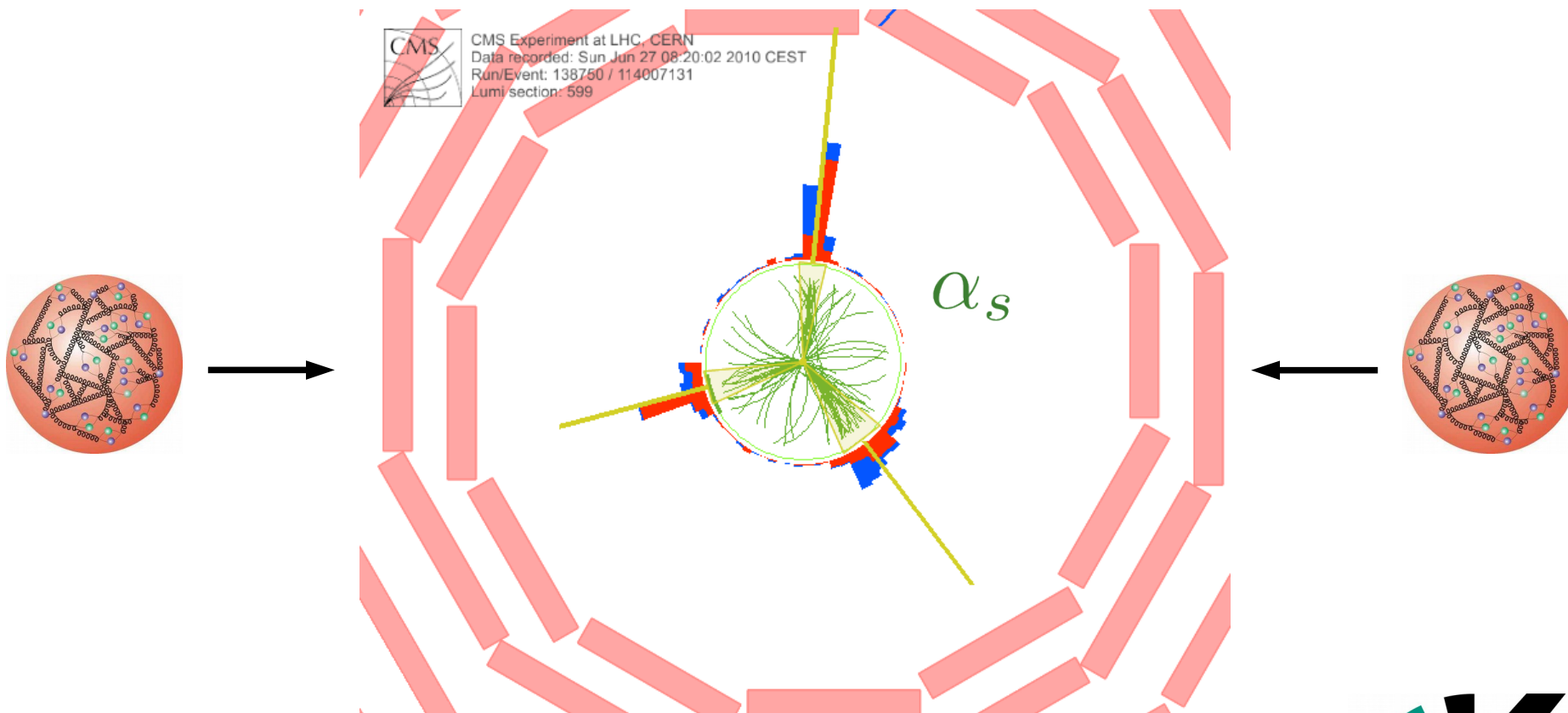




α_s Determinations from CMS Status & Plans



Klaus Rabbertz, KIT



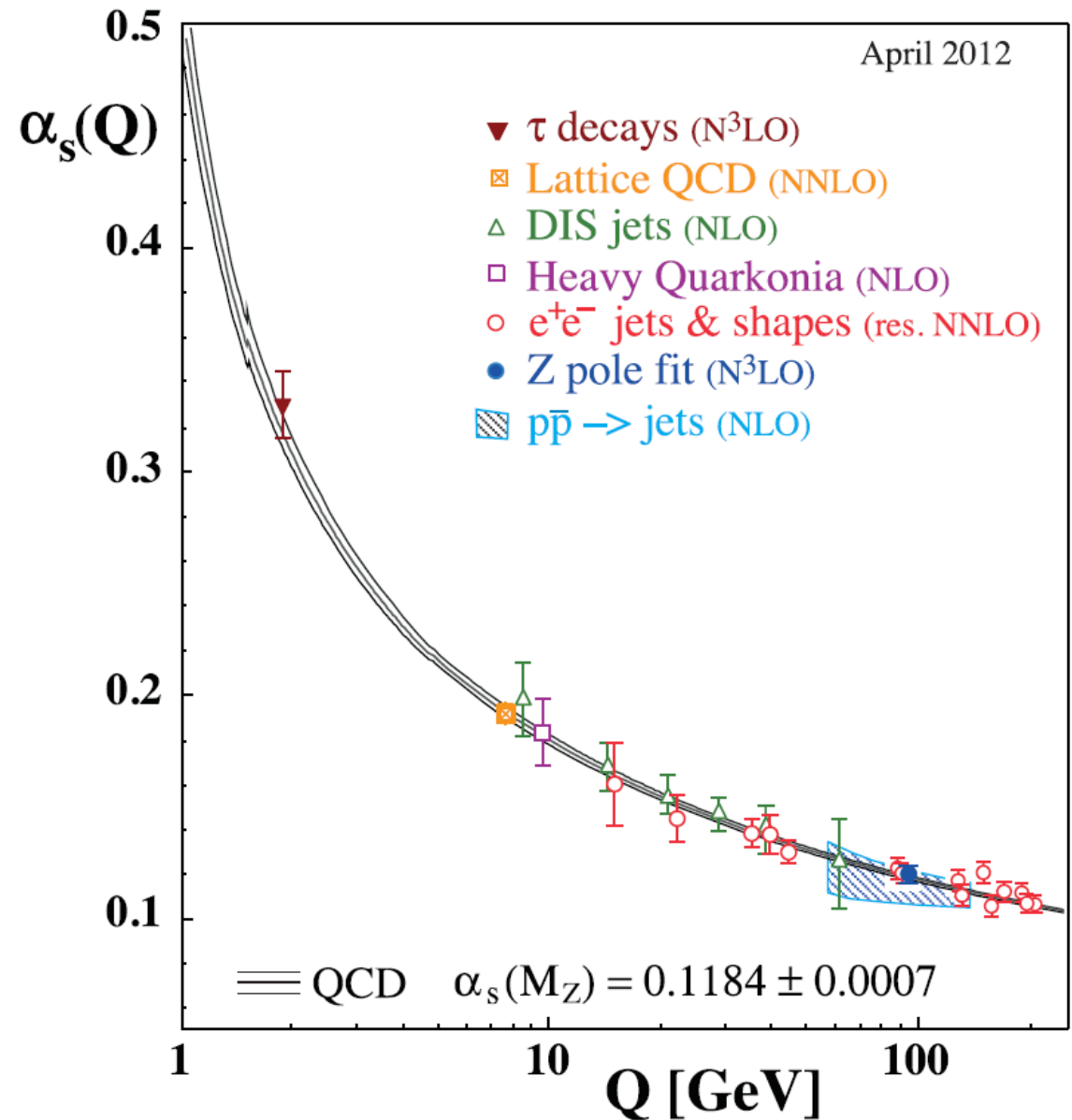
Outline



- Motivation
- Jet Measurements
 - + Inclusive Jets
 - + Multi-Jets
 - + Multi-Jet Ratios
- top-antitop Production
- α_s Summaries
- Perspectives with CMS and Beyond
- Summary

2012: No LHC results yet

PDG2012



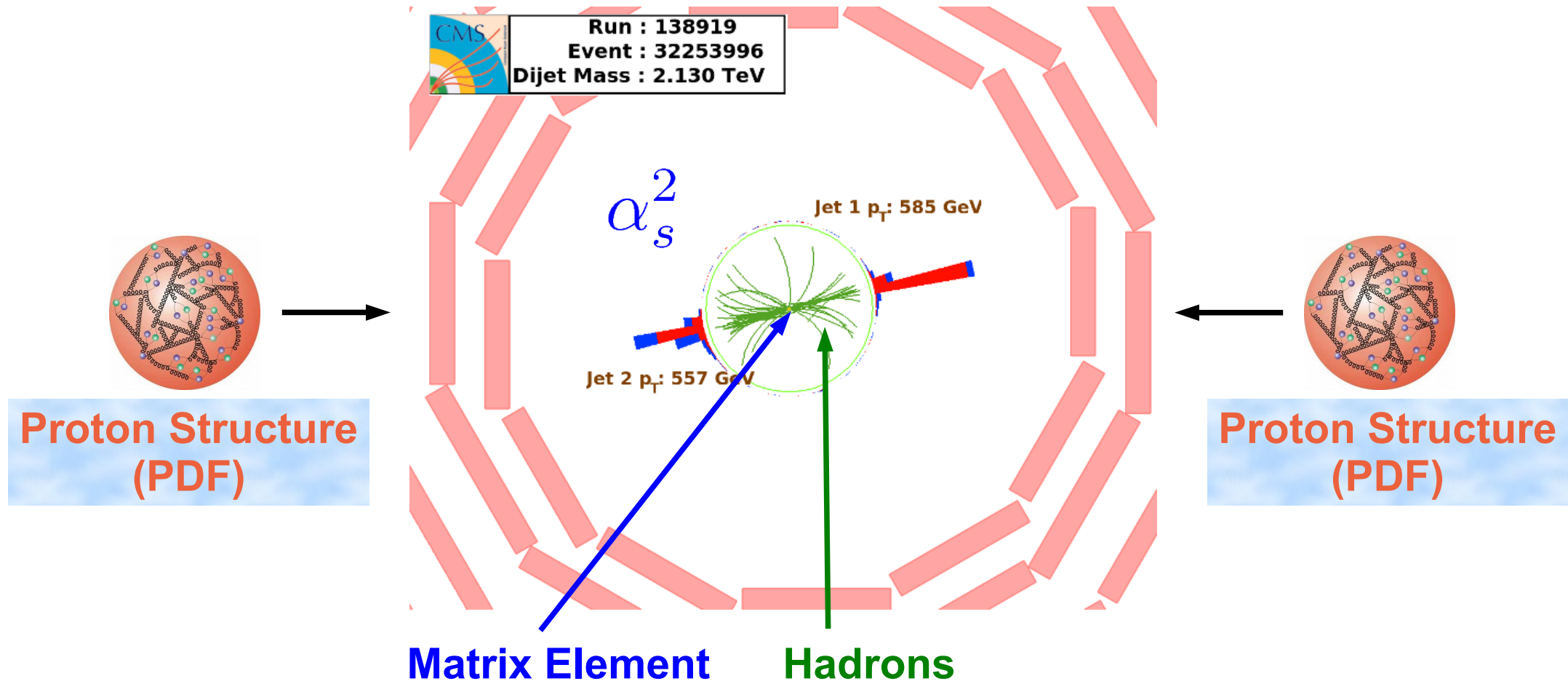


Jets at the LHC



Abundant production of jets:

- Jets at hadron colliders provide the highest reach ever to determine the strong coupling constant at high scales Q
- Also learn about hard QCD, the proton structure, non-perturbative effects, and electroweak effects at high Q



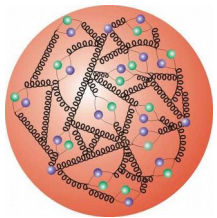
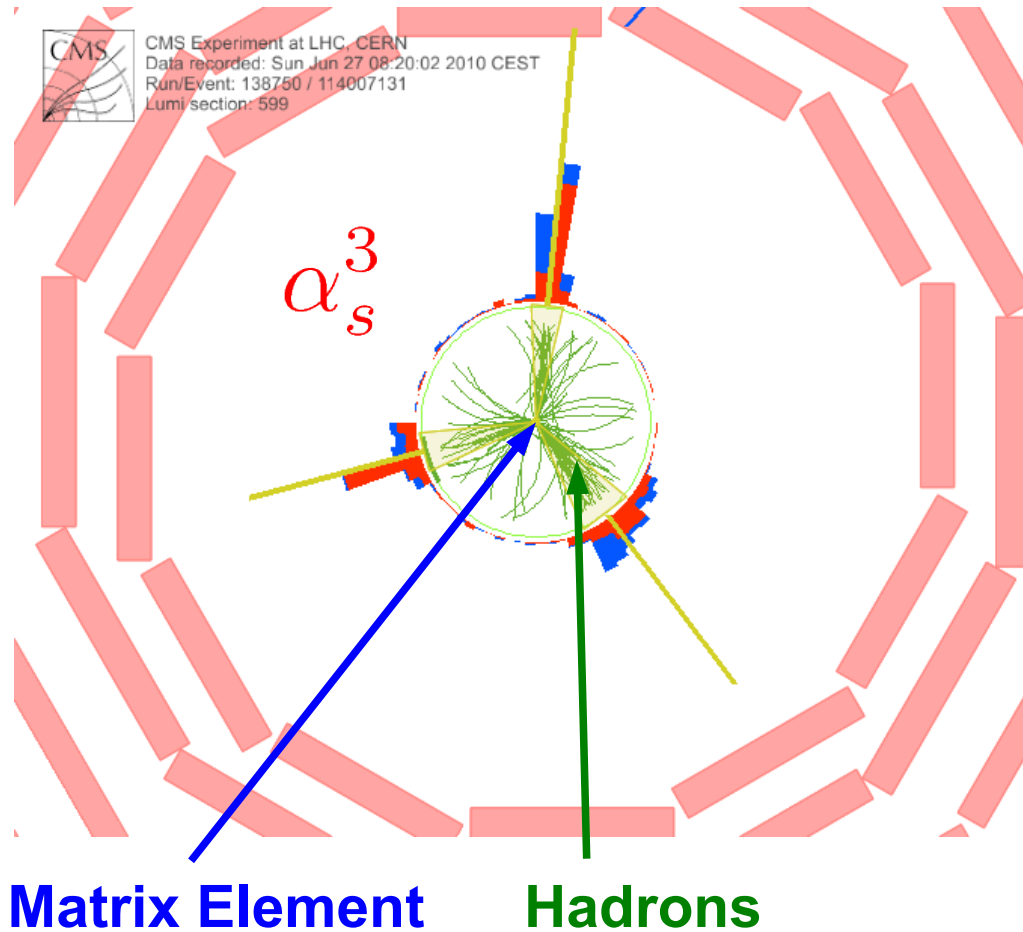


Jets at the LHC

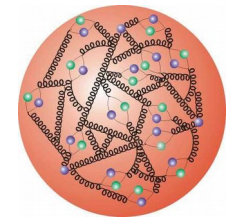


Abundant production of jets:

- Extract $\alpha_s(M_Z)$, the least precisely known fundamental constant!
- No multi-scale problem in this “Mercedes” start event :-)



