Determinations of α_s from $e^+e^- \to$ Hadrons

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1) Status: BESS (BaBar, Belle)

2) Status and Perspectives for $e^+e^- \to$ hadrons at Z

3) M_W from G_F, M_Z, α and the rest

4) Perspectives for $e^+e^- \to$ hadrons above Z

5) Perspectives for $e^+e^- \to Z + H$ ($\to$ hadrons)

1) Status: BESS (BaBar, Belle)

e^+e^- at low energies

BESS (PLB 641 (2006) 145)

$R(3.650 \text{ GeV and } 3.6648 \text{ GeV}) = 2.224 \pm 0.019 \pm 0.089$

$$R = 3\left(Q_u^2 + Q_d^2 + Q_s^2\right)\left(1 + a_s + 1.40923 a_s^2 - 12.7671 a_s^3 - 79.9806 a_s^4\right)$$
$$+ 3\underbrace{(Q_u + Q_d + Q_c)^2}_{=0}\left(-0.4138 a_s^3 - 4.9841 a_s^4\right)$$

present experimental precision at BESS: $\frac{\delta R}{R} \approx 4\%$

$\Rightarrow \alpha_s \approx 0.31^{+0.13}_{-0.14}$

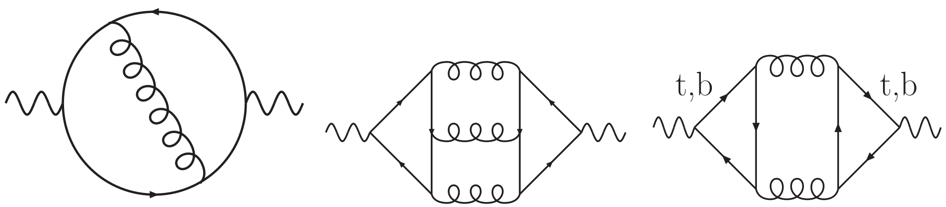
BaBar **Belle** ?

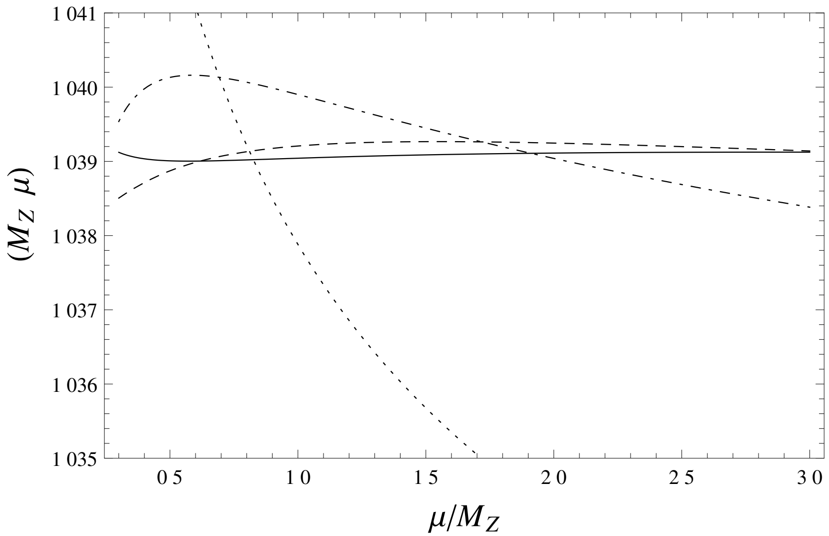
2) Status and Perspectives for $e^+e^- \to$ Hadrons at the Z

