Global fits to $bs\ell\ell$ data

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Rare B decays in 2015 - experiment and theory
Higgs Centre for Theoretical Physics, Edinburgh

11-13 May 2015

Inclusive decays

- $B \to X_s \gamma$
  Improved theory calculations (Misiak et al. 1503.01789)
  Excellent agreement with the measurements
- $B \to X_s \ell^+ \ell^-$
  Still waiting for the final words from Belle and Babar!
  High expectation from Belle II!

Exclusive decays

- $B \to K^* \gamma$
- First measurements of $B_s \to \mu^+ \mu^-$
- Angular distributions of $B \to K^* \mu^+ \mu^-$
  $\to$ large variety of experimentally accessible observables
- Also: $B \to K \mu^+ \mu^-$ and $B_s \to \phi \mu^+ \mu^-$

Issue of hadronic uncertainties in exclusive modes

Differential decay distribution:

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_V\, d\phi} = \frac{9}{32\pi} J(q^2, \theta_\ell, \theta_V, \phi)$$

$J(q^2, \theta_\ell, \theta_V, \phi) = \sum_i J_i(q^2)\, f_i(\theta_\ell, \theta_V, \phi)$

↘ angular coefficients J_{1-9}

↘ functions of the spin amplitudes A_0, $A_\parallel$, $A_\perp$, A_t, and A_S

Spin amplitudes: functions of Wilson coefficients and form factors

Standard Observables:

Dilepton invariant mass spectrum: $\frac{d\Gamma}{dq^2} = \frac{3}{4}\left(J_1 - \frac{J_2}{3}\right)$

Forward backward asymmetry:

$$A_{\rm FB}(q^2) \equiv \left[\int_{-1}^{0} - \int_{0}^{1}\right] d\cos\theta_l\, \frac{d^2\Gamma}{dq^2\, d\cos\theta_l} \bigg/ \frac{d\Gamma}{dq^2} = \frac{3}{8} J_6 \bigg/ \frac{d\Gamma}{dq^2}$$

Forward backward asymmetry zero-crossing: $q_0^2 \simeq -2 m_b m_B \frac{C_9^{\rm eff}(q_0^2)}{C_7} + O(\alpha_s, \Lambda/m_b)$

Polarization fraction: $F_L(q^2) = \frac{|A_0|^2}{|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2}$,

Optimised observables: form factor uncertainties cancel at leading order

$$\langle P_1\rangle_{\rm bin} = \frac{1}{2}\frac{\int_{\rm bin} dq^2[J_3+\bar{J}_3]}{\int_{\rm bin} dq^2[J_{2s}+\bar{J}_{2s}]} \qquad \langle P_2\rangle_{\rm bin} = \frac{1}{8}\frac{\int_{\rm bin} dq^2[J_{6s}+\bar{J}_{6s}]}{\int_{\rm bin} dq^2[J_{2s}+\bar{J}_{2s}]}$$

$$\langle P'_4\rangle_{\rm bin} = \frac{1}{\mathcal{N}'_{\rm bin}}\int_{\rm bin} dq^2[J_4+\bar{J}_4] \qquad \langle P'_5\rangle_{\rm bin} = \frac{1}{2\mathcal{N}'_{\rm bin}}\int_{\rm bin} dq^2[J_5+\bar{J}_5]$$

$$\langle P'_6\rangle_{\rm bin} = \frac{-1}{2\mathcal{N}'_{\rm bin}}\int_{\rm bin} dq^2[J_7+\bar{J}_7] \qquad \langle P'_8\rangle_{\rm bin} = \frac{-1}{\mathcal{N}'_{\rm bin}}\int_{\rm bin} dq^2[J_8+\bar{J}_8]$$

with

$$\mathcal{N}'_{\rm bin} = \sqrt{-\int_{\rm bin} dq^2[J_{2s}+\bar{J}_{2s}]\int_{\rm bin} dq^2[J_{2c}+\bar{J}_{2c}]}$$

+ CP violating clean observables and other combinations

U. Egede et al., JHEP 0811 (2008) 032, JHEP 1010 (2010) 056

J. Matias et al., JHEP 1204 (2012) 104

S. Descotes-Genon et al., JHEP 1305 (2013) 137

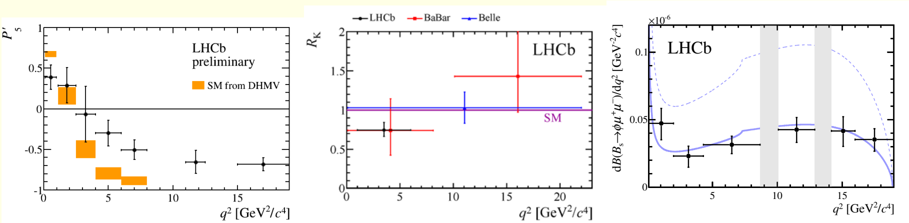
Or alternatively:

$$S_i = \frac{J_{i(s,c)} + \bar{J}_{i(s,c)}}{\frac{d\Gamma}{dq^2} + \frac{d\bar{\Gamma}}{dq^2}}$$

W. Altmannshofer, P. Ball, A. Bharucha, A.J. Buras, D.M. Straub, M. Wick, JHEP 0901 (2009) 019

3 main LHCb anomalies:

- P_5'
- R_K
- $\mathrm{BR}(B_s \to \phi\mu^+\mu^-)$



Possible explanations:

- Statistical fluctuations
- Theoretical issues
- New Physics!

Global analysis of the latest LHCb data

Relevant $\mathcal{O}$perators:

$$\mathcal{O}_7,\ \mathcal{O}_8,\ \mathcal{O}_{9\mu,e}^{(\prime)},\ \mathcal{O}_{10\mu,e}^{(\prime)} \quad \text{and} \quad \mathcal{O}_{S-P} \propto (\bar{s}P_R b)(\bar{\mu}P_L\mu) \equiv \mathcal{O}_0^\prime$$

NP manifests itself in the shifts of the individual coefficients with respect to the SM values:

$$C_i(\mu) = C_i^{\rm SM}(\mu) + \delta C_i$$

→ Scans over the values of δC_i

→ Calculation of flavour observables

→ Comparison with experimental results

→ Constraints on the Wilson coefficients C_i

Evaluations uncertainties and correlations:

- Experimental errors and correlations
  - 3 fb^{-1} LHCb data for $B \to K^{*0}\mu^+\mu^-$: provided in LHCb-CONF-2015-002
- Theoretical uncertainties and correlations
  - study of more than 100 observables
    (at a later stage, selection of the relevant observables for each fit)
  - Monte Carlo analysis
  - variation of the "standard" input parameters: masses, scales, CKM, ...
  - for $B_s \to \phi\mu^+\mu^-$, mixing effects taken into account
  - decay constants taken from the latest lattice results
  - use for the $B_{(s)} \to V$ form factors of the lattice+LCSR combinations from 1503.05534, including correlations (Cholesky decomposition method)
  - use for the $B \to K$ form factors of the lattice+LCSR combinations from 1411.3161, including correlations
  - two approaches for the exclusive decays: soft form factors, full form factors
  - two sets of hypotheses for the uncertainties associated to the factorisable and non-factorisable power corrections

$\Rightarrow$ Computation of a (theory + exp) correlation matrix

Global fits

For the exclusive semi-leptonic decays, two approaches and two evaluations of the uncertainties for each decay.

