Strong Coupling from the C-Parameter

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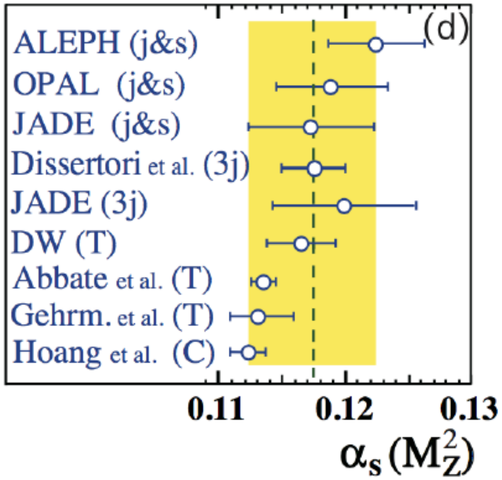
Outline

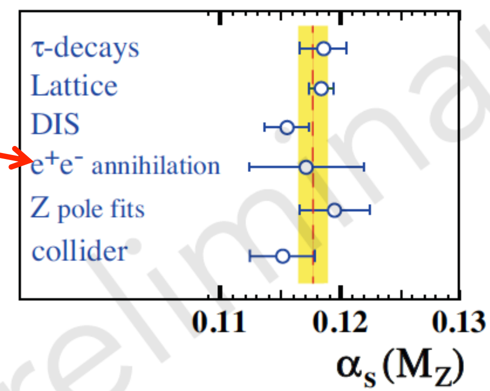
Preliminary 2015 World Average

α_s from jets and event shapes in e^+e^- annihilation



–> $\alpha_s(M_Z)$= 0.1174 ± 0.0051

2015 summary of α_s



$\alpha_s(M_Z)$ = 0.1177 ± 0.0013

Outline

- Introduction
- Anatomy of SCET description
- Our analysis
- Cross checks
- Why is our $\alpha_s(m_Z)$ so small?
- Conclusions

AHH, Kolodrubetz, Mateu, Stewart; PRD 91 (2015) 9, 094018

AHH, Kolodrubetz, Mateu, Stewart; PRD 91 (2015) 9, 094017

C-parameter distribution

Abbate, Fickinger, AHH, Mateu, Stewart; PRD 86 (2012) 094002

Thrust-moments

Abbate, Fickinger, AHH, Mateu, Stewart; PRD 83 (2011) 074021

Thrust distribution

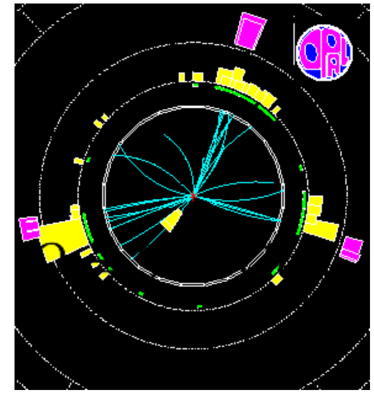
Event Shapes

→ Classic method for determining $\alpha_s(M_z)$

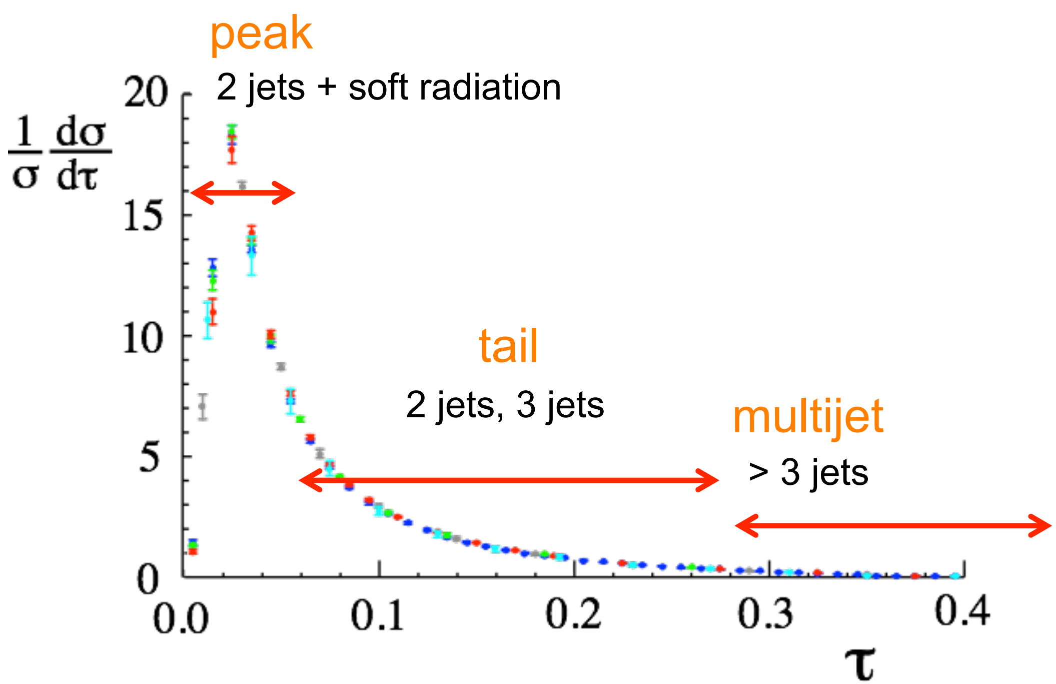
Single-variable jet distributions

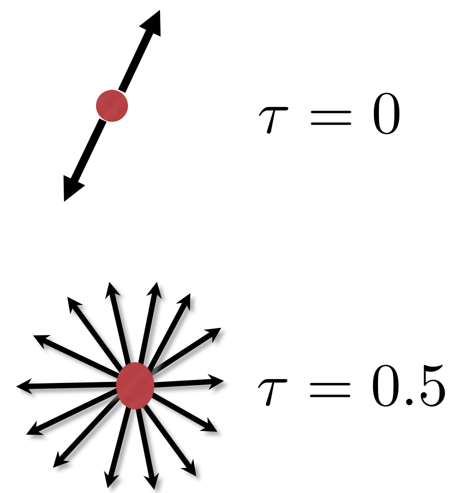
e.g. Thrust $$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

C-parameter $$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$$



OPAL 3 jet event





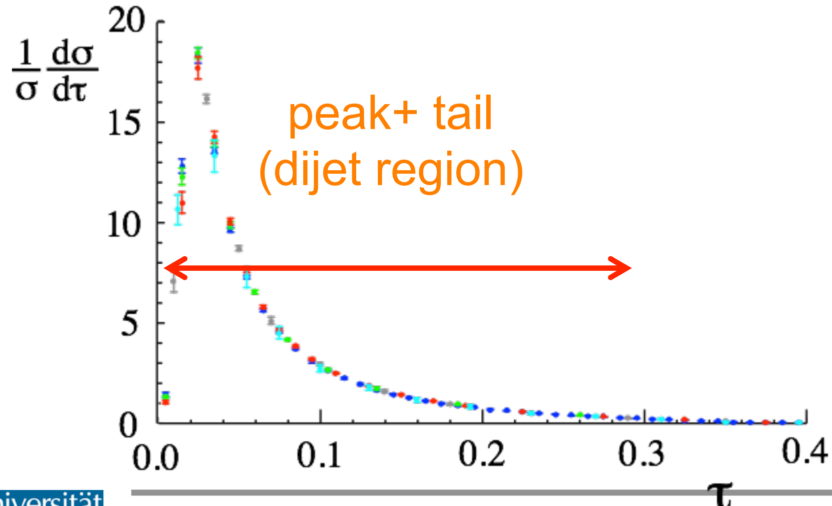
Issues in the Dijet Limit

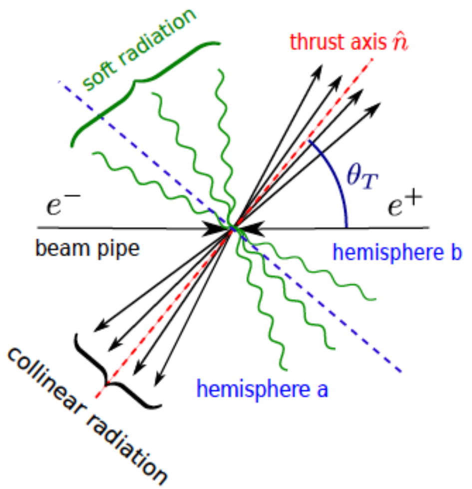
Singular Cross section

$$\frac{1}{\sigma_{\rm tot}^{\rm Born}}\frac{d\sigma}{d\tau} = \delta(\tau) + \frac{C_F\alpha_s}{\pi}\Bigg[\Big(\frac{\pi^2}{6}-\frac{1}{2}\Big)\delta(\tau) + \frac{-3+9\tau+3\tau^2-9\tau^3}{2\tau(1-\tau)} - \frac{2-3\tau+3\tau^2}{(1-\tau)}\left(\frac{\ln(\frac{\tau}{1-2\tau})}{\tau}\right)_+\Bigg]$$

$$= \boxed{\delta(\tau) + \frac{C_F\alpha_s}{\pi}\Bigg[\Big(\frac{\pi^2}{6}-\frac{1}{2}\Big)\delta(\tau) - \frac{3}{2}\Big(\frac{1}{\tau}\Big)_+ - 2\Big(\frac{\ln(\tau)}{\tau}\Big)_+} + \{\text{non-sing. terms}\}\Bigg]$$

- Completely determined from soft and collinear radiation in the dijet limit: $\tau \ll 1$
- Perturbative resummation of logarithmic terms to all orders required: $\alpha_s \ln(\tau) \sim 1$





Structure of Perturbative Corrections

Singular Cross section

$$\frac{1}{\sigma_{\rm tot}^{\rm Born}}\frac{d\sigma}{d\tau} = \delta(\tau) + \frac{C_F\alpha_s}{\pi}\Big[\big(\frac{\pi^2}{6}-\frac{1}{2}\big)\delta(\tau) + \frac{-3+9\tau+3\tau^2-9\tau^3}{2\tau(1-\tau)} - \frac{2-3\tau+3\tau^2}{(1-\tau)}\Big(\frac{\ln(\frac{\tau}{1-2\tau})}{\tau}\Big)_+\Big]$$

$$= \boxed{\delta(\tau) + \frac{C_F\alpha_s}{\pi}\Big[\big(\frac{\pi^2}{6}-\frac{1}{2}\big)\delta(\tau) - \frac{3}{2}\big(\frac{1}{\tau}\big)_+ - 2\big(\frac{\ln(\tau)}{\tau}\big)_+} + \{\text{non-sing. terms}\}\Big]$$

$$\log\Sigma(\tau_c) = \alpha_s(\log^2\tau_c + \log\tau_c + 1)$$ LO

[Catani, Seymour] $$\alpha_s^2(\log^3\tau_c + \log^2\tau_c + \log\tau_c + 1)$$ NLO

State of the art $$\alpha_s^3(\log^4\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1)$$ NNLO

$$\alpha_s^4(\log^5\tau_c + \log^3\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1)$$

$\dots$ not known!