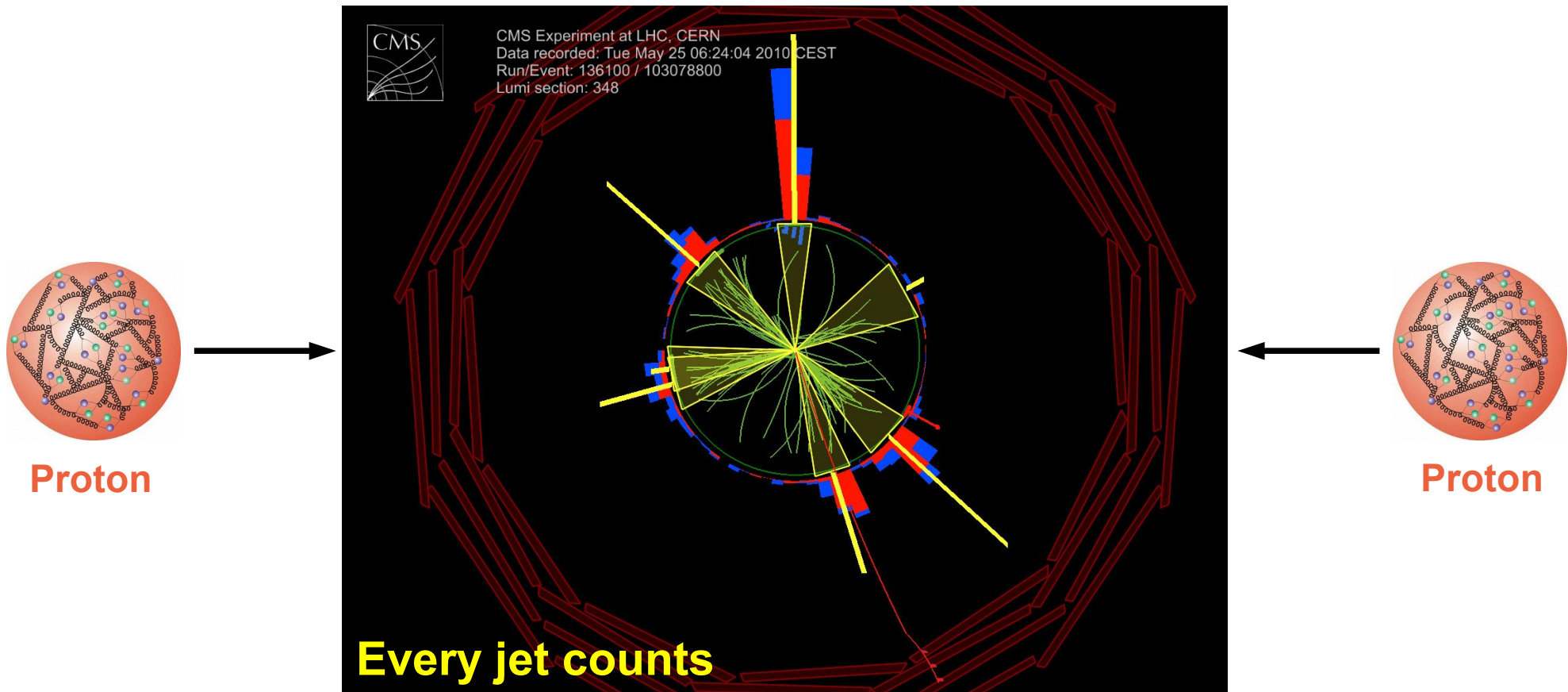
Proton Structure (PDF)



Proton Structure (PDF)



High transverse Momenta





Inclusive Jets



Agreement with predictions of QCD at NLO over many orders of magnitude in cross section and even beyond 2 TeV in jet p_T and for rapidities $|y|$ up to ~ 5
 Similar picture at 7 TeV, 8 TeV (left) or NEW 2.76 TeV (right)

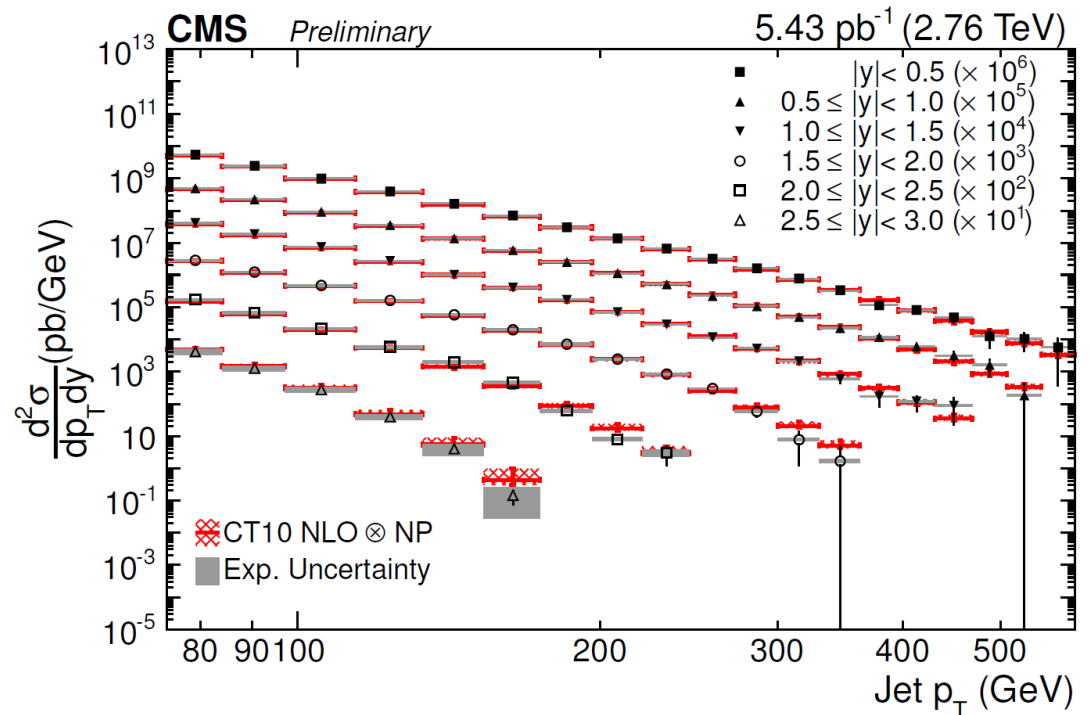
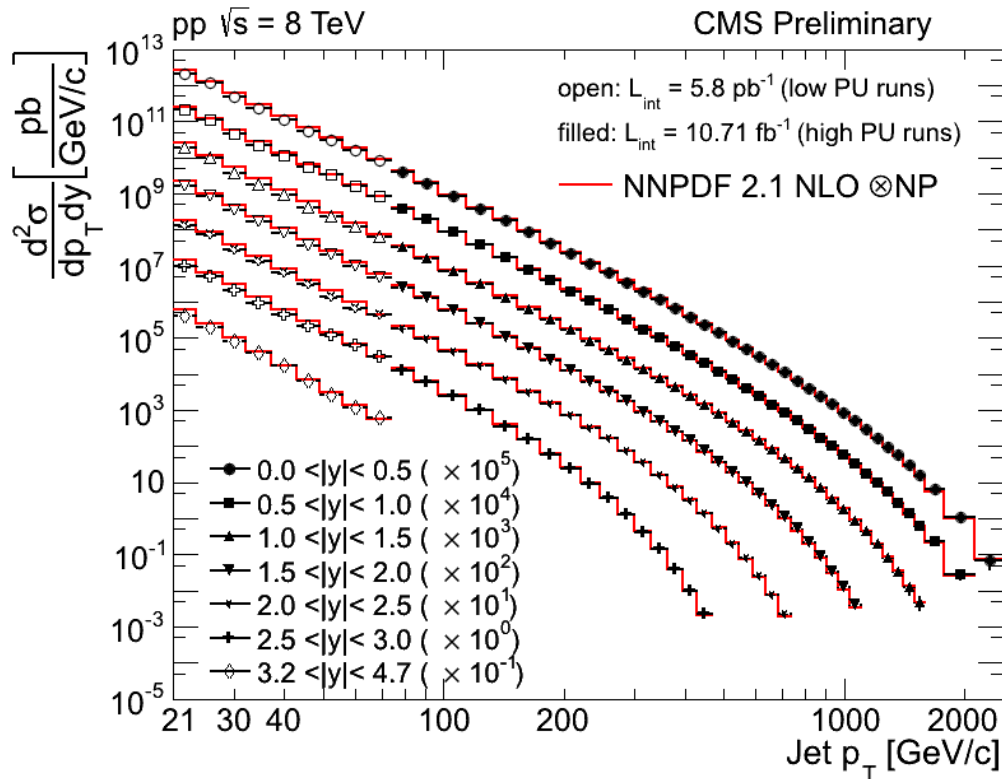
$$\frac{d^2\sigma}{dp_T dy} \propto \alpha_s^2$$

CMS-PAS-SMP-12-012 (2013),
 CMS-PAS-FSQ-12-031 (2013),
 CMS-PAS-SMP-14-017 (2015).

anti-kT, R=0.7, 8 TeV, 2012

Data vs. NLO pQCD
 ⊗ non-perturbative corrections

anti-kT, R=0.6, 2.76 TeV, 2012





Inclusive Jets + α_s & PDFs



Simultaneous fit of α_s & PDFs possible combining HERA 1 DIS & CMS jet data using HERAFitter Tool

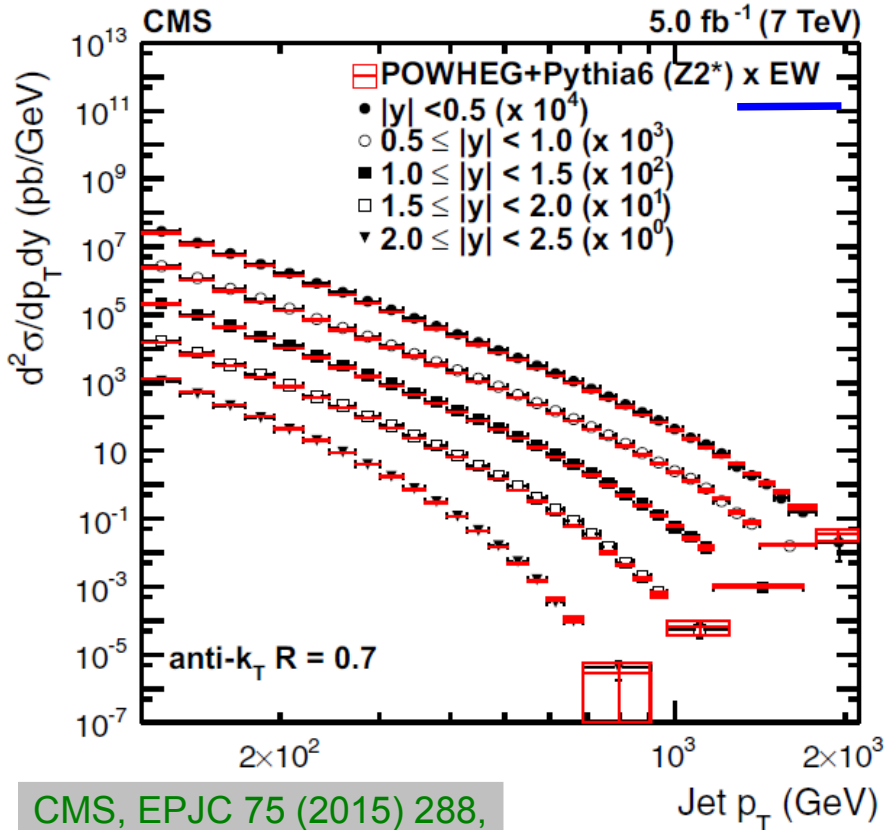
$$Q = p_{T,\text{jet}}$$

$$\frac{d^2\sigma}{dp_T dy} \propto \alpha_s^2$$

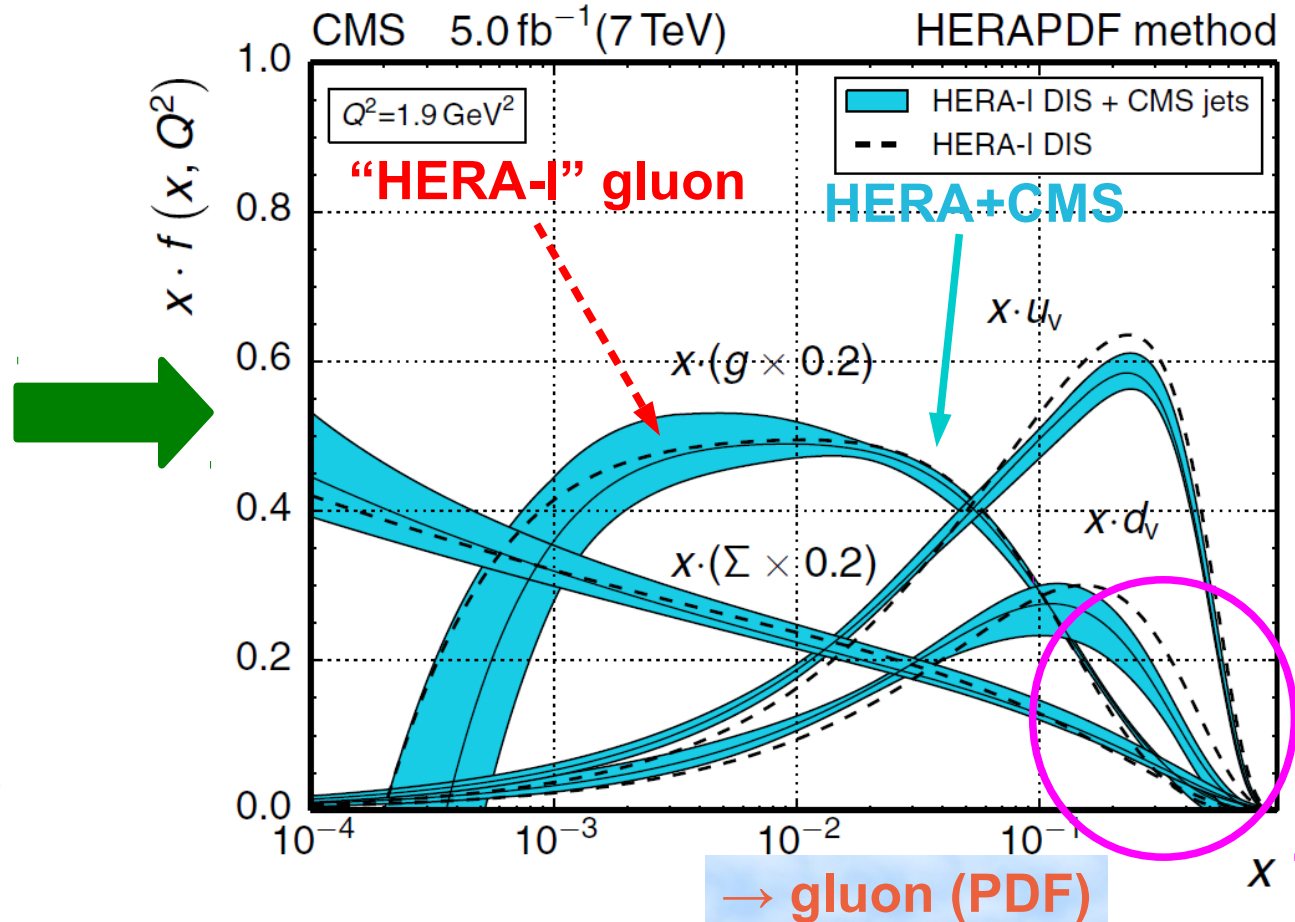
Data vs. NLO+PS \otimes EW corrections
→ impact visible in norm. dijet angular obs.