At **low** q^2:

- Soft form factor approach

  Uncertainties of the factorisable and non-factorisable corrections parametrised as

$$A_k \to A_k \left(1 + a_k \exp(i\phi_k) + \frac{q^2}{6 \text{ GeV}^2} b_k \exp(i\theta_k)\right)$$

  where A_k are the helicity amplitudes.

$$a_k \text{ in } [-10\%, +10\%] \text{ or } [-20\%, +20\%] \qquad \phi_k, \theta_k \text{ in } [-\pi, +\pi]$$
$$b_k \text{ in } [-25\%, +25\%] \text{ or } [-50\%, +50\%]$$

- Full form factor approach

  Uncertainties of the **non-factorisable power corrections only** parametrised in a similar way:

$$a_k \text{ in } [-5\%, +5\%] \text{ or } [-10\%, +10\%] \qquad \phi_k, \theta_k \text{ in } [-\pi, +\pi]$$
$$b_k \text{ in } [-10\%, +10\%] \text{ or } [-25\%, +25\%]$$

At **high** q^2, uncertainties parametrised as

$$A_k \to A_k (1 + a_k \exp(i\phi_k))$$

$$a_k \text{ in } [-10\%, +10\%] \text{ or } [-20\%, +20\%] \qquad \phi_k \text{ in } [-\pi, +\pi]$$

Global fits

Global fits of the observables by minimization of

$$\chi^2 = (\vec{O}^{\rm th} - \vec{O}^{\rm exp}) \cdot (\Sigma_{\rm th} + \Sigma_{\rm exp})^{-1} \cdot (\vec{O}^{\rm th} - \vec{O}^{\rm exp})$$

$(\Sigma_{\rm th} + \Sigma_{\rm exp})^{-1}$ is the inverse covariance matrix.

58 observables relevant for leptonic and semileptonic decays:

- $\text{BR}(B \to X_s\gamma)$
- $\text{BR}(B \to X_d\gamma)$
- $\Delta_0(B \to K^*\gamma)$
- $\text{BR}^{\text{low}}(B \to X_s\mu^+\mu^-)$
- $\text{BR}^{\text{high}}(B \to X_s\mu^+\mu^-)$
- $\text{BR}^{\text{low}}(B \to X_s e^+e^-)$
- $\text{BR}^{\text{high}}(B \to X_s e^+e^-)$
- $\text{BR}(B_s \to \mu^+\mu^-)$
- $\text{BR}(B_d \to \mu^+\mu^-)$
- $\text{BR}(B \to K^{*+}\mu^+\mu^-)$
- $\text{BR}(B \to K^0\mu^+\mu^-)$
- $\text{BR}(B \to K^+\mu^+\mu^-)$
- $\text{BR}(B \to K^* e^+e^-)$
- R_K
- $B \to K^{*0}\mu^+\mu^-$: F_L, A_{FB}, S_3, S_4, S_5 in five low q^2 and two high q^2bins
- $B_s \to \phi\mu^+\mu^-$: BR, F_L in three low q^2 and two high q^2bins

Statistical approaches:

- $\Delta\chi^2 = \chi^2 - \chi^2_{\rm min}$ method
  1. Determination of the minimum of $\chi^2 \to$ best fit point
  2. Computation for each point of the scan of the difference of χ^2 with the best fit point
  3. Find the $1-2\sigma$ regions corresponding to the number of d.o.f.

  Interpretation: considering the best fit point gives the "real" description, which variations of the parameters are allowed $\to$ *relative* global fit

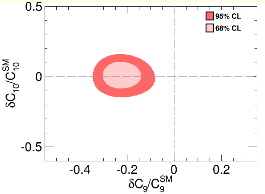
- Absolute χ^2 method
  1. Computation of the χ^2 for each point
  2. Find the $1-2\sigma$ regions corresponding to N d.o.f. where $N =$ (N_o observables - n_v variables)
  3. If an observable is relatively insensitive to the variation of the Wilson coefficients, remove it from the fit

  Interpretation: global fit assessing if each point is *globally* in agreement with all the measurements

$\rightarrow$ Using soft form factors

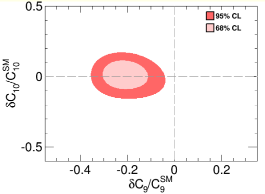
with 10% power correction errors:

$\Delta\chi^2$ method



with 20% power correction errors:

$\Delta\chi^2$ method



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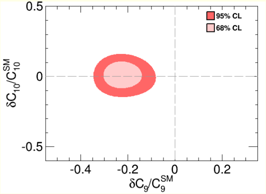

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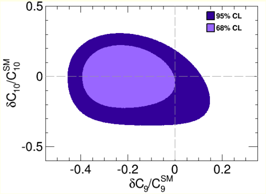

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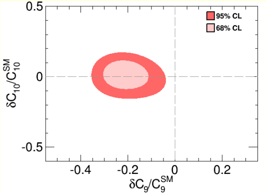

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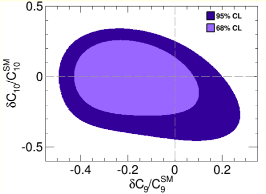


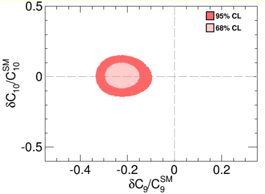
1σ agreement for C_9 is possible even in the 2 operator basis!