[Weinzierl]

[Gehrmann-De Rider, Gehrmann, Glover, Heinrich]

Structure of Perturbative Corrections

Singular Cross section

$$\frac{1}{\sigma_{\rm tot}^{\rm Born}}\frac{d\sigma}{d\tau} = \delta(\tau) + \frac{C_F\alpha_s}{\pi}\Big[\big(\tfrac{\pi^2}{6}-\tfrac{1}{2}\big)\delta(\tau) + \frac{-3+9\tau+3\tau^2-9\tau^3}{2\tau(1-\tau)} - \frac{2-3\tau+3\tau^2}{(1-\tau)}\Big(\frac{\ln(\frac{\tau}{1-2\tau})}{\tau}\Big)_+\Big]$$

$$= \boxed{\delta(\tau) + \frac{C_F\alpha_s}{\pi}\Big[\big(\tfrac{\pi^2}{6}-\tfrac{1}{2}\big)\delta(\tau) - \tfrac{3}{2}\big(\tfrac{1}{\tau}\big)_+ - 2\big(\tfrac{\ln(\tau)}{\tau}\big)_+} + \{\text{non-sing. terms}\}\Big]$$

$$\Sigma(\tau_c) \equiv \int_0^{\tau_c} \mathrm{d}\tau \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}$$

$$\begin{aligned}\log\Sigma(\tau_c) = {} & \alpha_s(\log^2\tau_c + \log\tau_c + 1)\\ & \alpha_s^2\,(\log^3\tau_c + \log^2\tau_c + \log\tau_c + 1)\\ & \alpha_s^3\,(\log^4\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1)\\ & \alpha_s^4\,(\log^5\tau_c + \log^3\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1)\\ & \vdots\end{aligned}$$

LL | NLL | N^{2}LL | N^{3}LL | not known!

State of the art

[Hoang, VM, Schwartz, Stewart]
[Becher, Schwartz]
[Chien, Schwartz]
[Abbate, Fickinger, Hoang, VM, Stewart]
[Hoang, Kolodrubetz, VM, Stewart]

Anatomy of SCET Prediction

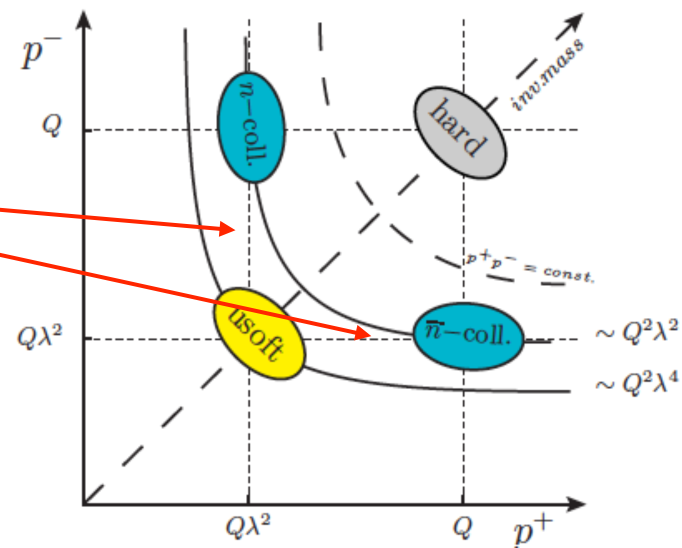
Singular Cross section

Korchemsky, Sterman; Bauer etal.
Fleming, Mantry, Stewart, AHH
Schwartz

$$\left(\frac{d\sigma}{dC}\right)^{\rm sing}_{\rm part} \sim \sigma_0 H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int d\ell d\ell' \, U_J\Big(\frac{QC}{6} - \ell - \ell', \mu_J, \mu_S\Big) \, J_\tau(Q\ell', \mu_J) \, S_{\rm C}(\ell - \Delta, \mu_S)$$

- Describe soft and collinear radiation by different quantum modes
- Effective field theory description

Convolution



$$\lambda \sim C$$

Anatomy of SCET Prediction

Matrix element and hard matching terms (fixed-order)

$$\left(\frac{d\sigma}{dC}\right)^{\rm sing}_{\rm part} \sim \sigma_0 H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int d\ell d\ell' \, U_J\left(\frac{QC}{6}-\ell-\ell',\mu_J,\mu_S\right) J_\tau(Q\ell',\mu_J)\, S_{\rm C}(\ell-\Delta,\mu_S)$$

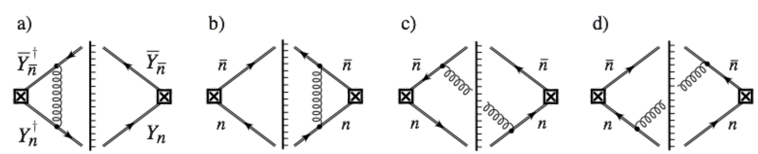
Hard function

order-by-order renormalon subtraction

Soft function

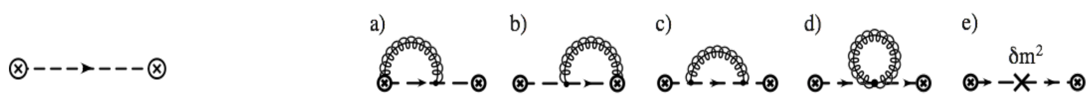
Each factor gauge invariant !

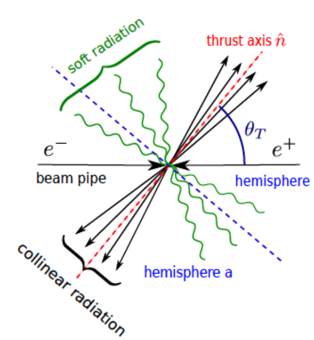
$$S_{\rm hemi}(\ell^+,\ell^-,\mu) = \frac{1}{N_c}\sum_{X_s} \delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\langle 0|\overline{Y}_{\bar n}\, Y_n(0)|X_s\rangle\langle X_s|Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger(0)|0\rangle$$



Jet function

$$J_n(Qr_n^+,\mu) = \frac{-1}{8\pi N_c Q}\,\mathrm{Disc}\int d^4x\, e^{ir_n\cdot x}\,\langle 0|\mathrm{T}\,\bar\chi_{n,Q}(0)\,\bar{n}\!\!\!/\,\chi_n(x)|0\rangle$$





Anatomy of SCET Prediction

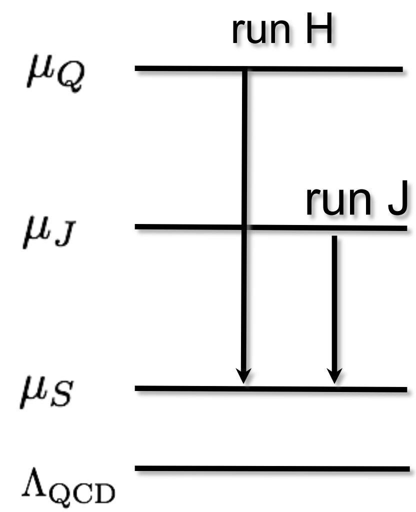
Summation of large logarithms

$$\left(\frac{d\sigma}{dC}\right)^{\rm sing}_{\rm part} \sim \sigma_0 H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int d\ell d\ell' \, U_J\left(\frac{QC}{6}-\ell-\ell',\mu_J,\mu_S\right) J_\tau(Q\ell',\mu_J)\, S_{\rm C}(\ell-\Delta,\mu_S)$$

2-jet production current

$$\mu\frac{d}{d\mu}H_Q(Q,\mu) = \gamma_{H_Q}(Q,\mu)\, H_Q(Q,\mu)$$

$$\gamma_{H_Q}(Q,\mu) = \Gamma_{H_Q}[\alpha_s] \ln\left(\frac{\mu^2}{Q^2}\right) + \gamma_{H_Q}[\alpha_s]$$



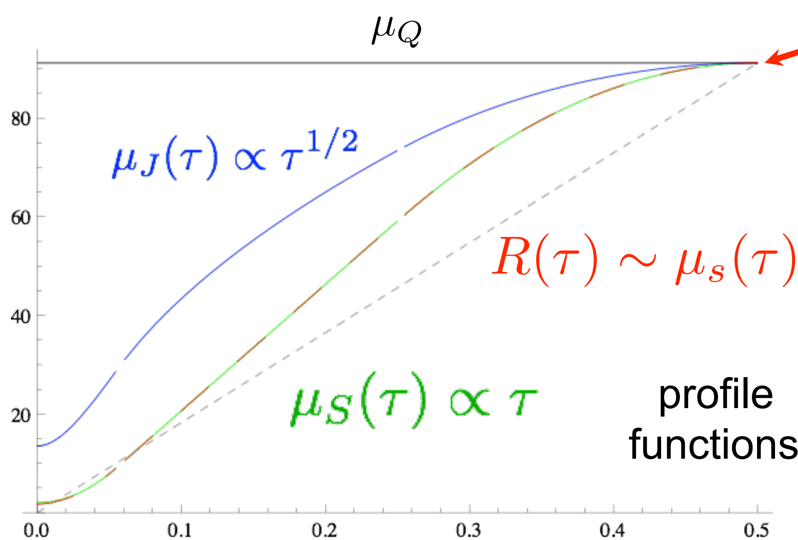
Jet function evolution

$$\mu\frac{d}{d\mu}J(y,\mu) = \gamma_J(y,\mu)\, J(y,\mu) = \left[2\Gamma^{\rm cusp}(\alpha_s)\ln(iy\mu^2 e^{\gamma_E}) + \gamma_J(\alpha_s)\right] J(y,\mu)$$

