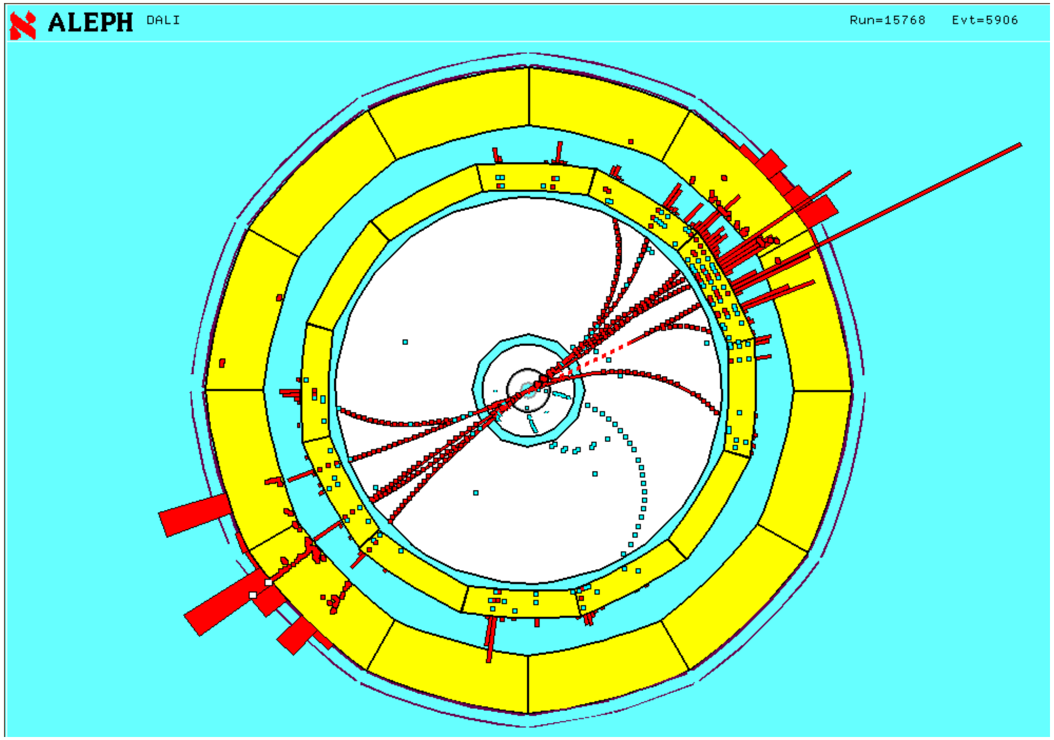
# $\alpha_s$ from $e^+e^-$ jet cross sections




***Andrea Banfi***
*– University of Sussex*

**Workshop on high-precision $\alpha_s$ measurements: from LHC to FCC-ee**

Thanks to P.F Monni and G. Luisoni for invaluable help

# Outline

- State-of-the-art theoretical predictions
- Existing measurements of $\alpha_s$
- EW corrections
- Expected theoretical developments

# Jet rates

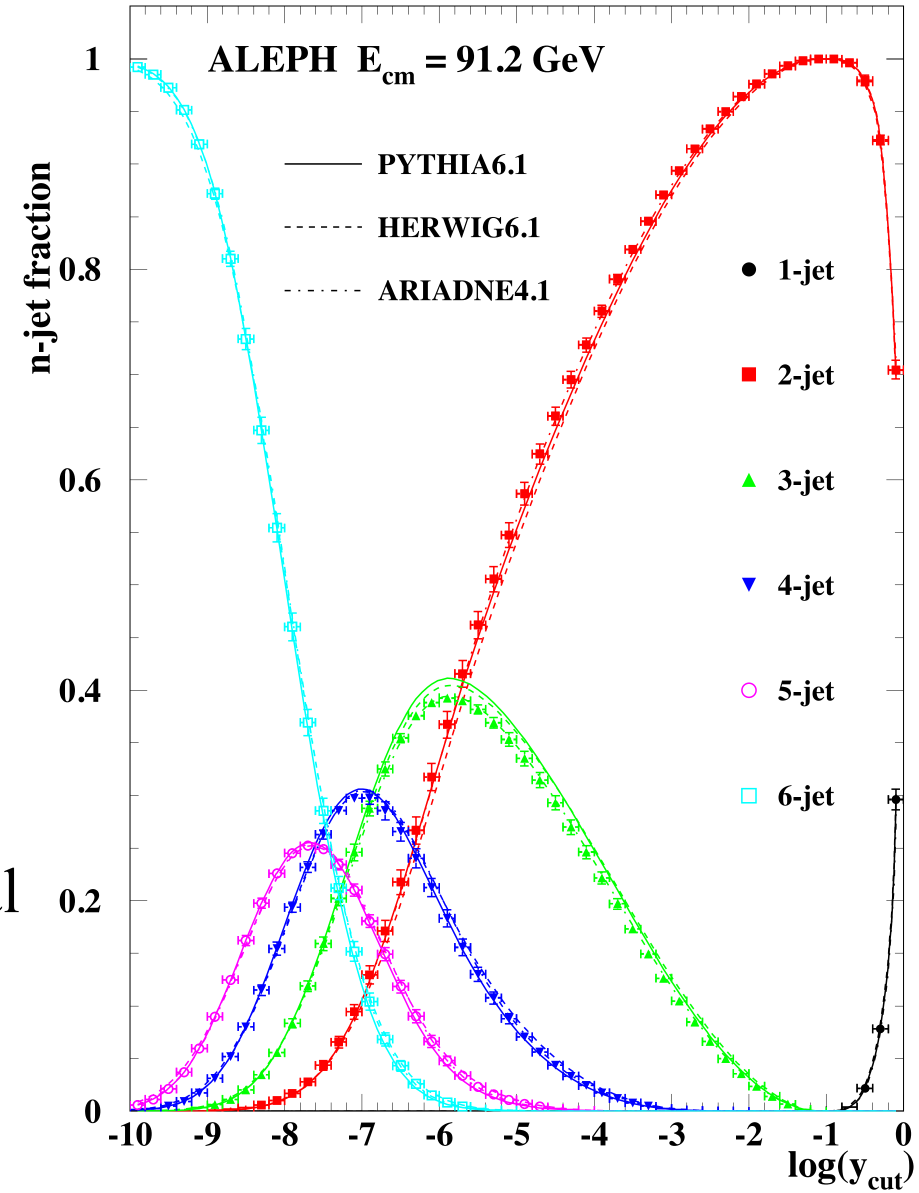
- For a given $y_{\text{cut}}$, the number of jets is the number of pseudo-particles left, when the sequential jet algorithm stops. This happens when all

$$y_{ij}^{(D)} = 2\frac{\min\{E_i^2, E_j^2\}}{Q^2}(1-\cos\theta_{ij}) > y_{\text{cut}}$$

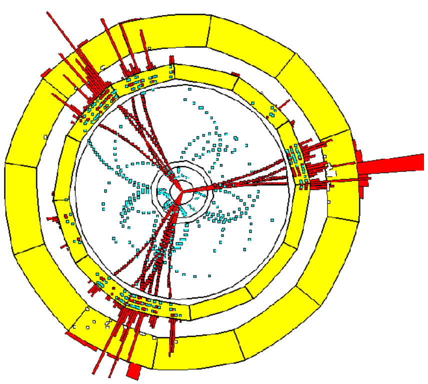
- $R_n(y_{\text{cut}})$: fraction of events having *n* jets

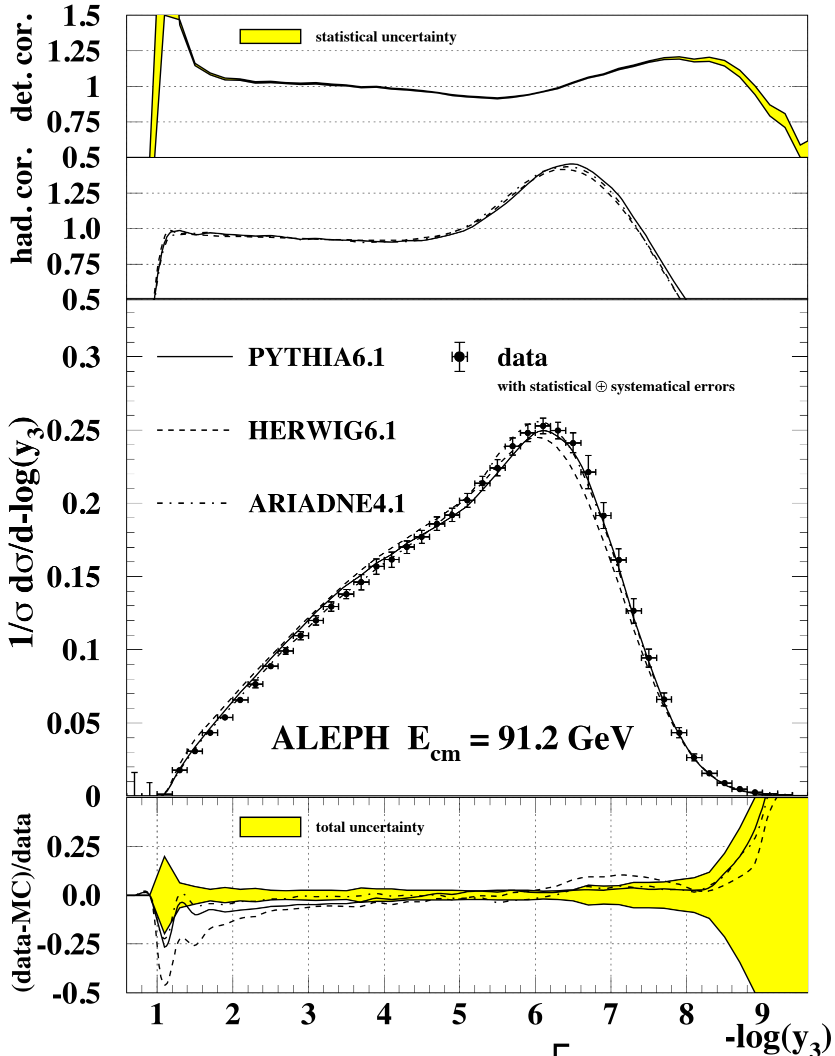
- Jet rates have been measured extensively by all experimental collaborations at LEP

