$B \to X_s ee$ and $B \to X_s \mu\mu$: Status, prospects and lepton non-universality

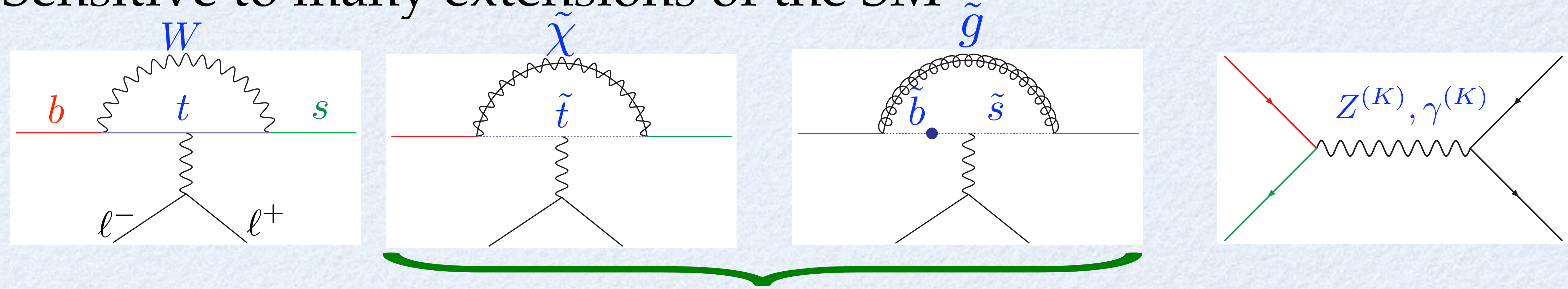
Enrico Lunghi
Indiana University

Rare B decays in 2015: Experiment and Theory

Jun 12, 2015

WHY $b \to s\ell^+\ell^-$

- Sensitive to many extensions of the SM



Central values of exclusive global fit require non-MFV models

- Exclusive modes are experimentally easier (LHCb) but harder to bring under theoretical control (factorization, power corrections, ...)

- Inclusive modes require a super-B machine to be fully exploited but the theoretical outlook is very impressive

- Some references (inclusive):

  Misiak; Buras, Munz, Bobeth, Urban, Asatryan, Asatrian, Greub, Walker, Ghinculov, Hurth, Isidori, Yao, Gambino, Gorbahn, Haisch, Huber, Lunghi, Wyler, Lee, Ligeti, Stewart, Tackmann, …

- Some references (exclusive):

  Beneke, Feldmann, Seidel, Grinstein, Pirjol, Bobeth, Hiller, Dyk, Wacker, Piranishvili, Altmannshofer, Ball, Bharucha, Buras, Wick, Straub, Matias, Lunghi, Virto, Descotes-Genon, Hofer, Hurth, Mahmoudi, …

WHY $b \to s\ell^+\ell^-$

SM operator basis:

$$\mathcal{L}_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb}V_{tq}^* \left[\sum_{i=1}^{10} C_i Q_i + \frac{V_{ub}V_{uq}^*}{V_{tb}V_{tq}^*} \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + \underbrace{\sum_{i=3}^{6} C_{iQ} Q_{iQ} + C_b Q_b}_{\text{for QED corrections}} \right]$$

- Magnetic & chromo-magnetic

$$Q_7 = \frac{e}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$

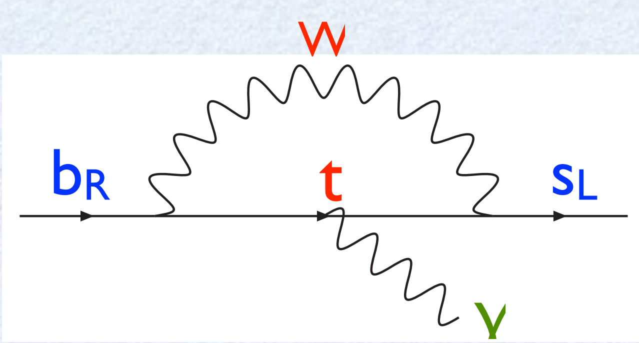
$$Q_8 = \frac{g}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$

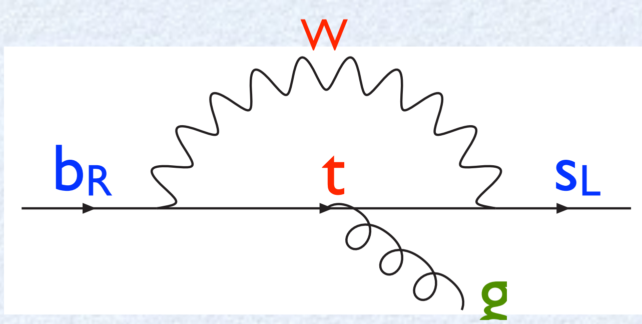
- Semileptonic

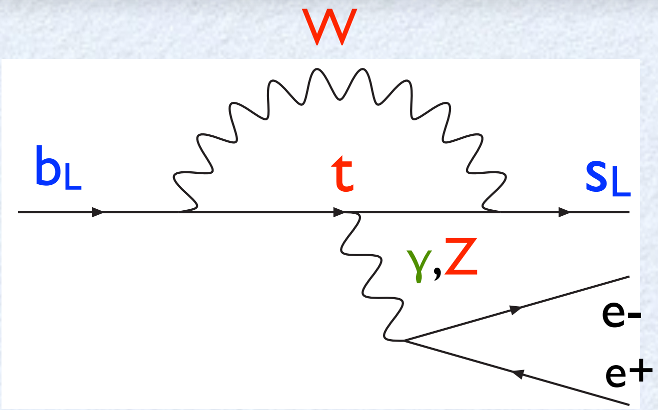
$$Q_9 = (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma^\mu \ell)$$

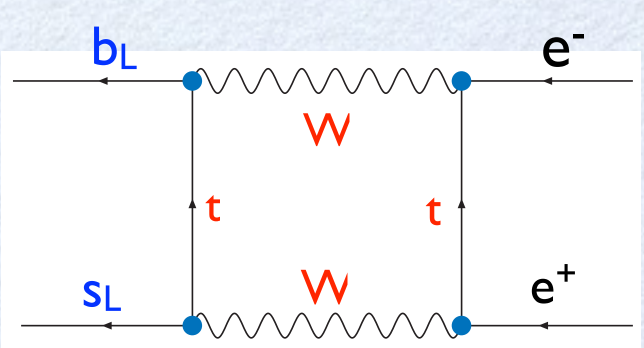
$$Q_{10} = (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

Everything is known very well ($V_{ub}V_{uq}$ contribution is small for b→sll but important for b→dll)









WHY $b \to s\ell^+\ell^-$

In NP extensions we get more structures (V+A, scalar, tensor)

- Right-handed (V+A):

$$Q_7' = \frac{e}{16\pi^2} m_b [\bar{s}_R \sigma^{\mu\nu} b_L] F_{\mu\nu}$$
$$Q_8' = \frac{g}{16\pi^2} m_b [\bar{s}_R \sigma^{\mu\nu} T^a b_L] G^a_{\mu\nu}$$
$$Q_9' = [\bar{s}_R \gamma_\mu b_R][\bar{\ell}\gamma^\mu \ell]$$
$$Q_{10}' = [\bar{s}_R \gamma_\mu b_R][\bar{\ell}\gamma^\mu \gamma_5 \ell]$$

- Scalar:

$$Q_S = [\bar{s}_L b_R][\bar{\ell}\bar{\ell}]$$
$$Q_S' = [\bar{s}_R b_L][\bar{\ell}\bar{\ell}]$$
$$Q_P = [\bar{s}_L b_R][\bar{\ell}\gamma_5 \bar{\ell}]$$
$$Q_P' = [\bar{s}_R b_L][\bar{\ell}\gamma_5 \bar{\ell}]$$

- Tensor

$$Q_T = [\bar{s}\sigma_{\mu\nu} b][\bar{e}ll\sigma^{\mu\nu}\ell]$$
$$Q_{T5} = \frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}[\bar{s}\sigma_{\mu\nu} b][\bar{\ell}\sigma_{\alpha\beta}\ell]$$

Q: is it possible to disentangle all these contributions?

A: With a little luck.

WHY $b \to s\ell^+\ell^-$

- Multi-objects in the final state (3 for B→K/X_s, 4 for B→K^*→Kπ) allows to isolate contributions from various operators
- $B \to K\ell\ell$

$$\frac{d^2\Gamma^K}{dq^2\ d\cos\theta_\ell} = a + b\ \cos\theta_\ell + c\ \cos\theta_\ell^2$$

$$\begin{aligned} a\ &\sim C_7 + C_7',\ C_9 + C_9',\ C_{10} + C_{10}', \\ &\quad C_S + C_S',\ C_P + C_P',\ m_\ell\, C_T \\ b\ &\sim C_S + C_S',\ C_P + C_P',\ C_T,\ C_{T5},\ m_\ell\ (C_{10} + C_{10}') \\ c\ &\sim C_7 + C_7',\ C_9 + C_9',\ C_{10} + C_{10}',\ C_T,\ C_{T5} \end{aligned}$$

- In the SM ***b*** is suppressed by the lepton mass: huge sensitivity to scalar, pseudoscalar, tensor operators (e.g. forward-backward asymmetry)
- **We have three observables and those related by CP and isospin**

WHY $b \to s\ell^+\ell^-$

- Multi-objects in the final state (3 for B→K/X_s, 4 for B→K^*→Kπ) allows to isolate contributions from various operators
- $B \to X_s \ell\ell$

$$\frac{d^2\Gamma^{X_s}}{dq^2\, d\cos\theta_\ell} = \frac{3}{8}\left[(1+\cos^2\theta_\ell)\, H_T + 2(1-\cos^2\theta_\ell)\, H_L + 2\cos\theta_\ell\, H_A\right]$$

$$H_T \sim 2\hat{s}(1-\hat{s})^2\left[|C_9 + \frac{2}{\hat{s}}C_7|^2 + |C_{10}|^2\right] \qquad \hat{s} = q^2/m_b^2$$

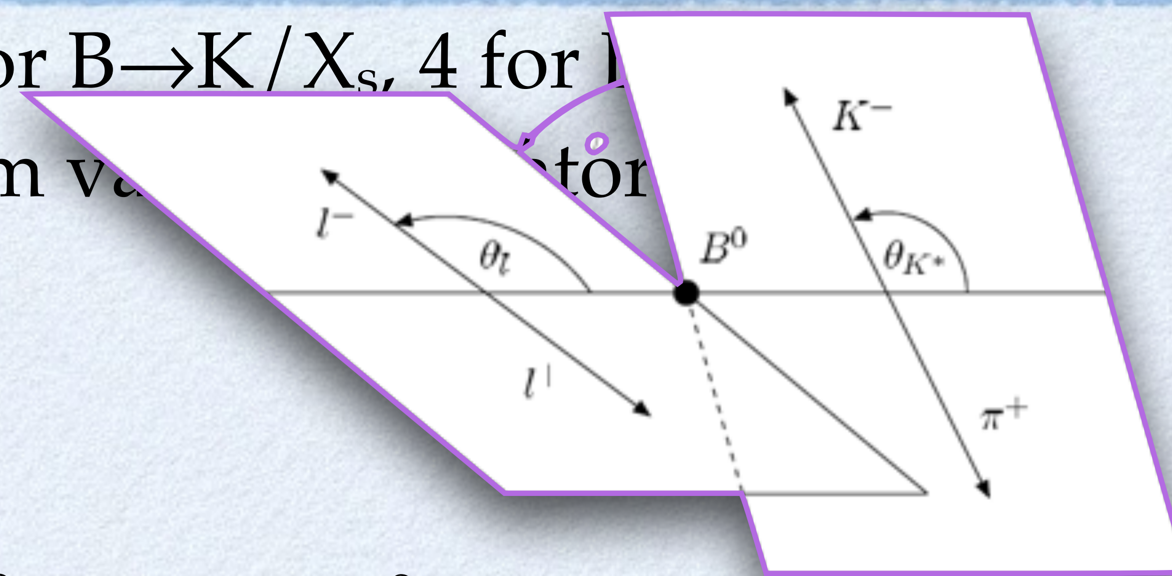
$$H_L \sim (1-\hat{s})^2\left[|C_9 + 2C_7|^2 + |C_{10}|^2\right]$$

$$H_A \sim -4\hat{s}(1-\hat{s})^2 \mathrm{Re}\left[C_{10}(C_9 + 2\frac{m_b^2}{q^2}C_7)\right]$$

- H_A is not suppressed by the lepton mass
- There are similar contributions from non-SM operators ***but there is no interference between V+A and V-A structures***
- We have three observables and those related by CP and isospin

WHY $b \to s\ell^+\ell^-$

- Multi-objects in the final state (3 for B→K/X$_s$, 4 for B
  allows to isolate contributions from va   tor



- $B \to K^*\ell\ell \to K\pi\ell\ell$

$$\frac{d^4\Gamma^{K^*}}{dq^2\; d\cos\theta_l\; d\cos\theta_{K^*}\; d\phi} \simeq$$

$$J_1^s \sin^2\theta_{K^*} + J_1^c \cos^2\theta_{K^*} + (J_2^s \sin^2\theta_{K^*} + J_2^c \cos^2\theta_{K^*}) \cos 2\theta_l$$
$$+ J_3 \sin^2\theta_{K^*} \sin^2\theta_l \cos 2\phi + J_4 \sin 2\theta_{K^*} \sin 2\theta_l \cos\phi + J_5 \sin 2\theta_{K^*} \sin\theta_l \cos\phi$$
$$+ J_6 \sin^2\theta_{K^*} \cos\theta_l + J_7 \sin 2\theta_{K^*} \sin\theta_l \sin\phi$$
$$+ J_8 \sin 2\theta_{K^*} \sin 2\theta_l \sin\phi + J_9 \sin^2\theta_{K^*} \sin^2\theta_l \sin 2\phi,$$

- ***We have 11 observables and those related by CP and isospin!***
- The J_a observables are functions of all the Wilson coefficients (V+A and V-A operators do interfere)
- In the literature one finds various combinations of these J_a

What to expect?

