α_s from lattice QCD

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Workshop on high-precision alpha_s measurements: from LHC to FCC-ee

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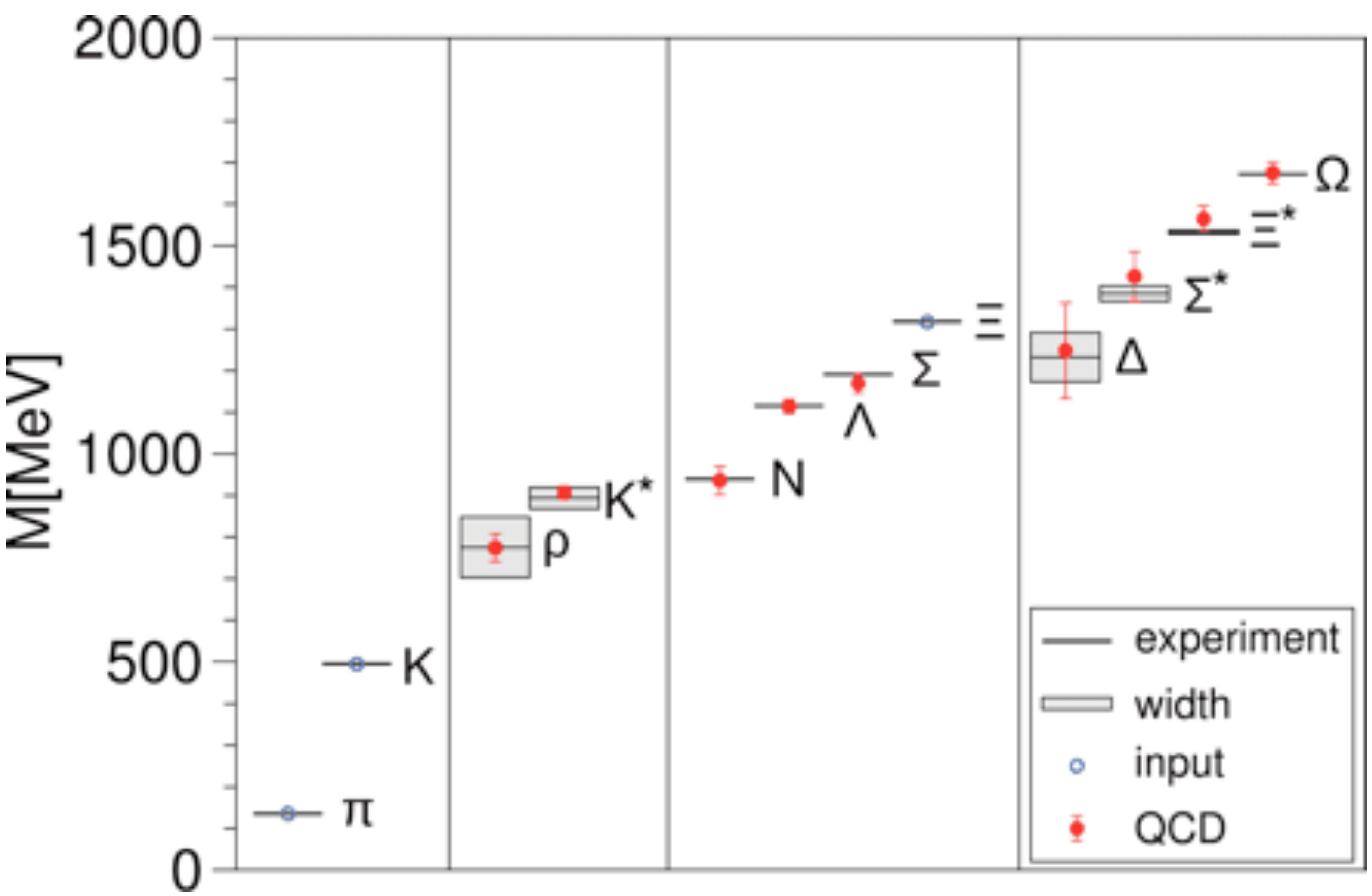
Lattice in the 21st century



BMW Collaboration,
Science, 21 November 2008.

For the past ~ten years, it has been possible to use lattice QCD Monte Carlo methods to calculate simple quantities with understood error budgets that are complete, including the effects of quark-antiquark pairs.

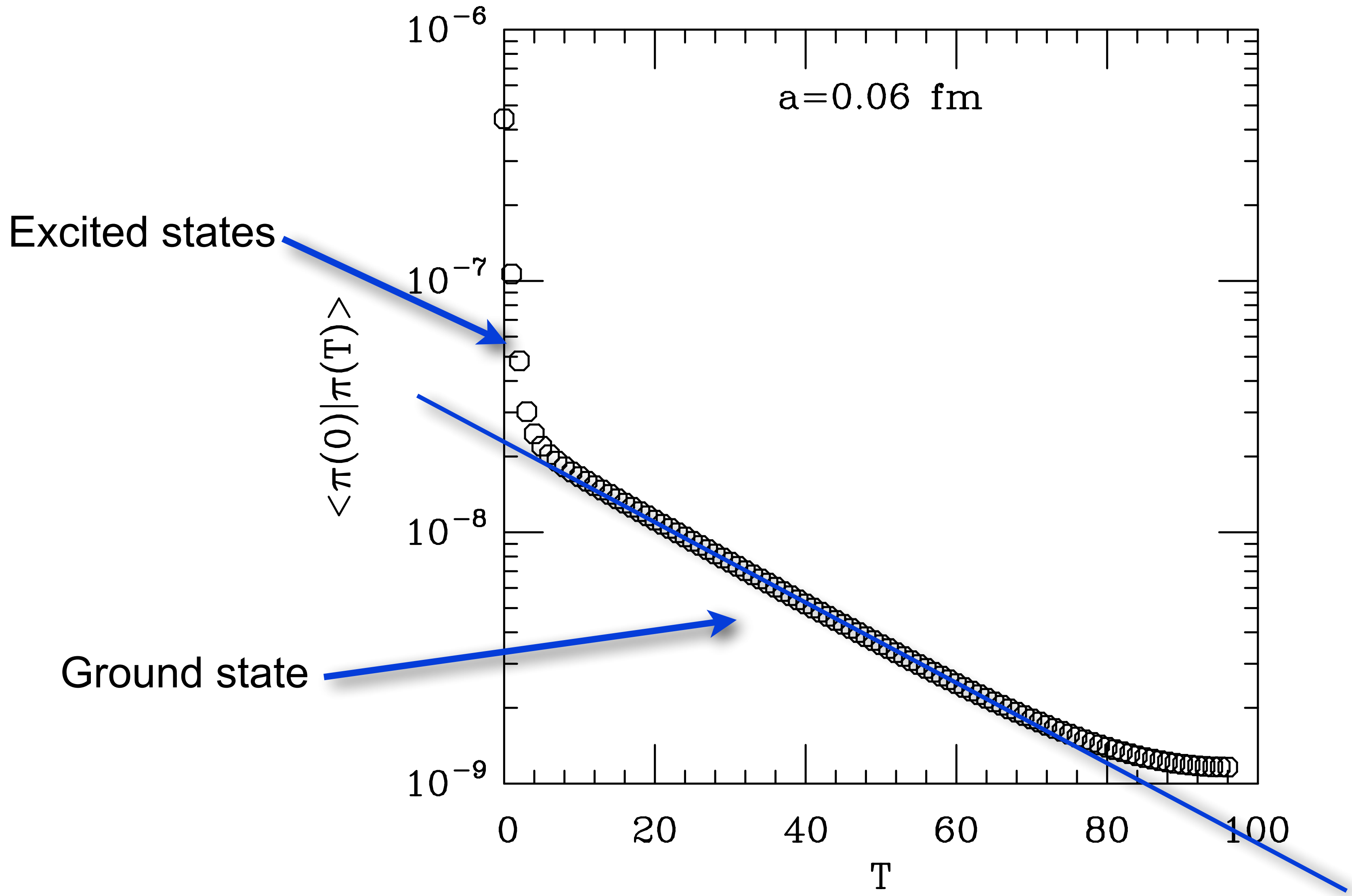
What is “simple”?

