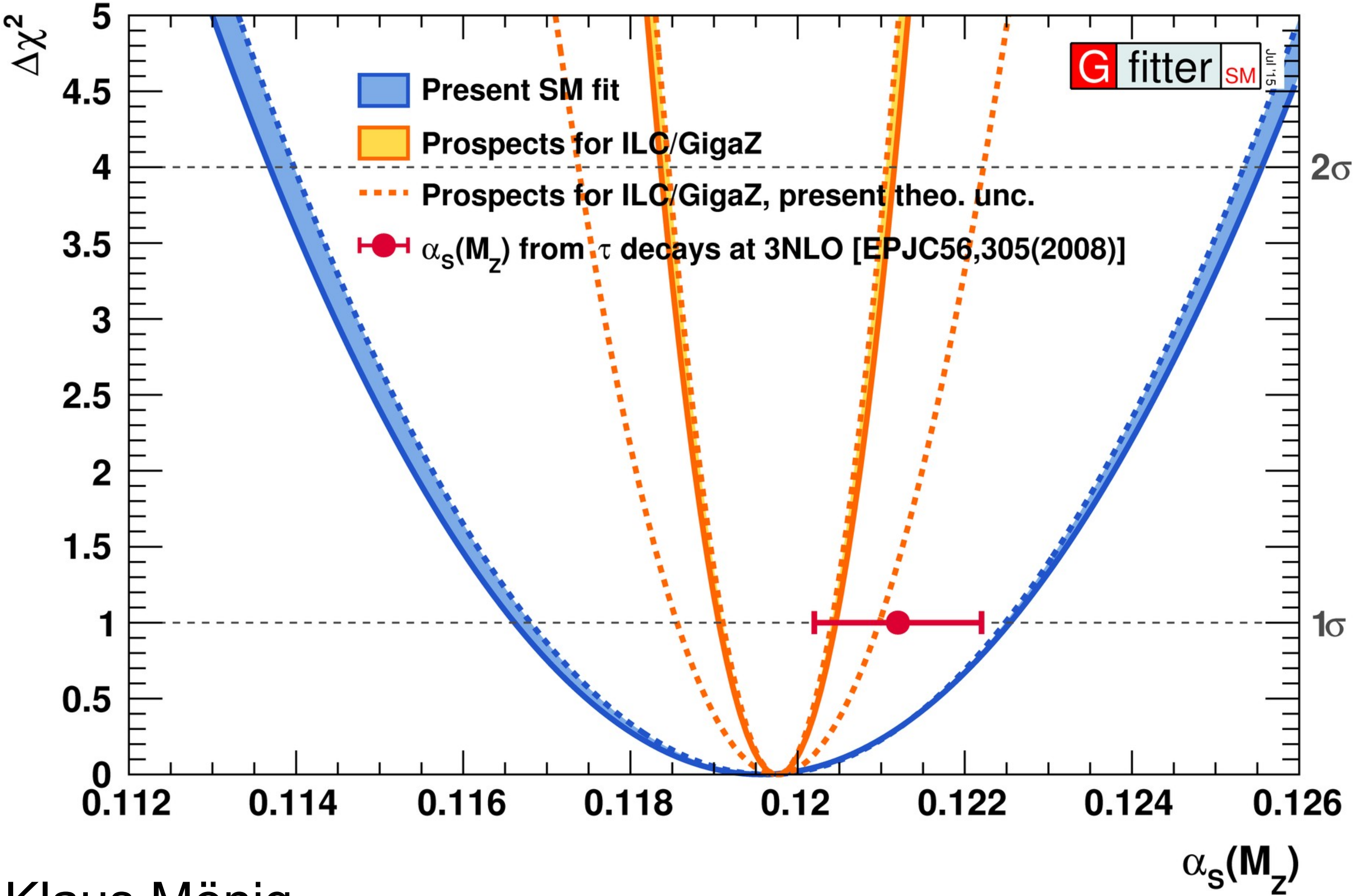
α_s from Z decays and from the full electroweak fit



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Introduction

- The hadronic width of the Z receives (almost) the same QCD corrections as $e^+e^- \rightarrow$ hadrons

$$\Gamma_{\mathrm{had}} = \Gamma_{\mathrm{had,\,no\,QCD}} \left(1 + \frac{\alpha_s}{\pi} + ...\right)$$

