α_s from scaling violations of hard parton-to-hadron FFs

Bernd Kniehl

II. Institut für Theoretische Physik
Universität Hamburg

Workshop on high-precision α_s measurements: from LHC to FCC-ee
CERN, 12–13 October 2015

In collaboration with S. Albino, P. Bolzoni, A.V. Kotikov, G. Kramer, and B. Pötter

Outline

1. **Basic formalism**
2. **Light-hadron FFs**
3. **Improvements**
4. **Jet multiplicities**
5. **Conclusions**

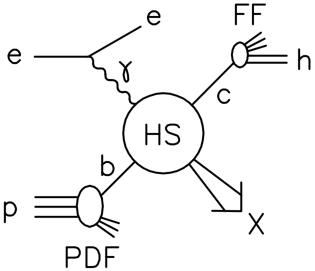
Basics

- QCD Lagrangian,

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}F^{(a)}_{\mu\nu}F^{(a)\mu\nu} + \sum_q \bar{q}(\not{D} - m_q)q,$$
$$F^{(a)}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f_{abc} A^b_\mu A^c_\nu,$$
$$D_\mu = \partial_\mu - \frac{i}{2} g_s \lambda^a A^a_\mu,$$

  contains quarks q and gluons A^a_μ as elementary fields.
- Asymptotic states (particle beams, targets and produced particles) correspond to hadrons.
- Hadron formation genuinely nonperturbative
- Lattice gauge theory unfeasible for high-energy collisions
- Solution: QCD-improved parton model based on factorization

QCD-improved parton model



- Initial state: PDFs $F_a^A(x, M^2)$, where $p_a = x p_A$ and M is factorization scale; nonperturbative input.
- Hard scattering: Partonic cross sections; amenable to perturbative QCD.
- Final state:
  - Inclusive jet production: Partons are clustered according to jet algorithm (parton-hadron duality).
  - Inclusive particle production: FFs $D_c^h(x, M^2)$, where $p_c = p_h/x$ and M is factorization scale; nonperturbative input if $m_h \lesssim \Lambda_{\mathrm{QCD}}$, (partly) calculable in QCD if $m_h \gg \Lambda_{\mathrm{QCD}}$. Field, Feynman, NPB136(1978)1

Next-to-leading-order formalism

$$\frac{d^3\sigma(AB\to h+X)}{dy\,d^2p_T}=\frac{1}{\pi}\sum_{a,b,c}\int dx_A\,dx_B\,\frac{dx_h}{x_h^2}\,F_a^A(x_A,M_A^2)F_b^B(x_B,M_B^2)$$
$$\times D_c^h(x_h,M_h^2)\left[\frac{d\sigma^0_{ab\to c}}{dt}(s,t,\mu^2)\,\delta\left(1+\frac{t+u}{s}\right)\right.$$
$$\left.+\,a_s(\mu^2)K_{ab\to c}(s,t,u,\mu^2,M_A^2,M_B^2,M_h^2)\,\theta\left(1+\frac{t+u}{s}\right)\right],$$

where $a_s=\frac{\alpha_s}{2\pi}, s=(p_a+p_b)^2, t=(p_a-p_c)^2, u=(p_b-p_c)^2$.
For hadroproduction, NLO corrections[1] $K_{ab\to c}$ comprise 16 channels and their $t\leftrightarrow u$ crossed counterparts.
Significance of NLO corretions:

- Reduction of dependence on μ, M_A, M_B, M_h (theor. uncertainty).
- Sizeable shifts in certain regions of phase space (K factor).

[1] F. Aversa, P. Chiappetta, M. Greco, J.Ph. Guillet, Nucl. Phys. B **327** (1989) 105.

DGLAP equations

μ^2 evolution of $D_a^h(x,\mu^2)$ is ruled by DGLAP equations:

$$\frac{\mu^2 d}{d\mu^2} D_a^h(x,\mu^2) = \sum_b \int_x^1 \frac{dz}{z} P_{a\to b}^{(T)}\left(\frac{x}{z}, a_s(\mu^2)\right) D_b^h(x,\mu^2),$$

with timelike $a \to b$ splitting functions

$$P_{a\to b}^{(T)}\left(x, a_s(\mu^2)\right) = a_s(\mu^2) P_{a\to b}^{(0,T)}(x) + a_s^2(\mu^2) P_{a\to b}^{(1,T)}(x) + \mathcal{O}(a_s^3).$$

Factorization theorem:

- Form of collinear divergence just depends on splitting $a \to b$, but not on process $AB \to h + X$.
- $\leadsto$ PDFs and FFs universal
- $\leadsto$ QCD-improved parton model predictive.

