Extraction of the Strong Coupling α_s Through W-boson Hadronic Decays

[Workshop on high-precision α_s measurements: From LHC to FCC-ee]

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October 2015

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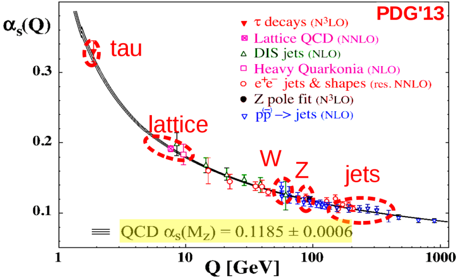
Motivation

Extraction of α_s through comparison of various experimental observables to different perturbative QCD predictions.



1. Hadronic τ decays: $R_\tau = \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = S_{\rm EW} N_C (1 + \sum_{n=1}^{4} c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_{np})$ ($\mathrm{N^3LO}$)
2. Lattice QCD: Various short-distance quantities: $\mathrm{K^{NP}} = \mathrm{K^{PT}} = \sum_{i=0}^{n} c_i \alpha_s^i$ (NNLO)
3. Hadronic Z decays: $R_Z = \frac{\Gamma(Z \to \text{hadrons})}{\Gamma(Z \to \ell)} = R_Z^{\rm EW} N_C (1 + \sum_{n=1}^{4} c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_{\rm m} + \delta_{np})$ ($\mathrm{N^3LO}$)
4. $e^+e^- \to q\bar{q}(g)$: Event-shapes, jet rates: $\frac{1}{\sigma}\frac{d\sigma}{dY} = \frac{dA}{dY}\hat{\alpha}_s + \frac{dB}{dY}\hat{\alpha}_s^2 + \frac{dC}{dY}\hat{\alpha}_s^3$ (NNLO)
5. $e^\pm p \to$ hadrons (PDF): $\sigma(\text{jet})$, $\frac{\partial}{\partial \ln Q^2} D_i^h(x, Q^2) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_s}{4\pi} P_{ji}(\frac{x}{z}, Q^2) D_j^h(z, Q^2)$ (NLO, NNLO)
6. $pp, p\bar{p} \to t\bar{t}$, jets (NNLO, NLO)
7. Hadronic W decays:
   - What is the theoretical and experimental status?
   - Can we extract α_s through hadronic W decays? With which precision?

Theoretical Calculation of $\Gamma_{\mathrm{W}}(\text{hadronic})$

The hadronic W-boson decay width has not been used so far for α_s extraction because:

- a complete N^{3}LO/NNLO formula with all computed corrections [1] was not available until recently [2] (albeit with a few approximations),
- the 2% relative experimental uncertainty on $\Gamma_W(\text{hadronic})$ was significantly large compared to 0.1% of $\Gamma_Z(\text{hadronic})$.

We recalculated $\Gamma_W(\text{hadronic})$ through implementation in *MATHEMATICA* the $\mathcal{O}(\alpha_s^4)$ or N^{3}LO formula using [2]:

$$\Gamma_{\mathrm{W}}(\text{hadronic}) = \sum_{i=0}^{4} \Gamma^{(i)}_{\mathrm{QCD}} + \Gamma^{(1)}_{\mathrm{EW}} + \Gamma^{(2)}_{\mathrm{Mixed}} = \frac{\sqrt{2} G_F m_W^3}{4\pi} \left[\sum_{i,j} |V_{i,j}|^2 \right] \left(1 + \sum_{k=1}^{4} \left(\frac{\alpha_s}{\pi} \right)^k + \delta_{\mathrm{EW}} + \delta_{\mathrm{Mixed}} \right),$$

where

- $\Gamma^{(k)}_{\mathrm{QCD}}$ is the leading order decay width and QCD corrections of order $\mathcal{O}(\alpha_s^k)$ and $k = 1, \ldots, 4$,
- $\Gamma^{(1)}_{\mathrm{EW}}$ electroweak corrections of order $\mathcal{O}(\alpha)$,
- $\Gamma^{(2)}_{\mathrm{Mixed}}$ mixed corrections of order $\mathcal{O}(\alpha \alpha_s)$.