“Harder” gluon at high x compared to DIS

anti-k_T, R=0.7, 7 TeV, 2011



CMS, EPJC 75 (2015) 288,
JHEP 2011, 095 (2012).





Inclusive Jets + α_s & PDFs



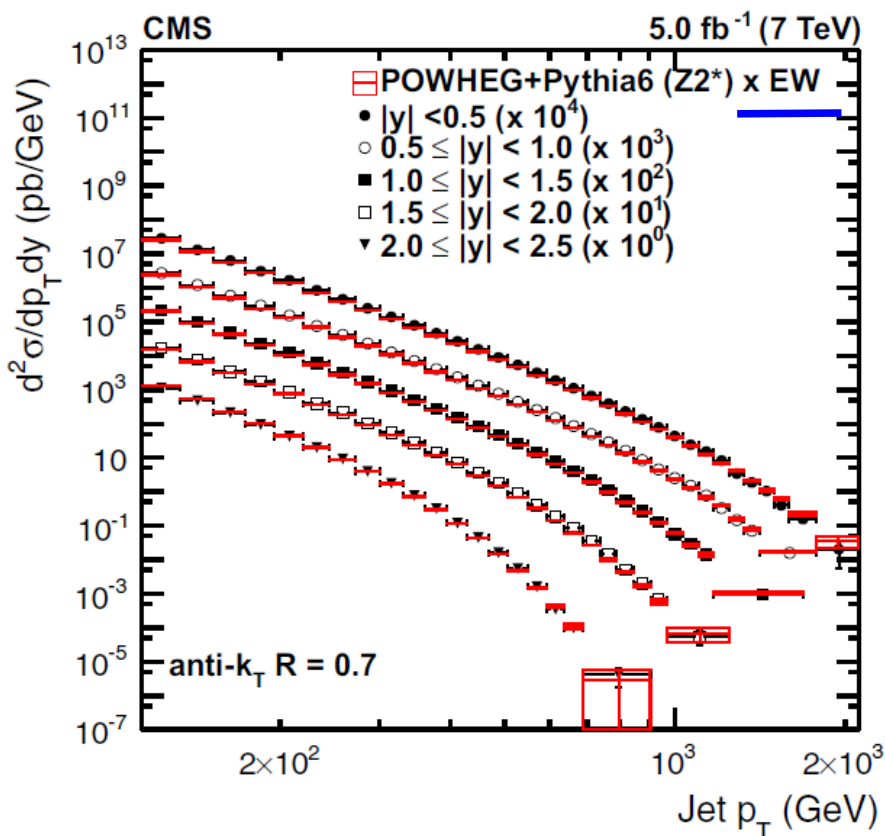
Simultaneous fit of α_s & PDFs possible combining HERA 1 DIS & CMS jet data using HERAFitter Tool

$$Q = p_{T,\text{jet}}$$

$$\frac{d^2\sigma}{dp_T dy} \propto \alpha_s^2$$

Data vs. NLO+PS \otimes EW corrections
→ impact visible in norm. dijet angular obs.

anti-k_T, R=0.7, 7 TeV, 2011



→ α_s

CT10-NLO: $\alpha_s(M_Z) = 0.1180$

NLO



$$\alpha_s(M_Z) = 0.1185 \pm 0.0019 \text{ (exp)}$$

$$\pm 0.0028 \text{ (PDF)} \pm 0.0004 \text{ (NP)} \pm 0.0053 \text{ (scale)}$$

$$= 0.1185 \pm 0.0035 \text{ (all w/o scale)}$$

→ α_s & gluon (PDF)



$$\alpha_s(M_Z) = 0.1192^{+0.0023}_{-0.0019} \text{ (all w/o scale)}$$

JHEP 2011, 095 (2012).



Details: α_s from inclusive Jets



$ y $ range	No. of data points	$\alpha_s(M_Z)$	χ^2/n_{dof}
$ y < 0.5$	33	0.1189 ± 0.0024 (exp) ± 0.0030 (PDF) ± 0.0008 (NP) $^{+0.0045}_{-0.0027}$ (scale)	16.2/32
$0.5 \leq y < 1.0$	30	0.1182 ± 0.0024 (exp) ± 0.0029 (PDF) ± 0.0008 (NP) $^{+0.0050}_{-0.0025}$ (scale)	25.4/29
$1.0 \leq y < 1.5$	27	0.1165 ± 0.0027 (exp) ± 0.0024 (PDF) ± 0.0008 (NP) $^{+0.0043}_{-0.0020}$ (scale)	9.5/26
$1.5 \leq y < 2.0$	24	0.1146 ± 0.0035 (exp) ± 0.0031 (PDF) ± 0.0013 (NP) $^{+0.0037}_{-0.0020}$ (scale)	20.2/23
$2.0 \leq y < 2.5$	19	0.1161 ± 0.0045 (exp) ± 0.0054 (PDF) ± 0.0015 (NP) $^{+0.0034}_{-0.0032}$ (scale)	12.6/18
$ y < 2.5$	133	0.1185 ± 0.0019 (exp) ± 0.0028 (PDF) ± 0.0004 (NP) $^{+0.0053}_{-0.0024}$ (scale)	104.1/132

Fit results in separate $|y|$ bins
PDF: CT10-NLO
(best consistency between fit and PDF preferred $\alpha_s(M_Z)$)

Fit results for all $|y|$ bins with other PDFs

	$\alpha_s(M_Z)$	χ^2/n_{dof}
CT10-NLO	0.1185 ± 0.0019 (exp) ± 0.0028 (PDF) ± 0.0004 (NP) $^{+0.0053}_{-0.0024}$ (scale)	104.1/132
NNPDF2.1-NLO	0.1150 ± 0.0015 (exp) ± 0.0024 (PDF) ± 0.0003 (NP) $^{+0.0025}_{-0.0025}$ (scale)	103.5/132
MSTW2008-NLO	0.1159 ± 0.0012 (exp) ± 0.0014 (PDF) ± 0.0001 (NP) $^{+0.0024}_{-0.0030}$ (scale)	107.9/132
CT10-NNLO	0.1170 ± 0.0012 (exp) ± 0.0024 (PDF) ± 0.0004 (NP) $^{+0.0044}_{-0.0030}$ (scale)	105.7/132
NNPDF2.1-NNLO	0.1175 ± 0.0012 (exp) ± 0.0019 (PDF) ± 0.0001 (NP) $^{+0.0018}_{-0.0020}$ (scale)	103.0/132
MSTW2008-NNLO	0.1136 ± 0.0010 (exp) ± 0.0011 (PDF) ± 0.0001 (NP) $^{+0.0019}_{-0.0024}$ (scale)	108.8/132

CMS, EPJC 75 (2015) 288.



Inclusive Jet Ratios: "2.76 / 8.0"



New from CMS:

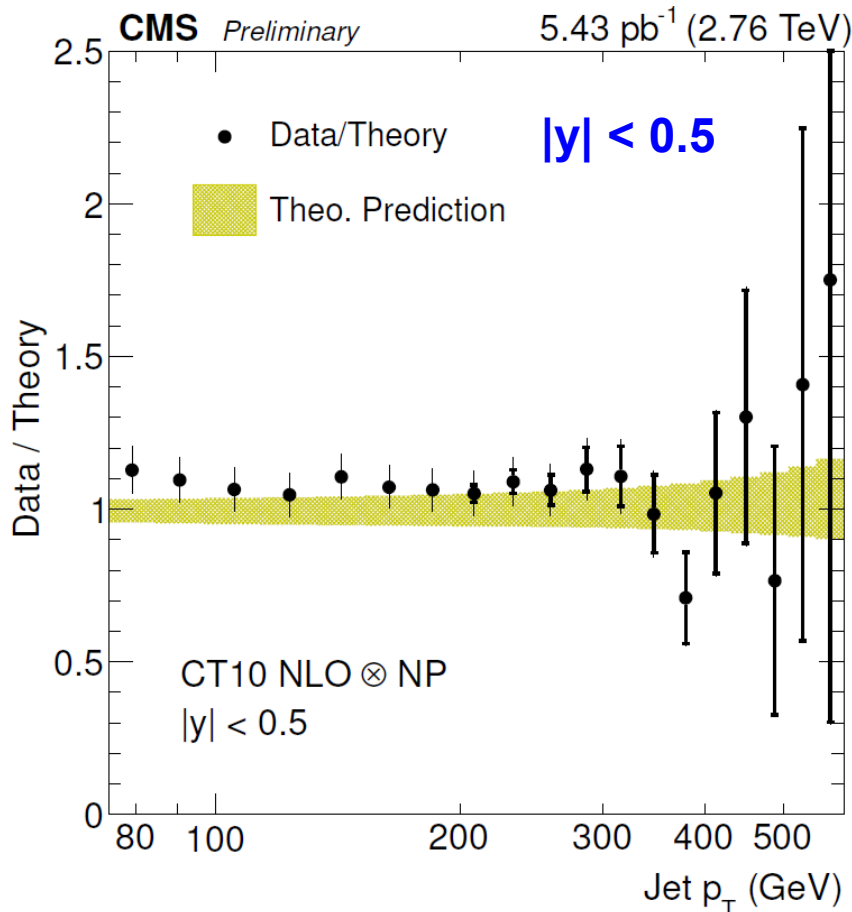
- cross sections at 2.76 TeV
- ratios to 8 TeV

Shown

- double ratio to theory

Ratio at $E_{\text{cms}} = 2.76$ and 8.0 TeV

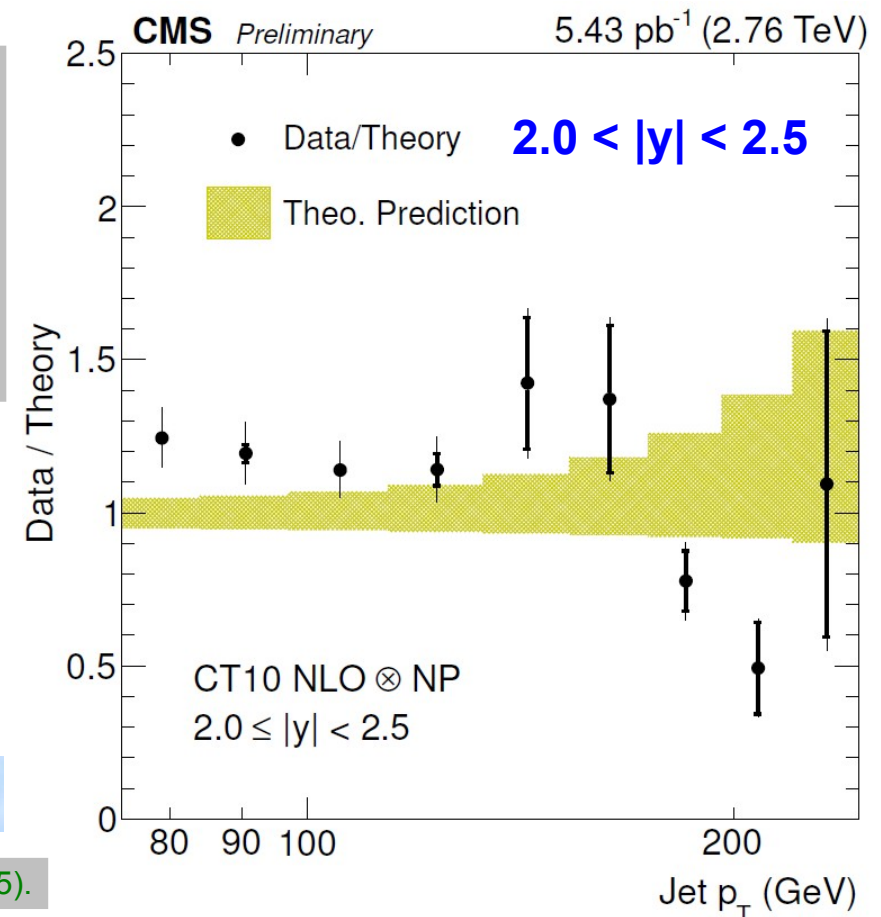
- at least partial cancellation of uncertainties
- more precise comparisons



Disentangle α_s & $g(x)$ by fitting in wide phase space of p_T , y , and \sqrt{s} ?

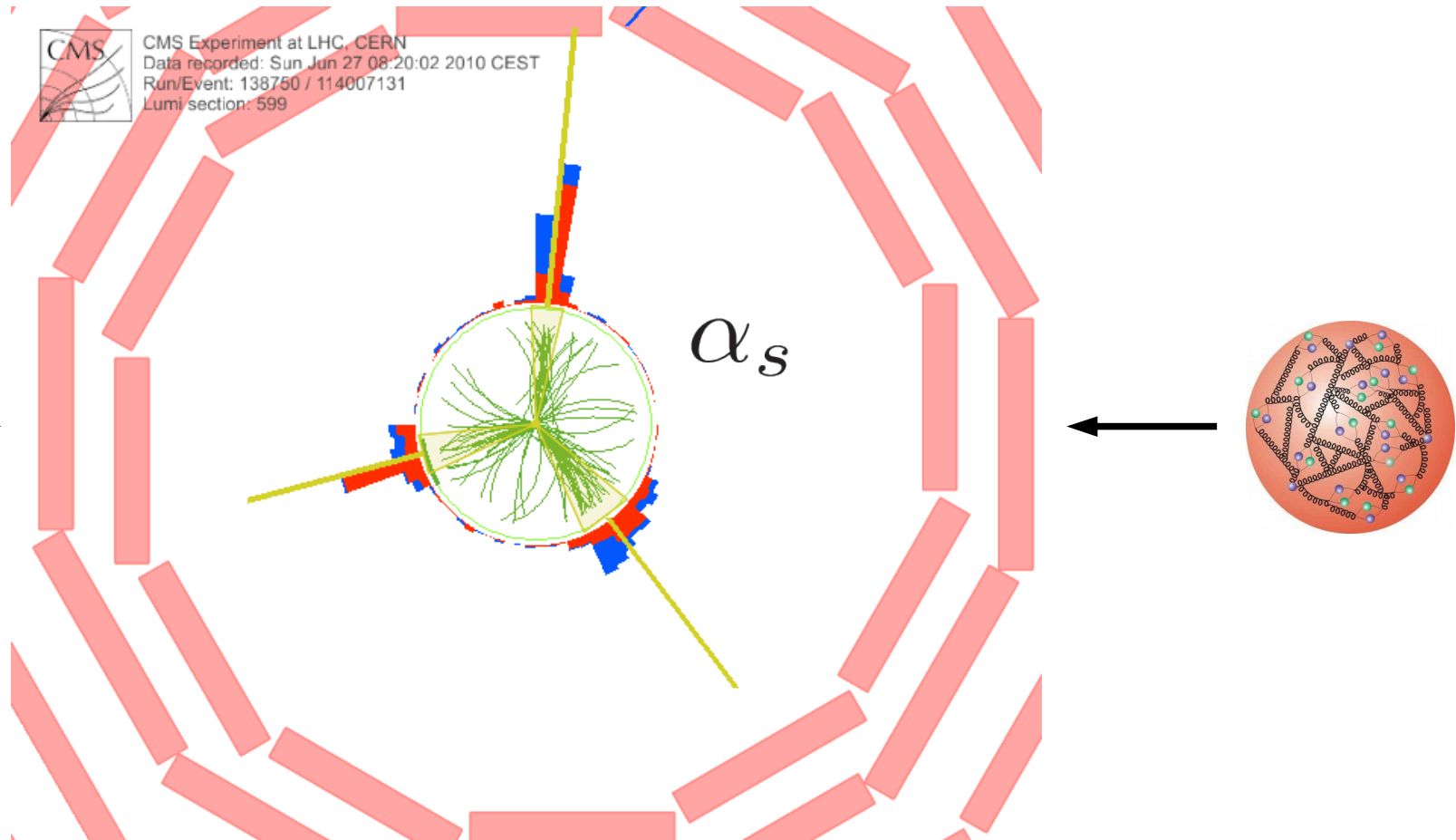
→ gluon (PDF)

CMS-PAS-SMP-14-017 (2015).