- **Inclusive branching ratios** have been measured at the **20% level** by Babar and Belle and there is discrete agreement
- **Exclusive modes** are accessible to LHCb and have been measured with greater accuracy
- In ***P_5'*** (one of K^* observable) there is a **4σ discrepancy at low-q^2**
- There is a **3σ discrepancy between B→Kμμ and B→Kee**
- **The elephant in the room: size of power corrections, their q^2 dependence and breakdown of the theoretical approach (e.g. resonant charm effects)**
- **What is the ultimate theoretical precision on K, K^* and Xs quantities?**

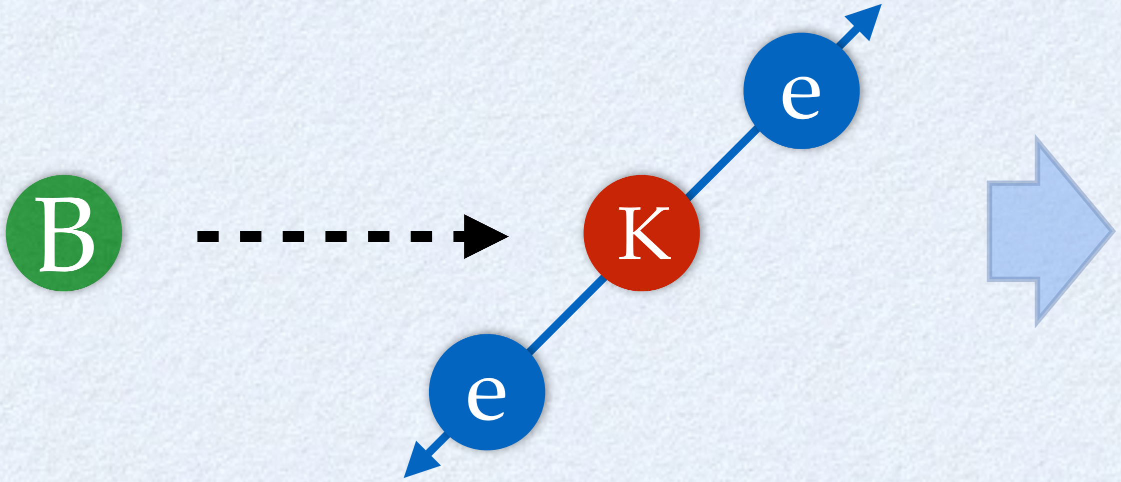
Theory: Exclusive

- The central problem is the calculation of matrix elements:

$$\langle K^{(*)}\ell\ell|O(y)|B\rangle \approx \langle K^{(*)}|T\, J_\mu^{\rm em}(x)\, O(y)|B\rangle$$

if O contains a leptonic current (i.e. $O_{7,9,10}$) the matrix elements reduces to a form factor (lattice, QCD sum rules)

- At high-q^2 the $K^{(*)}$ doesn't recoil:



$$(x-y)^2 \sim \frac{1}{q^2} \sim \frac{1}{m_b^2}$$

Grinstein & Pirjol showed how to write a simple OPE in which **all matrix elements** are given in terms of calculable hard coefficients and **form factors** (up to power corrections)

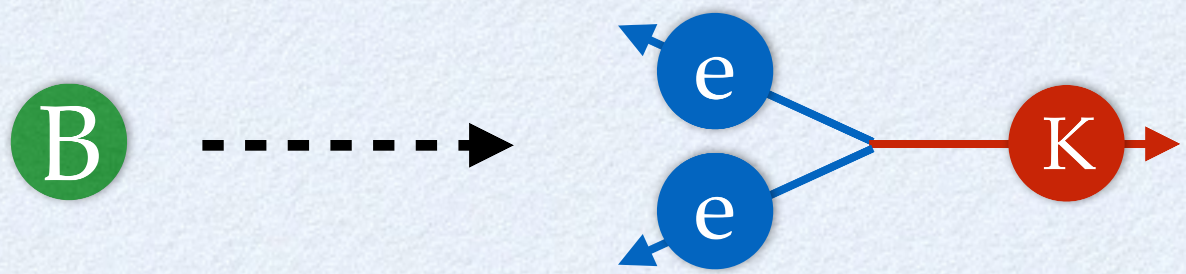
Theory: Exclusive

- The central problem is the calculation of matrix elements:

$$\langle K^{(*)}\ell\ell|O(y)|B\rangle \;\approx\; \langle K^{(*)}|T\; J_\mu^{\rm em}(x)\; O(y)|B\rangle$$

if O contains a leptonic current (i.e. $O_{7,9,10}$) the matrix elements reduces to a form factor (lattice, QCD sum rules)

- At low-q^2 the $K^{(*)}$ recoils strongly:



- The large energy of the $K^{(*)}$ introduces three scales: m_b^2, Λm_b and Λ^2:

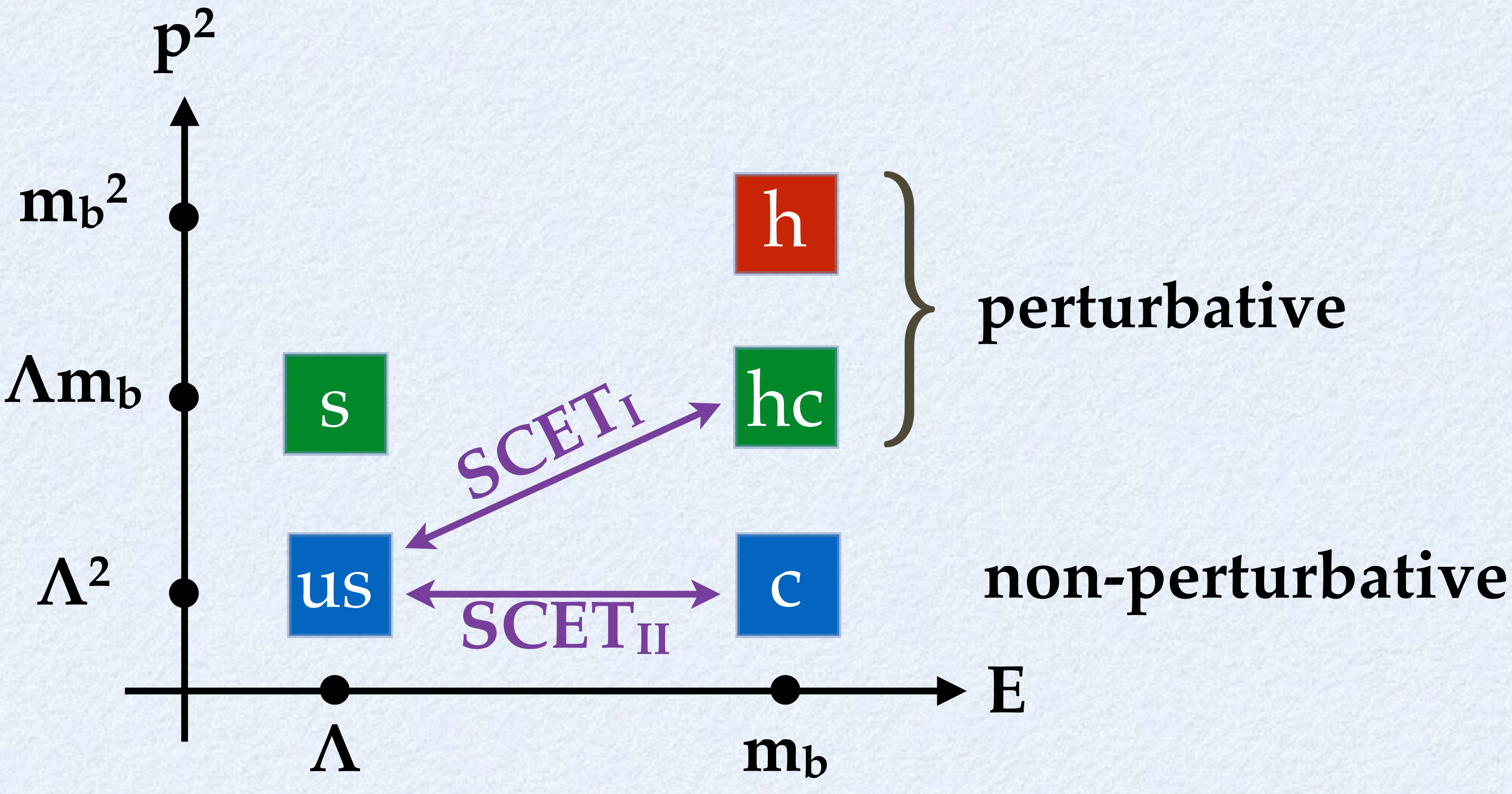
$$\langle K^{(*)}|T\; J_\mu^{\rm em}(x)\; O(y)|B\rangle \quad \sim \quad C \times [\text{Form Factor} + \phi_B \star J \star \phi_K] + O\left(\frac{\Lambda}{m_b}\right)$$

m_b^2 $\quad \Lambda^2 \quad$ Λ^2 $\;\Lambda m_b\;$ Λ^2

$\mathrm{SCET_{II}}$

THEORY: EXCLUSIVE

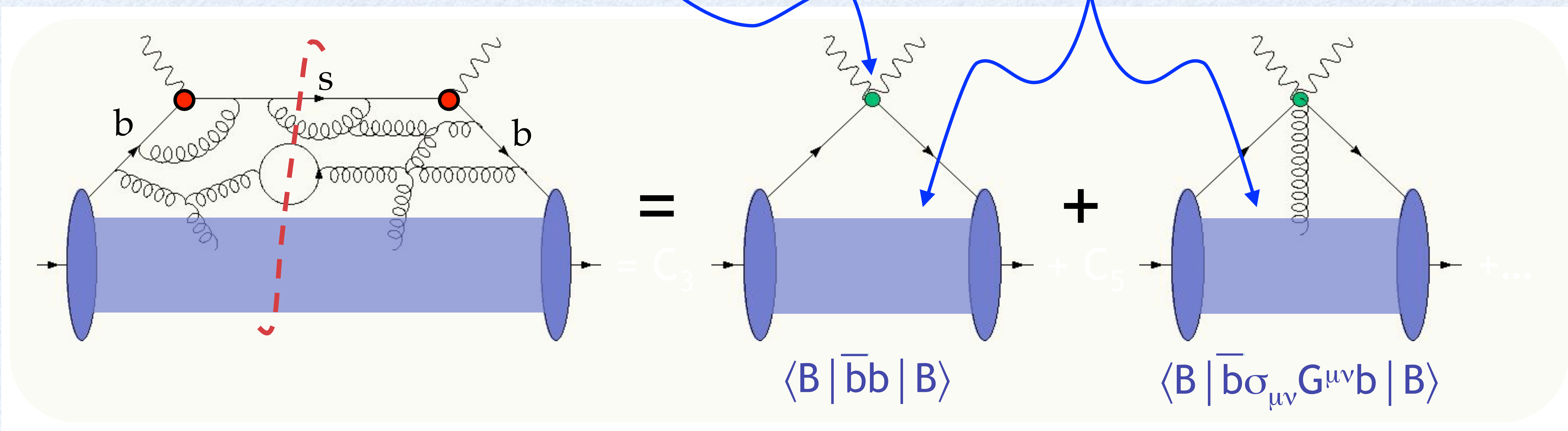
- **Soft Collinear Effective Theory**

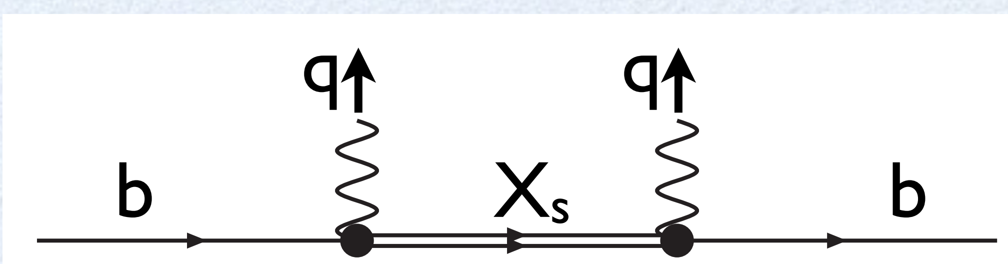


- us-hc factorization is rock solid (inclusive modes, collider physics)
- us-c factorization is more problematic (exclusive modes) because both collinear and ultrasoft modes have $p^2 \sim \Lambda^2$ and sometimes they don't factorize (zero-bin, messenger modes …)

Power corrections

$$\Gamma\left[\bar{B} \to X_s \ell^+ \ell^-\right] = \Gamma\left[\bar{b} \to X_s \ell^+ \ell^-\right] + O\left(\frac{\Lambda_{QCD}^2}{m_b^2}, \frac{\Lambda_{QCD}^3}{m_b^3}, \frac{\Lambda_{QCD}^2}{m_c^2}, \ldots\right)$$





$$p_{X_s}^2 = (p_b - q)^2 = m_b^2 + q^2 - 2 m_b q_0$$
$$< m_b^2 + q^2 - 2 m_b \sqrt{q^2} = \left(m_b - \sqrt{q^2}\right)^2$$

OPE is an expansion in $\Lambda_{QCD}/(m_b - \sqrt{q^2})$ and breaks down at $q^2 \sim m_b^2$