- In the Z observables Γ_{had} enters in several ways:

$$\begin{aligned}
\Gamma_Z &= 3\Gamma_\ell + 3\Gamma_\nu + \Gamma_{\mathrm{had}} \\
R_\ell^0 &= \frac{\Gamma_{\mathrm{had}}}{\Gamma_\ell} \\
\sigma_{\mathrm{had}}^0 &= \frac{12\pi}{m_Z} \frac{\Gamma_e \Gamma_{\mathrm{had}}}{\Gamma_Z^2} \\
\sigma_\ell^0 &= \frac{12\pi}{m_Z} \frac{\Gamma_\ell^2}{\Gamma_Z^2}
\end{aligned}$$

(From the last 3 only 2 are statistically independent)

Definition of observables

- All LEP observables are pseudo-observables, i.e. they are corrected for initial state radiation, photon exchange and γ-Z interference
- The observables are defined as
  - Γ_Z : total width of the Z boson
  - $\sigma^0_{\mathrm{had}(\ell)}$: hadronic (leptonic) pole cross section
  - R^0_ℓ :ratio of hadronic to leptonic pole cross section (almost identical to ratio of hadronic to leptonic event count)
- They are extracted from a fit to the Z-lineshape
- However $\sigma^0_{\mathrm{had}(\ell)}$ and R^0_ℓ can basically be obtained from the Z-peak only

Introduction (ii)

- Due to cancellations and squares the sensitivities are different:

$$\begin{aligned}
\Delta\alpha_s(m_Z) &\approx 4.4\,\Delta\Gamma_Z/\Gamma_Z \\
\Delta\alpha_s(m_Z) &\approx 3.1\,\Delta R^0_\ell/R^0_\ell \\
\Delta\alpha_s(m_Z) &\approx 7.4\,\Delta\sigma^0_{\mathrm{had}}/\sigma^0_{\mathrm{had}} \\
\Delta\alpha_s(m_Z) &\approx 2.2\,\Delta\sigma^0_\ell/\sigma^0_\ell
\end{aligned}$$

- Experimentally relevant quantities
  - very accurate beam energy for Γ_Z
  - relative event counting (hadrons) for Γ_Z
  - absolute event counting (leptons and hadrons) for cross sections and R
  - relative luminosity for Γ_Z
  - absolute luminosity for cross sections

α_s and electroweak corrections

- The partial widths are given by $\Gamma_f \propto |g_{Af}|^2 + |g_{Vf}|^2$
  with:

$$g_{A,f} = \sqrt{1 + \Delta\rho_f}\, i_3$$
$$g_{Vf} = g_{Af}\left(1 - 4|Q_f|\sin^2\theta_{\rm eff}^f\right)$$

- $\Delta\rho, \sin^2\theta_{\rm eff}$ depend on m_t, m_H and new physics
- the flavour dependence is independent of new physics and can be ignored here
- For the Z observables this means:
  - Γ_Z receives the full corrections from $\Delta\rho$
  - for the other observables $\Delta\rho$ cancels completely and $\sin^2\theta_{\rm eff}$ partially

Extraction of α_s

- The experiments provide Γ_Z, R^0_ℓ, $\sigma^0_{\rm had}$ with the full covariance matrix
- In principle this is sufficient to measure α_s
- However normally α_s is extracted from a global fit to all electroweak observables
- For the electroweak observables assumptions must be made:
  - if the SM is assumed $\Delta\rho, \sin^2\theta_{\rm eff}$ can be calculated from m_t, m_H and all α_s-sensitive observables can be used
  - if new physics is allowed in the most general way $\sin^2\theta_{\rm eff}$ can be obtained from the asymmetries and Γ_Z receives ~zero weight for α_s because it is needed to constrain $\Delta\rho$
  - there are intermediate possibilities eg. assuming U=0 in an STU framework adding some constraints on $\Delta\rho$ giving some weight back to Γ_Z