Multi-Jets and α_s





Azimuthal Decorrelations at 8 TeV

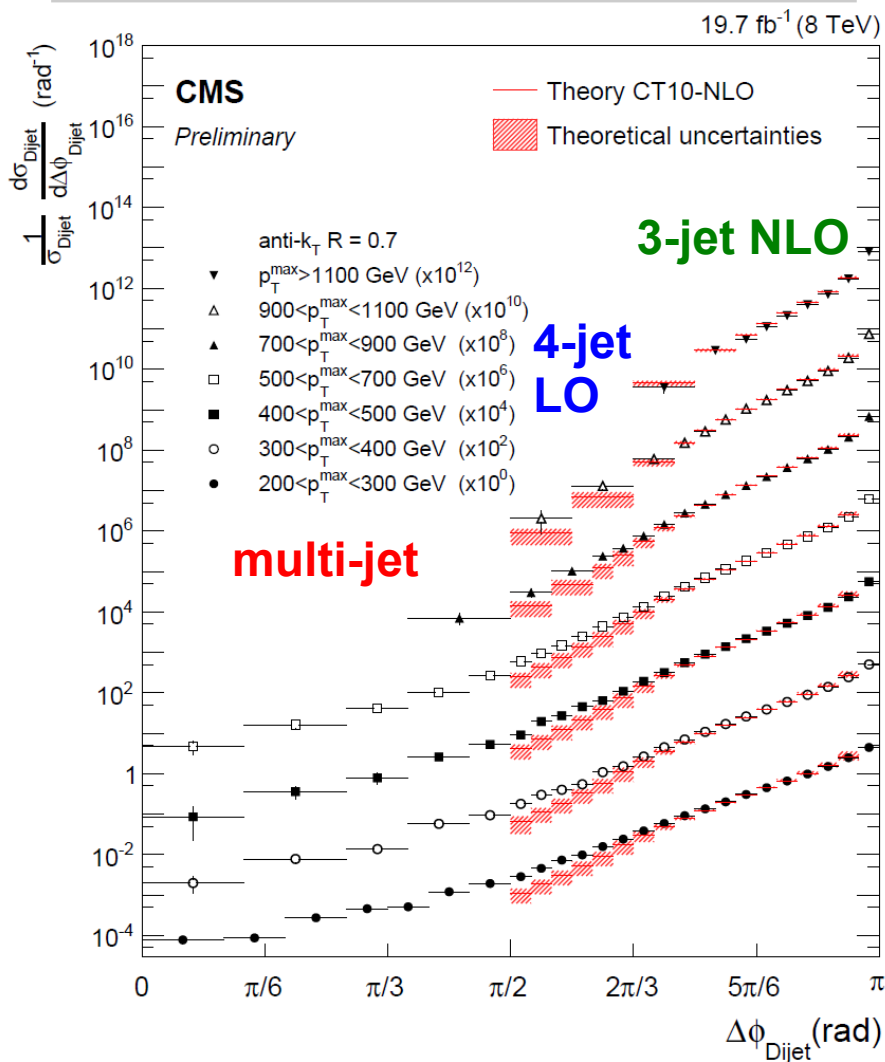


$\Delta\phi_{jj}$ in bins of p_{T1}

- dijet LO has always $\Delta\phi_{jj} = \pi$
- deviations through multi-jets

Related ratio observable $R_{\Delta\phi}$ proposed for α_s det.

Wobisch et al., JHEP01 (2013) 172, D0, PLB 721 (2013) 212.



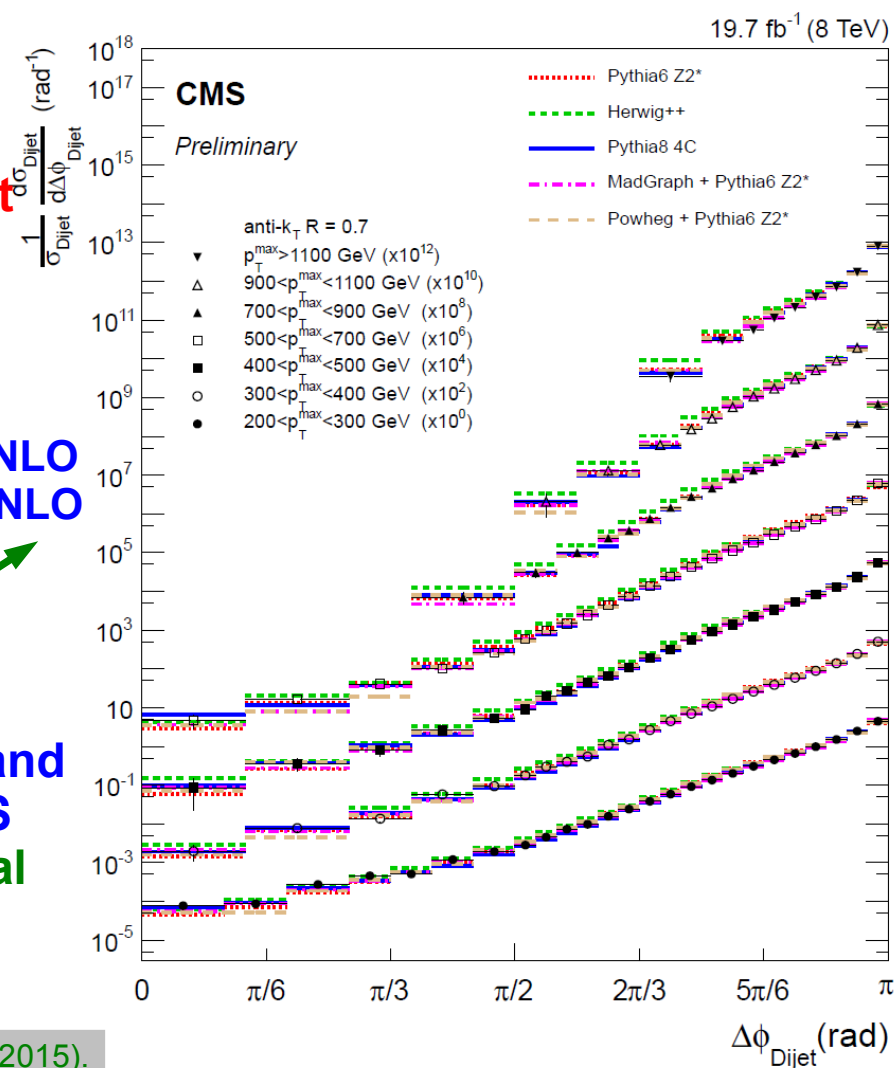
Comparison to fixed-order PQCD

→ need multijet NLO

Sherpa + BlackHat → 4-jet NLO
Njet → 5-jet NLO
to be checked

Comparison to LO ME+PS and multijet ME+PS
→ good general description

CMS-PAS-SMP-14-015 (2015).





3-Jet Mass

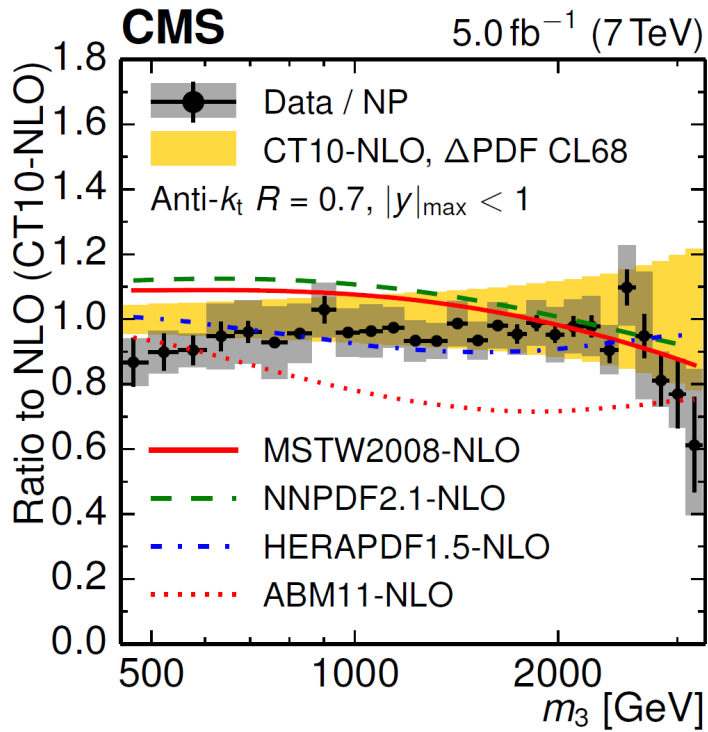


Sensitive to α_s beyond 2→2 process

NLO with 3-4 partons (NLOJet++)

Sensitive to PDFs

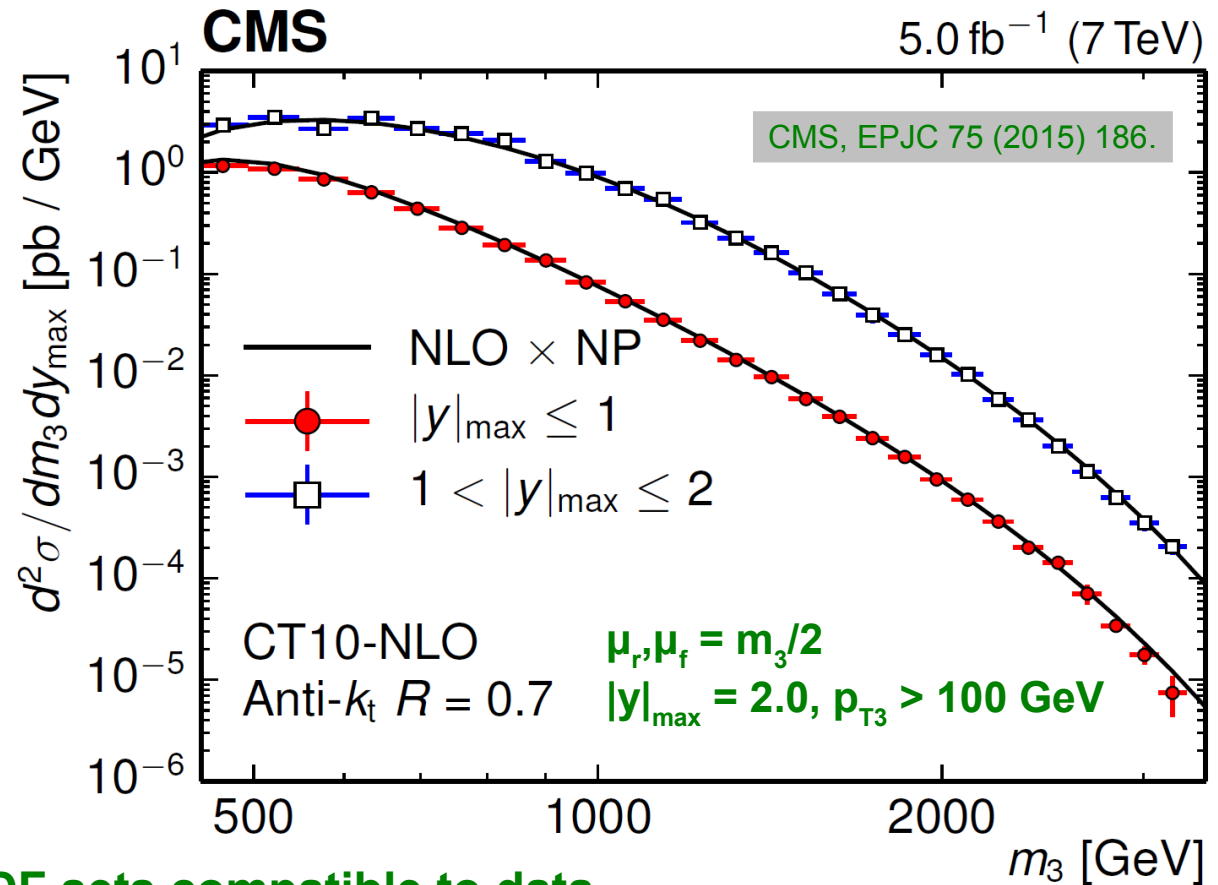
Involves additional "scale" $p_{T,3}$



Most PDF sets compatible to data

Extraction of $\alpha_s(M_Z)$: → α_s

Dominated by theory uncertainty (NLO)!



$$Q = m_3/2 \quad \frac{d\sigma_{3jet}}{dm_{3jet}} \propto \alpha_s^3$$

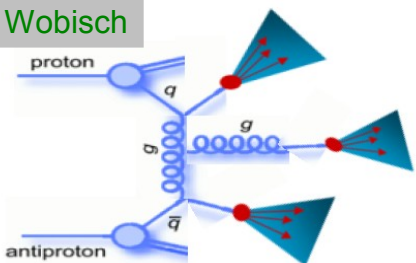
$$\alpha_s(M_Z) = 0.1171 \pm 0.0013(\text{exp}) \pm 0.0024(\text{PDF}) \pm 0.0008(\text{NP}) \pm 0.0069 \text{ (scale)} \text{ } ^{-0.0040}$$