[1] - A. Denner, B. Kniehl, J. Kühn, K. Chetyrkin, . . .

[2] - D. Kara, Nucl. Phys. B 877, 3 (2013)

Improvements with Respect to Previous Γ_W(hadronic) Calculations

In our calculations we carry out the following improvements compared to previous works:

1. implement finite quark masses in the dominant Γ_W(hadronic) terms: Born and first-order QCD corrections,
2. use NNLO α_s running instead of LO (between m_W and m_Z),
3. use current PDG world average values for parameters of the Standard Model (α_{QED}, G_{F}, m_{q}, m_ℓ, m_{W}, m_{Z}, m_{H}, *CKM matrix elements* $|V_{i,j}|$),
4. determination of associated theoretical and parametric uncertainties.

Hadronic W-boson Decay Width - Numerical Results

Previous state-of-the-art [2]: $\Gamma_{\mathrm{W}}(\text{hadronic}) = (1458.820 \pm 6 \times 10^{-3})\ \mathrm{MeV}$,

$$\Gamma_{\mathrm{W}}(\text{hadronic}) = (1428.803 \pm 0.030_{\mathrm{theor.}} \pm 22.608_{\mathrm{param.}})\ \mathrm{MeV},$$

$$\Gamma_{\mathrm{W}}(\text{hadronic}, V_{ij}V_{kj} = \delta_{ik}) = (1411.546 \pm 0.030_{\mathrm{theor.}} \pm 0.742_{\mathrm{param.}})\ \mathrm{MeV}.$$

Partial width	$\Gamma^{(0)}_{\mathrm{QCD}}$	$\Gamma^{(1)}_{\mathrm{QCD}}$	$\Gamma^{(2)}_{\mathrm{QCD}}$	$\Gamma^{(3)}_{\mathrm{QCD}}$	$\Gamma^{(4)}_{\mathrm{QCD}}$	$\Gamma^{(1)}_{\mathrm{EW}}$	$\Gamma^{(2)}_{\mathrm{Mixed}}$
Γ_W(hadronic) of [2]	1408.980	54.087	2.927	-1.018	-0.245	-5.132	-0.779
Γ_W(hadronic)	1379.851	53.080	2.873	-1.000	-0.241	-5.002	-0.757
$\Gamma_W(\text{hadronic}, V_{ij}V_{kj} = \delta_{ik})$	1363.186	52.439	2.838	-0.988	-0.238	-4.942	-0.749

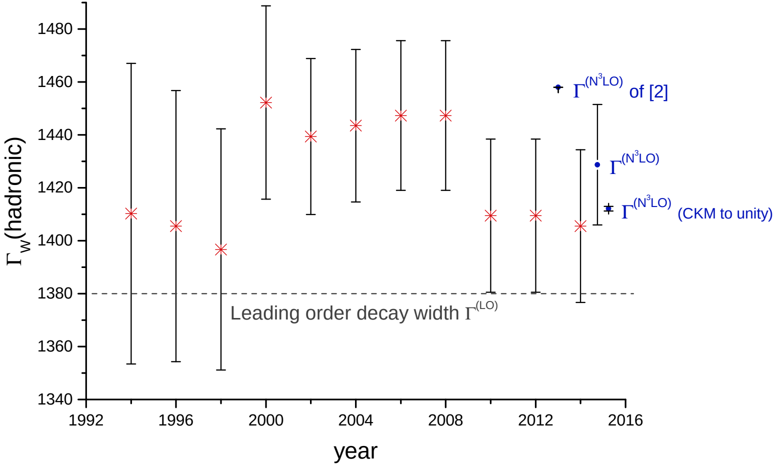
Numerical values of the partial decay widths. All values given in MeV.