Fit results for two operators: $\{C_9, C_{10}\}$

$\rightarrow$ Using full form factors

with 5% power correction errors:

$\Delta\chi^2$ method



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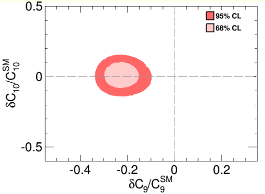

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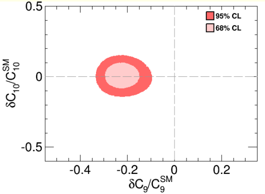

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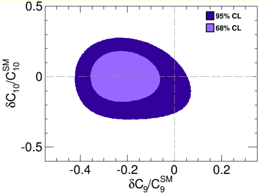

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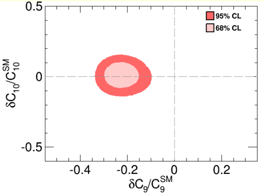

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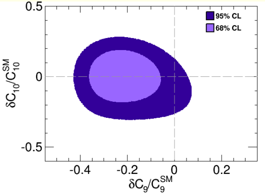

Using the full form factors, only 2σ agreement for C_9 could be possible.

Fit results for two operators: $\{C_9, C_9'\}$

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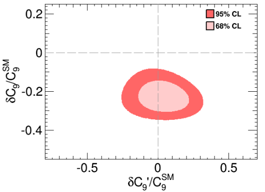

with 20% power correction errors:

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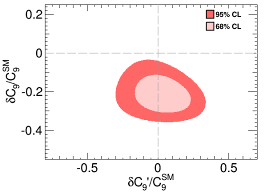

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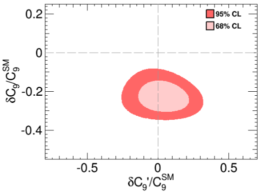

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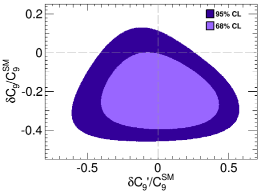

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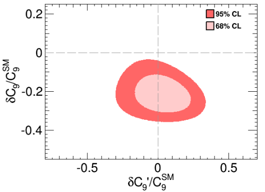

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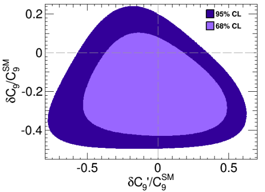
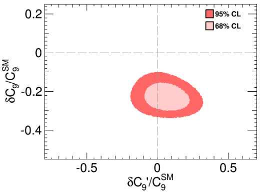


1σ agreement for C_9 is possible even in the 2 operator basis!

$\rightarrow$ Using full form factors

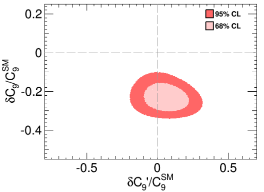
with 5% power correction errors:

$\Delta\chi^2$ method



with 10% power correction errors:

$\Delta\chi^2$ method



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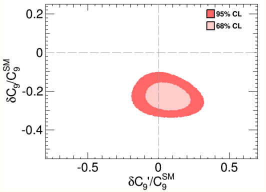

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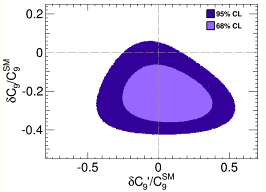

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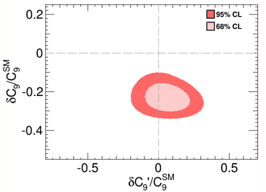

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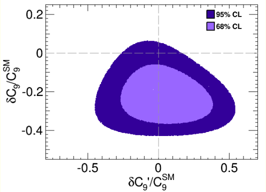


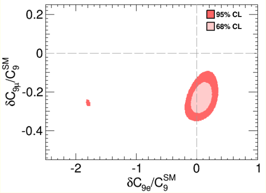
Using the full form factors, only 2σ agreement for C_9 could be possible.

Fit results for two operators: $\{C_9^e, C_9^\mu\}$

$\rightarrow$ Using soft form factors

with 10% power correction errors:

$\Delta\chi^2$ method



with 20% power correction errors:

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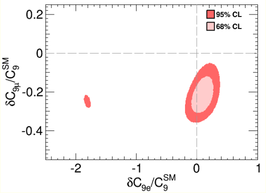

→ Using soft form factors

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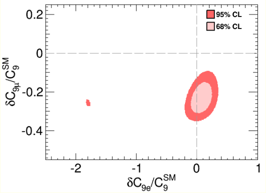
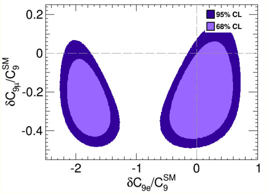


Absolute χ^2 method



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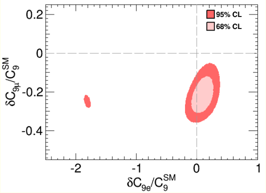

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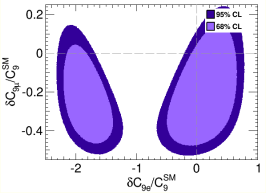
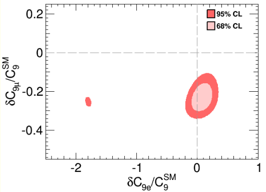


No tension in $C_{9\mu}$ with 20% errors for power corrections!

$\rightarrow$ Using full form factors

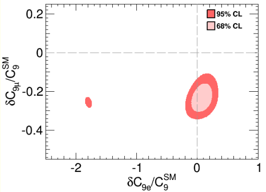
with 5% power correction errors:

$\Delta\chi^2$ method



with 10% power correction errors:

$\Delta\chi^2$ method

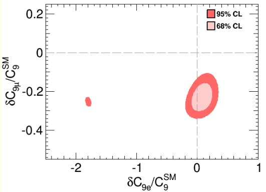


Fit results for two operators: $\{C_9^e, C_9^\mu\}$

$\rightarrow$ Using full form factors

with 5% power correction errors:

$\Delta\chi^2$ method



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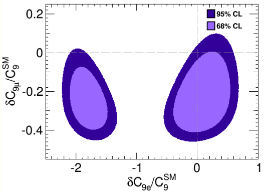
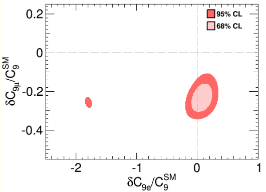


with 10% power correction errors:

$\Delta\chi^2$ method



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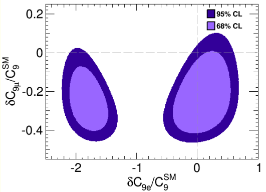
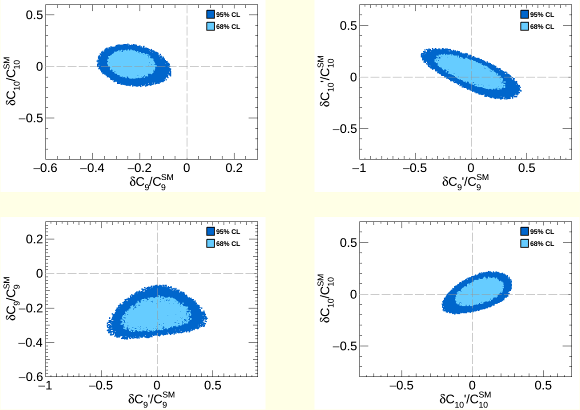


2σ agreement still possible!

