α_s determination at NNLO*+NNLL from a global fit of the jet frag. functions in e^+e^- & DIS

High-precision α_s measurements: from LHC to FCC-ee

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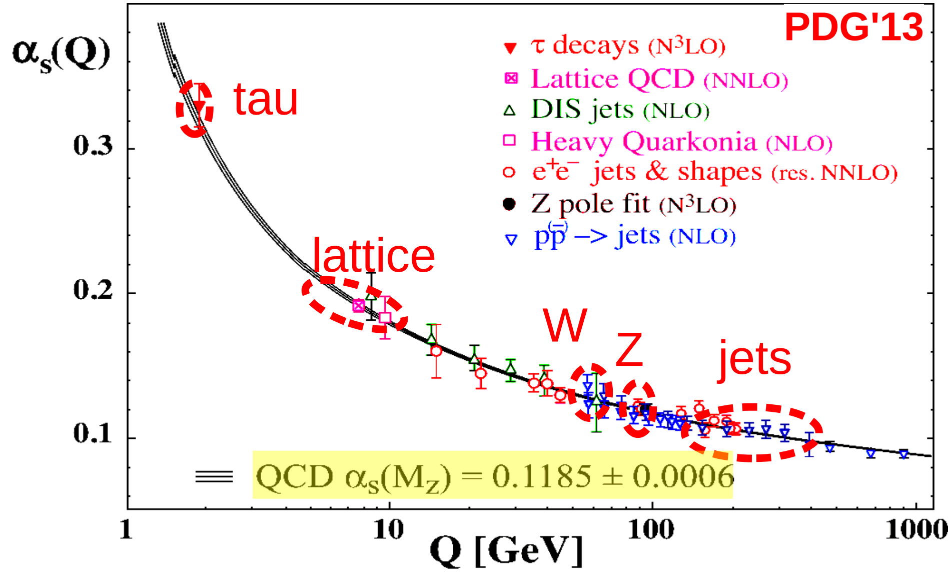
Determination of the QCD coupling α_s

α_s = Single free parameter in QCD (in the $m_q \to 0$ limit). Determined at a given ref. scale (e.g. m_Z). Decreases as $\sim\ln(Q^2/\Lambda^2)$, with $\Lambda\sim0.25$ GeV.



- Measured by comparing various experimental observables to different pQCD predictions:

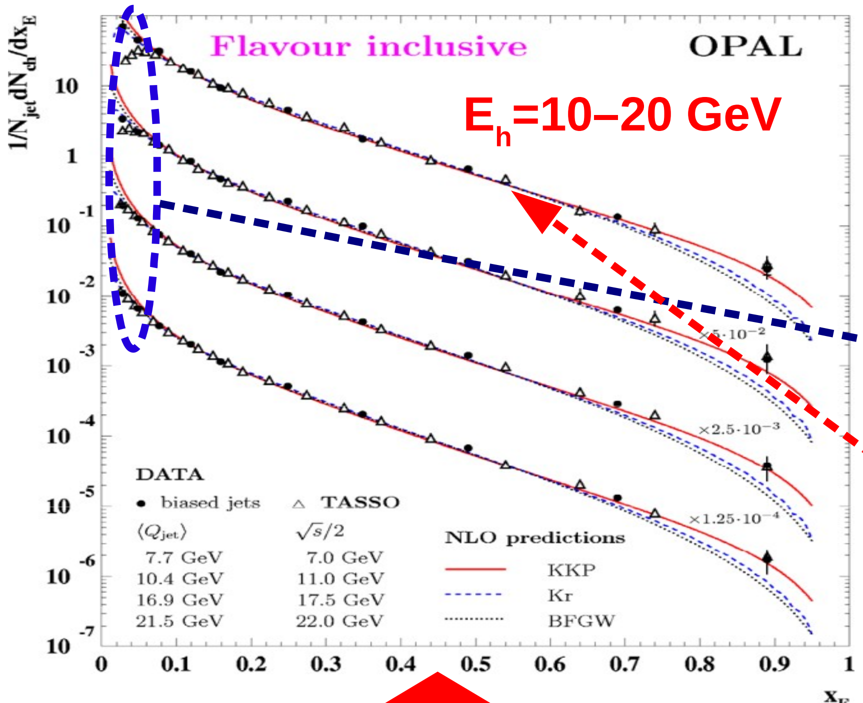
1. Hadronic τ decays: $R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = S_{\rm EW} N_C (1 + \sum_{n=1}^{4} c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_{\rm np})$ **(N³LO)**
2. Lattice QCD: Various short-distance quantities: $K^{\rm NP} = K^{\rm PT} = \sum_{i=0}^{n} c_i \alpha_s^i$ **(NNLO)**
3. Hadronic Z,W decays: $R_Z \equiv \frac{\Gamma(Z \to {\rm h})}{\Gamma(Z \to l)} = R_Z^{\rm EW} N_C (1 + \sum_{n=1}^{4} c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_{\rm m} + \delta_{\rm np})$ **(N³LO)**
4. DIS had. observables: PDFs, σ(jet): $\frac{\partial}{\partial \ln Q^2} D_i^h(x,Q^2) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_S}{4\pi} P_{ji}\left(\frac{x}{z}, Q^2\right) D_j^h(z,Q^2)$ **(NLO,NNLO)**
5. e^+e^- had. observables: Event-shapes, jet rates: $\frac{1}{\sigma}\frac{d\sigma}{dY} = \frac{dA}{dY}\hat{\alpha}_S + \frac{dB}{dY}\hat{\alpha}_S^2 + \frac{dC}{dY}\hat{\alpha}_S^3$ **(NNLO)**
6. Other hadronic observables: σ(ttbar),σ(jets) in p-p, $Q\bar{Q}$ rad. decays **(NLO,NNLO)**

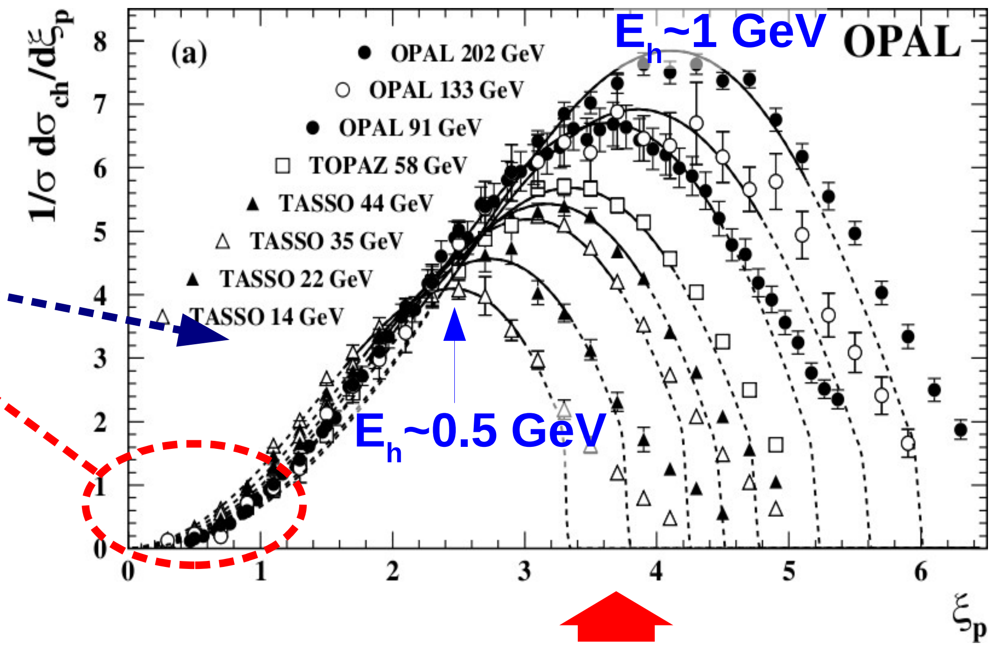
- Direct way to reduce α_s world-average uncertainty: Add new independent extractions

Parton-to-hadron fragmentation functions

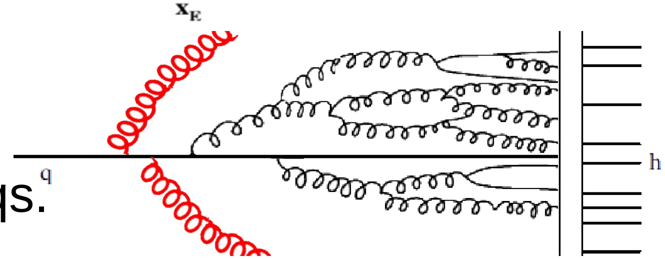
- **Hard** fragmentation function
  $z = p_{had}/p_{jet} > 0.1$
  High-p_T hadrons in jets

- **Soft** fragmentation function
  $\xi = \log(1/z) = \log(p_{jet}/p_{had}) > 1$
  Bulk hadron production in jets





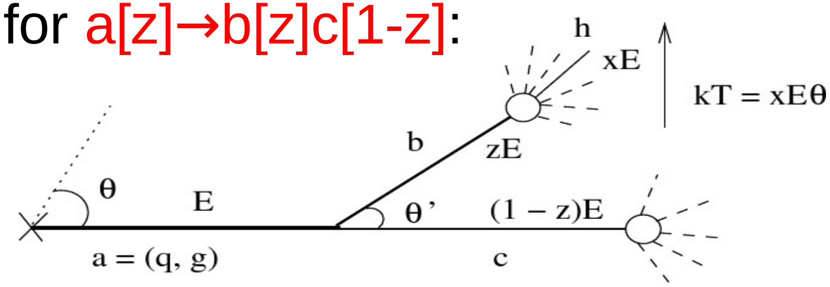
- Hard emission
- Ordered in k_T
- DGLAP evolution eqs.
  $\ln(k_T)$ evolution



- Soft/collinear emission
- Angular ordering
- (N)MLLA evolution eqs.
  $\ln(1/x)$ & $\ln(\theta)$ resummations

Combined QCD evolution eqs. for the FFs

- DGLAP+MLLA evolution equations for a[z]→b[z]c[1-z]:
  - z: energy fraction of intermediate parton
  - ω: energy of radiated gluon
  - x: energy fraction of final hadron

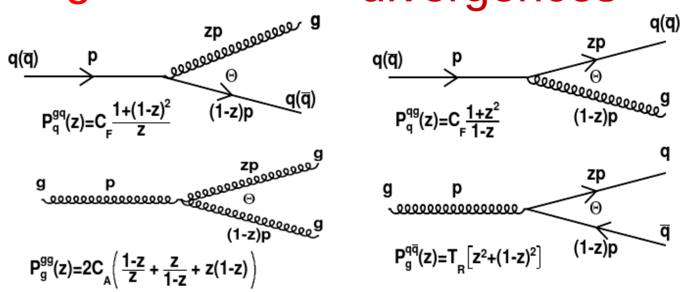


$$\frac{\partial}{\partial \ln \theta} x D_a^b(x, \ln E\theta) = \sum_c \int_0^1 dz \frac{\alpha_s(k_\perp^2)}{2\pi} P_{ac}(z) \left[\frac{x}{z} D_c^b \left(\frac{x}{z}, \ln z E\theta \right) \right]$$

QCD coupling — DGLAP splitting functions — soft&collinear divergences

$$\alpha_s(q^2) = \frac{4\pi}{\beta_0 \ln q^2} \left[1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln \ln q^2}{\ln q^2} \right], \text{ for } \quad q^2 = \frac{k_\perp^2}{\Lambda_{\rm QCD}^2}$$

$$\beta_0 = \frac{11}{3} N_c - \frac{4 n_f T_R}{3}, \quad \beta_1 = \frac{51}{3} N_c - \frac{38 n_f T_R}{3}$$



- Solution via Mellin moments transform:

$$\mathcal{D}(\omega, Y) = \int_0^\infty d\xi e^{-\omega\xi} D(\xi, Y), \; \hat{\xi} = \ln \frac{1}{z}, \; \hat{y} = \ln \frac{k_\perp}{Q_0}, \; \hat{\xi} + \hat{y} = \ln \frac{E\theta}{Q_0} \equiv Y$$

$$\Rightarrow \frac{\partial}{\partial Y} \mathcal{D}(\omega, Y) = \int_0^\infty d\hat{\xi} e^{-\omega\hat{\xi}} P(\hat{\xi}) \frac{\alpha_s(Y - \hat{\xi})}{2\pi} \mathcal{D}(\omega, Y - \hat{\xi}),$$

Solution of evolution eqs. via anomalous dim.

- Expressing the Mellin-transformed hadron distribution in terms of the anomalous dimension: $D \simeq C(\alpha_s(t)) \exp\left[\int^t \gamma(\alpha_s(t'))dt\right], \; t = \ln Q$ one solves evolution equations for an expansion in (half) orders of α_s:

$\gamma \sim O_{DLA}(\sqrt{\alpha_s}) + O_{MLLA}(\alpha_s) + O_{NMLLA}(\alpha_s^{3/2}) + O(\alpha_s^2) + O(\alpha_s^{5/2}) + ...$

DLA: $\alpha_s \log(1/x) \log\Theta$: resummation of soft and collinear gluons:

- main ingredient to the estimation of inclusive observables in jets,
- neglects the energy balance.

Single Logs (SL): $\alpha_s \log\Theta$:

- collinear splittings (i.e. LLA FFs, PDFs at large $x \sim 1$),
- running of $\alpha_s(k_\perp \to Q_0)$ ($\propto \beta_0$).

MLLA: $\underbrace{\alpha_s \log\log}_{\mathcal{O}(1)} + \underbrace{\alpha_s \log}_{\mathcal{O}(\sqrt{\alpha_s})}$: the SL corrections to DLA:

- "restore" the energy balance,
- take into account the running of $\alpha_s(k_\perp)$.