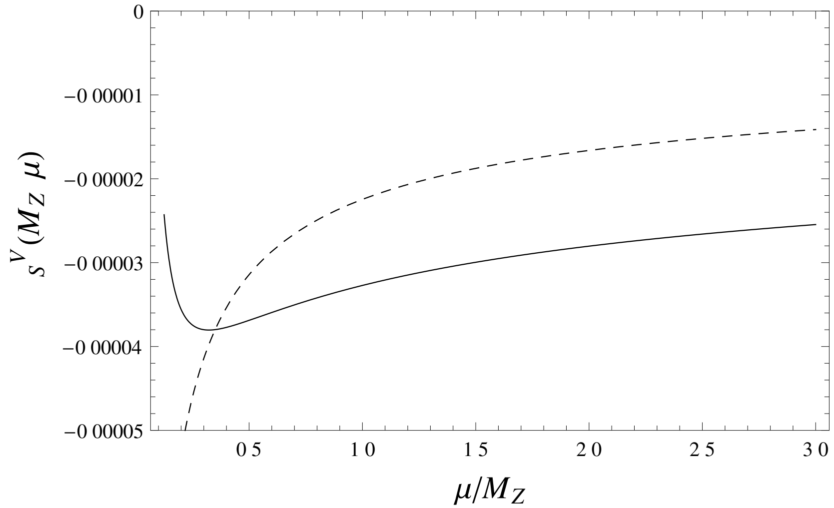
Γ_{had} and $\Gamma_{had}/\Gamma_{lept}$ corrections known to $O(\alpha_s^4)$, N^{3}LO

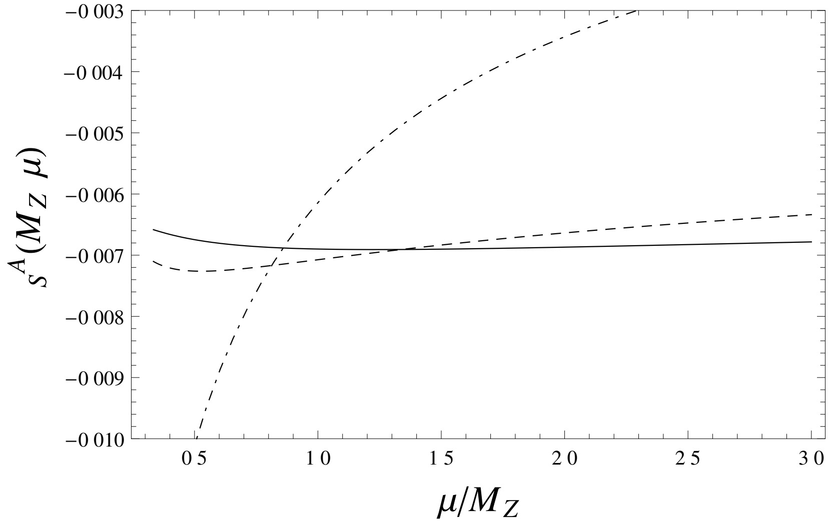
(Baikov, Chetyrkin, JK, Rittinger, arxiv: 0801.1821, 1201.5804)

non-singlet & singlet, vector & axial correlators









- theory uncertainty from $M_Z/3 < \mu < 3M_Z$

$$\left.\begin{aligned} \Rightarrow \quad \delta\Gamma_{NS} &= 101\text{keV}; \\ \delta\Gamma_S^V &= 2.7\text{keV}; \\ \delta\Gamma_S^A &= 42\text{keV}; \end{aligned}\right\} \quad \begin{gathered} \Sigma = 145.7\text{keV} \\ (\text{corresponds to } \delta\alpha_s \sim 3\times 10^{-4}) \end{gathered}$$

TLEP: $\delta\Gamma_{had} \widehat{=} 100$ keV

- similar analysis of $\Gamma(W \to$ had) only affected by non-singlet contributions!

- b-mass corrections under control: $m_b^2\alpha_s^4$; $m_b^4\alpha_s^3$; ...

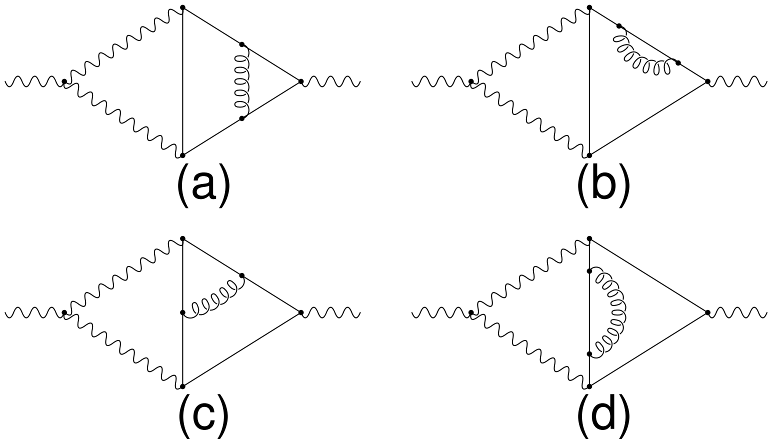
- one more loop?