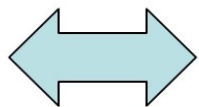
3- to 2-Jet Ratios



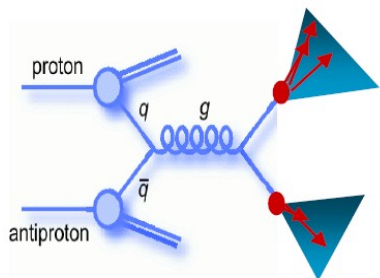
M. Wobisch



$R_{3/2}$



α_s



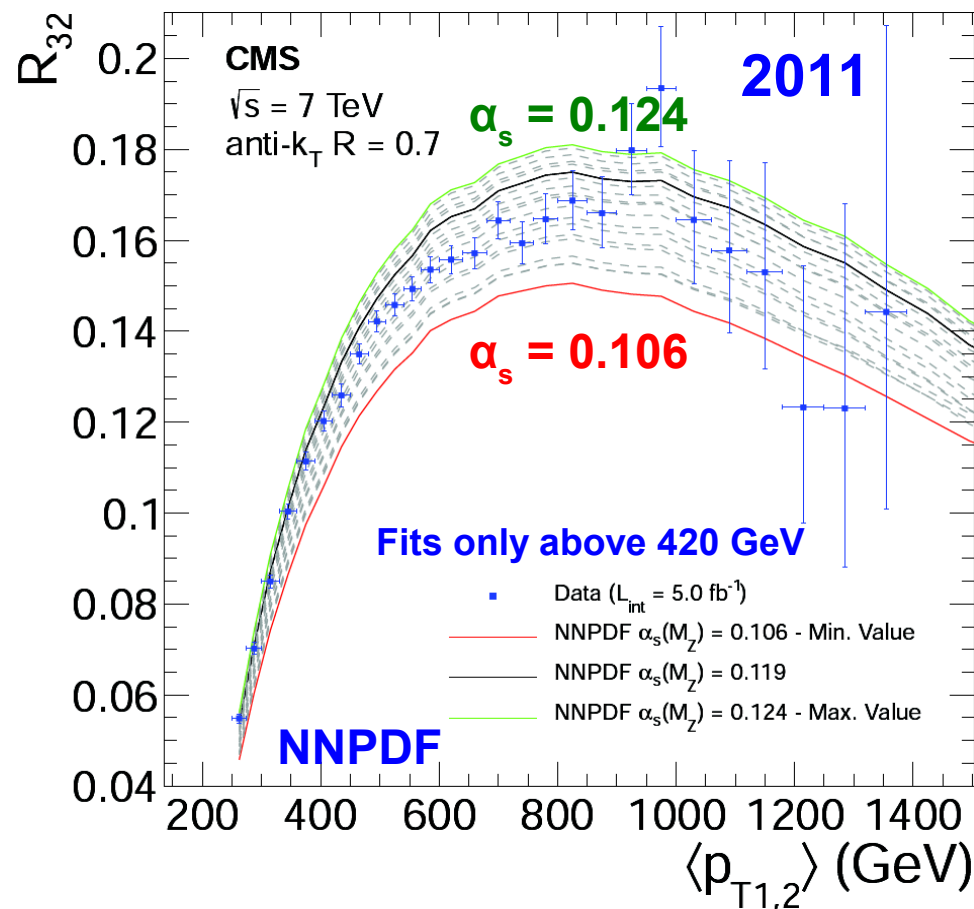
$$\frac{\sigma_{3+jet}}{\sigma_{2+jet}} \propto \alpha_s^1$$

$$Q = \langle p_{T1,2} \rangle$$

CMS: $R_{3/2}$

- Ratio of inclusive 3- to inclusive 2-jet events
- anti-kT R=0.7
- Min. jet pT: 150 GeV
- Max. rap.: $|y| < 2.5$
- Data 2011, 5/fb

$\rightarrow \alpha_s$



Similarly described by CT10 or MSTW2008

$$\alpha_s(M_Z) = 0.1148 \pm 0.0014 \text{ (exp)}$$

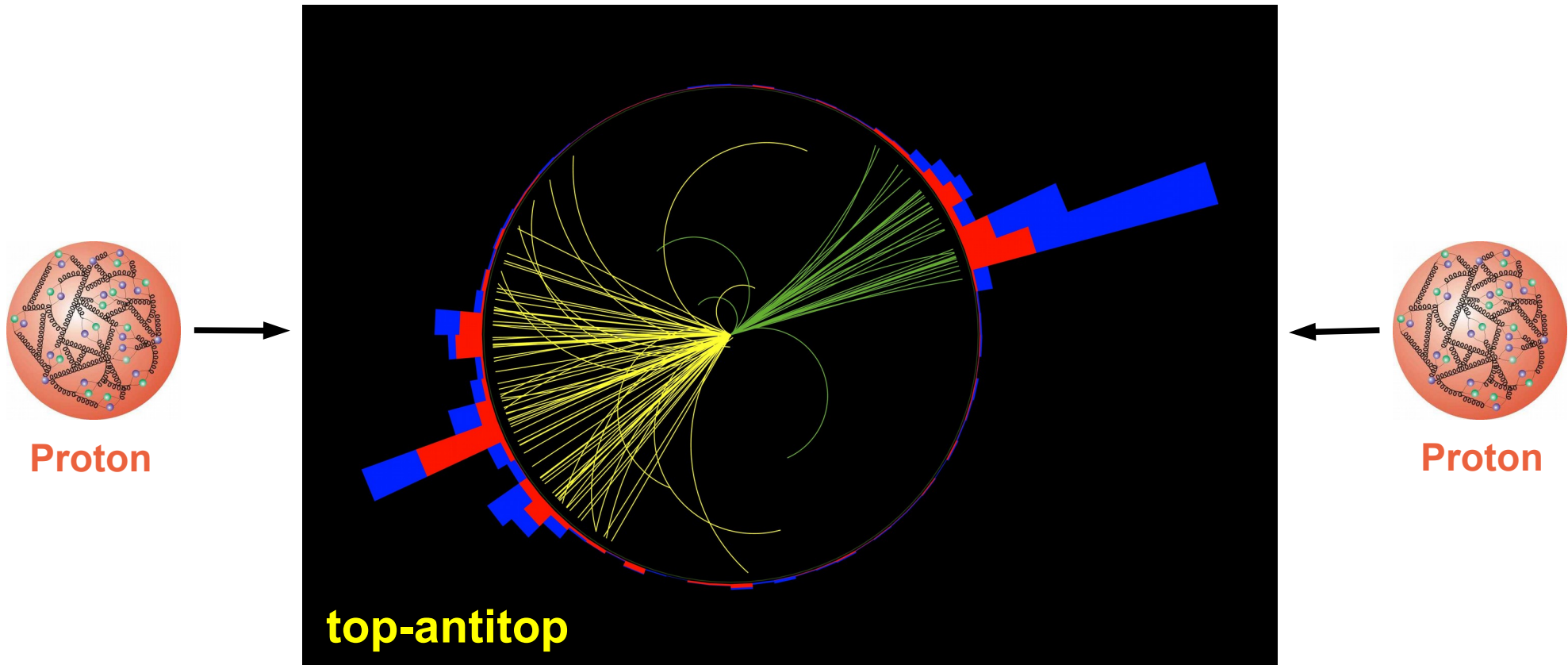
$$\pm 0.0018 \text{ (PDF)} \pm 0.0050 \text{ (theory)}$$

Dominated by NLO theory uncertainty!

CMS, EPJC 73 (2013) 2604.



High Masses





Fits with top-pair Production



Top-pair production is especially sensitive to:
 m_t^{pole} and α_s and $g(x, \mu_f^2)$ as the main production process at LHC is from gg
 Using only the $t\bar{t}$ cross section measurement (dilepton channel) combined fits are not possible. **Fixing the gluon** to one of 5 PDF sets, however, it is possible to extract m_t^{pole} while fixing α_s or vice versa.

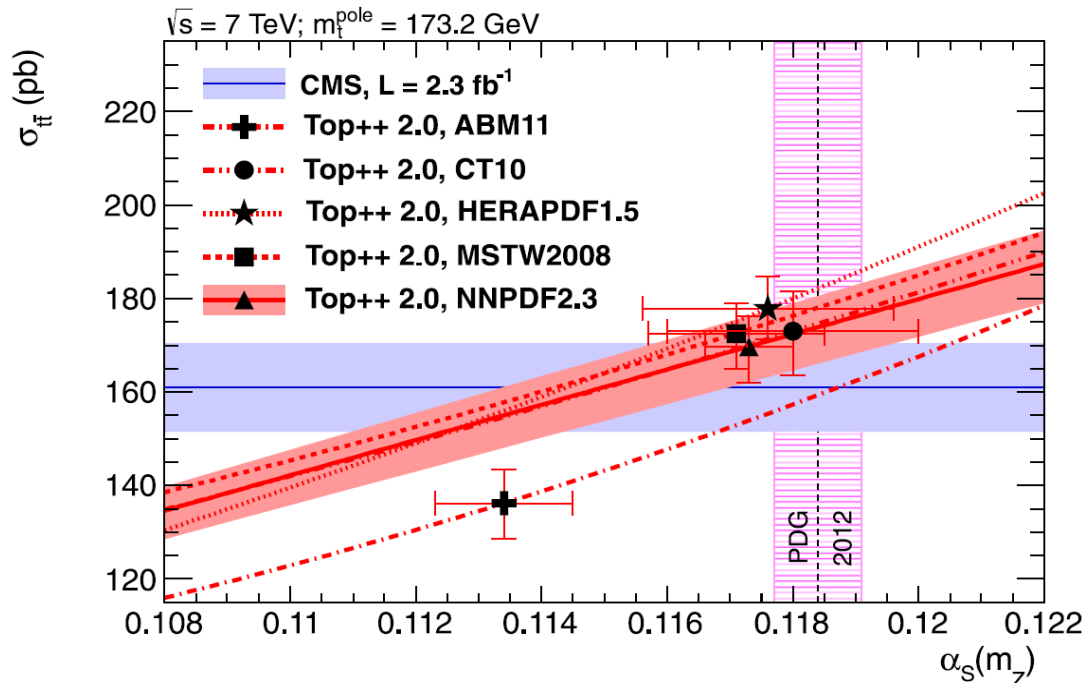
$$\alpha_s(M_Z) = 0.1151 \pm 0.0025(\text{exp})_{-0.0011}^{+0.0013}(\text{PDF})$$

NNLO + NNLL

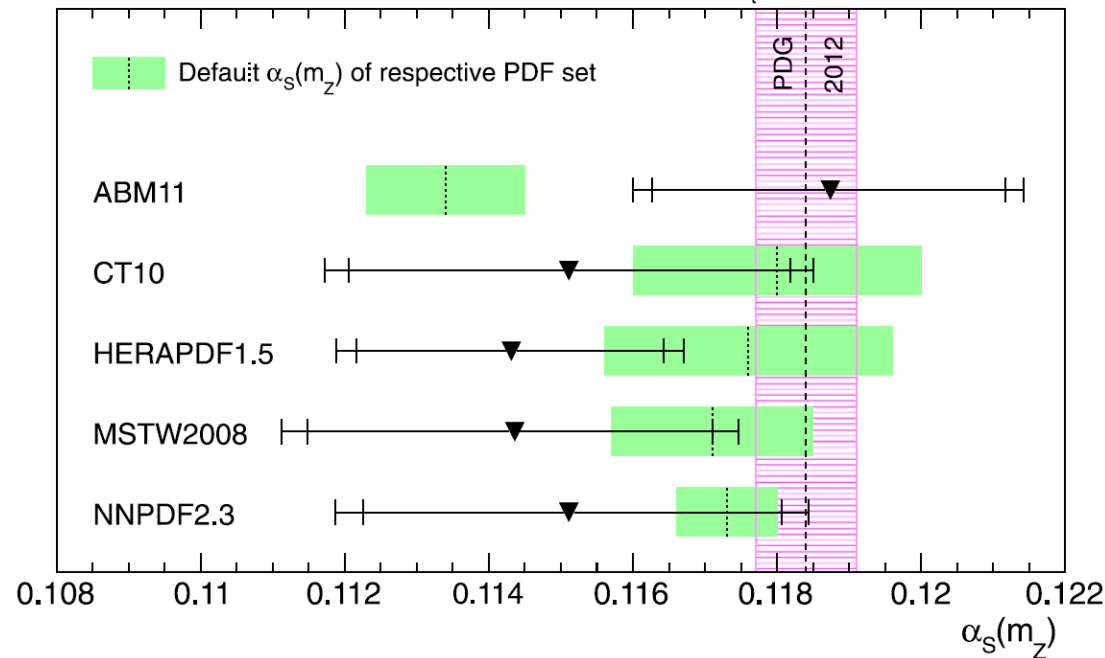
$$+0.0009_{-0.0008}(\text{scale}) \pm 0.0013(m_t^{\text{pole}}) \pm 0.0008(E_{\text{LHC}})$$

new top related

Fix m_t^{pole} → constrain α_s



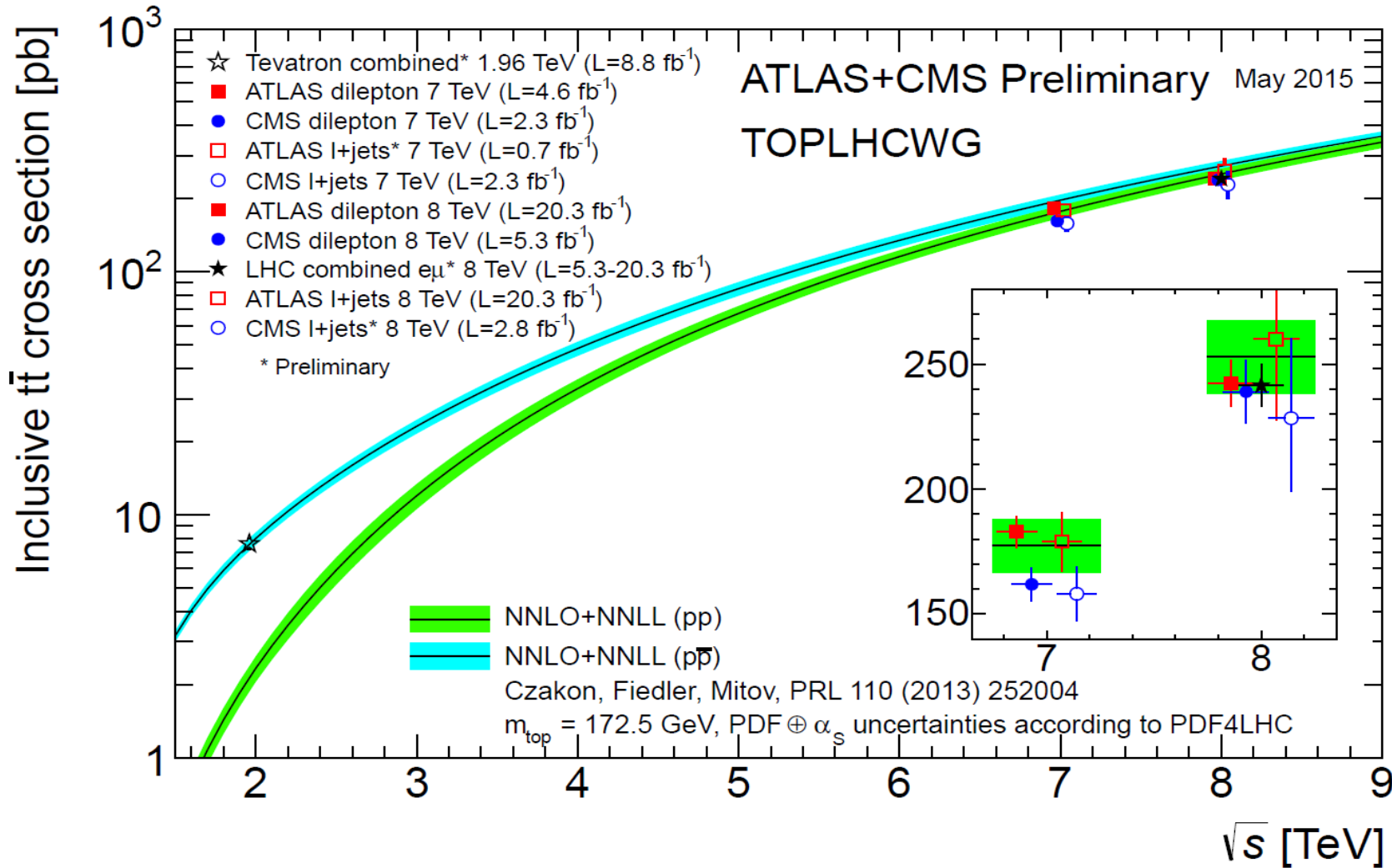
CMS, $\sqrt{s} = 7 \text{ TeV}, L = 2.3 \text{ fb}^{-1}$; NNLO+NNLL for $\sigma_{t\bar{t}}$; $m_t^{\text{pole}} = 173.2 \pm 1.4 \text{ GeV}$



CMS, PLB 728, 496 (2013), JHEP 11, 067 (2012).



ttbar Dilepton X Section in Comparison



New CMS prelim. results move up somewhat, but within uncertainty. 2 X (@ 7 TeV) and 4 X (@ 8 TeV) more data, improved reconstruction, plus further refinements.