Next-to-MLLA ("NNLL"): $\underbrace{\alpha_s \log\log}_{\mathcal{O}(1)} + \underbrace{\alpha_s \log}_{\mathcal{O}(\sqrt{\alpha_s})} + \underbrace{\alpha_s \log\log^{-1}}_{\mathcal{O}(\alpha_s)}$:

- improve the restoration of the energy balance,
- NLO running coupling effects ($\propto \beta_1$)

Anomalous FF dimension at NLO*+NNLL

- Expressing the Mellin-transformed hadron distribution in terms of the anomalous dimension: $D \simeq C(\alpha_s(t)) \exp\left[\int^t \gamma(\alpha_s(t'))dt\right], \ t = \ln Q$
  one solves evolution equations for an expansion in (half) orders of α_s:
  $\gamma \sim O_{DLA}(\sqrt{\alpha_s}) + O_{MLLA}(\alpha_s) + O_{NMLLA}(\alpha_s^{3/2}) + O(\alpha_s^2) + O(\alpha_s^{5/2}) + \ldots$

- Introducing running α_s at NLO, splitt. fcts at LO, NNLL soft-g. terms one gets the NLO*+NNLL solution for anomalous dim. evolution:

$$\gamma_\omega^{\rm NLO^*+NNLL} = \frac{1}{2}\omega(s-1) + \frac{\gamma_0^2}{4N_c}\left[-\frac{1}{2}a_1(1+s^{-1}) + \frac{\beta_0}{4}(1-s^{-2})\right]$$
$$+ \frac{\gamma_0^4}{256N_c^2}(\omega s)^{-1}\left[4a_1^2(1-s^{-2}) + 8a_1\beta_0(1-s^{-3}) + \beta_0^2(1-s^{-2})(3+5s^{-2})\right.$$
$$\left. - 64N_c\frac{\beta_1}{\beta_0}\ln 2(Y+\lambda)\right] + \frac{1}{4}\gamma_0^2\omega \ a_2(2+s^{-1}+s)$$

new higher-order terms computed for 1st time in JHEP08 (2014) 068

Anomalous FF dimension at NLO+NNLL

- Expressing the Mellin-transformed hadron distribution in terms of the anomalous dimension: $D \simeq C(\alpha_s(t)) \exp\left[\int^t \gamma(\alpha_s(t'))dt\right], \ t = \ln Q$ one solves evolution eqs. for an expansion in (half) orders of α_s:

$\gamma \sim O_{DLA}(\sqrt{\alpha_s}) + O_{MLLA}(\alpha_s) + O_{NMLLA}(\alpha_s^{3/2}) + O(\alpha_s^{2}) + O(\alpha_s^{5/2}) + ...$

- Introducing running α_s at NLO, splitt. fcts at NLO, NNLL soft-g. terms one gets the full-NLO+NNLL solution for anomalous dim. evolution:

$$\begin{aligned}
\gamma_\omega^{\mathrm{NLO+NNLL}} &= \frac{1}{2}\omega(s-1) + \frac{\gamma_0^2}{4N_c}\left[-\frac{1}{2}a_1(1+s^{-1}) + \frac{\beta_0}{4}(1-s^{-2})\right] \\
&+ \frac{\gamma_0^4}{256N_c^2}(\omega s)^{-1}\left[4a_1^2(1-s^{-2}) + 8a_1\beta_0(1-s^{-3}) + \beta_0^2(1-s^{-2})(3+5s^{-2})\right. \\
&\left. - 64N_c\frac{\beta_1}{\beta_0}\ln 2(Y+\lambda)\right] \\
&+ \frac{1}{4}\gamma_0^2\omega\left[a_2(2+s^{-1}+s) + a_3(s-1) - a_4(1-s^{-1}) - a_5(1-s^{-3}) - a_6\right].
\end{aligned}$$

new higher-order terms computed (to be published)

Anomalous FF dimension at NNLO*+NNLL

- Expressing the Mellin-transformed hadron distribution in terms of the anomalous dimension: $D \simeq C(\alpha_s(t)) \exp\left[\int^t \gamma(\alpha_s(t'))dt\right], \; t = \ln Q$ one solves evolution eqs. for an expansion in (half) orders of α_s:

$\gamma \sim O_{DLA}(\sqrt{\alpha_s}) + O_{MLLA}(\alpha_s) + O_{NMLLA}(\alpha_s^{3/2}) + O(\alpha_s^2) + O(\alpha_s^{5/2}) + ...$

- Introducing running α_s at NNLO, splitt. fcts at NLO, NNLL soft-g. terms one gets the NNLO*+NNLL solution for anomalous dim. evolution:

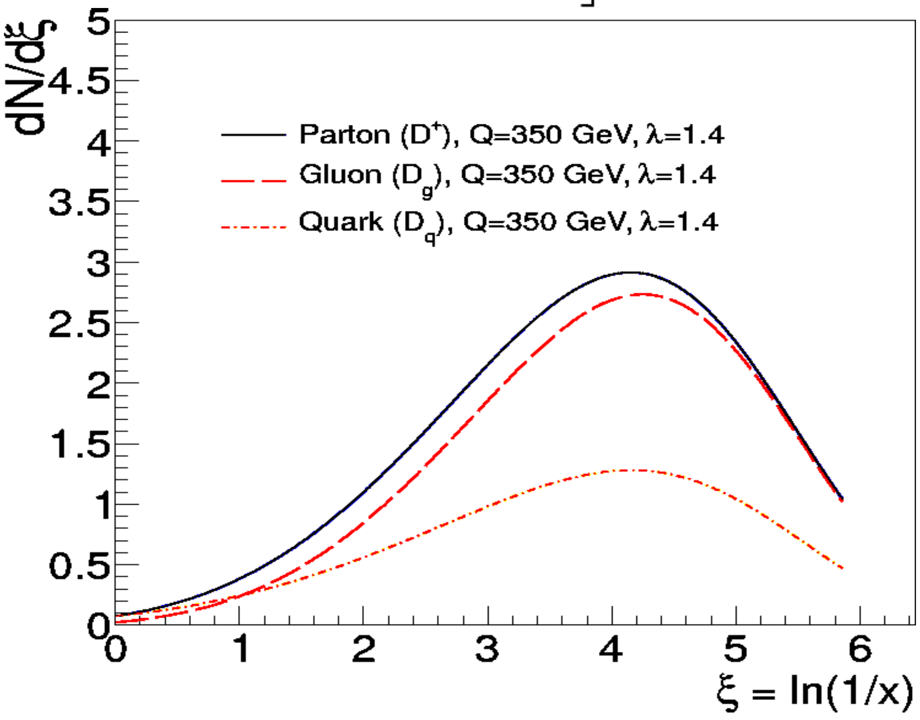
$$\gamma_\omega^{\rm NNLO*+NNLL} = \gamma_\omega^{\rm NLO+NNLL} + \Delta\gamma_{++}^{\rm NNLO*}$$

$$\begin{aligned}
\Delta\tilde{\gamma}_{++}^{\rm NNLO*}(\omega,\gamma_0) = & \frac{\beta_1}{16N_c^2}(\omega s)^{-2}\gamma_0^6 + \frac{\beta_0}{4N_c}(\omega s)^{-2}G_1(\omega,\gamma_0)\gamma_0^6 + \frac{\beta_0}{8N_c^2}(\omega s)^{-4}C_1(Y,\lambda)\gamma_0^8 \\
& - \frac{\beta_0^3}{16N_c^3}(\omega s)^{-6}\gamma_0^{10} + \frac{5\beta_0^3}{32N_c^3}(\omega s)^{-8}\gamma_0^{12} \\
& + \frac{1}{16N_c^2}(\omega s)^{-1}C_2(Y,\lambda)\gamma_0^6 + \frac{\beta_0\beta_1}{32N_c^3}(\omega s)^{-3}\gamma_0^8 - \frac{1}{16N_c^2}(\omega s)^{-3}C_1^2(Y,\lambda)\gamma_0^8 \\
& + \frac{\beta_0}{4N_c}(\omega s)^{-3}G_2(\omega,\gamma_0)\gamma_0^8 + \frac{\beta_0^2}{16N_c^3}(\omega s)^{-5}C_1(Y,\lambda)\gamma_0^{10} - \frac{3\beta_0\beta_1}{32N_c^3}(\omega s)^{-5}\gamma_0^{10} \\
& - \frac{\beta_0^2}{8N_c^2}(\omega s)^{-5}G_1(\omega,\gamma_0)\gamma_0^{10} - \frac{7\beta_0^2}{32N_c^3}(\omega s)^{-7}C_1(Y,\lambda)\gamma_0^{12} - \frac{\beta_0^4}{64N_c^4}(\omega s)^{-7}\gamma_0^{12} \\
& + \frac{7\beta_0^4}{64N_c^4}(\omega s)^{-9}\gamma_0^{14} - \frac{45\beta_0^4}{256N_c^4}(\omega s)^{-11}\gamma_0^{16} + \mathcal{O}(\gamma_0^6),
\end{aligned}$$

new higher-order terms computed now (preliminary)

NLO+NNLL parton/g/q single inclusive FFs

- Quark FF: $D_q(\xi,Y,\lambda) \approx \frac{C_F}{N_c}\left[1 + c_q^{(0)}\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial Y}\right) + c_q^{(1)}\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial Y}\right)^2\right] D^+(\xi,Y,\lambda)$
- Gluon FF: $D_g(\xi,Y,\lambda) \approx \left[1 + c_g^{(0)}\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial Y}\right) + c_g^{(1)}\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial Y}\right)^2\right] D^+(\xi,Y,\lambda)$



- Jet evolved from δ-function at scale $\lambda = \ln(Q_0/\Lambda_{QCD}) = 1.4$ i.e. $Q_0 \sim 0.8$ GeV, up to a virtuality Q = 350 GeV, yields Gaussian-like FF

- Ratio of g/q hadron multiplicities:

$$r_+ = \frac{N_c}{C_F}\frac{1 + c_g^{(0)}\frac{1}{\mathcal{N}(Y)}\frac{\partial\mathcal{N}(Y)}{\partial Y} + c_g^{(1)}\frac{1}{\mathcal{N}(Y)}\frac{\partial^2\mathcal{N}(Y)}{\partial Y^2} + c_g^{(2)}\frac{1}{\mathcal{N}(Y)}\frac{\partial^3\mathcal{N}(Y)}{\partial Y^3}}{1 + c_q^{(0)}\frac{1}{\mathcal{N}(Y)}\frac{\partial\mathcal{N}(Y)}{\partial Y} + c_q^{(1)}\frac{1}{\mathcal{N}(Y)}\frac{\partial^2\mathcal{N}(Y)}{\partial Y^2} + c_q^{(2)}\frac{1}{\mathcal{N}(Y)}\frac{\partial^3\mathcal{N}(Y)}{\partial Y^3}} = \frac{N_c}{C_F}\left(1 - r_1\gamma_0 - r_2\gamma_0^2 - r_3\gamma_0^3\right)$$

[Consistent with Bolzoni/Kniel/Kotikov 2013, fixed order approach]

Distorted Gaussian FF parametrization

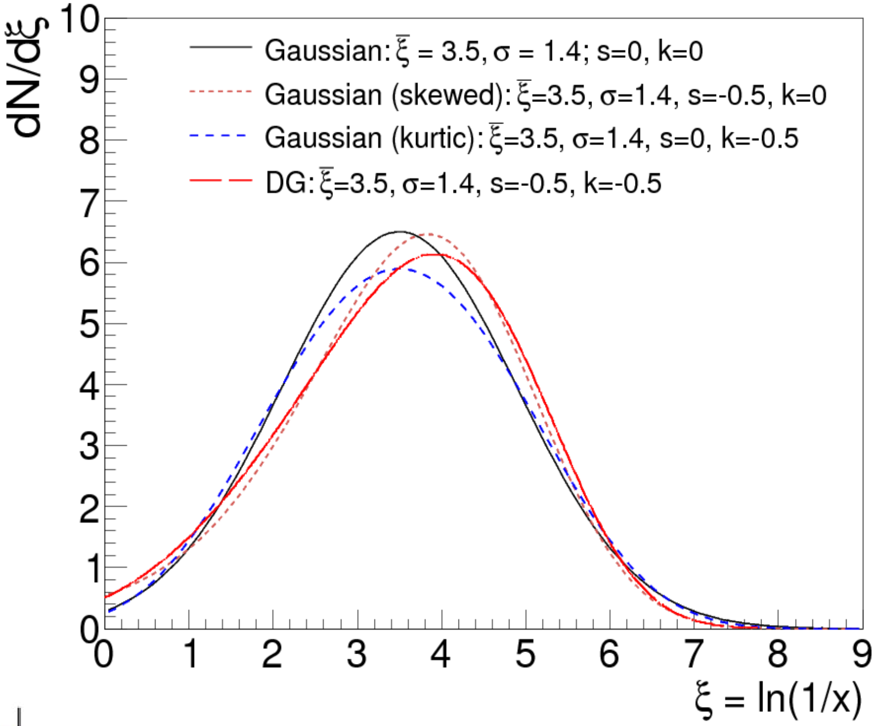
- The hadron distribution in jets can be, without loss of generality, expressed as a Distorted Gaussian:

$$D^+(\xi, Y, \lambda) = \frac{\mathcal{N}}{\sigma\sqrt{2\pi}} \exp\left[\frac{1}{8}k - \frac{1}{2}s\delta - \frac{1}{4}(2+k)\delta^2 + \frac{1}{6}s\delta^3 + \frac{1}{24}k\delta^4\right], \ \delta = \frac{(\xi-\bar{\xi})}{\sigma}$$

- FF moments:
  - ➔ Mean multiplicity: $\mathcal{N} = \mathcal{D}^+(\omega = 0, Y, \lambda)$
  - ➔ Peak position: $\bar{\xi}$ (mean)

$$\xi_{\max} - \bar{\xi} = -\frac{1}{2}\sigma s\left(1 - \frac{1}{4}\frac{k_5}{s} + \frac{5}{6}k\right)$$

