QCD factorisation in B decays

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Outline

Basic idea

Virtues

QCDF for B->Vll: status

QCDF for B->Vll: possible improvements

QCDF for B->Vll: limitations

Some numerics

Similarity and differences to light-cone sum rules

Complementarity with LCSR

Example: helicity hierarchies and right-handed current BSM searches

Basic idea

In the limit $m_b >> \Lambda$, B decay amplitudes factorise into:

- partonic decay amplitudes multiplied by/convoluted with

- form factors or meson wave functions (light-cone distribution amplitudes, LCDA)

QCD factorisation for B decays is a merger of

- collinear factorisation in hard exclusive processes (Brodsky-Lepage; Efremov-Radyushkin early 1980's) such as pion electromagnetic form factor, in turn a sibling of the standard QCD factorisation in inclusive (collider) processes

- heavy-quark spin symmetry as in heavy-quark effective theory (Eichten/Hill/Grinstein/Georgi/Isgur/Wise/... ca 1990)

reduces number of nonperturbative objects

relates complex objects (eg amplitudes) to simpler ones (form factors)

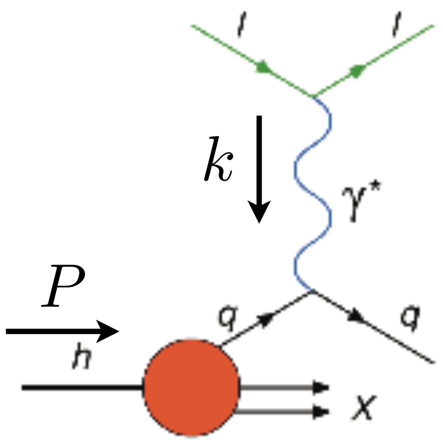
formalisation in terms of soft-collinear effective theory (SCET), now increasingly applied in high-p_T physics. **Not** a different approach, ie gives identical results; but a device for book-keeping (esp in proofs)

Origin: QCD factorization in DIS

In deep-inelastic scattering there is a large energy scale $Q^2 >> \Lambda^2$

As a result cross sections (or structure functions)

$$d\sigma(Q^2, x) \propto \mathrm{Im\ F.T.}\ T(j_{\rm em}(r) j_{\rm em}(0))\rangle$$



have an OPE

$$\mathrm{F.T.}\ \ T(j_{\rm em}(r) j_{\rm em}(0))\rangle = \frac{1}{Q^2} \sum_i C_i Q_i(0) + \mathcal{O}(\Lambda^2/Q^2)$$

Hadronic matrix elements of Q_i give moments of PDFs

$$d\sigma_A(Q^2, x) = \mathrm{pdf}_A(x; \mu) * T(x; Q^2/\mu^2) + \mathcal{O}(\Lambda^2/Q^2)$$

QCD (collinear) factorisation theorem

PDFs carry all dependence on hadron; nonperturbative; enter as universal building block in more general factorization theorems.

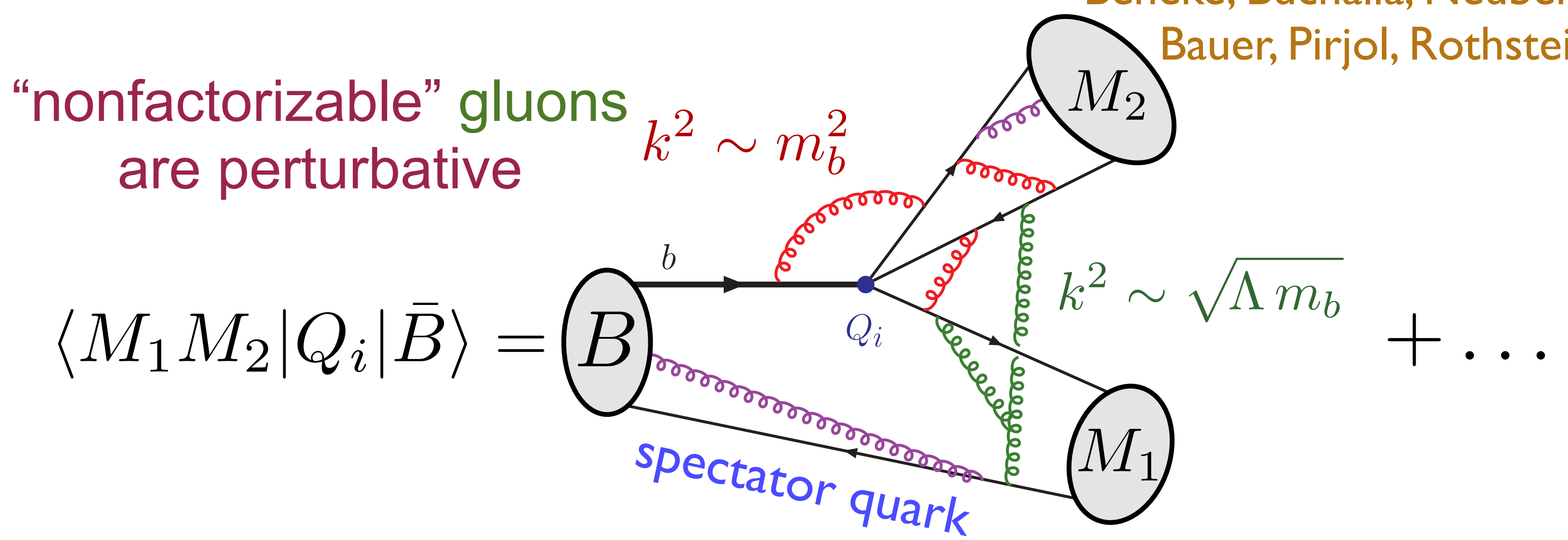
$$d\Gamma = d\hat{\sigma}(gg \to H + X) * pdf_g * pdf_g + \ldots + \mathcal{O}(\Lambda/\sqrt{\hat{s}})$$

power correction

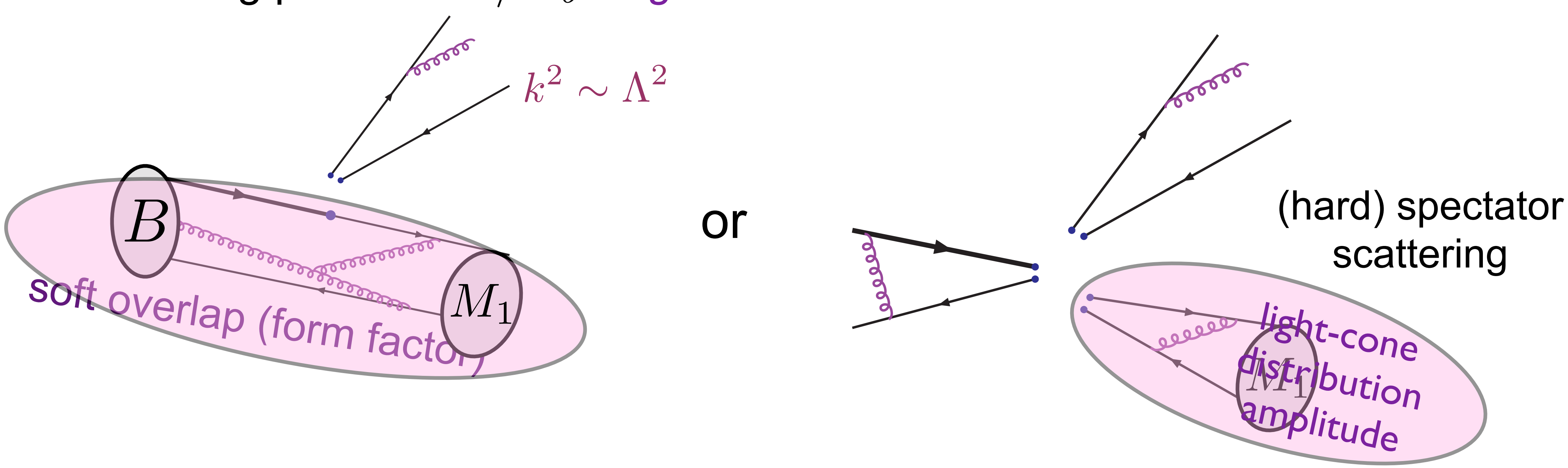
In most cases no OPE; diagrammatic arguments used to establish factorisation

Example: nonleptonic decay

Beneke, Buchalla, Neubert, Sachrajda 1999
Bauer, Pirjol, Rothstein, Stewart (SCET)



To leading power in Λ/m_b *long-distance* interactions look like

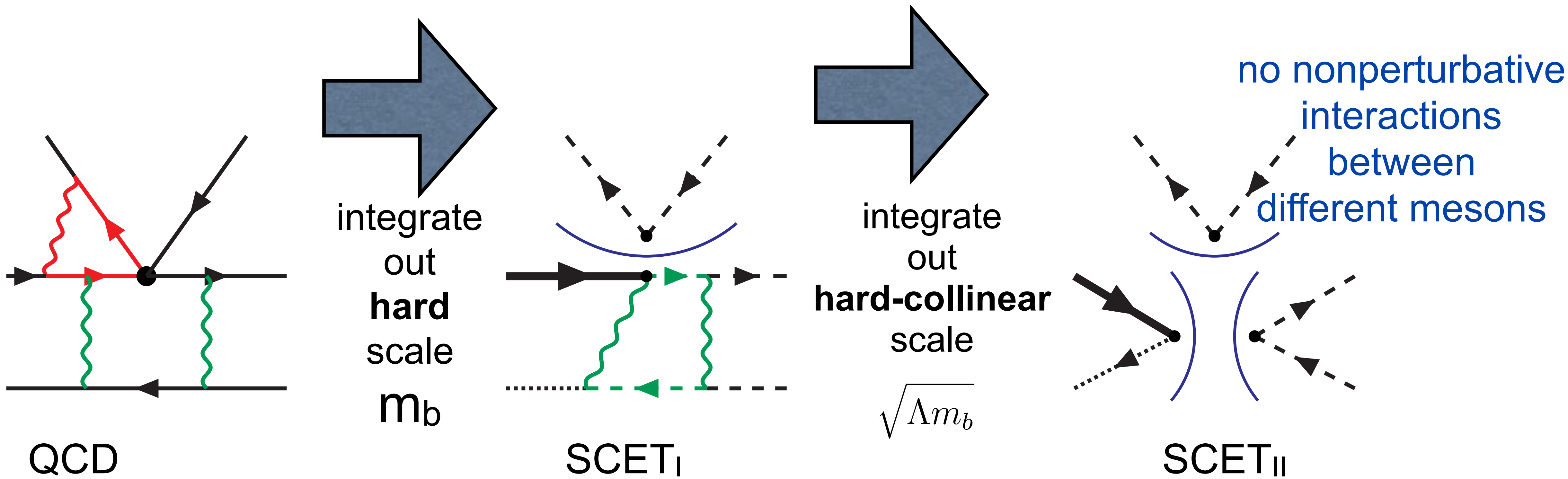


model dependence (only) at subleading power, because factorization breaks for some amplitudes

SCET picture

SCET = effective theory where 4-momenta of large (perturbative) virtualities have been removed

Organize as a two-step matching



The kernels now become Wilson coefficients, calculable order by order; IR finite; perturbative (must show SD dominance)