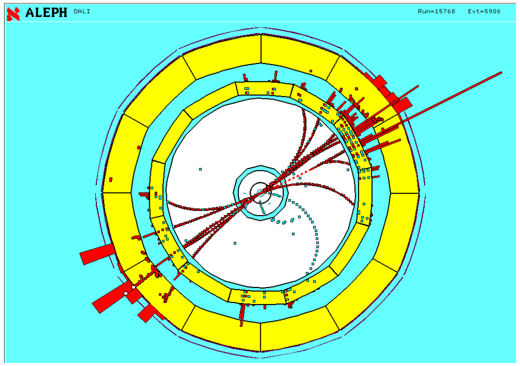
- Smaller hadronisation corrections wrt event shapes



# Main features of jet-parameter distributions



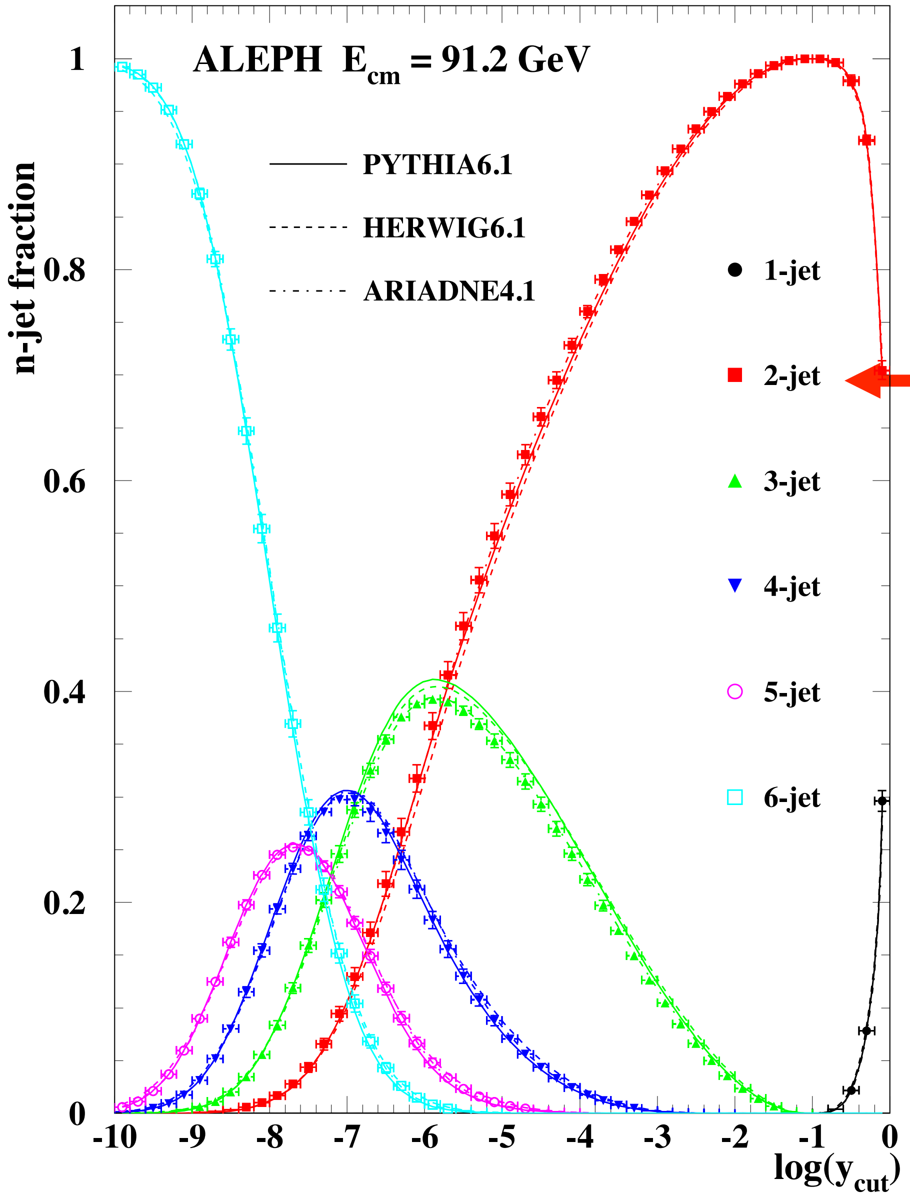



$$\sim \underbrace{\alpha_s}_{\text{LO}} + \underbrace{\alpha_s^2}_{\text{NLO}} + \underbrace{\alpha_s^3}_{\text{NNLO}} \ldots$$

$$\sim \exp\left[\underbrace{Lg_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} + \ldots\right]$$

# Fixed-order QCD predictions

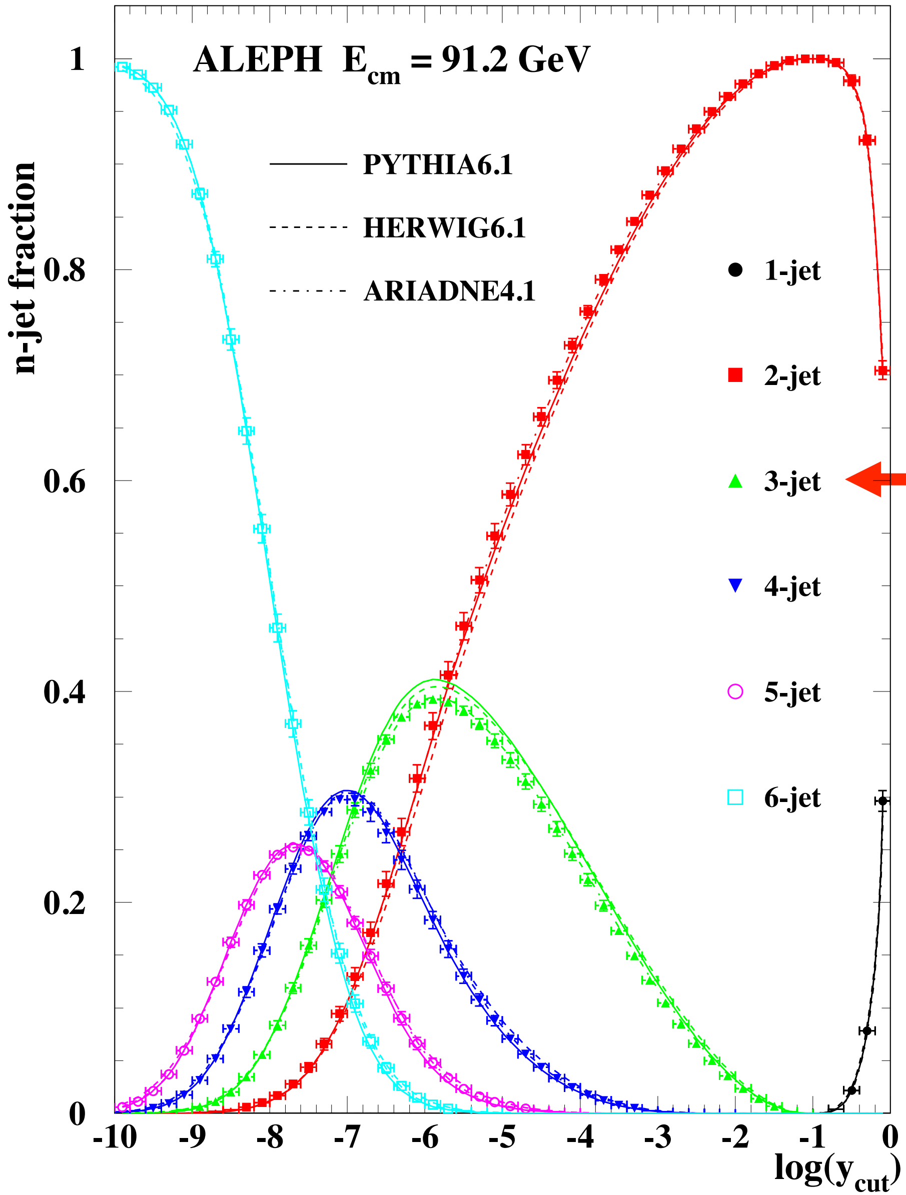



NNLO, i.e. $1 + \alpha_s + \alpha_s^2 + \alpha_s^3$

[Gehrmann Gehrmann Glover Heinrich '08]

[Weinzierl '09]