Present data

- Main ingredient: Z-scan from LEP $\Rightarrow \Gamma_Z,\ \sigma^0_{\mathrm{had}},\ R^0_\ell$ (+ correlation matrix)
- Parameters to calculate the partial widths on Born level $\alpha_{QED},\ G_F,\ m_Z$
- Observables to fix the loop level predictions for the partial widths:
  - asymmetries from LEP and SLD ($\Rightarrow \sin^2\theta_{\mathrm{eff}}$) and W-mass from LEP, Tevatron
  - direct measurements of m_H, m_t from LHC, Tevatron together with an input value for $\alpha_{\mathrm{QED}}(m_Z)$

Present Data (ii)

Parameter	Input value	Free in fit	Fit Result	w/o exp. input in line	w/o exp. input in line, no theo. unc
M_H [GeV]$^{(\circ)}$	125.14 ± 0.24	yes	125.14 ± 0.24	93^{+25}_{-21}	93^{+24}_{-20}
M_W [GeV]	80.385 ± 0.015	–	80.364 ± 0.007	80.358 ± 0.008	80.358 ± 0.006
Γ_W [GeV]	2.085 ± 0.042	–	2.091 ± 0.001	2.091 ± 0.001	2.091 ± 0.001
M_Z [GeV]	91.1875 ± 0.0021	yes	91.1880 ± 0.0021	91.200 ± 0.011	91.2000 ± 0.010
Γ_Z [GeV]	2.4952 ± 0.0023	–	2.4950 ± 0.0014	2.4946 ± 0.0016	2.4945 ± 0.0016
σ^0_{had} [nb]	41.540 ± 0.037	–	41.484 ± 0.015	41.475 ± 0.016	41.474 ± 0.015
R^0_ℓ	20.767 ± 0.025	–	20.743 ± 0.017	20.722 ± 0.026	20.721 ± 0.026
$A^{0,\ell}_{\mathrm{FB}}$	0.0171 ± 0.0010	–	0.01626 ± 0.0001	0.01625 ± 0.0001	0.01625 ± 0.0001
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	–	0.1472 ± 0.0005	0.1472 ± 0.0005	0.1472 ± 0.0004
$\sin^2\theta^\ell_{\mathrm{eff}}(Q_{\mathrm{FB}})$	0.2324 ± 0.0012	–	0.23150 ± 0.00006	0.23149 ± 0.00007	0.23150 ± 0.00005
A_c	0.670 ± 0.027	–	0.6680 ± 0.00022	0.6680 ± 0.00022	0.6680 ± 0.00016
A_b	0.923 ± 0.020	–	0.93463 ± 0.00004	0.93463 ± 0.00004	0.93463 ± 0.00003
$A^{0,c}_{\mathrm{FB}}$	0.0707 ± 0.0035	–	0.0738 ± 0.0003	0.0738 ± 0.0003	0.0738 ± 0.0002
$A^{0,b}_{\mathrm{FB}}$	0.0992 ± 0.0016	–	0.1032 ± 0.0004	0.1034 ± 0.0004	0.1033 ± 0.0003
R^0_c	0.1721 ± 0.0030	–	$0.17226^{+0.00009}_{-0.00008}$	0.17226 ± 0.00008	0.17226 ± 0.00006
R^0_b	0.21629 ± 0.00066	–	0.21578 ± 0.00011	0.21577 ± 0.00011	0.21577 ± 0.00004
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	yes	$1.27^{+0.07}_{-0.11}$	–	–
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	yes	$4.20^{+0.17}_{-0.07}$	–	–
m_t [GeV]	173.34 ± 0.76	yes	$173.81 \pm 0.85^{(\bigtriangledown)}$	$177.0^{+2.3}_{-2.4}{}^{(\bigtriangledown)}$	177.0 ± 2.3
$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)^{(\dagger\triangle)}$	2757 ± 10	yes	2756 ± 10	2723 ± 44	2722 ± 42
$\alpha_s(M_Z^2)$	–	yes	0.1196 ± 0.0030	0.1196 ± 0.0030	0.1196 ± 0.0028