  - ➔ Dispersion (width): σ
  - ➔ Skewness: s
  - ➔ Kurtosis: k



- FF moments from anomalous dim.:

$$K_{n\geq 0} = \int_0^Y dy \left(-\frac{\partial}{\partial\omega}\right)^n \gamma_\omega(\alpha_s(y+\lambda))\Big|_{\omega=0}$$

$$\mathcal{N} = K_0, \quad \bar{\xi} = K_1, \quad \sigma = \sqrt{K_2}, \quad s = \frac{K_3}{\sigma^3}, \quad k = \frac{K_4}{\sigma^4}$$

FF moments evolution: NNLO*+NNLL formulas

Final expressions as a function of $Y = \ln(E\theta/Q_0)$ and $\lambda = \ln(Q_0/\Lambda_{QCD})$:
(N_f=5) initial jet energy — shower energy cutoff

- **Multiplicity**:

$$\mathcal{N}(Y) = \mathcal{K}^{\rm ch} \exp\left[2.50217\left(\sqrt{Y+\lambda}-\sqrt{\lambda}\right) - 0.491546\ln\frac{Y+\lambda}{\lambda} + (0.0153206 + 0.41151\ln(Y+\lambda))\frac{1}{\sqrt{Y+\lambda}} - (0.0153206+0.41151\ln\lambda)\frac{1}{\sqrt{\lambda}}\right]. \quad (71)$$

- **Average**:

$$\bar{\xi}(Y) = 0.5Y + 0.592722\left(\sqrt{Y+\lambda}-\sqrt{\lambda}\right) + 0.0763404\ln\frac{Y+\lambda}{\lambda}. \quad (73)$$

- **Peak position**:

$$\xi_{\max}(Y) = 0.5Y + 0.592722\left(\sqrt{Y+\lambda}-\sqrt{\lambda}\right) + 0.0763404\ln\frac{Y+\lambda}{\lambda} - 0.355325. \quad (74)$$

- **Width**:

$$\sigma(Y,\lambda) = \left(\frac{\beta_0}{144N_c}\right)^{1/4}\sqrt{(Y+\lambda)^{3/2}-\lambda^{3/2}}\left\{1 - \frac{\beta_0}{64}f_1(Y,\lambda)\sqrt{\frac{16N_c}{\beta_0(Y+\lambda)}} + \left[\frac{3}{16}(3a_2+a_3+2a_4)f_2(Y,\lambda) - \frac{3}{64}\left(\frac{3a_1^2}{16N_c^2}f_2(Y,\lambda) + \frac{a_1\beta_0}{8N_c^2}f_2(Y,\lambda) - \frac{\beta_0^2}{64N_c^2}f_2(Y,\lambda) + \frac{3\beta_0^2}{128N_c^2}f_1^2(Y,\lambda)\right) + \frac{\beta_1}{64\beta_0}(\ln 2(Y+\lambda)-2)f_3(Y,\lambda)\right]\frac{16N_c}{\beta_0(Y+\lambda)}\right\}, \quad (75)$$

- **Skewness**:

$$\sigma(Y) = 0.36499\sqrt{(Y+\lambda)^{3/2}-\lambda^{3/2}}\left\{1 - 0.299739f_1(Y,\lambda)\frac{1}{\sqrt{Y+\lambda}} - [1.61321f_2(Y,\lambda) + 0.0449219f_1^2(Y,\lambda) + (0.32239 - 0.246692\ln(Y+\lambda))f_3(Y,\lambda)]\frac{1}{Y+\lambda}\right\}. \quad (76)$$

$$s(Y) = -\frac{1.94704}{\sqrt{(Y+\lambda)^{3/2}-\lambda^{3/2}}}\left[1 - 0.299739f_1(Y,\lambda)\frac{1}{\sqrt{Y+\lambda}}\right]. \quad (78)$$

- **Kurtosis**:

$$k(Y) = -\frac{2.15812}{\sqrt{Y+\lambda}}\frac{1-\left(\frac{\lambda}{Y+\lambda}\right)^{5/2}}{\left[1-\left(\frac{\lambda}{Y+\lambda}\right)^{3/2}\right]^2}\left\{1 + [1.19896f_1(Y,\lambda) - 1.99826f_4(Y,\lambda)]\frac{1}{\sqrt{Y+\lambda}} + [1.07813f_1^2(Y,\lambda) + 6.45283f_2(Y,\lambda) + 1.28956f_3(Y,\lambda) - 2.39583f_1(Y,\lambda)f_4(Y,\lambda) - 7.13372f_5(Y,\lambda) + 0.0217751f_6(Y,\lambda) - (0.986767f_3(Y,\lambda) - 0.822306f_6(Y,\lambda))\ln(Y+\lambda)]\frac{1}{Y+\lambda}\right\}. \quad (80)$$

Evolution of FF moments: limiting spectrum

- Final expressions evolved down to Λ_{QCD}: $Y = \ln(2\sqrt{s}/\Lambda_{QCD})$, $Q_0 = \Lambda_{QCD}$

(N_f=5)

$$\mathcal{N}(Y) = \mathcal{K}^{\rm ch} \exp\left[2.50217\sqrt{Y} - 0.491546\ln Y + (0.0153206 + 0.41151\ln Y)\frac{1}{\sqrt{Y}} + (0.00068 - 0.161658\ln Y)\frac{1}{Y} - (0.0447232 + 0.0222627\ln Y + 0.0338388\ln^2 Y)\frac{1}{Y^{3/2}}\right]$$

$$\bar{\xi}(Y) = 0.5Y + 0.592722\sqrt{Y} + 0.0763404\ln Y$$

$$\xi_{\max}(Y) = 0.5Y + 0.592722\sqrt{Y} - 0.351319 + 0.0763404\ln Y$$

$$\sigma(Y) = 0.36499Y^{3/4}\left[1 - 0.299739\frac{1}{\sqrt{Y}} - (1.98052 - 0.246692\ln Y)\frac{1}{Y} + (1.98667 - 0.098591\ln Y)\frac{1}{Y^{3/2}} + (0.121925 + 0.0330471\ln Y + 0.0202856\ln^2 Y)\frac{1}{Y^2}\right]$$

$$s(Y) = -\frac{1.94704}{Y^{3/4}}\left[1 - 0.299739\frac{1}{\sqrt{Y}} - \frac{1.64393}{Y}\right]$$

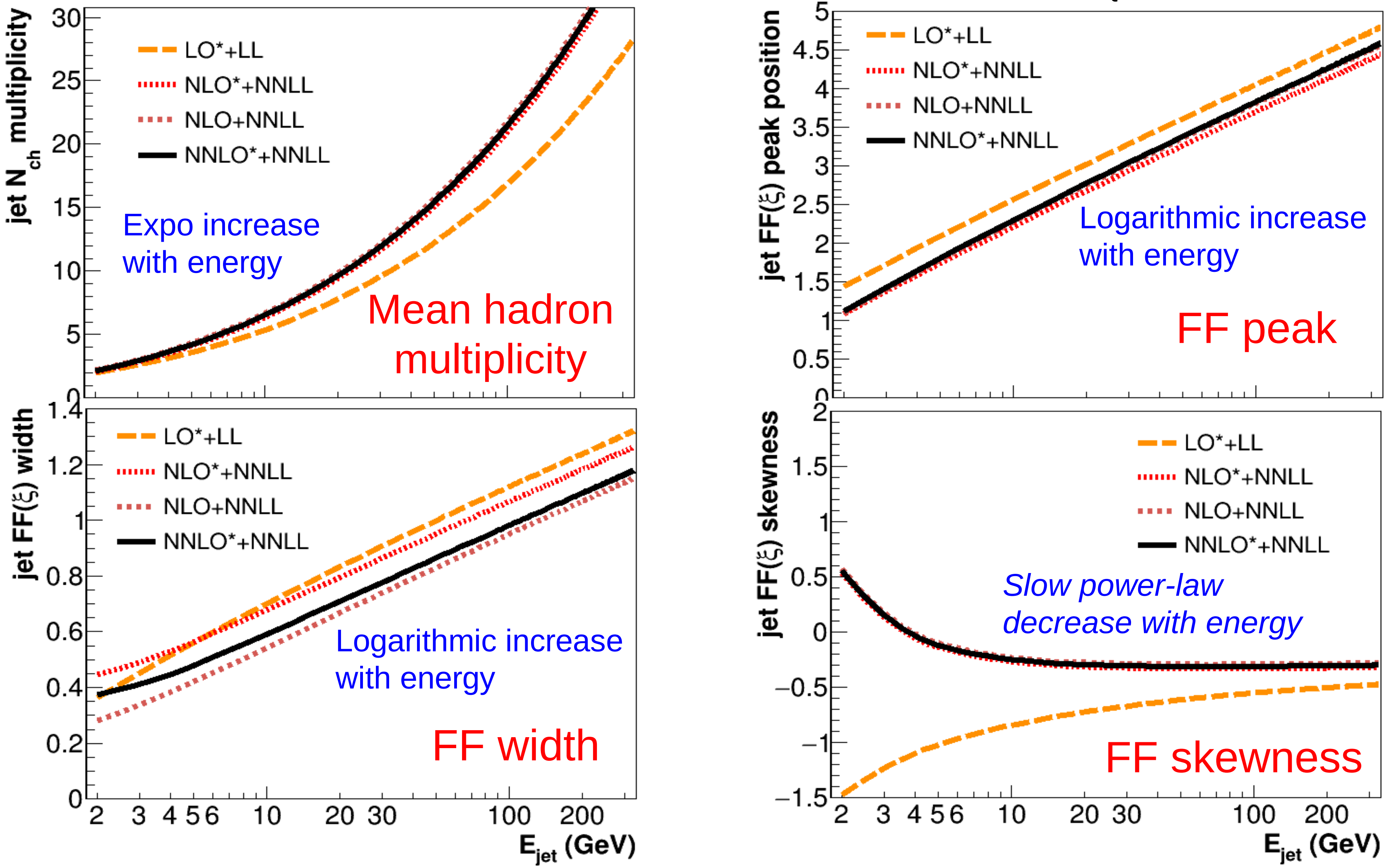
$$k(Y) = -\frac{2.15812}{\sqrt{Y}}\left[1 - 0.799305\frac{1}{\sqrt{Y}} - (0.687266 + 0.164461\ln Y)\frac{1}{Y} - (9.92639 + 0.90185\ln Y)\frac{1}{Y^{3/2}} + (0.272679 + 1.052\ln Y + 0.121714\ln^2 Y)\frac{1}{Y^2}\right]$$

- Evolution of all moments depends on 1 single free parameter: Λ_{QCD}
- Theoretical expressions depend on number of active flavours (N_f).

  N_f= 5 used for fit. Small corrections in evolution applied at thresholds:
  Moments for $\sqrt{s} < m_c$ = 1.3 GeV scaled by (N_f=3)/(N_f= 5) expectation
  Moments for $m_c < \sqrt{s} < m_b$ = 4.2 GeV scaled by (N_f=4)/(N_f= 5) expectation

Evolution of FF moments: LO,NLO*,NLO,NNLO*

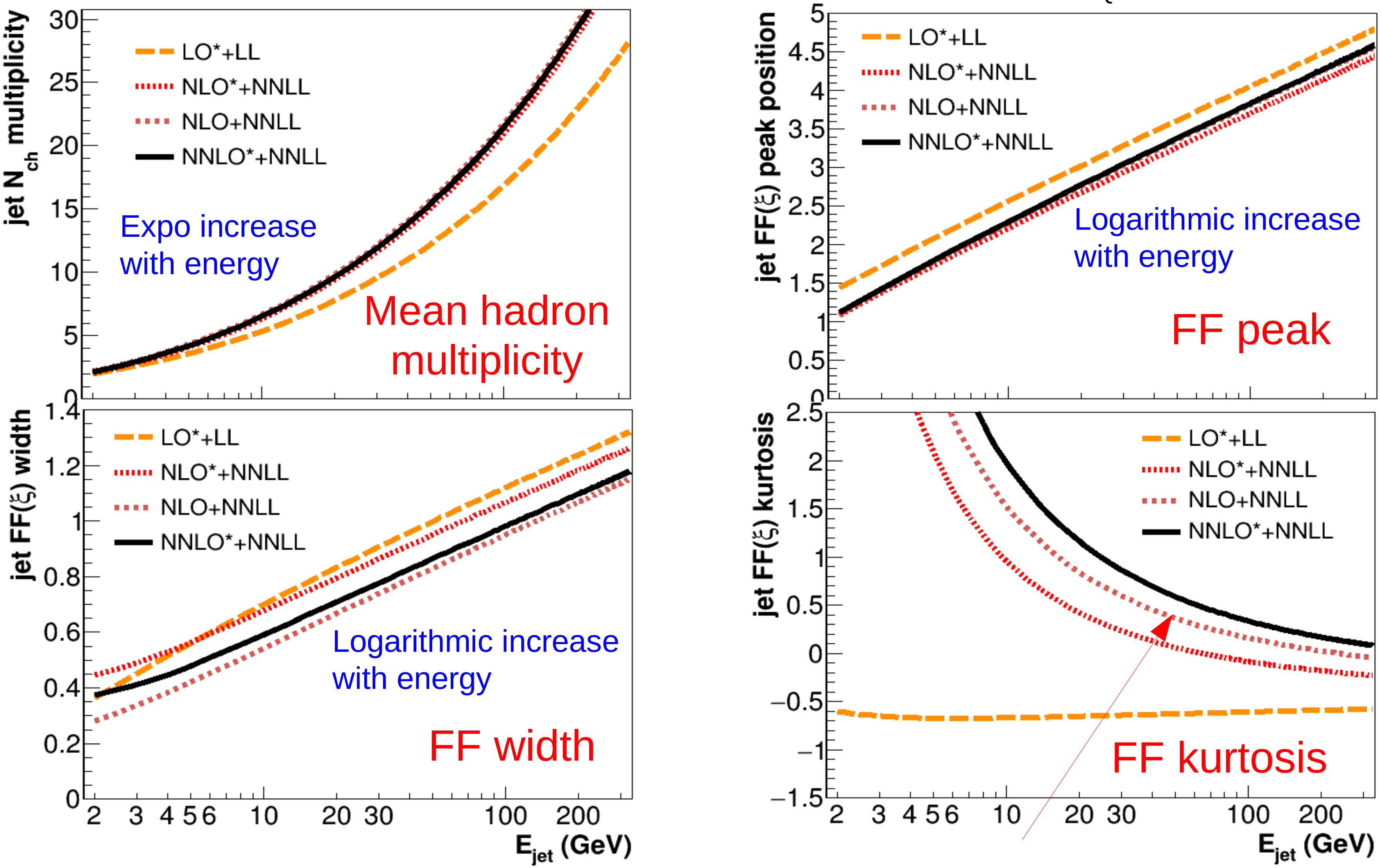
- Evolutions for "limiting spectrum" shower cutoff ($Q_0=\Lambda_{QCD}$):



➔ Small impact (±10% max) corrections beyond NLO* for first 4 FF moments

Evolution of FF moments: LO,NLO*,NLO,NNLO*

- Evolutions for "limiting spectrum" shower cutoff ($Q_0=\Lambda_{QCD}$):



➔ However, kurtosis (4th derivative of anomalous dim.) does not converge yet ...