Tests:

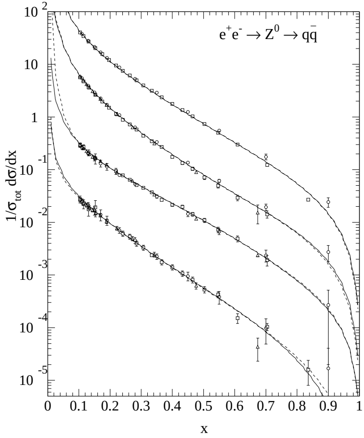
- Universality of FFs
- Scaling violations of FFs $\leadsto \alpha_s$ determination

Light-hadron FFs 2000 BKK, NPB582(2000)514, PRL85(2000)5288

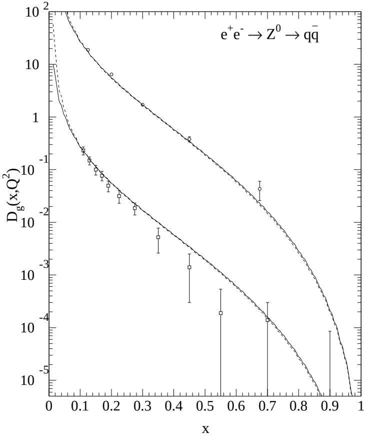
$\sqrt{s}$ [GeV]	Data type	$\chi^2_{\rm DF}$ in NLO (LO)	
29.0	σ^π (all)	0.64 (0.71) [TPC]	
	σ^K (all)	1.86 (1.40) [TPC]	
	σ^p (all)	0.79 (0.70) [TPC]	
91.2	σ^h (all)	1.28 (1.40) [DELPHI]	1.32 (1.44) [SLD]
	σ^h (uds)	0.20 (0.20) [DELPHI]	
	σ^h (b)	0.43 (0.41) [DELPHI]	
	σ^π (all)	1.28 (1.65) [ALEPH]	
		0.58 (0.60) [DELPHI]	3.09 (3.13) [SLD]
	σ^π (uds)	0.72 (0.73) [DELPHI]	1.87 (2.17) [SLD]
	σ^π (c)		1.36 (1.16) [SLD]
	σ^π (b)	0.57 (0.58) [DELPHI]	1.00 (0.99) [SLD]
	σ^K (all)	0.30 (0.32) [ALEPH]	
		0.86 (0.79) [DELPHI]	0.44 (0.45) [SLD]
	σ^K (uds)	0.53 (0.60) [DELPHI]	0.65 (0.64) [SLD]
	σ^K (c)		2.11 (1.90) [SLD]
	σ^K (b)	0.14 (0.14) [DELPHI]	1.21 (1.23) [SLD]
	σ^p (all)	0.93 (0.80) [ALEPH]	
		0.09 (0.06) [DELPHI]	0.79 (0.70) [SLD]
	σ^p (uds)	0.11 (0.14) [DELPHI]	1.29 (1.28) [SLD]
	σ^p (c)		0.92 (0.89) [SLD]
	σ^p (b)	0.56 (0.62) [DELPHI]	0.97 (0.89) [SLD]
$E_{\rm jet}$ [GeV]			
26.2	D_g^h	1.19 (1.18) [ALEPH]	
40.1	D_g^h	1.03 (0.90) [OPAL]	

Light-hadron FFs 2000 (cont.) BKK, NPB582(2000)514, PRL85(2000)5288

$h^\pm$, $\pi^\pm$, $K^\pm$, $p/\bar{p}$ production



gluon-tagged $h^\pm$ production



Light-hadron FFs 2000 (cont.) BKK, NPB582(2000)514, PRL85(2000)5288

	$\Lambda^{(5)}_{\overline{\mathrm{MS}}}$	$\alpha_s^{(5)}(M_Z)$
LO	$88^{+34\ +3}_{-31\ -23}$ MeV	$0.1181^{+0.0058\ +0.0006}_{-0.0069\ -0.0049}$
NLO	$213^{+75\ +22}_{-73\ -29}$ MeV	$0.1170^{+0.0055\ +0.0017}_{-0.0069\ -0.0025}$

Experimental error: vary $\Lambda^{(5)}_{\overline{\mathrm{MS}}}$ so that χ^2_{DF} is increased by 1