Anatomy of SCET Prediction

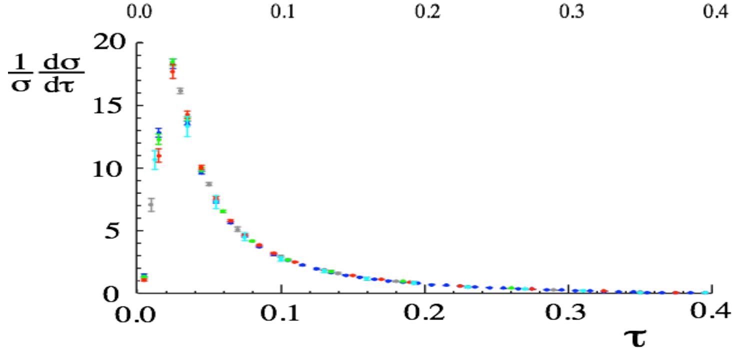
Summation of large logarithms

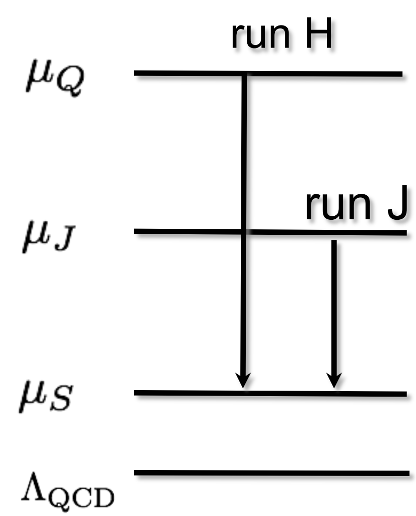
$$\left(\frac{d\sigma}{dC}\right)^{\rm sing}_{\rm part} \sim \sigma_0 H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int d\ell d\ell'\, U_J\!\left(\frac{QC}{6}-\ell-\ell',\mu_J,\mu_S\right) J_\tau(Q\ell',\mu_J)\, S_{\rm C}(\ell-\Delta,\mu_S)$$



scales become equal for multijet region

profile functions



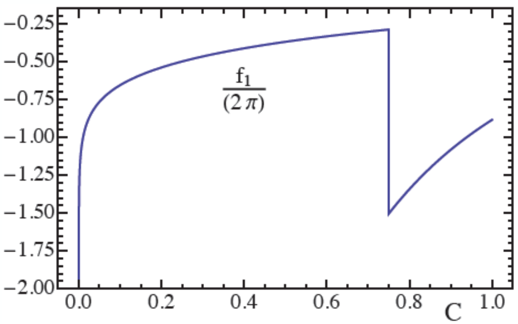


Anatomy of SCET Prediction

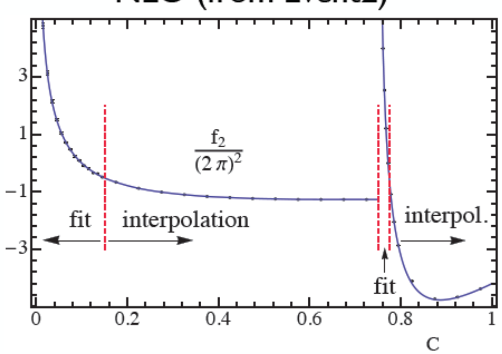
Combination for hadron level prediction

$$\left(\frac{d\sigma}{dC}\right) = \int d\ell \left[\left(\frac{d\sigma}{dC}\right)^{\text{sing}}_{\text{part}}\left(C - \frac{\ell}{Q}\right) + \left(\frac{d\sigma}{dC}\right)^{\text{nonsing}}_{\text{part}}\left(C - \frac{\ell}{Q}\right)\right] S^{\text{mod}}(\ell, \Delta(R))$$

LO (analytically)

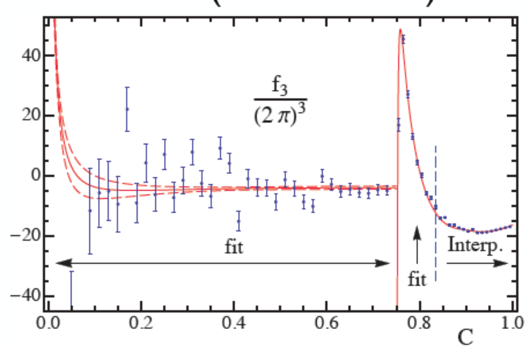


NLO (from Event2)

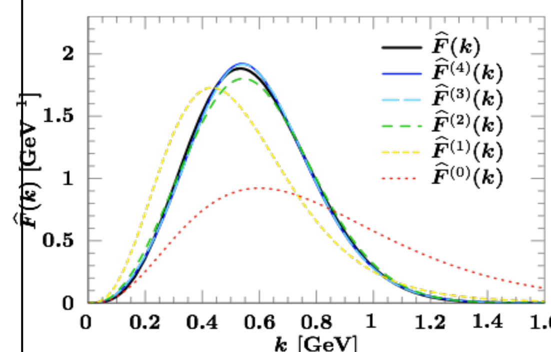


Fixed-order minus terms already resummed

NNLO (from EERAD3)



Soft matrix element model function (renormalon subtrated)



Anatomy of SCET Prediction

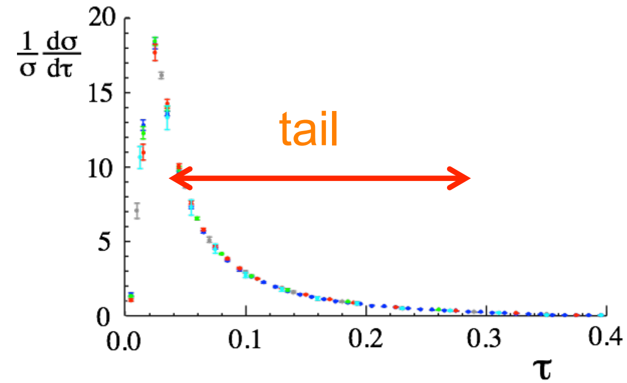
OPE for non perturbative power correction

- For C ≫ Λ_{QCD}/Q, in the tail region, the soft model function can be expanded in an OPE.

$$S_C^{\rm mod}(\ell) = \delta(\ell) + \Omega_1^C\,\delta'(\ell) + \ldots$$

$$\left(\frac{d\sigma}{dC}\right)^{\rm tail} \approx \frac{d\hat\sigma}{dC} - \frac{\Omega_1^C}{Q}\frac{d^2\hat\sigma}{dC^2} \approx \frac{d\hat\sigma}{dC}\left(C - \frac{\Omega_1^C}{Q}\right)$$

Only two fit parameters: α_s and Ω_1^C



- Universality of powercorrections: $\Omega_1^C = \frac{3\pi}{2}\,\Omega_1^\tau = 4.2\,\Omega_1^\tau$ Lee, Sterman

- We use the thrust normalization for the parametrization of the leading power correction:

$$\Omega_1 = \frac{2}{3\pi}\,\Omega_1^C$$

- Hadron mass effects break the relation, but as small in the relation between thrust and C-parameter. Mateu, Stewart, Thaler

Strong Coupling Determination

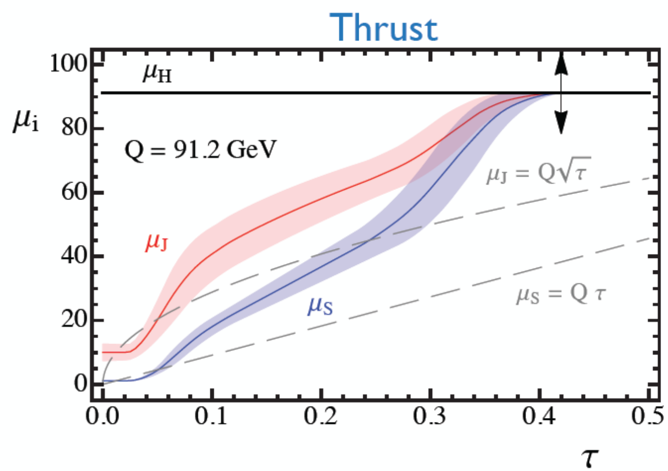
Scale and uncertainty parameter variations: **"Random Scan"**

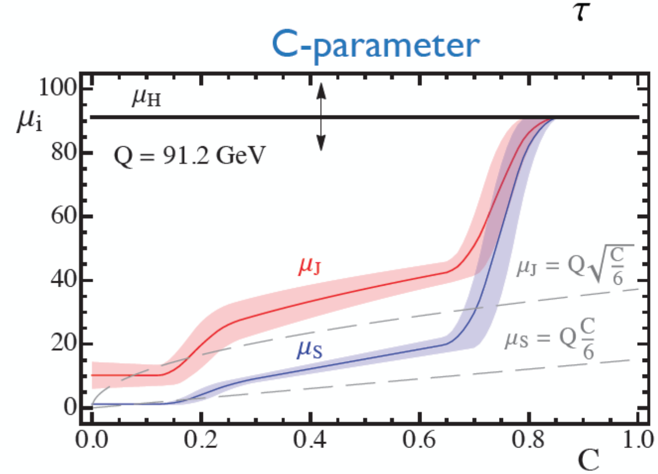
- Pick 500 random points and fit for each choice separately (numerically costly!).
- More conservative than error band method OR qudratic sum of individual variations.

	parameter	default value	range of values
scale variation	μ_0	1.1 GeV	1 to 1.3 GeV
	R_0	0.7 GeV	0.6 to 0.9 GeV
	n_0	12	10 to 16
	n_1	25	22 to 28
	t_2	0.67	0.64 to 0.7
	t_s	0.83	0.8 to 0.86
	r	0.33	0.26 to 0.38
	e_J	0	− 0.5 to 0.5
	e_H	1	0.5 to 2.0
	n_s	0	− 1, 0, 1
unknowns	Γ_3^{cusp}	1553.06	− 1553.06 to + 4659.18
	s_2	− 43.2	− 44.2 to − 42.2
	j_3	0	−3000 to +3000
	s_3	0	−500 to +500
non-singular	$\epsilon_{2,\mathrm{low}}$	0	− 1, 0, 1
	$\epsilon_{2,\mathrm{high}}$	0	− 1, 0, 1
	$\epsilon_{3,\mathrm{low}}$	0	− 1, 0, 1
	$\epsilon_{3,\mathrm{high}}$	0	− 1, 0, 1

4-loop cusp anomalous dimension → Γ_3^{cusp}

non-log $O(\alpha_s^3)$ corrections → s_2, j_3, s_3



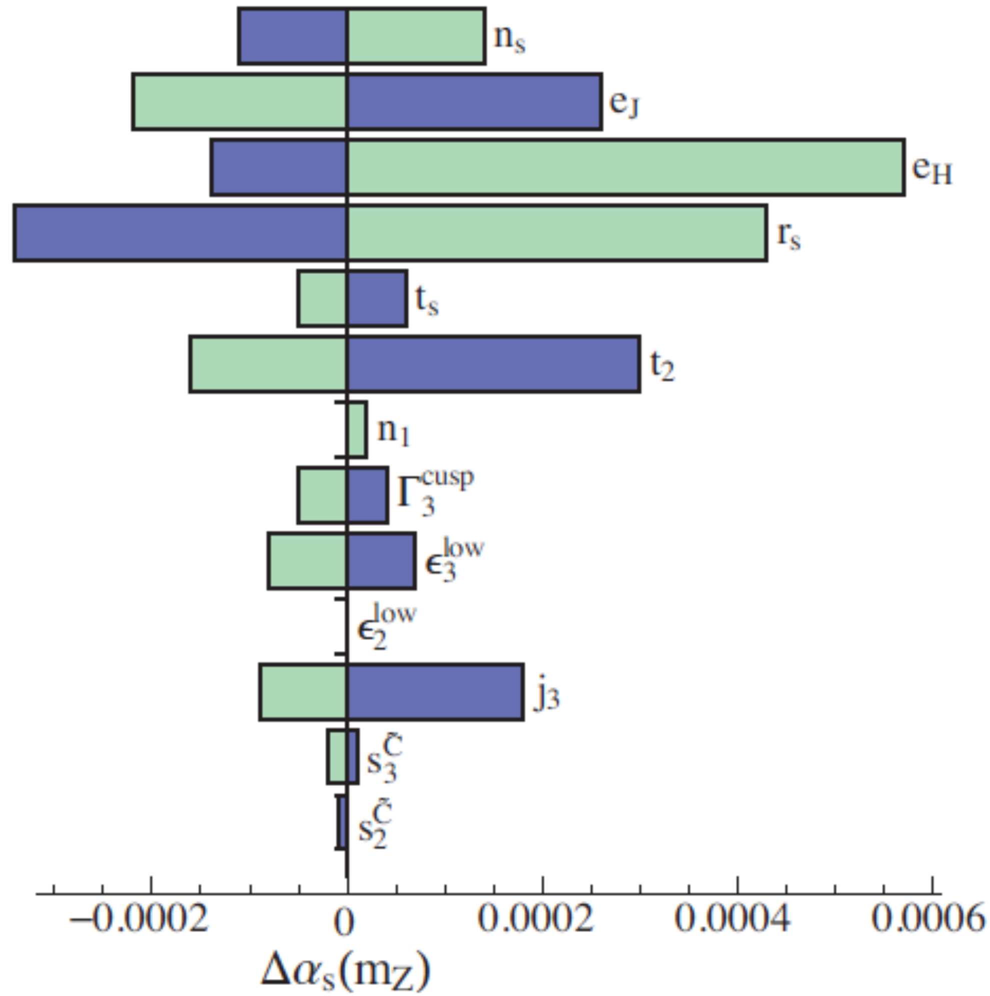


Strong Coupling Determination

Scale and uncertainty parameter variations:

Renormalization scale parameters

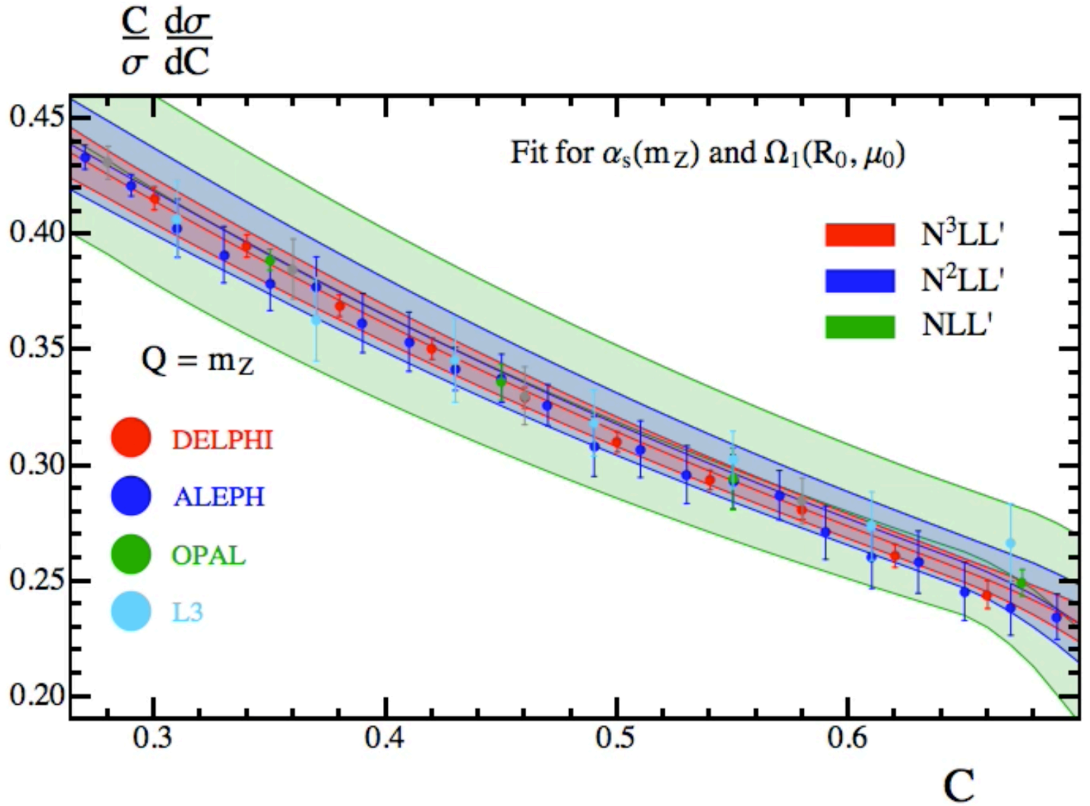
Unknowns and numerical uncertainties



Strong Coupling Determination

Convergence (using Random Scan scale variation)

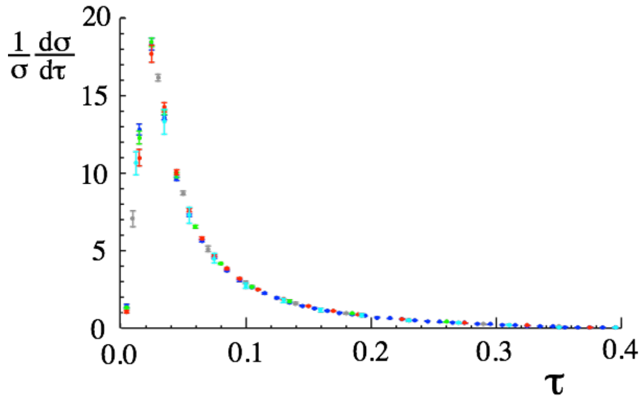
→ Excellent convergence when order of description is increased.
(Picture for best fit)



Strong Coupling Determination

Experimental Data:

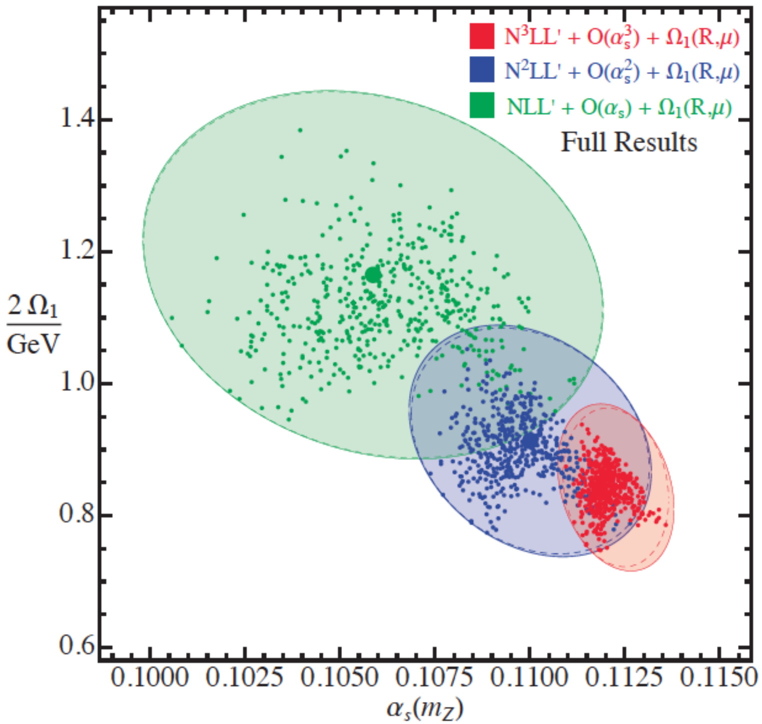
Experiment:	Values of Q :
ALEPH	[91.2, 133.0, 161.0, 172.0, 183.0, 189.0, 200.0, 206.0]
DELPHI	[45.0, 66.0, 76.0, 89.5, 91.2, 93.0, 133.0, 161.0, 172.0, 183.0, 189.0, 192.0, 196.0, 200.0, 202.0, 205.0, 207.0]
OPAL	[91.0, 133.0, 177.0, 197.0]
L3	[41.4, 55.3, 65.4, 75.7, 82.3, 85.1, 91.2, 130.1, 136.1, 161.3, 172.3, 182.8, 188.6, 194.4, 200.0, 206.2]
SLD	[91.2]
TASSO	[14.0, 22.0, 35.0, 44.0]
JADE	[35.0, 44.0]
AMY	[55.2]