- Simplest: stable mesons.
  - Over the last ten years, many key quantities. Hadronically stable mesons are simplest (baryons improving rapidly):
    - Heavy and light meson decay constants,
    - Semileptonic decays,
    - Meson-antimeson mixing.
  - Make possible important determinations of 8 CKM matrix elements, 5 quark masses, the strong coupling constant.
- Now: $\pi\pi$ systems, other multihadron states, …

Coming HEP experimental program

- Next five years: lattice calculations are needed *throughout* the entire future US experimental program.
  - g-2
  - mu2e, LBNE, Nova: nucleon matrix elements.
  - Underground LBNE: proton decay matrix elements.
  - LHCb, Belle-2: continued improvement of CKM results
  - LHC, Higgs decays: lattice provides the most accurate α_s and m_c now, and m_b in the future

How?



$$\langle \overline{\psi}\gamma_5\psi(t=0) \mid \overline{\psi}\gamma_5\psi(t)\rangle = C\exp(-Mt) + \text{excited states.}$$

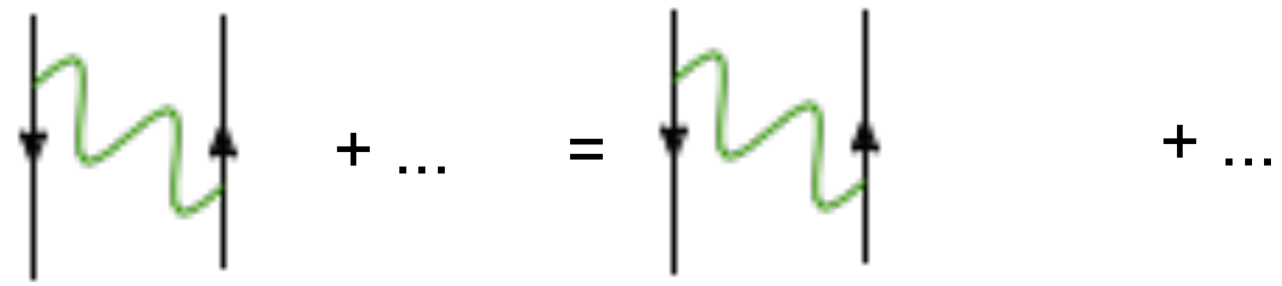
If the two quarks were a u and a $\overline{u}$, the slope would give M_π, C would be proportional to $F_\pi{}^2$.

α_s with lattice QCD

To determine α_s with a lattice calculation:

1. Determine the lattice spacing be comparing any physical quantity with experiment.
   - Use any quantity that can be determined precisely in experiment and with the lattice. Many possibilities available that fix the lattice spacing to fraction of a percent accuracy.
     - E.g., the proton mass would be OK.
   - ⇒ Experimental uncertainties in lattice determinations are usually negligible.
2. Use lattice perturbation theory or nonperturbative calculations to determine $\alpha_s^{\overline{MS}}$.

In principle, can get α_{MS} from $\alpha_{\rm latt}$ by equating Green's functions calculated in perturbation theory in the two regulators:



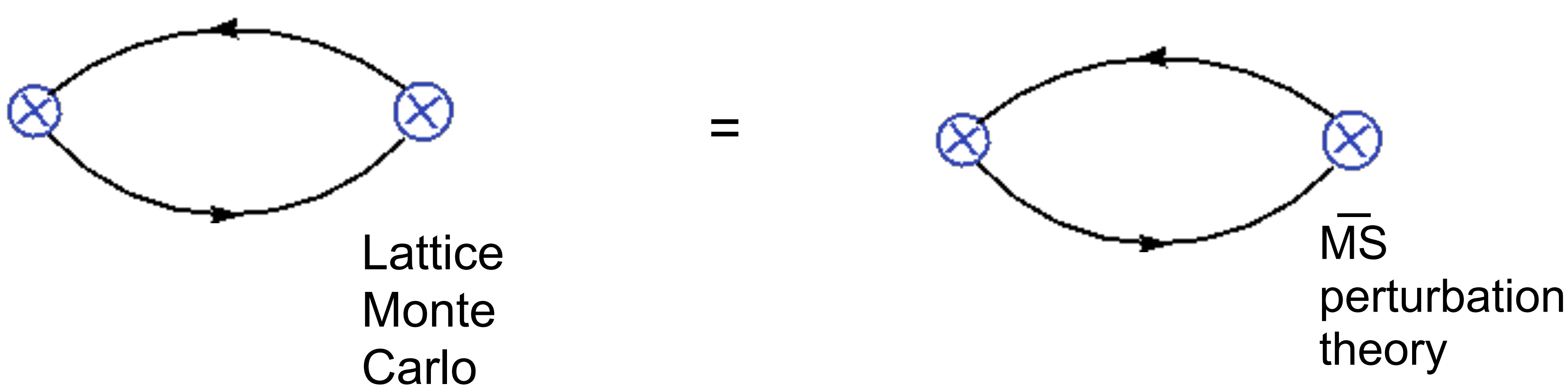
$$\alpha_{\overline{MS}} + c_2 \alpha^2_{\overline{MS}} + \ldots = \alpha_{\rm latt} + d_2 \alpha^2_{\rm latt} + \ldots$$

Expansions in regulated but unrenormalized coupling constants are poorly convergent. Better to express expansions with couplings inspired by calculations of physical quantities, like $\alpha_{\overline{MS}}$ and α_V.