Theory inputs

- The QCD corrections to Γ_q and derived quantities are known to $\mathcal{O}(\alpha_s^4)$ (Baikov et al. arxiv:1201.5804, see talk by H. Kühn in this session)
- Electroweak corrections to Γ_f are known to 2nd order for the fermionic corrections plus some higher order terms (A. Freytas arxiv:1310.2256)
- $\sin^2\theta_{\text{eff}}$ is known to full 2-loop order with leading 3- and 4-loop corrections $\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}((\alpha m_t)^2\alpha_s)$, $\mathcal{O}((\alpha m_t)^3)$, $\mathcal{O}(\alpha m_t\alpha_s^3)$ (Awramik et al. hep-ph/0608099)
- m_W is known to the same precision as $\sin^2\theta_{\text{eff}}$ (Awramik et al. hep-ph/0608099)

Theory errors

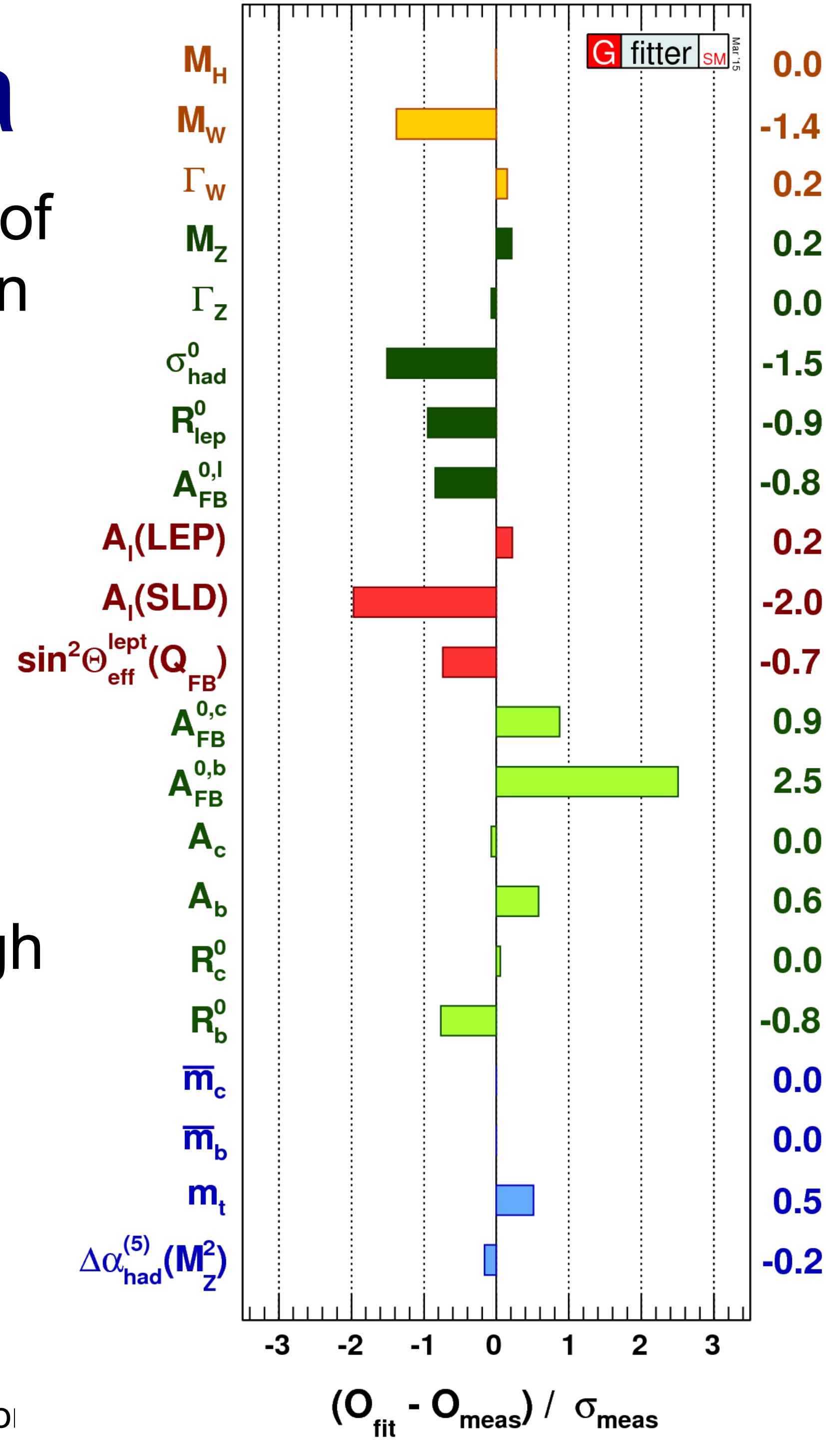
- The errors on the electroweak parameters are taken from the original literature
- They are estimated from missing higher order corrections