# Fixed-order QCD predictions

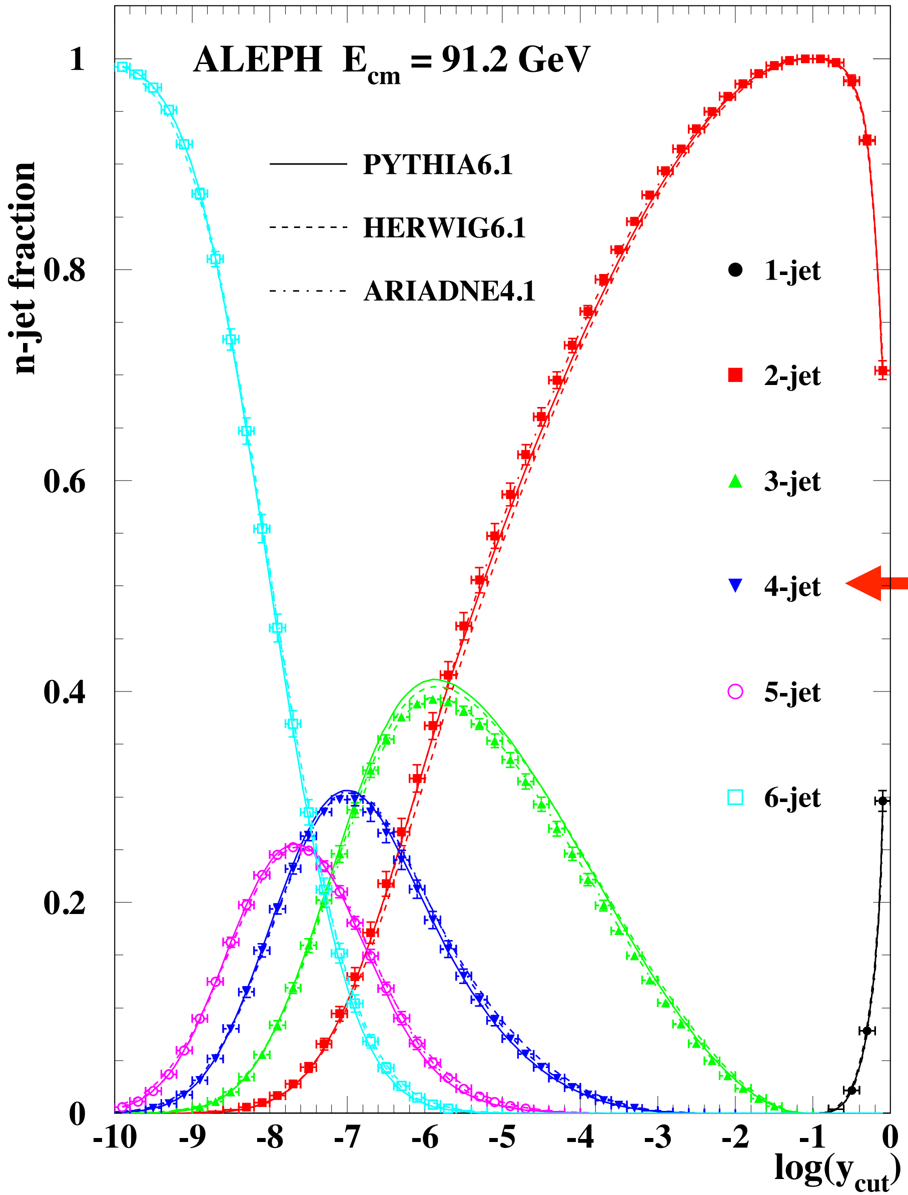



NNLO, i.e. $\alpha_s + \alpha_s^2 + \alpha_s^3$

[Gehrmann Gehrmann Glover Heinrich ‘08]

[Weinzierl ‘09]

# Fixed-order QCD predictions

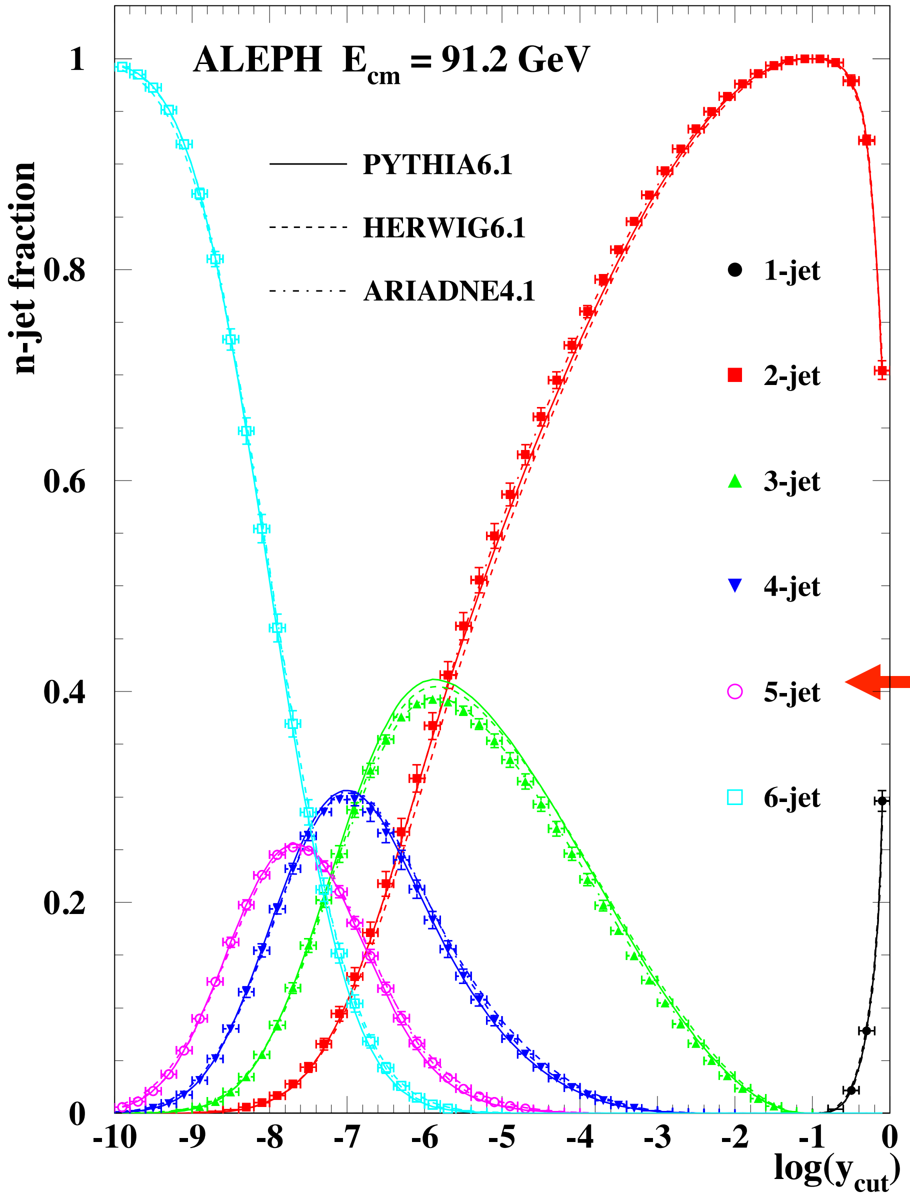



NLO, i.e. $\alpha_s^2 + \alpha_s^3$

[Nagy Trocsanyi '99, Kosower Weinzierl '99]

[Campbell Cullen Glover '99]

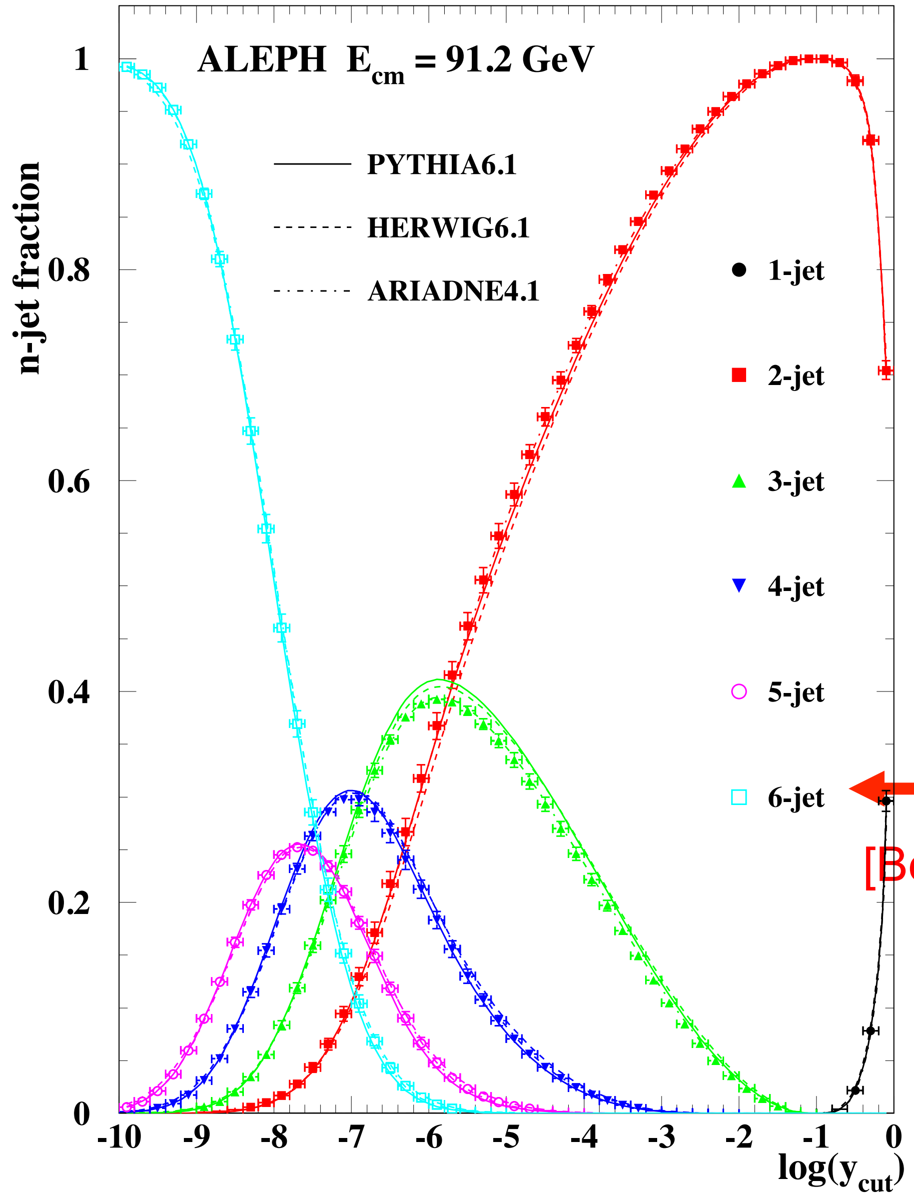
# Fixed-order QCD predictions




NLO, i.e. $\alpha_s^3 + \alpha_s^4$

[Frederix Frixione Melnikov Zanderighi '10]