The following uncertainties are present in Γ_{W}(hadronic):

- parametric uncertainty (modifying all PDG parameters by $\pm\sigma$, adding changes in quadrature)
  - ± 22.608 MeV (the dominant parametric uncertainty is V_{cs}),
  - ± 0.742 MeV for $V_{ij}V_{kj} = \delta_{ik}$ (dominated by m_W),
- higher-order corrections (assumed equal to Γ_Z, from N^{3}LO to N^4LO): ± 0.0195 MeV,
- non-perturbative uncertainties (assuming power-corrections of order $\mathcal{O}\left(\frac{\Lambda^4}{m_W^4}\right)$): $\pm 7 \times 10^{-8}$ MeV,
- finite quark masses beyond LO corrections: ± 0.0042 MeV,
- mixed corrections from [2]: ± 0.006 MeV.

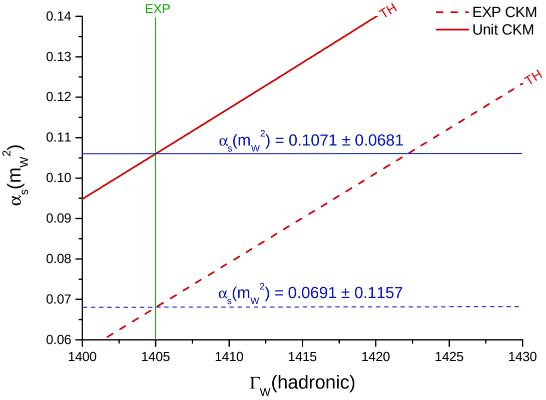
Γ_W(hadronic) : Historical Data Versus Theory

Evolution of the PDG world average value of $\Gamma_{\mathrm{W}}^{\mathrm{EXP}}$(hadronic) (current value is $(1405 \pm 29)\ \mathrm{MeV}$) by year compared to theoretically predicted decay widths.



Extraction of α_s from Γ_W(hadronic)

Using the $\mathcal{O}(\alpha_s^4)$ W-boson decay width formula we can extract α_s by comparing it to the experimental value which is $\Gamma_{\mathrm{W}}^{\mathrm{EXP}}$(hadronic) $= (1405 \pm 29)$ MeV.



$\Rightarrow$ Current large parametric ($\pm$ 23 MeV) and experimental ($\pm$29 MeV) uncertainties on Γ_{W}(hadronic) propagate into a huge α_s uncertainty $\sim$ 60%.

Experimental priorities should be:

- measure $|V_{cs}|$ with better precision (current 1.6%),
- significantly reduce uncertainty of Γ_W(hadronic) measurement to a few MeV,
- reduce m_W uncertainty (now it propagates to $\pm$0.8 MeV on Γ_W(hadronic)).

	$\alpha_s(m_W^2)$	$\alpha_s(m_Z^2)$
Experimental CKM	$0.0691 \pm 0.0513_{\text{param.}} \pm 0.0644_{\text{exp.}}$	$0.0683 \pm 0.0509_{\text{param.}} \pm 0.0638_{\text{exp.}}$
Unit CKM, $V_{ij}V_{kj} = \delta_{ik}$	$0.1071 \pm 0.0017_{\text{param.}} \pm 0.0664_{\text{exp.}}$	$0.1053 \pm 0.0016_{\text{param.}} \pm 0.0657_{\text{exp.}}$

Alternative Approach: α_s Extraction via $\mathrm{BR}_W(\text{hadronic}) = \frac{\Gamma_W(\text{hadronic})}{\Gamma_W(\text{total})}$

Γ_W(hadronic) has a 2% experimental uncertainty, BR_W(hadronic) has an uncertainy of 0.4%.

$$\Rightarrow \text{ We try } \mathrm{BR}_W(\text{hadronic}) = \frac{\Gamma_W(\text{hadronic})}{\Gamma_W(\text{total})} \text{ to extract } \alpha_s \text{ instead of } \Gamma_W(\text{hadronic}).$$