  $\alpha_s^2(1979)$, $\alpha_s^3(1991)$, $\alpha_s^4(2008)$, $\alpha_s^5(\,?\,)$,

  guesses on α_s^5 based on .... .

Mixed electroweak and QCD: light quarks (u,d,c,s)

terms of $O(\alpha\alpha_s)$, Czarnecki, JK; hep-ph/9608366



$\Delta\Gamma \equiv \Gamma(\text{two loop (EW} \star \text{QCD)}) - \Gamma_{\text{Born}}\delta^{\text{NLO}}_{\text{EW}}\delta^{\text{NLO}}_{\text{QCD}} = -0.59(3)$ MeV

aim (T-LEP paper): $\delta\Gamma \approx 0.1$ MeV

three loop: reduction by $\# \cdot \frac{\alpha_s}{\pi} = \#0.04$

should not exceed 5!

corrections of $O(\alpha_w \alpha_s^2)$ (three loop) difficult

$\Gamma(Z \to b\bar{b}) \equiv \Gamma_b$

aim: $\delta R_b \equiv \frac{\delta\Gamma_b}{\Gamma_Z} = 2 - 5 \times 10^{-5}$ (LEP: $R_b = 0.21629 \pm 0.00066$, corresponds to $\delta\Gamma_b \approx 1.6$ MeV)

2×10^{-5} corresponds to 0.05 MeV!

corrections specific for $b\bar{b}$:

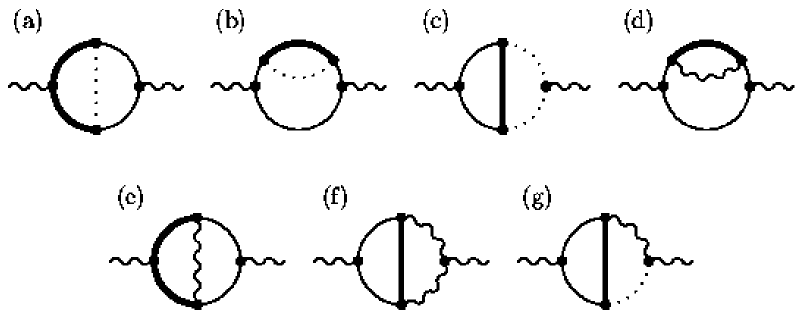
m_t^2-enhancement: order $G_F m_t^2$ and $G_F m_t^2 \alpha_s$

$\Delta\Gamma = \frac{G_F M^3}{16\pi^3} G_f m_t^2 (1 - \frac{2}{3} s_w^2)(1 - \frac{\pi^2 - 3}{3} \frac{\alpha_s}{\pi})$ (Fleischer et al 1992)

Complete $\alpha_w \alpha_s$ result:

$$\Gamma_b - \Gamma_q = (-5.69 - 0.79 \quad O(\alpha)$$

$$+0.50 + 0.06 \quad O(\alpha\alpha_s))\ \text{MeV}$$



dressed with gluons

separated into m_t^2-enhanced and rest

(Harlander, Seidensticker, Steinhauser

hep-ph/9712228)

motivates the evaluation of m_t^2-enhanced corrections of $O(G_F m_t^2 \alpha_s^2)$

(Chetyrkin, Steinhauser, hep-ph/990480)

$\delta\Gamma_b(G_F m_t^2 \alpha_s^2) \approx 0.1$MeV (non-singlet)

(absent in Z-fitter, G-fitter!)

General observation:

many top-induced corrections become significantly smaller, if m_t is expressed in $\overline{MS}$ convention

$$\bar{m}_t(\bar{m}_t) = m_{pole}\left(1 - 1.33\left(\frac{\alpha_s}{\pi}\right) - 6.46\left(\frac{\alpha_s}{\pi}\right)^2 - 60.27\left(\frac{\alpha_s}{\pi}\right)^3 - 704.28\left(\frac{\alpha_s}{\pi}\right)^4\right)$$

(Karlsruhe, 1999) (Marquard, Smirnov, Smirnov, Steinhauser, 2015)

$$= (173.34 - 7.96 - 1.33 - 0.43 - 0.17)\text{GeV}$$

$$= \left(163.45 \pm 0.72|_{m_t} \pm 0.19|_{\alpha_s} \pm ?|_{th}\right)\text{GeV}$$

top scan $\Rightarrow$ m(potential subtracted)