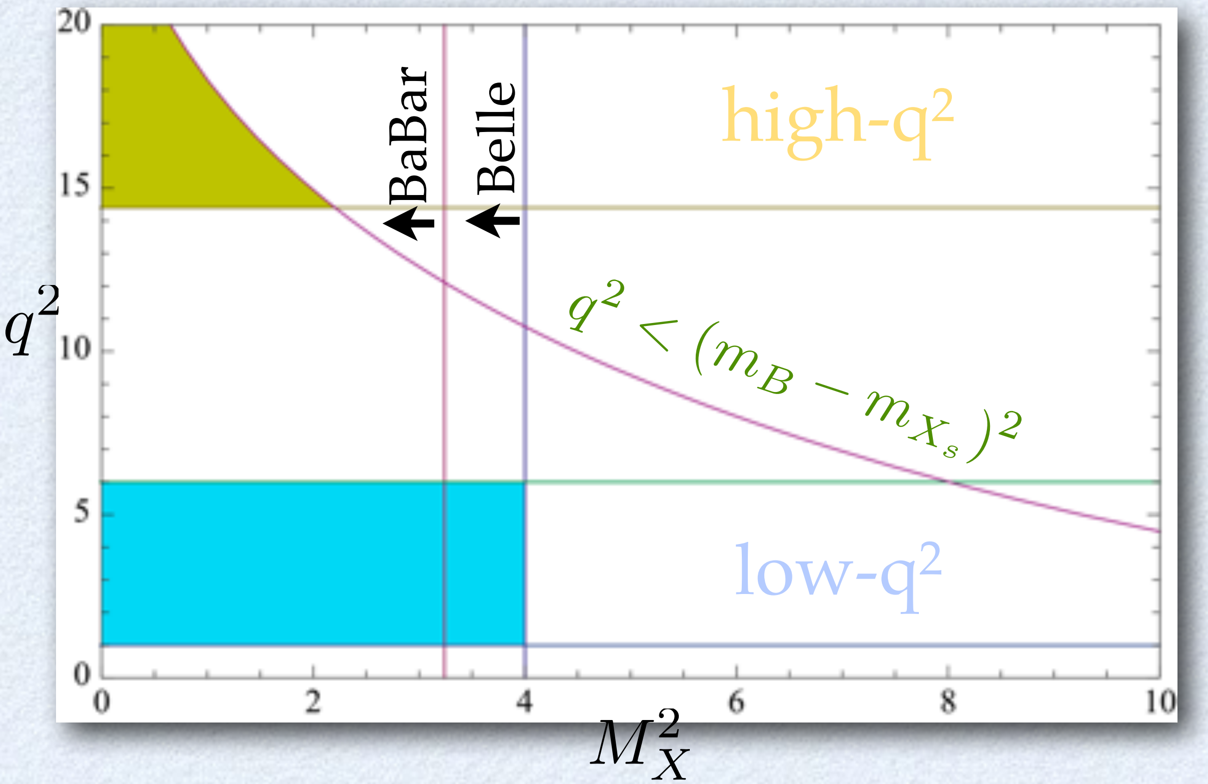
Theory: Inclusive

$$\Gamma\left[\bar{B} \to X_s \ell^+ \ell^-\right] = \Gamma\left[\bar{b} \to X_s \ell^+ \ell^-\right] + O\left(\frac{\Lambda_{QCD}^2}{m_b^2}, \frac{\Lambda_{QCD}^3}{m_b^3}, \frac{\Lambda_{QCD}^2}{m_c^2}, \ldots\right)$$

local OPE, optical theorem
quark-hadron duality

HQET

Phase space cuts introduce sensitivity to new scales, the rate becomes less inclusive and new non-perturbative effects appear



$M_{X_s} < [1.8, 2]$ GeV cut to remove double semileptonic decay background

- High-q^2 region unaffected
- Experiments correct using Fermi motion model
- $SCET_I$ suggests cuts are universal (same for b→sll and b→ulν)

Effect of cc resonances can be included using data from ee→hadrons

Theory: inclusive

$$\Gamma\left[\bar{B}\to X_s\ell^+\ell^-\right]=\Gamma\left[\bar{b}\to X_s\ell^+\ell^-\right]+O\left(\frac{\Lambda_{QCD}^2}{m_b^2},\frac{\Lambda_{QCD}^3}{m_b^3},\frac{\Lambda_{QCD}^2}{m_c^2},...\right)$$

local OPE, optical theorem
quark-hadron duality

HQET

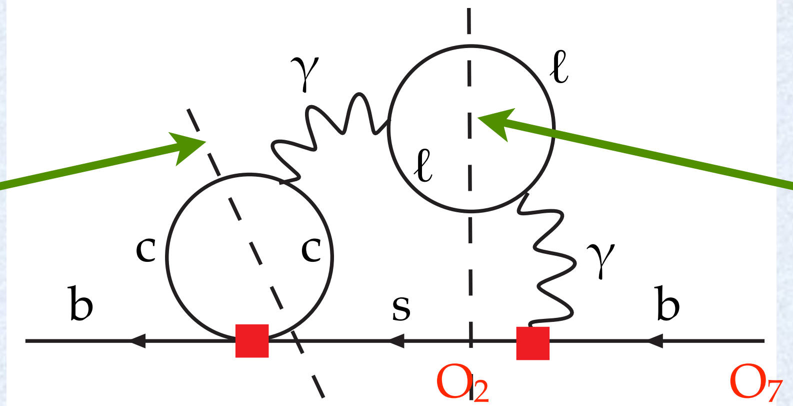
- Low-q^2: theory in excellent shape
- High-q^2: the OPE starts to break down and only integrated quantities are reliable
  - mismatch between partonic and hadronic phase space
  - power corrections are larger
  - higher charmonium resonances must be integrated over
  - things improve dramatically by normalizing the rate to the semileptonic rate with the same q^2 cut [Ligeti et al.]

$$\mathcal{R}(s_0)=\int_{s_0}^1 \mathrm{d}\hat{s}\,\frac{\mathrm{d}\Gamma(\bar{B}\to X_s\ell^+\ell^-)}{\mathrm{d}\hat{s}}\Big/\int_{s_0}^1 \mathrm{d}\hat{s}\,\frac{\mathrm{d}\Gamma(\bar{B}^0\to X_u\ell\nu)}{\mathrm{d}\hat{s}}$$

Charmonium troubles

- Quark-hadron duality breaks down when the rate is dominated by charmonium resonances:

$b \to c\bar{c}s$

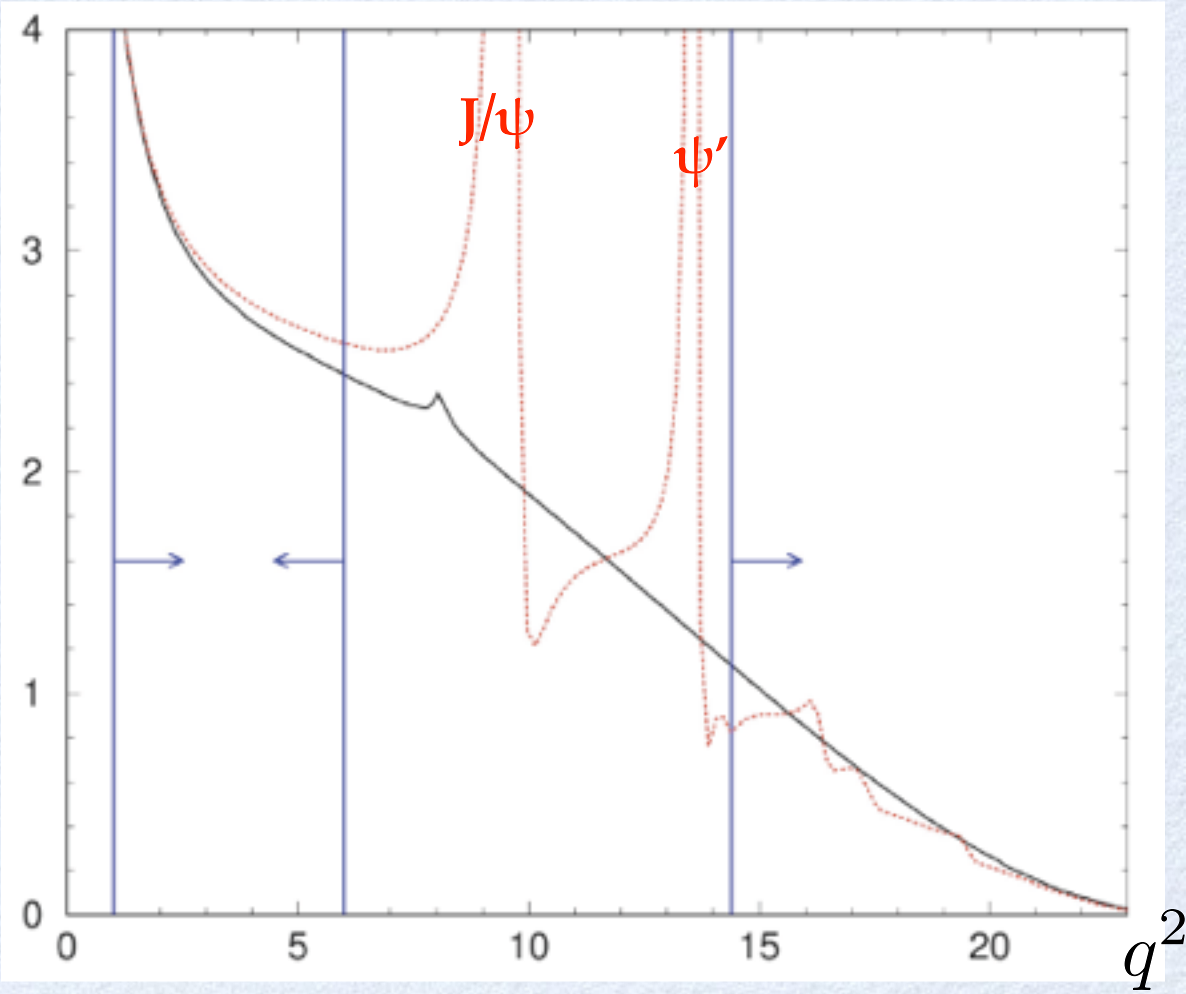


$b \to s\ell^+\ell^-$

- Three regions:
  - $0.04\ \mathrm{GeV}^2 < q^2 < 1\ \mathrm{GeV}^2$
  - ***$1\ \mathrm{GeV}^2 < q^2 < 6\ \mathrm{GeV}^2$***
  - ***$q^2 > 14.4\ \mathrm{GeV}^2$***

  dominated by the photon pole ($b \to s\gamma$)

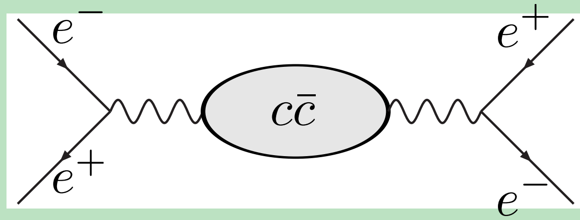
- Resonances model using data:
  - ★ Krüger-Sehgal (e+e- data)
  - ★ Breit-Wigner ansatz

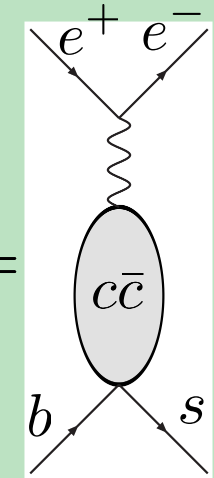


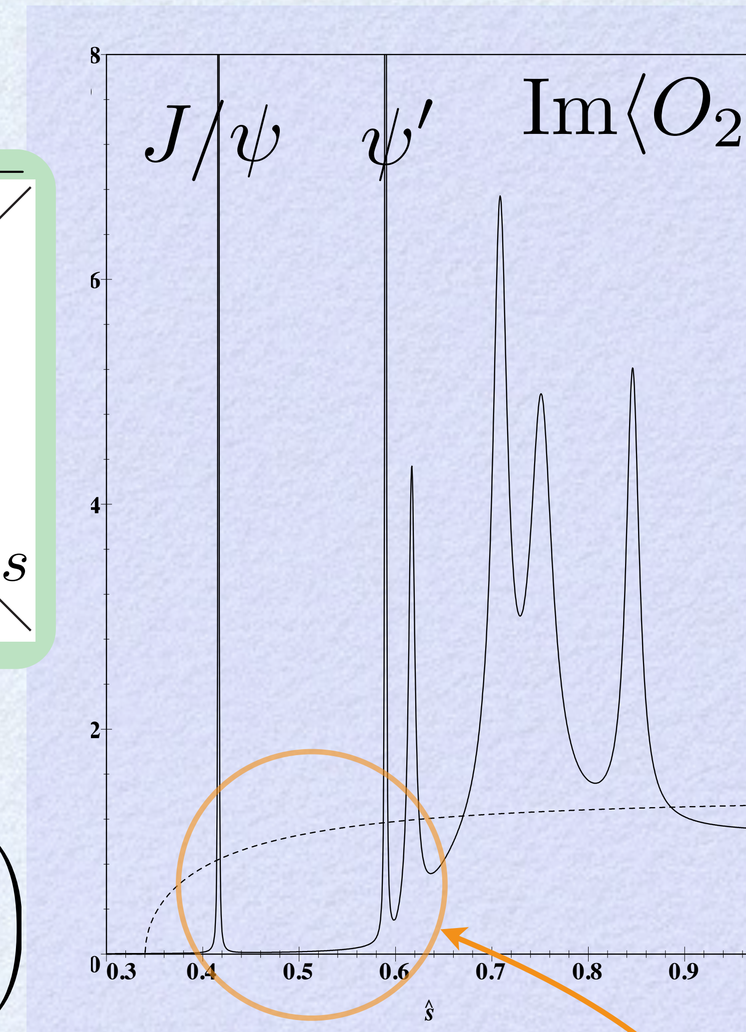
Q^2 cuts

- Kruger-Sehgal mechanism:

$$R_{\rm had}^{c\bar c} = \frac{\sigma(e^+e^- \to c\bar c \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$=$ 

$\langle O_2 \rangle =$ 



$$\mathrm{Im}\langle O_2\rangle \to \langle O_9\rangle_{\rm tree}\left(\frac{\pi}{3}R_{\rm had}^{c\bar c}(\hat s)\right)$$

$$\mathrm{Re}\langle O_2\rangle \to \langle O_9\rangle_{\rm tree}\left(-\frac{8}{9}\log m_c/m_b - \frac{4}{9} + \frac{\hat s}{3} P\int_{4\hat m_D^2}^{\infty} \frac{R_{\rm had}^{c\bar c}(\hat s')}{\hat s'(\hat s' - \hat s)} d\hat s'\right)$$