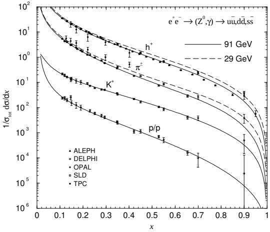
Theoretical error: repeat fits for $\mu = \xi\sqrt{s},\ \xi E_{\mathrm{jet}}$ with $\xi = 0.5, 2$

Light-hadron FFs 2005 AKK, NPB725(2005)181

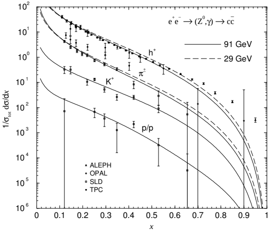
$\sqrt{s}$ [GeV]	Data type	$\chi^2_{\rm DF}$		
29.0	$F_{uds}^{\{h\}}$	3.44 [TPC]*		
	$F_c^{\{h\}}$	2.56 [TPC]*		
	$F_b^{\{h\}}$	1.74 [TPC]*		
	$F_{\{q\}}^{\pi}$	0.80 [TPC]		
	F_{uds}^{π}	1.01 [TPC]		
	F_c^{π}	2.51 [TPC]		
	F_b^{π}	2.14 [TPC]		
	$F_{\{q\}}^{K}$	0.37 [TPC]		
	$F_{\{q\}}^{p}$	0.80 [TPC]		
91.2	$F_{\{q\}}^{\{h\}}$	2.61 [DELPHI]*	105 [ALEPH]*	22.0 [OPAL]*
		4.99 [SLD]*		
	$F_{\{uds\}}^{\{h\}}$	1.10 [DELPHI]*	64.8 [ALEPH]*	2.08 [OPAL]*
	$F_c^{\{h\}}$		34.4 [ALEPH]*	0.57 [OPAL]*
	$F_b^{\{h\}}$	0.21 [DELPHI]*	183.6 [ALEPH]*	5.90 [OPAL]*
	$F_{\{q\}}^{\pi}$	0.98 [ALEPH]	1.13 [DELPHI]	1.82 [SLD]
	$F_{\{uds\}}^{\pi}$		1.82 [DELPHI]	1.12 [SLD]
	F_c^{π}			1.08 [SLD]
	F_b^{π}		0.40 [DELPHI]	0.67 [SLD]
	$F_{\{q\}}^{K}$	0.52 [ALEPH]	0.31 [DELPHI]	0.52 [SLD]
	$F_{\{uds\}}^{K}$		0.31 [DELPHI]	0.83 [SLD]
	F_c^{K}			1.79 [SLD]
	F_b^{K}		0.10 [DELPHI]	1.17 [SLD]
	$F_{\{q\}}^{p}$	0.65 [ALEPH]	0.11 [DELPHI]	0.69 [SLD]
	$F_{\{uds\}}^{p}$		0.22 [DELPHI]	1.43 [SLD]
	F_c^{p}			0.74 [SLD]
	F_b^{p}		0.52 [DELPHI]	1.14 [SLD]
$E_{\rm jet}$ [GeV]				
26.2	$D_g^{\{h\}}$	23.6 [ALEPH]*		
40.1	$D_g^{\{h\}}$	4.36 [OPAL]*		

Light-hadron FFs 2005 (cont.) AKK, NPB725(2005)181

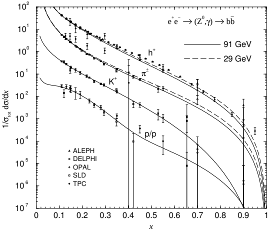
$u\bar{u}$, $d\bar{d}$, $s\bar{s}$



$c\bar{c}$

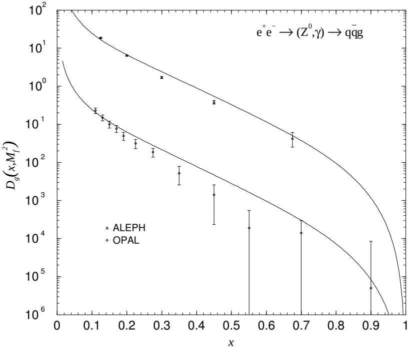


$b\bar{b}$

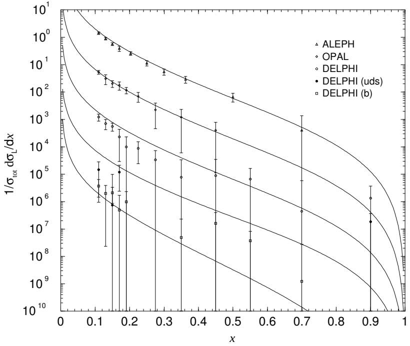


Light-hadron FFs 2005 (cont.) AKK, NPB725(2005)181

g-tagged 3*j* data



longitudinal cross section



$$\Lambda^{(5)}_{\overline{\mathrm{MS}}} = 221 \pm 74\,^{+9}_{-10}\ \mathrm{MeV} \rightsquigarrow \alpha_s^{(5)}(M_Z) = 0.1176\,^{+0.0053}_{-0.0067}\,^{+0.0007}_{-0.0009}$$