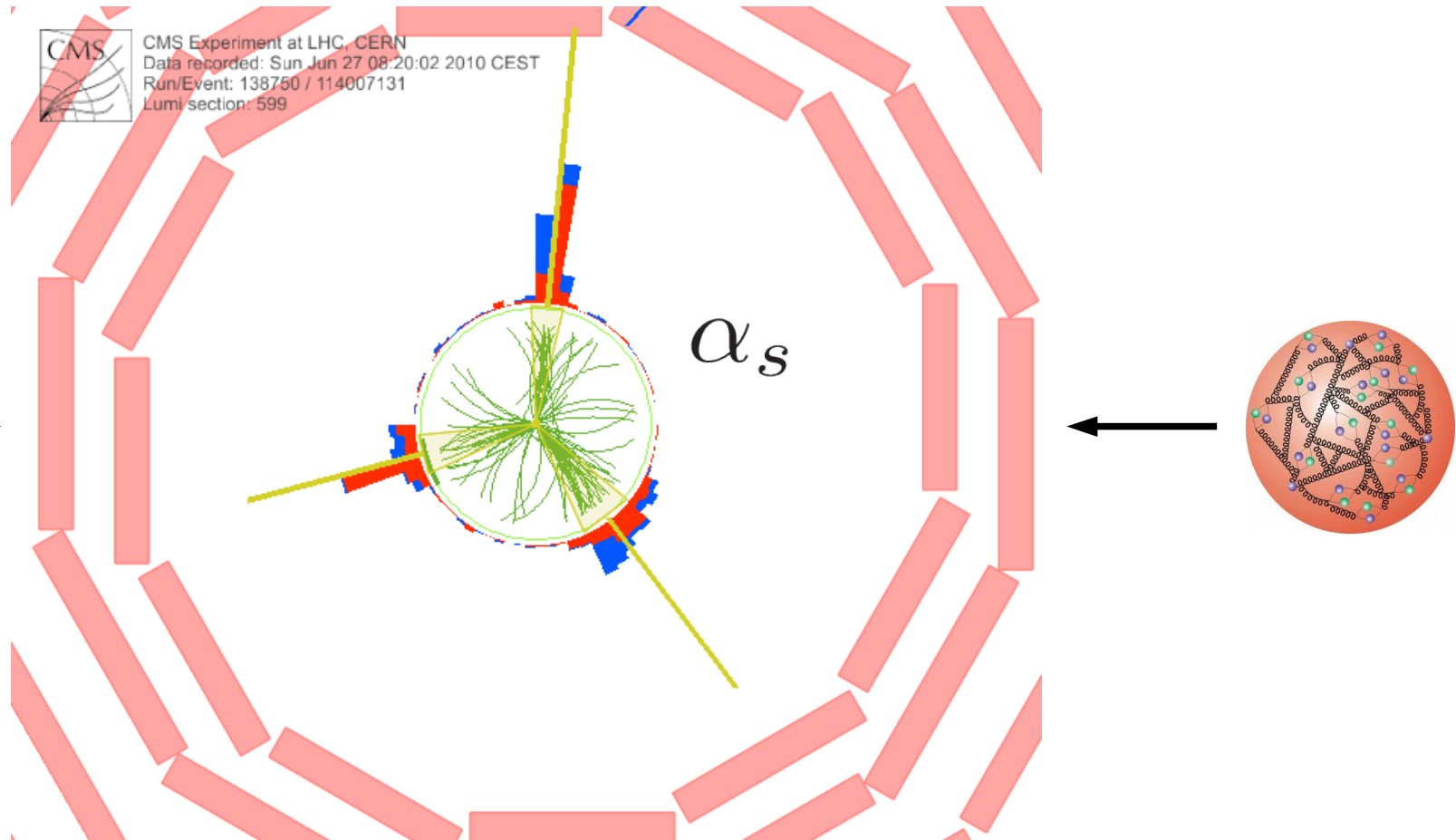
$$\sigma_{t\bar{t}} = 174.5 \pm 2.1(\text{stat}) \pm_{4.0}^{4.5}(\text{syst}) \pm 3.8(\text{lumi}) \text{ pb} \quad \text{at } \sqrt{s} = 7 \text{ TeV and}$$

$$\sigma_{t\bar{t}} = 245.6 \pm 1.3(\text{stat}) \pm_{5.5}^{6.6}(\text{syst}) \pm 6.5(\text{lumi}) \text{ pb} \quad \text{at } \sqrt{s} = 8 \text{ TeV,}$$

CMS-TOP-13-004 (2012).

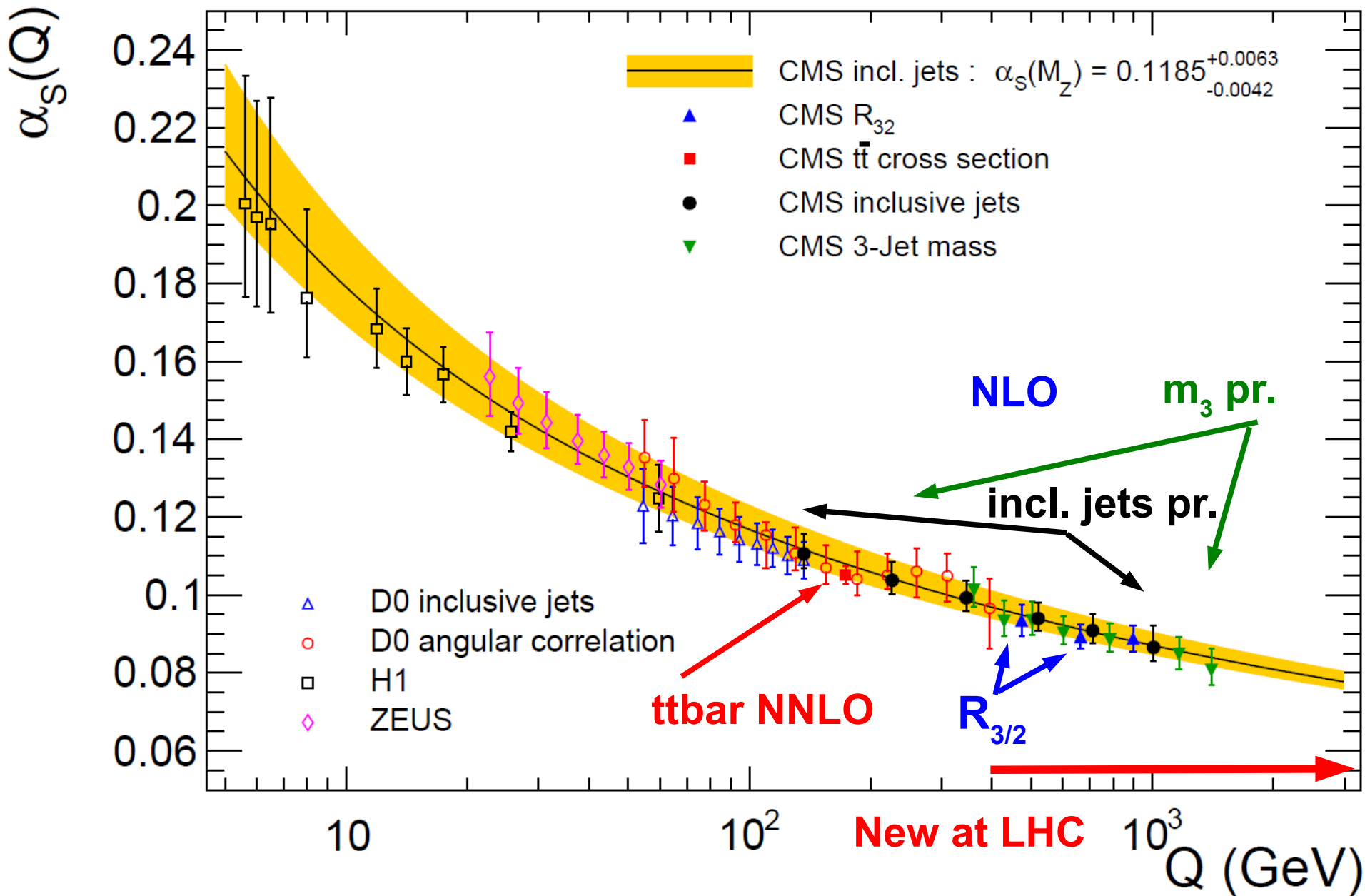


$\alpha_s (1 \text{ TeV}) ?$



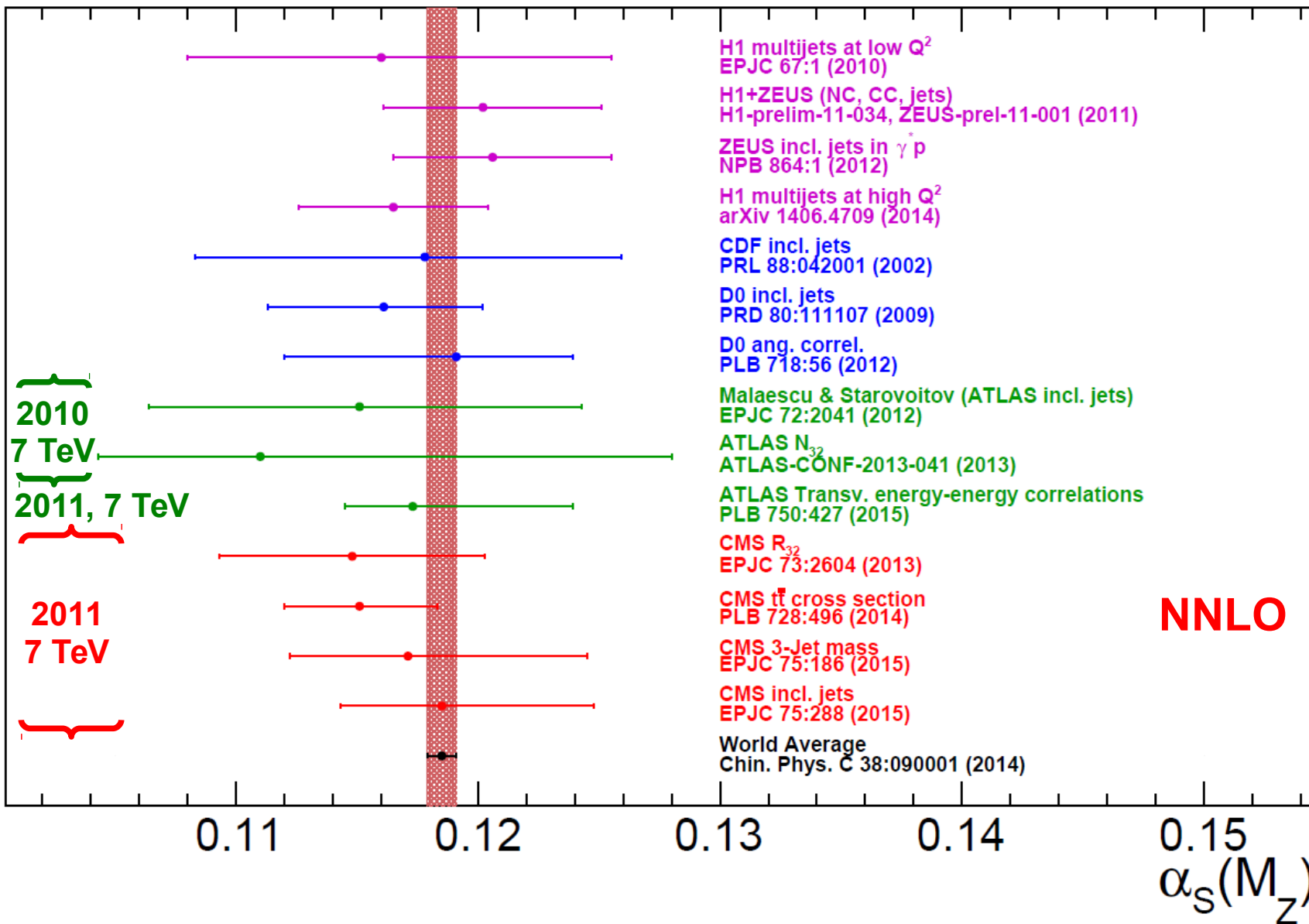


CMS Summary





Hadron Collider Summary



$\Delta\alpha_s/\alpha_s / \%$
exp PDF scale

1.2 1.4 7.3

no final publ.

1.9 1.9 2.5

0.7 0.8 3.1

7.5 5.0 5.0

2.9 1.0 2.3

0.7 1.2 4.7

4.1 1.8 2.4

no final publ.