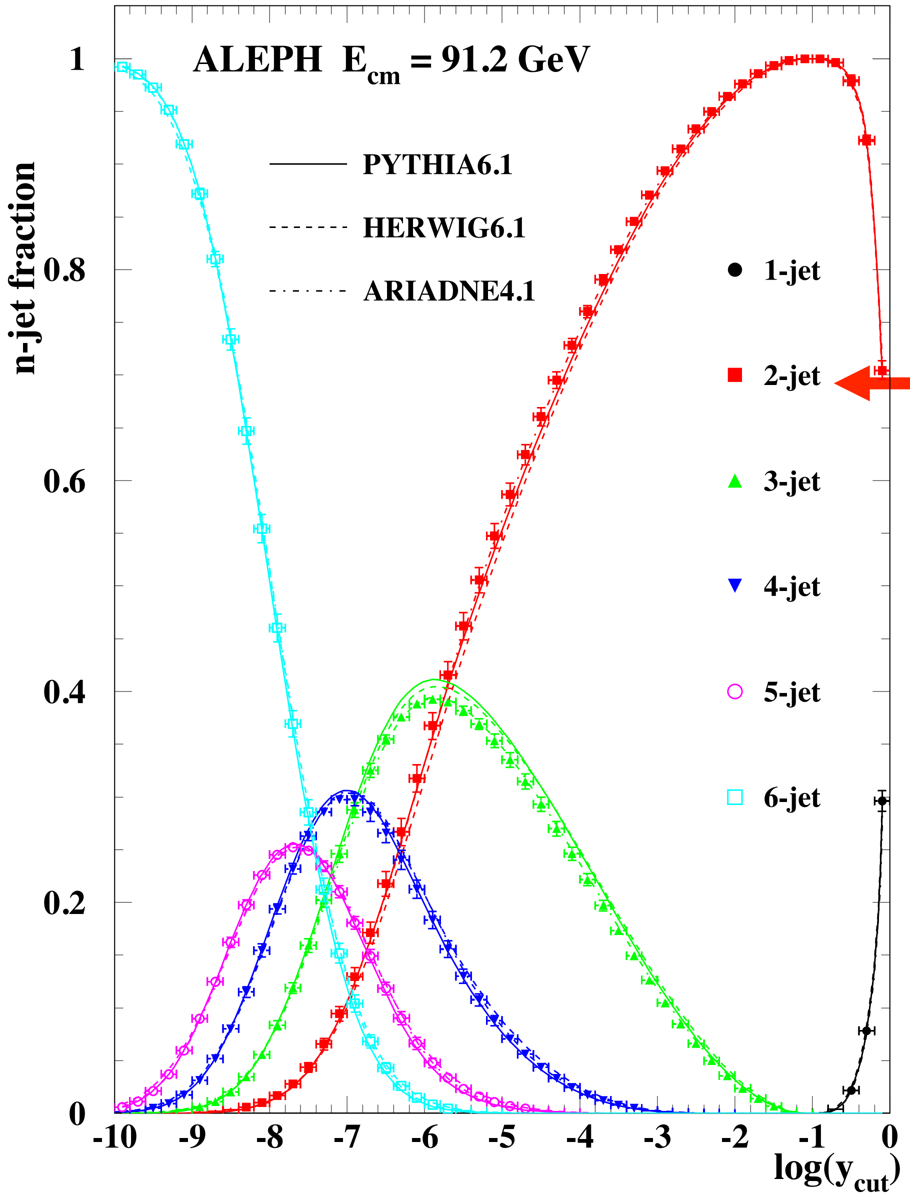
# Fixed-order QCD predictions




NLO for 6 and 7 jets (leading color only)

[Becker Goetz Reuschle Schwan Weinzierl '12]

# Resummed QCD predictions

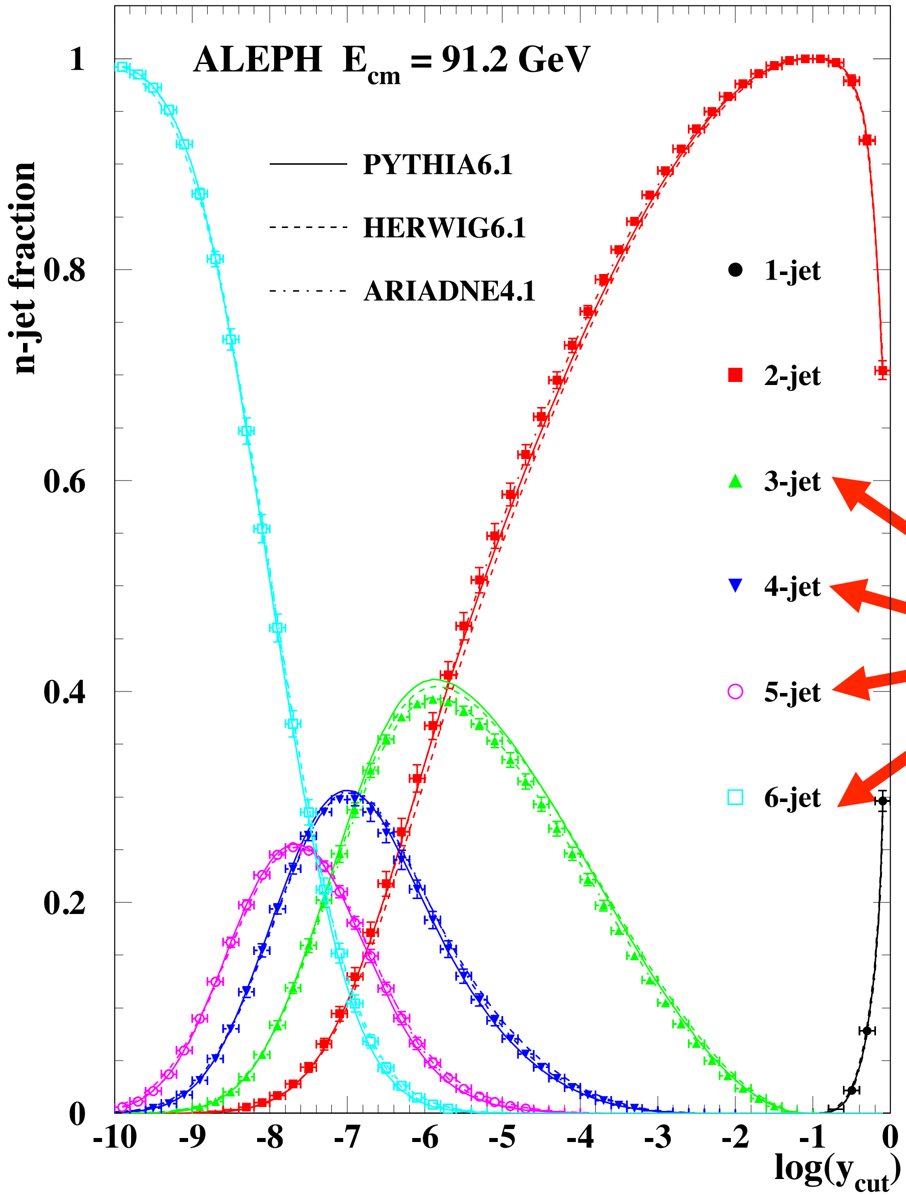



$$L = \ln\left(\frac{1}{y_{\mathrm{cut}}}\right)$$

NLL, i.e. $\exp\left[\alpha_s^n L^{n+1} + \alpha_s^n L^n\right]$

[Banfi Salam Zanderighi '01]

# Resummed QCD predictions




$$L = \ln\left(\frac{1}{y_{\rm cut}}\right)$$

NLL$_\Sigma$, i.e. $\exp\left[\alpha_s^n L^{2n} + \alpha_s^n L^{2n-1}\right]$

[Catani Dokshitzer Olsson Turnock Webber '91]

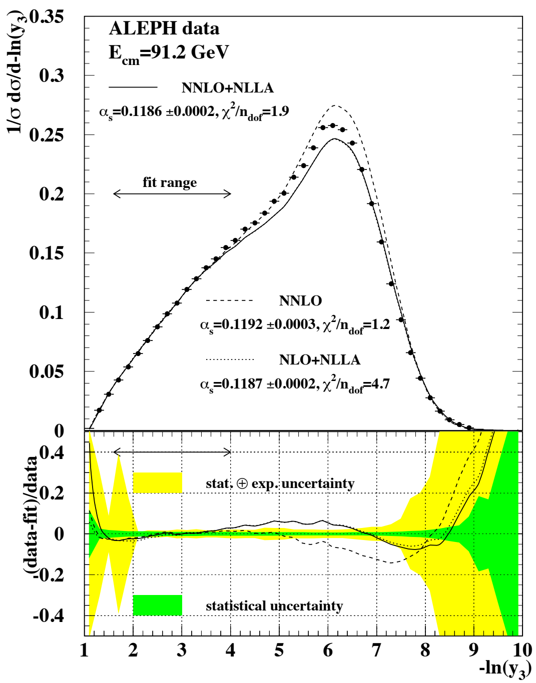
# Two-jet rate

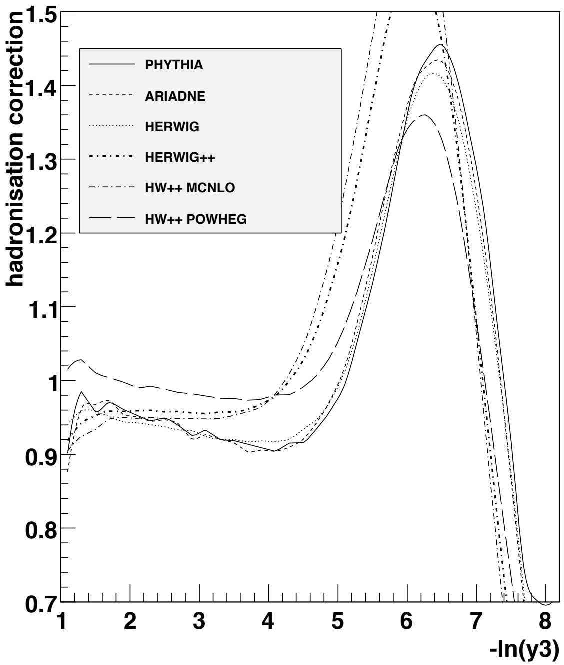
- Most accurate determinations use NNLO+NLL

[Bethke Kluth Pahl Schieck and JADE collaboration ‘08]

[OPAL collaboration ‘11]

[Dissertori Gehrmann Gehrmann Glover Heinrich Luisoni Stenzel ‘09]

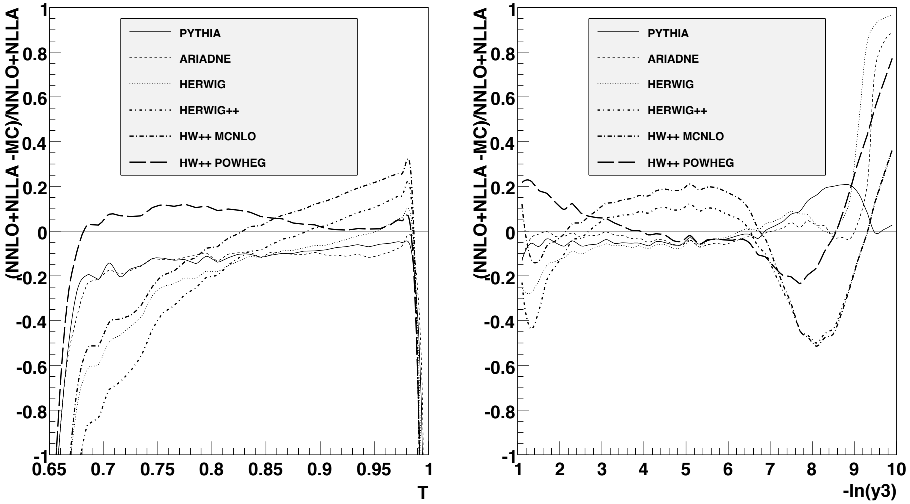



- No analytic model for hadronisation power-suppressed corrections (taken from MC) $\Rightarrow$ raise the energy!