K_S^0 and $\Lambda/\bar{\Lambda}$ FFs with this $\alpha_s^{(5)}(M_Z)$ as input AKK, NPB734(2006)50

Light-hadrons FFs 2008 AKK, NPB803(2008)42

Fit to charge-sign unidentified quantities $O^{h/\bar{h}} = O^h + O^{\bar{h}}$ and charge-sign asymmetries $O^{\Delta_c h^\pm} = O^{h^+} - O^{h^-}$ for $h = \pi^\pm, K^\pm, p/\bar{p}, K_S^0, \Lambda/\overline{\Lambda}$ using ansaetze

$$
\begin{aligned}
D_i^{h/\bar{h}}(x, M_0^2) &= N_i^{h/\bar{h}} x^{a_i^{h/\bar{h}}} (1-x)^{b_i^{h/\bar{h}}} \left[1 + c_i^{h/\bar{h}} (1-x)^{d_i^{h/\bar{h}}}\right], \\
D_i^{\Delta_c h^\pm}(x, M_0^2) &= N_i^{\Delta_c h^\pm} x^{a_i^{\Delta_c h^\pm}} (1-x)^{b_i^{\Delta_c h^\pm}}
\end{aligned}
$$

for $i = g, u, d, s, c, b$ and $M_0^2 = 2$ GeV^2 with constraints

$$
\begin{aligned}
D_{\bar{q}}^{h^\pm}(x, M_0^2) &= D_q^{h^\pm}(x, M_0^2), \qquad \text{(exact)} \\
D_{\bar{q}}^{\Delta_c h^\pm}(x, M_0^2) &= -D_q^{\Delta_c h^\pm}(x, M_0^2), \qquad \text{(exact)} \\
D_u^{\pi^\pm/\Delta_c \pi^\pm} &= D_{\bar{d}}^{\pi^\pm/\Delta_c \pi^\pm}. \qquad \text{(SU(2))}
\end{aligned}
$$

$\rightsquigarrow$ 5 fits w/ 6(5)×5 parameters & 3 fits w/ 2(1)×3 parameters; $\alpha_s^{(5)}(M_Z)$ fixed because of inclusion of hadropropduction data

Experimental data NPB803(2008)42

Philosophy: Include data with parton and hadron identification

Type	$\sqrt{s}$ [GeV]	Collider	Collaboration
e^+e^-	10.4	PEP-II	BaBar
		KEKB	Belle
	29.0	PEP	TPC
	91.2	LEP-I	ALEPH, DELPHI, OPAL
		SLC	SLD
pp	200	RHIC	BRAHMS, PHENIX, STAR
$p\bar{p}$	630	Tevatron	CDF
	1800		

Hadron mass effects AKK+Ochs, PRD73(2006)054020; AKK+Sandoval, PRD75(2007)034018

In e^+e^- annihilation, $\frac{d\sigma}{dx_p} = \frac{d\xi}{dx_p}\frac{d\sigma}{d\xi}$, where $x_p = \frac{2|\mathbf{p}_h|}{\sqrt{s}} = \xi\left(1 - \frac{m_h^2}{s\xi^2}\right)$ and ξ is light-cone scaling variable.

Particle	Fitted mass (MeV)	True mass (MeV)
$\pi^\pm$	154.6	139.6
$K^\pm$	337.0	493.7
$p/\bar{p}$	948.8	938.3
K_S^0	343.0	497.6
$\Lambda/\overline{\Lambda}$	1127.0	1115.7

- $\pi^\pm$: large excess, due to admixture of $\rho(770) \to \pi^+\pi^-$
- $K^\pm, K_S^0$: large undershoot, due to complicated decay channels, e.g. K resonance $\to \pi + K$?
- $p/\bar{p}, \Lambda/\overline{\Lambda}$: $\simeq$ 1% excess, due to decays from slightly heavier resonances?

Large-x resummation AKK, PRL100(2008)192002

For $x \to 1$, hard-scattering cross sections and DGLAP evolution kernels develop divergences $a_s^n \left[\frac{\ln^{n-r}(1-x)}{1-x}\right]_+$ of class $r = 0, \ldots, n$.