Using full form factors with 5% power correction errors

$\rightarrow$ with $\Delta\chi^2$ method

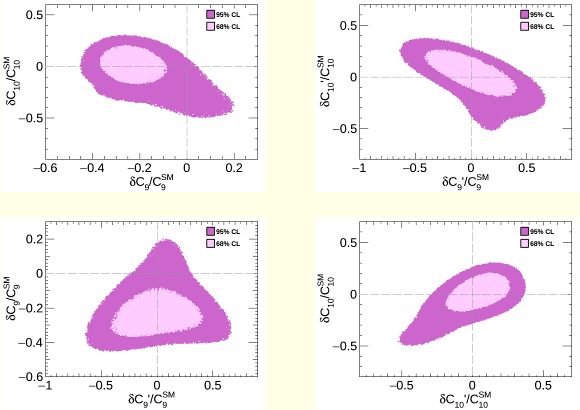


Adding C_{10} or primed coefficients doesn't improve the fit

Fit results for four operators: $\{C_9, C_9', C_{10}, C_{10}'\}$

Using full form factors with 5% power correction errors

$\rightarrow$ with absolute χ^2 method



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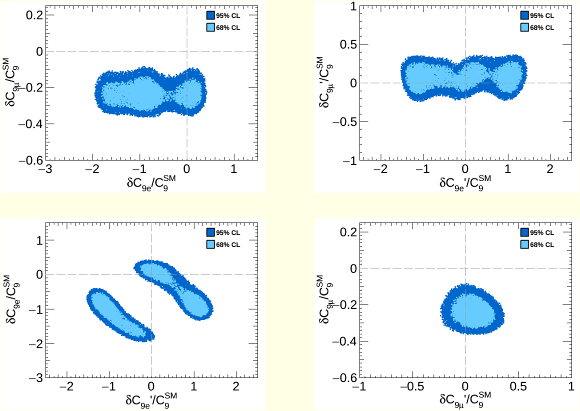
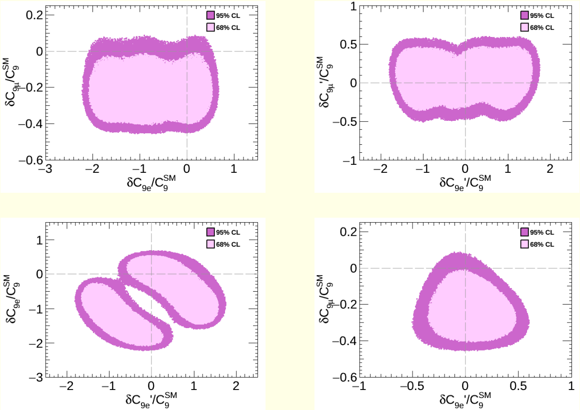


Separating electron and muon coefficients improves the fit by more than 2σ

Fit results for four operators: $\{C_9^\mu, C_9'^\mu, C_9^e, C_9'^e\}$

Using full form factors with 5% power correction errors

$\rightarrow$ with absolute χ^2 method

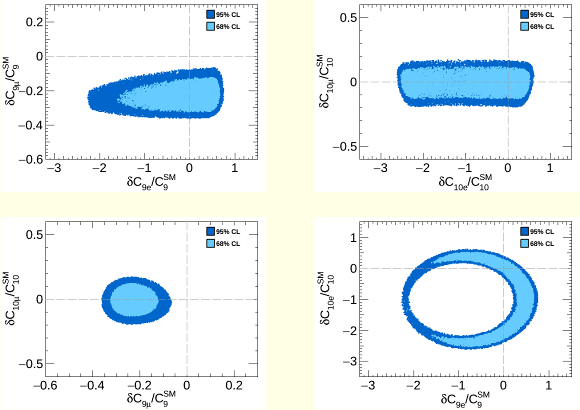


Separating electron and muon coefficients improves the fit by more than 2σ

Fit results for four operators: $\{C_9^\mu, C_9^e, C_{10}^\mu, C_{10}^e\}$

Using full form factors with 5% power correction errors

$\rightarrow$ with $\Delta\chi^2$ method

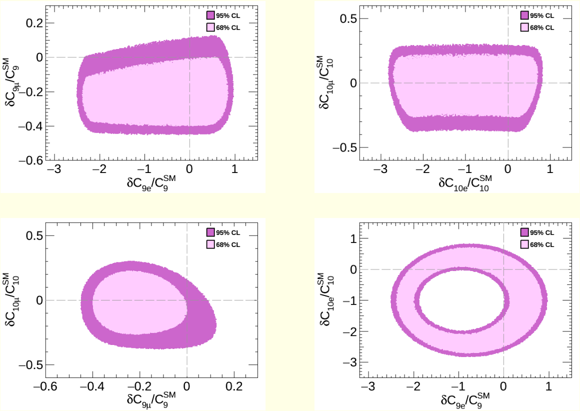


Again the non universal solutions are favoured

Fit results for four operators: $\{C_9^\mu, C_9^e, C_{10}^\mu, C_{10}^e\}$

Using full form factors with 5% power correction errors

$\rightarrow$ with absolute χ^2 method

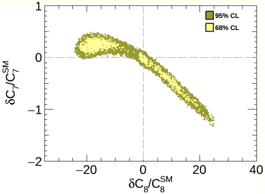


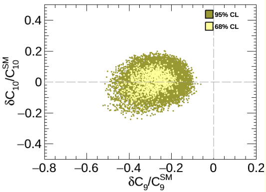
Again the non universal solutions are favoured

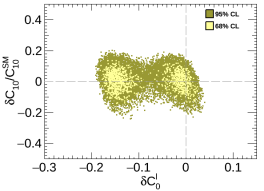
Using full form factors with 5% power correction errors

Preliminary

$\rightarrow$ with $\Delta\chi^2$ method







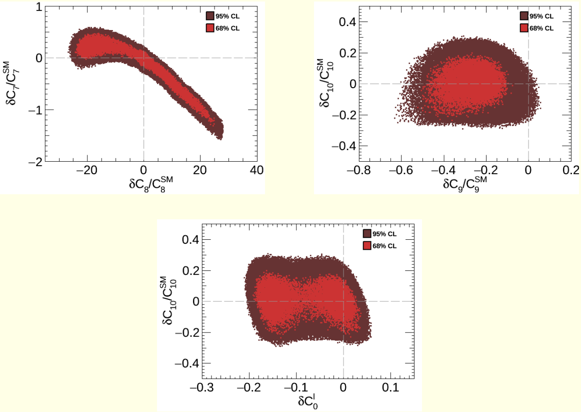
The tension in C_9 is present even in the MFV fit!