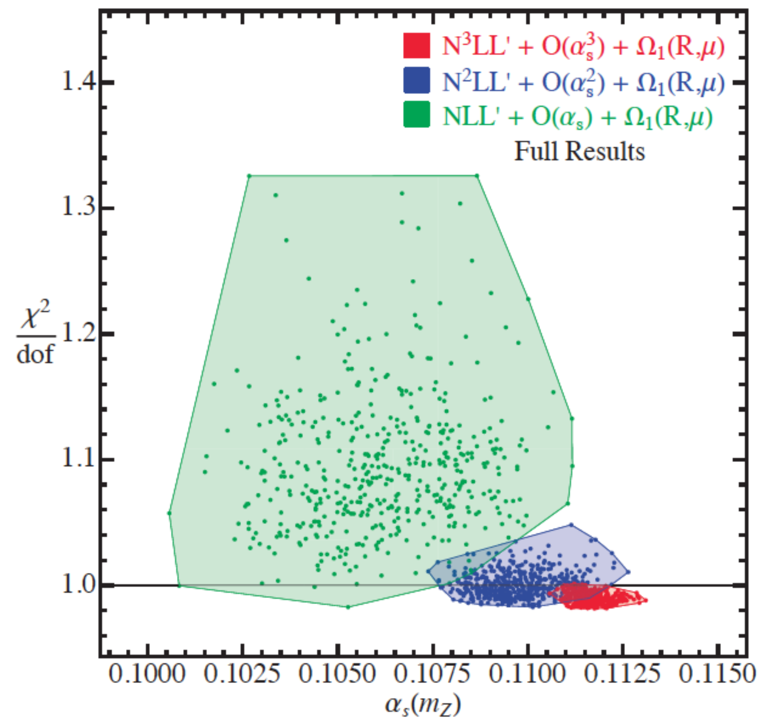
Lot of data available: 807 bins in τ

Strong Coupling Determination

$\alpha_s - \Omega_1$ plane



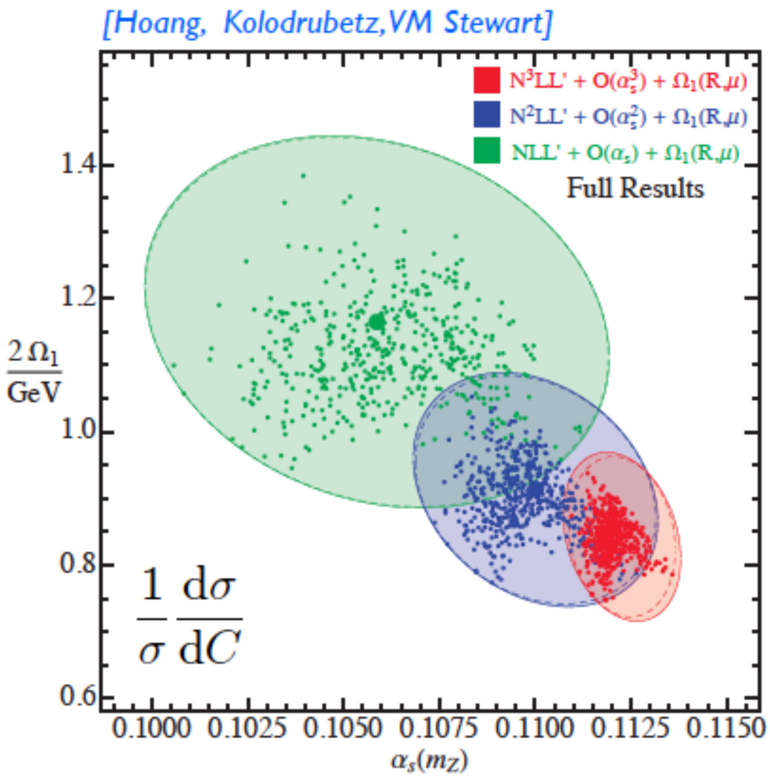
$\alpha_s - \chi^2$ plane



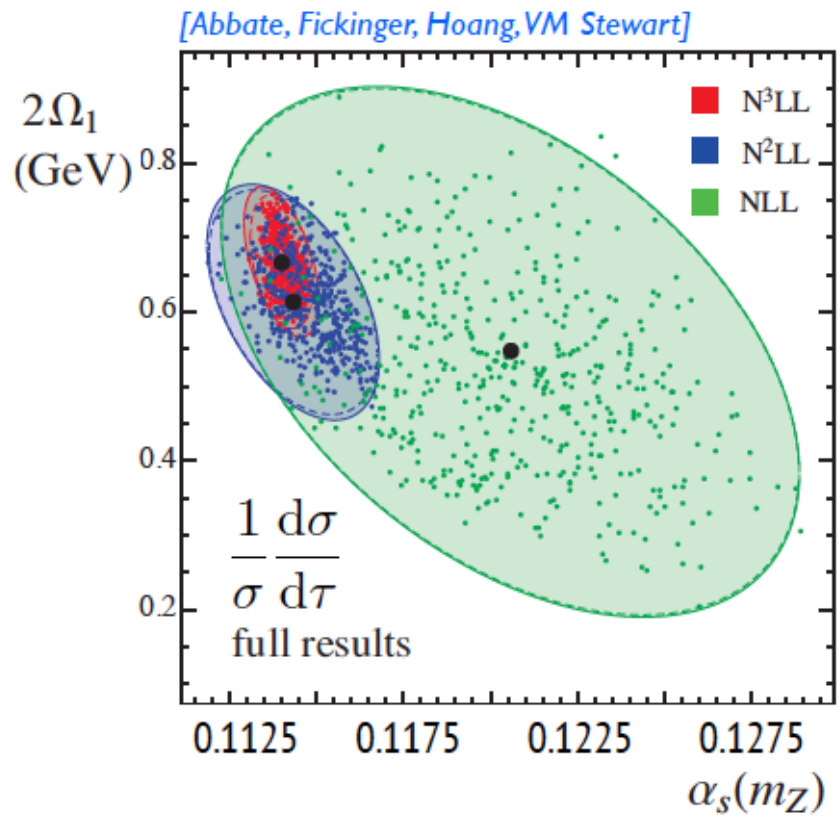
- Good convergence of the fit when approaching higher order.
- Improved quality of theoretical description with increasing order.

Strong Coupling Determination

C-parameter Tail Global Fit



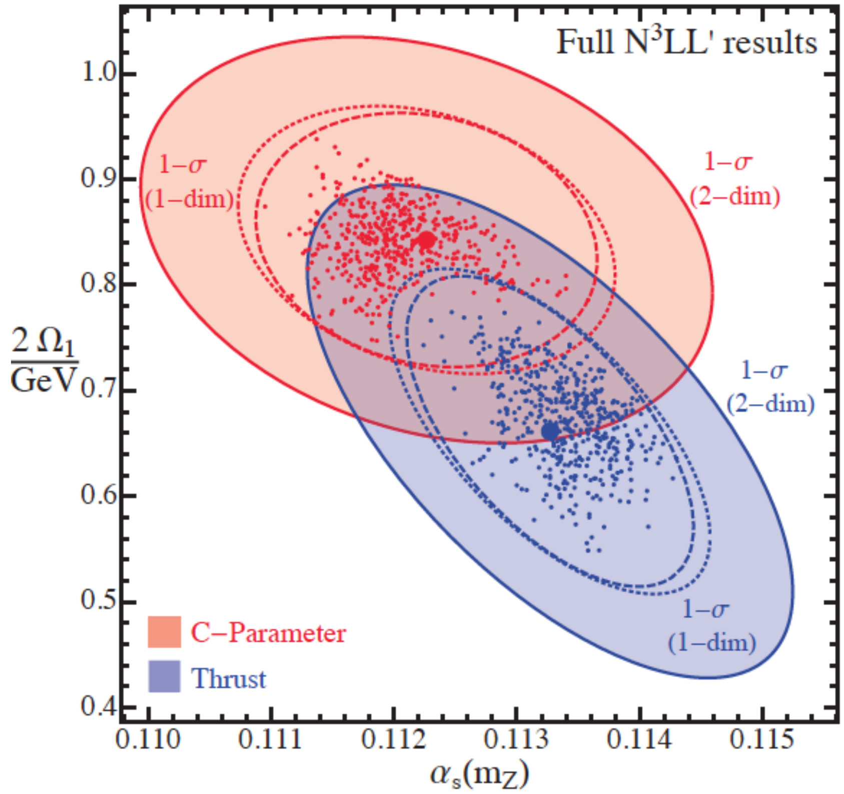
Thrust Tail Global Fit



- Different behavior of fits with increase order
- Very good agreement at $N^3LL + O(\alpha_s^3)$ with renormalon subtraction.

Strong Coupling Determination

C-parameter versus Thrust Tail Global Fit

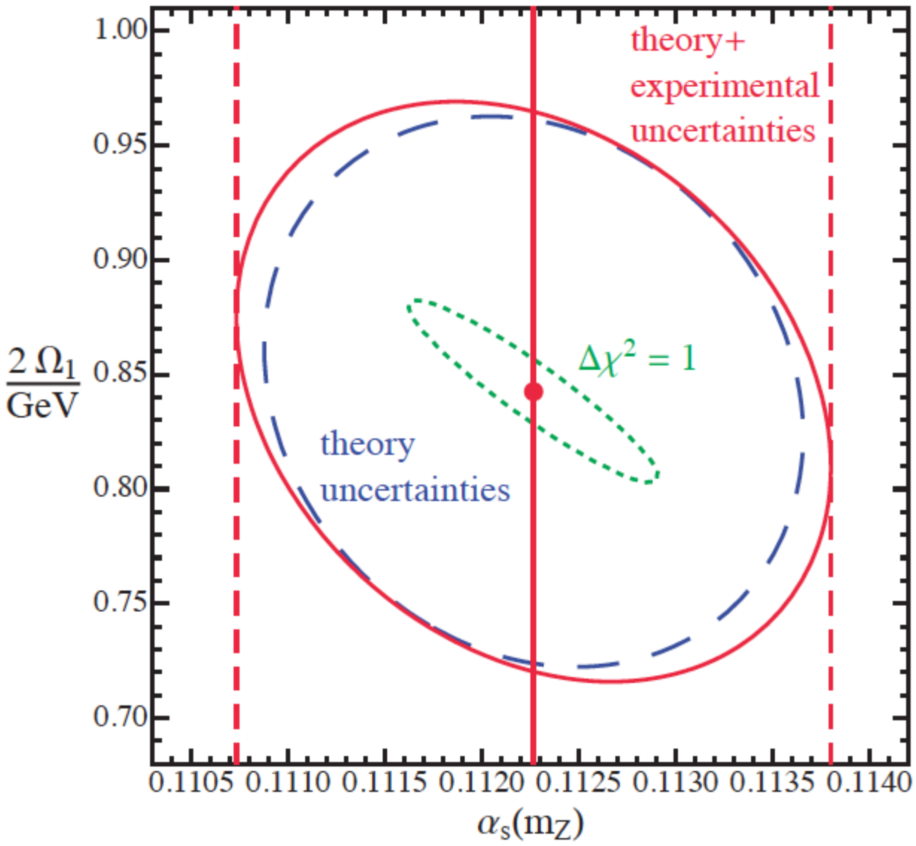


→ Very good agreement at $N^3LL + O(\alpha_s^3)$ with renormalon subtraction.

Strong Coupling Determination

Theoretical vs. Experimental vs. Hadronization Uncertainties:

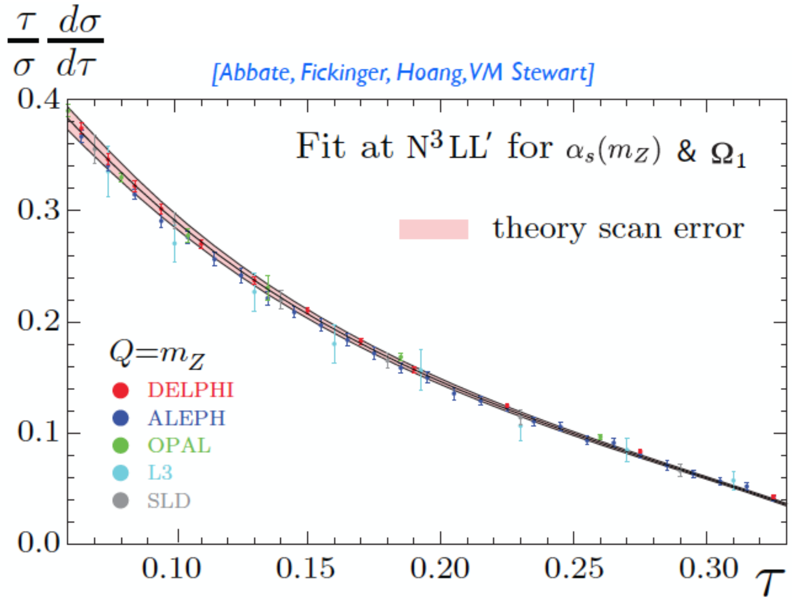
C-parameter Tail Global Fit

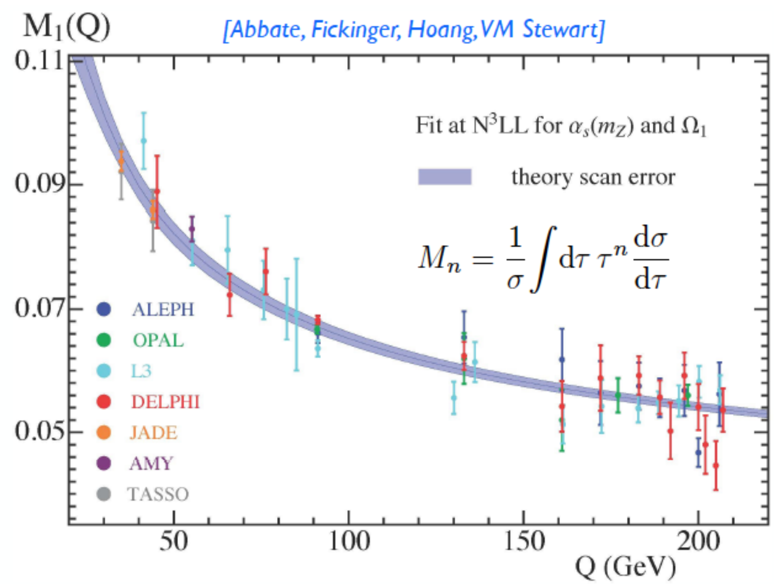


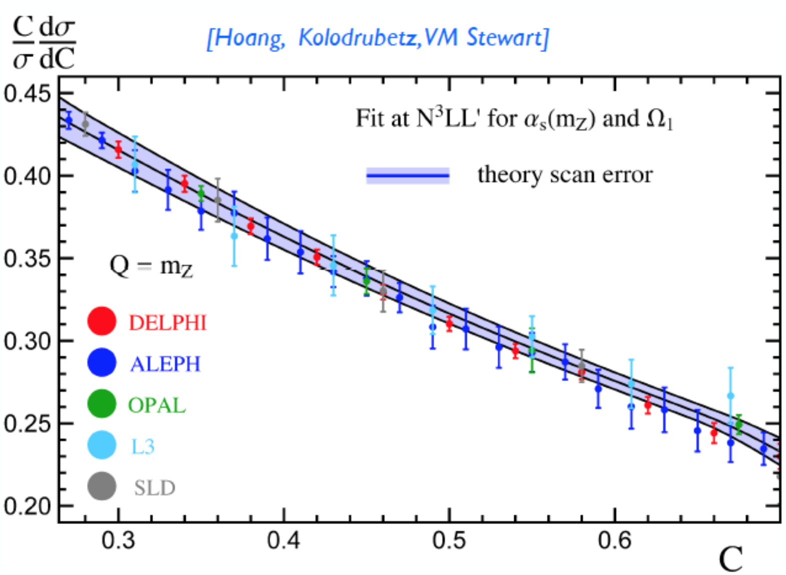
$$\alpha_s(m_Z) = 0.1123 \pm 0.0002_{\text{exp}} \pm 0.0007_{\text{hadr}} \pm 0.0014_{\text{pert}}$$

- Perturbative errors dominate
- Experimental errors smallest
- Similar pattern for other eventshape analyses.