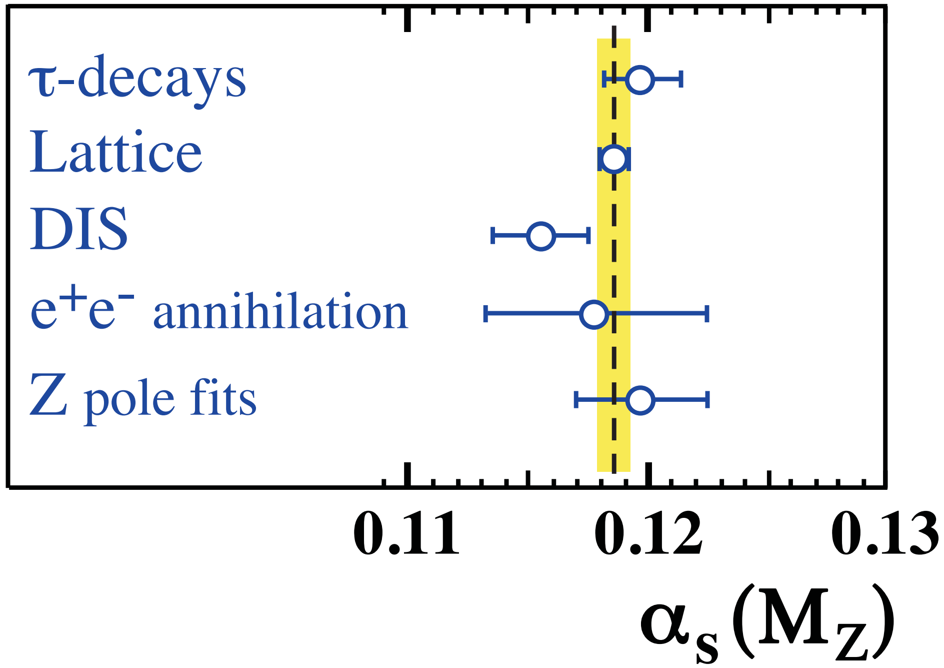
Dimensional regularization has ln(4π) - γ; lattice PT has tadpoles, neither of which affects physics.

Even better, lattice calculations of short distance quantities can sometimes be done with Monte Carlo methods, eliminating the need for lattice perturbation theory.

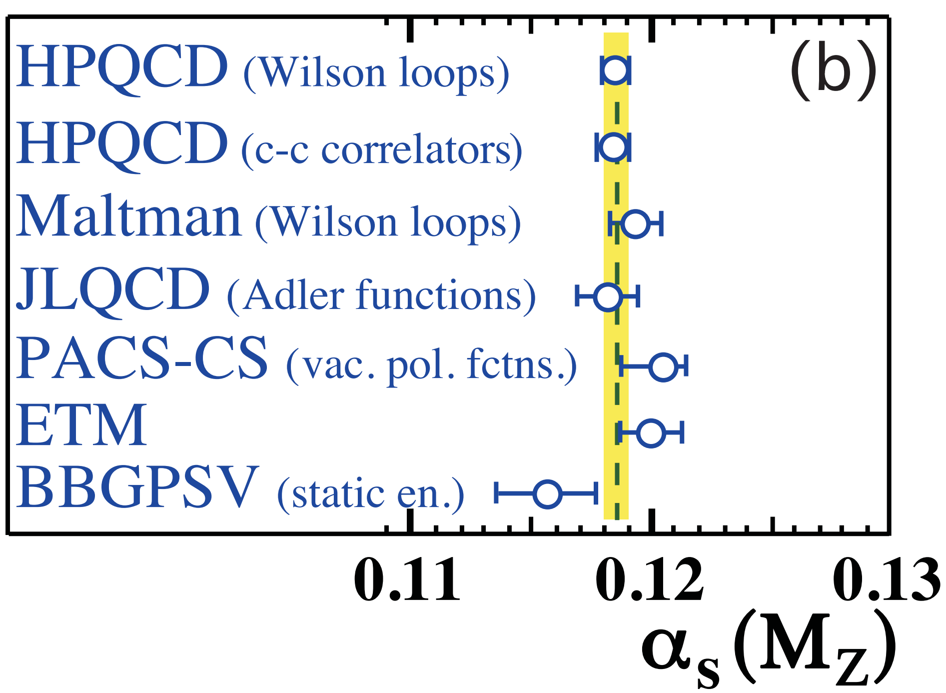
E.g., the current-current correlation functions used by the Karlsruhe group and other to determine the heavy quark masses from e+e- data can also be calculated with lattice QCD entirely nonperturbatively and compared with $\overline{\text{MS}}$ PT.



α_s in the Particle Data Group QCD review



There are multiple ways of determining α_s, both with and without the lattice.