- Alternatively use a Breit-Wigner ansatz to parametrize <O$_2$>

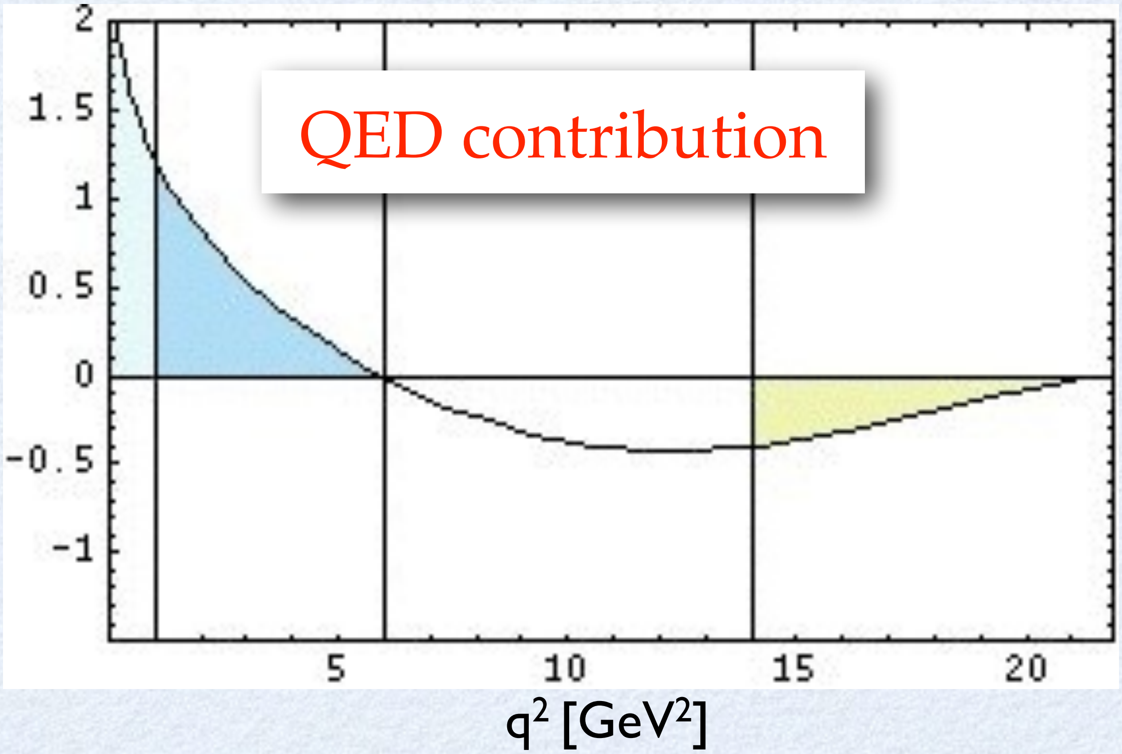
$$Y_{\rm amm}(\hat s) = Y_{\rm pert}(\hat s) + \frac{3\pi}{\alpha^2} C^{(0)} \sum_{V_i=\psi(1s),\ldots,\psi(6s)} \kappa_i \frac{\Gamma(V_i \to \ell^+\ell^-)\, m_{V_i}}{{m_{V_i}}^2 - \hat s\, {m_B}^2 - i m_{V_i}\Gamma_{V_i}}$$

Fudge factors

- The two approaches agree well above and below the resonances but not in between: interference between resonances is tricky
- The impact in the low q^2 region is +1.8%, in the high q^2 region is -10%

QED Logs: overview

- The ***rate is proportional to*** $\alpha_{\rm em}^2(\mu^2)$. Without QED corrections the scale μ is undetermined $\to \pm\, 4\%$ uncertainty
- Focus on corrections enhanced by large logarithms:
  - $\alpha_{\rm em} \log(m_W/m_b) \sim \alpha_{\rm em}/\alpha_s$ [WC, RG running] [Bobeth,Gambino,Gorbahn,Haisch]
  - $\alpha_{\rm em} \log(m_\ell/m_b)$ [Matrix Elements]
- The differential rate is not IR safe with respect to photon emission the results in the presence of a physical collinear logarithm, $\log(m_\ell/m_b)$



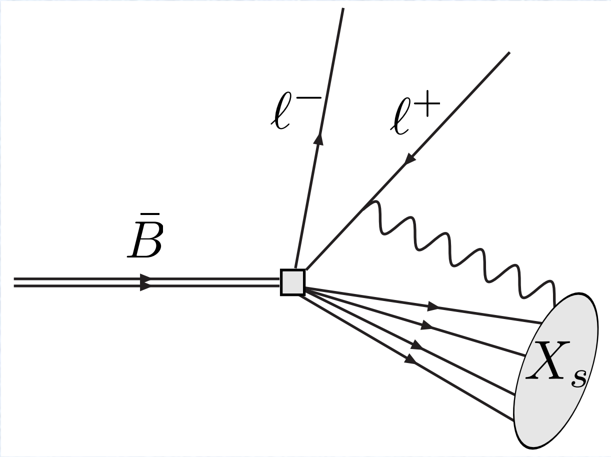
$$\text{virtual} = \frac{A_{\rm soft+collinear}}{\epsilon^2} + \frac{B_{\rm collinear} + B_{\rm soft}}{\epsilon} + C$$

$$\text{real} = -\frac{A_{\rm soft+collinear}}{\epsilon^2} - \frac{B'_{\rm collinear} + B_{\rm soft}}{\epsilon} + C'$$

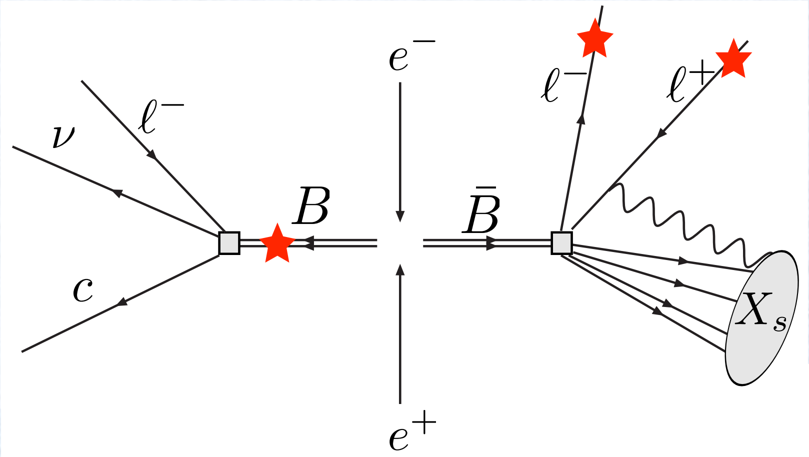
$$\int dq^2 \left(B_{\rm collinear} - B'_{\rm collinear}\right) = 0$$

QED Logs: Thery vs Experiment

- *Theory*
  include all bremsstrahlung photons into the X_s system:



- *Experiment (fully inclusive, Super-B only)*
  One B is identified; on the other side only the two leptons are reconstructed:



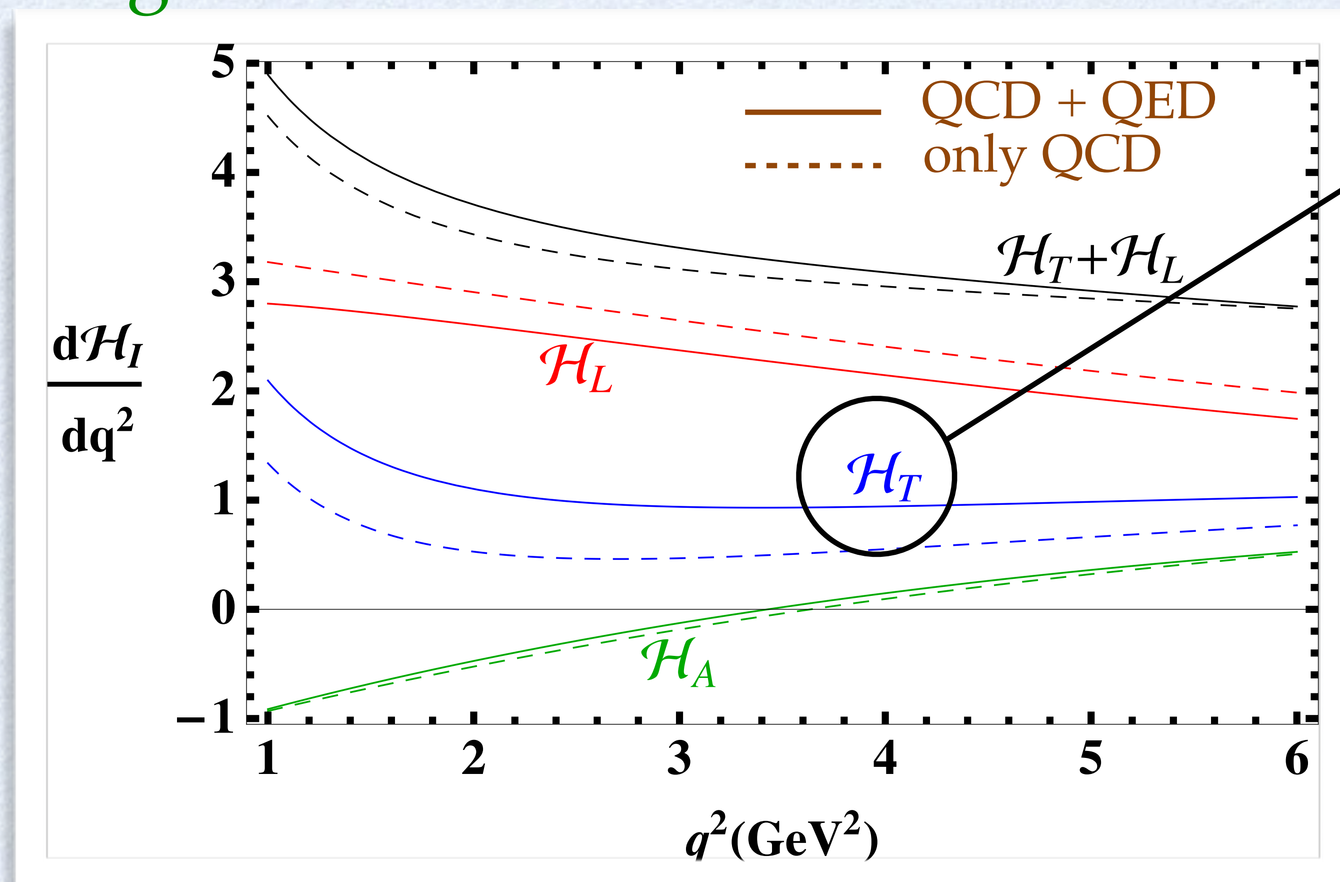
- *Experiment (Xs system reconstructed as a sum over exclusive states):*
  At BaBar (Belle) photons with energies smaller than 30 (20) MeV are not resolved. Photons emitted inside a small cone (35x50 mrad) around the electrons are identified and included in the event reconstruction. Events with any other photon (E > 30 (20) MeV and outside of the cone) are vetoed.

  Note: at BaBar (Belle) photons inside the cone are (are not) included in the definition of the q^2

- *Measured rates are sensitive to the soft photon cutoff and to the size of the cone*

QED Logs: size of the effect

- We calculated the effect of collinear photon radiation and found large effects on some observables



Shift on H_T is ~70%!

H_T is smaller than H_L ($\hat{s} \lesssim 0.3$):

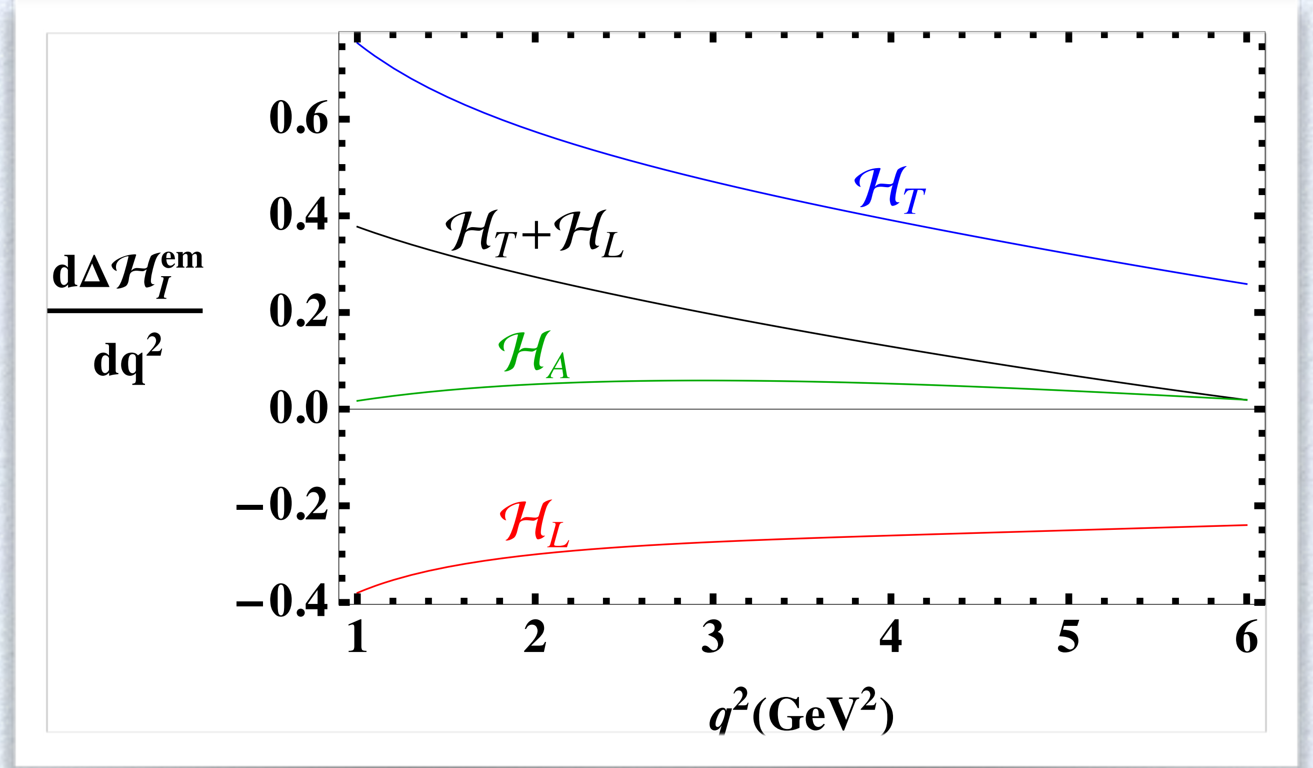
$$H_T \sim 2\hat{s}(1-\hat{s})^2 \left[|C_9 + \frac{2}{\hat{s}} C_7|^2 + |C_{10}|^2 \right]$$

$$H_L \sim (1-\hat{s})^2 \left[|C_9 + 2C_7|^2 + |C_{10}|^2 \right]$$

	$q^2 \in [1,6]$ GeV^2			$q^2 \in [1,3.5]$ GeV^2			$q^2 \in [3.5,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$	$\frac{O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{O_{[1,3.5]}}$	$\frac{O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{O_{[3.5,6]}}$
$\mathcal{B}$	100	5.1	5.1	54.6	3.7	6.8	45.4	1.4	3.1
$\mathcal{H}_T$	19.5	14.1	72.5	9.5	8.8	92.1	10.0	5.4	53.6
$\mathcal{H}_L$	80.0	-8.7	-10.9	44.7	-4.7	-10.6	35.3	-4.0	-11.3
$\mathcal{H}_A$	-3.3	1.4	-43.6	-7.2	0.8	-10.7	4.0	0.6	16.2