For the total decay width Γ_W(total) we use the ZFitter NNLO (includes up to $\mathcal{O}(\alpha_s^3)$ QCD, $\mathcal{O}(\alpha)$ electroweak and $\mathcal{O}(\alpha\alpha_s)$ mixed corrections) fitted result by [3] which is parametrized as

$$\Gamma_{\mathrm{W}}(\text{total}) = G_W^0 m_W^3 \quad \text{and} \quad G_W^0 = 4.0279 \times 10^{-6}(1 + 0.00095x_H - 0.0024x_H^2 + 0.0016x_H^3 + 000065x_s)\ \mathrm{GeV}^{-2},$$

where $x_s = f(\alpha_s)$, $x_H = f(m_H)$.

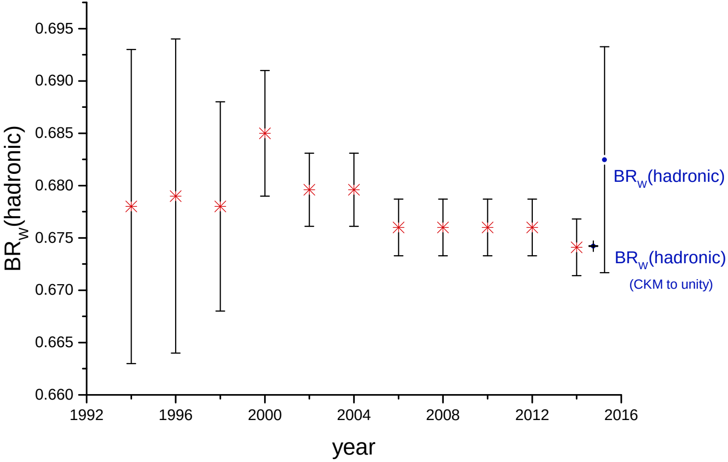
We also computed the associated parametric uncertainties as done for Γ_W(hadronic).

Γ_W(hadronic)	(1428.803 ± 22.638) MeV
$\Gamma_W(\text{hadronic}, V_{ij}V_{kj} = \delta_{ik})$	(1411.546 ± 0.772) MeV
Γ_W(total)	$(2093.591 \pm 1.172_{\text{param.}})$ MeV
BR_W(hadronic)	$0.6825 \pm 0.0108_{\text{param.}}$
$\mathrm{BR}_W(\text{hadronic}, V_{ij}V_{kj} = \delta_{ik})$	$0.67422 \pm 0.00003_{\text{param.}}$
$\mathrm{BR}_W(\text{hadronic})^{\mathrm{EXP}}$	0.6741 ± 0.0027

[3] - G. C. Cho, K. Hagiwara, Y. Matsumoto and D. Nomura, JHEP 111, 068 (2011)

BR_W(hadronic) : Historical Data Versus Theory

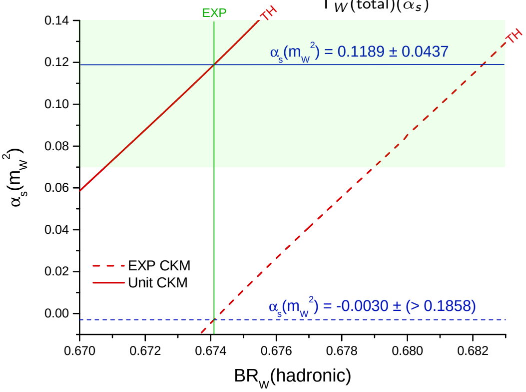
Evolution of the PDG world average value of BR_W(hadronic) (current value is 0.6741 ± 0.0027) by year compared to theoretically predicted decay widths.



