for the MS-bar scheme



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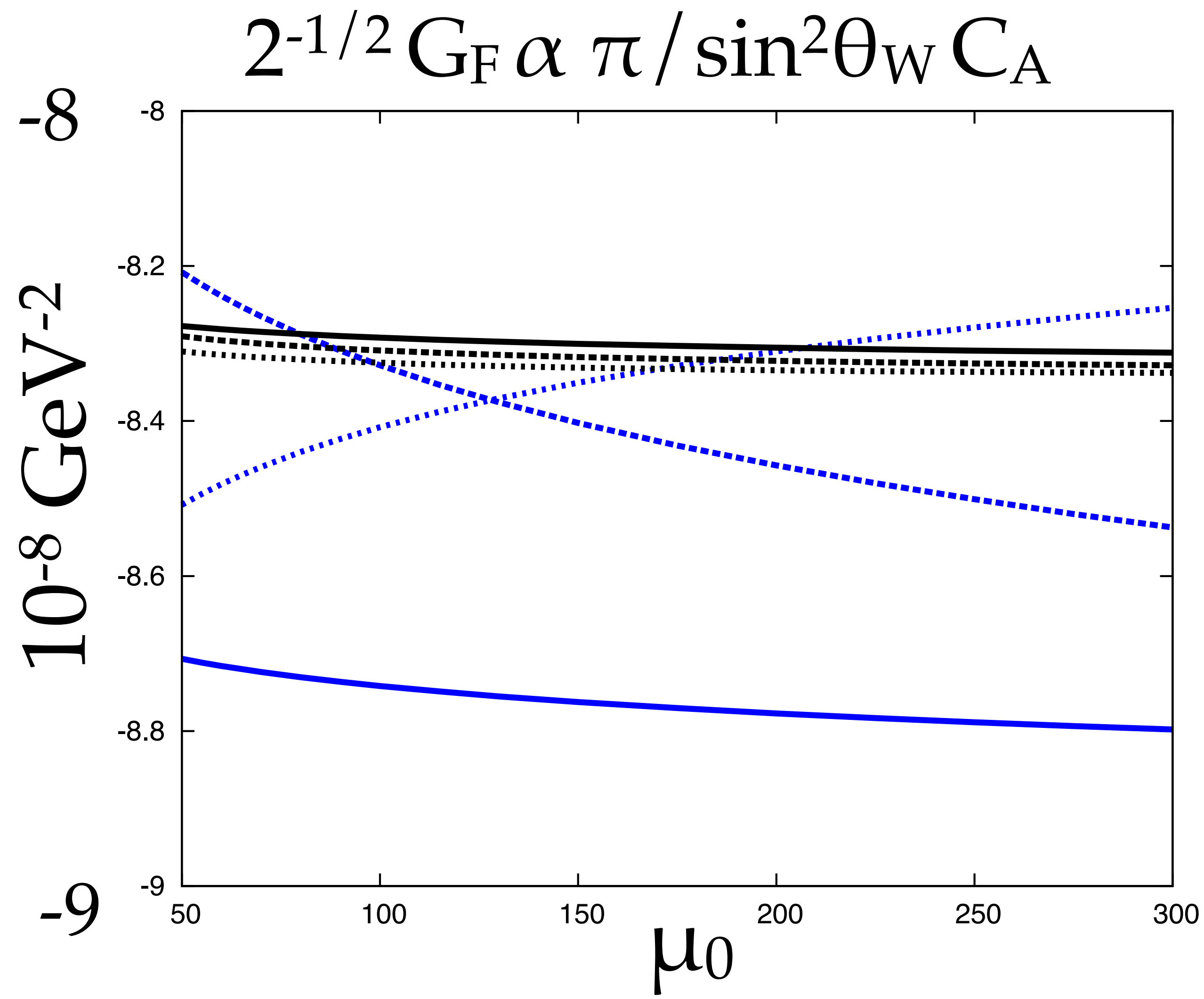
3. and significant shift for the hybrid scheme at MZ.