QED LOGS: SIZE OF THE EFFECT

- We calculated the effect of collinear photon radiation and found large effects on some observables

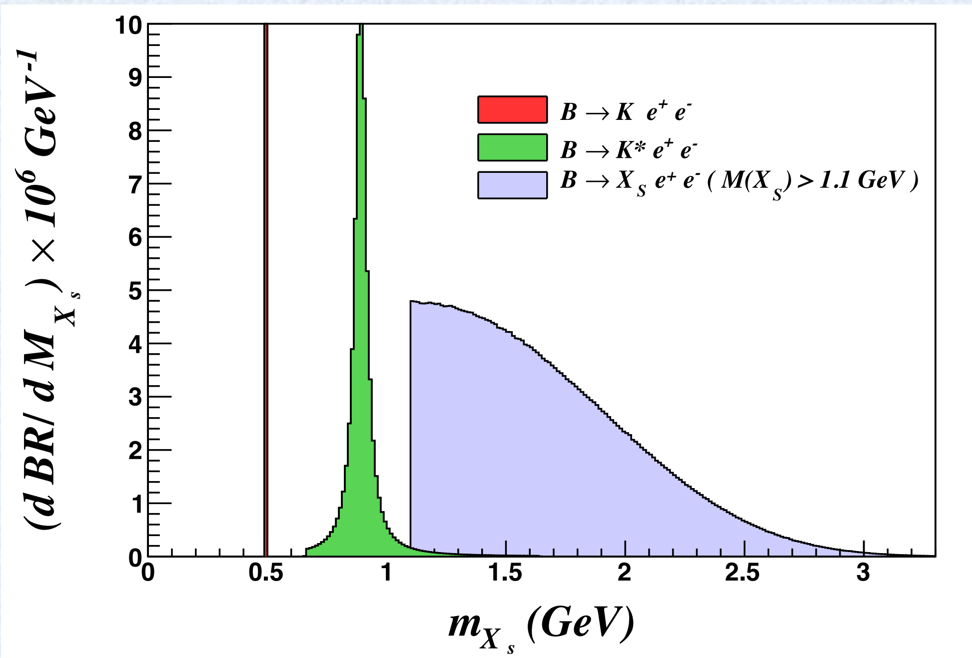


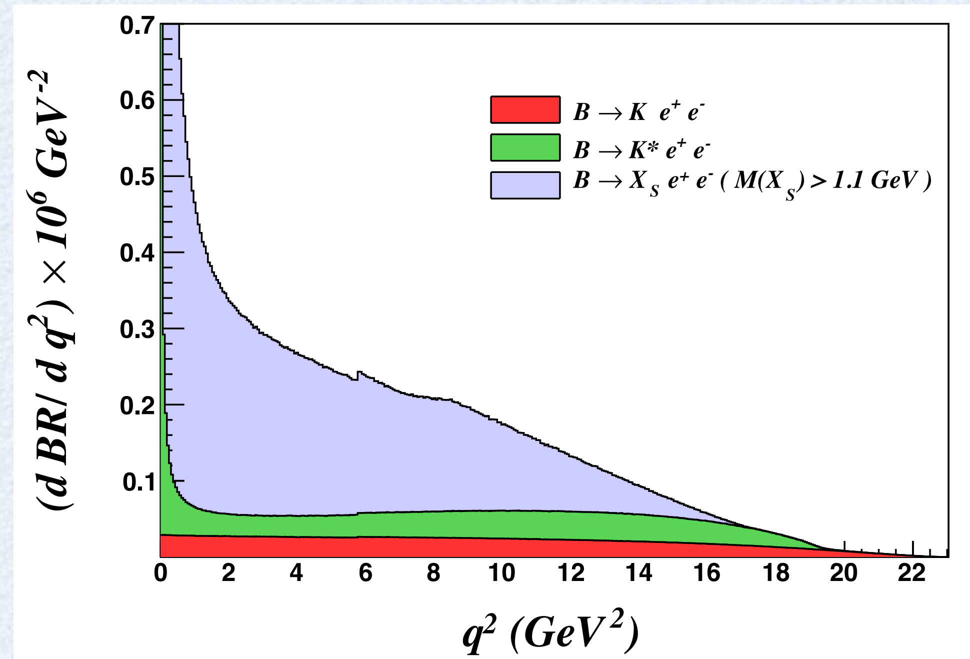
Size of QED contributions to the H_T and H_L is similar

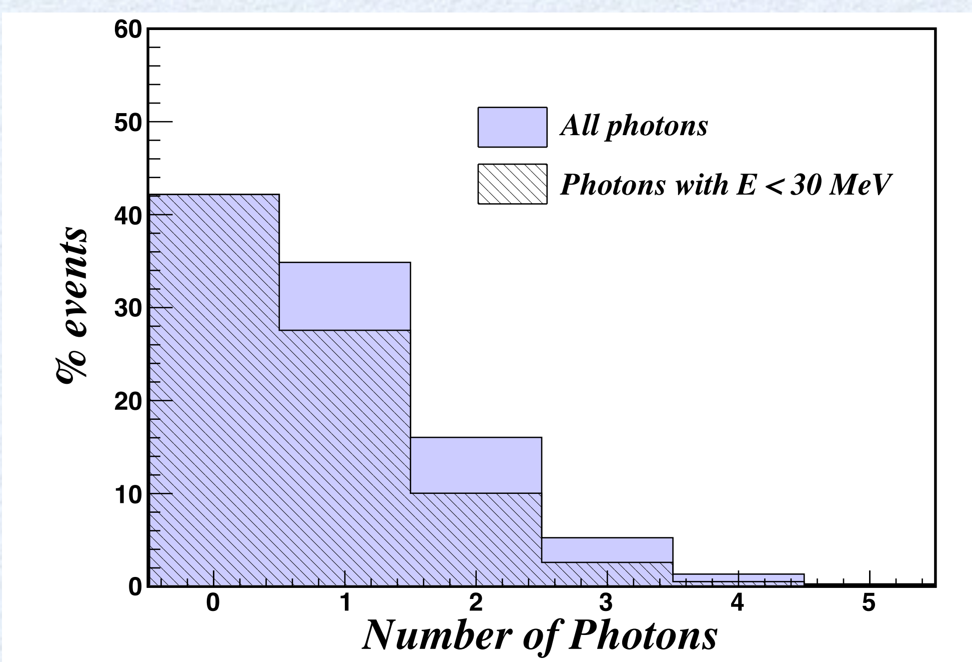
	$q^2 \in [1,6]$ GeV^2			$q^2 \in [1,3.5]$ GeV^2			$q^2 \in [3.5,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$	$\frac{O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{O_{[1,3.5]}}$	$\frac{O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{O_{[3.5,6]}}$
$\mathcal{B}$	100	5.1	5.1	54.6	3.7	6.8	45.4	1.4	3.1
$\mathcal{H}_T$	19.5	14.1	72.5	9.5	8.8	92.1	10.0	5.4	53.6
$\mathcal{H}_L$	80.0	-8.7	-10.9	44.7	-4.7	-10.6	35.3	-4.0	-11.3
$\mathcal{H}_A$	-3.3	1.4	-43.6	-7.2	0.8	-10.7	4.0	0.6	16.2

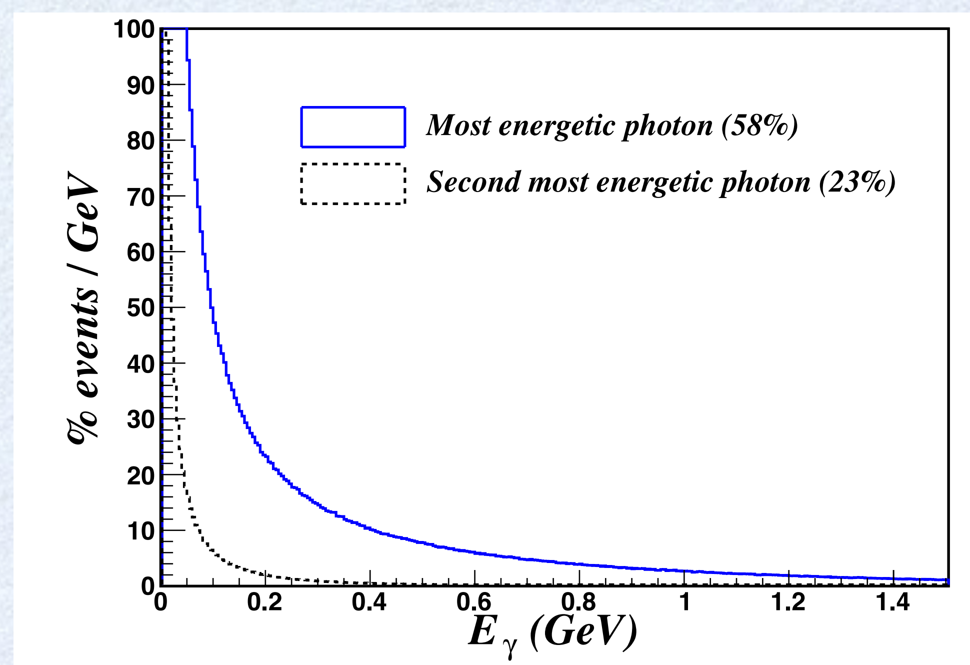
QED Logs: Monte Carlo

- EM effects have been calculated analytically and cross checked against Monte Carlo generated events (EVTGEN + PHOTOS)



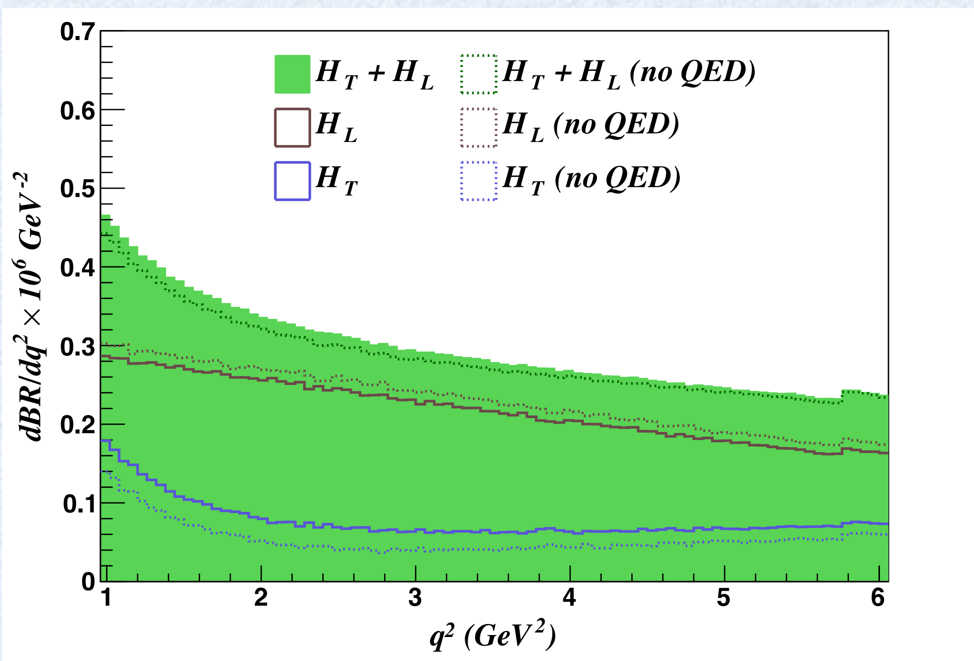


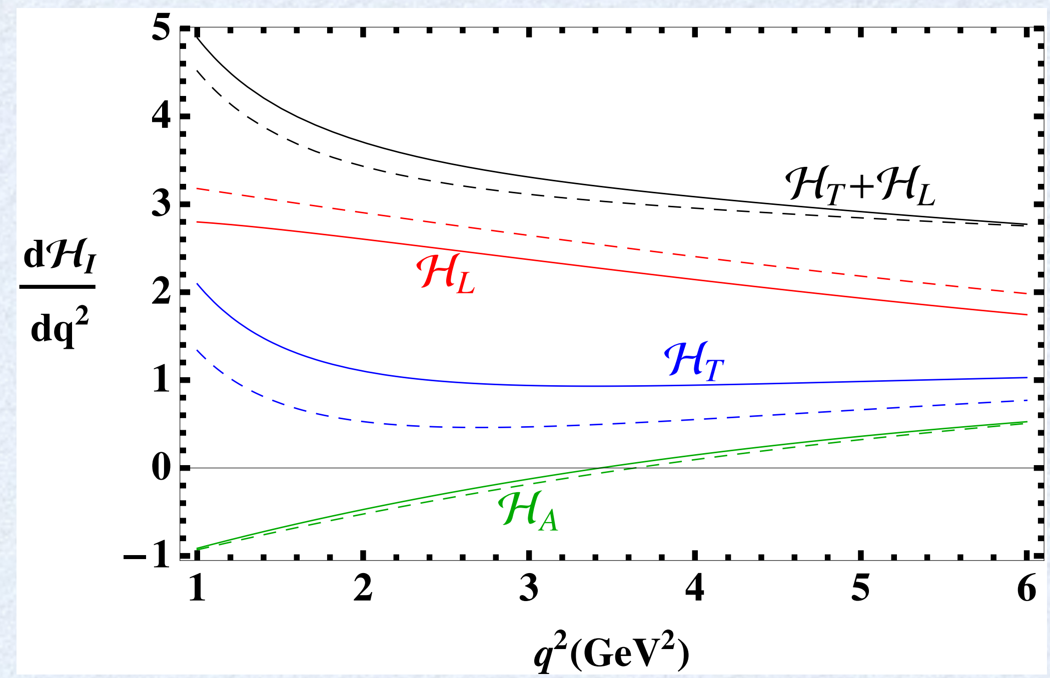


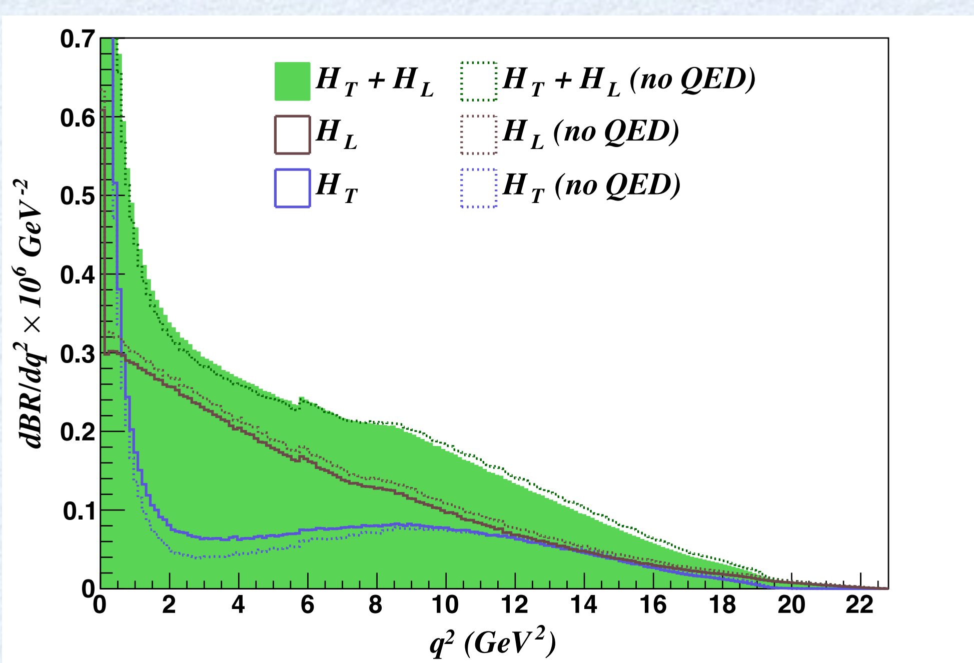


QED Logs: Monte Carlo

- The Monte Carlo study reproduces the main features of the analytical results







Monte Carlo:

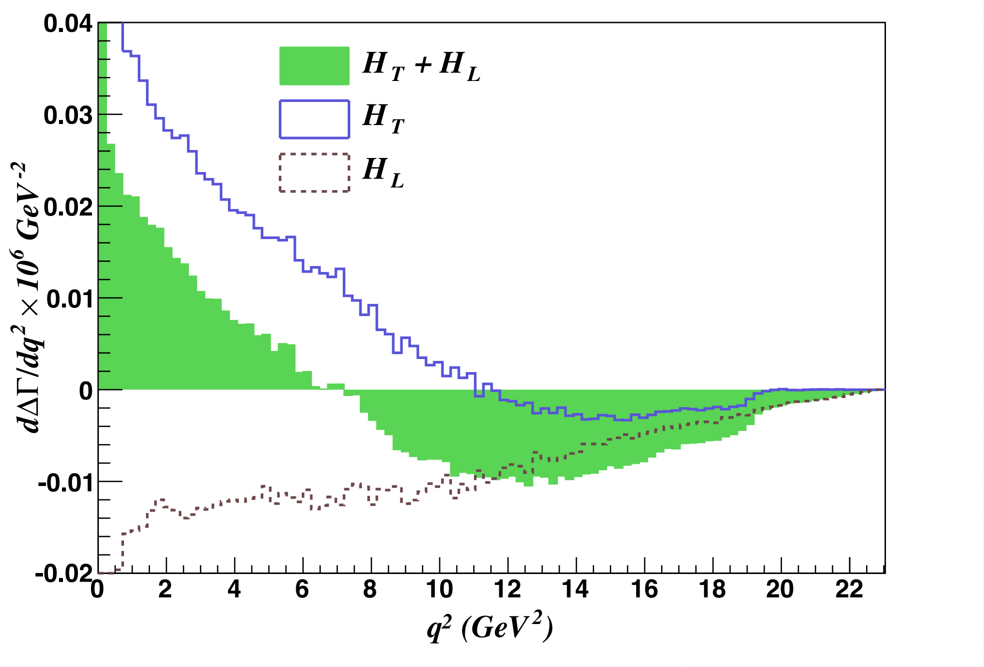
	$q^2 \in [1,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$
$\mathcal{B}$	100	3.5	3.5
$\mathcal{H}_T$	19.0	8.0	43.0
$\mathcal{H}_L$	81.0	-4.5	-5.5

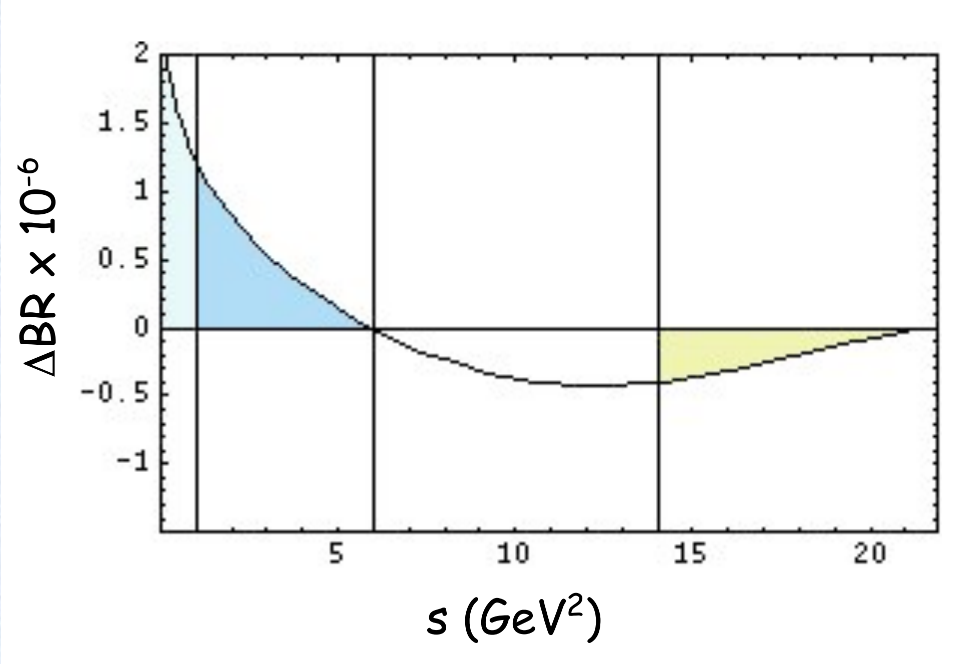
Analytical:

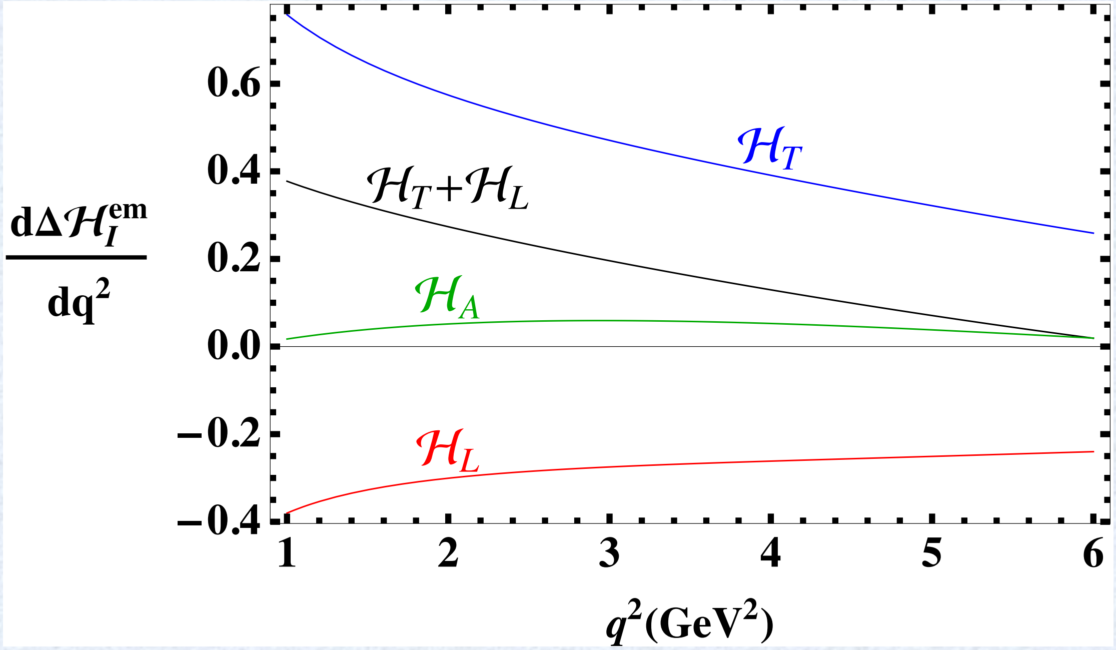
	$q^2 \in [1,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$
$\mathcal{B}$	100	5.1	5.1
$\mathcal{H}_T$	19.5	14.1	72.5
$\mathcal{H}_L$	80.0	-8.7	-10.9

QED LOGS: MONTE CARLO

- The Monte Carlo study reproduces the main features of the analytical results







Definition of observables

- At leading order in QED and at all orders in QCD, the double differential width is a quadratic polynomial: $\Gamma \sim a \cos^2\theta + b \cos\theta + c$.
- Γ receives non polynomial log-enhanced QED corrections
- Best strategy: **measure individual observables (BR, A_{FB}) and use Legendre polynomial as projectors**

$$H_I(q^2) = \int_{-1}^{+1} \frac{d^2\Gamma}{dq^2 dz} W_I(z) dz$$

$$W_T = \frac{2}{3} P_0(z) + \frac{10}{3} P_2(z)\,, \qquad W_3 = P_3(z)$$

$$W_L = \frac{1}{3} P_0(z) - \frac{10}{3} P_2(z)\,, \qquad W_4 = P_4(z)$$

$$W_A = \frac{4}{3} \mathrm{sign}(z)\,.$$

new observables

$$\frac{d\Gamma}{dq^2} = \int_{-1}^{+1} \frac{d^2\Gamma}{dq^2 dz} dz = H_T + H_L$$

$$\frac{dA_{\mathrm{FB}}}{dq^2} = \int_{-1}^{+1} \frac{d^2\Gamma}{dq^2 dz} \mathrm{sign}(z) dz = \frac{3}{4} H_A$$

$$\frac{d\bar{A}_{\mathrm{FB}}}{dq^2} = \frac{\int_{-1}^{+1} \frac{d^2\Gamma}{dq^2 dz} \mathrm{sign} dz}{\int_{-1}^{+1} \frac{d^2\Gamma}{dq^2 dz} dz} = \frac{3}{4} \frac{H_A}{H_T + H_L}$$

Inclusive: present status

	δ_{th}		R(μ/e)
$\mathcal{H}_T[1,6]_{\mu\mu} = (4.03 \pm 0.28) \cdot 10^{-7}$	±7%	$\mathcal{H}_T[1,6]_{ee} = (5.34 \pm 0.38) \cdot 10^{-7}$	0.75
$\mathcal{H}_L[1,6]_{\mu\mu} = (1.21 \pm 0.07) \cdot 10^{-6}$	±6%	$\mathcal{H}_L[1,6]_{ee} = (1.13 \pm 0.06) \cdot 10^{-6}$	1.07
$\mathcal{H}_A[1,3.5]_{\mu\mu} = (-1.10 \pm 0.05) \cdot 10^{-7}$	±5%	$\mathcal{H}_A[1,3.5]_{ee} = (-1.03 \pm 0.05) \cdot 10^{-7}$	1.07
$\mathcal{H}_A[3.5,6]_{\mu\mu} = (+0.67 \pm 0.12) \cdot 10^{-7}$	±18%	$\mathcal{H}_A[3.5,6]_{ee} = (+0.73 \pm 0.12) \cdot 10^{-7}$	0.92
$\mathcal{H}_3[1,6]_{\mu\mu} = (3.71 \pm 0.50) \cdot 10^{-9}$	±13%	$\mathcal{H}_3[1,6]_{ee} = (8.92 \pm 1.20) \cdot 10^{-9}$	0.42
$\mathcal{H}_4[1,6]_{\mu\mu} = (3.50 \pm 0.32) \cdot 10^{-9}$	±9%	$\mathcal{H}_4[1,6]_{ee} = (8.41 \pm 0.78) \cdot 10^{-9}$	0.42
$\mathcal{B}[1,6]_{\mu\mu} = (1.62 \pm 0.09) \cdot 10^{-7}$	±5%	$\mathcal{B}[1,6]_{ee} = (1.67 \pm 0.10) \cdot 10^{-7}$	0.97
$\mathcal{B}[> 14.4]_{\mu\mu} = (2.53 \pm 0.70) \cdot 10^{-7}$	±28%	$\mathcal{B}[> 14.4]_{ee} = (2.20 \pm 0.70) \cdot 10^{-7}$	1.15

- Scale uncertainties dominate at low-q^2
- Power corrections and scale uncertainties dominate at high-q^2
- Log-enhanced QED corrections at low and high q^2 are correlated

Inclusive R_X

- Actual results for the *μ/e* ratios:

$$\mathcal{R}_T[1,3.5] = (0.72 \pm 0.01) \qquad \mathcal{R}_{\rm BR}[1,3.5] = (0.959 \pm 0.004)$$
$$\mathcal{R}_T[3.5,6] = (0.80 \pm 0.01) \qquad \mathcal{R}_{\rm BR}[3.5,6] = (0.983 \pm 0.002)$$
$$\mathcal{R}_T[1,6] = (0.75 \pm 0.01) \qquad \mathcal{R}_{\rm BR}[1,6] = (0.970 \pm 0.003)$$
$$\mathcal{R}_L[1,3.5] = (1.069 \pm 0.006) \qquad \mathcal{R}_A[1,3.5] = (1.07 \pm 0.006)$$
$$\mathcal{R}_L[3.5,6] = (1.074 \pm 0.006) \qquad \mathcal{R}_A[3.5,6] = (0.92 \pm 0.02)$$
$$\mathcal{R}_L[1,6] = (1.072 \pm 0.006) \qquad \mathcal{R}_A[1,6] = (1.45 \pm 0.34)$$
$$\mathcal{R}_3 = 0.42$$
$$\mathcal{R}_4 = 0.42$$

- There are additional uncertainties stemming from genuine $O(\alpha_e)$ corrections that we have not calculated

HIGH-Q^2: REDUCING THE ERRORS

- Normalize the decay width to the semileptonic B→$X_u l\nu$ rate with the ***same dilepton invariant mass cut***:

$$\mathcal{R}(s_0) = \frac{\displaystyle\int_{\hat{s}_0}^{1} \mathrm{d}\hat{s}\, \frac{\mathrm{d}\Gamma(\bar{B} \to X_s \ell^+ \ell^-)}{\mathrm{d}\hat{s}}}{\displaystyle\int_{\hat{s}_0}^{1} \mathrm{d}\hat{s}\, \frac{\mathrm{d}\Gamma(\bar{B}^0 \to X_u \ell \nu)}{\mathrm{d}\hat{s}}}$$

[Ligeti,Tackmann]

- ***Impact of*** $1/m_b^2$ ***and*** $1/m_b^3$ ***power corrections drastically reduced:***

$$\begin{aligned}\mathcal{R}(14.4)_{\mu\mu} =& (2.62 \pm 0.09_{\text{scale}} \pm 0.03_{m_t} \pm 0.01_{C,m_c} \pm 0.01_{m_b} \pm 0.01_{\alpha_s} \pm 0.23_{\text{CKM}} \\ & \pm 0.0002_{\lambda_2} \pm 0.09_{\rho_1} \pm 0.04_{f_u^0+f_s} \pm 0.12_{f_u^0-f_s}) \cdot 10^{-3} \\ =& (2.62 \pm 0.30) \cdot 10^{-3}\end{aligned}$$

$$\begin{aligned}\mathcal{R}(14.4)_{ee} =& (2.25 \pm 0.12_{\text{scale}} \pm 0.03_{m_t} \pm 0.02_{C,m_c} \pm 0.01_{m_b} \pm 0.01_{\alpha_s} \pm 0.20_{\text{CKM}} \\ & \pm 0.02_{\lambda_2} \pm 0.14_{\rho_1} \pm 0.08_{f_u^0+f_s} \pm 0.12_{f_u^0-f_s}) \cdot 10^{-3} \\ =& (2.25 \pm 0.31) \cdot 10^{-3}\end{aligned}$$

- ***The largest source of uncertainty is*** V_{ub}

Present Status

BaBar: 471x10[6] BB pairs (424 fb[-1])
Belle: 152x10[6] BB pairs (140 fb[-1])
711 fb[-1] on tape!!