# Two-jet rate

- Most accurate determinations use NNLO+NLL
  [Dissertori Gehrmann Gehrmann Glover Heinrich Luisoni Stenzel '09]




| $\alpha_s(M_Z)$ | $T$ | $C$ | $M_H$ | $B_W$ | $B_T$ | $-\ln y_3$ |
|---|---|---|---|---|---|---|
| PYTHIA | 0.1266 | 0.1252 | 0.1211 | 0.1196 | 0.1268 | 0.1186 |
| $\chi^2/N_{dof}$ | 0.16 | 0.47 | 4.4 | 4.4 | 0.84 | 1.89 |
| HW++ POWHEG | 0.1189 | 0.1179 | 0.1236 | 0.1169 | 0.1224 | 0.1142 |
| $\chi^2/N_{dof}$ | 1.46 | 2.55 | 3.8 | 3.9 | 1.54 | 0.56 |

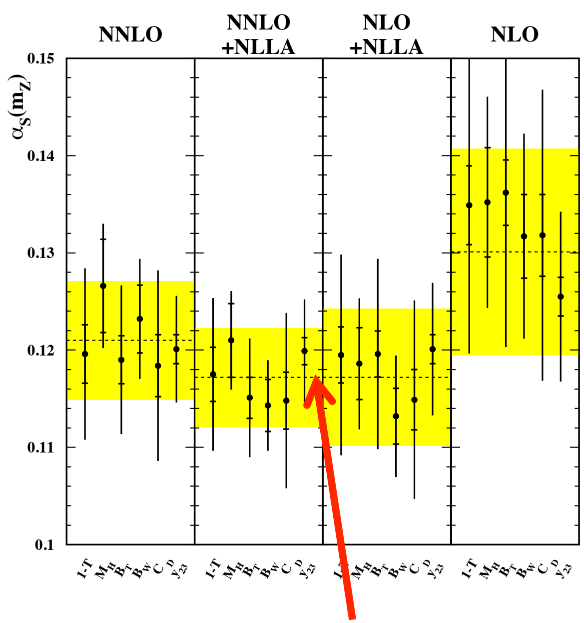
- For $y_3$, reasonable agreement between various MC and NNLO +NLL $\Rightarrow$ sensible estimate of hadronisation from MC

Thanks to G. Luisoni for pointing this out

# Two-jet rate

- Most accurate determinations use NNLO+NLL
- Perturbative uncertainties (scales, matching) are of order 3%
- Largest contribution: scale uncerts in NLL ⇒ NNLL needed!

$$14\,\mathrm{GeV} < \sqrt{s} < 44\,\mathrm{GeV}$$



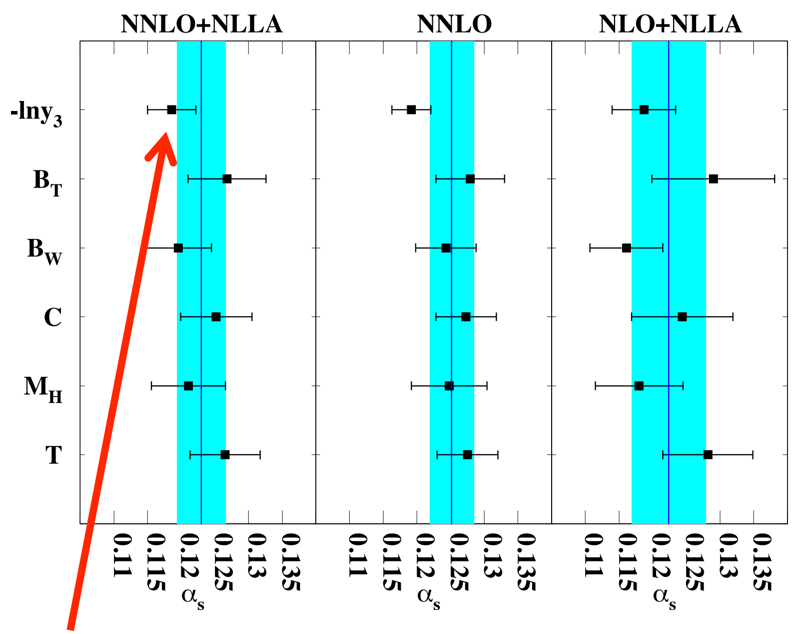

$$\alpha_s(M_Z) = 0.1199 \pm 0.0005(\mathrm{stat.}) \pm 0.0013(\mathrm{exp.}) \pm 0.0046(\mathrm{had.}) \pm 0.0023(\mathrm{theo.})$$

[Bethke Kluth Pahl Schieck and JADE collaboration ‘08]

# Two-jet rate

- Most accurate determinations use NNLO+NLL
- Perturbative uncertainties (scales, matching) are of order 3%
- Largest contribution: scale uncerts in NLL ⇒ NNLL needed!

$\sqrt{s} = M_Z$ ALEPH



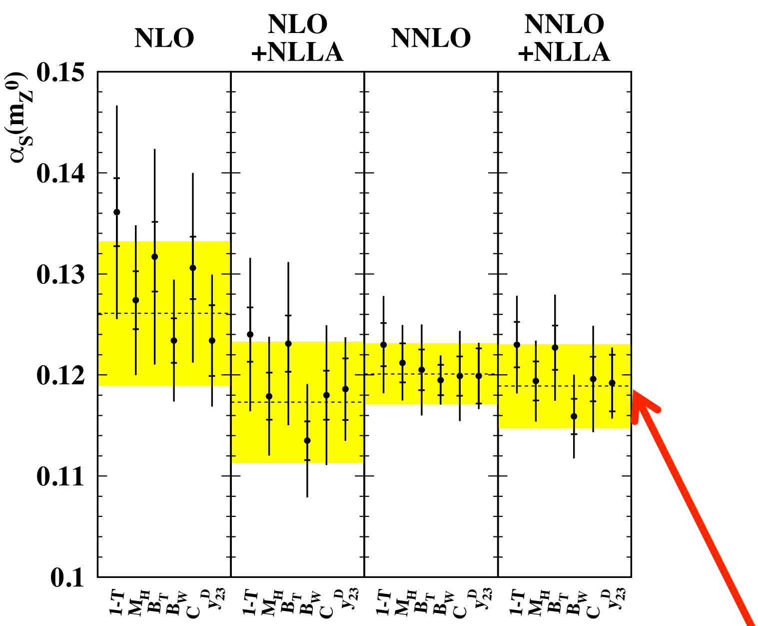

$$\alpha_s(M_Z) = 0.1186 \pm 0.0002(\text{stat.}) \pm 0.0011(\text{exp.}) \pm 0.0017(\text{had.}) \pm 0.0029(\text{theo.})$$

[Dissertori Gehrmann Gehrmann Glover Heinrich Luisoni Stenzel '09]

# Two-jet rate

- Most accurate determinations use NNLO+NLL
- Perturbative uncertainties (scales, matching) are of order 3%
- Largest contribution: scale uncerts in NLL ⇒ NNLL needed!

$91\,\mathrm{GeV} < \sqrt{s} < 209\,\mathrm{GeV}$




$$\alpha_s(M_Z) = 0.1192 \pm 0.0010(\mathrm{stat.}) \pm 0.0026(\mathrm{exp.}) \pm 0.0004(\mathrm{had.}) \pm 0.0021(\mathrm{theo.})$$

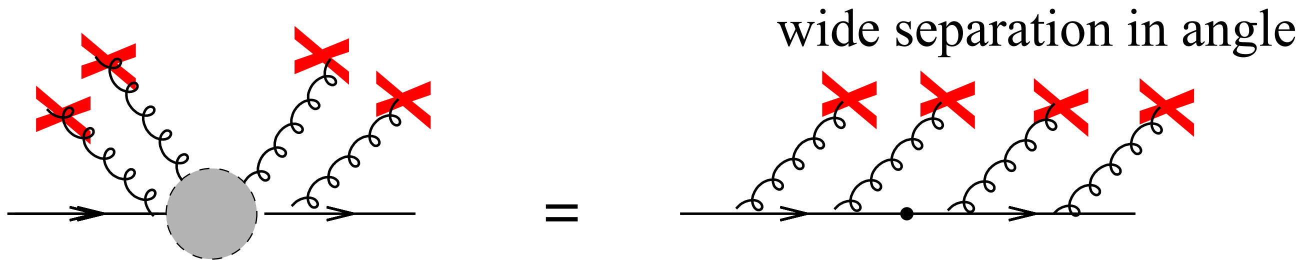
[OPAL collaboration ‘11]

# Two-jet rate at NLL howto

- NLL resummation of the two-jet rate with a semi-numerical approach

[Banfi Salam Zanderighi '01 and '04]

$$R_2^{\mathrm{NLL}}(y_{\mathrm{cut}}) = e^{-R_{\mathrm{NLL}}(y_{\mathrm{cut}})}\mathcal{F}(R'_{\mathrm{NLL}})$$




$$\mathcal{F}(\mathcal{R}') = \int d\mathcal{Z}[R', \{k_i\}]\Theta\left(1 - \lim_{y_{\mathrm{cut}}\to 0}\frac{y_3(\{\tilde{p}\}, \{k_i\})}{y_{\mathrm{cut}}}\right)$$

- The function $\mathcal{F}(R')$ contains the details of the clustering
- No clustering for Cambridge or AO Durham at NLL $\Rightarrow \mathcal{F}(R') = 1$

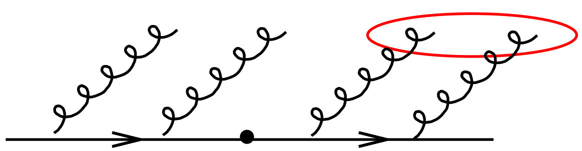
# Two-jet rate NNLL howto

- NNLL resummation requires computing a number of corrections from various phase space regions

[Banfi Monni McAslan Zanderighi ’15]