Strong Coupling Determination







Thrust distribution (2010):

$\alpha_s(M_Z) = 0.1135 \pm 0.0010$

- Updated profiles
- 2-loop soft function

Thrust distribution (2015 update!):

$\alpha_s(M_Z) = 0.1128 \pm 0.0012$

Thrust 1st moment (2012):

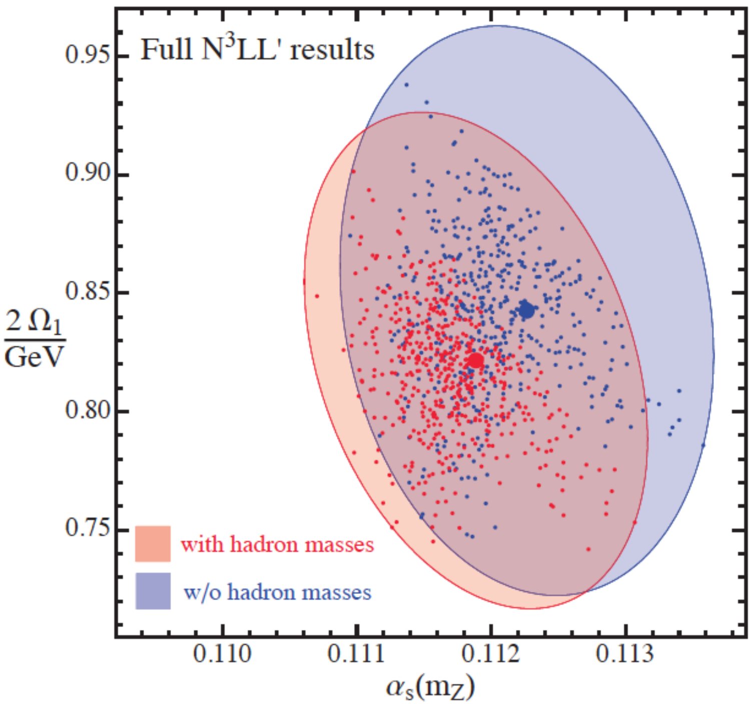
$\alpha_s(M_Z) = 0.1140 \pm 0.0016$

C-parameter distribution (2015):

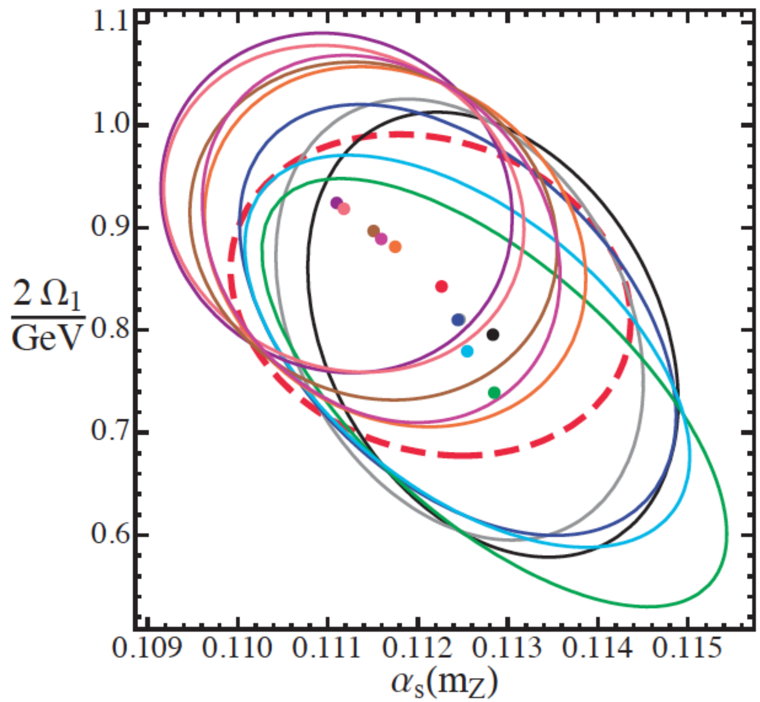
$\alpha_s(M_Z) = 0.1230 \pm 0.0015$

Cross Checks

Hadron mass effects



Data set dependence



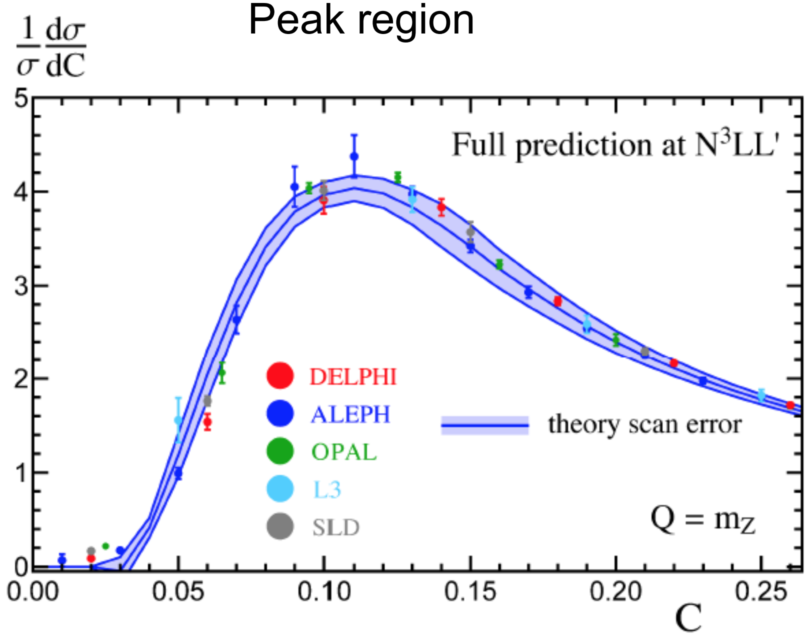
- Hadron mass effects modify the way how the soft function enters the theory prediction.
- Effect is very small and

- Dependence on the upper and lower boundary of fit intervals.
- Dependence compatible with theory uncertainty. (NOT ADDITIONAL ERROR!)

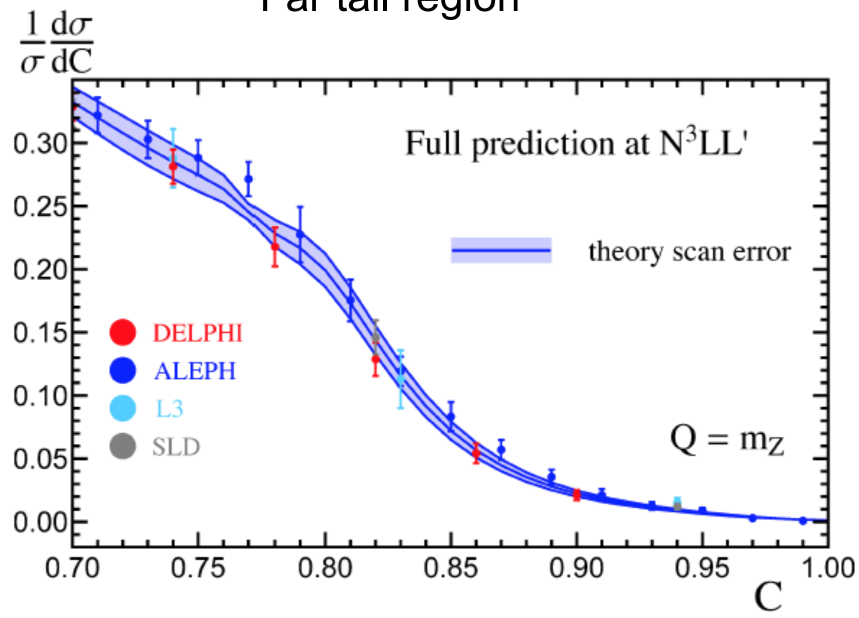
Cross Checks

Agreement with data outside fit region:

Peak region

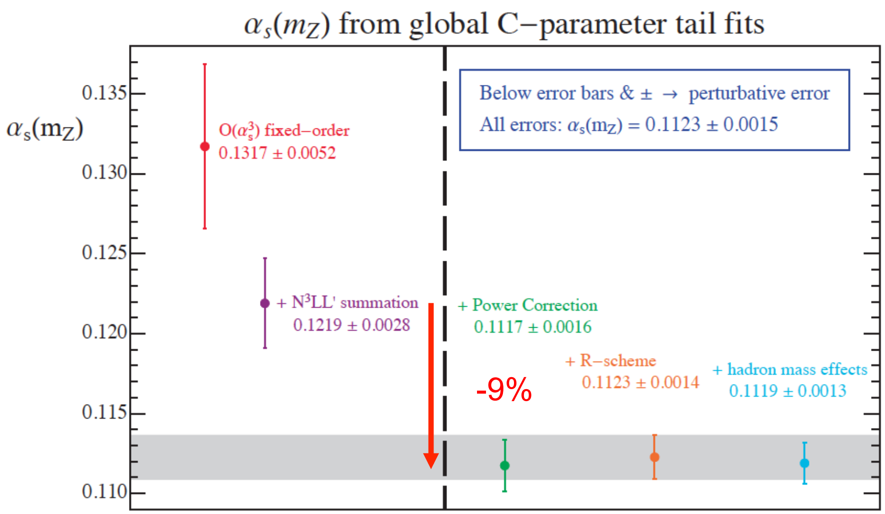


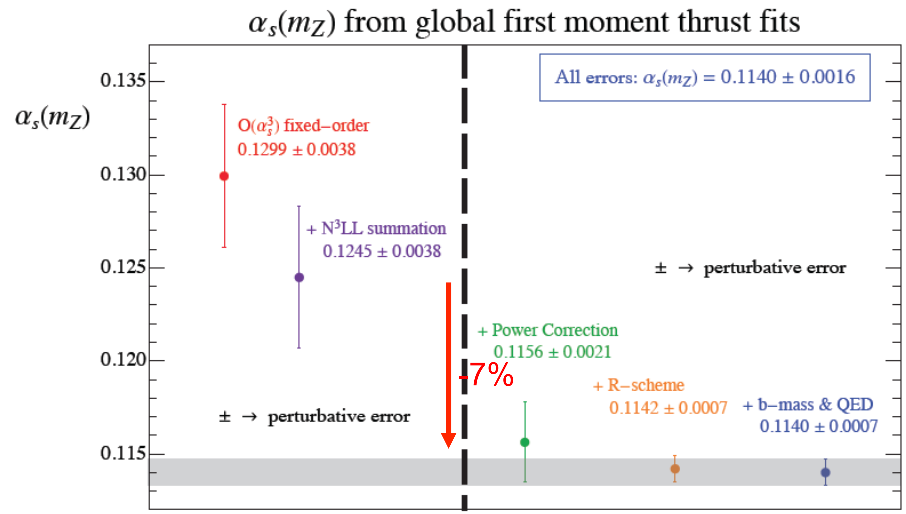
Far tail region

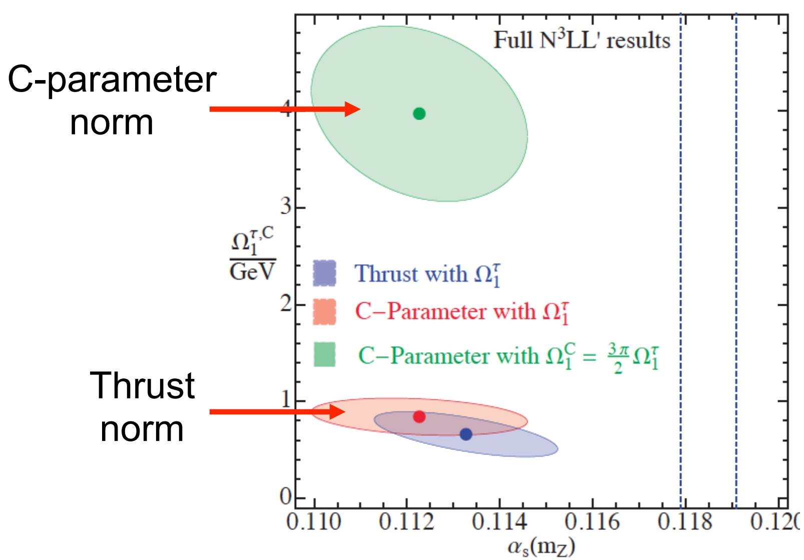


- Very good agreement with data for the entire spectrum.
- Demonstrates ability of or approach to cover the whole spectrum.

Size of Non-Perturbartive Effects



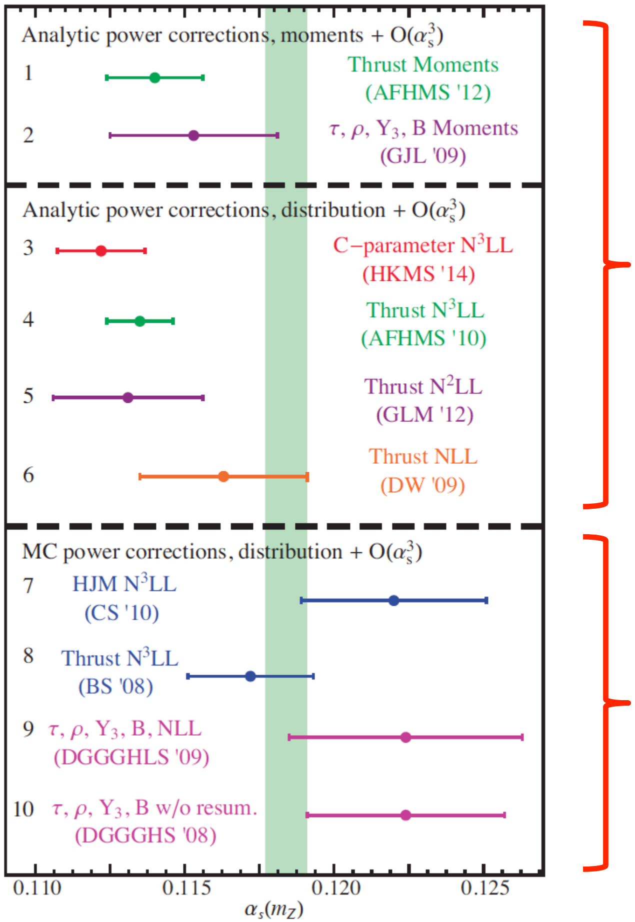




- Finite non-perturbative effects drive strong coupling small.
- Perfect consistency between fitted size of Ω_1 between C-parameter and Thrust
- Predicted universality relation confirmed from experimental data.
- Parametrization of non-perturbative effects compatible with experimental data.

Size of Non-Perturbartive Effects

Monte-Carlo estimate vs. fits of non-perturbative powercorrection:

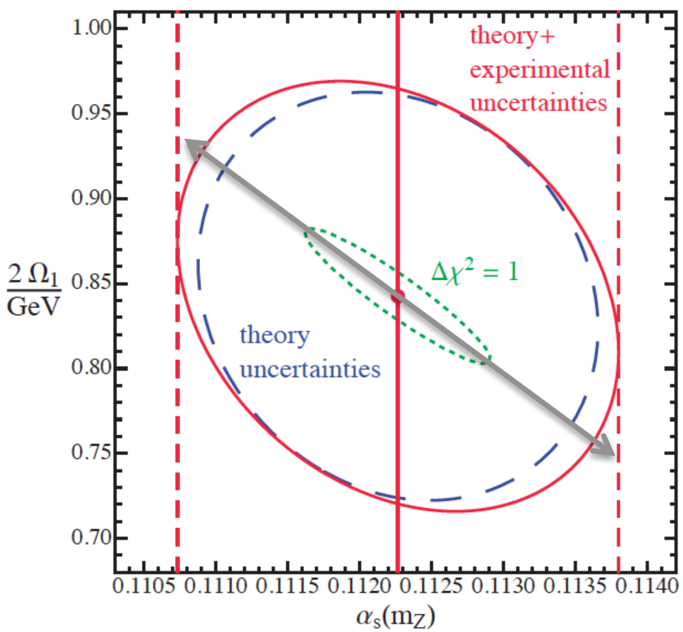


- Simultaneous fit of power corrections and the strong coupling.
- Sizeable power correction and strong coupling smaller than world average.

- Power corrections taken from difference $MC_{\text{parton level}}$ - $MC_{\text{hadron level}}$
- Small power correction and strong generically larger than world average.
- <u>Problem:</u> $MC_{\text{parton level}}$ is only LO/LL description: $MC_{\text{parton level}}$ - $MC_{\text{hadron level}}$ is LO/LL !
- Should not be used in event shape averages.

Strong Coupling vs. Non-Perturbative Effects

→ Degeneracy between α_s and Ω_1 :



$$\frac{\Omega_1}{41.26\,\text{GeV}} = 0.1221 - \alpha_s(m_Z)$$

We would obtain

$$\alpha_s(m_Z) \sim 0.118 \quad \text{for} \quad \Omega_1 \sim 0.170\ \text{GeV}$$

The measured Ω_1 value is 2.5 times larger:

$$\Omega_1 \ = \ 0.421 \pm 0.063\ \text{GeV}$$

There might be an issue in the data away from the Z pole coming from poor experimental precision.

Recall: α_s and Ω_1 enter differently depending on Q: $\left(\frac{d\sigma}{dC}\right)^{\text{tail}} \approx \frac{d\hat\sigma}{dC} - \frac{\Omega_1^C}{Q}\frac{d^2\hat\sigma}{dC^2} \approx \frac{d\hat\sigma}{dC}\left(C - \frac{\Omega_1^C}{Q}\right)$

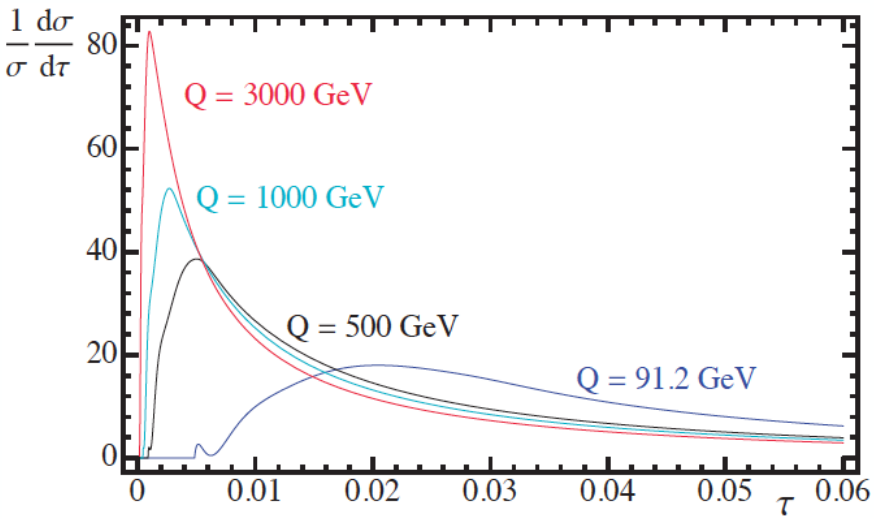
Need: Data from widely different Qs needed to resolve degeneracy.

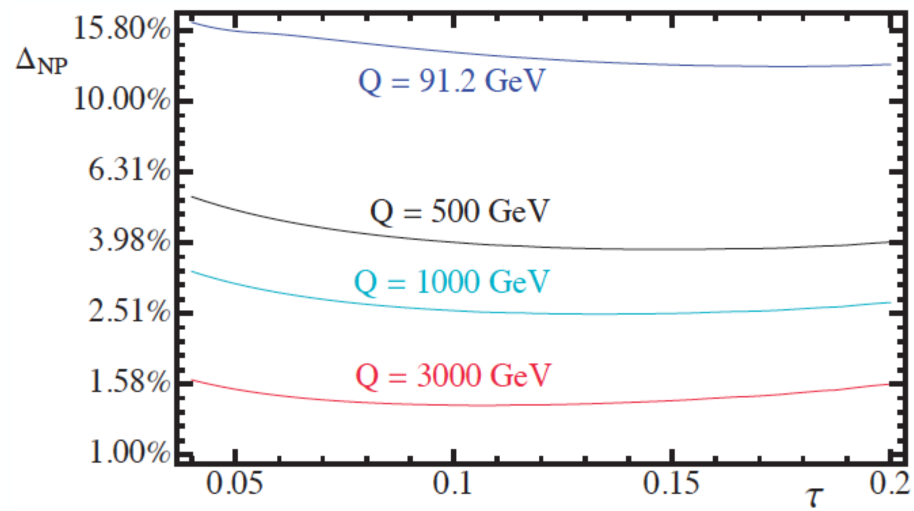
→ Need: High precision data from low energies !!

→ B-factories have this event shape data: PLEASE PUBLISH THEM !