observable	error	% of exp error	source
R_ℓ^0	$5 \cdot 10^{-3}$	20	arXiv:1401.2447
$\sigma_{\rm had}^0$	$6\,\mathrm{pb}$	16	
Γ_Z	$0.5\,\mathrm{MeV}$	17	
$\sin^2\theta_{\rm eff}$	$4.7 \cdot 10^{-5}$	29	hep-ph/0608099

- For the QCD series we assign the full $O(\alpha_s{}^4)$ term as error
- For the top mass from direct reconstruction we assign 500 MeV error because of the mass definition

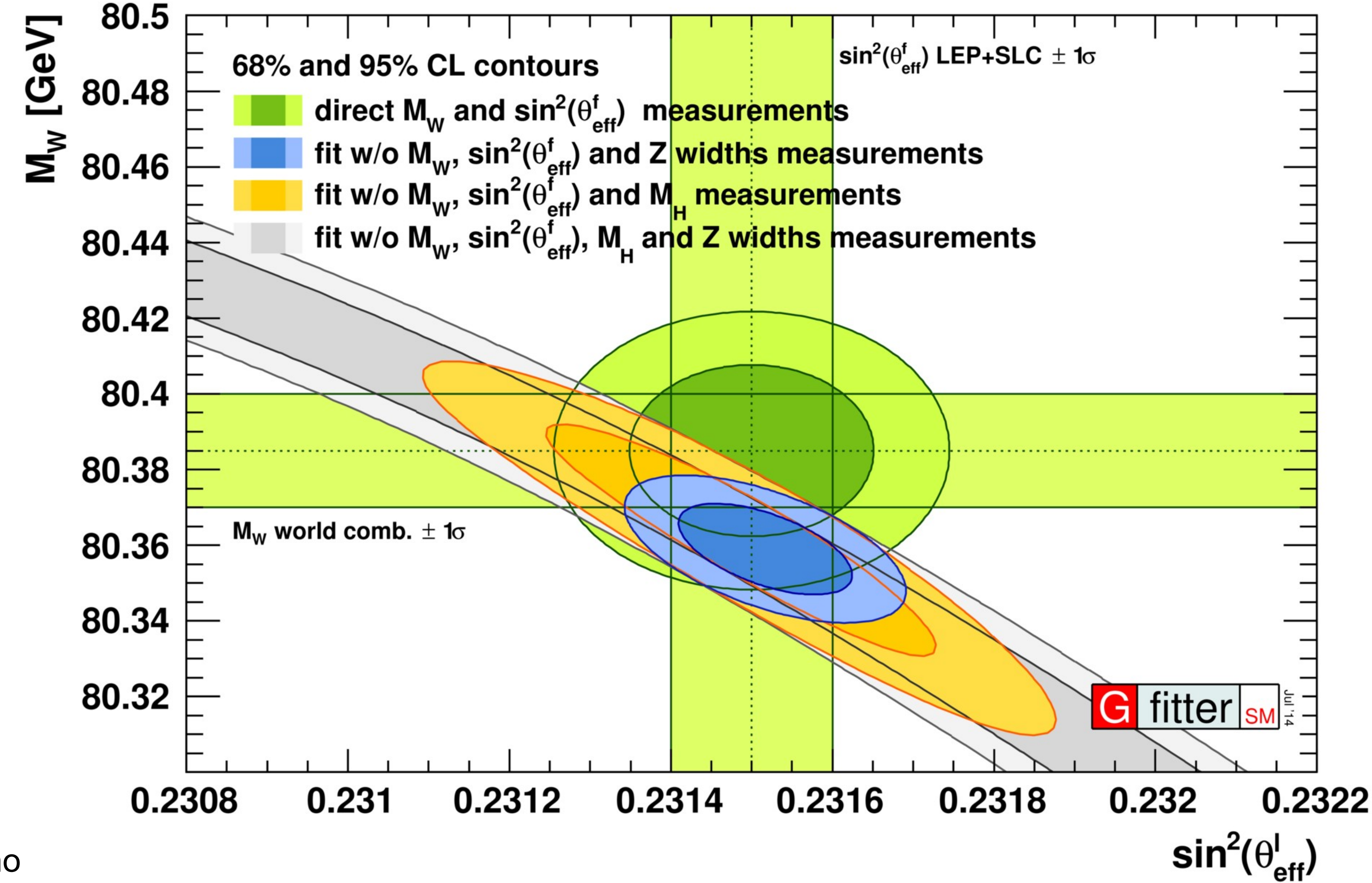
Global fit to data

- A global fit shows the consistency of all data and validates the extraction of Standard Model parameters
- After the Higgs discovery α_s is the only real free parameter of the fit
- The data are consistent with a χ^2/ndf=17.8/14 (Prob=22%)