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Matching Correction for C_A

There are sizeable shifts and reduction of scale dependence if we go from 1-loop to 2-loop

1. We find largest shift in the on-shell scheme,

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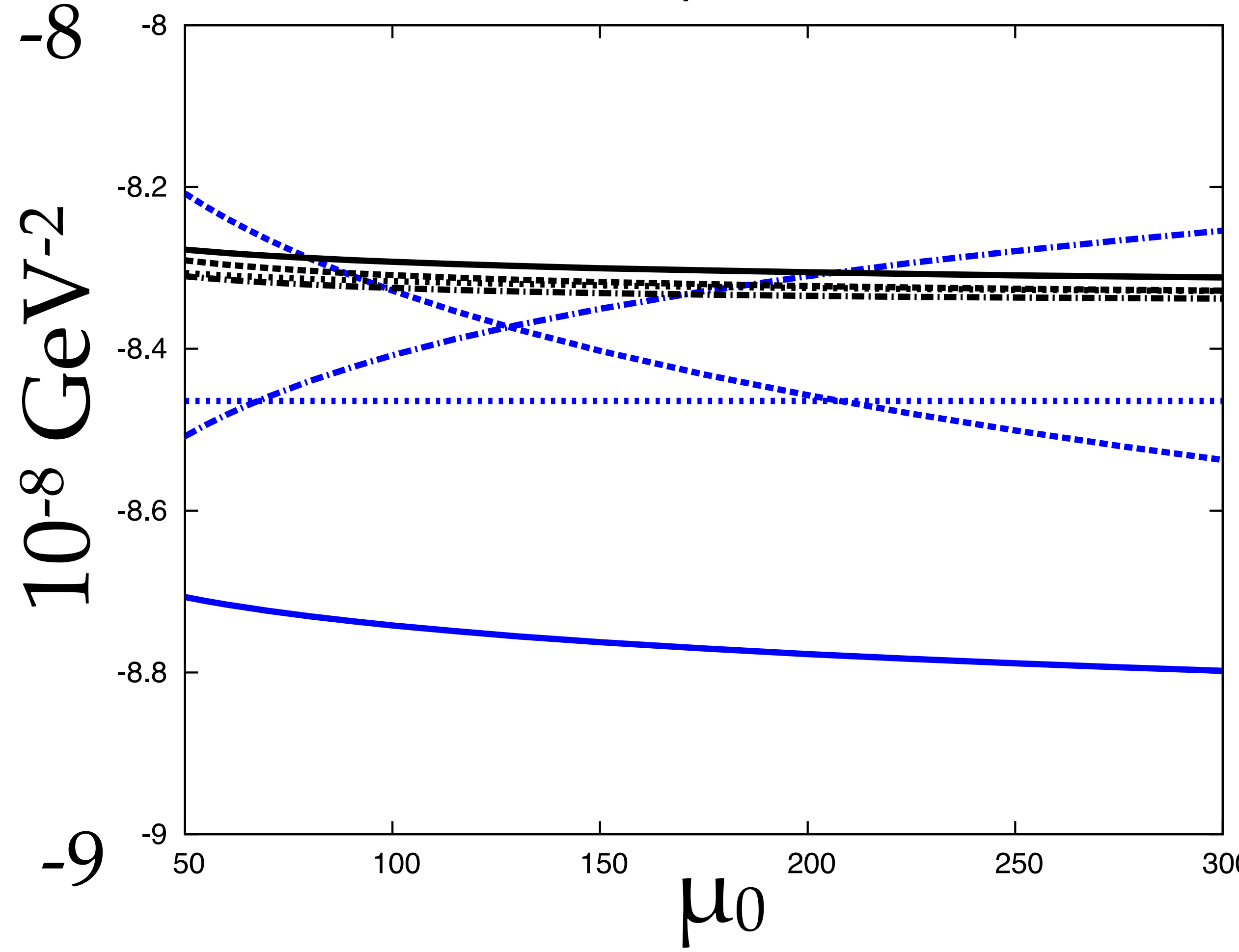
3. and significant shift for the hybrid scheme at MZ.