MFV fit results: $\{C_7, C_8, C_9, C_{10}, C_0^l\}$

Using full form factors with 5% power correction errors

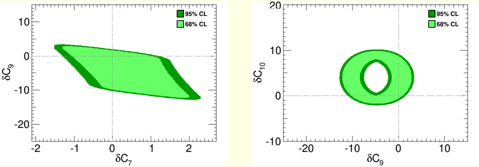
Preliminary

$\rightarrow$ with absolute χ^2 method

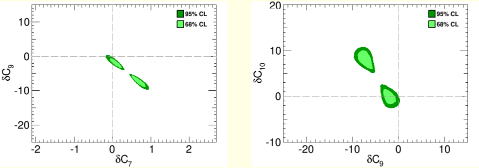


The tension in C_9 is present even in the MFV fit!

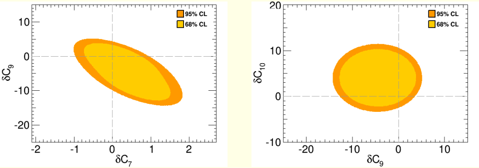
using only BR($B \to K^0\mu^+\mu^-$), BR($B^+ \to K^+\mu^+\mu^-$), R_K



using only $B \to K^*\mu^+\mu^-$ observables



using only BR($B \to X_s\mu^+\mu^-$) at low- and high-q^2



T. Hurth, FM, S. Neshatpour, JHEP 1412 (2014) 053

All three sets of exclusion plots nicely compatible with each other

$\to$ non-trivial consistency check

Summary

- There are two possible statistical approaches: $\Delta\chi^2$ and absolute χ^2
- With $\Delta\chi^2$, the error on power corrections has a smaller impact than with absolute χ^2
- With $\Delta\chi^2$, in the two operator fits $\{C_9, C_{10}\}$, $\{C_9, C_9'\}$, $\{C_9^\mu, C_9^e\}$, SM shows more than 2σ tension

  $\rightarrow$ In principle there is no reason to consider only 2 operators!

- In the 4 operator fits, the tension in C_9 weakens but still exists at the 2σ level
- The tension in C_9 is also seen in the MFV fit

The tensions are almost GONE with abs χ^2 and 20% error for the power corrections!

The largest remaining tension is for C_9^μ but it's less than 2σ

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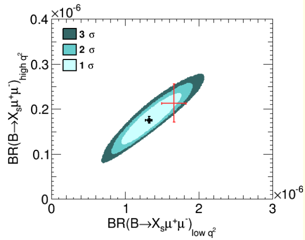
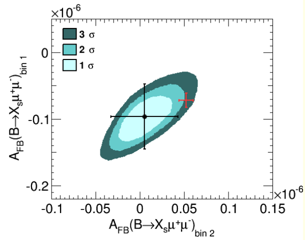
- Important to correctly choose the statistical method, depending on which question is asked
- There is a small tension of about 2σ, in the global fits in the absence of lepton flavour violation
- We should be cautious not over interpreting the tension
- To claim new physics, the use of $\Delta\chi^2$ is NOT appropriate, one needs to use the absolute χ^2
- The ideal would be to consider properly the Look Elsewhere Effect
- The cross check with the updated results in particular for $B_s \to \phi\mu^+\mu^-$ is awaited
- The cross check with the inclusive results is also of importance

Backup

At Belle-II, for inclusive $b \to s\ell\ell$:
expected uncertainty of 2.9% (4.1%) for the branching fraction in the low- (high-)q^2 region, absolute uncertainty of 0.050 in the low-q^2 bin 1 ($1 < q^2 < 3.5$ GeV^2), 0.054 in the low-q^2 bin 2 ($3.5 < q^2 < 6$ GeV^2) for the *normalised* A_{FB}





T. Hurth, FM, JHEP 1404 (2014) 097
T. Hurth, FM, S. Neshatpour, JHEP 1412 (2014) 053

Predictions based on our model-independent analysis

black cross: future measurements at Belle-II assuming the best fit solution

red cross: SM predictions

$\rightarrow$ inclusive mode will lead to very strong constraints

Observable	SM prediction	Measurement
$10^4 \times \mathrm{BR}(B \to X_s\gamma)$	3.37 ± 0.19	3.43 ± 0.22
$10^2 \times \Delta_0(B \to K^*\gamma)$	6.9 ± 3.0	5.2 ± 2.6
$10^9 \times \mathrm{BR}(B_s \to \mu^+\mu^-)$	3.54 ± 0.27	2.9 ± 0.7
$10^{10} \times \mathrm{BR}(B_d \to \mu^+\mu^-)$	1.07 ± 0.27	3.6 ± 1.6
$R_{K\ q^2\in[1.0,6.0](\mathrm{GeV})^2}$	1.0006 ± 0.0004	0.745 ± 0.097
$10^6 \times \mathrm{BR}\left(B \to X_s e^+ e^-\right)_{q^2\in[1,6](\mathrm{GeV})^2}$	$1.73^{+0.12}_{-0.12}$	1.93 ± 0.55
$10^6 \times \mathrm{BR}\left(B \to X_s e^+ e^-\right)_{q^2>14.2(\mathrm{GeV})^2}$	$0.20^{+0.06}_{-0.06}$	0.56 ± 0.19
$10^6 \times \mathrm{BR}\left(B \to X_s \mu^+\mu^-\right)_{q^2\in[1,6](\mathrm{GeV})^2}$	$1.67^{+0.12}_{-0.12}$	0.66 ± 0.88
$10^6 \times \mathrm{BR}\left(B \to X_s \mu^+\mu^-\right)_{q^2>14.2(\mathrm{GeV})^2}$	$0.23^{+0.07}_{-0.06}$	0.60 ± 0.31