Experimental e^+e^- & DIS FF moments

- 200 e+e- data points
- 140 e-p data points

Table 1: List of experimental measurements of jet FF moments in e^+e^- collisions.

e^+e^- collisions $\sqrt{s}$ (GeV)	Experiment(s)	Number of FF moments data-points $\langle N_{ch}\rangle$	peak	width	skewness
1.77	PDG τ hadronic decay [?]	1	–	–	–
2.2	BES [?]	1	1	1	1
2.6	BES [?]	1	1	1	1
3.0	BES [?]	1	1	1	1
3.2	BES [?]	1	1	1	1
4.6	BES [?]	1	1	1	1
4.8	BES [?]	1	1	1	1
5.2	MARK-II [?]	1	1	1	1
6.5	MARK-II [?]	1	1	1	1
10.54	BaBar [35]	1	1	1	1
12.0	JADE, PLUTO, TASSO [?]	3	1	–	–
13.0	PLUTO [?]	1	–	–	–
14.0	TASSO [37]	1	1	1	1
17.0	PLUTO [?]	1	–	–	–
22.0	TASSO [37]+JADE, PLUTO [?]	1+2	1	1	1
25.0	TASSO [37]	1	1	1	1
27.6	JADE, PLUTO [?]	2	–	–	–
29.0	MARK-II [?], TPC [38], HRS [?]	3	3	3	3
30.0–30.7	JADE, PLUTO, TASSO [?]	4	1	–	–
31.3	PLUTO [?]	1	–	–	–
31.6	JADE [?]	1	–	–	–
35.0	TASSO [37]+JADE [?]	1+1	1+1	1	1
41.5	TASSO [?]	1	–	–	–
43.7	TASSO [37]	1	1	1	1
50.0	AMY [?]	1	–	–	–
52.0	AMY [?]	1	–	–	–
55.0	AMY [?]	1	–	–	–
56.0	AMY [?]	1	–	–	–
57.0	AMY [?]	1	–	–	–
57.8	TOPAZ [?, 39]	1+1	–	–	–
60.0	AMY [?]	1	–	–	–
60.8	AMY [?]	1	–	–	–
61.4	AMY [?]	1	–	–	–
80.4	PDG $W^\pm$ hadronic decay [?]	1	–	–	–
91.2	ALEPH [?, 42], L3 [?], OPAL [5, 40]	4	4	4	4
91.2	ALEPH, DELPHI, L3, OPAL [?]	7	–	–	–
91.2	PDG Z hadronic decay [?]	1	–	–	–
130.0	L3, DELPHI [?]	2	–	–	–
133.0	ALEPH [41, 42], DELPHI [43], OPAL [44]	4	4	4	4
136.0	L3 [?]	1	–	–	–
161.3	ALEPH [42], OPAL [45] + DELPHI [?]	2+1	2	2	2
172.2	ALEPH [42], OPAL [46] + DELPHI [?]	2+1	2	2	2
182.7	ALEPH [42], OPAL [46]	2	2	2	2
183.0	DELPHI [?]	1	–	–	–
188.7	ALEPH [42], OPAL [46]	2	2	2	2
189.0	DELPHI [?]	1	–	–	–
196.0	ALEPH [42]	1	1	1	1
200.0	ALEPH [42] + DELPHI [?]	1+1	1	1	1
201.7	OPAL [47]	1	1	1	1
206.0	ALEPH [42]	1	1	1	1
TOTAL		83	41	38	38

Table 2: List of experimental measurements of jet FF moments in DIS e,ν-p collisions.

DIS e-p collisions $\langle Q\rangle$ (GeV)	Experiment(s)	Number of FF moments data-points Nch	peak	width	skewness
2.9	ZEUS [?]	1	–	–	–
3.7	H1 [?, ?]	2	2	2	–
3.8	ZEUS [?]	1	1	1	1
4.2	H1 [?, ?]	2	2	2	–
5.3	ZEUS [?],H1 [?, ?]	2	2	2	1
5.5	H1 [?]	1	1	1	–
5.9	ZEUS [?]	1	–	–	–
6.9	H1 [?]	1	1	1	–
7.1	H1 [?]	1	1	1	1
7.3	ZEUS [?]	1	1	1	1
8.3	H1 [?, ?]	2	2	2	–
9.3	H1 [?]	1	1	1	–
9.6	ZEUS [?]	1	–	–	–
10.4	ZEUS [?]	1	1	1	1
11.7	H1 [?]	1	1	1	–
12.3	H1 [?, ?]	2	1	1	–
14.5	ZEUS [?],H1 [?, ?, ?]	4	3	3	2
14.7	ZEUS [?]	1	1	1	1
14.8	ZEUS [?]	1	–	–	–
18.0	H1 [?, ?]	2	1	1	–
18.7	H1 [?]	1	1	1	–
20.4	ZEUS [?]	1	1	1	1
21.0	ZEUS [?]	1	1	1	1
23.8	ZEUS [?]	1	–	–	–
25.0	H1 [?, ?]	2	1	1	–
26.9	H1 [?]	1	1	1	–
29.2	ZEUS [?]	1	1	1	1
29.5	ZEUS [?]	1	1	1	1
35.6	ZEUS [?]	1	–	–	–
36.6	H1 [?]	1	–	–	–
41.2	H1 [?]	1	1	1	–
42.0	ZEUS [?]	1	1	1	1
58.1	ZEUS [?]	1	–	–	–
58.5	H1 [?]	1	–	–	–
59.4	ZEUS [?]	1	1	1	1
67.1	H1 [?]	1	1	1	–
82.9	ZEUS [?]	1	1	1	1
102.5	H1 [?]	1	–	–	–
115.7	ZEUS [?]	1	1	1	1
160.3	ZEUS [?]	1	1	1	1
TOTAL		50	36	36	15

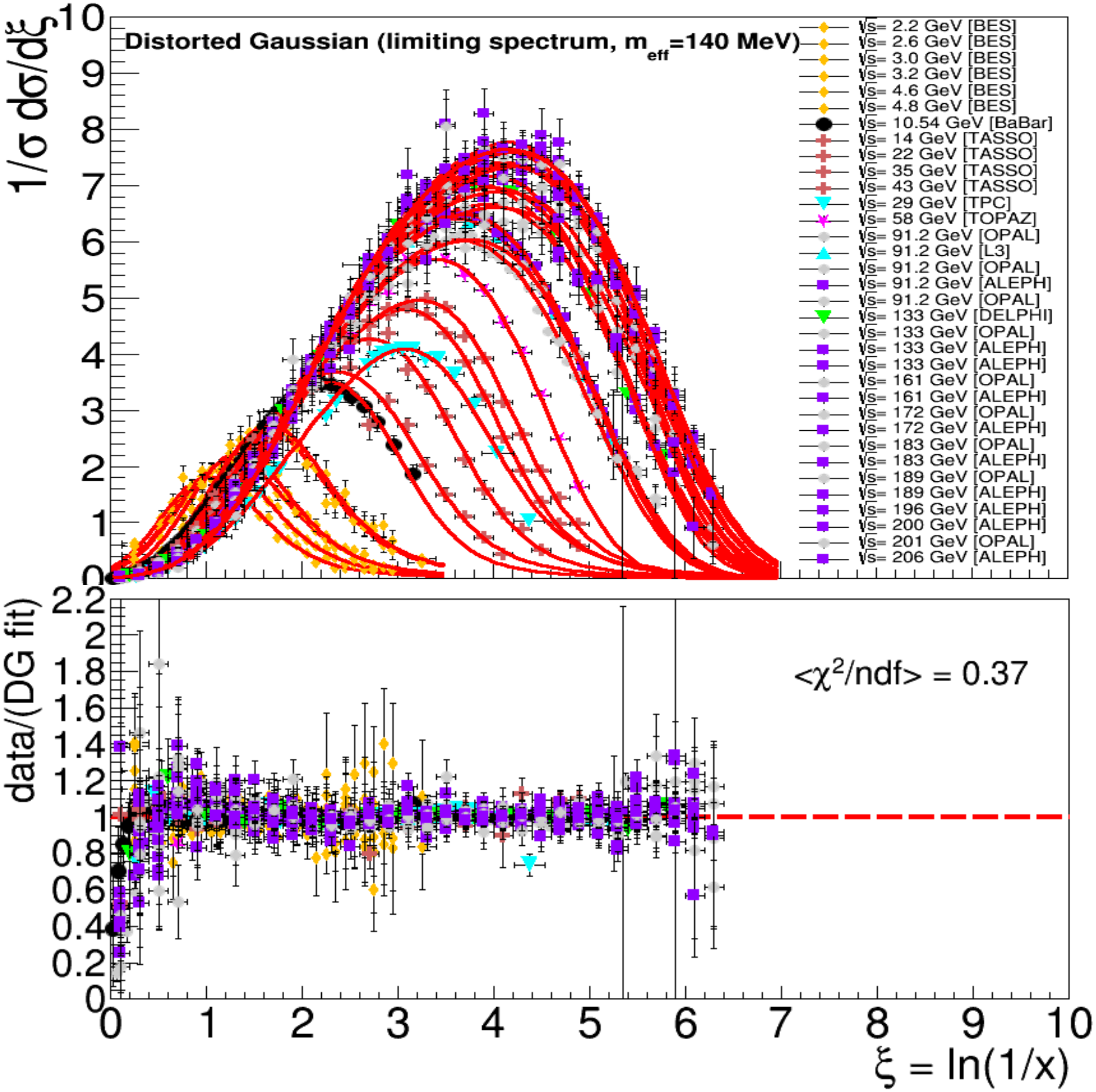
Distorted Gaussian fits to e^+e^- FFs

- 34 e^+e^- data-sets at $\sqrt{s} = 2.2 – 206$ GeV

  ~1200 data points

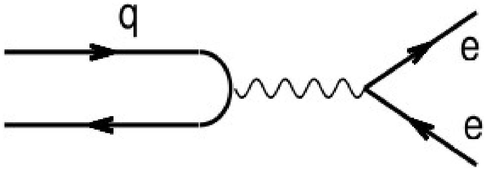
- Peak shifts to right, width increases, moderate non-Gaussian tails

- Excellent fit to DG at all energies, with 5 free parameters: N_{ch}, ξ_{max}, σ, s, k



Distorted Gaussian fits to DIS FFs

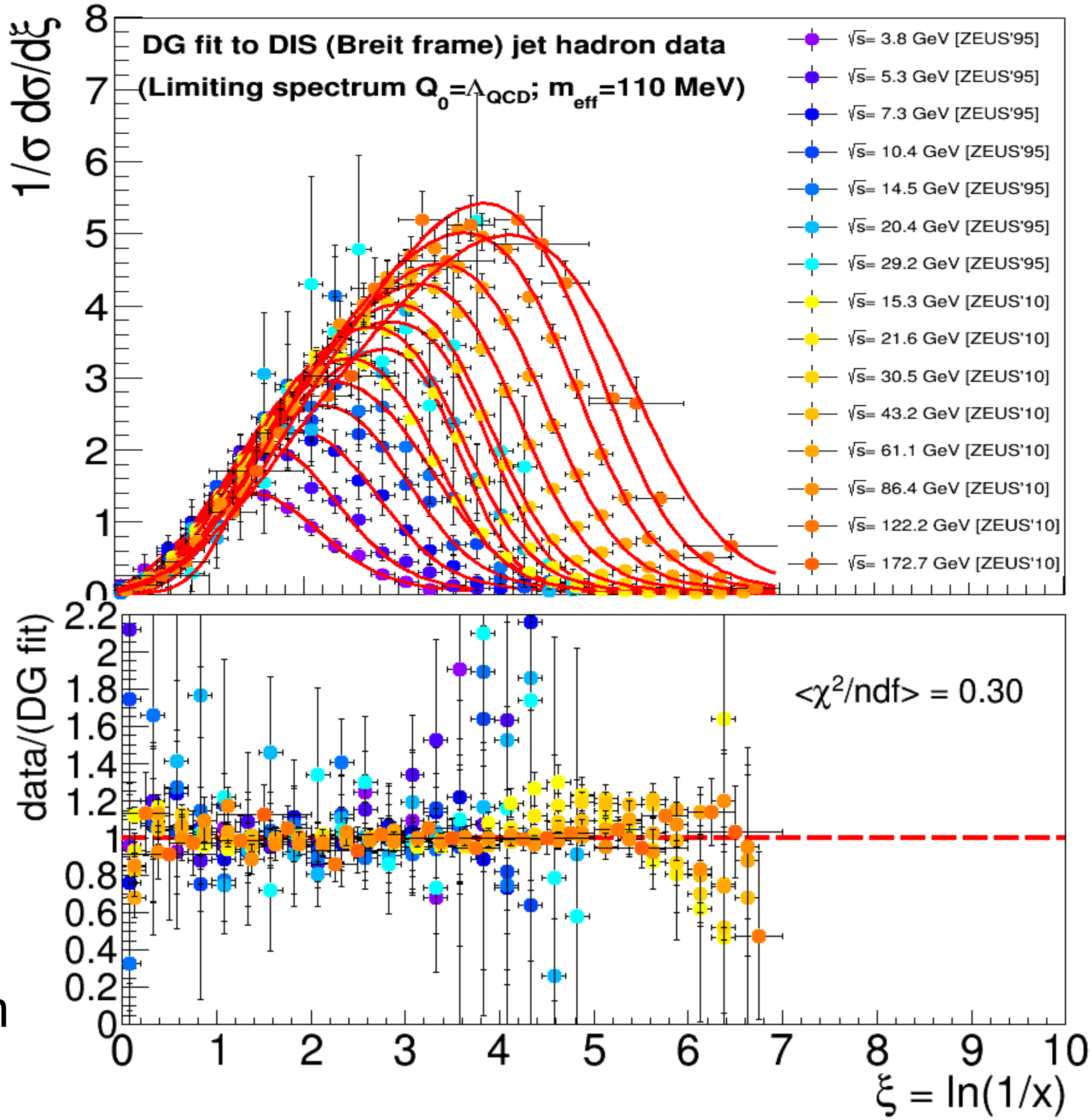
- Breit frame in DIS:



"Brick wall" frame: Incoming quark scatters off photon & returns along same axis

- 15 ZEUS data-sets at $\sqrt{s} = 3.8 - 173$ GeV
  ~250 data points
  (other measured H1, ZEUS moments added to global fit)

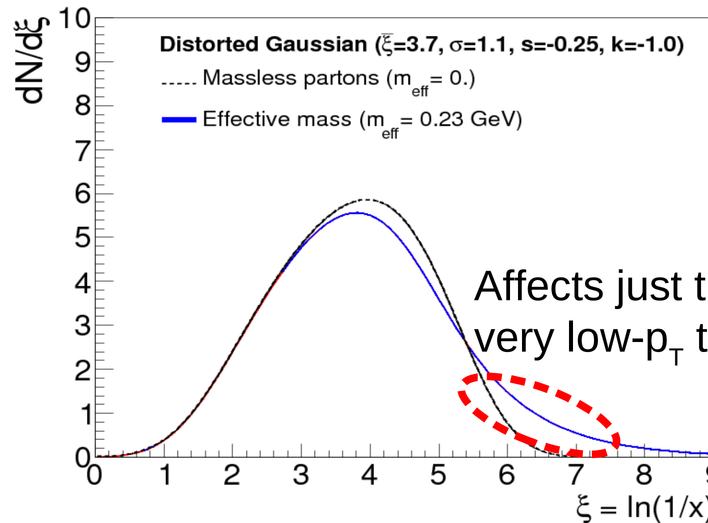
- Good fits to DG but larger uncertainties than e^+e^- measurements



Hadron-level corrections: finite mass & decays

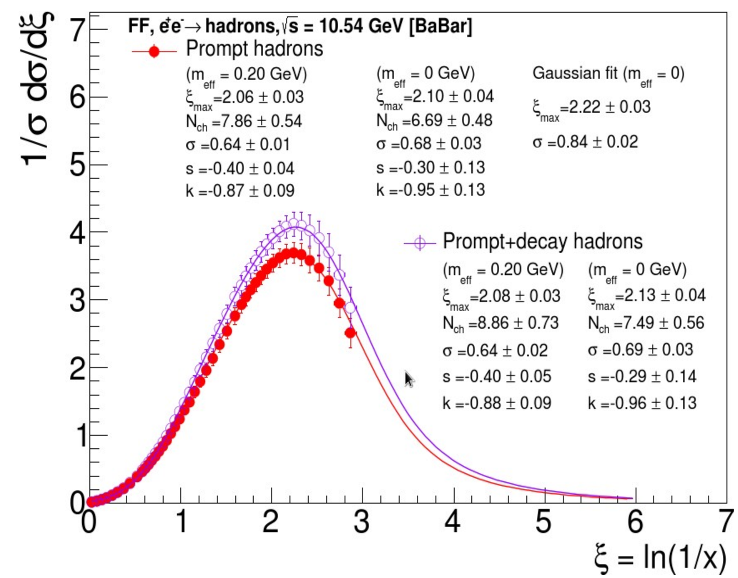
- Difference between FF measurement (massive hadrons, ξ_p) and theory (massless partons, $\xi = \xi_E$) accounted for via effective mass in DG fit:

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma^{\rm h}}{d\xi_p} \propto \frac{p_h}{E_h} D^{+}(\xi, Y) \qquad E_h = \sqrt{p_h^2 + m_{\rm eff}^2}$$



Data fits varied within $m_{\rm eff}$ = 0–0.36 GeV. Best χ^2/ndf for $m_{\rm eff} \sim m_\pi$.

- Differences in hadron-level definitions among measurements (weak-decays) assessed with fits to BaBar prompt vs. inclusive hadrons. Only mult. affected: N_{ch}(incl)=0.92·N_{ch}(prompt)



➔ All associated uncertainties propagated to final FF moments.

N_{ch} & FF peak vs. √s: Data vs. NLO*+NNLL

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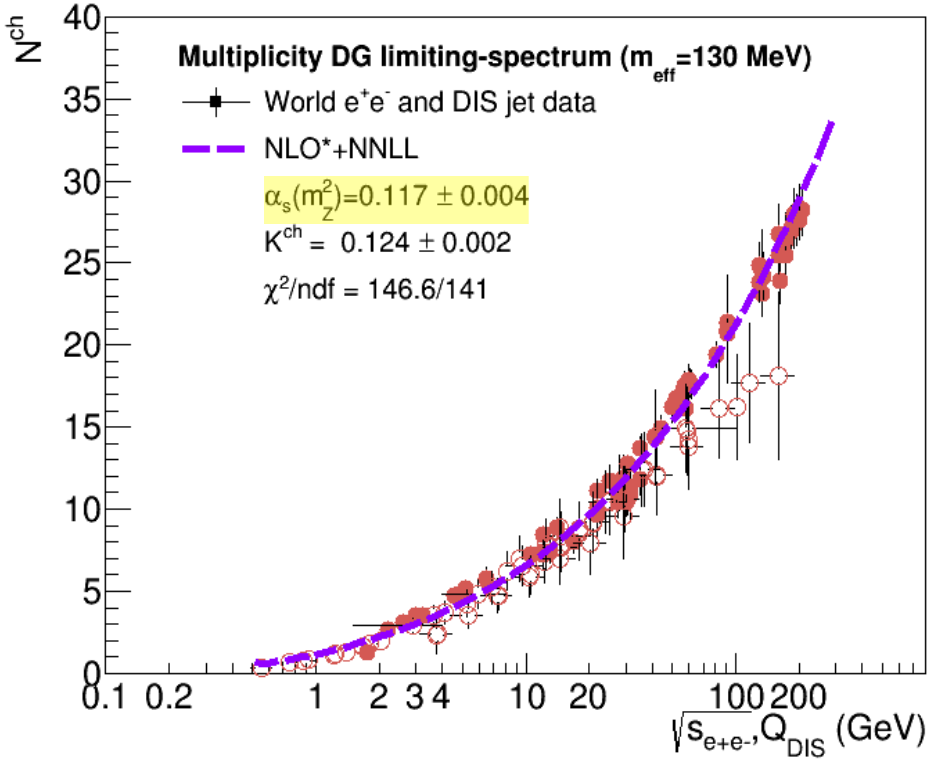

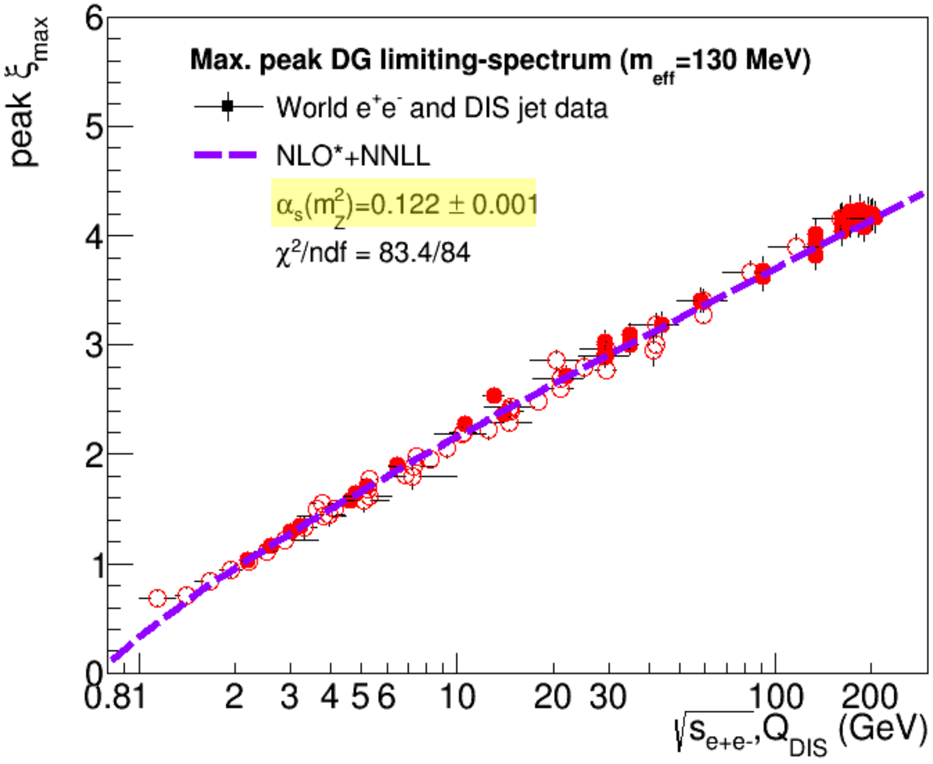

Theoretical N_{ch} absolutely normalized to match data (local hadron-parton duality).

- DIS multiplicity lower than e^+e^- but with larger uncertainties.

- Very good agreement between e^+e^-,DIS and theory for the FF peak position.