- World averages (Babar, Belle):

$$\mathrm{BR}^{\rm exp} = (1.58 \pm 0.37) \times 10^{-6} \qquad q^2 \in [1,6]$$

$$\mathrm{BR}^{\rm exp} = (0.48 \pm 0.10) \times 10^{-6} \qquad q^2 > 14.4$$

$$\overline{A}_{\rm FB}^{\rm exp} = \begin{cases} 0.34 \pm 0.24 & q^2 \in [0.2, 4.3] \\ 0.04 \pm 0.31 & q^2 \in [4.3, 7.3(8.1)] \end{cases}$$

δ_{exp} ≈ 23%
δ_{exp} ≈ 21%

non-optimal binning

- Theory:

$$\mathrm{BR}^{\rm th} = (1.65 \pm 0.10) \times 10^{-6} \qquad q^2 \in [1,6]$$

$$\mathrm{BR}^{\rm th} = (0.237 \pm 0.070) \times 10^{-6} \qquad q^2 > 14.4$$

$$\overline{A}_{\rm FB}^{\rm th} = \begin{cases} -0.077 \pm 0.006 & q^2 \in [0.2, 4.3] \\ 0.05 \pm 0.02 & q^2 \in [4.3, 7.3(8.1)] \end{cases}$$

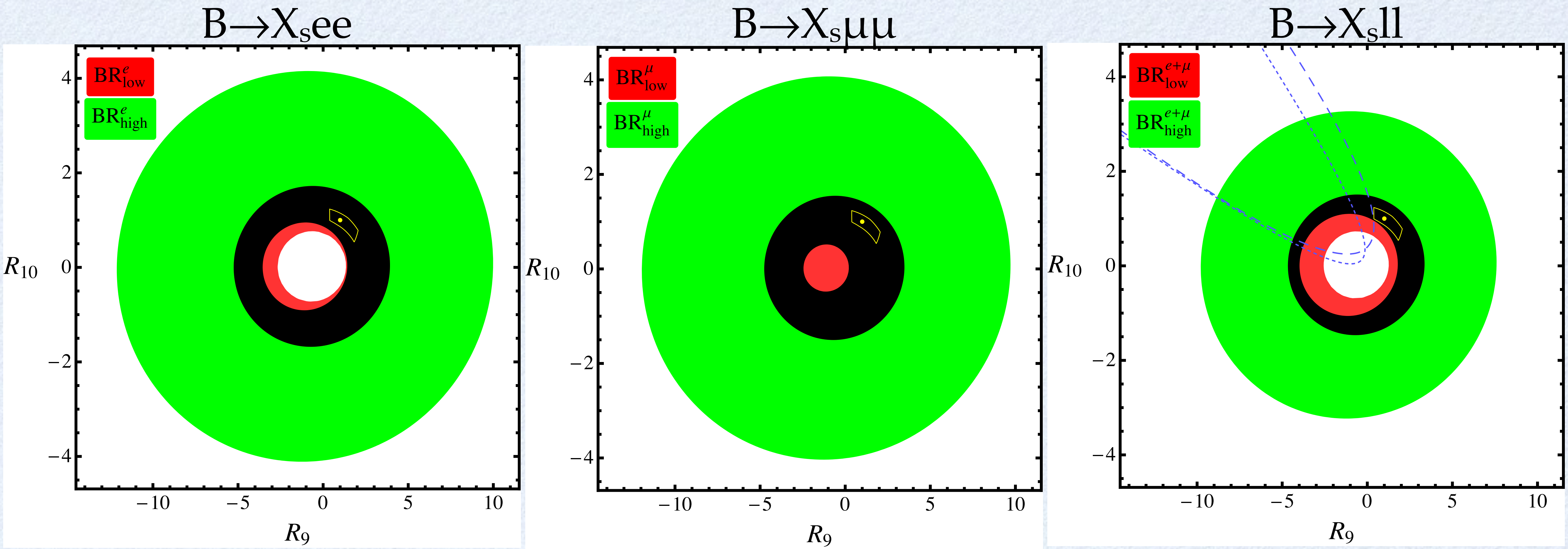
δ_{th} ≈ 6%
δ_{th} ≈ 30%

non-optimal binning

- $\mathrm{BR} = H_T + H_L \qquad \overline{A}_{\rm FB} = \frac{3}{4}\frac{H_A}{H_T + H_L}$

Present Status

- Constraints in the [R_9,R_{10}] plane ($R_i = C_i(\mu_0)/C_i^{SM}(\mu_0)$):



- Note that $C_9^{SM}(\mu_0) = 1.61$ and $C_{10}^{SM}(\mu_0) = -4.26$
- Best fits from the exclusive anomaly translate in R_9 ~ 0.3 (for the single WC fit) or R_9 ~ 0.65 and R_{10} ~ 0.9 (for the $C_9^{NP} = -C_{10}^{NP}$ scenario)

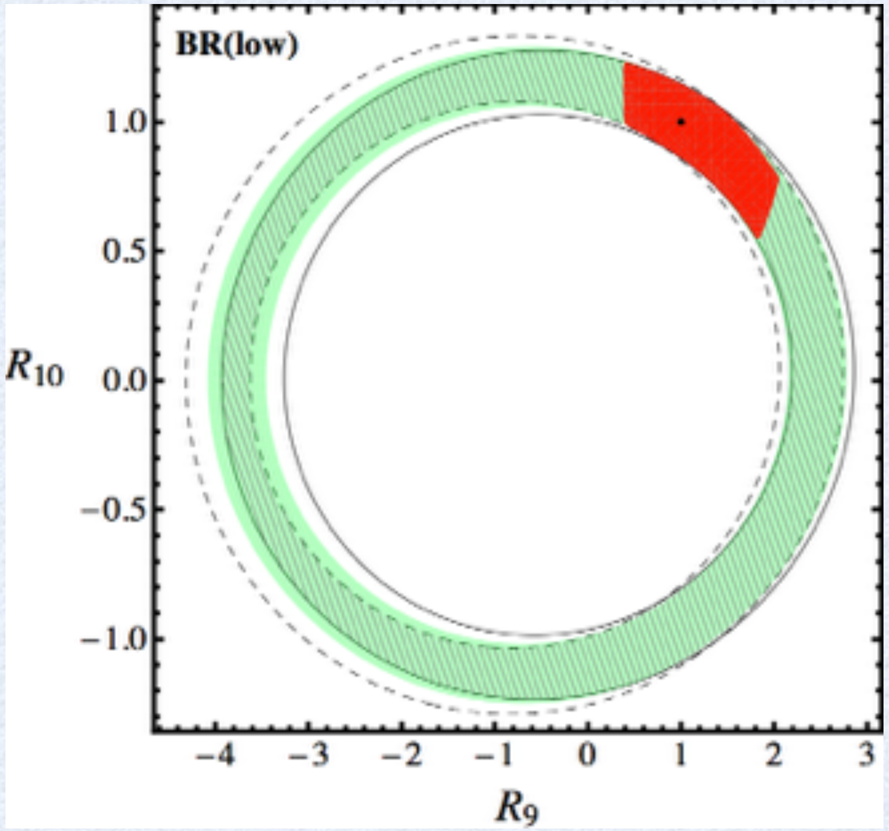
Projections

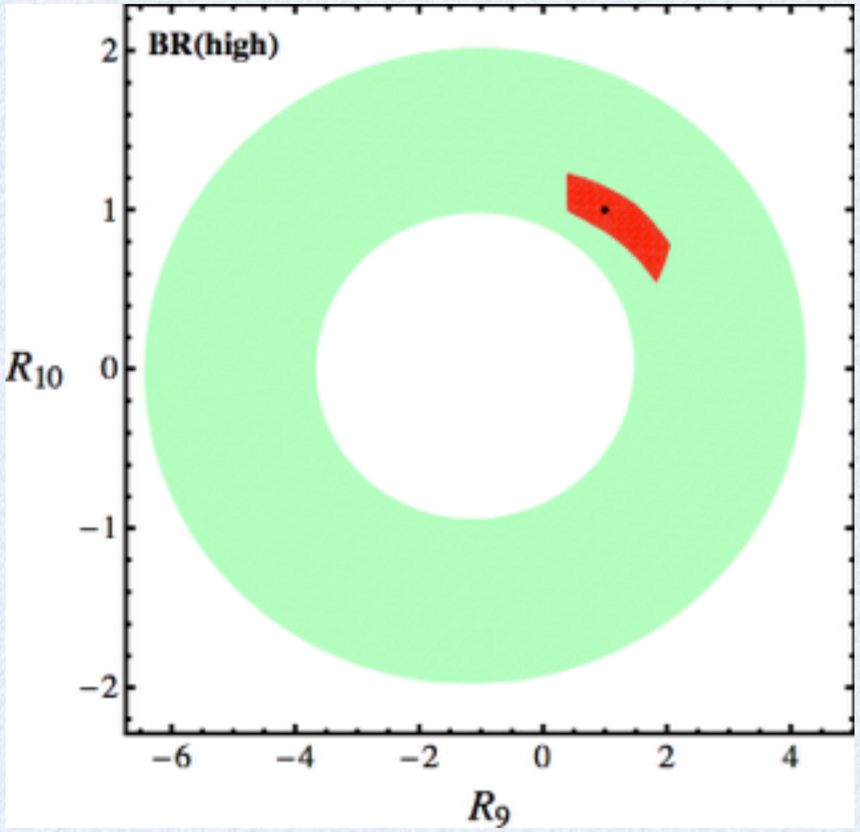
- Projected reach with 50 ab^{-1} of integrated luminosity

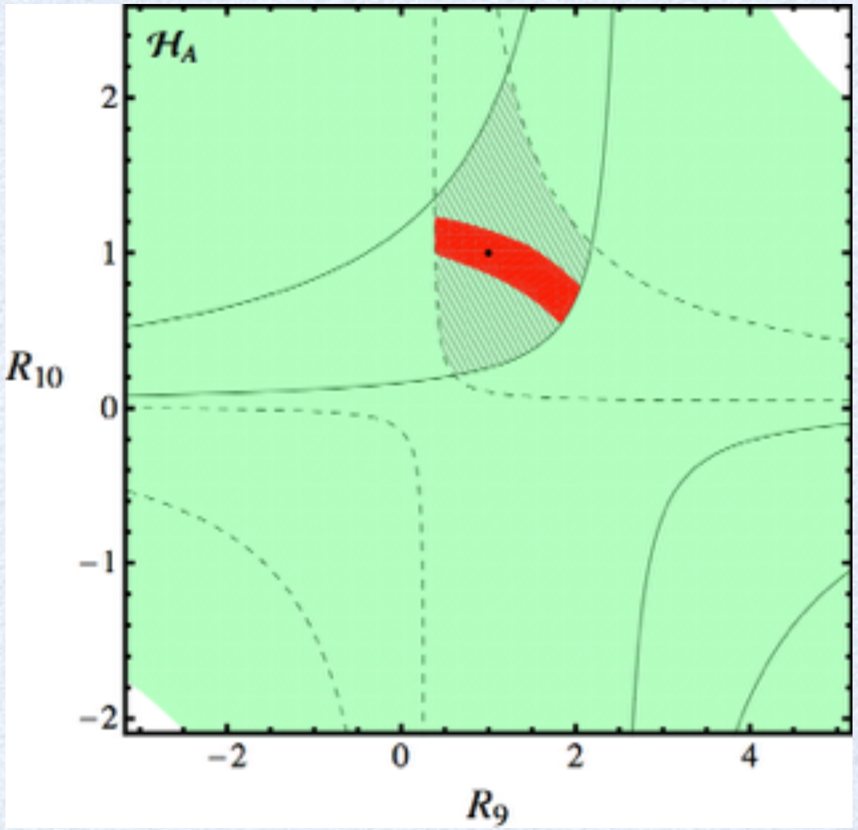
$$\mathcal{O}_{\rm exp} = \int \frac{d^2\mathcal{N}}{d\hat{s}dz} W[\hat{s}, z]\, d\hat{s}\, dz\ ,$$

$$\delta\mathcal{O}_{\rm exp} = \left[\int \frac{d^2\mathcal{N}}{d\hat{s}dz} W[\hat{s}, z]^2\, d\hat{s}\, dz\right]^{\frac{1}{2}}$$