In spite of appearances the hard-collinear scale only generates even powers, so the overall expansion is still in Λ/m_b

Virtues

pushes model dependence to the level $O(\Lambda/m_b)$ ("power corrections")

within this accuracy (more on power corrections later), **unambiguous, scale-and scheme-independent,** expressions for

- hadronic B decay amplitudes (including direct CP phases) calculable in terms of α_s, form factors, LCDAs (prev slide)

  to lowest order naive factorisation recovered

- radiative and semileptonic decay amplitudes calculable

  to lowest order naive factorisation recovered

- ratios of form factors of same helicity calculable

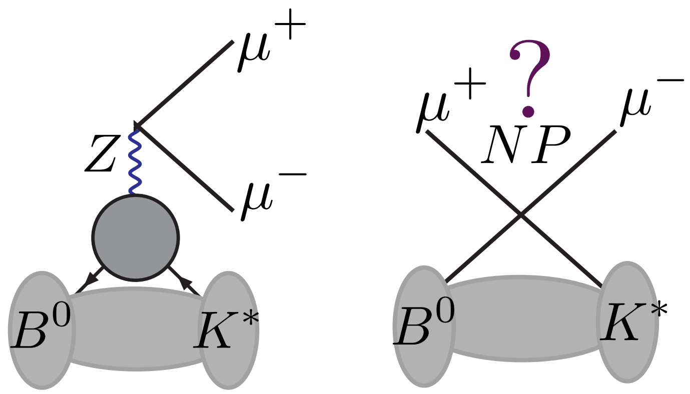
  to lowest order, large-energy symmetry relations of Charles et al recovered

In all cases, QCD factorisation provides **both** the (necessary) justification of the lowest-order result, and a systematic prescription to go beyond it.
Natural, unique (ie unambiguous) reference point to expand about.

Application to B->Vll decay

Two mechanisms to produce dilepton in & beyond SM

- via axial lepton current (in SM: Z, boxes)



K* helicity

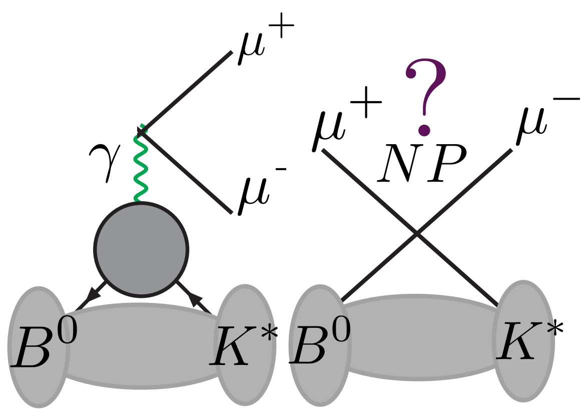
$$H_A(\lambda) \propto \tilde{V}_\lambda(q^2)C_{10} - V_{-\lambda}(q^2)C'_{10}$$

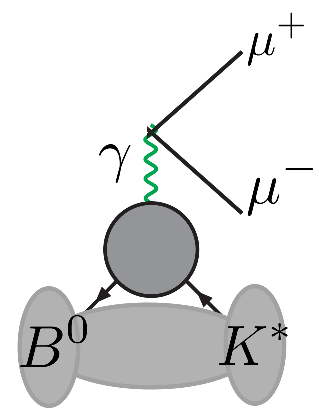
one form factor (nonperturbative) per helicity

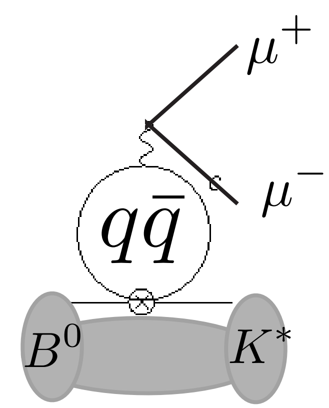
amplitudes factorize naively

[nb - one more amplitude if not neglecting lepton mass]

- via vector lepton current (in SM: (mainly) photon)







$$H_V(\lambda) \propto \tilde{V}_\lambda(q^2)C_9 - V_{-\lambda}(q^2)C'_9 + \frac{2\,m_b m_B}{q^2}\left(\tilde{T}_\lambda(q^2)C_7 - \tilde{T}_{-\lambda}(q^2)C'_7\right) - \frac{16\,\pi^2 m_B^2}{q^2} h_\lambda(q^2)$$

photon pole at $q^2=0$

two form factors interfere for each helicity

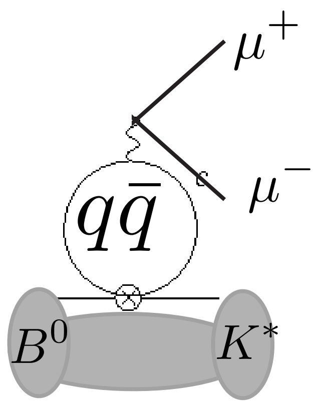
nonlocal "quark loops"
do **not** factorize naively

natural and transparent discussion in terms of 6 (7 if m_l != 0) helicity amplitudes

SJ, Martin Camalich 2012

Vector amplitude: nonlocal term

$$H_V(\lambda) \propto \tilde{V}_\lambda(q^2) C_9 - V_{-\lambda}(q^2) C_9' + \frac{2\, m_b m_B}{q^2} \left(\tilde{T}_\lambda(q^2) C_7 - \tilde{T}_{-\lambda}(q^2) C_7' \right) - \frac{16\, \pi^2 m_B^2}{q^2} h_\lambda(q^2)$$



+ strong interactions!

more properly:

$$\frac{e^2}{q^2} L_V^\mu a_\mu^{\rm had} = -i \frac{e^2}{q^2} \int d^4x e^{-iq\cdot x} \langle \ell^+\ell^- | j_\mu^{\rm em,lept}(x) | 0 \rangle \int d^4y\, e^{iq\cdot y} \langle M | j^{{\rm em,had},\mu}(y) \mathcal{H}_{\rm eff}^{\rm had}(0) | \bar{B} \rangle$$

$$h_\lambda \equiv \frac{i}{m_B^2} \epsilon^{\mu *}(\lambda) a_\mu^{\rm had}$$

nonlocal, nonperturbative, large normalisation ($V_{cb}^* V_{cs}\ C_2$)

traditional "ad hoc fix" : $C_9 \to C_9 + Y(q^2) = C_9^{\rm eff}(q^2)$, $C_7 \to C_7^{\rm eff}$ "taking into account the charm loop"

* for $C_7^{\rm eff}$ this seems ok at lowest order (pure UV effect; scheme independence)
* for $C_9^{\rm eff}$ amounts to factorisation of scales ~ m_b (, m_c ,q^2) and Λ (soft QCD)
* not justified in large-N limit (broken already at leading logarithmic order)
* what about QCD corrections?
* not a priori clear whether this even gets one closer to the true result!

only known justification is a heavy-quark expansion in Λ/m_b (just like inclusive decay is treated !)

Beneke, Feldmann, Seidel 2001, 2004

Nonlocal term - another look

traditional “ad hoc fix” : $C_9 \to C_9 + Y(q^2) = C_9^{eff}(q^2)$, $C_7 \to C_7^{eff}$

dominant effect: charm loop, proportional to ($z = 4\, m_c^2/q^2$)

$$-\frac{4}{9}\left(\ln\frac{m_q^2}{\mu^2} - \frac{2}{3} - z\right) - \frac{4}{9}(2+z)\sqrt{|z-1|}\begin{cases}\arctan\dfrac{1}{\sqrt{z-1}}, & z>1,\\ \ln\dfrac{1+\sqrt{1-z}}{\sqrt{z}} - \dfrac{i\pi}{2}, & z\leqslant 1\end{cases}$$

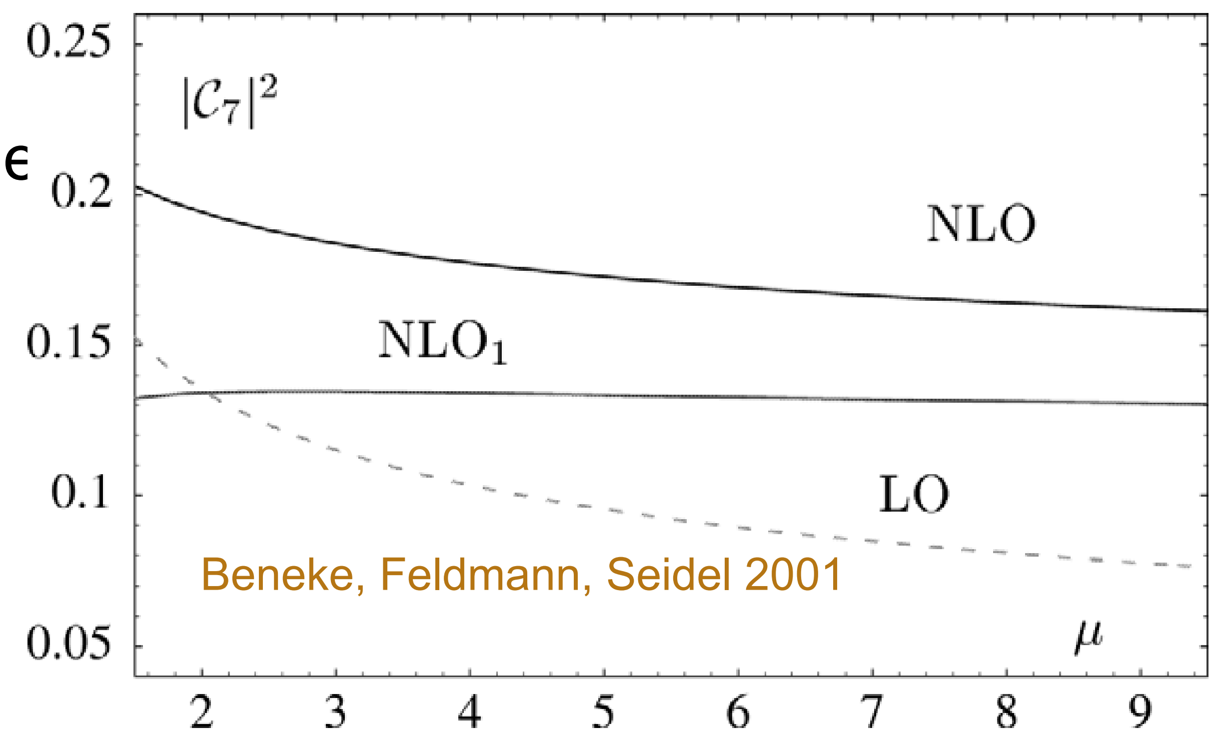
$$C_9^{\rm eff} = \begin{cases} 4.18|_{C_9} + (0.22+0.05i)|_Y & (m_c = m_c^{\rm pole} = 1.7{\rm GeV}) \\ 4.18|_{C_9} + (0.40+0.05i)|_Y & (m_c = m_c^{\overline{\rm MS}} = 1.2{\rm GeV}). \end{cases}$$

ie a 5% mass scheme ambiguity

separately, one has a residual scale ambiguity of order 30% at the level of the decay amplitude