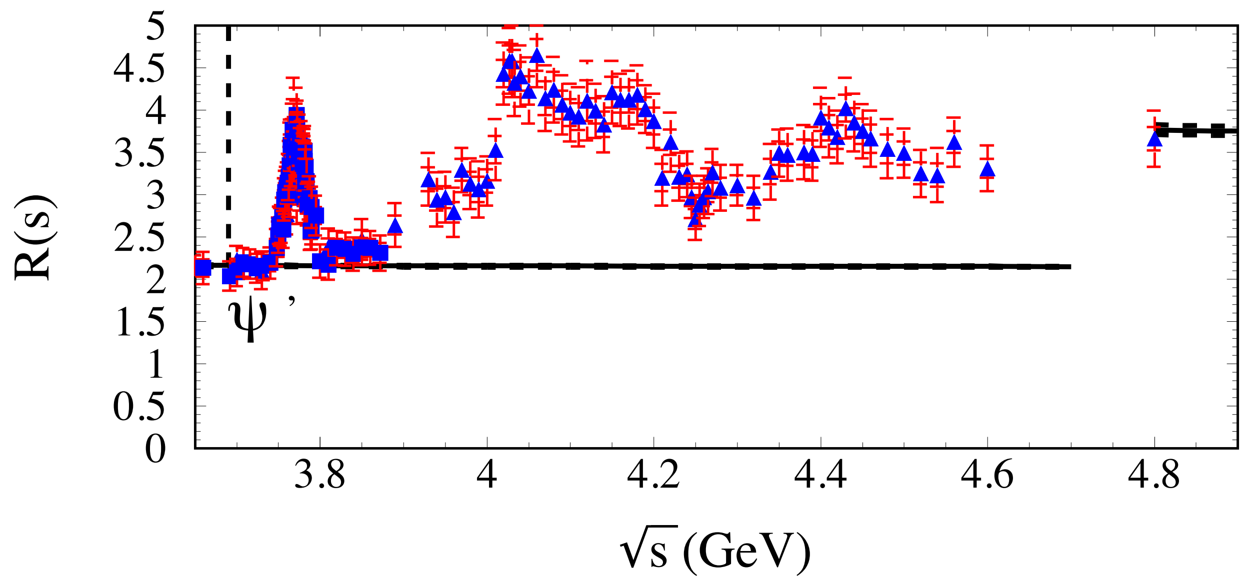


PDG, QCD review, 2014.

There are several lattice determinations equal to or more precise than all the non-lattice determinations together.

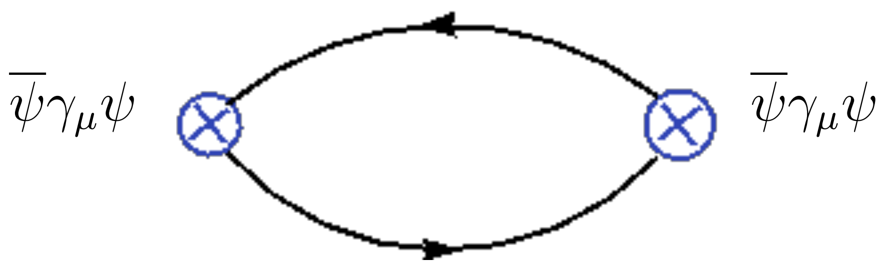
α_s from correlation functions

The most precise non-lattice determinations of m_c use e^+e^- annihilation data and ITEP sum rules. (Karlsruhe group, Chertyrkin et al.)



$$\mathcal{M}_n \equiv \int \frac{ds}{s^{n+1}} R_Q(s)$$

Moments of the heavy quark production cross section in e⁺e⁻ annihilation can be related to the derivatives of the vacuum polarization at q^2=0.



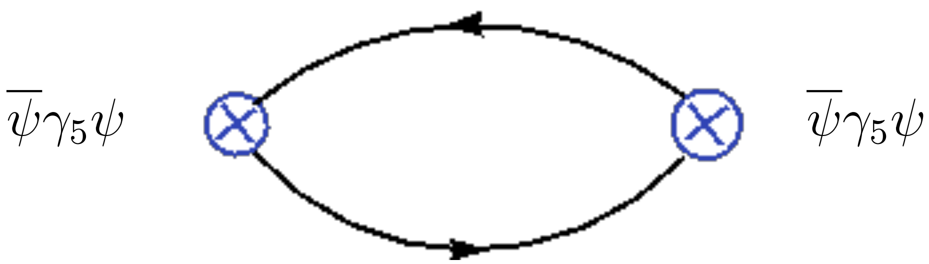
$$\mathcal{M}_n = \frac{12\pi^2}{n!}\left(\frac{d}{dq^2}\right)^n \Pi_Q(q^2)|_{q^2=0}$$

Can be calculated in perturbation theory.
Known to O(α_s^3) (Chetyrkin et. al.)

Lattice QCD

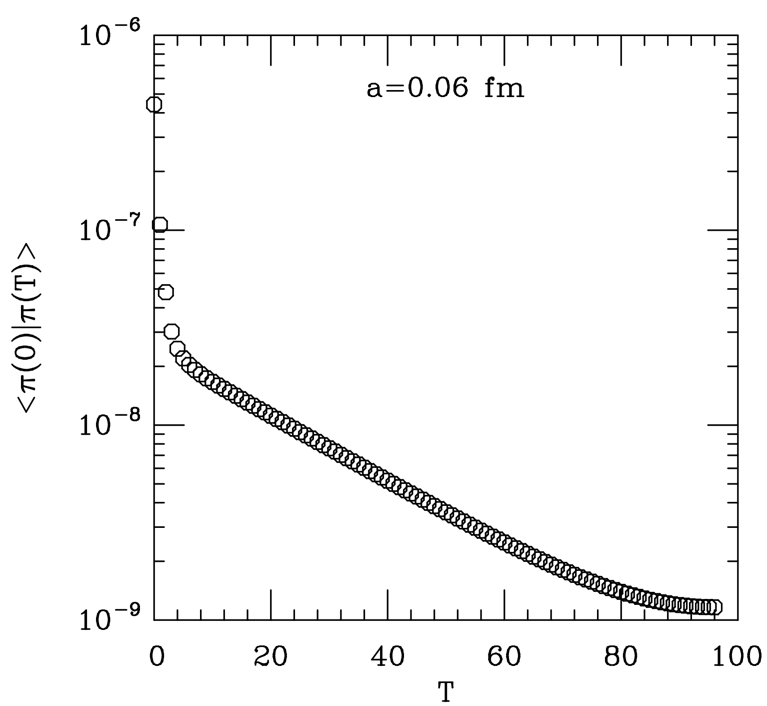
can also compute such correlation functions with high accuracy (HPQCD).