SM predictions and experimental values of the $B^0 \to K^{*0}\mu^+\mu^-$ observables

Observable	Soft FF (10%)	Soft FF (20%)	Full FF (5%)	Full FF (10%)	Measurement
			$q^2 \in [0.1\,,\ 0.98]\,\mathrm{GeV}^2$		
$\langle BR\rangle \times 10^7$	1.082 ± 0.157	1.082 ± 0.197	1.071 ± 0.148	1.071 ± 0.148	$- - - - - - - -$
$\langle F_L\rangle$	0.244 ± 0.042	0.244 ± 0.050	0.247 ± 0.037	0.247 ± 0.037	$0.263^{+0.046}_{-0.044} \pm 0.017$
$\langle A_{FB}\rangle$	-0.088 ± 0.019	-0.088 ± 0.036	-0.088 ± 0.006	-0.088 ± 0.008	$-0.003^{+0.057}_{-0.059} \pm 0.008$
$\langle S_3\rangle$	0.000 ± 0.011	0.000 ± 0.023	0.007 ± 0.002	0.007 ± 0.003	$-0.036^{+0.063}_{-0.062} \pm 0.005$
$\langle S_4\rangle$	-0.097 ± 0.007	-0.097 ± 0.010	-0.096 ± 0.005	-0.096 ± 0.005	$0.082^{+0.070}_{-0.066} \pm 0.009$
$\langle S_5\rangle$	0.239 ± 0.014	0.239 ± 0.022	0.242 ± 0.010	0.242 ± 0.010	$0.170^{+0.060}_{-0.059} \pm 0.018$
$\langle S_7\rangle$	0.022 ± 0.014	0.022 ± 0.026	0.022 ± 0.006	0.022 ± 0.006	$0.015^{+0.059}_{-0.057} \pm 0.006$
$\langle S_8\rangle$	-0.004 ± 0.006	-0.004 ± 0.012	-0.004 ± 0.003	-0.004 ± 0.003	$0.079^{+0.077}_{-0.078} \pm 0.007$
$\langle S_9\rangle$	-0.001 ± 0.011	-0.001 ± 0.023	-0.001 ± 0.000	-0.001 ± 0.001	$-0.083^{+0.060}_{-0.059} \pm 0.004$
$\langle P'_5\rangle$	0.657 ± 0.024	0.657 ± 0.049	0.665 ± 0.008	0.665 ± 0.011	$0.387^{+0.141}_{-0.131} \pm 0.052$
			$q^2 \in [1.1\,,\ 2.5]\,\mathrm{GeV}^2$		
$\langle BR\rangle \times 10^7$	0.658 ± 0.078	0.658 ± 0.101	0.656 ± 0.069	0.656 ± 0.069	$- - - - - - - -$
$\langle F_L\rangle$	0.721 ± 0.045	0.721 ± 0.060	0.722 ± 0.037	0.722 ± 0.037	$0.660^{+0.088}_{-0.075} \pm 0.022$
$\langle A_{FB}\rangle$	-0.158 ± 0.029	-0.158 ± 0.038	-0.156 ± 0.024	-0.156 ± 0.024	$-0.191^{+0.069}_{-0.078} \pm 0.012$
$\langle S_3\rangle$	0.000 ± 0.008	0.000 ± 0.016	0.003 ± 0.001	0.003 ± 0.002	$-0.077^{+0.089}_{-0.104} \pm 0.005$
$\langle S_4\rangle$	-0.012 ± 0.009	-0.012 ± 0.009	-0.008 ± 0.008	-0.008 ± 0.009	$-0.077^{+0.112}_{-0.112} \pm 0.005$
$\langle S_5\rangle$	0.106 ± 0.015	0.106 ± 0.017	0.108 ± 0.015	0.108 ± 0.015	$0.137^{+0.094}_{-0.098} \pm 0.009$
$\langle S_7\rangle$	0.035 ± 0.008	0.035 ± 0.010	0.034 ± 0.008	0.034 ± 0.008	$-0.219^{+0.093}_{-0.105} \pm 0.003$
$\langle S_8\rangle$	-0.012 ± 0.004	-0.012 ± 0.006	-0.011 ± 0.004	-0.011 ± 0.004	$-0.098^{+0.107}_{-0.122} \pm 0.005$
$\langle S_9\rangle$	-0.001 ± 0.008	-0.001 ± 0.016	-0.001 ± 0.001	-0.001 ± 0.001	$-0.119^{+0.087}_{-0.101} \pm 0.005$
$\langle P'_5\rangle$	0.252 ± 0.028	0.252 ± 0.035	0.258 ± 0.030	0.258 ± 0.032	$0.289^{+0.216}_{-0.200} \pm 0.023$
			$q^2 \in [2.5\,,\ 4.0]\,\mathrm{GeV}^2$		
$\langle BR\rangle \times 10^7$	0.637 ± 0.081	0.637 ± 0.117	0.637 ± 0.065	0.637 ± 0.065	$- - - - - - - -$
$\langle F_L\rangle$	0.808 ± 0.036	0.808 ± 0.056	0.807 ± 0.028	0.807 ± 0.028	$0.877^{+0.089}_{-0.096} \pm 0.017$
$\langle A_{FB}\rangle$	-0.053 ± 0.017	-0.053 ± 0.026	-0.051 ± 0.011	-0.051 ± 0.012	$-0.118^{+0.075}_{-0.088} \pm 0.007$
$\langle S_3\rangle$	-0.011 ± 0.008	-0.011 ± 0.014	-0.010 ± 0.003	-0.010 ± 0.003	$0.035^{+0.101}_{-0.086} \pm 0.006$
$\langle S_4\rangle$	0.124 ± 0.016	0.124 ± 0.022	0.127 ± 0.013	0.127 ± 0.013	$-0.234^{+0.132}_{-0.144} \pm 0.006$
$\langle S_5\rangle$	-0.146 ± 0.021	-0.146 ± 0.031	-0.144 ± 0.017	-0.144 ± 0.017	$-0.022^{+0.110}_{-0.104} \pm 0.008$
$\langle S_7\rangle$	0.026 ± 0.024	0.026 ± 0.047	0.026 ± 0.006	0.026 ± 0.006	$0.068^{+0.119}_{-0.112} \pm 0.005$
$\langle S_8\rangle$	-0.011 ± 0.009	-0.011 ± 0.017	-0.010 ± 0.003	-0.010 ± 0.003	$0.030^{+0.123}_{-0.127} \pm 0.006$
$\langle S_9\rangle$	-0.001 ± 0.007	-0.001 ± 0.013	-0.001 ± 0.000	-0.001 ± 0.001	$-0.092^{+0.108}_{-0.125} \pm 0.007$
$\langle P'_5\rangle$	-0.386 ± 0.050	-0.386 ± 0.077	-0.382 ± 0.037	-0.382 ± 0.039	$-0.066^{+0.341}_{-0.360} \pm 0.023$