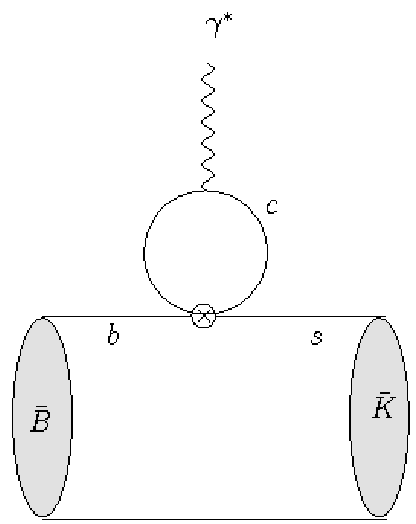
resolved in the heavy-quark expansion (to leading power)

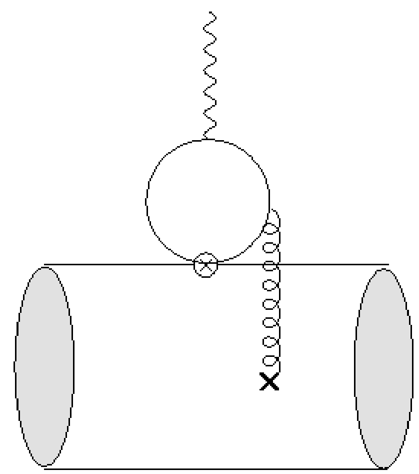
Beneke, Feldmann, Seidel 2001, 2004



Beneke, Feldmann, Seidel 2001

Nonlocal terms:heavy-quark expansion





leading-power: factorises into perturbative kernels, form factors, LCDA's (including hard/hard-collinear gluon corrections to all orders)

α_s^0 : $C_7 \rightarrow C_7^{eff}$
$C_9 \rightarrow C_9^{eff}(q^2)$
+ 1 annihilation diagram
α_s^1 : further corrections to $C_7^{eff}(q^2)$ and $C_9^{eff}(q^2)$

(convergent) convolutions of hard-scattering kernels with meson light cone-distribution amplitudes
Beneke, Feldmann, Seidel 2001

state-of-the-art in phenomenology

unambigous (save for parametric uncertainties)

at subleading powers: breakdown of factorisation

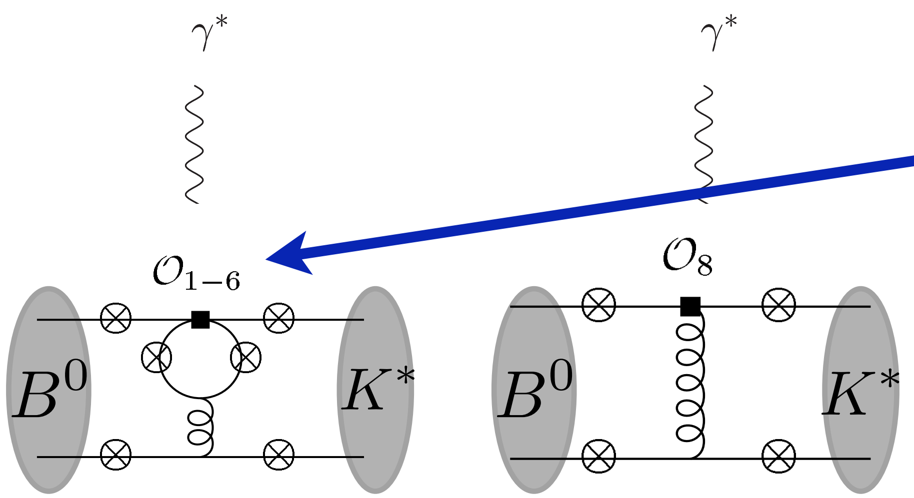
some contributions have been estimated as end-point divergent convolutions with a cut-off Kagan&Neubert 2001, Feldmann&Matias 2002

can perform light-cone OPE of charm loop & estimate resulting (nonlocal) operator matrix elements

Khodjamirian et al 2010

effective shifts of helicity amplitudes as large as ~10%

New effect: spectator scattering



includes Q_1^c, Q_2^c - large Wilson coefficients

+ annihilation (+ "vertex corrections")

Beneke, Feldmann, Seidel 2001

leading-power: everything factorises into perturbative kernels, form factors, meson light-cone distribution amplitudes (including hard/hard-collinear gluon corrections to all orders)

$$h_\lambda = \int_0^1 du \phi_K^*(u) T(u, \alpha_s) + \mathcal{O}(\Lambda/m_b)$$

- leading power in the heavy quark limit - same as the vertex corrections going into C_7^{eff}, C_9^{eff}
- sensitivity to substructure of mesons, via light-cone distribution amplitudes: leading twist for K*, two two-particle LCDAs for B-meson

Form factors

Helicity amplitudes naturally involve helicity form factors

$$
\begin{aligned}
-im_B\tilde{V}_{L(R)\lambda}(q^2) &= \langle M(\lambda)|\bar{s}\not{\epsilon}^*(\lambda)P_{L(R)}b|\bar{B}\rangle, \\
m_B^2\tilde{T}_{L(R)\lambda}(q^2) &= \epsilon^{*\mu}(\lambda)q^\nu\langle M(\lambda)|\bar{s}\sigma_{\mu\nu}P_{R(L)}b|\bar{B}\rangle \\
im_B\tilde{S}_{L(R)}(q^2) &= \langle M(\lambda=0)|\bar{s}P_{R(L)}b|\bar{B}\rangle.
\end{aligned}
$$

~ Bharucha/Feldmann/Wick 2010

definitions here:
SJ, Martin Camalich 2012

(S is essentially A_0 in the traditional nomenclature.)

- **directly relevant** to B->V l l including the LHCb anomaly
in particular, **V_-/T_- determines of the zero crossing of both A_{FB} and of S_5/P_5', as far as form factors are concerned**

- helicity+ vanishes at q^2=0, in particular

(Burdman; Beneke/Feldmann/Seidel)
SJ, Martin Camalich 2012,2014, this talk and WIP

implying several clean null tests of the SM

$$T_+(q^2=0) = 0$$

Burdman, Hiller 2000
SJ, Martin Camalich 2012

difficult to calculate - lattice cannot cover small q^2 (plus other issues)
best shot: light-cone sum rules with continuum subtractions

Form factor relations

Once one accepts the heavy-quark limit as necessary evil (?) for dealing with the nonleptonic Hamiltonian ("charm loops" etc) one takes note that it also predicts simple relations between the (helicity) form factors, for instance:

Charles et al 1999
Beneke, Feldmann 2000
...

$$\frac{T_-(q^2)}{V_-(q^2)} = 1 + \frac{\alpha_s}{4\pi} C_F \left[\ln \frac{m_b^2}{\mu^2} - L \right] + \frac{\alpha_s}{4\pi} C_F \frac{1}{2} \frac{\Delta F_\perp}{V_-}$$

where

$$L = -\frac{2E}{m_B - 2E} \ln \frac{2E}{m_B}$$

"vertex" correction: no new parameter

"spectator scattering": mainly dependent on B meson LCDA but a_s suppressed

Eliminates form factor dependence from some observables (eg P_2') almost completely, up to power corrections

Descotes-Genon, Hofer, Matias, Virto

(earlier: Egede et al; Becirevic and Schneider; Bobeth et al, ...)

Limitations

While at leading power one has an unambiguous result (though still dependent on nonperturbative parameters which must be separately determined, or fit to data), not much is known about $O(\Lambda/m_b)$ corrections.

- only partial calculations exist (particularly in connection to isospin asymmetry)

Kagan, Neubert; application to b->sll: Feldmann, Matias

- most seriously, attempts at factorising power corrections leads fails in some cases: the “nonfactorisable” gluon-exchanges receive O(1) corrections

- note that the latter is already true at leading power for the helicity-zero amplitude if $q^2 < \Lambda m_b$. This implies, for instance, that F_L and P_5’ cannot be calculated in the lowest bin (S_5 fine).

Endpoint divergences

Attempting to factorize the soft form factor results in divergent factors such as

$$\int_0^1 du \frac{(1+\bar{u})\phi_\pi(u)}{\bar{u}^2}$$

diverges at $u = 1$
not regularized in SCET

``hard-scattering'' kernel picks out an exceptional configuration
exchanged gluon then has *low* virtuality

Parameterization (model) (cf. BBNS)

phenomenological [ϕ=0 here]

hadronic scale

$$\int_0^1 du \frac{\phi_\pi(u)}{\bar{u}^2} \to \int_0^{1-\Lambda/m_b} du \frac{\phi_\pi(u)}{\bar{u}^2} = 6(1+\rho e^{i\phi}) \ln \frac{\Lambda_h}{m_b} + \text{finite}$$

Proposed modification of SCET (0-bin subtraction) [Manohar, Stewart 06]

$$\int_0^1 du \frac{\phi_\pi(u)}{\bar{u}^2} \to \int_0^1 du \frac{\phi_\pi(u) - \bar{u}\, \phi'_\pi(1)}{\bar{u}^2} + \phi'_\pi(1) \ln \frac{m_b}{\mu_-}$$

new non-perturbative object - no expansion in Gegenbauer moments

new factorization scale

Unfortunately, nobody has been able to give a definition (or show the existence) of a suitable form of the object $\phi'(1)$

Possible improvements

Some parts of the calculations, in particular form factor ratios, have been known to NNLO (α_s^2) for some time.

Beneke&Yang; Becher&Hill;...

Resummations of logarithms of hard and hard-collinear scale (based on experience from nonleptonic effect, unlikely to be very important)

The true limitation are power corrections that do not factorise. Progress within the heavy-quark expansion (such as establishing a factorisation theorem) would require a conceptual breakthrough.