Upon Mellin transformation $f(N) = \int_0^1 dx\, x^{N-1} f(x)$, one has $C \simeq 1 + A a_s \ln N + B a_s^2 \ln^2 N + \cdots$

At NLO, resum $r = 0, 1$ terms as $C \to C_{\rm res}[1 + a_s(C^{(1)} - C_{\rm res}^{(1)})]$.

Effects:

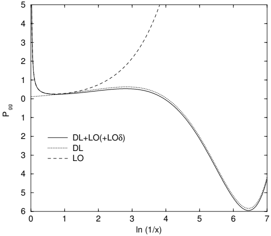
- Enhances cross sections at large x (more so at small $\sqrt{s}$)
- Reduces theoretical uncertainty
- Reduces χ^2

Particle	Unresummed	Resummed
$\pi^\pm$	519.0	518.7
$K^\pm$	439.4	416.6
$p/\bar{p}$	538.0	525.2
K_S^0	318.7	317.2
$\Lambda/\bar{\Lambda}$	325.7	273.1

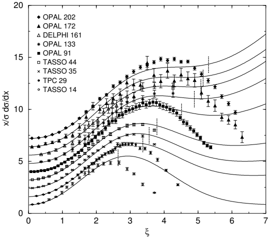
Small-x resummation AKK+Ochs, PRL95(2005)232002, PRD73(2006)054020

For $x \to 0$, timelike DGLAP splitting functions develop soft-gluon logarithms $\frac{a_s^n}{x} \ln^{2n-1-m} x$ of class $m = 1, \ldots, 2n-1$.
Resum $m = 1$ terms via double logathithmic approximation
$a_s P^{(0)}(x) \to P_{\rm DL}(x, a_s) + a_s [P^{(0)}(x) - P^{(0)}_{\rm DL}(x)]$ with
$P_{\rm DL}(x, a_s) = \frac{A \sqrt{C_A a_s}}{x \ln \frac{1}{x}} J_1\left(4\sqrt{C_A a_s} \ln \frac{1}{x}\right)$.

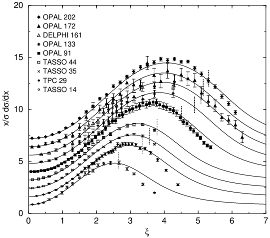
P_{gg} vs. $\ln(1/x)$



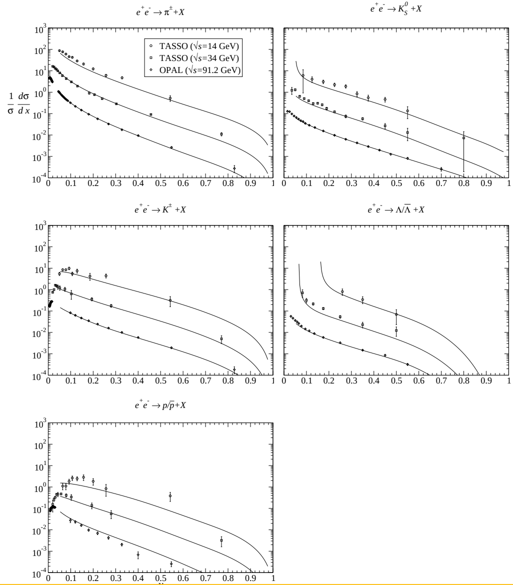
fixed order



low-x resummation



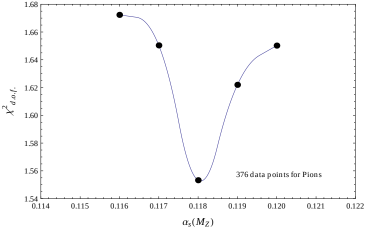
Test of scaling violations AKK, NPB803(2008)42



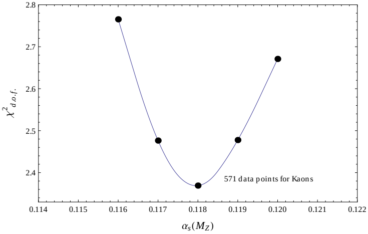
New global α_s fit KK, in preparation

Includes pp data from BRAHMS, PHENIX, STAR; $p\bar{p}$ data from CDF; and new e^+e^- data from BaBar & Belle