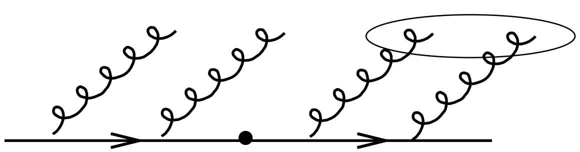
$$R_2^{\mathrm{NNLL}}(y_{\mathrm{cut}}) = e^{-R_{\mathrm{NNLL}}(y_{\mathrm{cut}})}\left[\mathcal{F}(R'_{\mathrm{NLL}}) + \frac{\alpha_s}{\pi}\left(\delta\mathcal{F}_{\mathrm{evshp}}(R'_{\mathrm{NLL}}) + \delta\mathcal{F}_{\mathrm{clust}}(R'_{\mathrm{NLL}})\right)\right]$$

$\delta\mathcal{F}_{\mathrm{clust}} =$



exact clustering

$-$



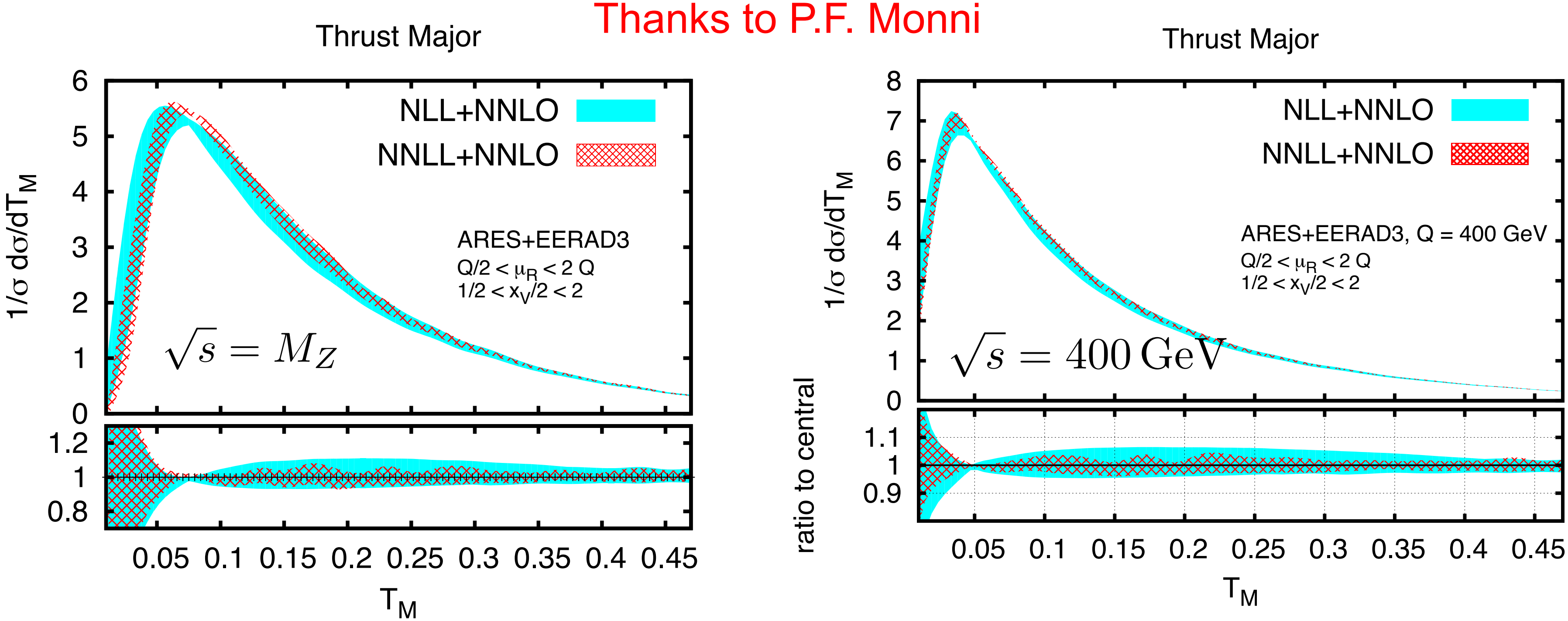
NLL clustering

- It is not clear if $\delta\mathcal{F}_{\mathrm{clust}}$ can be separated from $\delta\mathcal{F}_{\mathrm{evshp}}$
- Cambridge and AO Durham easier than Durham due to $\mathcal{F}(R') = 1$

# Theoretical uncertainty: NLL vs NNLL

- NNLL resummation for the 2-jet in progress; we use the thrust-major for projections on uncertainties when increasing $\sqrt{s}$

Thanks to P.F. Monni

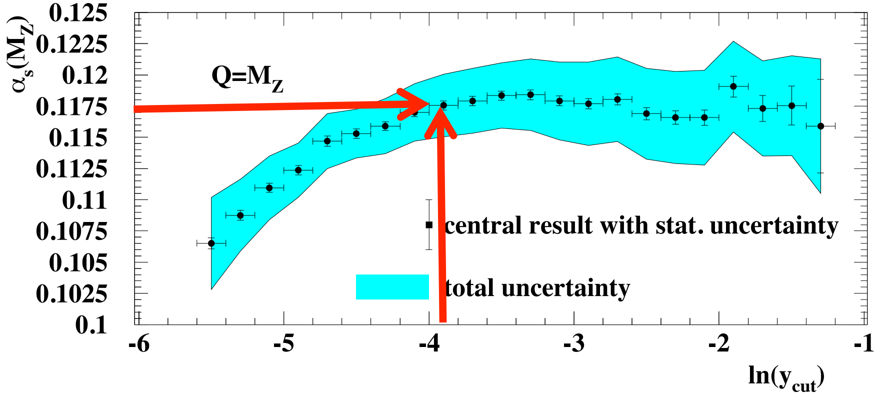


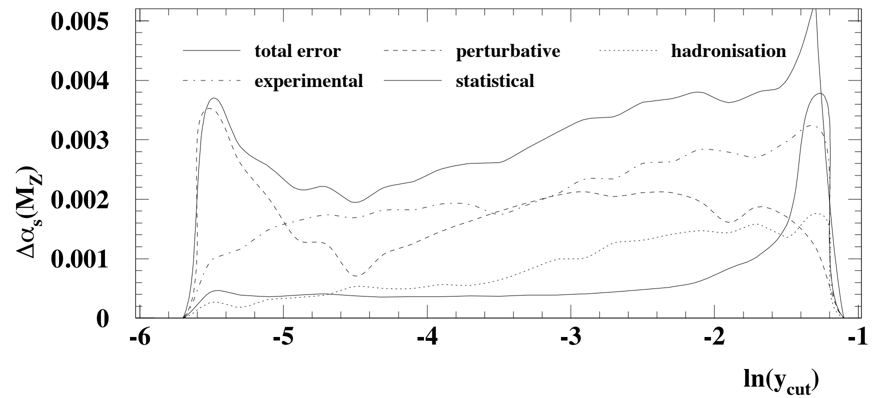

- Uncertainty of NLL and NNLL halved from 91GeV to 400GeV
- Resummation uncertainty also halved from NLL to NNLL

# Three-jet rate (most precise determination)

- NNLO with ALEPH data at LEP1, for $\ln y_{\rm cut} = -3.9$

[Dissertori Gehrmann Gehrmann Glover Heinrich Stenzel '09]




$$\alpha_s(M_Z) = 0.1175 \pm 0.0020\,(\mathrm{exp}) \pm 0.0015\,(\mathrm{theo})$$

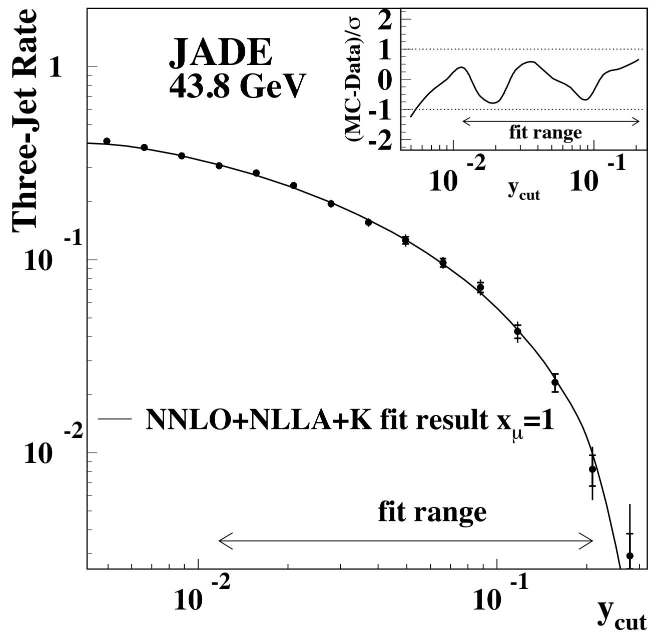
- Hadronisation corrections are very small in the fit range
- The theoretical error is 1.3%, and less than the exp error
- No NLL resummation, only $\mathrm{NLL}_\Sigma$ ,not used in the fit, relevant for $\ln y_{\rm cut} \lesssim -4.5$

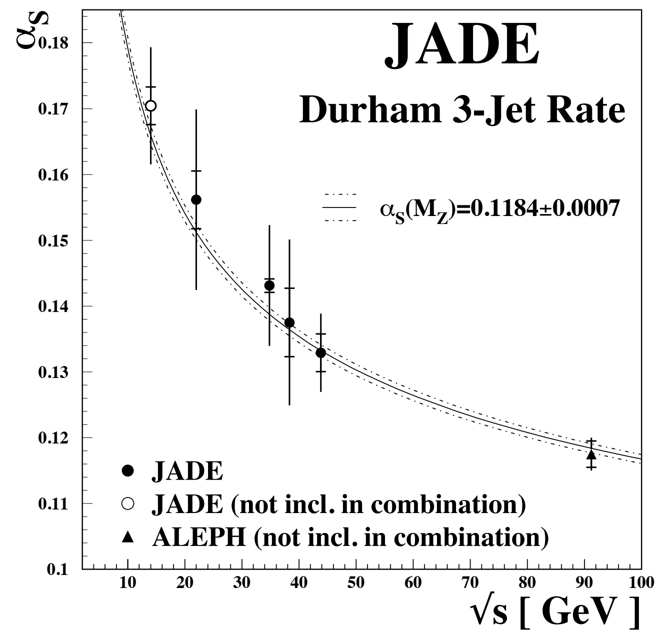
# Three-jet rate

- NNLO + resummation at (improved) $NLL_{\Sigma}$

[Schieck Bethke Kluth Pahl Trocsanyi and JADE collaboration '12]

$$22\,\mathrm{GeV} < \sqrt{s} < 43.8\,\mathrm{GeV}$$




$$\alpha_S(M_{Z^0}) = 0.1199 \pm 0.0010(\mathrm{stat.}) \pm 0.0021(\mathrm{exp.}) \pm 0.0054(\mathrm{had.}) \pm 0.0007(\mathrm{theo.})$$