What can a future lepton collider help?

What would a precise measurement of event shapes at higher Q values contribute?





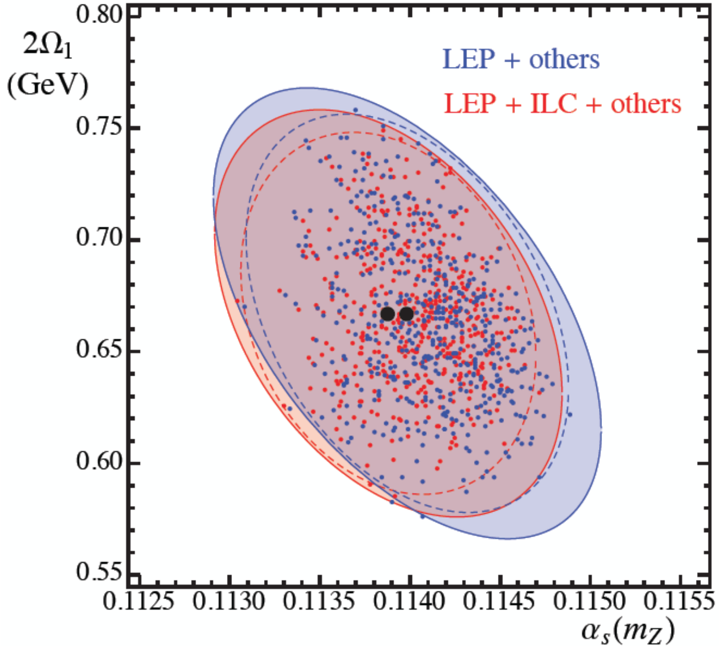
- Event accumulate in very small region at small values.
- High precision needed.
- Background tricky (γγ)

- Non-perturbative effects decrease with Q
- At some point smaller than experimental uncertainty and negligible !!

Strong Coupling from a Future Lepton Collider

What would a precise measurement of event shapes at higher Q values contribute?

Exercise: Make up fictitious ILC data at 500 GeV, with assumed 1% statistical and 1% systematical uncertainties. Repeat fits.



- Limited impact concerning precision because high-energy uncertainties blown up in the evolution to Z mass
- Nevertheless important impact in lifting degeneracy between α_s and Ω_1.

Conclusions

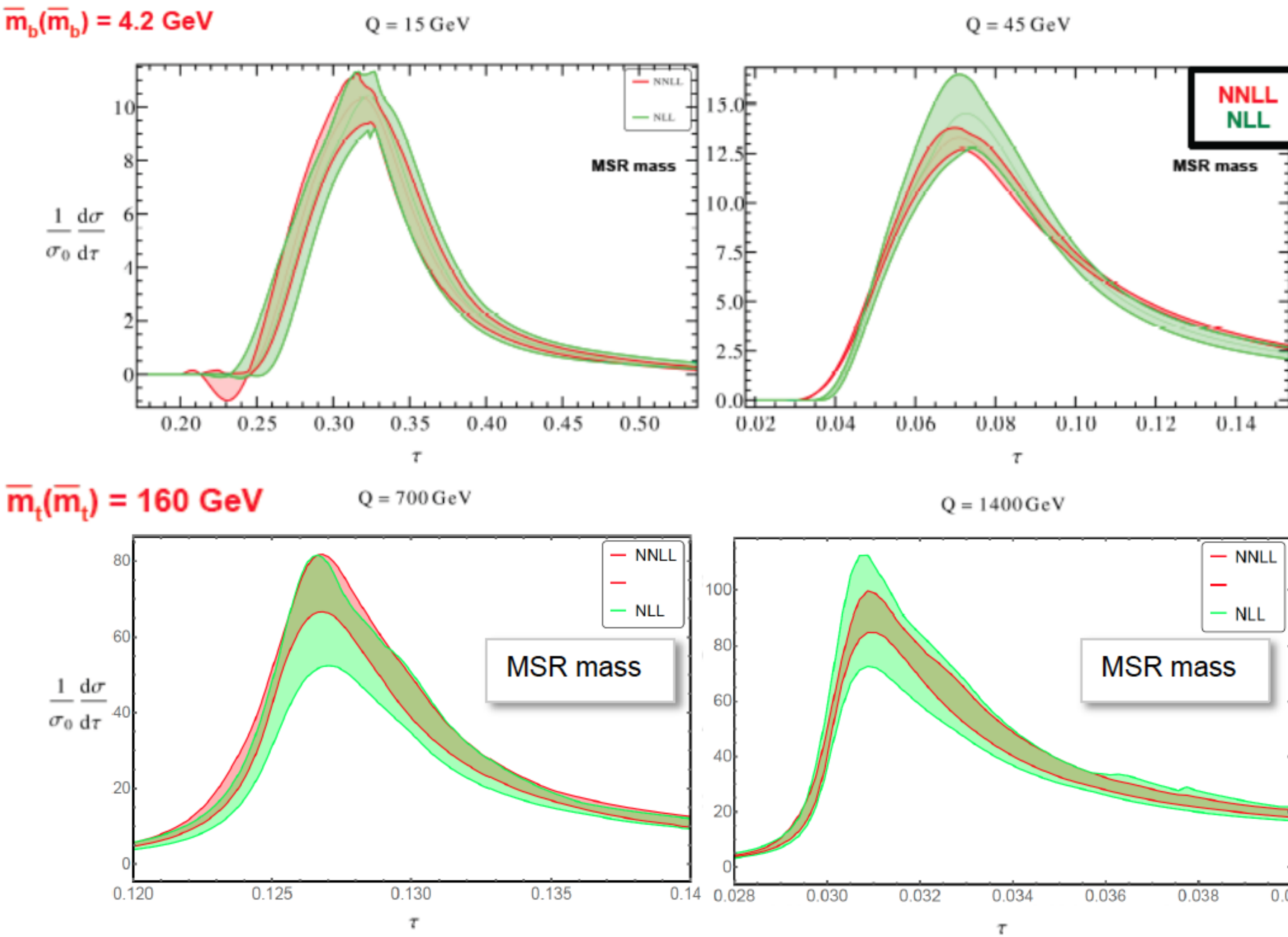
- Event shapes are a high-precision tool to extract the strong coupling.
- SCET allows for high-precision calculations.
- More $N^3LL + O(\alpha_s^3)$ analyses on the way:

  - ☑ Tail of thrust *[Abbate, Fickinger, Hoang, VM Stewart]*
  - ☑ Moments of thrust distribution *[Abbate, Fickinger, Hoang, VM Stewart]*
  - ☑ Tail of C-parameter *[Hoang, Kolodrubetz, VM Stewart]*
  - ☐ Tail of Heavy Jet Mass (w.i.p) *[Hoang, VM, Schwartz, Stewart]*
  - ☐ Moments of C-parameter (preliminary) and HJM

- Theory tools to describe data from B factories exists (VFNS for final state jets, fully quark mass dependent event shapes, full control over quark mass scheme): opportunity to learn more !
- B-factories: Please publish the measurements !
- P_T-dependent event shapes (broadening): Bern group
- Hadronic event shapes: intensely studied

Full Mass Dependent Event Shapes

Butenschön, Dehnadi, AHH, Mateu, Stewart, to be published soon



(preliminary)

(preliminary)

Full Mass Dependent Event Shapes

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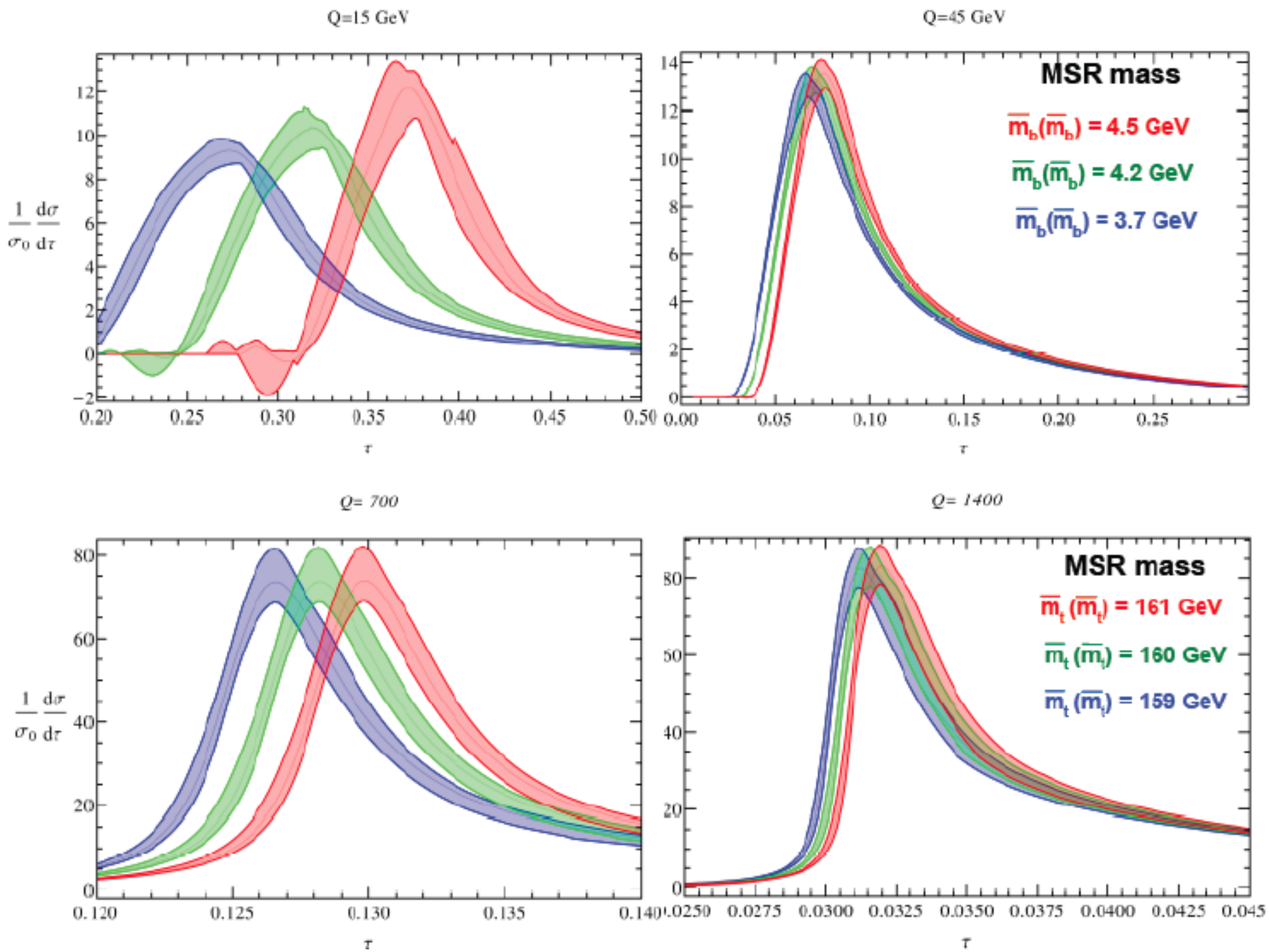

(preliminary)

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