SM predictions and experimental values of the $B^0 \to K^{*0}\mu^+\mu^-$ observables

Observable	Soft FF (10%)	Soft FF (20%)	Full FF (5%)	Full FF (10%)	Measurement
			$q^2 \in [6.0\,,\ 8.0]\,\mathrm{GeV}^2$		
$\langle BR\rangle \times 10^7$	1.059 ± 0.105	1.059 ± 0.177	1.065 ± 0.065	1.065 ± 0.065	− − − − − − − −−
$\langle F_L\rangle$	0.625 ± 0.073	0.625 ± 0.126	0.624 ± 0.041	0.624 ± 0.041	$0.579^{+0.043}_{-0.047} \pm 0.015$
$\langle A_{FB}\rangle$	0.228 ± 0.049	0.228 ± 0.083	0.230 ± 0.026	0.230 ± 0.026	$0.152^{+0.040}_{-0.040} \pm 0.008$
$\langle S_3\rangle$	-0.044 ± 0.029	-0.044 ± 0.055	-0.045 ± 0.011	-0.045 ± 0.011	$-0.042^{+0.057}_{-0.058} \pm 0.011$
$\langle S_4\rangle$	0.260 ± 0.021	0.260 ± 0.039	0.262 ± 0.009	0.262 ± 0.009	$-0.296^{+0.065}_{-0.065} \pm 0.011$
$\langle S_5\rangle$	-0.393 ± 0.041	-0.393 ± 0.077	-0.391 ± 0.013	-0.391 ± 0.013	$-0.249^{+0.062}_{-0.061} \pm 0.012$
$\langle S_7\rangle$	0.010 ± 0.079	0.010 ± 0.149	0.009 ± 0.003	0.009 ± 0.004	$-0.047^{+0.066}_{-0.062} \pm 0.003$
$\langle S_8\rangle$	-0.005 ± 0.031	-0.005 ± 0.060	-0.005 ± 0.002	-0.005 ± 0.002	$-0.085^{+0.072}_{-0.073} \pm 0.006$
$\langle S_9\rangle$	-0.001 ± 0.026	-0.001 ± 0.052	-0.001 ± 0.001	-0.001 ± 0.002	$-0.024^{+0.059}_{-0.062} \pm 0.005$
$\langle P_5'\rangle$	-0.819 ± 0.083	-0.819 ± 0.160	-0.814 ± 0.025	-0.814 ± 0.025	$-0.505^{+0.118}_{-0.177} \pm 0.024$
			$q^2 \in [15.0\,,\ 17.0]\,\mathrm{GeV}^2$		
$\langle BR\rangle \times 10^7$	1.258 ± 0.073	1.258 ± 0.092	1.258 ± 0.068	1.258 ± 0.073	− − − − − − − −−
$\langle F_L\rangle$	0.339 ± 0.039	0.339 ± 0.055	0.339 ± 0.034	0.339 ± 0.039	$0.349^{+0.040}_{-0.039} \pm 0.009$
$\langle A_{FB}\rangle$	0.409 ± 0.025	0.409 ± 0.037	0.409 ± 0.022	0.409 ± 0.026	$0.411^{+0.040}_{-0.035} \pm 0.008$
$\langle S_3\rangle$	-0.181 ± 0.024	-0.181 ± 0.037	-0.181 ± 0.020	-0.181 ± 0.024	$-0.142^{+0.046}_{-0.047} \pm 0.007$
$\langle S_4\rangle$	0.294 ± 0.008	0.294 ± 0.013	0.294 ± 0.007	0.294 ± 0.008	$-0.321^{+0.053}_{-0.078} \pm 0.007$
$\langle S_5\rangle$	-0.315 ± 0.024	-0.315 ± 0.037	-0.315 ± 0.019	-0.315 ± 0.024	$-0.316^{+0.051}_{-0.058} \pm 0.009$
$\langle S_7\rangle$	0.000 ± 0.034	0.000 ± 0.067	0.000 ± 0.017	0.000 ± 0.034	$0.061^{+0.058}_{-0.060} \pm 0.005$
$\langle S_8\rangle$	0.000 ± 0.009	0.000 ± 0.018	0.000 ± 0.005	0.000 ± 0.009	$0.003^{+0.060}_{-0.060} \pm 0.003$
$\langle S_9\rangle$	0.000 ± 0.016	0.000 ± 0.032	0.000 ± 0.008	0.000 ± 0.016	$-0.019^{+0.055}_{-0.057} \pm 0.004$
$\langle P_5'\rangle$	-0.666 ± 0.041	-0.666 ± 0.065	-0.666 ± 0.033	-0.666 ± 0.042	$-0.662^{+0.112}_{-0.126} \pm 0.017$
			$q^2 \in [17.0\,,\ 19.0]\,\mathrm{GeV}^2$		
$\langle BR\rangle \times 10^7$	0.866 ± 0.055	0.866 ± 0.069	0.866 ± 0.051	0.866 ± 0.054	− − − − − − − −−
$\langle F_L\rangle$	0.322 ± 0.042	0.322 ± 0.057	0.322 ± 0.037	0.322 ± 0.042	$0.354^{+0.048}_{-0.048} \pm 0.025$
$\langle A_{FB}\rangle$	0.321 ± 0.023	0.321 ± 0.033	0.321 ± 0.021	0.321 ± 0.024	$0.305^{+0.048}_{-0.046} \pm 0.013$
$\langle S_3\rangle$	-0.256 ± 0.025	-0.256 ± 0.034	-0.256 ± 0.021	-0.256 ± 0.024	$-0.188^{+0.076}_{-0.086} \pm 0.017$
$\langle S_4\rangle$	0.309 ± 0.010	0.309 ± 0.014	0.309 ± 0.009	0.309 ± 0.010	$-0.266^{+0.065}_{-0.071} \pm 0.010$
$\langle S_5\rangle$	-0.224 ± 0.022	-0.224 ± 0.032	-0.224 ± 0.019	-0.224 ± 0.022	$-0.323^{+0.062}_{-0.069} \pm 0.009$
$\langle S_7\rangle$	0.000 ± 0.035	0.000 ± 0.071	0.000 ± 0.018	0.000 ± 0.035	$0.044^{+0.072}_{-0.073} \pm 0.013$
$\langle S_8\rangle$	0.000 ± 0.007	0.000 ± 0.013	0.000 ± 0.003	0.000 ± 0.007	$0.013^{+0.067}_{-0.071} \pm 0.005$
$\langle S_9\rangle$	0.000 ± 0.013	0.000 ± 0.025	0.000 ± 0.006	0.000 ± 0.013	$-0.094^{+0.067}_{-0.069} \pm 0.004$
$\langle P_5'\rangle$	-0.481 ± 0.039	-0.481 ± 0.057	-0.481 ± 0.033	-0.481 ± 0.039	$-0.675^{+0.138}_{-0.152} \pm 0.017$