4. $G_F^2 M_W^2$ normalisation removes `artificial´ scale and parameter dependence

Note: α(nq=6) used for plot

$2^{-1/2} G_F \alpha \pi / \sin^2\theta_W C_A$



EW corrections reduce modulus of Wilson Coefficient and remove 7 % scale uncertainty in the BR

Theory Prediction $B_s \to \mu^+\mu^-$

We find for the time integrated BR @ NNLO & EW

[Bobeth MG, Hermann, Misiak, Steinhauser, Stamou `13]

$Br_{the} = (3.65 \pm 0.23)\ 10^{-9}$ $Br_{exp} = (\ 2.8 + 0.7 - 0.6)\ 10^{-9}$

LHCb CMS Combination

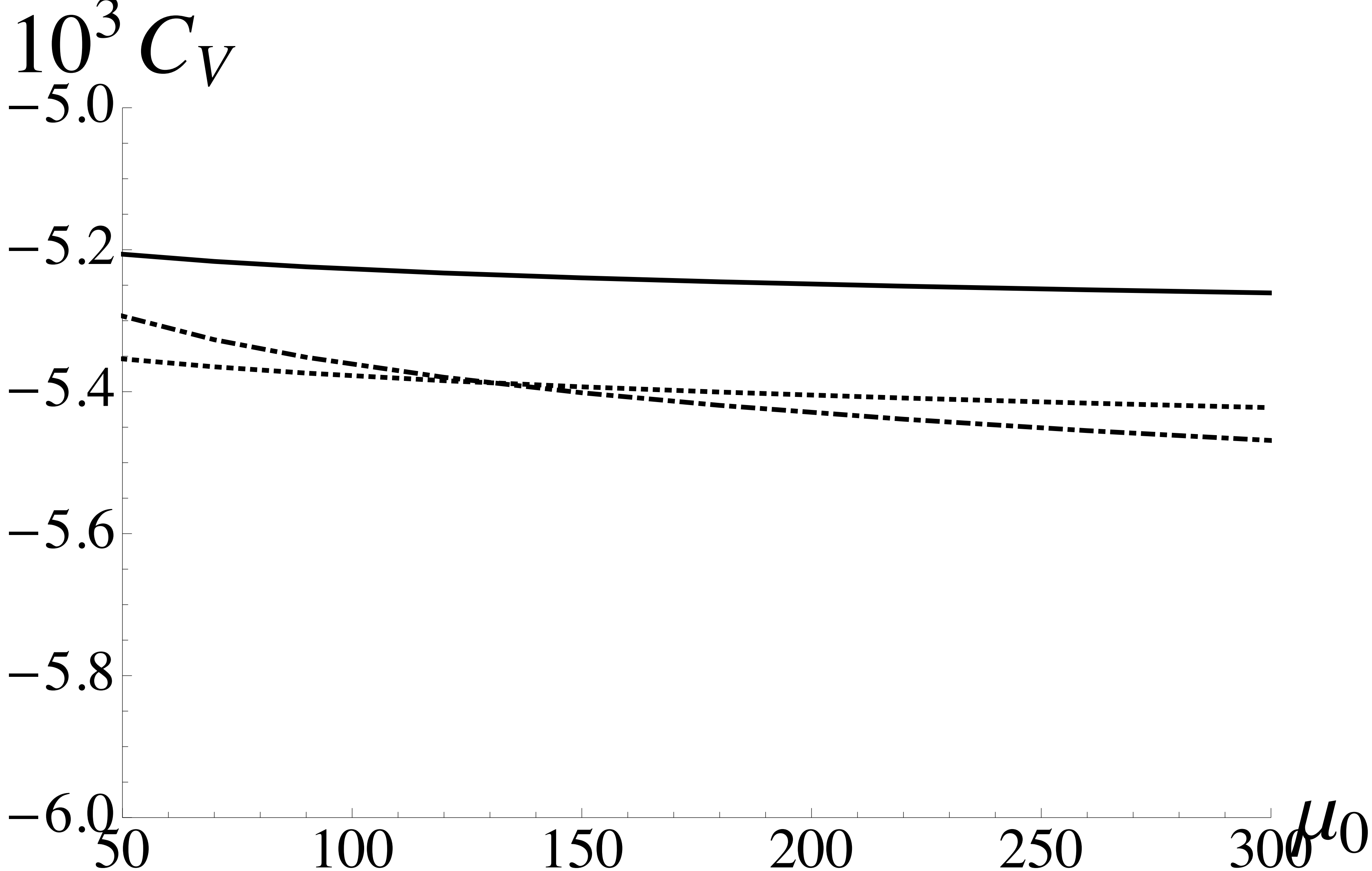
	f_{B_q}	CKM	τ_H^q	M_t	α_s	other param.	non-param.	$\sum$
$\overline{\mathcal{B}}_{s\ell}$	4.0%	4.3%	1.3%	1.6%	0.1%	$< 0.1\%$	1.5%	6.4%