Alternative Approach: α_s Extraction via $\text{BR}_W(\text{hadronic}) = \frac{\Gamma_W(\text{hadronic})}{\Gamma_W(\text{total})}$

We can extract α_s by comparing the theoretical hadronic branching ratio formula to the experimental world average value

$$\frac{\Gamma_W(\text{hadronic})(\alpha_s)}{\Gamma_W(\text{total})(\alpha_s)} = \text{BR}_W(\text{hadronic})^{\text{EXP}}.$$



$\Rightarrow$ Setting the CKM matrix to unit matrix instead of using experimental values, we can extract α_s with 35% uncertainy.

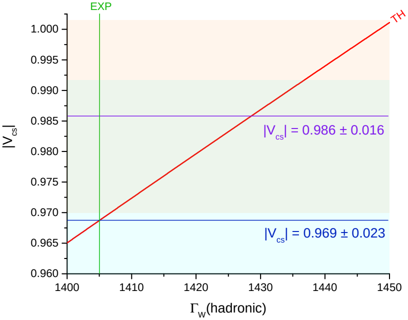
To extract α_s with a higher precision:

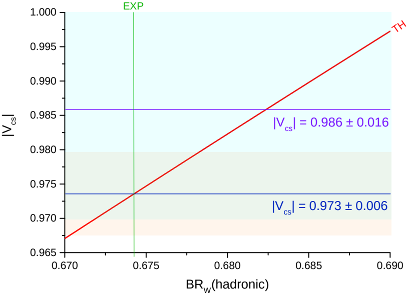
- reduce the uncertainty of $|V_{cs}|$ as mentioned earlier,
- reduce the uncertainty of m_W (measured to 0.02%) which becomes dominant once the $|V_{cs}|$ uncertainty is reduced below 0.05%,
- measure $\text{BR}_W(\text{hadronic})$ with a better precision than today (0.4% now).

	$\alpha_s(m_W^2)$	$\alpha_s(m_Z^2)$	$\Delta\alpha_s(m_W^2)$
Experimental CKM	$-0.0030 \pm (> 0.1505_{\text{param.}}) \pm 0.0353_{\text{exp.}}$	/	/
Unit CKM	$0.1189 \pm 0.0004_{\text{param.}} \pm 0.0433_{\text{exp.}}$	$0.1167 \pm 0.0004_{\text{param.}} \pm 0.0430_{\text{exp.}}$	$\pm 37\%$

Extra: Indirect Determination of $|V_{cs}|$

The large experimental $\mathrm{BR}_W(\text{hadronic})^{\mathrm{EXP}}$ uncertainty precludes an accurate extraction of α_s, but we can use $\mathrm{BR}_W(\text{hadronic})^{\mathrm{EXP}}$ to determine $|V_{cs}|$ (fixing α_s to world average).





Extraction method	$|V_{cs}|$ value
$\Gamma_W(\text{hadronic})$	$0.969 \pm 0.002_{\mathrm{param.}} \pm 0.021_{\mathrm{exp.}}$
$\mathrm{BR}_W(\text{hadronic})$	$0.973 \pm 0.002_{\mathrm{param.}} \pm 0.004_{\mathrm{exp.}}$

$\Rightarrow$We can extract $|V_{cs}|$ with an uncertainty of 0.6% compared to 1.6% of the experimental measurement $|V_{cs}|^{\mathrm{EXP}} = 0.986 \pm 0.016$.

Future Perspectives: LHC & FCC-ee

To determine α_s with a higher precision we need more precise measurement of Γ_W(total) and/or BR_W(hadronic) with reduced uncertainties.

- Uncertainties at LHC (5×10^5 high-m_T W's at $\sqrt{s} = 8$ TeV, 20 fb^{-1}):
  -Statistical: $\sim$ 3 MeV (30 MeV at Tevatron, with 5×10^3 high-m_T W's)
  -Systematics: $\sim$ 15 MeV (down from $\sim$ 40 MeV at Tevatron, reduced PDF uncertainties)

  Improved result: Γ_W(hadronic) $\sim (1429 \pm 12)$ MeV (i.e. 0.8% uncertainty instead of 2%)

	$\alpha_s(m_W^2)$	$\Delta\alpha_s(m_W^2)$
Unit CKM	$0.1208 \pm 0.0004_{\mathrm{param.}} \pm 0.0275_{\mathrm{exp.}}$	$\pm 23\%$

$\Rightarrow$Improved Γ_W(hadronic) at LHC allows to extract α_s with $\sim$ 23% uncertainty.