0.9 1.4 3.5

1.2 1.6 4.4

2.3 1.0 0.7

1.1 2.0 4.7

1.6 2.4 3.2



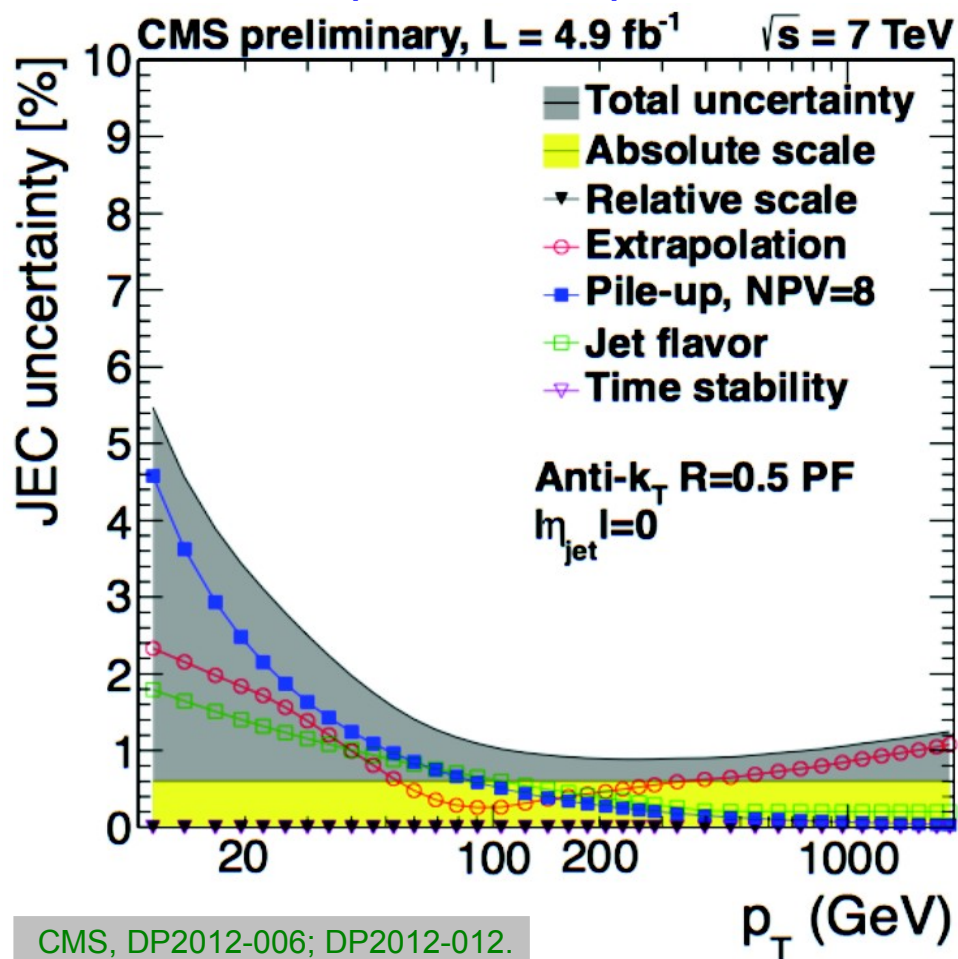
Perspectives with CMS and Beyond



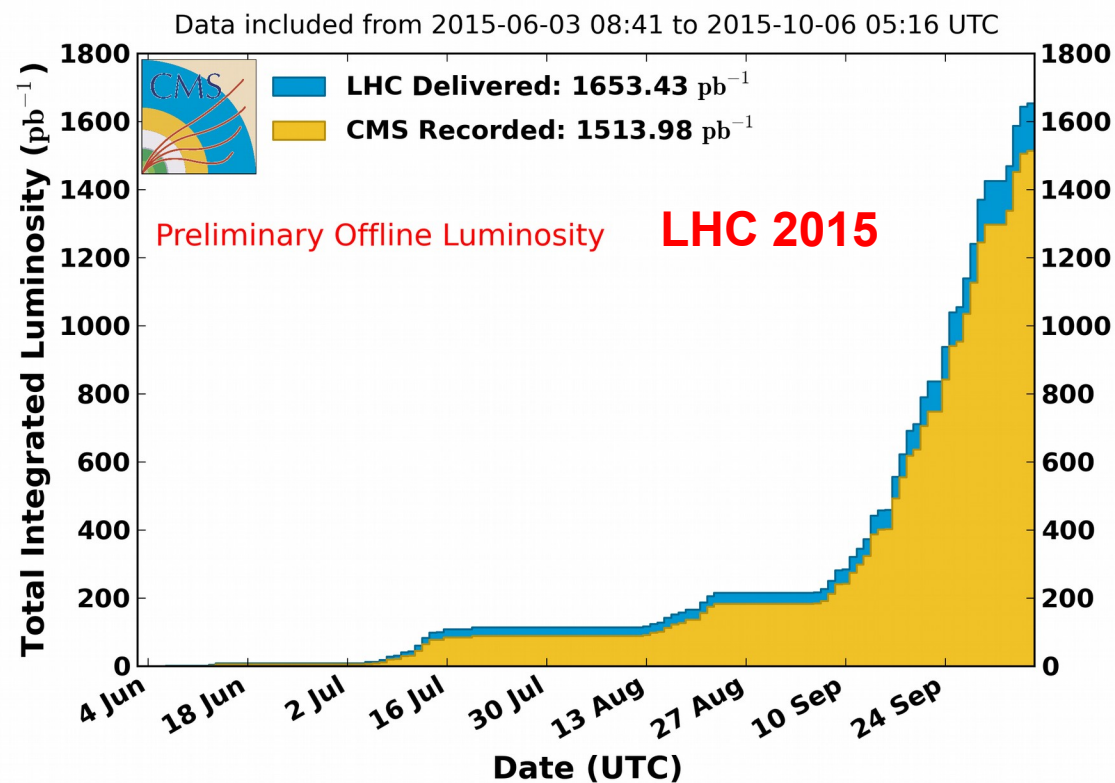
CMS: Jet energy scale 1 – 3 % prel. (Run 1)
 → percent level precision at medium p_T
 → more precise $\alpha_s(M_Z)$

LHC: $E_{\text{cms}} = 0.9, 2.36, 2.76, 7, 8,$ and now 13 TeV
 → much higher reach to check $\alpha_s(Q)$

CMS from 5/fb (7 TeV, 2011)



CMS Integrated Luminosity, pp, 2015, $\sqrt{s} = 13 \text{ TeV}$





Jet Energy Scale and α_s



Two goals for α_s :

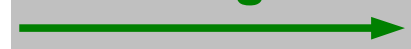
1. Measure the running of $\alpha_s(Q)$ up to the highest scales possible
→ In CMS mostly looked into $\alpha_s(Q)$!
2. Measure $\alpha_s(M_Z)$ as precisely as possible
→ For $\alpha_s(M_Z)$ might want to stay at minimal JEC uncertainty:
200 – 800 GeV, central rapidity

Better in:

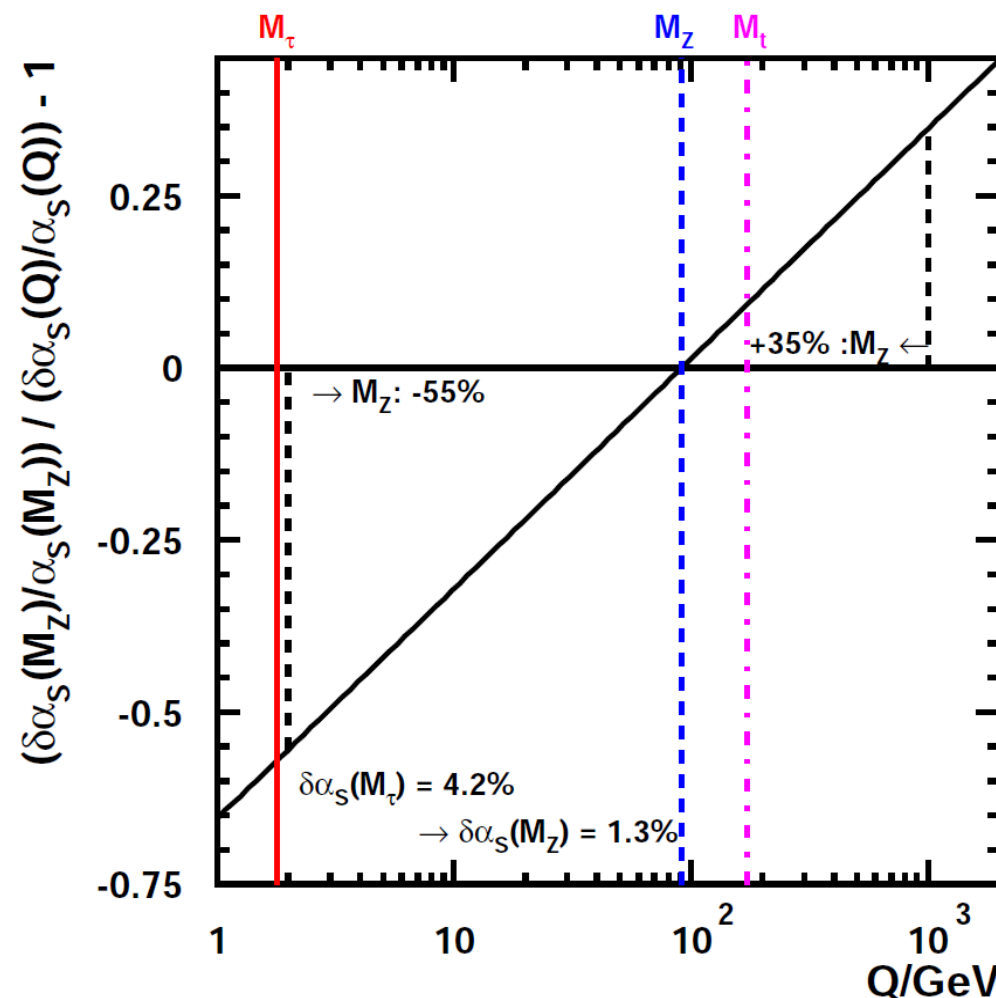
- JEC uncertainty
- PDF uncertainty
- Evolution to M_Z

Worse in: NP effects

Incredibly shrinking error



Uncomfortably growing error





● Experiment:

- ➔ Done: Observables $\sigma \sim \alpha_s^2, \alpha_s^3$; $R_{3/2} \sim \alpha_s$; 7 TeV; full phase space
- ➔ 8 TeV data: Reduce experimental uncertainty by some permille?
- ➔ Best JEC phase space: Another reduction by some permille?
- ➔ Other observables: Ratios $(n+m) / n$ jets (incl. γ, W, Z), $R_{\Delta\Phi}, R_{\Delta R}$ (\rightarrow D0)
Normalized cross sections

● Theory:

- ➔ Scales: NNLO important (see Joao's Talk) \rightarrow reduction by some percent!?
- ➔ PDFs: Much improved after LHC I, also HERA 2 data available
 - ➔ Better known gluon (**Attention circularity jets $\rightarrow g(x)$ & jets $\rightarrow \alpha_s$**)
 - ➔ Fits combining observables at various \sqrt{s} to disentangle $g(x), M_t, \alpha_s$
- ➔ NNLO ratios?



Summary



- LHC at 7 TeV and 8 TeV enables measurements up to scales of 2 TeV
- 13 TeV data yet to come
- Theory at NNLO QCD + electroweak corrections are a must!
- Typical uncertainties on $\alpha_s(M_Z)$:
 - ➔ Experimental: $\sim 1 - 2 \%$
 - ➔ PDF: $\sim 1 - 2 \%$
 - ➔ Scale: $3 - 5 \%$
 - ➔ Nonpert. Effects: $< 1 \%$
- Beyond CMS:
 - ➔ Combined fits of ATLAS & CMS (LHC) measurements
 - ➔ Combined fits of HERA, Tevatron & LHC measurements
- ➔ **CHALLENGE**



Triple Five:

- Within the next **FIVE** years
- Check running of $\alpha_s(Q)$ up to **FIVE TeV** and
- Determine $\alpha_s(M_Z)$ to **FIVE permille** accuracy

**Thank you for your attention
and the invitation to speak here!**



Backup Slides





PDF Sets



Base set	Refs.	Evol.	N_f	M_t (GeV)	M_Z (GeV)	$\alpha_S(M_Z)$	$\alpha_S(M_Z)$ range
ABM11	[17]	NLO	5	180	91.174	0.1180	0.110–0.130
ABM11	[17]	NNLO	5	180	91.174	0.1134	0.104–0.120
CT10	[18]	NLO	≤ 5	172	91.188	0.1180	0.112–0.127
CT10	[18]	NNLO	≤ 5	172	91.188	0.1180	0.110–0.130
HERAPDF1.5	[19]	NLO	≤ 5	180	91.187	0.1176	0.114–0.122
HERAPDF1.5	[19]	NNLO	≤ 5	180	91.187	0.1176	0.114–0.122
MSTW2008	[20,21]	NLO	≤ 5	10^{10}	91.1876	0.1202	0.110–0.130
MSTW2008	[20,21]	NNLO	≤ 5	10^{10}	91.1876	0.1171	0.107–0.127
NNPDF2.1	[22]	NLO	≤ 6	175	91.2	0.1190	0.114–0.124
NNPDF2.1	[22]	NNLO	≤ 6	175	91.2	0.1190	0.114–0.124

CMS, EPJC 75 (2015) 288.



Details: α_s from inclusive Jets



Fit results in separate $|y|$ bins
PDF: CT10-NNLO

$ y $ range	No. of data points	$\alpha_s(M_Z)$	χ^2/n_{dof}
$ y < 0.5$	33	0.1180 ± 0.0017 (exp) ± 0.0027 (PDF) ± 0.0006 (NP) $^{+0.0031}_{-0.0026}$ (scale)	15.4/32
$0.5 \leq y < 1.0$	30	0.1176 ± 0.0016 (exp) ± 0.0026 (PDF) ± 0.0006 (NP) $^{+0.0033}_{-0.0023}$ (scale)	23.9/29
$1.0 \leq y < 1.5$	27	0.1169 ± 0.0019 (exp) ± 0.0024 (PDF) ± 0.0006 (NP) $^{+0.0033}_{-0.0019}$ (scale)	10.5/26
$1.5 \leq y < 2.0$	24	0.1133 ± 0.0023 (exp) ± 0.0028 (PDF) ± 0.0010 (NP) $^{+0.0039}_{-0.0029}$ (scale)	22.3/23
$2.0 \leq y < 2.5$	19	0.1172 ± 0.0044 (exp) ± 0.0039 (PDF) ± 0.0015 (NP) $^{+0.0049}_{-0.0060}$ (scale)	13.8/18
$ y < 2.5$	133	0.1170 ± 0.0012 (exp) ± 0.0024 (PDF) ± 0.0004 (NP) $^{+0.0044}_{-0.0030}$ (scale)	105.7/132



Details: 3-Jet Mass



Fit results in separate $|y|$ bins (CT10-NLO) and with other PDFs

CMS, EPJC 75 (2015) 186.

m_3 (GeV)	$\langle Q \rangle$ (GeV)	χ^2/n_{dof}	$\alpha_S(M_Z)$	$\pm(\text{exp})$	$\pm(\text{PDF})$	$\pm(\text{NP})$	$\pm(\text{scale})$
664–794	361	4.5/3	0.1232	+0.0040 –0.0042	+0.0019 –0.0016	+0.0008 –0.0007	+0.0079 –0.0044
794–938	429	7.8/3	0.1143	+0.0034 –0.0033	+0.0019 –0.0016	± 0.0008	+0.0073 –0.0042
938–1098	504	0.6/3	0.1171	+0.0033 –0.0034	± 0.0022	± 0.0007	+0.0068 –0.0040
1098–1369	602	2.6/5	0.1152	± 0.0026	+0.0027 –0.0026	+0.0008 –0.0007	+0.0060 –0.0027
1369–2172	785	8.8/13	0.1168	+0.0018 –0.0019	+0.0030 –0.0031	+0.0007 –0.0006	+0.0068 –0.0034
2172–2602	1164	3.6/5	0.1167	+0.0037 –0.0044	+0.0040 –0.0044	± 0.0008	+0.0065 –0.0041
2602–3270	1402	5.5/7	0.1120	+0.0043 –0.0041	+0.0056 –0.0040	± 0.0001	+0.0088 –0.0050
$ y _{\text{max}} < 1$	413	10.3/22	0.1163	+0.0018 –0.0019	± 0.0027	± 0.0007	+0.0059 –0.0025
$1 \leq y _{\text{max}} < 2$	441	10.6/22	0.1179	+0.0018 –0.0019	± 0.0021	± 0.0007	+0.0067 –0.0037
$ y _{\text{max}} < 2$	438	47.2/45	0.1171	± 0.0013	± 0.0024	± 0.0008	+0.0069 –0.0040
PDF set		χ^2/n_{dof}	$\alpha_S(M_Z)$	$\pm(\text{exp})$	$\pm(\text{PDF})$	$\pm(\text{NP})$	$\pm(\text{scale})$
CT10-NLO		47.2/45	0.1171	± 0.0013	± 0.0024	± 0.0008	+0.0069 –0.0040
CT10-NNLO		48.5/45	0.1165	+0.0011 –0.0010	+0.0022 –0.0023	+0.0006 –0.0008	+0.0066 –0.0034
MSTW2008-NLO		52.8/45	0.1155	+0.0014 –0.0013	+0.0014 –0.0015	+0.0008 –0.0009	+0.0105 –0.0029
MSTW2008-NNLO		53.9/45	0.1183	+0.0011 –0.0016	+0.0012 –0.0023	+0.0011 –0.0019	+0.0052 –0.0050
HERAPDF1.5-NNLO		49.9/45	0.1143	± 0.0007	+0.0020 –0.0035	+0.0003 –0.0008	+0.0035 –0.0027
NNPDF2.1-NNLO		51.1/45	0.1164	± 0.0010	+0.0020 –0.0019	+0.0010 –0.0009	+0.0058 –0.0025