f_{Bs} [MeV]	τ_{Bs} [ps^{-1}]	$\vert V_{tb} V_{ts}\vert$	M_t [GeV]
227.7(45)	1.516(11)	0.0415(13)	173.1(9)

where we have used $V_{cb} = 0.0424(9)$ [Gambino, Schwanda `13]

EW corrections for Q_V ?

EW Uncertainties for LO matching below 5% level



Only the electroweak scheme dependence is plotted, while the effect of operator mixing is switched off

Conclusions

QED final state corrections in $B_s \to \mu^+ \mu^-$ under control

Uncertainty from initial state radiation

7% electroweak scheme ambiguity in $B_s \to \mu^+ \mu^-$ is removed

Only EW corrections to C_V missing to $\mathcal{L}_{\text{eff}}$

$B_s \to \mu^+ \mu^-$ and New Physics

Contribution of Q_S and Q_P are not helicity suppressed

Potentially large coefficients C_S and C_P in 2HDM

Yet, only if contribution to ΔM_s is suppressed,
i.e. type 2 Higgs potential, $\lambda_5 \ll 1$ and type 3 Yukawas

which is the MSSM at $\tan\beta \gg 1$, with the Branching Ratio

$$\mathrm{BR} \propto (\tan\beta)^6 \; M_A^{-4}$$

Non-zero $\Delta\Gamma_s$ allows for another untagged observable beyond the BR via an effective lifetime measurement.
[Bruyn, Fleischer, Knegjens et.al. `12]

Experimental Status

$$\mathcal{B}(B_s^0 \to \mu^+\mu^-) = \left(2.8\,^{+0.7}_{-0.6}\right) \times 10^{-9} \text{ and}$$
$$\mathcal{B}(B^0 \to \mu^+\mu^-) = \left(3.9\,^{+1.6}_{-1.4}\right) \times 10^{-10},$$

CMS & LHCb ArXiv:1411.4413v1

For $B_{(s)} \to \mu^+ \mu^-$ experiment and theory consistent within present accuracy (2 σ).

Reduce the (theory) uncertainty:

Either $B_{(s)} \to \mu^+ \mu^-$ will result in a signal of new physics or in a precision test of the standard model.

Either way we will get additional information on C_A, C_S and C_P (+ flipped Operators ...)

Theory Status at NLO

C_S & C_P can be neglected within the Standard Model

$C_A(m_t\ /\ M_W)^{NLO} = 1.0113\ C_A(m_t\ /\ M_W)^{LO}$

– for QCD MS-bar $m_t = m_t(m_t)$ [Buras, Buchalla; Misiak, Urban `99]

For pure QCD determine $< \mu^- \mu^+ | Q_A | B_s >$ from

$< 0\ | \bar{b}\, \gamma^\mu\, \gamma_5\, s\, | B_s > = i\, p^\mu\, f_{Bs}$ (f_{Bs}= 227.7(4.5)MeV [FLAG])

QED & Electroweak were so far only known at LO –

this leads to a ±2% & ±7% uncertainty