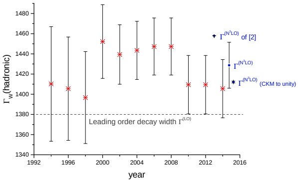
- Uncertainties at FCC-ee (5×10^8 W's at $\sqrt{s} = m_W$):
  -Statistical: $\sim$ 0.005% (0.4% at LEP with 8×10^4 W's)

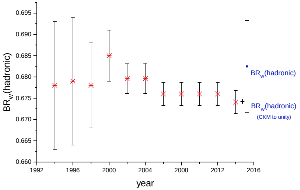
  Final result: BR_W(hadronic) $\sim 0.67410 \pm 0.00003$

	$\alpha_s(m_W^2)$	$\Delta\alpha_s(m_W^2)$
Unit CKM	$0.1208 \pm 0.0004_{\mathrm{exp.}}$	$\pm 0.3\%$

$\Rightarrow$ FCC-ee would provides us a value for α_s with a relative uncertainty of $\sim$ 0.3%.

Summary

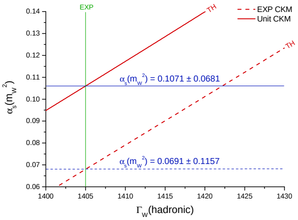


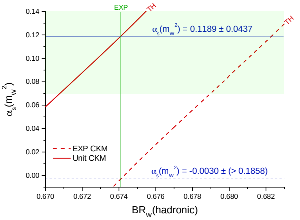


⇒We computed $\Gamma_W(\text{hdronic})$ and $\text{BR}_W(\text{hadronic})$ using state-of-the-art $\text{N}^3\text{LO}/\text{NNLO}$ calculations, removing some of the previously applied approximations:

$$\Gamma_W(\text{hadronic}) = (1411.546 \pm 0.772)\ \text{MeV},$$

$$\text{BR}_W(\text{hadronic}) = 0.67422 \pm 0.00003.$$





⇒Current experimental $\Gamma_W(\text{hadronic})$, $\text{BR}_W(\text{hadronic})$ and $|V_{cs}|$ uncertainties preclude precise extraction of α_s.

⇒Improvements at LHC, and in particular FCC-ee, will allow one to incorporate $\Gamma_W(\text{hadronic})$ and $\text{BR}_W(\text{hadronic})$ into the PDG α_s.

	$\alpha_s(m_W^2)$	$\alpha_s(m_Z^2)$	$\Delta\alpha_s(m_W^2)$
Today (Unit CKM)	$0.1189 \pm 0.0004_{\text{param.}} \pm 0.0433_{\text{exp.}}$	$0.1167 \pm 0.0004_{\text{param.}} \pm 0.0430_{\text{exp.}}$	$\pm 37\%$
LHC (Unit CKM)	$0.1208 \pm 0.0004_{\text{param.}} \pm 0.0271_{\text{exp.}}$	$0.1185 \pm 0.0004_{\text{param.}} \pm 0.0260_{\text{exp.}}$	$\pm 23\%$
FCC-ee	$0.1208 \pm 0.0004_{\text{exp.}}$	$0.1185 \pm 0.0004_{\text{exp.}}$	$\pm 0.3\%$

⇒FCC-ee will allow us to measure α_s with $\sim$ 0.3% uncertainty.

Thank you. Any questions?