SM predictions and experimental values for $B_s \to \phi\, \mu^+\mu^-$ and $B \to K\, \mu^+\mu^-$

$B_s \to \phi\, \mu^+\mu^-$ SM prediction					
Observable	Soft FF (10%)	Soft FF (20%)	Full FF (5%)	Full FF (10%)	Measurement
$q^2 \in [0.1\,,\ 2.0]\,\mathrm{GeV}^2$					
$\langle BR \rangle \times 10^7$	1.631 ± 0.134	1.631 ± 0.161	1.611 ± 0.095	1.611 ± 0.095	$0.90^{+0.21}_{-0.19} \pm 0.04 \pm 0.09$
$\langle F_L \rangle$	0.390 ± 0.043	0.390 ± 0.058	0.397 ± 0.034	0.397 ± 0.035	$0.37^{+0.19}_{-0.17} \pm 0.07$
$\langle S_3 \rangle$	-0.001 ± 0.010	-0.001 ± 0.020	0.006 ± 0.002	0.006 ± 0.003	$-0.11^{+0.28}_{-0.25} \pm 0.05$
$q^2 \in [2.0\,,\ 4.3]\,\mathrm{GeV}^2$					
$\langle BR \rangle \times 10^7$	1.013 ± 0.072	1.013 ± 0.112	1.017 ± 0.053	1.017 ± 0.054	$0.53^{+0.18}_{-0.16} \pm 0.03 \pm 0.05$
$\langle F_L \rangle$	0.802 ± 0.032	0.802 ± 0.053	0.803 ± 0.020	0.803 ± 0.020	$0.53^{+0.25}_{-0.23} \pm 0.10$
$\langle S_3 \rangle$	-0.012 ± 0.007	-0.012 ± 0.015	-0.011 ± 0.003	-0.011 ± 0.003	$-0.97^{+0.53}_{-0.03} \pm 0.17$
$q^2 \in [4.30\,,\ 8.68]\,\mathrm{GeV}^2$					
$\langle BR \rangle \times 10^7$	2.284 ± 0.095	2.284 ± 0.168	2.306 ± 0.058	2.306 ± 0.059	$1.38^{+0.25}_{-0.23} \pm 0.05 \pm 0.14$
$\langle F_L \rangle$	0.651 ± 0.063	0.651 ± 0.116	0.650 ± 0.029	0.650 ± 0.029	$0.81^{+0.11}_{-0.13} \pm 0.05$
$\langle S_3 \rangle$	-0.046 ± 0.025	-0.046 ± 0.049	-0.048 ± 0.010	-0.048 ± 0.010	$0.25^{+0.21}_{-0.24} \pm 0.05$
$q^2 \in [14.18\,,\ 16.0]\,\mathrm{GeV}^2$					
$\langle BR \rangle \times 10^7$	1.167 ± 0.072	1.167 ± 0.092	1.167 ± 0.066	1.167 ± 0.073	$0.76^{+0.19}_{-0.17} \pm 0.04 \pm 0.08$
$\langle F_L \rangle$	0.349 ± 0.036	0.349 ± 0.054	0.349 ± 0.030	0.349 ± 0.036	$0.34^{+0.18}_{-0.17} \pm 0.07$
$\langle S_3 \rangle$	-0.172 ± 0.022	-0.172 ± 0.036	-0.172 ± 0.017	-0.172 ± 0.022	$-0.03^{+0.29}_{-0.31} \pm 0.06$
$q^2 \in [16.0\,,\ 19.0]\,\mathrm{GeV}^2$					
$\langle BR \rangle \times 10^7$	1.280 ± 0.053	1.280 ± 0.068	1.280 ± 0.049	1.280 ± 0.054	$1.06^{+0.23}_{-0.21} \pm 0.06 \pm 0.11$
$\langle F_L \rangle$	0.325 ± 0.039	0.325 ± 0.056	0.325 ± 0.033	0.325 ± 0.039	$0.16^{+0.17}_{-0.10} \pm 0.07$
$\langle S_3 \rangle$	-0.248 ± 0.022	-0.248 ± 0.034	-0.248 ± 0.018	-0.248 ± 0.022	$0.19^{+0.30}_{-0.31} \pm 0.05$

BR($B \to K\mu^+\mu^-$) SM prediction					
bin	Soft FF (10%)	Soft FF (20%)	Full FF (5%)	Full FF (10%)	Measurement
$10^7 \times \langle BR \rangle(B^0 \to K^0\mu^+\mu^-)$					
$q^2 \in [1.1 - 6.0]\,\mathrm{GeV}^2$	1.353 ± 0.061	1.353 ± 0.100	1.350 ± 0.045	1.350 ± 0.045	$0.92^{+0.17}_{-0.16} \pm 0.04$
$q^2 \in [15.0 - 22.0]\,\mathrm{GeV}^2$	0.942 ± 0.014	0.942 ± 0.015	0.942 ± 0.014	0.942 ± 0.014	$0.67^{+0.11}_{-0.11} \pm 0.04$
$10^7 \times \langle BR \rangle(B^+ \to K^+\mu^+\mu^-)$					
$q^2 \in [1.1 - 6.0]\,\mathrm{GeV}^2$	1.481 ± 0.067	1.481 ± 0.110	1.477 ± 0.049	1.477 ± 0.049	$1.19 \pm 0.03 \pm 0.06$
$q^2 \in [15.0 - 22.0]\,\mathrm{GeV}^2$	1.024 ± 0.016	1.024 ± 0.016	1.024 ± 0.016	1.024 ± 0.016	$0.85 \pm 0.03 \pm 0.04$