Correlation functions of all currents can be calculated in perturbation theory (and with the lattice). The most precise α_s and m_c can be obtained by choosing the one that is most precise on the lattice: the pseudoscalar correlator.



$$G(t) = a^6 \sum_{\mathbf{x}} (am_{0h})^2 \langle 0 | j_5(\mathbf{x}, t) j_5(0, 0) | 0 \rangle,$$

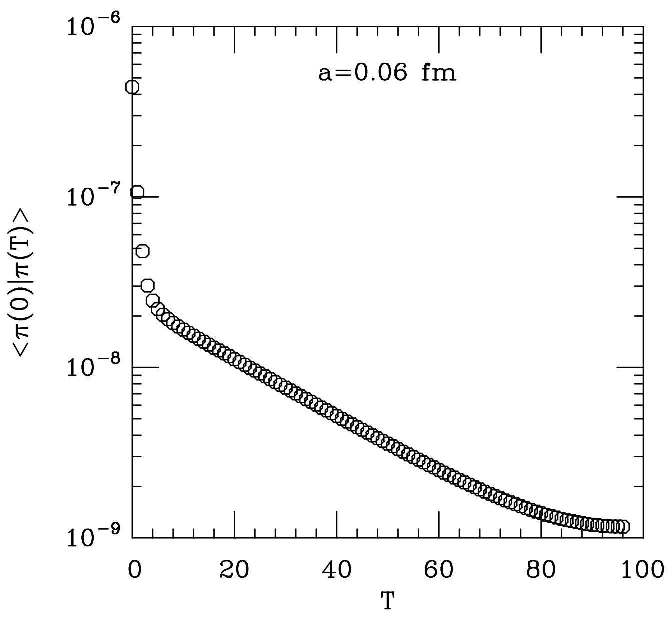
$$G_n \equiv \sum_t (t/a)^n G(t),$$



Perturbation theory to α_s^3 from the Karlsruhe group.

Technical tricks to make the lattice calculation more precise

Choose pseudoscalar (easiest) current correlator. (Easier to calculate than a pion or charmonium mass.)

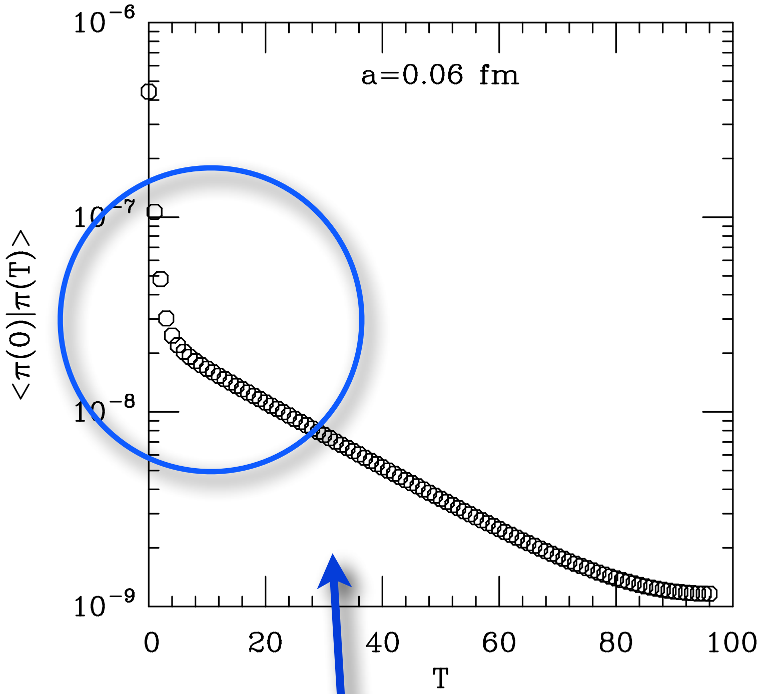


In matching perturbative and nonperturbative results, divide both by the tree level correlator. (Removes leading discretization errors.)

In the lattice calculation of, for example, the charm correlator, use $M\eta_c$ as experimental input to set the energy scale. (Removes sensitivity to the tuning of the lattice mass used.)

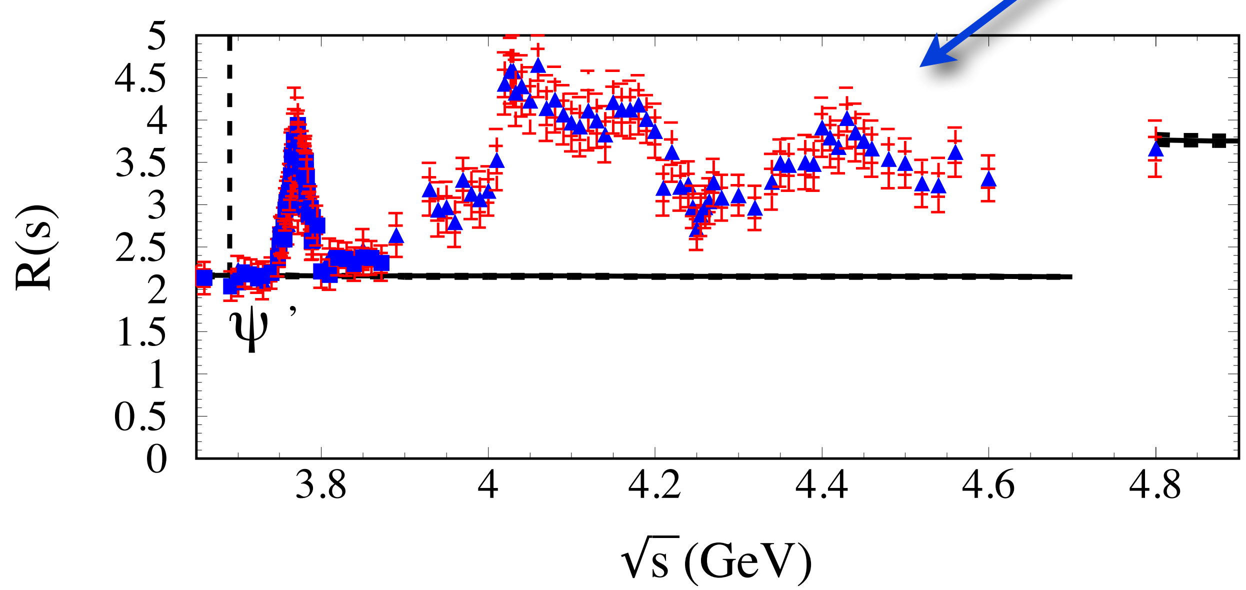
$$R_n \equiv \begin{cases} G_4/G_4^{(0)} & \text{for } n = 4 \\ \frac{am_{\eta_h}}{2am_{0h}} (G_n/G_n^{(0)})^{1/(n-4)} & \text{for } n \geq 6 \end{cases}$$

Why can lattice determinations of m_c from correlation functions be more precise than those from e^+e^-?