Backup slides

Experimental Hadronic Width Measurement

- Experimental hadronic Γ_{W}(hadronic) from Γ_{W}(total) via BR_W(hadronic)[2]:

$$\begin{aligned}\Gamma_{\mathrm{W}}(\text{hadronic}) &= \Gamma_{\mathrm{W}}(\text{total}) \times \mathrm{BR}_W(\text{hadronic}) \\ &= (2085 \pm 42) \times (0.6741 \pm 0.0027)\ \mathrm{MeV} \\ &= (1405 \pm 29)\ \text{MeV (i.e. 2\% uncertainty)}\end{aligned}$$

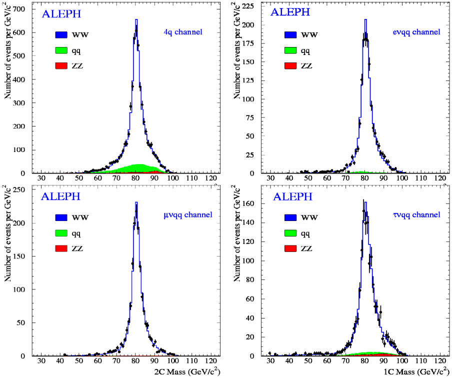
- Γ_{W}(total) in $e^+e^- \to W^+W^- \to 4q, 2q + \ell\nu$
  Where: LEP ($\sqrt{s} = 161 - 209$ GeV) & FCC-ee ($\sqrt{s} = 161, 240, 350$ GeV)
  How: Maximum-likelihood fit of m_{W} Breit-Wigner with Γ_{W}(total) as free parameter.

- Γ_{W}(total) in $p\bar{p}$, $pp \to W + X$, with $W \to e\nu$, $\mu\nu$
  Where: Tevatron($\sqrt{s} = 1.8, 1.96$ TeV) & LHC($\sqrt{s} = 7, 8, 13, 14$ TeV)
  How: Maximum likelihood fit of high $m_{\mathrm{T}}(W)$ tail with Γ_{W}(total) as free parameter (and via $\sigma(W)/\sigma(Z)$ ratios).

[2] 0.4 % uncertainty measured in e^+e^-

LEP-2 W-boson Width Measurement [arXiv:1302.3415]

- $e^+e^- \to W^+W^- \to 4q, 2q + \ell\nu$ at $\sqrt{s} = 161 - 209$ GeV
- Statistics:
  N(W's in all channels / experiments) $\sim$ 40.000 pairs
- Binned likelihood fit to m_W Breit-Wigner with Γ_W(total) as free parameter:



Source	Systematic Uncertainty in MeV			
	on m_W			on Γ_W
	$q\bar{q}\ell\nu_\ell$	$q\bar{q}q\bar{q}$	Combined	
ISR/FSR	8	5	7	6
Hadronisation	13	19	14	40
Detector effects	10	8	9	23
LEP energy	9	9	9	5
Colour reconnection	−	35	8	27
Bose-Einstein Correlations	−	7	2	3
Other	3	10	3	12
Total systematic	21	44	22	55
Statistical	30	40	25	63
Statistical in absence of systematics	30	31	22	48
Total	36	59	34	83

- Final result: $\Gamma_W(\text{total}) = (2495 \pm 63_{\text{stat.}} \pm 55_{\text{syst.}})$ MeV

CDF W-boson Width Measurement [PRL 100, 071801 (2008)]

- $p\bar{p} \to W + X$, with $W \to e\nu, \mu\nu$ at $\sqrt{s} = 1.96$ TeV ($L_{int.} \sim 350\ pb^{-1}$)
- Statistics:
  N(W's)=3.436+2.619 with $90 < M_T < 200$ GeV
- Binned likelihood fit to ($90 < M_T < 200$ GeV) spectra with Γ_W(total) as free parameter:

$$M_T = \sqrt{2(p_T^\ell p_T^\nu - \vec{p}_T^{\,\ell} \cdot \vec{p}_T^{\,\nu})}$$

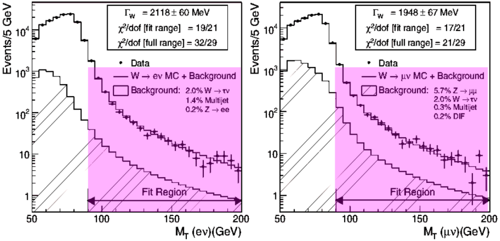


TABLE I. The sources of uncertainty (in MeV) on Γ_W for the $W \to e\nu$ and $W \to \mu\nu$ measurements. If there is a correlated source of error between the two measurements its contribution to each measurement is listed in the third column, labeled C.