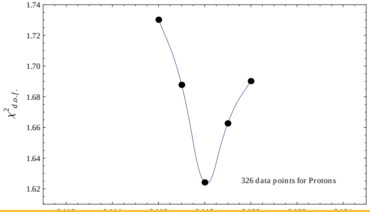
$\pi^\pm$



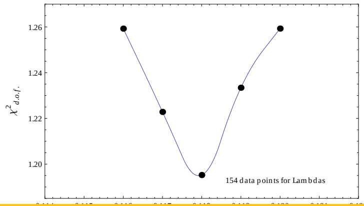
$K^\pm$



$p/\bar{p}$



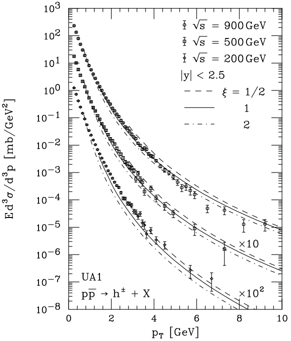
$\Lambda/\bar{\Lambda}$



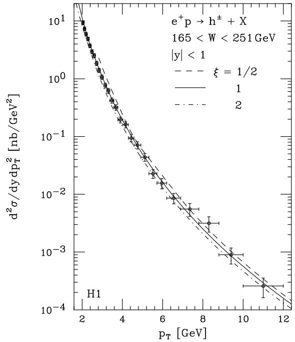
Test of universality KKP, NPB597(2001)337

KKP FFs fitted to e^+e^- data vs. other types of data

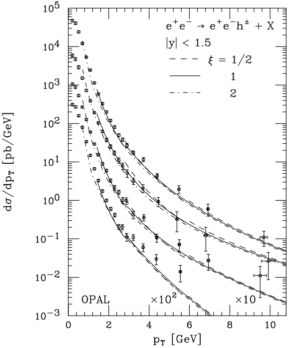
UA1 @ CERN SPS ($p\bar{p}$)



H1 @ DESY HERA (γp)

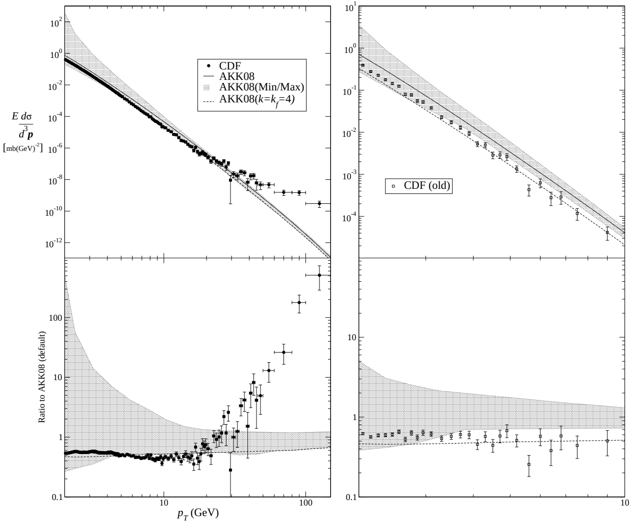


OPAL @ CERN LEP-II ($\gamma\gamma$)



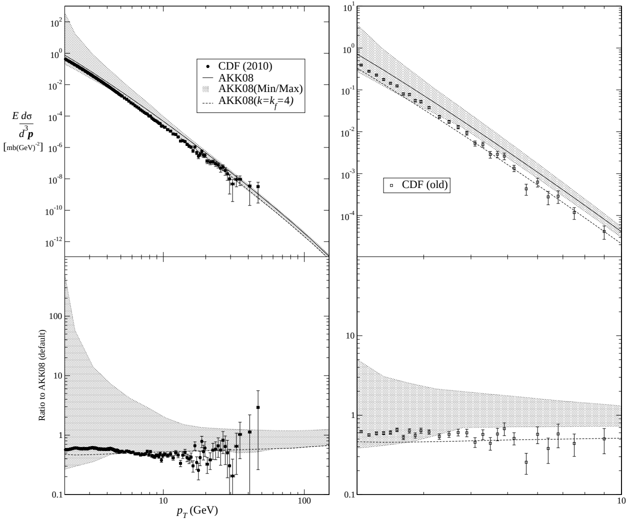
Test of universality (cont.) AKK, PRL04(2010)242001

CDF PRD79(2009)112005 vs. AKK08: Factorization breaking?