- This also means that the different assumptions for the α_s extraction will give consistent results, although with slightly different errors



A note on the fit

- At present the predictions of $\sin^2\theta$ and m_W are better than the measurements and thus dominate the Γ_f predictions
- The prediction errors are dominated by $\alpha_{\mathrm{QED}}(m_Z)$ and m_t



Results

- Global fit:

$$\alpha_s(m_Z^2) = 0.1196 \pm 0.0028_{\rm exp} \pm 0.0006_{\rm QCD} \pm 0.0006_{\rm EW}$$
$$= 0.1196 \pm 0.0030 \quad (0.00296)$$

(J. Kühn: $\Delta\alpha_s^{\rm QCD} = 0.0003$)

- Fit without asymmetries, m_W:

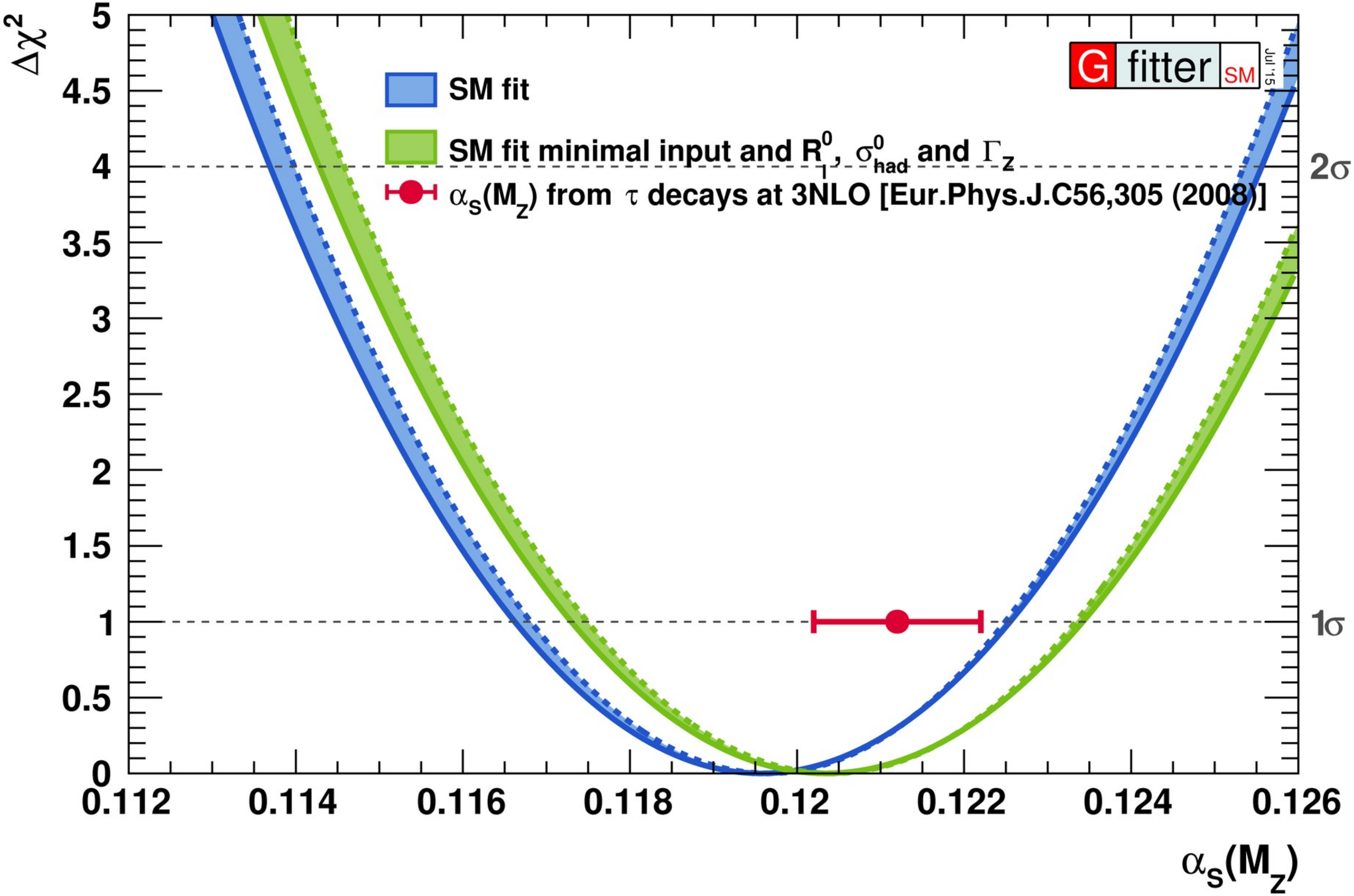
$$\alpha_s(m_Z^2) = 0.1202 \pm 0.0030 \quad (0.00305)$$