This leaves two strategies

- parameterise power corrections and fit to data

- combine QCD factorisation with other methods (such as LCSR)

Brief comparison of heavy-quark expansion to 2015 data

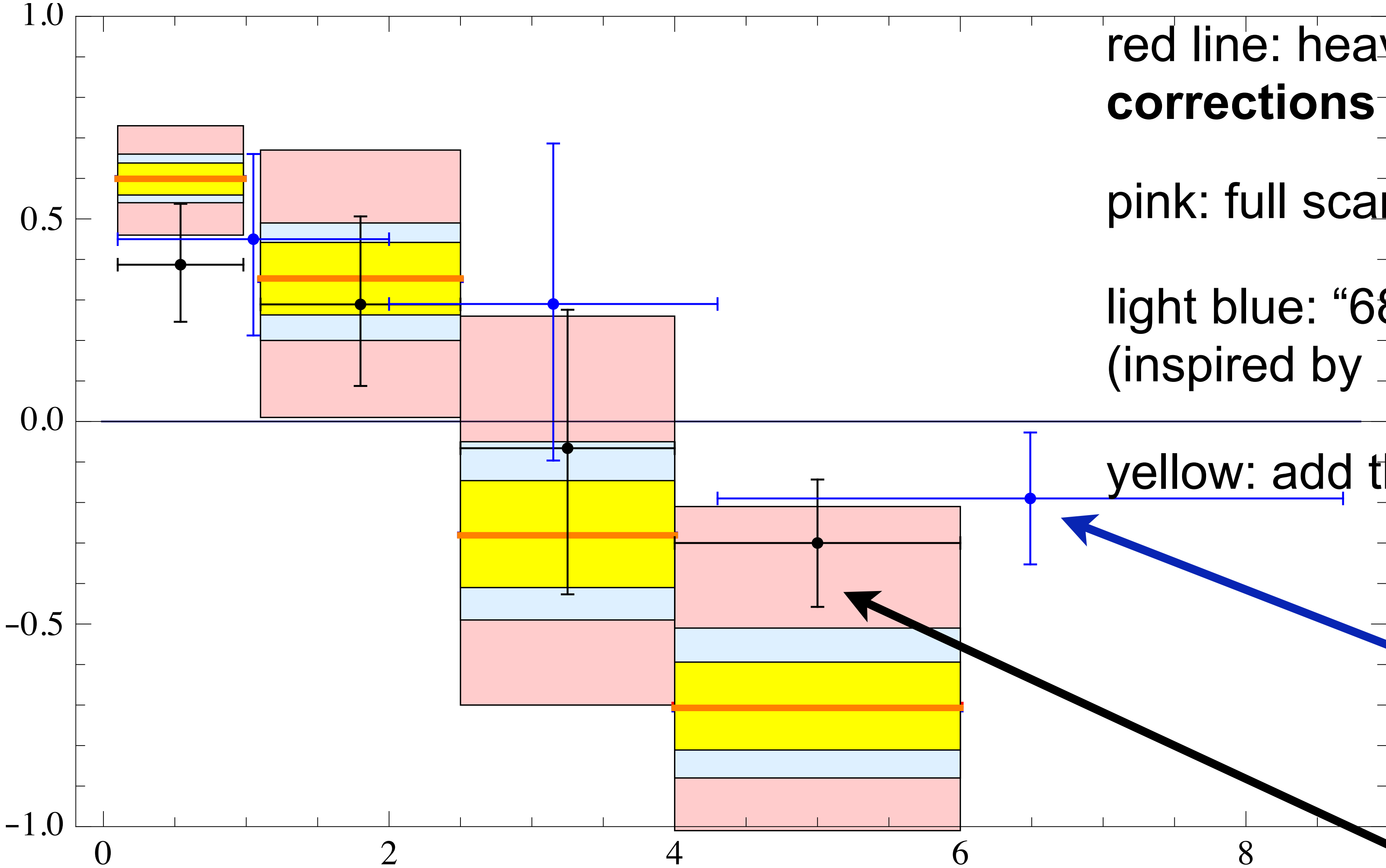
- methodology as in SJ and Martin Camalich 2012, 2014, parameter ranges as in 1412.3183

(In particular a certain model of power corrections.)

Central value lines in the following plots correspond to the pure heavy-quark limit, ie all power corrections set to zero. All numbers preliminary

P_5'

SJ, Martin Camalich, preliminary



red line: heavy-quark limit, **no power corrections**

pink: full scan over all theory errors

light blue: “68% Gaussian” theory error (inspired by Descotes-Genon, Hofer, Matias, Virto)

yellow: add theory errors in quadrature

LHCb 2013 (1 fb^{-1})

LHCb Moriond 2015 (3 fb^{-1})

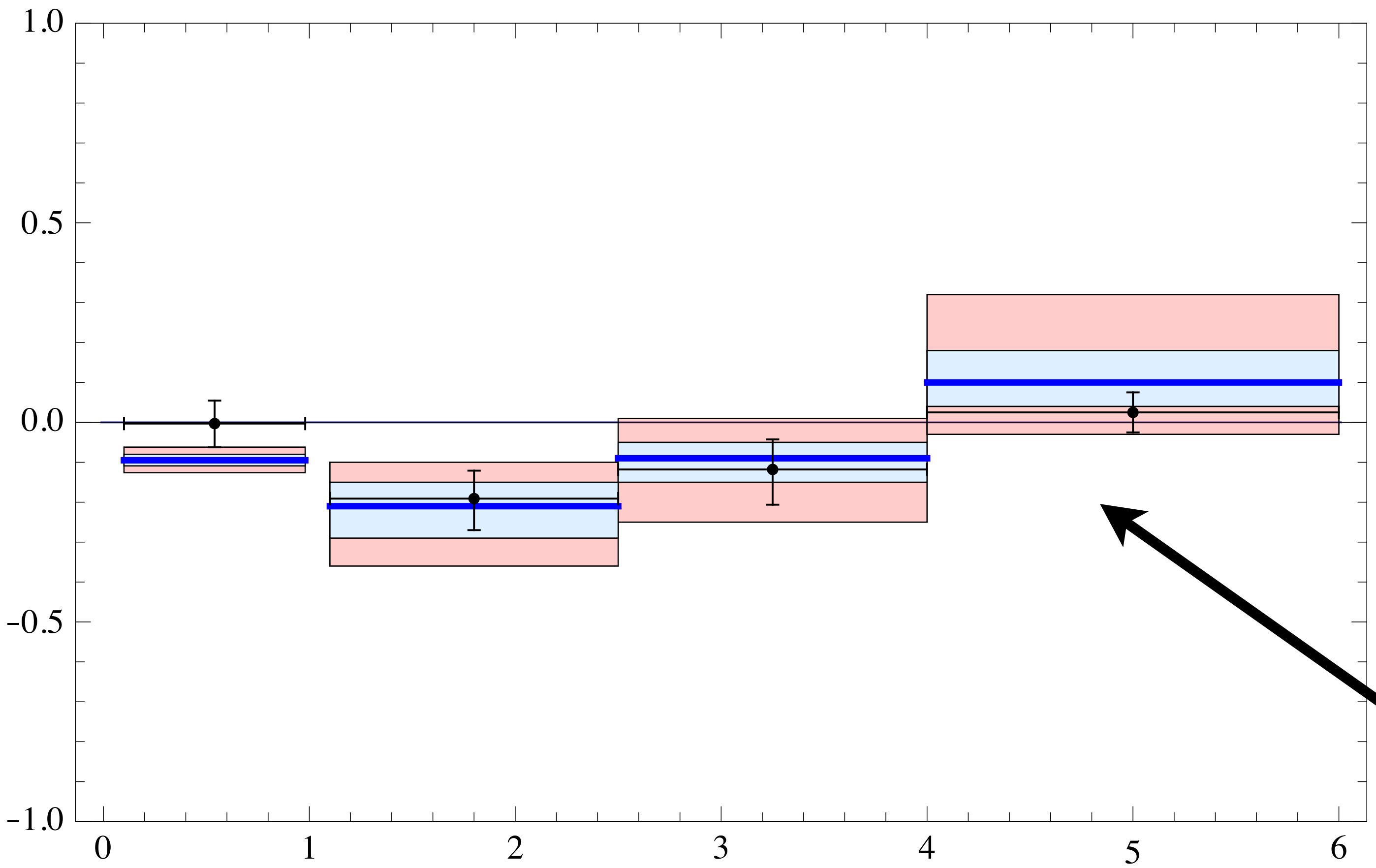
Pure heavy-quark limit (!) describes data surprisingly well.

Within errors there appears to be no significant discrepancy

Cannot support LHCb claim of 2.9 sigma effect in the 4..6 GeV^2 bin

Forward-backward asymmetry

SJ, Martin Camalich, preliminary



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pink: full scan over all theory errors

light blue: “68% Gaussian” theory error (a la Descotes-Genon, Hofer, Matias, Virto)

LHCb Moriond 2015 (3 fb^{-1})

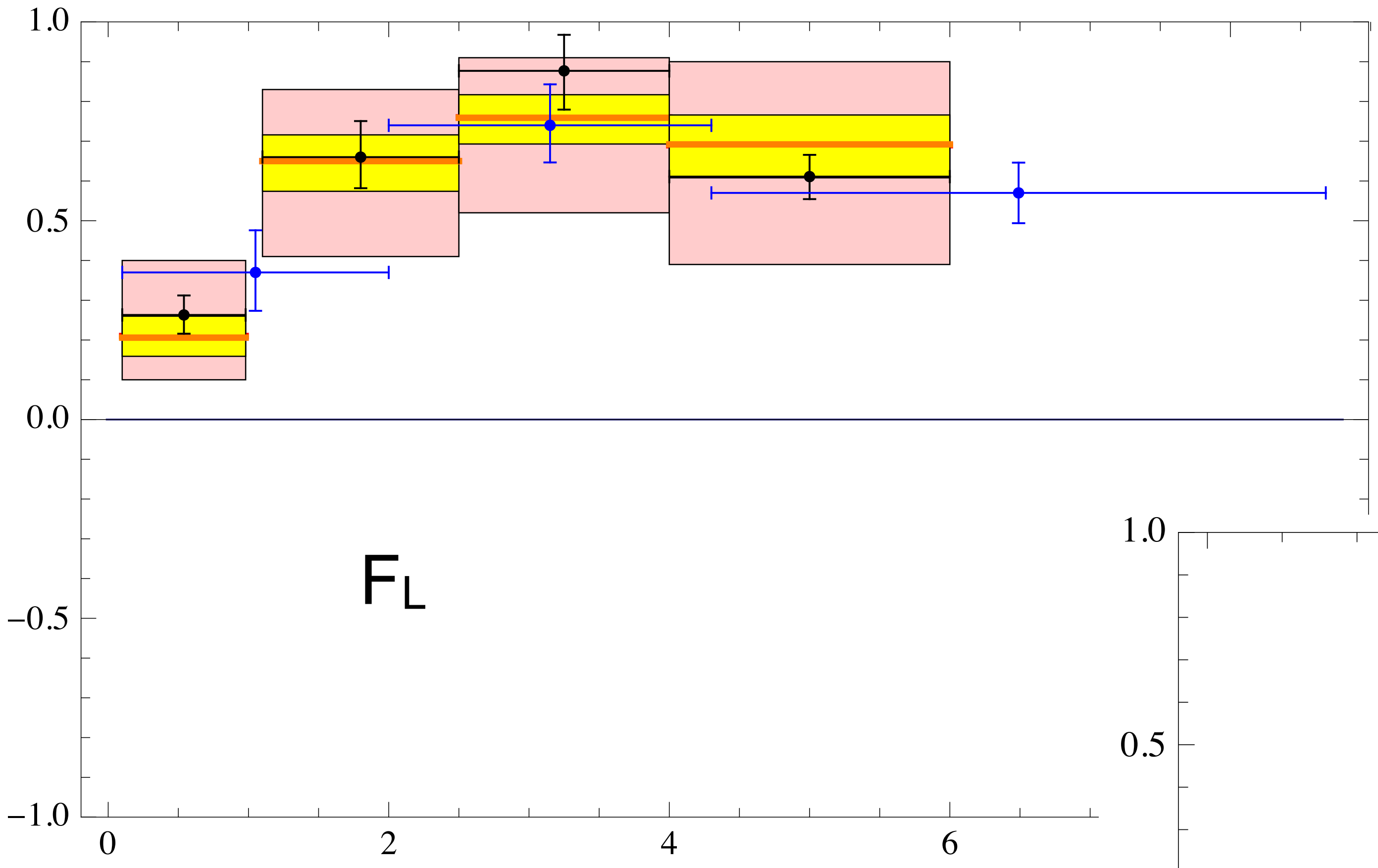
Pure heavy-quark limit (!) matches data. Even at central values nothing of significance.