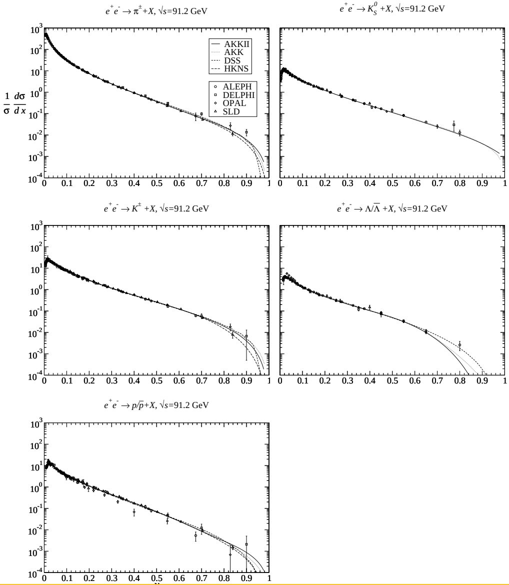


Test of universality (cont.) AKK, PRL04(2010)242001

CDF Erratum PRD82(2010)119903



Comparison with other FF sets AKK, NPB803(2008)42



M. Hirai *et al.*, PRD75(2007)094009
D. de Florian *et al.*, PRD75(2007)114010

Average gluon and quark jet multiplicities Bolzoni, BK, Kotikov, PRL109(2012)242002, NPB875(2013)18

Mellin moments: $D_a(\omega,Q^2)=\int_0^1 dx\, x^\omega D_a(x,Q^2)$
Average gluon and quark jet multiplicities:
$\langle n_h(Q^2)\rangle_a \equiv D_a(0,Q^2) \qquad (a=g,q)$
DGLAP equations:

$$\frac{\mu^2 d}{d\mu^2}\begin{pmatrix} D_s \\ D_g \end{pmatrix}=\begin{pmatrix} P_{qq} & P_{gq} \\ P_{qg} & P_{gg} \end{pmatrix}\begin{pmatrix} D_s \\ D_g \end{pmatrix}$$

First Mellin moments P_{ba} are ill defined and require resummation.
Use NNLL results [Kom, Vogt, Yeats, JHEP10(2012)033].
Diagonalize DGLAP kernel:

$$U^{-1}\begin{pmatrix} P_{qq} & P_{gq} \\ P_{qg} & P_{gg} \end{pmatrix}U=\begin{pmatrix} P_{--} & 0 \\ 0 & P_{++} \end{pmatrix}$$

Large and small components:

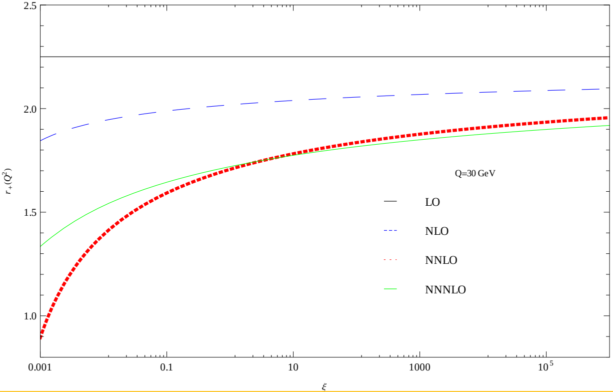
$$\begin{pmatrix} D_- \\ D_+ \end{pmatrix}=U^{-1}\begin{pmatrix} D_s \\ D_g \end{pmatrix}$$

Decompose: $D_a=D_a^+ + D_a^-$

Theoretical framework BKK, NPB875(2013)18

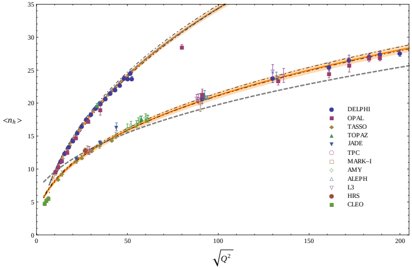
Define: $r_\pm = D_g^\pm / D_s^\pm$
Use ($n_f = 5$) [Capella et al. PRD61(2000)074009]:

$$r_+ = 2.25 - 2.18249\sqrt{a_s} - 27.54\, a_s + 10.8462\, a_s^{3/2} + \mathcal{O}(a_s^2) \lesssim C_A/C_F$$
$$r_- = -2.72166\sqrt{a_s} + \mathcal{O}(a_s)$$

