α_s from hadronic quarkonia decays

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Distribution

- Current Status (NLO extraction)
- Road map towards a NNLO extraction

Current Status

CLEO Collaboration (D. Besson et al., Phys.Rev. D74 (2006) 012003). Data on $\Upsilon(nS) \to X, \gamma + X$; $\quad n = 1, 2, 3$

$$\alpha_s(M_{\Upsilon(1S)}) = 0.1735 \pm 0.0005 \pm 0.0072 \pm 0.0133$$

$$\alpha_s(M_{\Upsilon(2S)}) = 0.151 \pm 0.002 \pm 0.009 \pm 0.017$$

$$\alpha_s(M_{\Upsilon(3S)}) = 0.172 \pm 0.003 \pm 0.018 \pm 0.021$$

$$\alpha_s^{M_Z,\Upsilon(1S)} = 0.1114 \pm 0.0002 \pm 0.0029 \pm 0.0053$$

$$\alpha_s^{M_Z,\Upsilon(2S)} = 0.1026 \pm 0.0007 \pm 0.0041 \pm 0.0077$$

$$\alpha_s^{M_Z,\Upsilon(3S)} = 0.113 \pm 0.001 \pm 0.007 \pm 0.008$$

Magenta errors are due to model dependence

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$$R_\gamma \equiv \frac{\Gamma(\Upsilon(1S) \to \gamma X)}{\Gamma(\Upsilon(1S) \to X)} = \frac{36}{5} \frac{e_b^2 \alpha}{\alpha_s} \frac{N}{D},$$

$$N = 1 + C_{gg\gamma} \frac{\alpha_s}{\pi} + C_{\mathcal{P}_1(^3S_1)} \mathcal{R}_{\mathcal{P}_1(^3S_1)}$$

$$D = 1 + C_{ggg} \frac{\alpha_s}{\pi} + C_{\mathcal{P}_1(^3S_1)} \mathcal{R}_{\mathcal{P}_1(^3S_1)}$$

Current Status: improving on CLEO's results

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$$\begin{aligned} N &= 1 + C_{gg\gamma}\frac{\alpha_{\rm s}}{\pi} + C_{\mathcal{P}_1(^3S_1)}\mathcal{R}_{\mathcal{P}_1(^3S_1)} \\ &\quad + \frac{\pi}{\alpha_{\rm s}} C_{\gamma\, O_8(^1S_0)}\mathcal{R}_{O_8(^1S_0)} + \frac{\pi}{\alpha_{\rm s}} C_{\gamma\, O_8(^3P_0)}\mathcal{R}_{O_8(^3P_0)}, \\ D &= 1 + C_{ggg}\frac{\alpha_{\rm s}}{\pi} + C_{\mathcal{P}_1(^3S_1)}\mathcal{R}_{\mathcal{P}_1(^3S_1)} + \frac{\pi}{\alpha_{\rm s}} C_{O_8(^3S_1)}\mathcal{R}_{O_8(^3S_1)} \\ &\quad + \frac{\pi}{\alpha_{\rm s}} C_{O_8(^1S_0)}\mathcal{R}_{O_8(^1S_0)} + \frac{\pi}{\alpha_{\rm s}} C_{O_8(^3P_0)}\mathcal{R}_{O_8(^3P_0)}, \end{aligned}$$

Current Status: estimating the CO matrix elements

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  - Continuum: $O_8(^1S_0)$, $O_8(^3P_0)$ (X. Garcia i Tormo, JS, PRD69 (2004)114006)

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- $O_8(^3S_1)$ contribution is small, according to the lattice estimates

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Uncertainties in the L determination:

$$\delta_{C_{ggg}}\alpha_s = 0.0026$$
$$\delta_{\mathcal{R}_{O_8(^1S_0)}}\alpha_s = 0.0040$$
$$\delta_{\mathcal{R}_{O_8(^3S_1)}}\alpha_s = 0.0026$$
$$\delta_{\mathcal{R}_{O_8(^3P_0)}}\alpha_s = 0.0027$$
$$\delta_{\mathcal{R}_{\mathcal{P}_1(^3S_1)}}\alpha_s = 0.0014$$
$$\delta_{\mathcal{R}_{\mathcal{O}_N(v^3)}}\alpha_s = 0.0044$$
$$\delta_{\mathcal{R}_{\mathcal{O}_D(v^3)}}\alpha_s = 0.0033$$
$$\delta_{R_\gamma^{\exp}}\alpha_s = 0.0085$$