- “STU” fit:

$$\alpha_s(m_Z^2) = 0.1200 \pm 0.0031 \quad (0.00307)$$

- All results are consistent
- Error changes only little with assumptions

Results (ii)



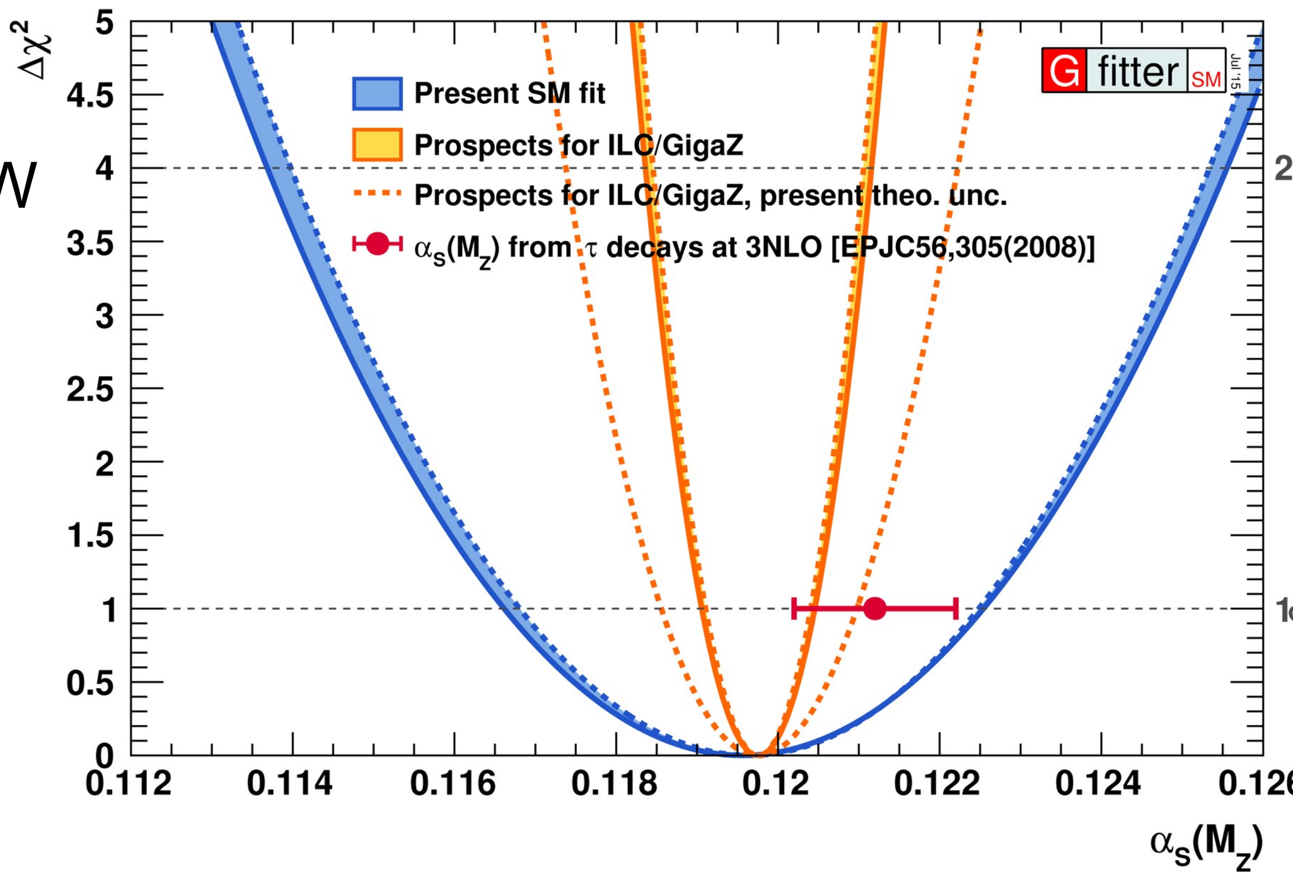
$\Delta\chi^2$
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0.112
0.114
0.116
0.118
0.12
0.122
0.124
0.126
$\alpha_S(M_Z)$
SM fit
SM fit minimal input and R^0_l, σ^0_{had} and Γ_Z
$\alpha_S(M_Z)$ from τ decays at 3NLO [Eur.Phys.J.C56,305 (2008)]
2σ
1σ
G fitter SM
Jul '15