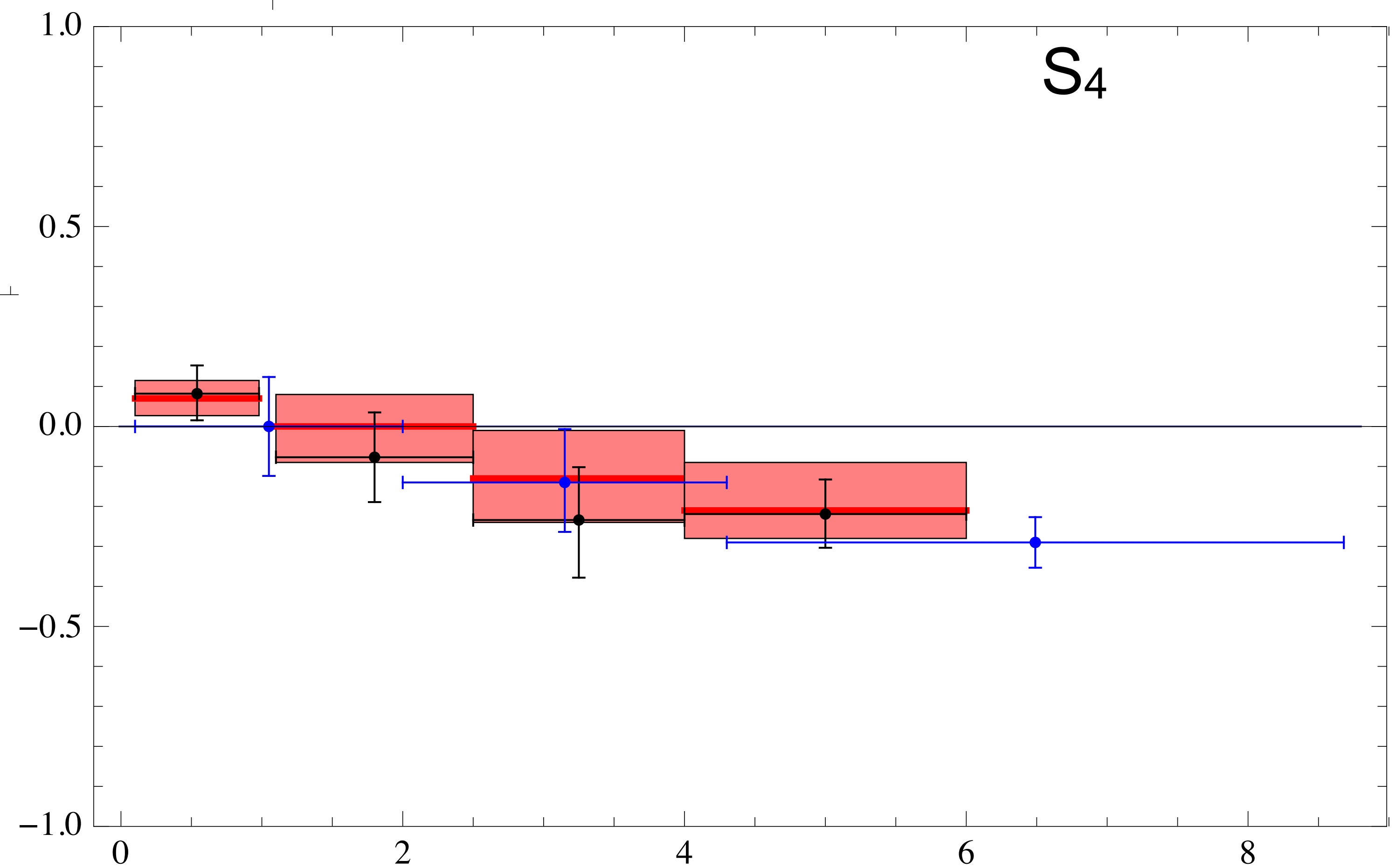
Data almost spot on our predictions -
cannot confirm systematic downward shift claimed by LHCb.

Similar conclusions F_L and S_4.

F_L and S_4

SJ, Martin Camalich, preliminary





“Null tests” S_3 not yet analysed with new data; A_9 no update by LHCb yet. Would be very useful!

Comparison to LCSR

LCSR relate nonperturbative objects to other nonperturbative objects (decay constants),

which are then taken from data or from further sum rules for phenomenology.

From a phenomenological perspective, in my view:

- advantage of LCSR: more nonperturbative objects are accessible

- price: No small parameter controlling the modelling uncertainties introduced (primarily through continuum subtractions)

Besides, there are technical issues such as establishing short distance dominance, and **everything that can be calculated is calculated perturbatively**. All this works quite similarly in LCSR and in QCDF.

Complementarity of LCSR, QCDF

A complementary approach will take advantage of both

- the model-independence of the leading-power QCDF results ie the power suppression of irreducible uncertainties

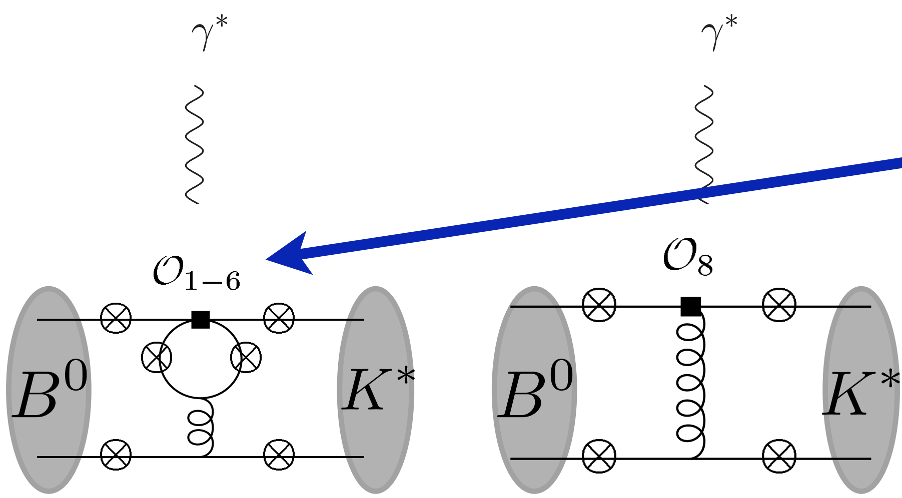
- the ability of LCSR to access quantities that do not factorise

In other words, LCSR should ideally focus on estimating power suppressed terms only, where only a modest relative uncertainty is required (and even O(1) may be enough).

Important difficulty: avoid double counting. Need to establish correspondence of LCSR results to particular power corrections

Example: size of the helicity-+ amplitude, crucial for the sensitivity to right-handed dipole transitions

Charm loop



includes Q_1^c, Q_2^c - large Wilson coefficients

\+ "vertex corrections" + annihilation

Beneke, Feldmann, Seidel 2001

leading-power: factorises into perturbative kernels, form factors, meson light-cone distribution amplitudes (including hard/hard-collinear gluon corrections to all orders)

$$h_\lambda = \int_0^1 du \phi_K^*(u) T(u, \alpha_s) + \mathcal{O}(\Lambda/m_b)$$

α_s^0 : $C_7 \rightarrow C_7^{eff}$ $C_9 \rightarrow C_9^{eff}(q^2)$ + 1 annihilation diagram

α_s^1 : (convergent) convolutions of hard- scattering kernels with meson LCDA

unambigous (save for parametric uncertainties)
state-of-the-art in phenomenology

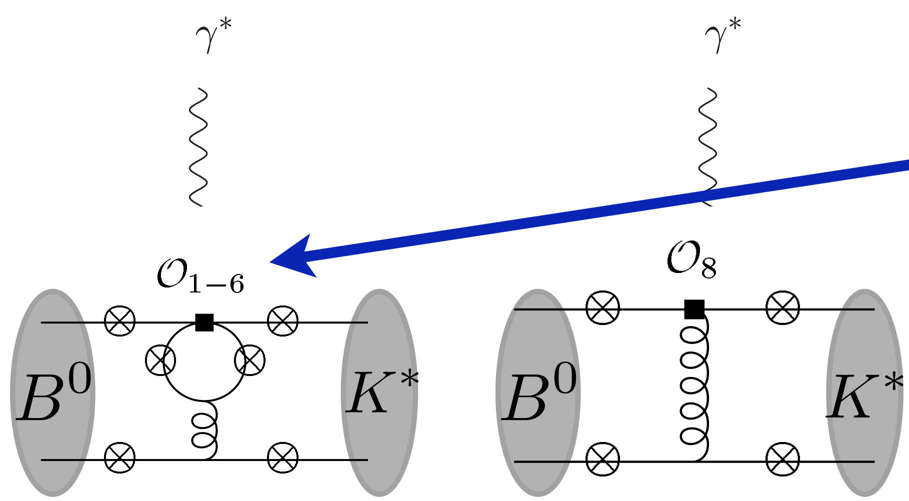
at subleading powers: breakdown of factorisation
some contributions have been estimated as end-point divergent convolutions with a cut-off

Kagan&Neubert 2001, Feldmann&Matias 2002

LCSR computation finds effective shifts of transversity amplitudes as large as ~10%

Khodjamirian, Mannel, Pivovarov, Wang 2010

Nonlocal terms: power corrections



includes Q_1^c , Q_2^c - large Wilson coefficients

+ “vertex corrections” + annihilation

Beneke, Feldmann, Seidel 2001

subleading power: breakdown of factorisation. Schematically for Q_1^c, Q_2^c :

$$r_\lambda^c = \int_{\Lambda_h}^1 du \phi_K^*(u) T(u, \alpha_s) + r_{\lambda,\text{soft}}^c$$

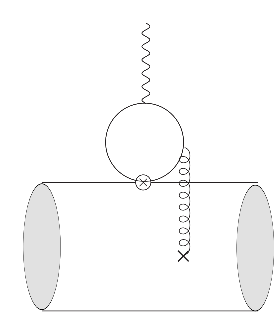
1) power corrections from: (i) higher-twist 2-particle LCDA; (ii) multi-particle LCDA, and from soft endpoint region (iii)

2) some endpoint-divergent contributions from hard-collinear gluon exchanges;

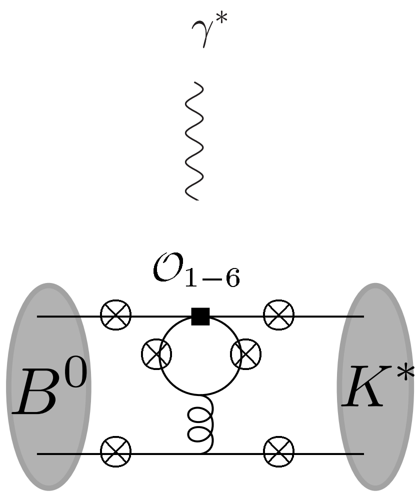
Kagan&Neubert 2001, Feldmann&Matias 2002

3) need to allow for “soft” remainder even if endpoint convergent: means only that endpoint region is power suppressed relative to “bulk” region!