Source	$\Delta\Gamma_W^{e\nu}$	$\Delta\Gamma_W^{\mu\nu}$	C
Statistics	60	67	
Lepton E or p scale	21	17	12
Lepton E or p resolution	31	26	
Electron energy loss simulation	13		
Recoil model	54	49	
p_T^W	7	7	7
Backgrounds	32	33	
PDFs	20	20	20
M_W	9	9	9
EW radiative corrections	10	6	6
Lepton ID/acceptance	10	7	
Total systematic	79	71	27
Total (statistic + systematic)	99	98	27

- Final result: $\Gamma_\mathrm{W}(\text{total}) = (2032 \pm 45_{\text{stat.}} \pm 57_{\text{syst.}})$ MeV

Tevatron W-boson Width Combined [arXiv: 1003.2826]

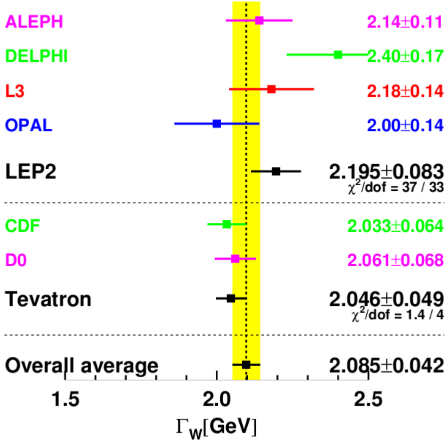
- CDF + D0 combined (BLUE method) $p\bar{p} \to W + X$, with $W \to e\nu, \mu\nu$ at 1.8 and 1.96 TeV.
- Improved likelihood fits to M_T spectra with updated underlying parameters:

	Run-I			Run-II	
	CDF-Ia	CDF-Ib	D0-Ib	CDF	D0
Γ_W (published)	2,110	2,042.5	2,231	2,032	2,028.3
Total uncertainty (published)	329	138.3	172.8	72.4	72
M_W used in publication	80,140	80,400	80,436	80,403	80,419
Correction to Γ_W from M_W	−78	0.3	11.1	1.2	6.0
Γ_W (corrected)	2,032	2,042.8	2,242.1	2,033.2	2,034.3
Total uncertainty(corrected)	329.3	138.3	172.4	72.4	71.9
Uncorrelated uncertainty (corrected)	327.6	136.8	167.4	68.7	68.5
PDF uncertainty(published)	0	15	39	20	20
PDF uncertainty (this analysis)	15	15	39	20	20
EWK RC uncertainty	28	10	10	6	7
M_W uncertainty (published)	0	10	15	9	5
M_W uncertainty (this analysis)	7	7	7	7	7
M_W extrapolation	26	0	4	0	2

- Final result: $\Gamma_W(\text{total}) = (2046 \pm 49)$ MeV ± 39 MeV (stat.) $\pm(20+7.4+7.4)$ MeV(PDF + m_W+EW. corr.)

W-boson Width: Tevatron + LEP Combined [PDG]

- World average of all LEP + Tevatron measurements:



Final result: $\Gamma_W(\text{total}) = (2.085 \pm 0.042)$ GeV (i.e. 2% uncertainty)

Note: $\Gamma_Z(\text{total}) = (2.4952 \pm 0.0023)$ GeV (i.e. 0.1 % uncertainty)

List of References

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