FF width & skewn. vs. √s: Data vs. NLO*+NNLL

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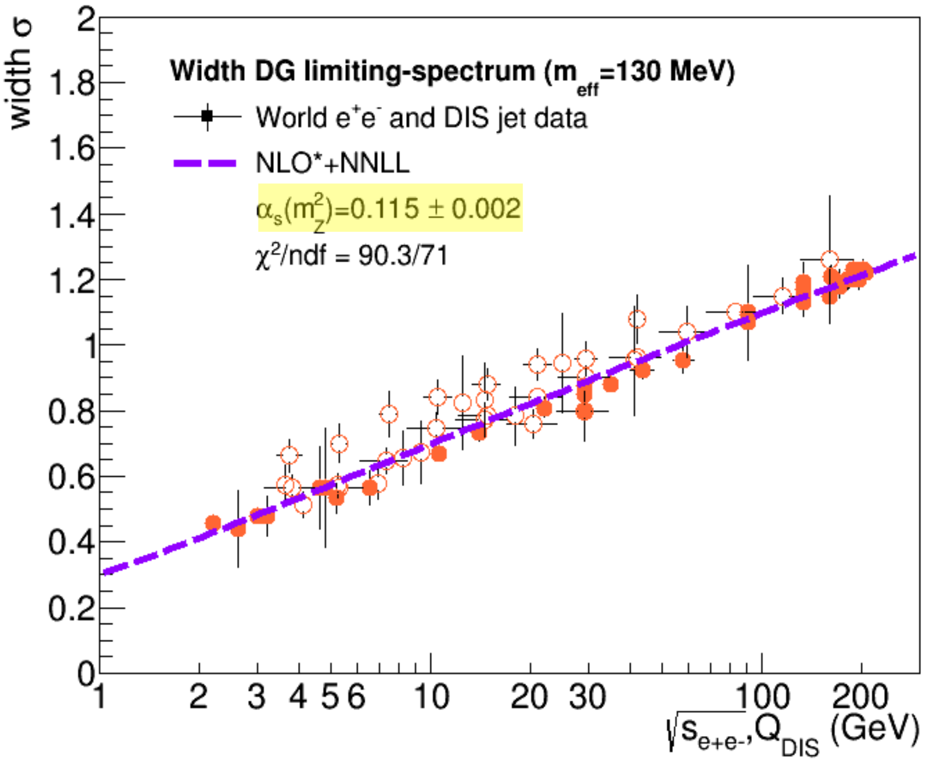
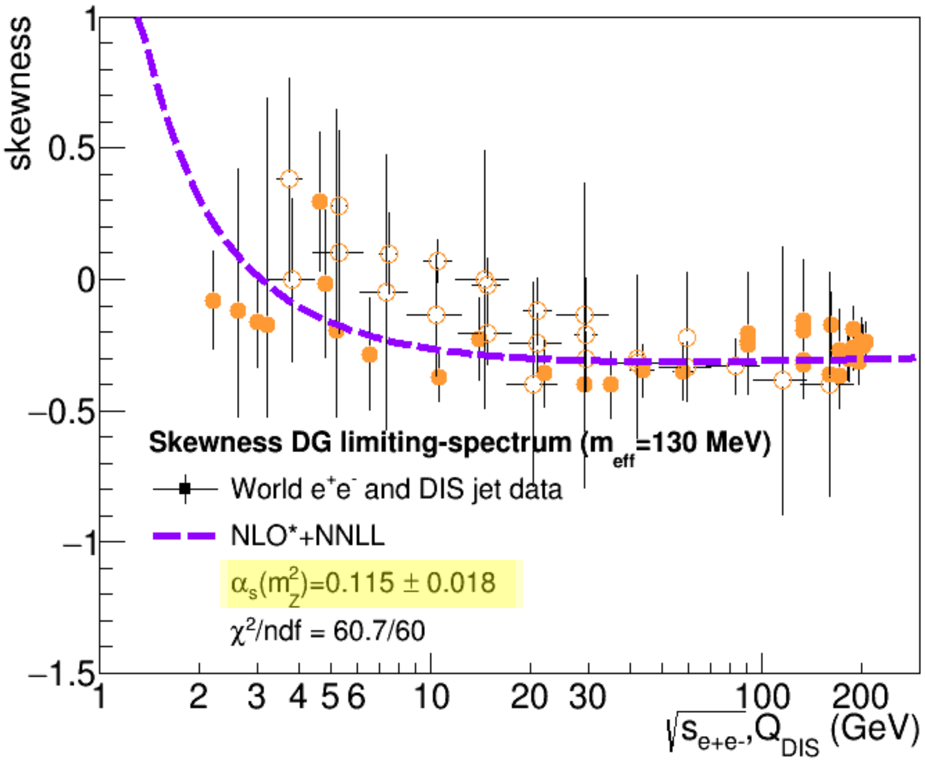
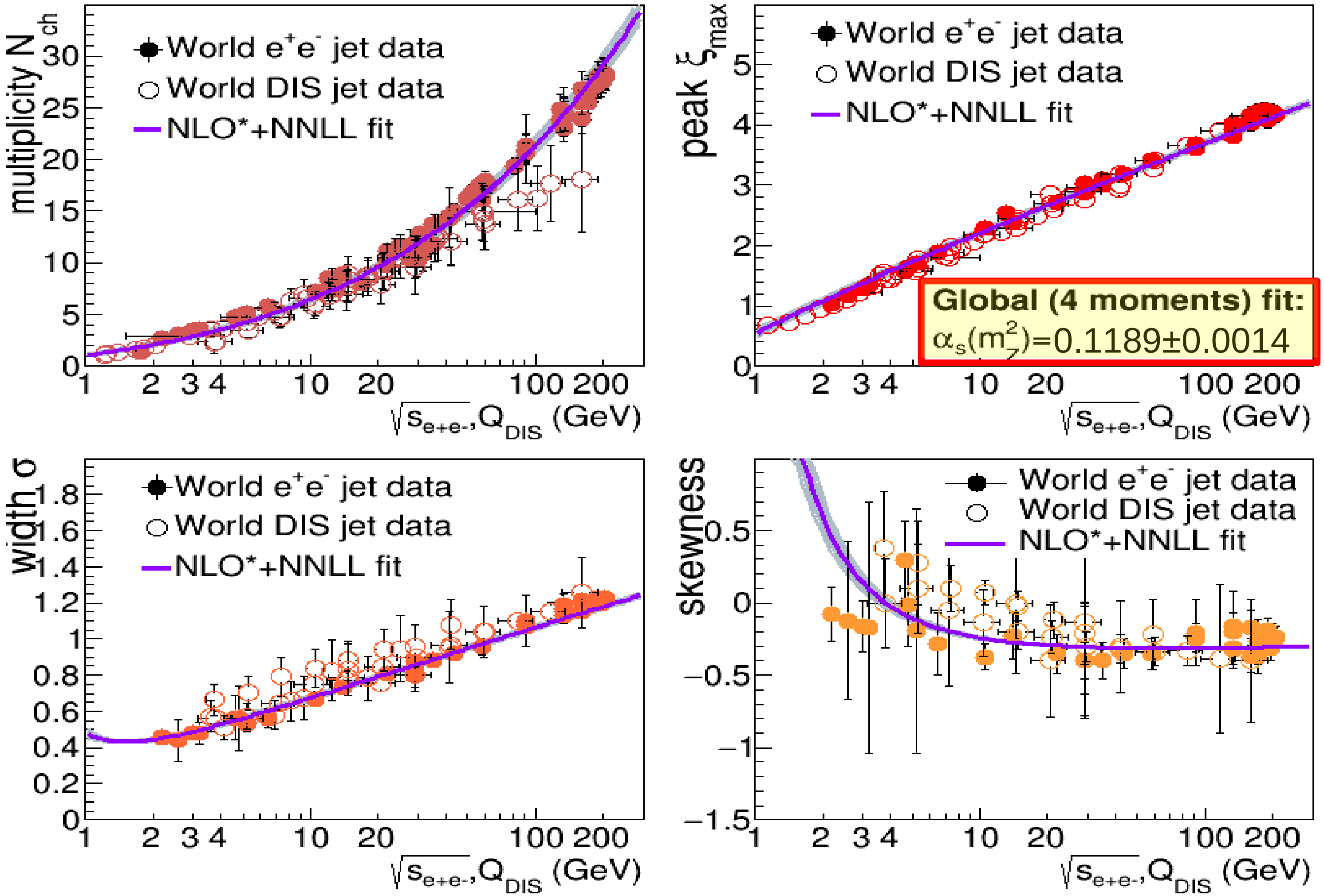




- Good data-theory agreement.
  Skewness has large experimental uncertainties
- Consistent e^+e^- & e-p moments
  (but larger DIS uncertainties).

Global fit of FF moments: Data vs. NLO*+NNLL

- Combined global fit of e^+e^-&DIS data to NLO*+NNLL:

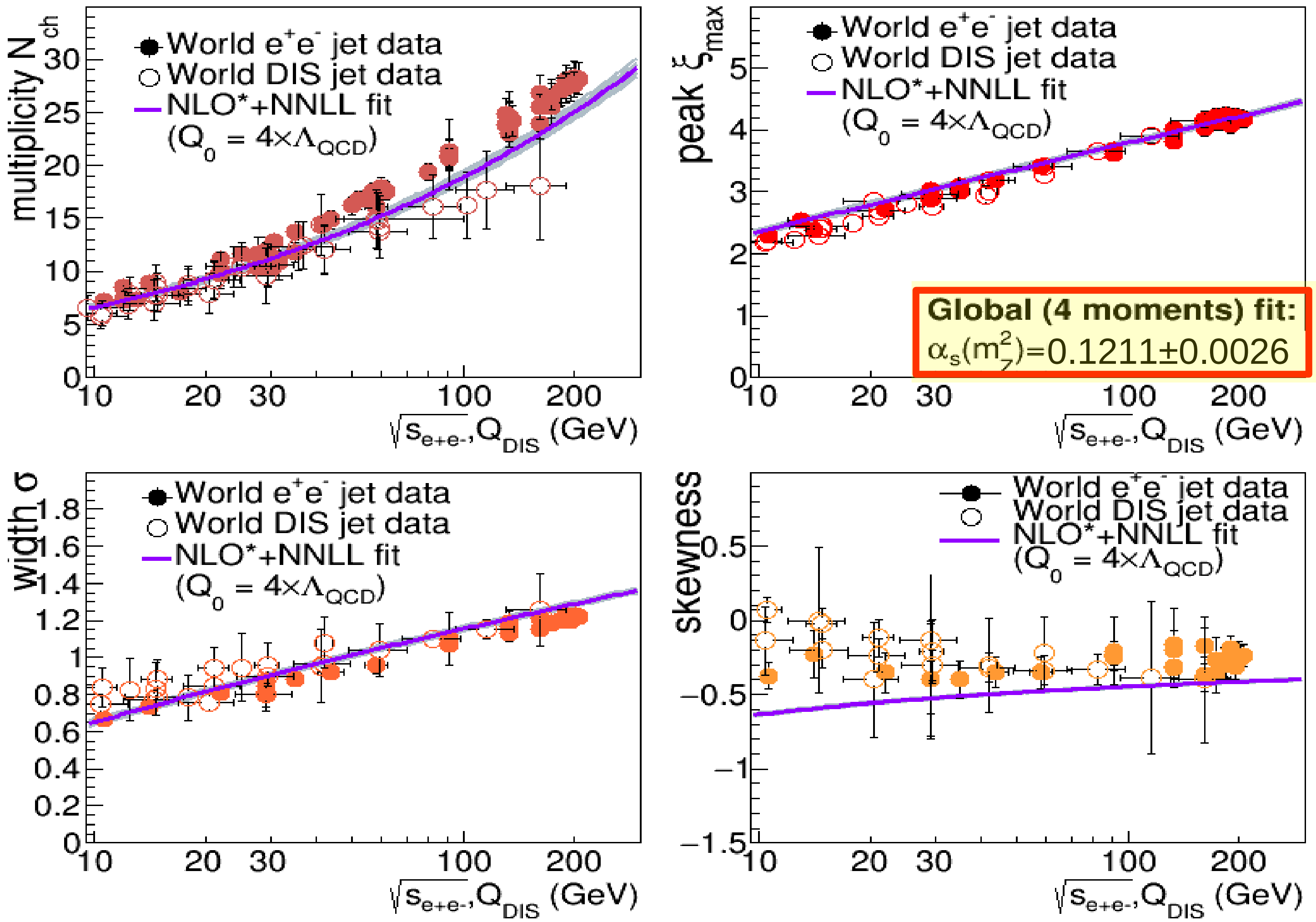


➔ χ^2 averaging: increased uncertainty for few points to reach χ^2/ndf~1.

- Final α_s uncertainty of ~1.2% includes m_{eff}, exp. fits, and correlations

α_s at NLO*+NNLL: Scale uncertainty

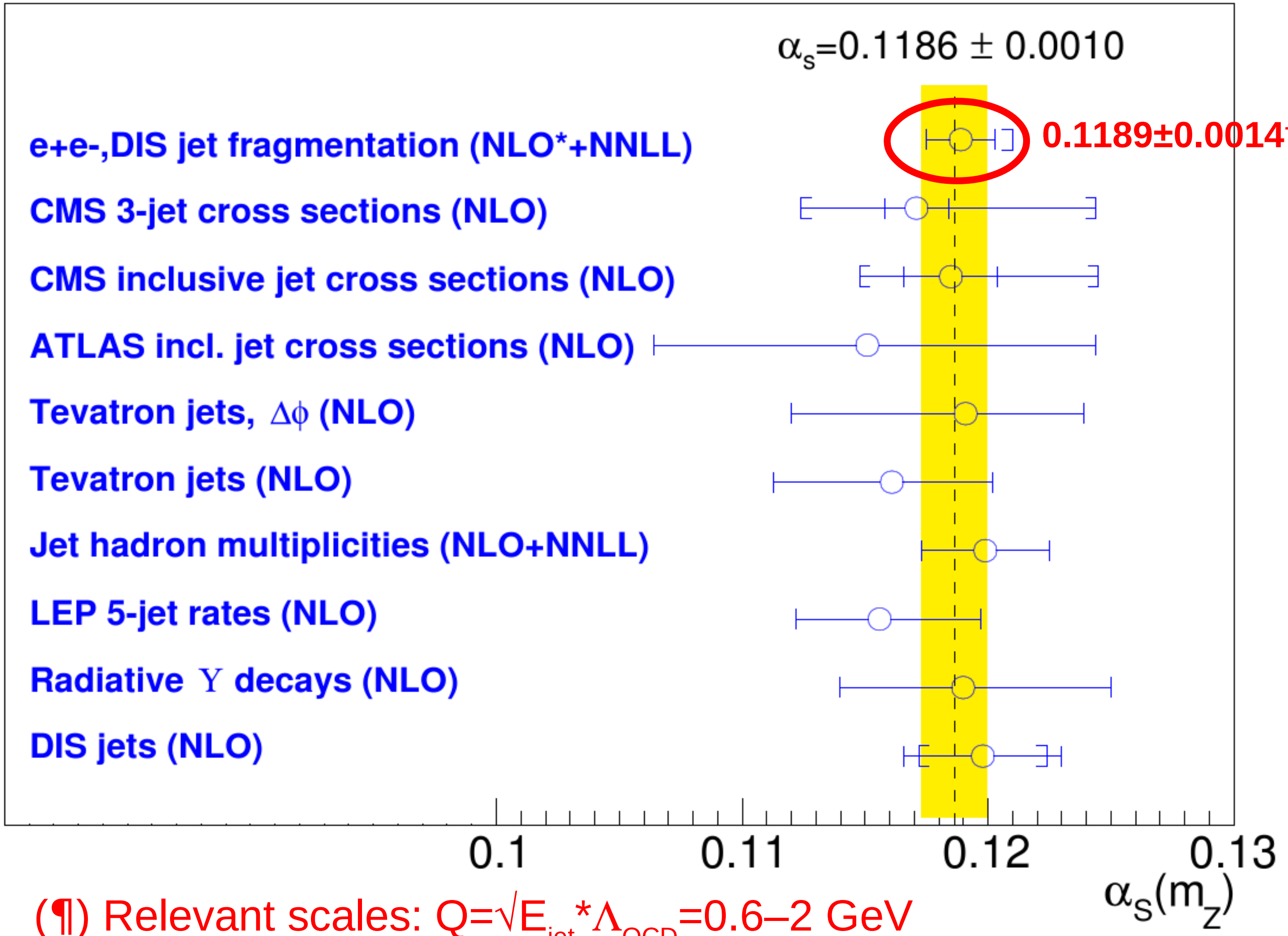
- Scale uncertainty determined by redoing global fit for $Q_0 = 4\times\Lambda_{QCD}$~1 GeV



- Extra uncertainty $\alpha_s(Q_0=\Lambda_{QCD})$-$\alpha_s(Q_0$=1GeV)=+2% due to scale variation.