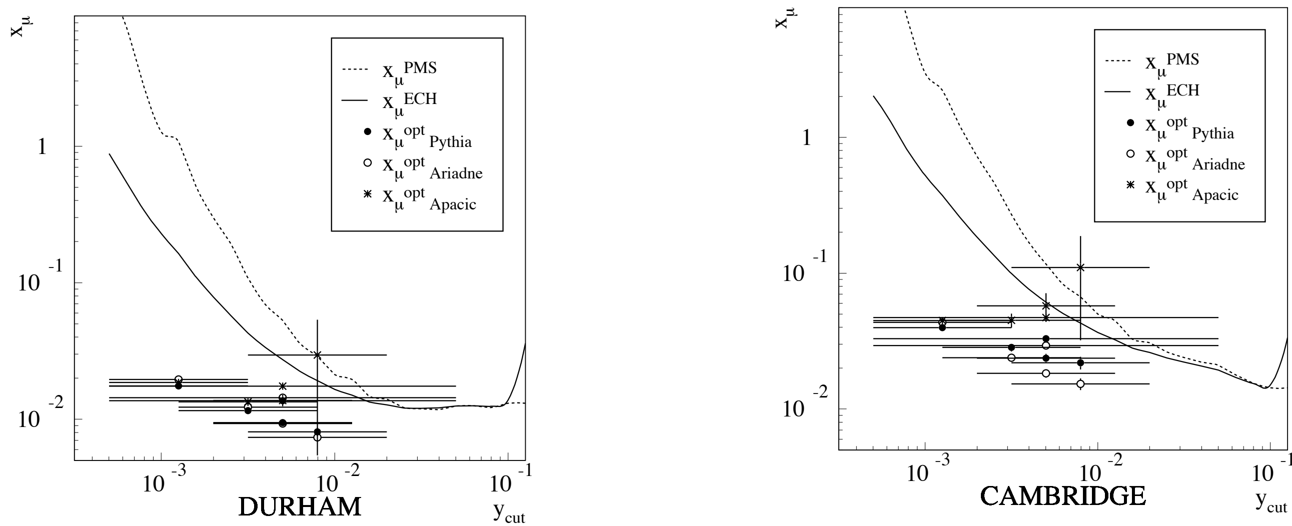
- Resummation plays an important role in correctly describing the shape of the three-jet rate

# Four-jet rate

- NLO (DEBRECEN=NLOJET++)

[DELPHI collaboration '04]

$$\sqrt{s} = M_Z \text{ and } y_{\rm cut} = 0.0063$$




| algorithm | fit range | $x_\mu^{\rm opt}$ |
|---|---|---|
| DURHAM | 0.001 – 0.01 | 0.015 |
| CAMBRIDGE | 0.001 – 0.01 | 0.042 |

| observable | $\alpha_s(M_Z^2)$ | $\pm$ | exp. | $\pm$ | hadr. | $\pm$ | scale |
|---|---|---|---|---|---|---|---|
| DURHAM | 0.1178 | $\pm$ | 0.0012 | $\pm$ | 0.0031 | $\pm$ | 0.0014 |
| CAMBRIDGE | 0.1175 | $\pm$ | 0.0010 | $\pm$ | 0.0027 | $\pm$ | 0.0007 |

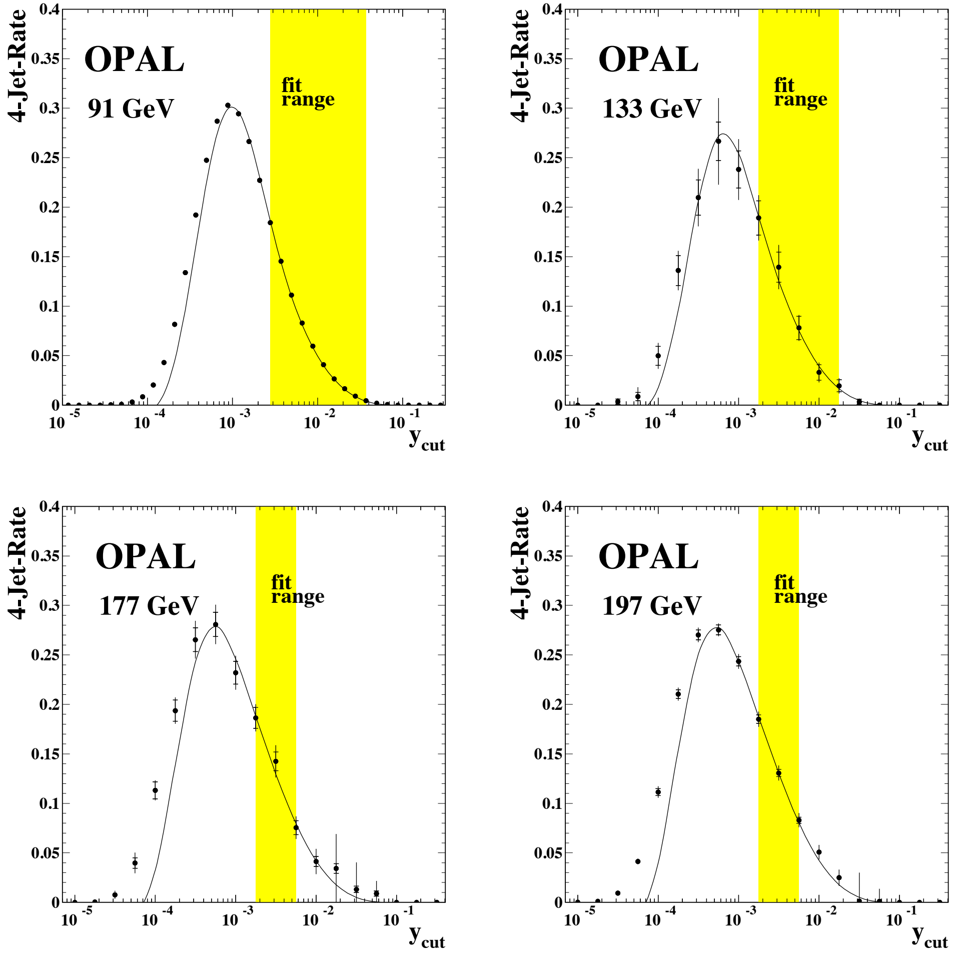
$\mu_R = 1.3\,\text{GeV}$

- Two-variable fit of $\alpha_s(M_Z)$ and $x_\mu = \mu_R/\sqrt{s}$

# Four-jet rate

- NLO (DEBRECEN=NLOJET++) + $\mathrm{NLL}_{\Sigma}$

[OPAL collaboration '06]

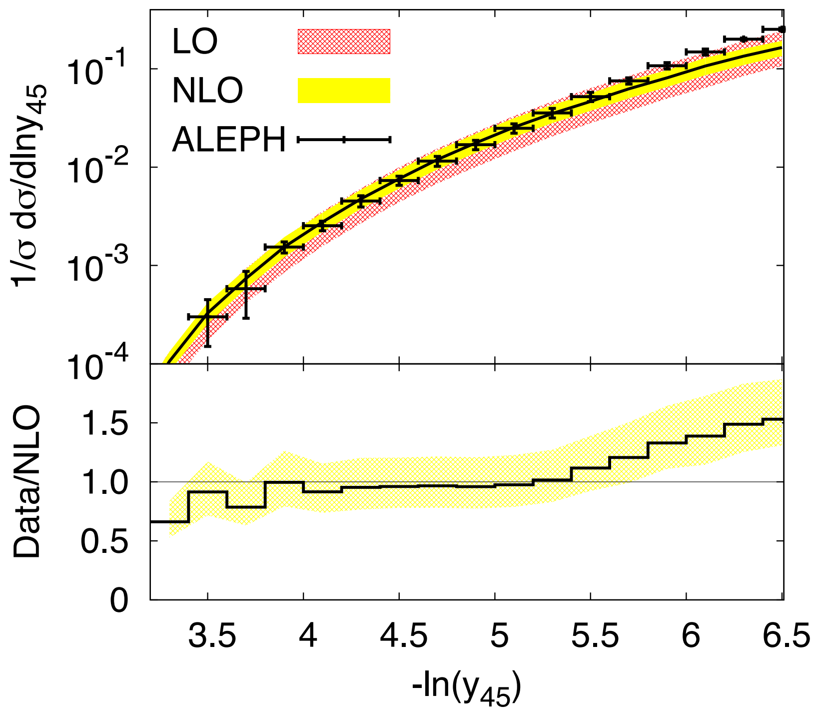



b-mass

$$\alpha_S(M_{Z^0}) = 0.1182 \pm 0.0003(\text{stat.}) \pm 0.0015(\text{exp.}) \pm 0.0011(\text{had.}) \pm 0.0012(\text{scale}) \pm 0.0013(\text{mass})$$

# Five-jet rate

- Determination uses NLO only, and LEP1 and LEP2 ALEPH data [Frederix Frixione Melnikov Zanderighi '10]




| | LEP1, hadr. $\sigma_{\rm tot}^{-1} {\rm d}\sigma/{\rm d}y_{45}$, $R_5$ | LEP1, no hadr. $\sigma_{\rm tot}^{-1} {\rm d}\sigma/{\rm d}y_{45}$, $R_5$ |
|---|---|---|
| stat. | $+0.0002$ $-0.0002$ | $+0.0002$ $-0.0002$ |
| syst. | $+0.0027$ $-0.0029$ | $+0.0027$ $-0.0029$ |
| pert. | $+0.0062$ $-0.0043$ | $+0.0068$ $-0.0047$ |
| fit range | $+0.0014$ $-0.0014$ | $+0.0005$ $-0.0005$ |
| hadr. | $+0.0012$ $-0.0012$ | – |
| $\alpha_s(M_Z)$ | $0.1159^{+0.0070}_{-0.0055}$ | $0.1163^{+0.0073}_{-0.0055}$ |