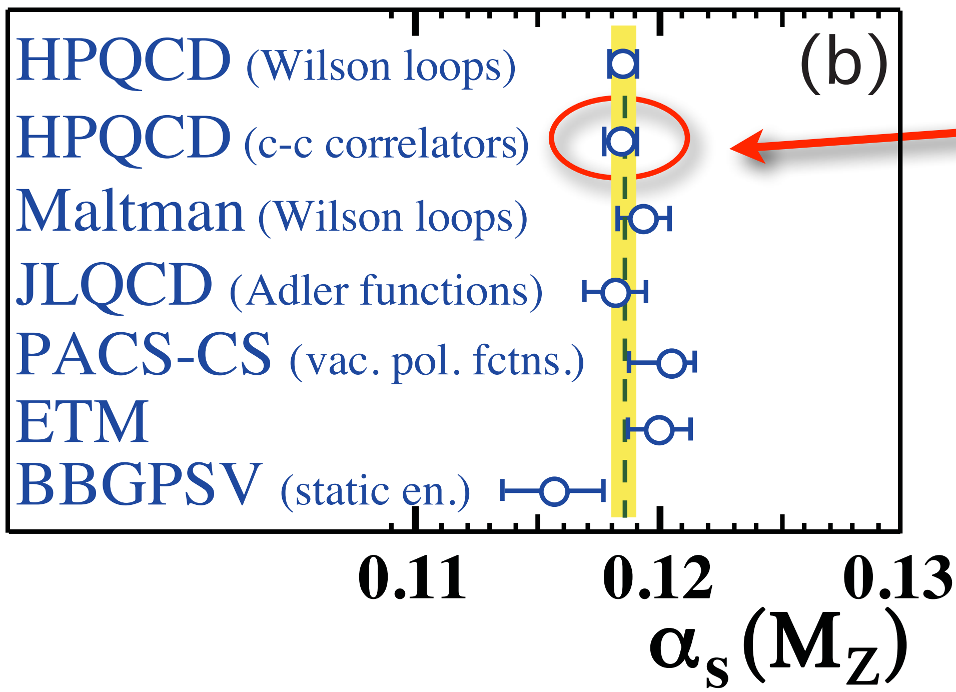


Moments of correlation functions are even easier than what I earlier told you have been considered the easiest quantities for the last ten years.
We need the correlation functions at finite *T*, and not their asymptotic form at large *T*.

Because this is cleaner data than this.



α_s results: correlator method



	$\alpha_{\overline{MS}}(M_Z)$
a^2 extrapolation	0.2%
Perturbation theory	0.4
Statistical errors	0.2
m_h extrapolation	0.0
Errors in r_1	0.1
Errors in r_1/a	0.1
Errors in m_{η_c}, m_{η_b}	0.0
α_0 prior	0.1
Gluon condensate	0.2
Total	0.6%

Results are dominated by perturbation theory. It will be a long calculation to add fourth order term, but higher coefficients can be bounded to some extent by data.

Perturbative coefficients for moments

Lattice and continuum data can be compared at a large number of fictitious quark masses between 1 and 5 GeV; ⇒ restricts unknown higher order PT coefficients.

TABLE III. Perturbation theory coefficients ($n_f = 3$) for r_n [2–6]. Coefficients are defined by $r_n = 1 + \sum_{j=1} r_{nj} \alpha^j_{\overline{\mathrm{MS}}}(\mu)$ for $\mu = m_h(\mu)$. The third-order coefficients are exact for $4 \leq n \leq 10$. The other coefficients are based upon estimates; we assign conservative errors to these.

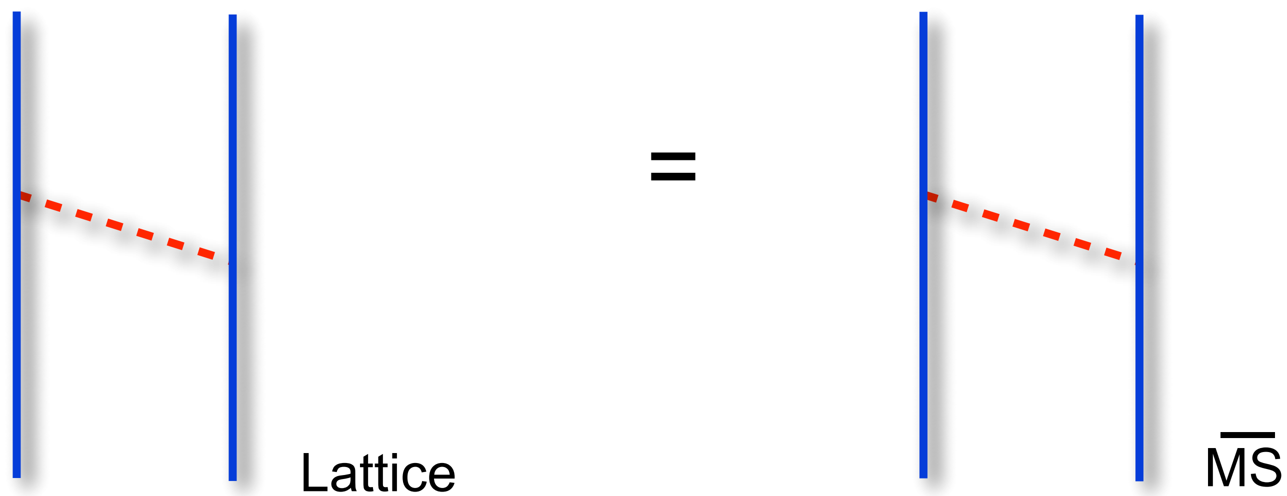
n	r_{n1}	r_{n2}	r_{n3}
4	0.7427	−0.0577	0.0591
6	0.6160	0.4767	−0.0527
8	0.3164	0.3446	0.0634
10	0.1861	0.2696	0.1238
12	0.1081	0.2130	0.1(3)
14	0.0544	0.1674	0.1(3)
16	0.0146	0.1293	0.1(3)
18	−0.0165	0.0965	0.1(3)

HPQCD take uncalculated coefficients in series for moments r_{nj} ~ $O(0.5\ \alpha_s(m_q)^j)$;
further constrain the possible sizes for coefficients by comparing nonperturbative results for many quark masses with perturbation theory using Baysian priors for higher order terms.

The data also restrict the fourth order coefficients (though not as tightly as the third order).