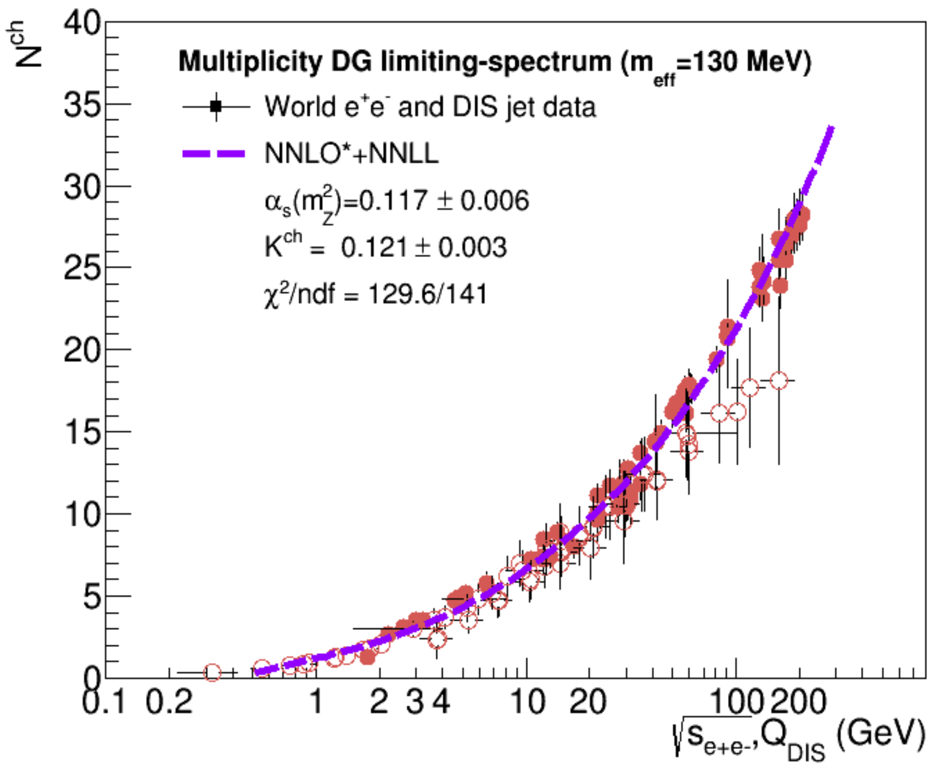
$\alpha_s(m_Z)$ at NLO*+NNLL from FFs evolution

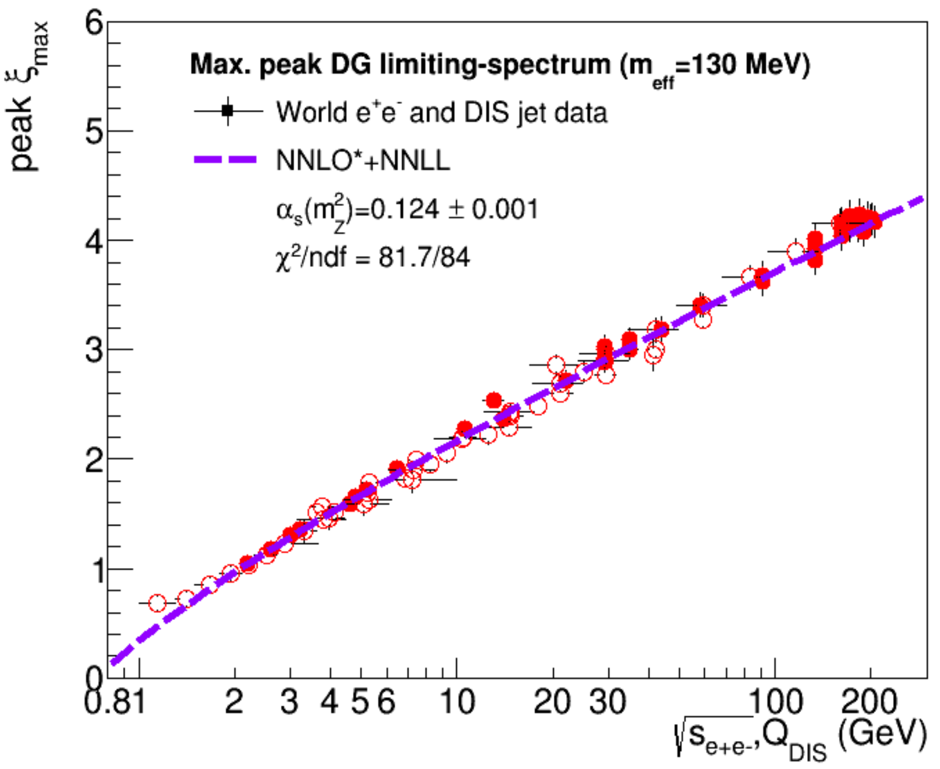
- Most precise measurement of α_s among those at NLO(*) accuracy (with a totally different set of systematic uncertainties):



(¶) Relevant scales: $Q=\sqrt{E_{jet}*\Lambda_{QCD}}$=0.6–2 GeV

Preliminary NNLO*+NNLL fits for N_{ch} & FF peak



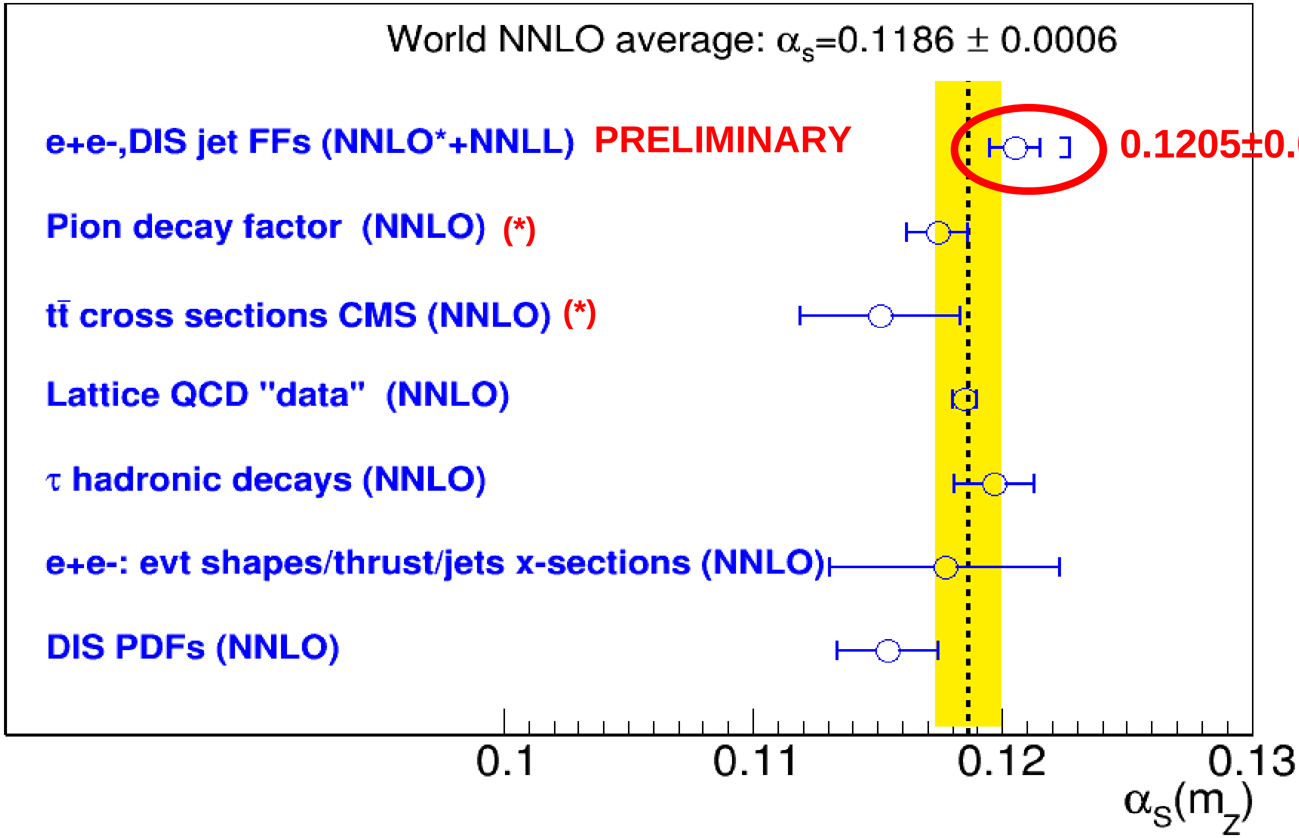


- Similar results to those from the individual NLO*+NNLL fit analysis.
  Simple average of 2 first moments would yield: $\alpha_s=0.1205\pm0.0010^{+0.0022}_{-0.0000}$
  Relevant scales: $Q=\sqrt{E_{jet}*\Lambda_{QCD}}$=0.6–2 GeV
- Work in progress to incorporate all moments & determine NNLO* scale uncertainty.

Summary: $\alpha_s(m_Z)$ at NNLO*+NNLL from jet FFs

- New approach developed to extract α_s at (N)NLO*+NNLL accuracy from jet FF with small uncertainties (in particular reduced npQCD) and with totally different systematics than rest of current methods:

(*) NEW results.
Not in PDG13
world average



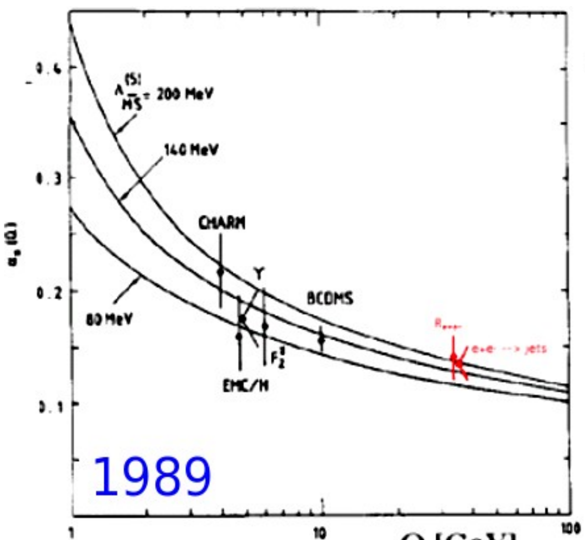
- Final goal: Full-NNLO for all moments. Incorporated into upcoming world QCD coupling average with combined <<1% uncertainty.

Backup slides

Determination of the QCD coupling α_s

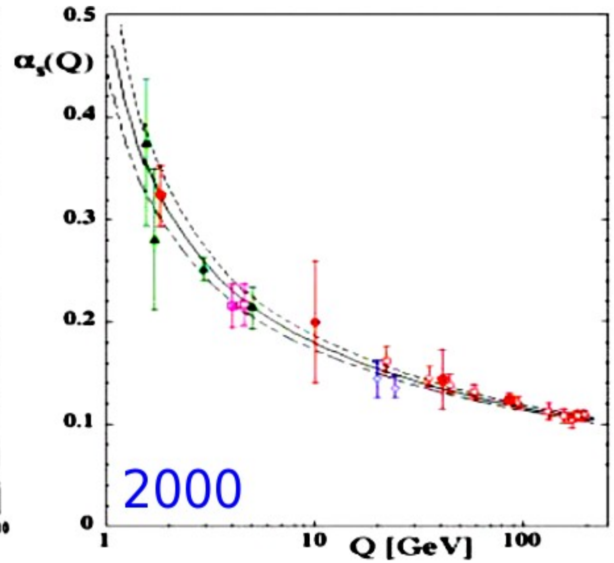
α_s = Single free parameter in QCD (in the $m_q \rightarrow 0$ limit). Determined at a given ref. scale (e.g. m_Z). Decreases as $\sim\ln(Q^2/\Lambda^2)$, with $\Lambda\sim0.25$ GeV

- Least precisely known of all couplings: $\delta\alpha \sim 3\cdot10^{-10}$, $\delta G_F \sim 5\cdot10^{-8}$, $\delta G \sim 10^{-5}$, $\delta\alpha_s \sim 5\cdot10^{-3}$
- Impacts all LHC cross-sections.
- Key for SM precision fits (e.g. uncertainties b,c Yukawa).



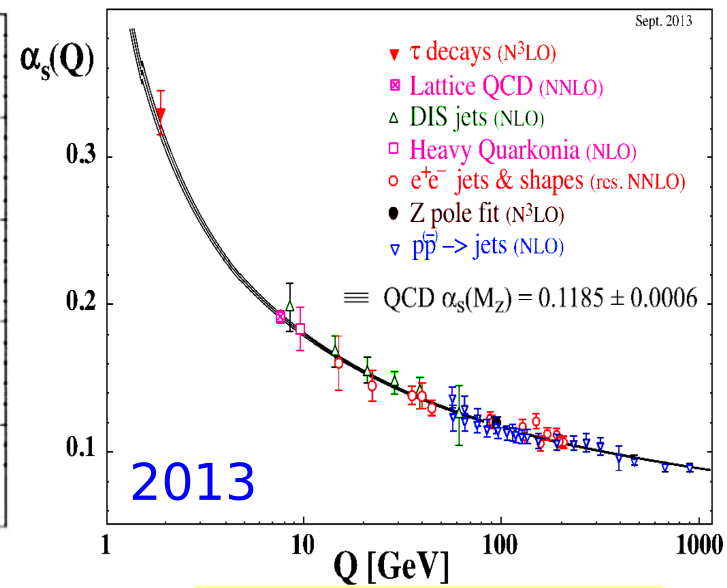
$\alpha_S(M_Z) = 0.110^{+0.006}_{-0.008}$ (NLO)

G. Altarelli, Ann. Rev. Nucl. Part. Sci. 39, 1989



$\alpha_S(M_Z) = 0.1184 \pm 0.0031$ (NNLO)