$R_{3/2}$ Details



Fit results in separate Q ranges (NNPDF21-NNLO) and with other PDFs

$\langle p_{T1,2} \rangle$ range (GeV)	Q (GeV)	$\alpha_S(M_Z)$	$\alpha_S(Q)$	No. of data points	χ^2/N_{dof}
420–600	474	0.1147 ± 0.0061	0.0936 ± 0.0041	6	4.4/5
600–800	664	0.1132 ± 0.0050	0.0894 ± 0.0031	5	5.9/4
800–1390	896	0.1170 ± 0.0058	0.0889 ± 0.0034	10	5.7/9

$\langle p_{T1,2} \rangle$ range (GeV)	Q (GeV)	$\alpha_S(M_Z)$	exp.	PDF	theory
420–600	474	0.1147	± 0.0015	± 0.0015	± 0.0057
600–800	664	0.1132	± 0.0018	± 0.0025	± 0.0039
800–1390	896	0.1170	± 0.0024	± 0.0021	± 0.0048

MSTW2008: $\alpha_S(M_Z) = 0.1141 \pm 0.0022$ (exp.),

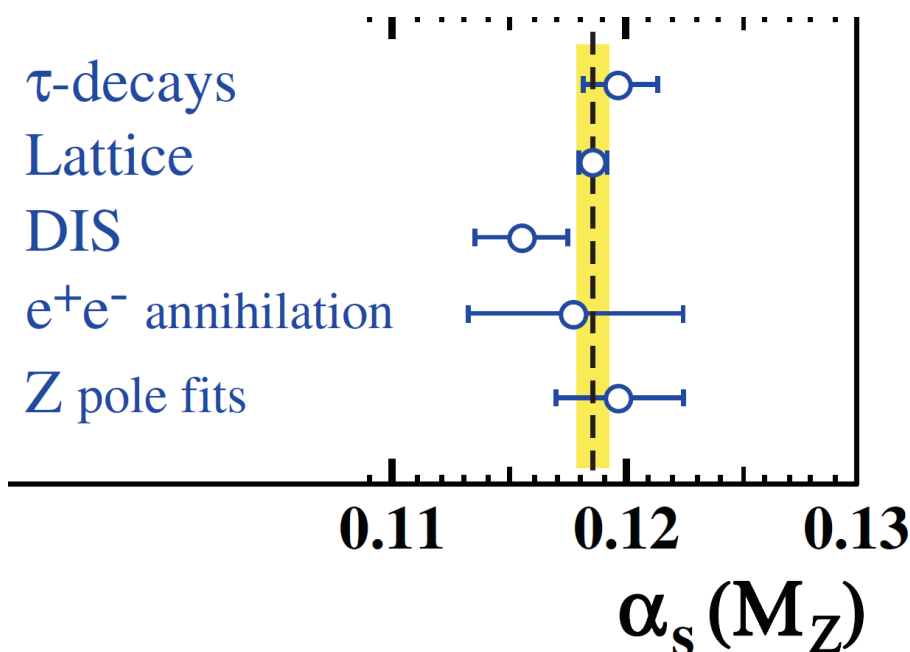
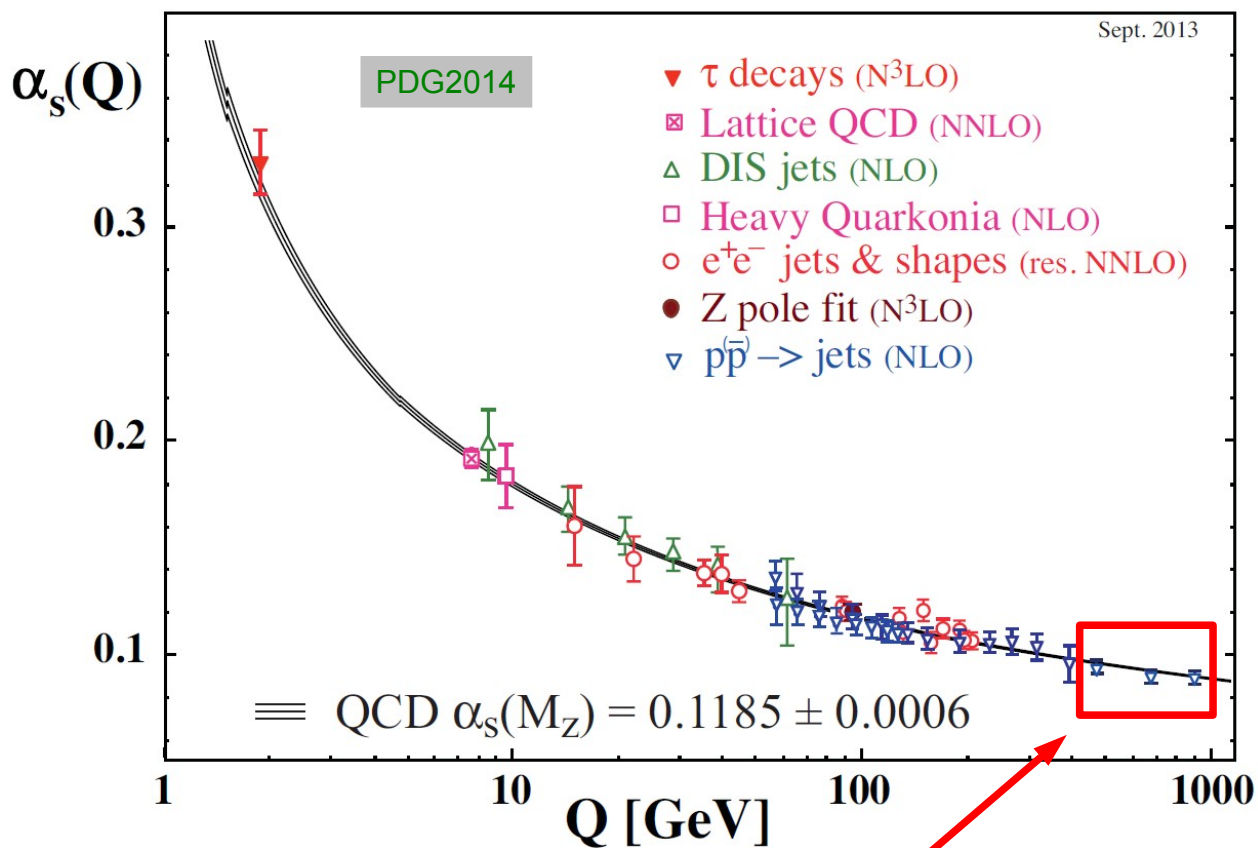
CT10: $\alpha_S(M_Z) = 0.1135 \pm 0.0019$ (exp.),

$\mu_r / \langle p_{T1,2} \rangle$	$\mu_f / \langle p_{T1,2} \rangle$	$\alpha_S(M_Z) \pm$ (exp.)	χ^2/N_{dof}
1	1	0.1148 ± 0.0014	22.0/20
1/2	1/2	0.1198 ± 0.0021	30.6/20
1/2	1	0.1149 ± 0.0014	22.2/20
1	1/2	0.1149 ± 0.0014	22.2/20
1	2	0.1150 ± 0.0015	21.9/20
2	1	0.1159 ± 0.0014	20.7/20
2	2	0.1172 ± 0.0018	21.3/20

CMS, EPJC 73 (2013) 2604.



PDG Summary



Dominated by Lattice Gauge Theory

CMS data, but not in average since only NLO theory!

$$\alpha_s(M_Z) = 0.1177 \pm 0.0013 ?$$

$$\frac{\Delta\alpha_s(M_Z)}{\alpha_s(M_Z)} = 1.1\%$$

PDG'92: 2.4%

PDG, Chin. Phys. C 38 (2014) 090001.



α_s Projections



Still at LHC:

Only jets probe running α_s at highest scales

< 1% uncertainty at M_Z challenging, but not impossible

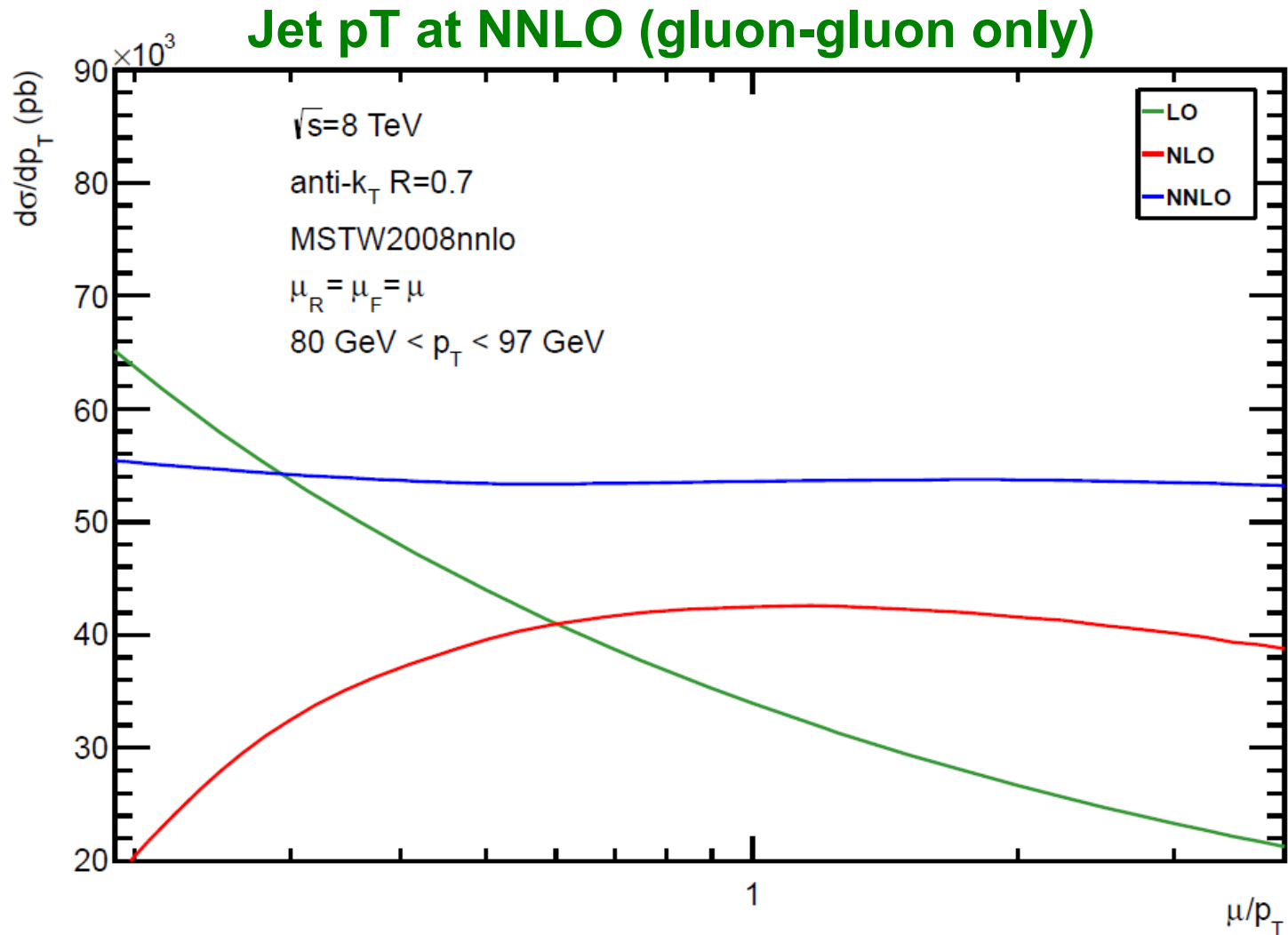
Need NNLO and improved PDFs (gluon) plus some experimental optimization

Method	Current relative precision	Future relative precision	
<u>e^+e^- evt shapes</u>	expt $\sim 1\%$ (LEP) thry $\sim 1-3\%$ (NNLO+up to N ³ LL, n.p. signif.) [27]	< 1% possible (ILC/TLEP) $\sim 1\%$ (control n.p. via Q^2 -dep.)	$\sim 1\%$
<u>e^+e^- jet rates</u>	expt $\sim 2\%$ (LEP) thry $\sim 1\%$ (NNLO, n.p. moderate) [28]	< 1% possible (ILC/TLEP) $\sim 0.5\%$ (NLL missing)	$\sim 1\%$
<u>precision EW</u>	expt $\sim 3\%$ (R_Z , LEP) thry $\sim 0.5\%$ (N ³ LO, n.p. small) [9, 29]	0.1% (TLEP [10]), 0.5% (ILC [11]) $\sim 0.3\%$ (N ⁴ LO feasible, ~ 10 yrs)	< 1%
τ decays	expt $\sim 0.5\%$ (LEP, B-factories) thry $\sim 2\%$ (N ³ LO, n.p. small) [8]	< 0.2% possible (ILC/TLEP) $\sim 1\%$ (N ⁴ LO feasible, ~ 10 yrs)	
<u>ep colliders</u>	$\sim 1-2\%$ (pdf fit dependent) [30, 31], (mostly theory, NNLO) [32, 33]	0.1% (LHeC + HERA [23]) $\sim 0.5\%$ (at least N ³ LO required)	< 1%
<u>hadron colliders</u>	$\sim 4\%$ (Tev. jets), $\sim 3\%$ (LHC $t\bar{t}$) (NLO jets, NNLO $t\bar{t}$, gluon uncert.) [17, 21, 34]	< 1% challenging (NNLO jets imminent [22])	$\sim 1\%$
<u>lattice</u>	$\sim 0.5\%$ (Wilson loops, correlators, ...) (limited by accuracy of pert. th.) [35-37]	$\sim 0.3\%$ (~ 5 yrs [38])	< 0.5%

Snowmass QCD Report, arXiv:1310.5189.



NNLO Scale Dependence



**Drastically reduced
scale dependence!**

$|y| < 4.4, 80 \text{ GeV} < p_T < 97 \text{ GeV}$

Gehrmann- de Ridder et al.,
PRL110 (2013), JHEP1302 (2013).