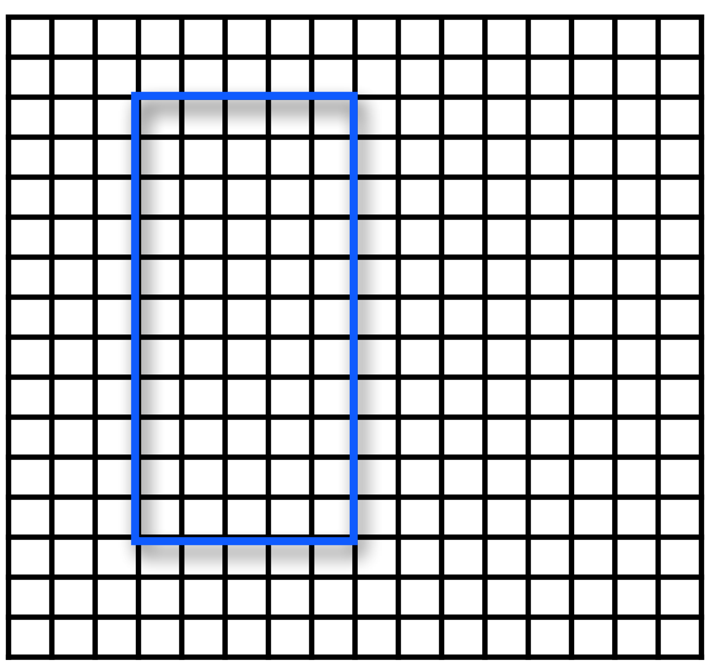
α_s results: Wilson loops

α_s can be determined with lattice calculations of many other quantities, e.g., the heavy quark potential.



to obtain α_s $\overline{\text{MS}}$.

Lattice calculates the heavy quark potential from Wilson loops and Creutz ratios (renormalizable combinations of Wilson loops).



HPQCD has determined α_s directly from Wilson loops.

Result compatible with their correlator result, similar precision: $\alpha_s = 0.1184(6)$, but totally different uncertainties, heavy use of lattice perturbation theory.

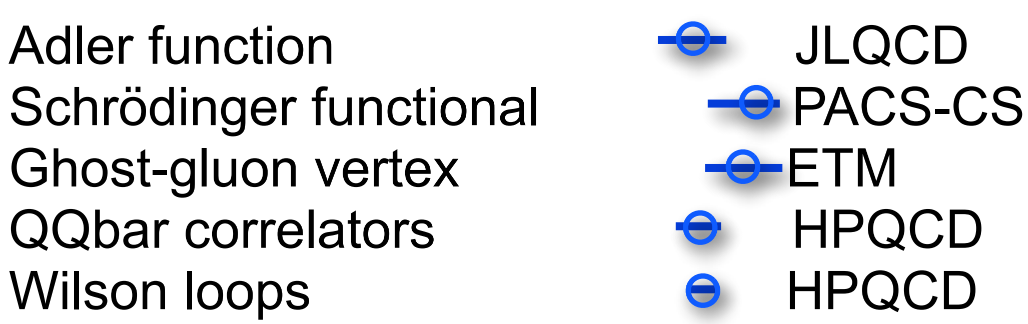
α_s, other lattice results

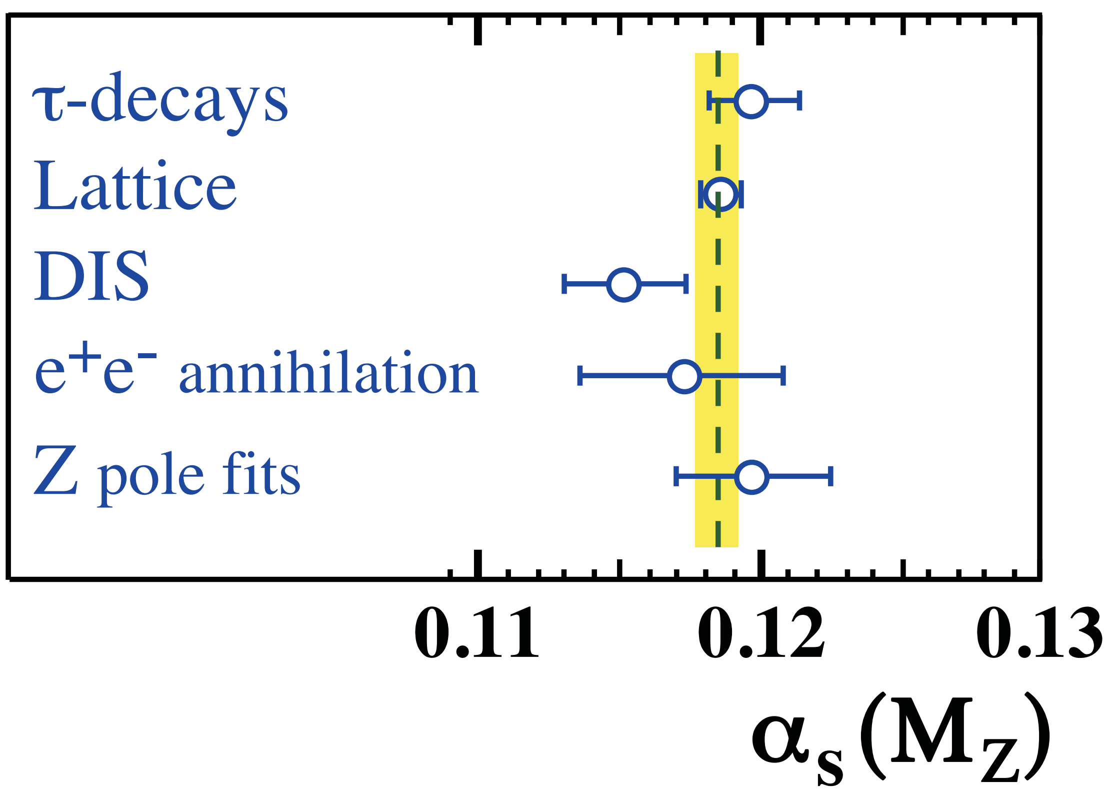
There are numerous good ways of determining α_s using lattice QCD.

- The Adler function, JLQCD. Phys.Rev. D82 (2010) 074505.
  - α_s = 0.1181 ± 0.0003+0.0014-0.0012
- The Schrödinger functional, PACS-CS. JHEP 0910:053,2009.
  - α_s = 0.1205(8)(5)(+0/–17)
- The ghost-gluon vertex, European Twisted Mass Collaboration (ETM). Phys.Rev.Lett. 108 (2012) 262002.
  - α_s = 0.1200(14)

PDG average

2014, combined the lattice numbers in a weighted average.
It takes a combined error of the most precise of the inputs.





The lattice results (2013) are dominated by the two most precise results from HPQCD, but there are several other lattice results from Europe and Japan, all of which agree with each other and all but one of which is more precise than any non-lattice result.