4) In endpoint region hard-collinear gluon becomes soft

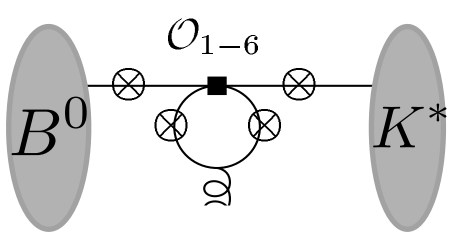


Long-distance “charm loop”



$$r^c_\lambda = \int_{\Lambda_h}^1 du \phi^*_K(u) T(u, \alpha_s) + r^c_{\lambda,\mathrm{soft}}$$

Q_1^c, Q_2^c insertions with hard-collinear gluon(s):
cannot generate λ=+ (left-handed strange quark) with two-particle LCDA
multi-particle LCDA contributions suppressed by extra α_s



Q_1^c, Q_2^c insertions with soft gluon: can still integrate out charm, but not the gluons   Grinstein, Grossmann, Ligeti, Pirjol 2004

for single soft gluon the two gluon attachments to the charm line give

$$r^c_{\lambda,\mathrm{soft}} = \epsilon^{\mu *}(\lambda) \langle M(k,\lambda)|\tilde{\mathcal{O}}_\mu|\bar{B}\rangle$$

where the light-cone operator (in notation of Khodjamirian, Mannel, Pivovarov, Wang 2010)

$$\tilde{\mathcal{O}}_\mu = \int d\omega I_{\mu\rho\alpha\beta}(q,\omega) \bar{s}_L \gamma^\rho \delta\Big(\omega - \frac{in_+ \cdot D}{2}\Big) \tilde{G}^{\alpha\beta} b_L$$

(corresponds to the two photon attachments to the charm loop, treating $\Lambda^2/(4\ m_c^2) \sim \Lambda/m_b$)

matrix element power counting: $\Lambda^2/(4\ m_c^2) \sim \Lambda/m_b$ per soft gluon   Khodjamirian et al 2010

power suppression as expected from heavy-quark power counting!
no double counting! - but 4 more photon attachments

Helicity hierarchies survive!

- LCSR helicity amplitudes

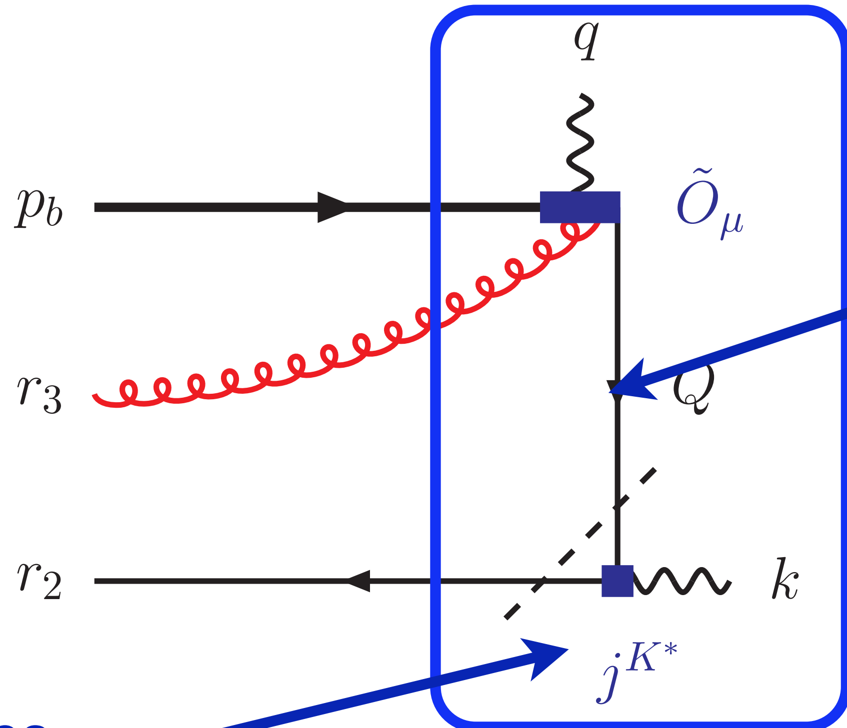
SJ, Martin Camalich 2012
(also for helicity-+ form factors!)

$$G_{h\lambda}(q^2;k^2) = -i\int d^4y e^{iky}\langle 0|T\{\epsilon^{\nu *}(\hat{z};\lambda) j_\nu^{K^*}(y)\epsilon^{\mu *}(-\hat{z};\lambda)\tilde{O}_\mu(0)\}|B\rangle$$

This has a hadronic representation containing the desired matrix elements

$$G_{h\lambda}(q^2;k^2) = \frac{f_{K^*\|}\, m_{K^*}}{m_{K^*}^2 - k^2}\langle K^*(\tilde{k};\lambda)|\epsilon^{\mu *}(-\hat{z};\lambda)\tilde{O}_\mu(0)|B\rangle \;+\text{continuum contributions.}$$

based on Khodjamirian et al 2010



for k^2 ~ - 1 GeV^2
this line is hard-collinear
(numerically only - no heavy-quark expansion!)

integrate out
(standard
LCSR step)

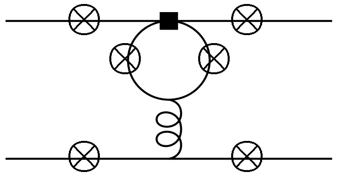
operator defining 3-particle
B-meson LCDA

key: project out helicities
through interpolating current

vanishes for + helicity, up to higher power of Λ/m_b

SJ, Martin Camalich 2012

(

1) further photon attachments: 

attachments to b or s quark quite local operator; simpler argument;
again helicity hierarchy

attachments to spectator lines should give nonlocal operator product of [s G b] operator and light-quark part of em current.
However as photon always hard, soft-gluon exchange appears kinematically impossible (more detailed investigation desirable)

2) earlier estimates of **long-distance** effecs in $h_\lambda(0)$

SCET-based Grinstein, Grossman, Ligeti, Pirjol 2004

identify $SCET_I$ operator ~ $\tilde{O}_\mu$
only power counting estimate of matrix element, misses helicity hierarchy (cannot match onto $SCET_{II}$ b/c endpoint divergences)

LCSR-based Ball, Jones, Zwicky 2006 (also Muheim, Xie, Zwicky 2008)

derive sum rule with external K* external (instead of B)
- does not single out the soft (endpoint) configuration
- moreover expand a light-cone operator in local operators; but the neglected higher-dimensional matrix elements scale like $m_B^2/(4\ m_c^2)$: not justified!
(different from somewhat analogous B -> X_s gamma case)

)

Conclusions

QCD factorisation

- is a well established consequence of the heavy-quark limit for many B decays

- allows partial calculations of many observables in the heavy-quark limit, providing an unambigous reference point.

- depends on certain nonperturbative normalisations and suffers from some incalculable power corrections

For B->Vll, predictions for eg the forward-backward zero crossing have been stable for many years, because of a very weak sensitivity to residual nonperturbative inputs.

There seems to be no significant effect in a bin-by-bin analysis of 2015 LHCb data, unless power corrections are such that they introduce a significance effect (which then requires new physics).

More complementarity between QCDF and LCSR would be desirable. Demonstrated this in detail for the case of the charm loop in the helicity+ amplitude, crucial for searches for right-handed currents.

BACKUP

Heavy-quark limit and corrections

At most 1-2% over entire 0..6 GeV^2 range -> ignore

$$F(q^2) = F^\infty(q^2) + a_F + b_F q^2/m_B^2 + \mathcal{O}([q^2/m_B^2]^2)$$

heavy quark limit

Power corrections - parameterise

$$F^\infty(q^2) = F^\infty(0)/(1 - q^2/m_B^2)^p + \Delta_F(\alpha_s; q^2)$$

(Charles et al)

(Beneke, Feldmann)

q^2 dependence in heavy-quark limit not known (model by a power p, and/or a pole model)

$V_+^\infty(0) = 0$ $T_+^\infty(0)=0$

$V_-^\infty(0) = T_-^\infty(0)$

$V_0^\infty(0) = T_0^\infty(0)$

from heavy-quark/ large energy symmetry

Corrections are calculable in terms of perturbation theory, decay constants, light cone distribution amplitudes

$V_+^\infty(q^2) = 0$ $T_+^\infty(q^2)=0$

hence

$$T_+(q^2) = \mathcal{O}(q^2) \times \mathcal{O}(\Lambda/m_b)$$
$$V_+(q^2) = \mathcal{O}(\Lambda/m_b).$$

- **"naively factorizing" part of the helicity amplitudes $H_{V,A}^+$ strongly suppressed** as a consequence of chiral SM weak interactions
- We see the suppression is **particularly strong** near low-q^2 endpoint
- Form factor relations imply reduced uncertainties in suitable observables

Burdman, Hiller 1999 (quark picture)

Beneke, Feldmann, Seidel 2001 (QCDF)

"Clean" angular observables

Useful to consider functions of the angular coefficients for which form factors drop out in the heavy quark limit if perturbative QCD corrections neglected.

Becirevic, Schneider 2011
Matias, Mescia, Ramon, Virto 2012
Descotes-Genon et al 2012
(also Krueger,Matias 2005; Egede et al 2008)

E.g.

neglecting strong phase differences
[tiny; take into account in numerics]

$$P_1 \equiv \frac{I_3 + \bar{I}_3}{2(I_{2s} + \bar{I}_{2s})} = \frac{-2\,\mathrm{Re}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2} = 0$$

$$P_3^{CP} \equiv -\frac{I_9 - \bar{I}_9}{4(I_{2s} + \bar{I}_{2s})} = -\frac{\mathrm{Im}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2} = 0$$

(Melikhov 1998)
Krueger, Matias 2002
Lunghi, Matias 2006
Becirevic, Schneider 2011

$$P_5' = \frac{\mathrm{Re}[(H_V^- - H_V^+)H_A^{0*} + (H_A^- - H_A^+)H_V^{0*}]}{\sqrt{(|H_V^0|^2 + |H_A^0|^2)(|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2)}} = \frac{C_{10}\left(C_{9,\perp} + C_{9,\parallel}\right)}{\sqrt{(C_{9,\parallel}^2 + C_{10}^2)(C_{9,\perp}^2 + C_{10}^2)}}$$

in SM, neglecting power corrections and pert. QCD corrections

where $C_{9,\perp} = C_9^{\mathrm{eff}}(q^2) + \frac{2\, m_b\, m_B}{q^2} C_7^{\mathrm{eff}}$

$C_{9,\parallel} = C_9^{\mathrm{eff}}(q^2) + \frac{2\, m_b\, E}{q^2} C_7^{\mathrm{eff}}$

C_7 and C_9 opposite sign
Interference maximal near zero-crossing
enhances vulnerability to anything that violates the large-energy form factor relations

much more relevant to P_5' (and others) than to P_1 or P_3^{CP}

Power corrections: analytical

SJ, Martin Camalich 1412.3183

Compare

$$P_5' = P_5'|_\infty \Bigg(1 + \frac{a_{V_-} - a_{T_-}}{\xi_\perp}\frac{m_B}{|\vec{k}|}\frac{m_B^2}{q^2} C_7^{\rm eff}\frac{C_{9,\perp}C_{9,\parallel} - C_{10}^2}{(C_{9,\perp}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\parallel})}$$
$$+\frac{a_{V_0} - a_{T_0}}{\xi_\parallel}\, 2\, C_7^{\rm eff}\frac{C_{9,\perp}C_{9,\parallel} - C_{10}^2}{(C_{9,\parallel}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\parallel})}$$
$$+ 8\pi^2\frac{\tilde{h}_-}{\xi_\perp}\frac{m_B}{|\vec{k}|}\frac{m_B^2}{q^2}\frac{C_{9,\perp}C_{9,\parallel} - C_{10}^2}{C_{9,\perp} + C_{9,\parallel}} + \text{further terms}\Bigg) + \mathcal{O}(\Lambda^2/m_B^2)$$

(truncated after 3 out of 11 independent power-correction terms!)
also, dependence on soft form factors reappears at PC level

and

$$P_1 = \frac{1}{C_{9,\perp}^2 + C_{10}^2}\frac{m_B}{|\vec{k}|}\Bigg(-\frac{a_{T_+}}{\xi_\perp}\frac{2\,m_B^2}{q^2}C_7^{\rm eff}C_{9,\perp} - \frac{a_{V_+}}{\xi_\perp}(C_{9,\perp}C_9^{\rm eff} + C_{10}^2) - \frac{b_{T_+}}{\xi_\perp}2C_7^{\rm eff}C_{9,\perp}$$
$$-\frac{b_{V_+}}{\xi_\perp}\frac{q^2}{m_B^2}(C_{9,\perp}C_9^{\rm eff} + C_{10}^2) + 16\pi^2\frac{h_+}{\xi_\perp}\frac{m_B^2}{q^2}C_{9,\perp}\Bigg) + \mathcal{O}(\Lambda^2/m_B^2).$$

(complete expression)

Further notice that a_{T+} vanishes as $q^2 \to 0$, h_+ helicity suppressed, and the other three terms lacks the photon pole.