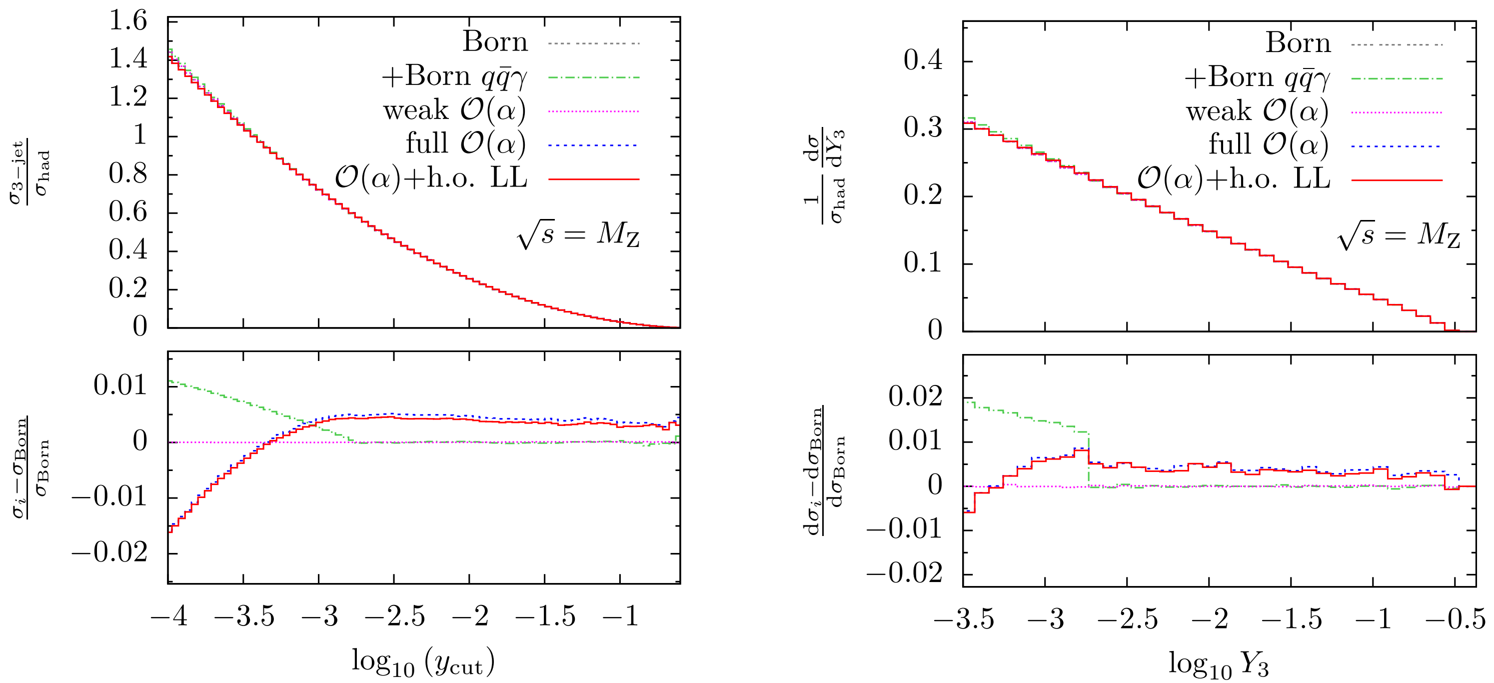
- Hadronisation corrections are small and not included in the fit

$$\alpha_s(M_Z) = 0.1156^{+0.0041}_{-0.0034}$$

# Electroweak corrections

- NLO EW effects potentially as big as NNLO $\alpha \approx \alpha_s^2$

[Denner Dittmaier Gehrmann Kurz '10]

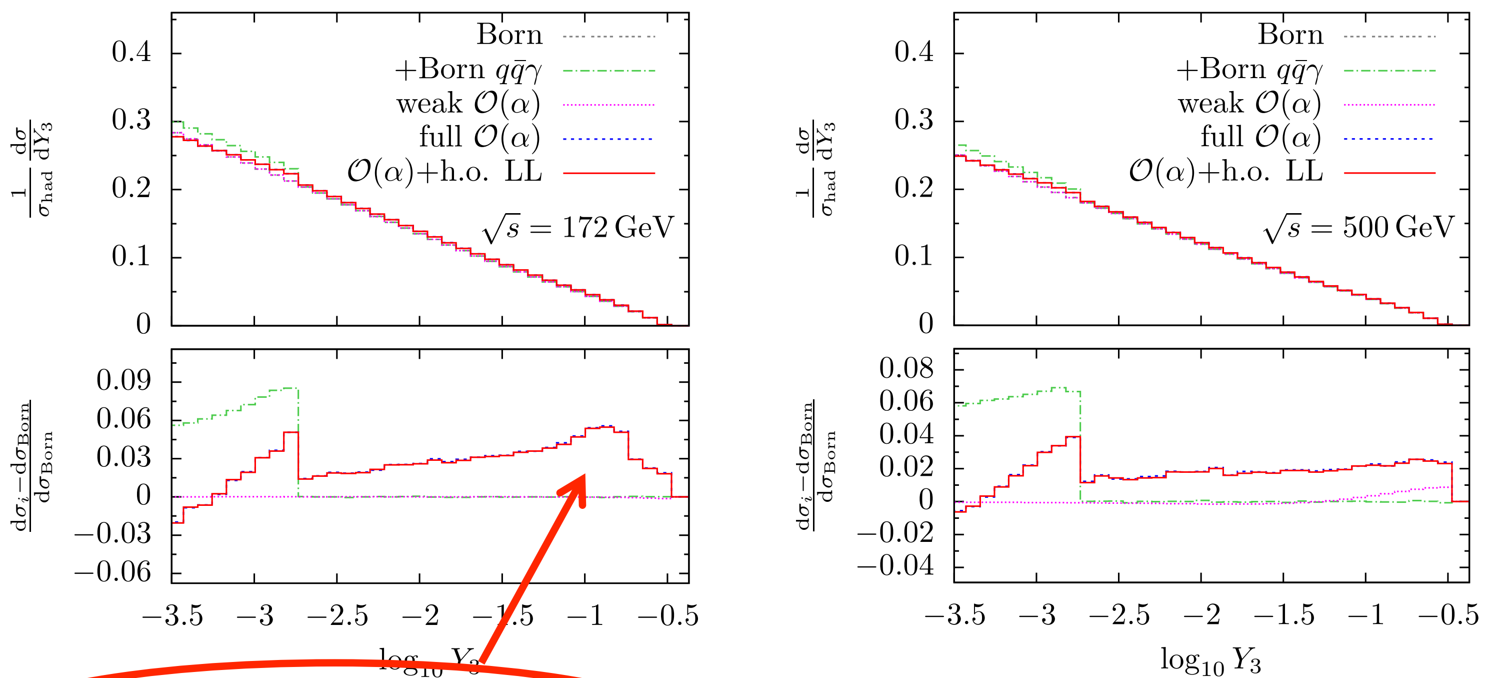



- ISR largely cancels in the ratio with the total cross-section
- Weak loop corrections at the per-mille level
- Result depends crucially on photon isolation cuts

# Electroweak corrections

- NLO EW effects potentially as big as NNLO $\alpha \approx \alpha_s^2$

[Denner Dittmaier Gehrmann Kurz '10]




- Additional peaky structure due to radiative return + clustering of a soft gluon with the hard photon

- The additional peak moves to lower $y_3$ with increasing energy and disappears at $\sqrt{s} = 500\,\mathrm{GeV}$

# Conclusions

- Jet rates have small hadronisation effects, suitable for precision measurements of $\alpha_s \Rightarrow 2.1\% < \Delta\alpha_s/\alpha_s < 4.3\%$
- No analytic model for hadronisation $\Rightarrow$ clear gain from increasing the energy (hadronisation suppressed + ½ QCD uncertainty?)
- NNLO QCD known up to three jets, not clear whether we'll have four jets in the near future
- EW corrections $\Rightarrow$ no need to correct for photons or subtract WW as a background
- Resummation for jet rates highly desirable
    - ✓ NNLL for two-jet rate
    - ✓ NLL for all other jet rates

# Extra slides

# Jet algorithms for $e^+e^-$ annihilation (I)

- The jet algorithm employed in existing analyses are of sequential type. Examples:

1. Find the pair of particles $p_i$ and $p_j$ with the minimum distance

$$y_{ij}^{(J)} = 2\frac{E_i E_j}{Q^2}(1-\cos\theta_{ij}) \qquad \text{(JADE)}$$

$$y_{ij}^{(D)} = 2\frac{\min\{E_i^2, E_j^2\}}{Q^2}(1-\cos\theta_{ij}) \qquad \text{(Durham)}$$

2. If $y_{ij} < y_{\rm cut}$, merge the two particles $p_i$ and $p_j$ in a single pseudo-particle $p_{ij}=p_i+p_j$, and go back to 1. Otherwise stop.

# Jet algorithms for $e^+e^-$ annihilation (II)

- Another known variant is the Cambridge algorithm

1. Find the pair of particles $p_i$ and $p_j$ with the minimum *angular* distance

   $$v_{ij} = 2(1 - \cos\theta_{ij})$$

2. If $y_{ij}^{(D)} < y_{\mathrm{cut}}$, merge the two particles $p_i$ and $p_j$ in a single pseudo-particle $p_{ij}=p_i+ p_j$, and go back to 1. Otherwise, eliminate the softer between $p_i$ and $p_j$ (soft freezing).

# Jet algorithms for $e^+e^-$ annihilation (II)

- A variant of the Cambridge algorithm is the angular ordered (AO) Durham

1. Find the pair of particles $p_i$ and $p_j$ with the minimum *angular* distance

$$v_{ij} = 2(1 - \cos\theta_{ij})$$

2. If $y_{ij}^{(D)} < y_{\mathrm{cut}}$, merge the two particles $p_i$ and $p_j$ in a single pseudo-particle $p_{ij}=p_i+ p_j$, and go back to 1. Otherwise, repeat step 2 with the next minimal $v_{ij}$. If no pair with $y_{ij}^{(D)} < y_{\mathrm{cut}}$ is found, the algorithm stops