Current Status: two determinations of α_s

Uncertainties in the C determination:

$$\delta_{C_{ggg}}\alpha_s = 0.0009$$

$$\delta_{\alpha_s(m_b v)}\alpha_s =^{+0.0006}_{-0.0064}$$

$$\delta_{\alpha_s(m_b v^2)}\alpha_s =^{+0.0083}_{-0.0076}$$

$$\delta_{\mathcal{R}_{O_8(^3S_1)}}\alpha_s = 0.0016$$

$$\delta_{\mathcal{R}_{\mathcal{O}_N(v^3)}}\alpha_s =^{+0.0035}_{-0.0034}$$

$$\delta_{\mathcal{R}_{\mathcal{O}_D(v^3)}}\alpha_s =^{+0.0026}_{-0.0025}$$

$$\delta_{R_\gamma^{\exp}}\alpha_s = 0.01$$

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- A precise value of α_s can be obtained from $\Gamma(\Upsilon(1S) \to \gamma\, X)/\Gamma(\Upsilon(1S) \to X)$:

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- Not competitive anymore: since 2012 only NNLO calculations in α_s are included in the PDG average

How to make it competitive

$$R_\gamma \equiv \frac{\Gamma(\Upsilon(1S) \to \gamma\, X)}{\Gamma(\Upsilon(1S) \to X)} = \frac{36}{5} \frac{e_b^2 \alpha}{\alpha_{\rm s}} \frac{N}{D},$$

$$N,\ D = 1 + \mathcal{O}(\alpha_{\rm s}) + \mathcal{O}(v^2)$$

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$$+\mathcal{O}({\alpha_{\rm s}}^2) + \mathcal{O}(\alpha_{\rm s} v^2) + \mathcal{O}(\alpha_{\rm s}\frac{v^4}{\alpha_{\rm s}}) + \mathcal{O}(v^4) + \mathcal{O}(\frac{v^6}{\alpha_{\rm s}})$$