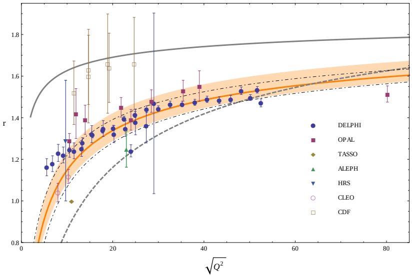


Fit results BKK, NPB875(2013)18

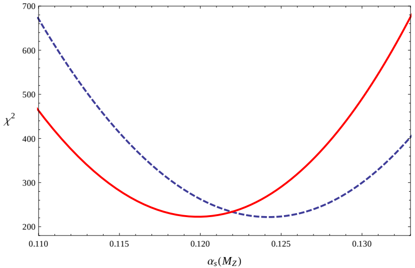
$\langle n_h \rangle_g, \langle n_h \rangle_q$



r



Fit results (cont.) BKK, NPB875(2013)18



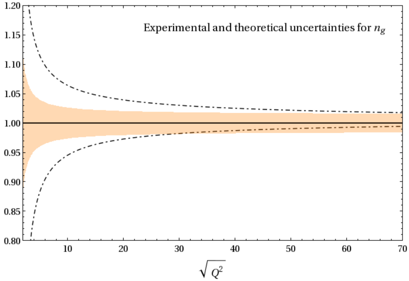
	$\mathrm{N^3LO_{approx}+NNLL}$	$\mathrm{N^3LO_{approx}+NLO+NNLL}$
$\langle n_h(Q_0)\rangle_g$	24.18 ± 0.32	24.22 ± 0.33
$\langle n_h(Q_0)\rangle_q$	15.86 ± 0.37	15.88 ± 0.35
$\alpha_s^{(5)}(M_Z)$	0.1242 ± 0.0046	0.1199 ± 0.0044
χ^2_{dof}	2.84	2.85

$Q_0 = 50$ GeV, 90% CL errors

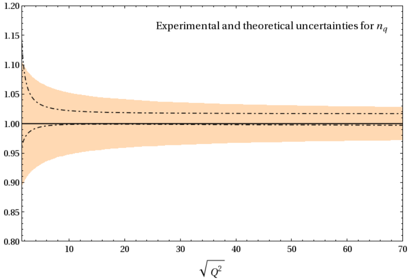
$\rightsquigarrow \alpha_s^{(5)}(M_Z) = 0.1199 \pm 0.0026$ 68% CL

Uncertainties BKK, NPB875(2013)18

$\langle n_h \rangle_g$



$\langle n_h \rangle_q$



Conclusions: α_s from scaling violations in FFs

- Determine Λ_{QCD} along with FF parameters in one global fit
- e^+e^- annihilation: theoretically clean, no nonperturbative input required
- x dependence of $D_a^h(x,\mu^2)$ nonperturbative, but μ dependence purely perturbative!
- Need fine binning in x, wide span in $\sqrt{s}$, quark-flavor and gluon-jet tagging, hadron identification
- Include hadron-mass effects
- Implement small-x and large-x resummation $\rightsquigarrow$ increases validity range
- Include QCD ISR in continuum Kneesch, BK, Kramer, Schienbein, NPB799(2008)34
- $\alpha_s^{(5)}(M_Z) = 0.1170\,^{+0.0055}_{-0.0069}\,^{+0.0017}_{-0.0025}$ BKK, PRL85(2000)5288
- $\alpha_s^{(5)}(M_Z) = 0.1176\,^{+0.0053}_{-0.0067}\,^{+0.0007}_{-0.0009}$ AKK, NPB725(2005)181
- Theoretical uncertainty $\ll$ experimental error $\rightsquigarrow$ High-precision measuments of inclusive hadron production at FCC-ee will significantly reduce $\delta\alpha_s$
- Inclusion of photo- and hadroproduction: need α_s-dependent PDFs, additional source of uncertainty

Conclusions: α_s from average gluon and quark jet multiplicities

- Determine $\Lambda_{\rm QCD}$ along with $\langle n_h(Q_0^2)\rangle_g$ and $\langle n_h(Q_0^2)\rangle_q$ in one global fit to e^+e^- data at different $\sqrt{s}$ values
- Jet algorithms used in experimental data analyses must be compatible
- Possible and competitive thanks to inclusion of minus components BKK, PRL109(2012)242002
- NNLO in $\sqrt{\alpha_s}$, NNLL in $\ln x$, RG improved in $\ln Q^2$
- $\alpha_s^{(5)}(M_Z) = 0.1199 \pm 0.0026$ BKK, NPB875(2013)18
- Hard to reduce theoretical uncertainty
- Theoretical uncertainty $\lesssim$ experimental error $\leadsto$ High-precision measuments of inclusive hadron production at FCC-ee will appreciably reduce $\delta\alpha_s$