S. B. , J. Phys. G 26, 2000

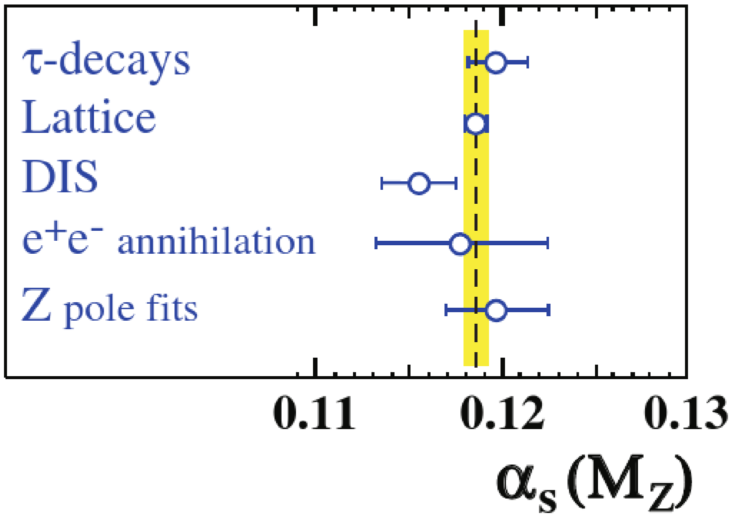


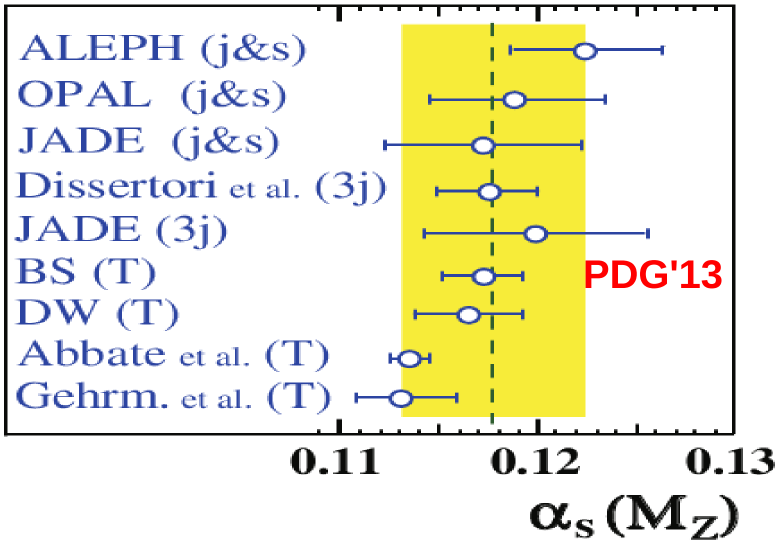
$\alpha_s(M_Z) = 0.1185 \pm 0.0006$ (NNLO)

Current uncertainty: ±0.6%
(Increased in 2015 to ±1%)

Multi-prong determination of α_s coupling

$$\alpha_s(M_Z^2) = 0.1185 \pm 0.0006$$



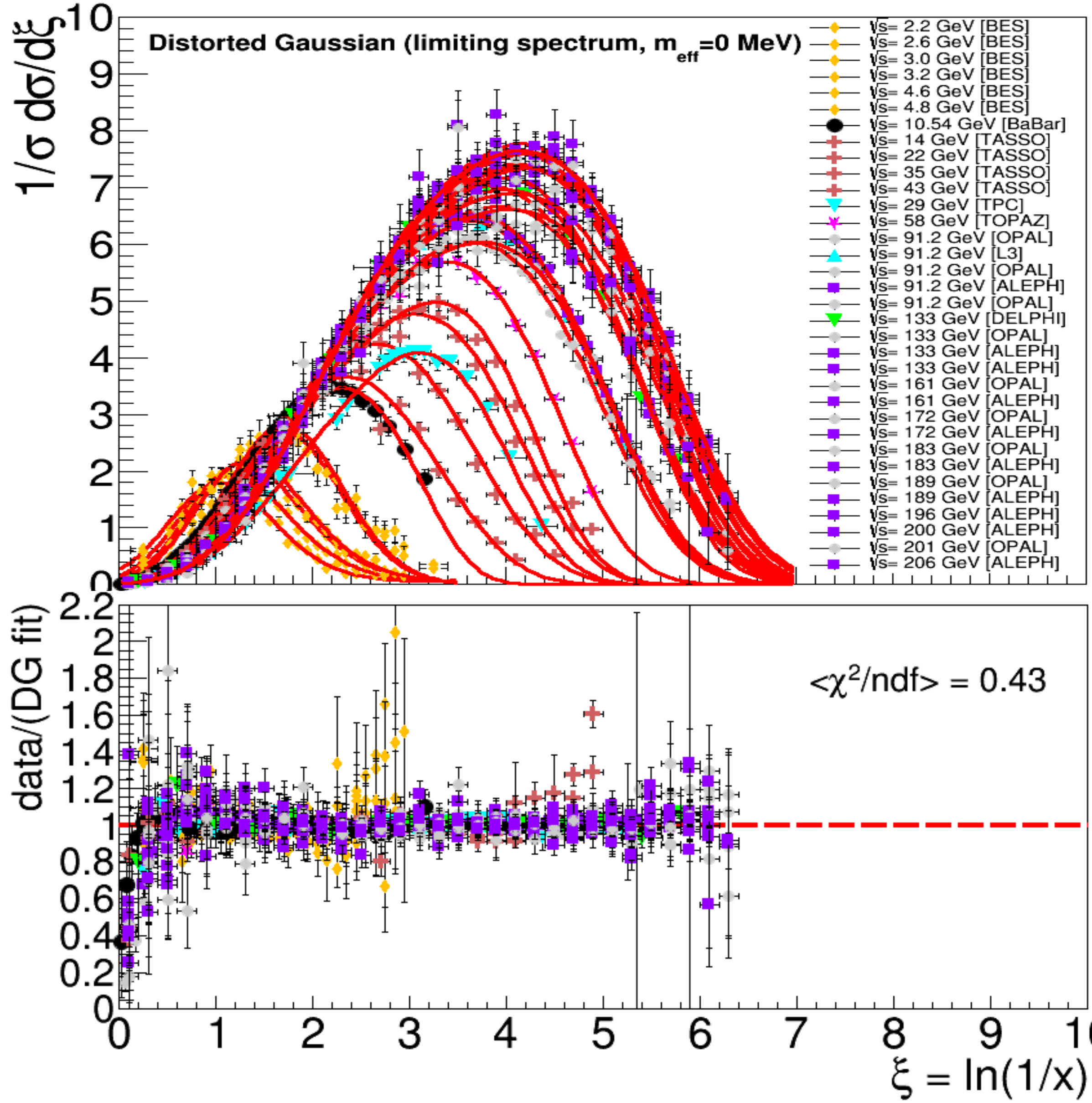


Snowmass'13, arXiv:1310.5189

Method	Current relative precision	Future relative precision
e^+e^- evt shapes	expt $\sim 1\%$ (LEP) thry ~ 1–3% (NNLO+up to N^{3}LL, n.p. signif.)	$< 1\%$ possible (ILC/TLEP) $\sim 1\%$ (control n.p. via Q^2-dep.)
e^+e^- jet rates	expt $\sim 2\%$ (LEP) thry $\sim 1\%$ (NNLO, n.p. moderate)	$< 1\%$ possible (ILC/TLEP) $\sim 0.5\%$ (NLL missing)
precision EW	expt $\sim 3\%$ (R_Z, LEP) thry $\sim 0.5\%$ (N^{3}LO, n.p. small)	0.1% (TLEP [10]), 0.5% (ILC [11]) $\sim 0.3\%$ (N^{4}LO feasible, ~ 10 yrs)
τ decays	expt $\sim 0.5\%$ (LEP, B-factories) thry $\sim 2\%$ (N^{3}LO, n.p. small)	$< 0.2\%$ possible (ILC/TLEP) $\sim 1\%$ (N^{4}LO feasible, ~ 10 yrs)

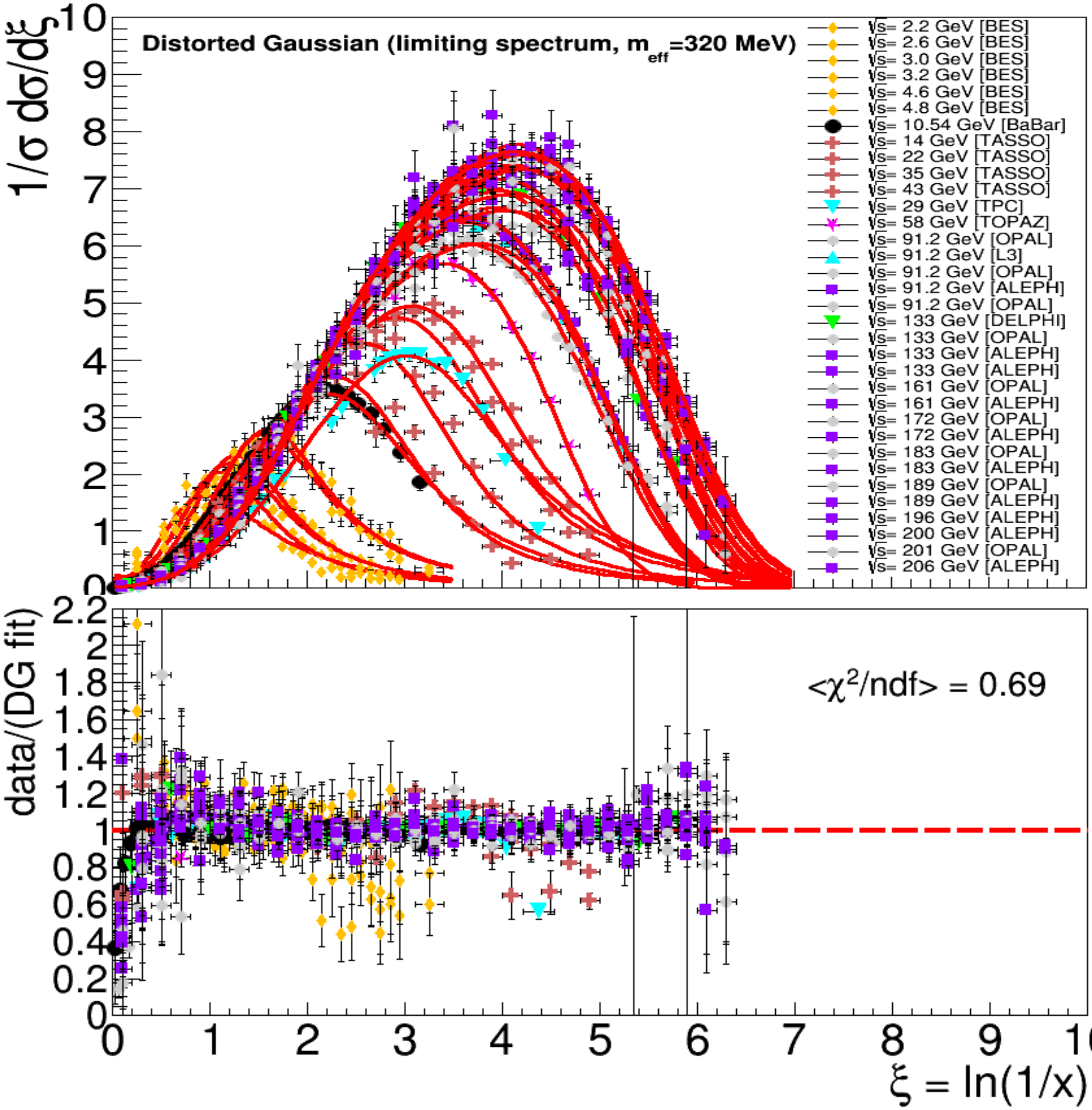
Distorted Gaussian fits to e^+e^- FFs (m_{eff} = 0 GeV)

- 34 e^+e^- data-sets at $\sqrt{s} = 2.2 - 206$ GeV ~1200 data points
- Peak shifts to right, width increases, moderate non-Gaussian tails
- Excellent fit to DG at all energies, with 5 free parameters: N_{ch}, ξ_{max}, σ, s, k



Distorted Gaussian fits to e^+e^- FFs (m_{eff}=0.36 GeV)

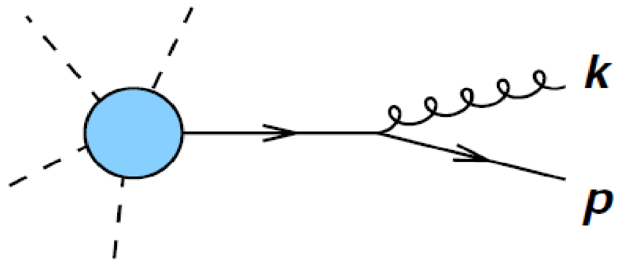
- 34 e^+e^- data-sets at $\sqrt{s}$ = 2.2 – 206 GeV ~1200 data points
- Peak shifts to right, width increases, moderate non-Gaussian tails
- Excellent fit to DG at all energies, with 5 free parameters: N_{ch}, ξ_{max}, σ, s, k



Parton shower evolution

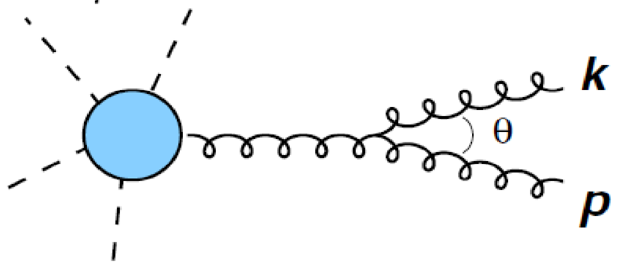
- Soft & collinear divergences are ubiquitous in all QCD processes:

Soft gluon emission from quark:



$$d\sigma \approx \frac{2\alpha_s C_F}{\pi}\frac{dE}{E}\frac{d\theta}{\theta}$$

Soft gluon emission from gluon:



$$d\sigma \approx \frac{2\alpha_s C_A}{\pi}\frac{dE}{E}\frac{d\theta}{\theta}$$

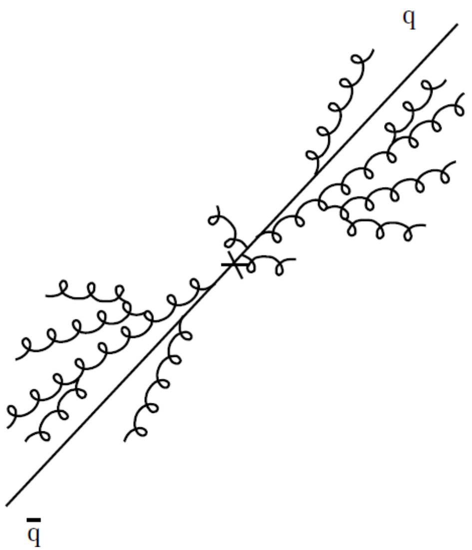
- Same divergence structures:

  For $E \to 0$: infrared (or soft) divergence

  For $\theta \to 0,\pi$: collinear divergence

  regardless of where gluon is emitted from

- Soft&collinear contributions dominate the radiation evolution of partons (jets).



Experimental e^+e^- & DIS jet FF data-sets