Future prospects

- Detailed studies for ILC exist
- Improvements relevant for α_s:
  - $\Delta R_\ell^0 : \ 25 \cdot 10^{-3} \rightarrow 4 \cdot 10^{-3}$ most important for α_s, optimistic systematics limit (M. Winter, LC-PHSM-2001-16)
  - $\Delta \sin^2 \theta_{\text{eff}} : 16 \cdot 10^{-5} \rightarrow 1.3 \cdot 10^{-5}$
  - $\Delta m_W : 15\,\text{MeV} \rightarrow 5\,\text{MeV}$
  - $\Delta m_t : 0.6\,\text{GeV} \rightarrow 0.1\,\text{GeV}$
  - further improvement of the last three will not help
  - Higgs mass already now better than needed
- External improvements:
  - $\Delta\alpha^5_{\text{had}}(m_Z^2) : 10 \cdot 10^{-5} \rightarrow 4 \cdot 10^{-5}$
  - all theory errors factor 4 smaller than today (and no error on m_t)

Future results

- Standard fit: $\Delta\alpha_s = 0.00065_{\mathrm{exp}} \oplus 0.00023_{\mathrm{QCD}} \oplus 0.00025_{\mathrm{EW}}$
  $= 0.00070$ (QCD almost there with errors from J.K.)
- Same result without asymmetries
- "STU" fit:
  $\Delta\alpha_s = 0.00075$
- Improvement in EW theory is essential!



FCCee improvements

- FCC might improve R_ℓ^0 by a factor 4
- A Z scan might allow a better measurement of m_Z, Γ_Z
- To estimate the effect of the scan assume arbitrarily

$$\Delta m_Z = \Delta\Gamma_Z = 0.5\mathrm{MeV}$$

(no improvement on σ^0_{had} assumed since projection of luminosity error is very difficult)

- α_s error reduces to $\Delta\alpha_s = 0.00060$
- Next step: implement current FCC estimates:

$$\begin{aligned} \Delta m_Z &= 0.1\mathrm{MeV} \\ \Delta\Gamma_Z &= 0.05\mathrm{MeV} \\ \Delta R_\ell^0 &= 1\cdot 10^{-3} \end{aligned}$$

Conclusions

- Present accuracy of strong coupling constant from the Z width:

$$\alpha_s(m_Z^2) = 0.1196 \pm 0.0030$$

- This result depends only little on the theory assumptions
- At ILC this accuracy can go down to

$$\Delta\alpha_s = 0.00070$$

(fits with FCC accuracy still need to be done)

- To make this happen also the theoretical uncertainties on the QCD corrections and especially on the electroweak corrections to the Z partial widths must decrease substantially