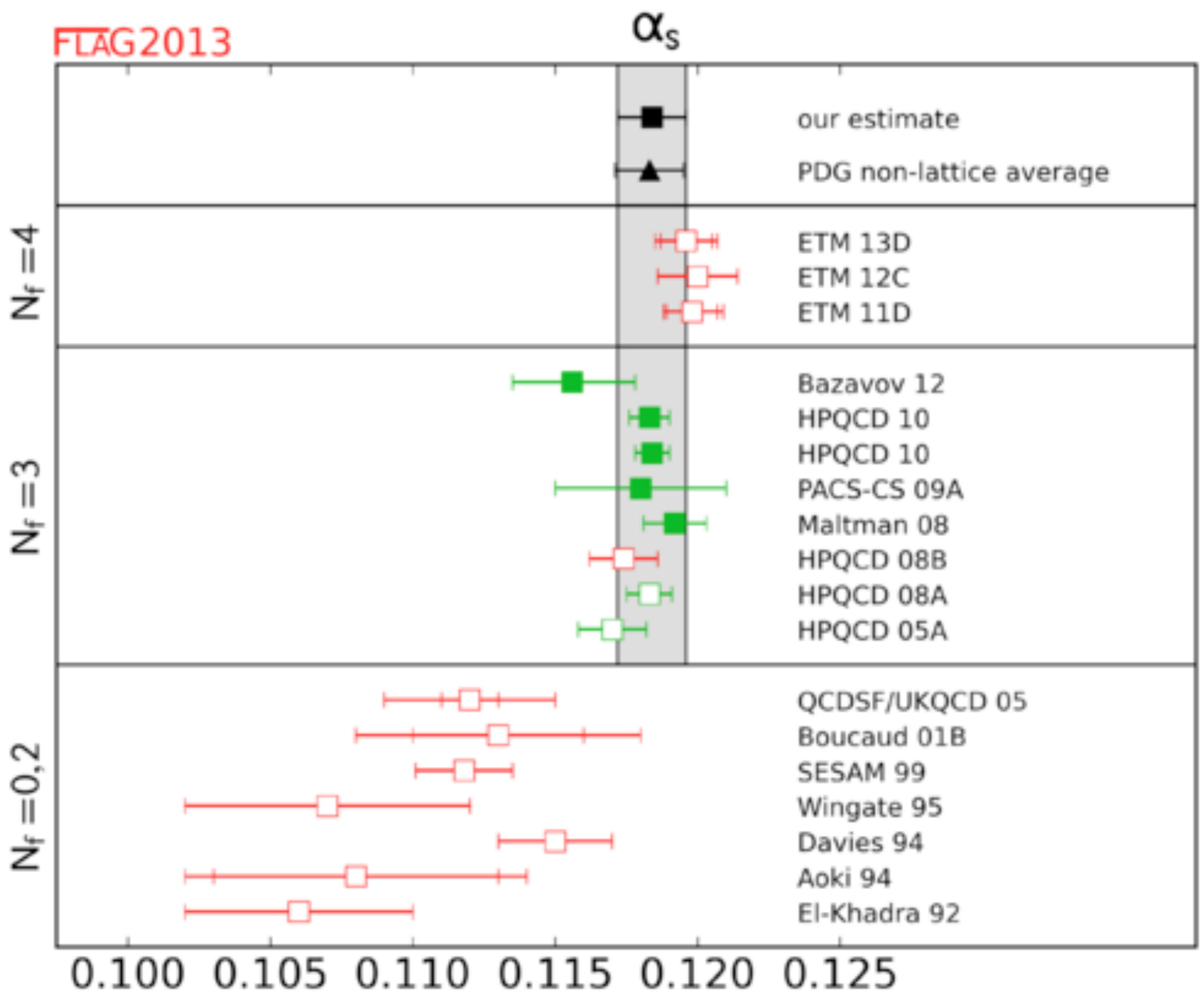
PDG lattice QCD average:

$$\alpha_s(M_Z^2) = 0.1185 \pm 0.0005$$

PDG world average:

$$\alpha_s(M_Z^2) = 0.1185 \pm 0.0006 \ ,$$

FLAG review of lattice results (2013)



http://itpwiki.unibe.ch/flag

The Flavor Lattice Averaging Group chapter contains many more useful details of lattice determinations of αs.
But N.B., the combined uncertainty is based on their opinion about perturbation theory in general, not about lattice QCD.

FLAG uncertainty in α_s.

Flag result for combined α_s: $$\alpha^{(5)}_{\overline{\rm MS}}(M_Z) = 0.1184(12)\,. \qquad (205)$$

"Taking the coefficient $|c_4/c_1| \approx 2$ in eq. (196) yields the estimate $\Delta\alpha^2 = 0.0012$ for $\alpha^{(5)}(M_Z)$." (Flag report)

"This is important to keep in mind when comparing our chosen range for $\alpha^{(5)}(M_Z)$ from lattice determinations in eq. (205) with the non-lattice average from the PDG." (Flag report)

FLAG's range for lattice determinations of αs is a statement of their opinion about perturbation theory, including continuum PT, not about lattice gauge theory. They note that it is not supported by the results they describe.

"We note that there is a diversity of opinion over the size of our range for $\alpha_{\overline{\rm MS}}(M_Z)$ in eq. (205) within FLAG. Some members are sufficiently convinced by the overall consistency of the results from various groups within their quoted errors, as well as by the internal tests performed by individual groups, to take the quoted errors at face value." (Flag report)

Prospects:

What uncertainties could we expect from the computers ten years in the future and perhaps a fourth order of perturbation theory?

Lepage, Mackenzie, and Peskin (arXiv:1404.0319v2).

(Motivated by the question of whether uncertainties in the knowledge of m_b will be small enough to make a high-precision study of Higgs branching fractions at a high-luminosity ILC possible. Answer: yes.)

- Estimate used HPQCD's fitting program for the correlation function calculations supplied with fake data with estimated future numerical precisions and/or a fourth order of PT.
- Result:
  - Uncertainty in α_s could be halved with (much) better numerical data.
  - It could be cut to ~0.1% with a fourth loop of perturbation theory + much better numerical data.
    - NB: fourth order of PT would be a ten-year project (Kuhn).

Summary

- The uncertainties in the Wilson loop and correlator determinations of α_s are dominated by perturbation theory and will improve somewhat with better numerical data.

- α_s can be determined well from lattice calculations of many different quantities. There is likely to be continued improvement in the apparent robustness of the lattice results as more quantities are calculated with increasing precision.

- As of now there are results from
  - seven different quantities,
  - six different groups on three continents,
  - four different fermion discretizations.
  - Results are completely independent and consistent, and all but one is more precise than the most precise non-lattice determination.