Hence P_1 **much** cleaner than P_5', especially at very low q^2

Probing right-handed currents

Extending to BSM Wilson coefficients, have

neglecting strong phase differences
[tiny; take into account in numerics]

close to $q^2 = 0$ (photon
pole dominance)

$$P_1 \equiv \frac{I_3 + \bar{I}_3}{2(I_{2s} + \bar{I}_{2s})} = \frac{-2\,\mathrm{Re}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2} \approx 2\frac{\mathrm{Re}(C_7 C_7'^*)}{|C_7|^2 + |C_7'|^2}$$

$$P_3^{CP} \equiv -\frac{I_9 - \bar{I}_9}{4(I_{2s} + \bar{I}_{2s})} = -\frac{\mathrm{Im}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2} \approx \frac{\mathrm{Im}(C_7 C_7'^*)}{|C_7|^2 + |C_7'|^2}$$

- recall **double** suppression of T_+ at (very) low q^2

- extends to the long-distance contribution to H_V^+
(discussed in great detail in 1212.2264 and 1412.3183)

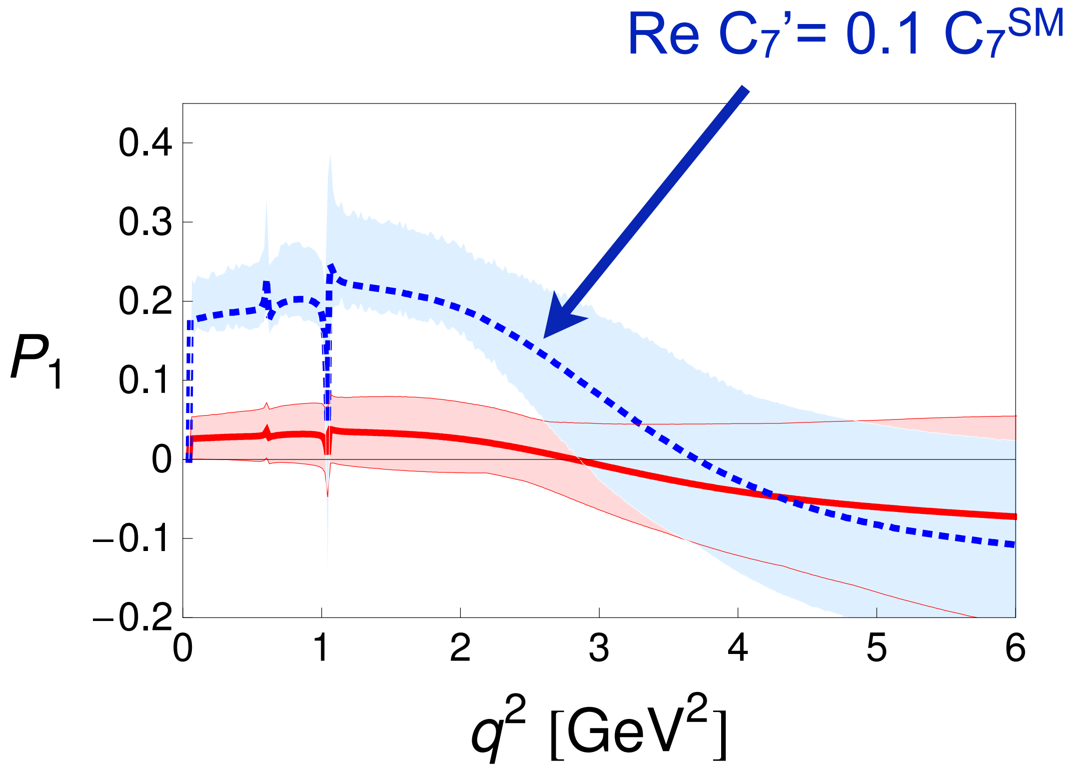
so very small nonperturbative QCD corrections to right-hand side

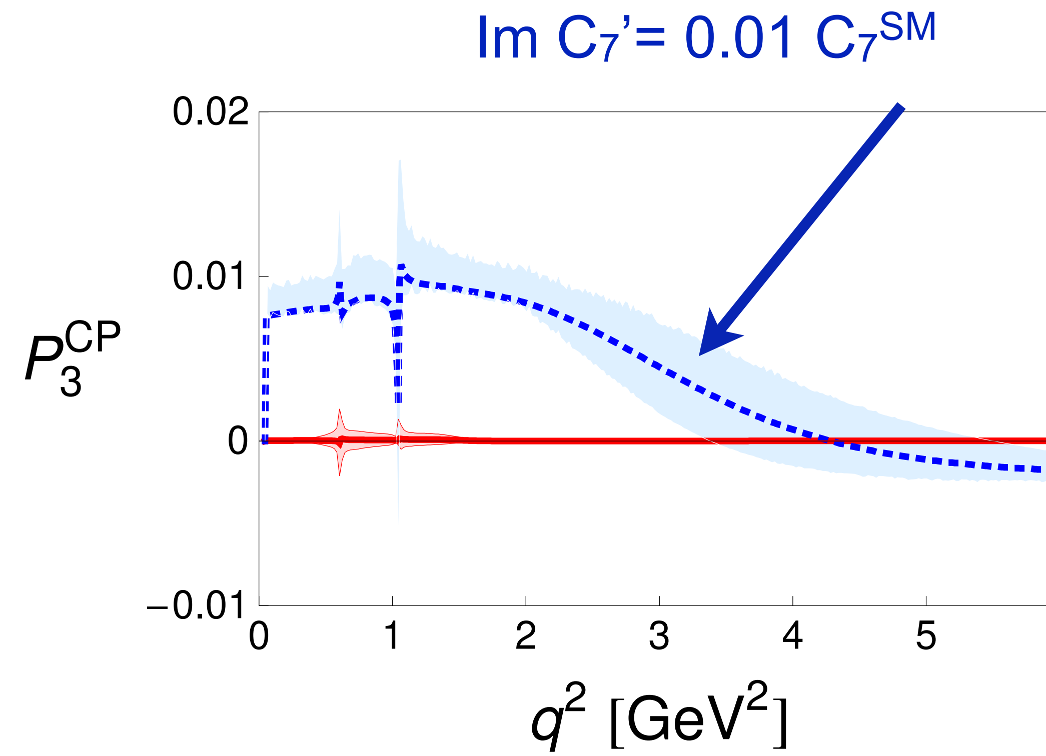
also, B->K* gamma is described in terms of the same λ=+/- 1
helicity amplitudes

$$\mathcal{A}(\bar{B} \to V(\lambda)\gamma(\lambda)) = \lim_{q^2 \to 0} \frac{q^2}{e} H_V(q^2 = 0; \lambda) \quad \text{exact (LSZ)}$$

$$= \frac{iN m_B^2}{e}\left[\frac{2\hat{m}_b}{m_B}(C_7 \tilde{T}_\lambda(0) - C_7' \tilde{T}_{-\lambda})(0) - 16\pi^2 h_\lambda(q^2 = 0)\right]$$

Sensitivity to C_7'(muonic mode)





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Two angular observables remain clean null tests of the SM in the presence of long-distance corrections

(theoretical limit on) sensitivity to Re C_7' at <10% (C_7^{SM}) level, to Im C_7' at <1%

sensitivity stems from $q^2 \in [0.1, 2]$ GeV^2

Predictions for electronic mode

$Br\ [10^{-8}]$	F_L	P_1	P_2	$P_3^{CP}\ [10^{-4}]$	P_4'	P_5'	P_6'	P_8'
26^{+12}_{-9}	$0.10^{+0.11}_{-0.05}$	$0.030^{+0.047}_{-0.044}$	$-0.073^{+0.020}_{-0.016}$	$0.1^{+0.6}_{-0.6}$	$0.18^{+0.06}_{-0.08}$	$0.55^{+0.11}_{-0.12}$	$0.06^{+0.07}_{-0.07}$	$0.01^{+0.09}_{-0.09}$

SJ, Martin Camalich
1412.3183

"Effective" bin $[0.0020^{+0.0008}_{-0.0008}, 1.12^{+0.06}_{-0.06}]$ to deal with acceptance issues (negligible impact on theory error)

Theoretically even cleaner than muonic mode at very low q^2 as tensor form factor / photon pole dominates more