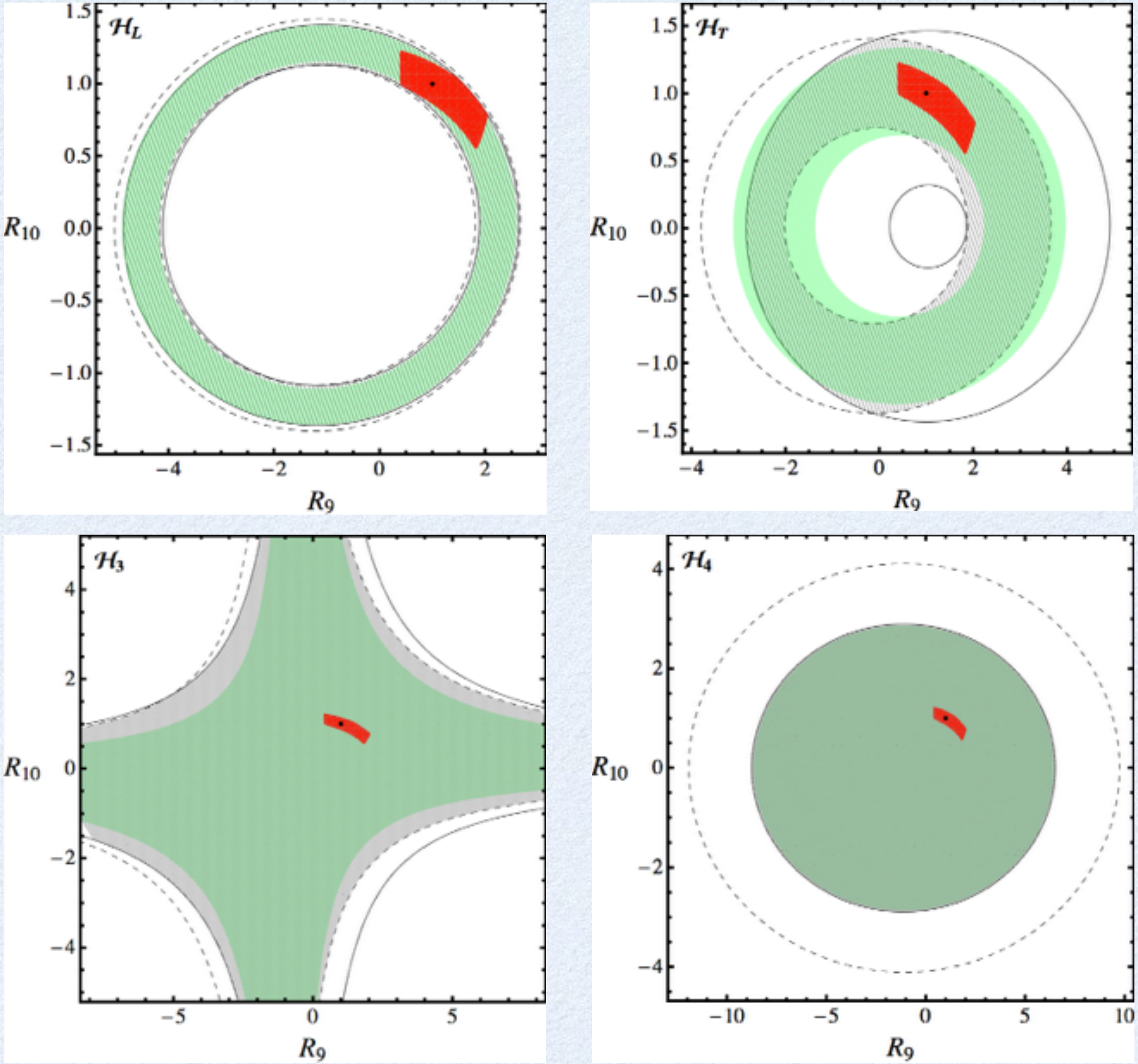
	[1, 3.5]	[3.5, 6]	[1, 6]	> 14.4
$\mathcal{B}$	3.7 %	4.0 %	3.0 %	4.1%
$\mathcal{H}_T$	24 %	21 %	16 %	-
$\mathcal{H}_L$	5.8 %	6.8 %	4.6 %	-
$\mathcal{H}_A$	37 %	44 %	200 %	-
$\mathcal{H}_3$	240 %	180 %	150 %	-
$\mathcal{H}_4$	140 %	360 %	140 %	-







Projections



Conclusions

- Inclusive calculations are almost at the "end-of-the-road", are clean but require Belle II
- Inclusive modes are sensitive to the treatment of QED radiation. The effect can be very large (depending on the observable) and can be exploited to test further combinations of Wilson coefficients
- Exclusive modes have a rich phenomenology but are plagued by form factor uncertainties (progress from lattice QCD expected), parametric uncertainties (light-cone wave functions, …) and power corrections
- LHCb data are in general agreement with the SM predictions with the exception of an angular distribution (P_5'), the BR at high-q^2 and a lepton flavor universality breaking ratio (R_K)

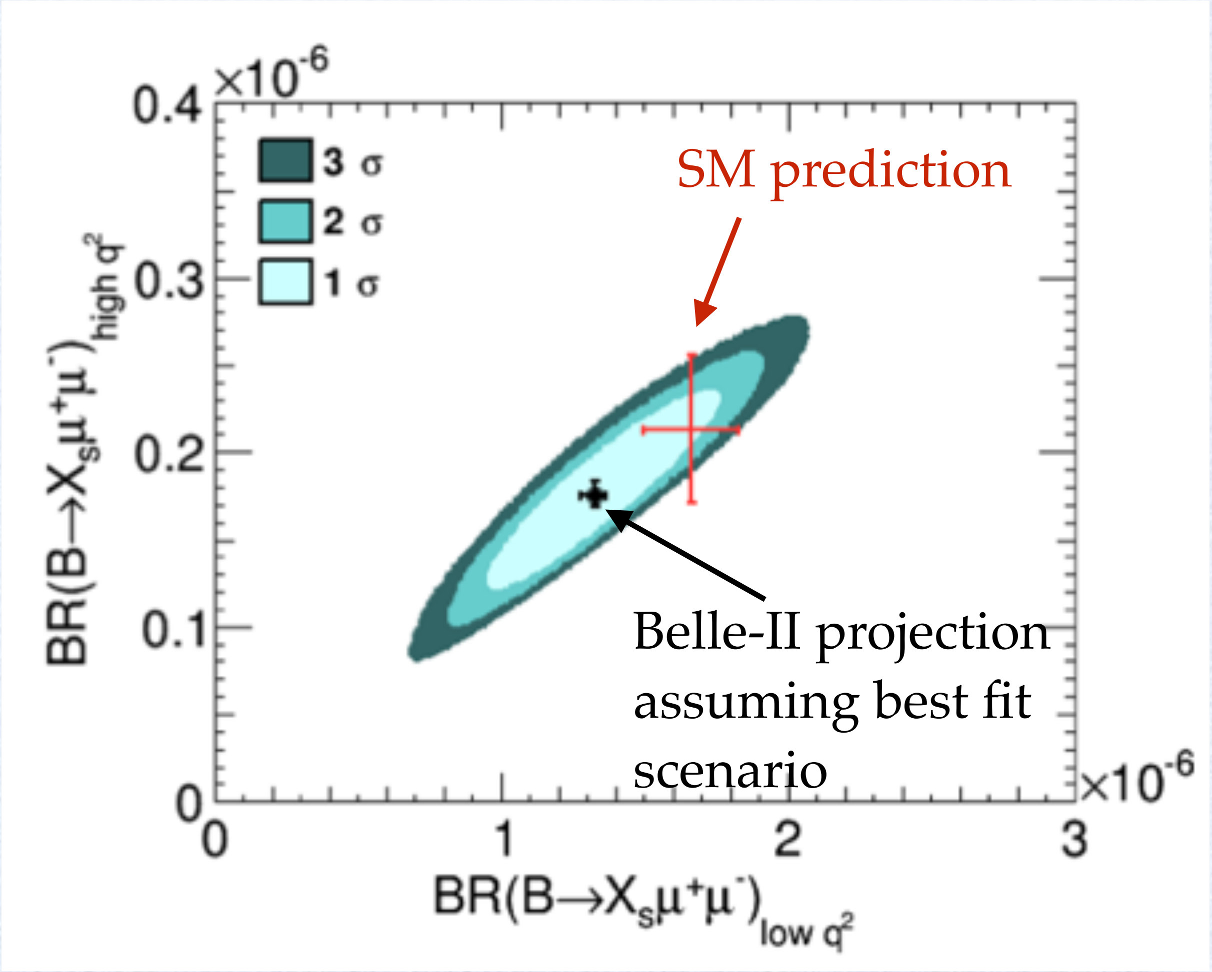
Backup Slides

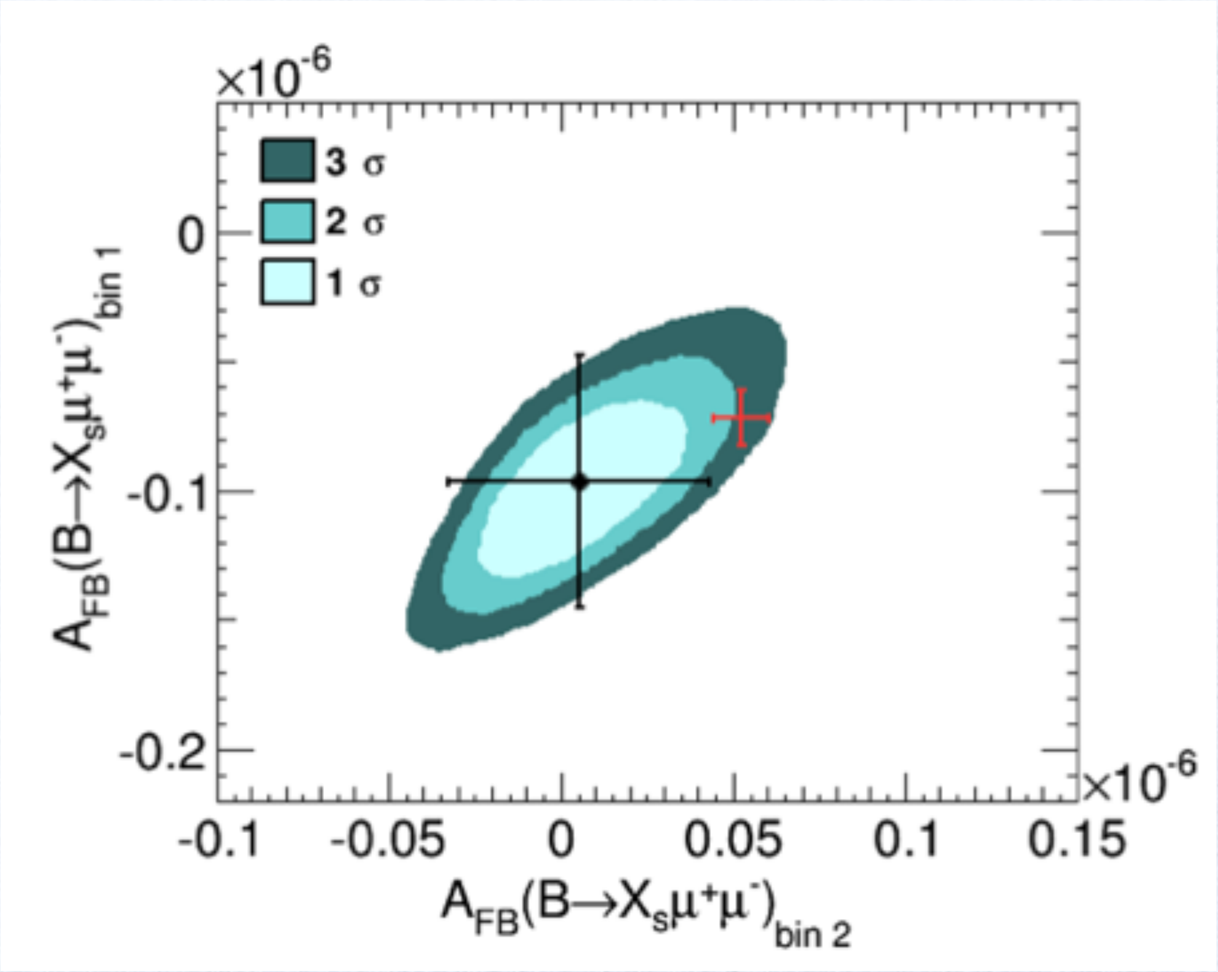
Inputs for B→sll

$\alpha_s(M_z) = 0.1184 \pm 0.0007$	$m_e = 0.51099892$ MeV
$\alpha_e(M_z) = 1/127.918$	$m_\mu = 105.658369$ MeV
$s_W^2 \equiv \sin^2\theta_W = 0.2312$	$m_\tau = 1.77699$ GeV
$\lvert V_{ts}^* V_{tb}/V_{cb}\rvert^2 = 0.9621 \pm 0.0027$ [85]	$m_c(m_c) = (1.275 \pm 0.025)$ GeV
$\lvert V_{ts}^* V_{tb}/V_{ub}\rvert^2 = 130.5 \pm 11.6$ [85]	$m_b^{1S} = (4.691 \pm 0.037)$ GeV [86, 87]
$BR(B \to X_c e\bar{\nu})_{\rm exp} = 0.1051 \pm 0.0013$ [86]	$m_{t,\rm pole} = (173.5 \pm 1.0)$ GeV
$M_Z = 91.1876$ GeV	$m_B = 5.2794$ GeV
$M_W = 80.385$ GeV	$C = 0.574 \pm 0.019$ [71]
$\mu_b = 5^{+5}_{-2.5}$ GeV	$\mu_0 = 120^{+120}_{-60}$ GeV
$\lambda_2^{\rm eff} = (0.12 \pm 0.02)$ GeV^2	$\rho_1 = (0.06 \pm 0.06)$ GeV^3 [88]
$\lambda_1^{\rm eff} = (-0.362 \pm 0.067)$ GeV^2 [86, 87]	$f_u^0 + f_s = (0 \pm 0.2)$ GeV^3 [52]
$f_u^0 - f_s = (0 \pm 0.04)$ GeV^3 [52]	$f_u^\pm = (0 \pm 0.4)$ GeV^3 [52]

INCL-EXCL INTERPLAY

- The effects on C_9 and C_9' are large enough to be easily checked at Belle II with inclusive decays (free of most uncertainties that plague the exclusive modes)





[Hurth, Mahmoudi 1411.2786]