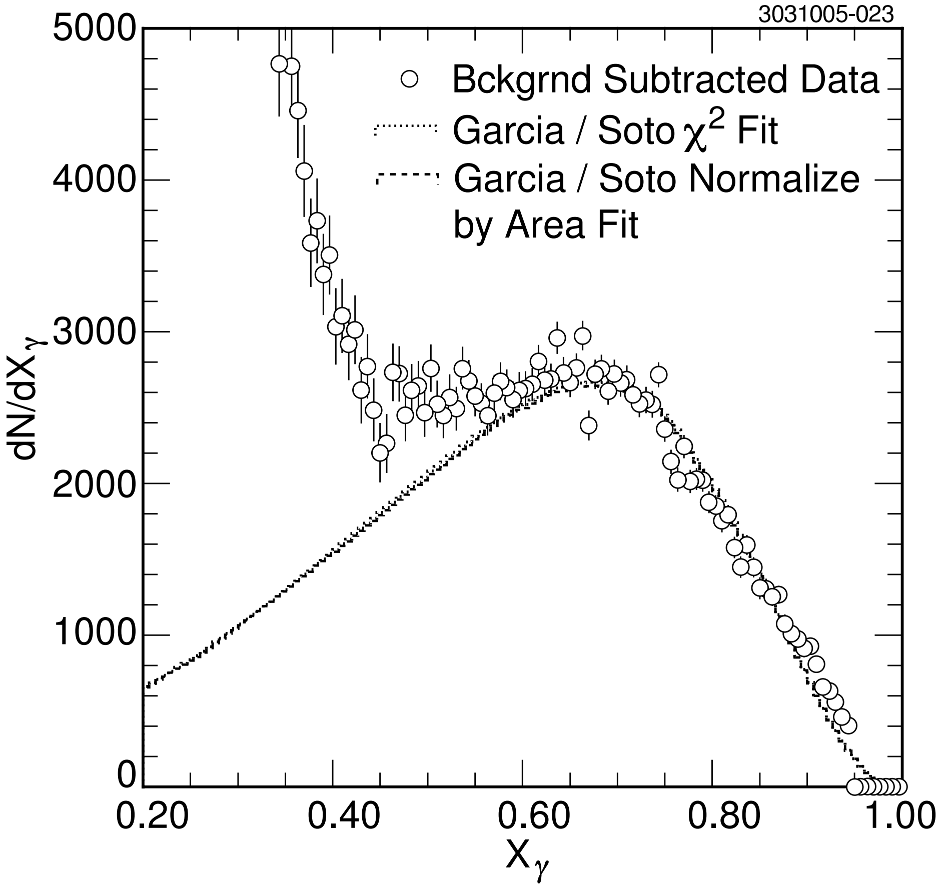
How to make it competitive

- $\mathcal{O}(\alpha_{\rm s}{}^2)$: unknown (Im 4-loop, 1 scale) [known numerically for QED: Caswell, Lepage, 79; Khriplovich, Yelkhovsky, 90; Adkins, Fell, Sapirstein, 00]
- $\mathcal{O}(\alpha_{\rm s} v^2)$: unknown (Im 3-loop, 1 scale)
- $\mathcal{O}(\alpha_{\rm s} \frac{v^4}{\alpha_{\rm s}})$: known: Petrelli, Cacciari, Greco, Maltoni, Mangano, 97 (D); Maltoni, Petrelli, 98 (N)
- $\mathcal{O}(v^4)$: known for D: Bodwin, Petrelli, 02, 13
- $\mathcal{O}(\frac{v^6}{\alpha_{\rm s}})$: known for D: Bodwin, Braaten, Lepage, 94, 97; Petrelli, Cacciari, Greco, Maltoni, Mangano, 97; Brambilla, Mereghetti, Vairo 08, 11

How to make it competitive

- Matrix Elements $\mathcal{O}(v^2)$ and $\mathcal{O}(\frac{v^4}{\alpha_s})$: needed with some precision ($\lesssim 10\%$)
  - Lattice NRQCD (demanding)
  - Continuum calculations (pNRQCD, limited by the small ultrasoft scale)
  - Fits from the photon spectrum (require precise data)
- Matrix Elements $\mathcal{O}(v^4), \mathcal{O}(\frac{v^6}{\alpha_s})$: estimated

The photon spectrum



From D. Besson *et al.* [CLEO Collaboration], (hep-ex/0512061)

The Photon Spectrum

$$\frac{d\Gamma}{dz} = \frac{d\Gamma^{frag}}{dz} + \frac{d\Gamma^{dir}}{dz}$$

- Direct: the photon is emitted from the heavy quarks
- Fragmentation: the photon is emitted from the decay products (light quarks)

(Garcia i Tormo, JS, 05)

Direct Contributions

$$
\begin{aligned}
\frac{d\Gamma^{dir}}{dz} &= z\frac{M_n}{16\pi^2}\mathrm{Im}T(z) \\
T(z) &= -i\int d^4x e^{-iq\cdot x}\left\langle\Upsilon(n)|T\{J_\mu(x)J_\nu(0)\}|\Upsilon(n)\right\rangle \eta_\perp^{\mu\nu}
\end{aligned}
$$

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Regions:

- The central region ($z \sim 0.5$)
- The lower end-point region ($z \to 0$)
- The upper end-point region ($z \to 1$)

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Regions:

- The central region ($z \sim 0.5$): depends only on $\mathcal{O}(v^2)$
- The lower end-point region ($z \to 0$)
- The upper end-point region ($z \to 1$)

Fragmentation contributions

$$\frac{d\Gamma^{frag}}{dz} = \sum_{a=q,\bar{q},g} \int_z^1 \frac{dx}{x} C_a(x) D_{a\gamma}\left(\frac{z}{x}, M\right),$$

C_a = partonic kernels, $D_{a\gamma}$ = fragmentation functions

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$$C_a = \sum_{\mathcal{Q}} C_a[\mathcal{Q}]$$

$\mathcal{Q}$ = NRQCD operators : $\mathcal{O}(v^2)$ and $\mathcal{O}(\frac{v^4}{\alpha_s})$ only

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  - More precise data for the $\Upsilon(1S)$ photon spectrum (and total hadronic width)
  - Non-trivial higher order perturbative calculations