Boost in BR due to lower q^2_{min}

for C_7' sensitivity, offsets disadvantages at LHCb

B→K*l l: angular distribution

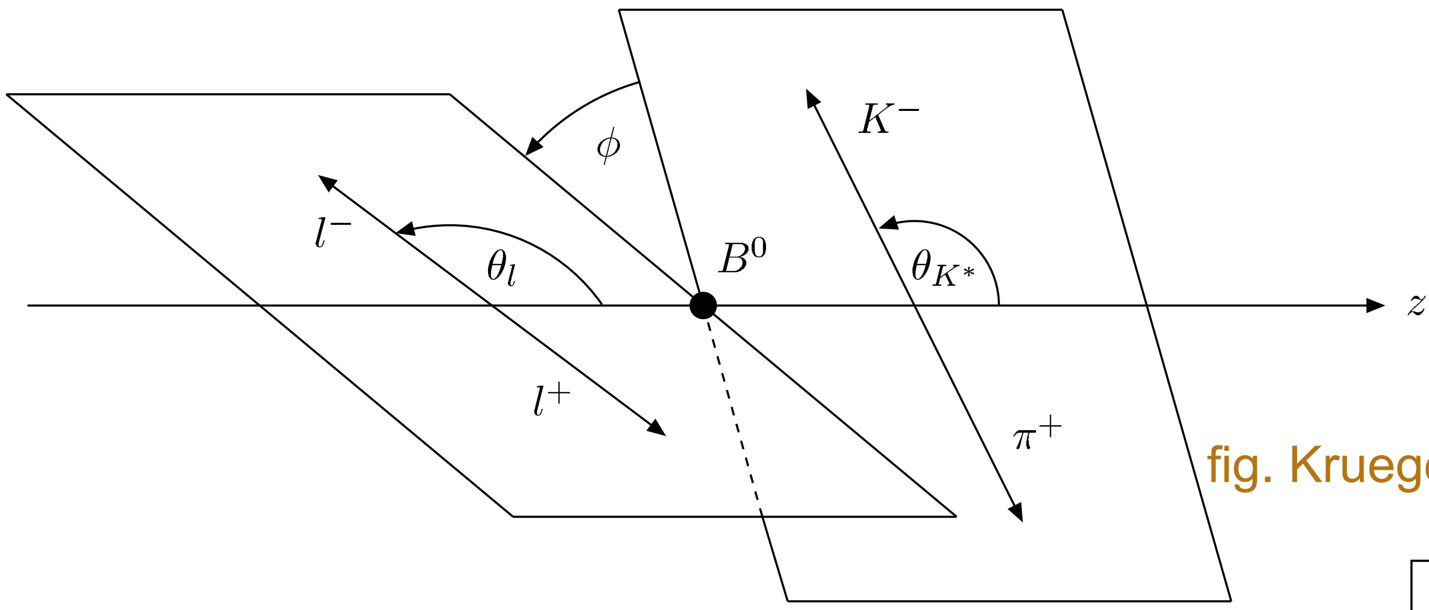

fig. Krueger, Matias 2002

θ$_K$ in K* rest frame

θ$_l$ in dilepton cm frame

ϕ boost-invariant (w.r.t. z axis)

Expt.	~# events
CDF	100 PRL106(2011)161801
BaBar	150 PRD86(2012)032012
Belle	200 PRL103(2009)171801
CMS	400 PLB727(2013)77
ATLAS	500 arXiv:1310.4213
LHCb (μ)	1000 (1 fb^{-1}) JHEP 1308 (2013) 131
LHCb (e)	128 ([0.0004, 1] GeV^2) M Borsato (LHCb)

$$\frac{d^{(4)}\Gamma}{dq^2\, d(\cos\theta_l)d(\cos\theta_k)d\phi} = \frac{9}{32\,\pi}$$

$$\times\Big(I_1^s \sin^2\theta_k + I_1^c \cos^2\theta_k + (I_2^s \sin^2\theta_k + I_2^c \cos^2\theta_k)\cos 2\theta_l$$
$$+ I_3 \sin^2\theta_k \sin^2\theta_l \cos 2\phi + I_4 \sin 2\theta_k \sin 2\theta_l \cos\phi$$
$$+ I_5 \sin 2\theta_k \sin\theta_l \cos\phi + (I_6^s \sin^2\theta_k + I_6^c \cos^2\theta_K)\cos\theta_l$$
$$+ I_7 \sin 2\theta_k \sin\theta_l \sin\phi + I_8 \sin 2\theta_k \sin 2\theta_l \sin\phi + I_9 \sin^2\theta_k \sin^2\theta_l \sin 2\phi\Big)$$

Each angular coefficient is a function of **Wilson coefficients** incorporating the weak interactions and any BSM effects, and of the dilepton invariant mass q^2

This can be used to probe for new physics in various bins

Form factors

Helicity amplitudes naturally involve helicity form factors

$$-\, i m_B \tilde{V}_{L(R)\lambda}(q^2) = \langle M(\lambda)|\bar{s}\not{\epsilon}^*(\lambda) P_{L(R)} b|\bar{B}\rangle,$$
$$m_B^2 \tilde{T}_{L(R)\lambda}(q^2) = \epsilon^{*\mu}(\lambda) q^\nu \langle M(\lambda)|\bar{s}\sigma_{\mu\nu} P_{R(L)} b|\bar{B}\rangle$$
$$i m_B \tilde{S}_{L(R)}(q^2) = \langle M(\lambda=0)|\bar{s} P_{R(L)} b|\bar{B}\rangle.$$

~ Bharucha et al 2010

(& rescale helicity-0 form factors by kinematic factor.)
Can be expressed in terms of traditional "transversity" FFs

$$V_\pm(q^2) = \frac{1}{2}\left[\left(1+\frac{m_V}{m_B}\right)A_1(q^2) \mp \frac{\lambda^{1/2}}{m_B(m_B+m_V)}V(q^2)\right],$$
$$V_0(q^2) = \frac{1}{2m_V\lambda^{1/2}(m_B+m_V)}\left[(m_B+m_V)^2(m_B^2-q^2-m_V^2)A_1(q^2) - \lambda A_2(q^2)\right]$$
$$T_\pm(q^2) = \frac{m_B^2-m_V^2}{2m_B^2}T_2(q^2) \mp \frac{\lambda^{1/2}}{2m_B^2}T_1(q^2),$$
$$T_0(q^2) = \frac{m_B}{2\,m_V\lambda^{1/2}}\left[(m_B^2+3m_V^2-q^2)T_2(q^2) - \frac{\lambda}{(m_B^2-m_V^2)}T_3(q^2)\right],$$
$$S(q^2) = A_0(q^2),$$

The form factors satisfy two exact relations:

$$T_+(q^2=0) = 0,$$
$$S(q^2=0) = V_0(0)$$

note - M can be multiparticle state. Eg for a two-pseudoscalar state

$$\tilde{V}_{L\lambda} = -\eta(-1)^L \tilde{V}_{R,-\lambda} \equiv \tilde{V}_\lambda,$$
$$\tilde{T}_{L\lambda} = -\eta(-1)^L \tilde{T}_{R,-\lambda} \equiv \tilde{T}_\lambda,$$
$$\tilde{S}_L = -\eta(-1)^L \tilde{S}_R \equiv \tilde{S},$$

L = angular momentum
η = intrinsic parity
+ invariant mass dependence

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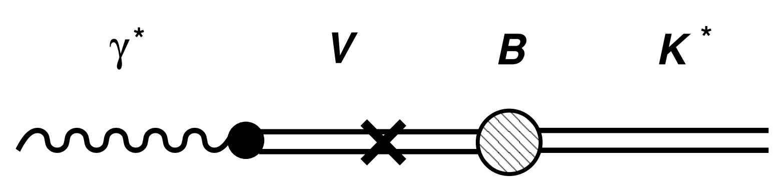
Light-quark contributions

Operators without charm have strong charm or CKM suppression; power corrections should be negligible.

However, they generate (mild) resonance structure even below the charm threshold, presumably "duality violation"
Presumably ρ,ω,φ most important; use vector meson dominance supplemented by heavy-quark limit B➔VK* amplitudes

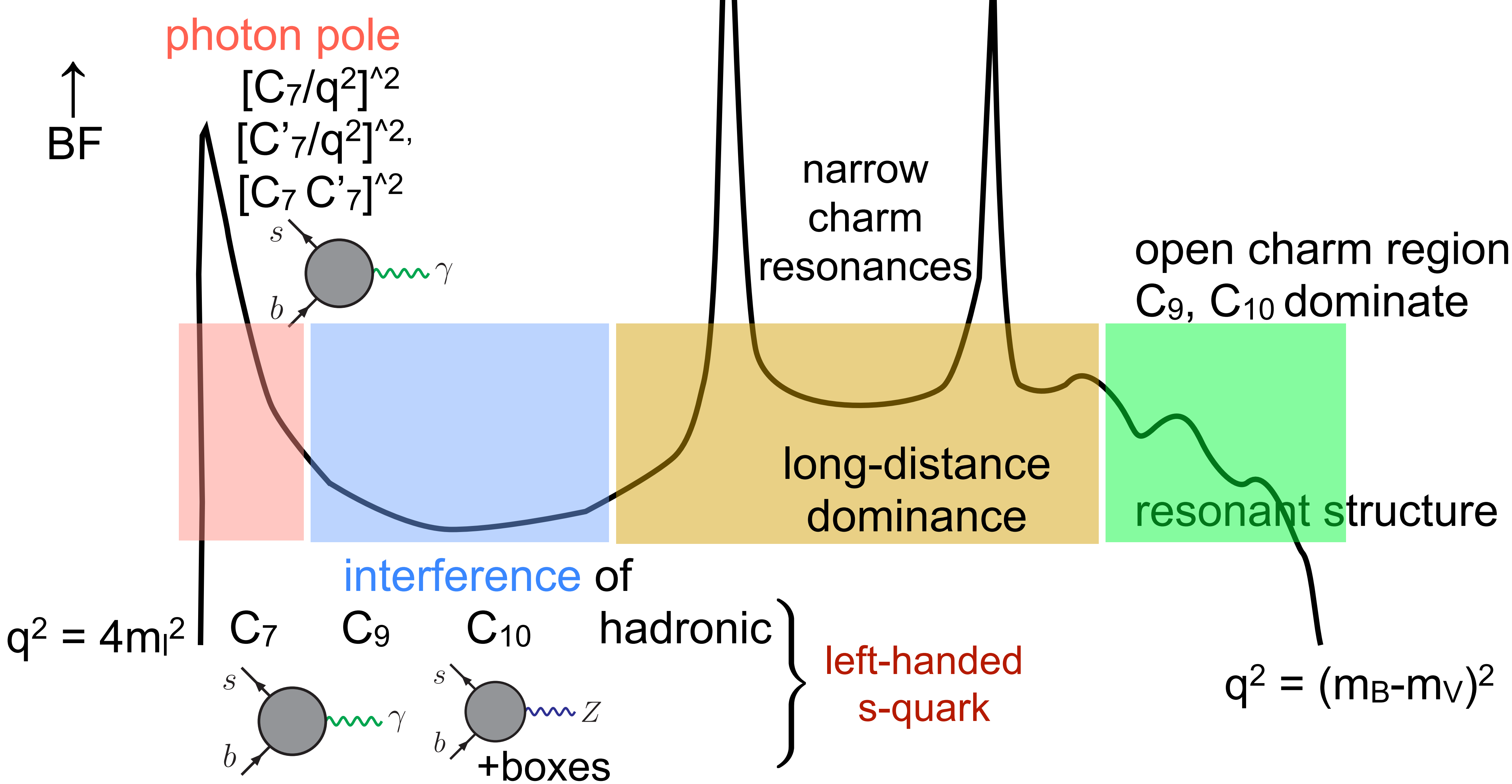


$$\tilde{a}_\mu^{\rm had,\,lq} = \int d^4x\, e^{-iq\cdot x} \sum_{P,P'} \langle 0|j_\mu^{\rm em,lq}(x)|P'\rangle\langle P'(x)|P(0)\rangle\langle \bar{K}^* P|\,\mathcal{H}_{\rm eff}^{\rm had}(0)|\bar{B}\rangle$$

estimate **uncertainty** from difference between VMD model and the subset of heavy-quark limit diagrams corresponding to intermediate V states.

Helicity hierarchies in **hadronic** B decays prevent large uncertainties in H_V^+ from this source, too.

Rate: q^2 dependence (qualitative)



BSM only:
C'_7 C'_9 C'_{10} (hadronic)
(may involve Z' etc)

right-handed s-quark **suppressed** in SM, including long-distance

"low q^2 / large recoil" "high q^2 / low recoil"