$\delta m_t \sim 20 - 30$ MeV

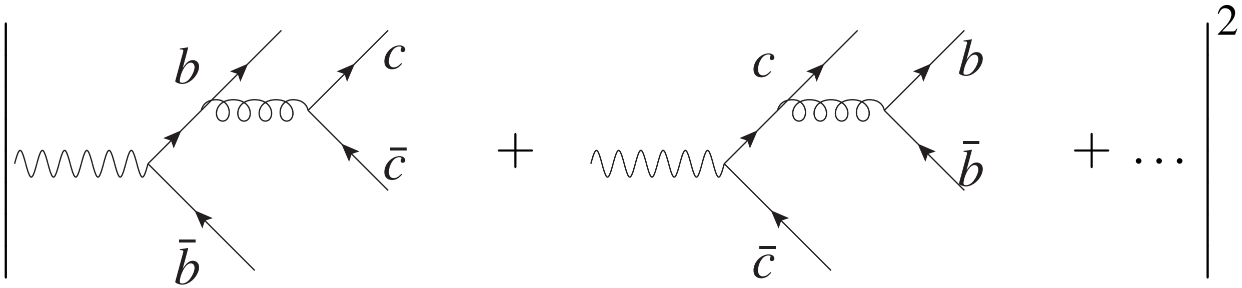
$\Gamma(Z \to b\bar{b})$

Can we isolate the $Zb\bar{b}$-vertex?

$R_b = 0.21629 \pm 0.00066$ (LEP); $\quad 3‰ \,\widehat{=}\, \Gamma(Z \to b\bar{b})/\Gamma_{had} \equiv 1.2$ MeV

TLEP: $\quad \delta R = 2 - 5 \times 10^{-5} \,\widehat{=}\, 50 - 120$ keV

conceptual problem: singlet-terms



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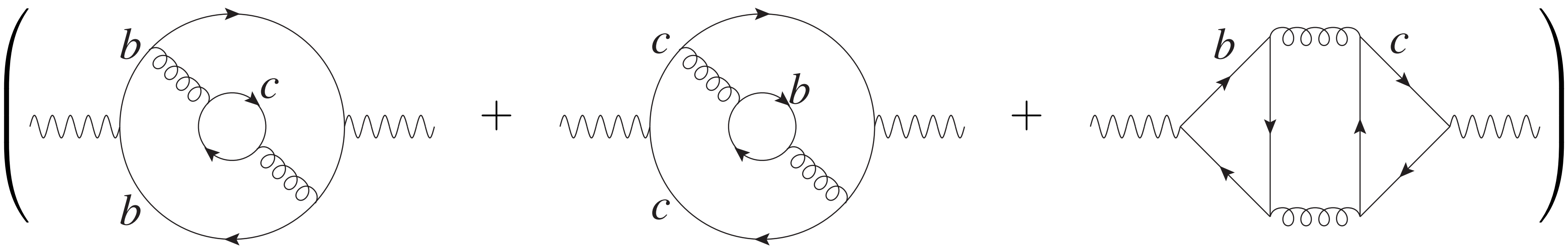


mixed contributions, “singlet”

$$\Gamma^{\text{singlet}}_{b\bar{b}c\bar{c}} = \left(\frac{G_F M_Z^3}{8\sqrt{2}\pi}\right) 0.31 \left(\frac{\alpha_s}{\pi}\right)^2 \approx 340 \text{ keV}$$

(total hadronic rate more robust!)

3) M_W from G_F, M_Z, α and the rest

LEP: $\delta M_W \simeq 30$ MeV; LEP+Tevatron: $\delta M_W \simeq 15$ MeV; Fcc-ee: $\delta M_W \simeq 0.5 - 1$ MeV

Theory

$$M_W^2 = f(G_F, M_Z, m_t, \Delta\alpha, \ldots) = \frac{M_Z^2}{2(1-\delta\rho)}\left(1 + \sqrt{1 - \frac{4\pi\alpha(1-\delta\rho)}{\sqrt{2}G_F M_Z^2}\left(\frac{1}{1-\Delta\alpha} + \ldots\right)}\right);$$

m_t-dependence through $\delta\rho_t$

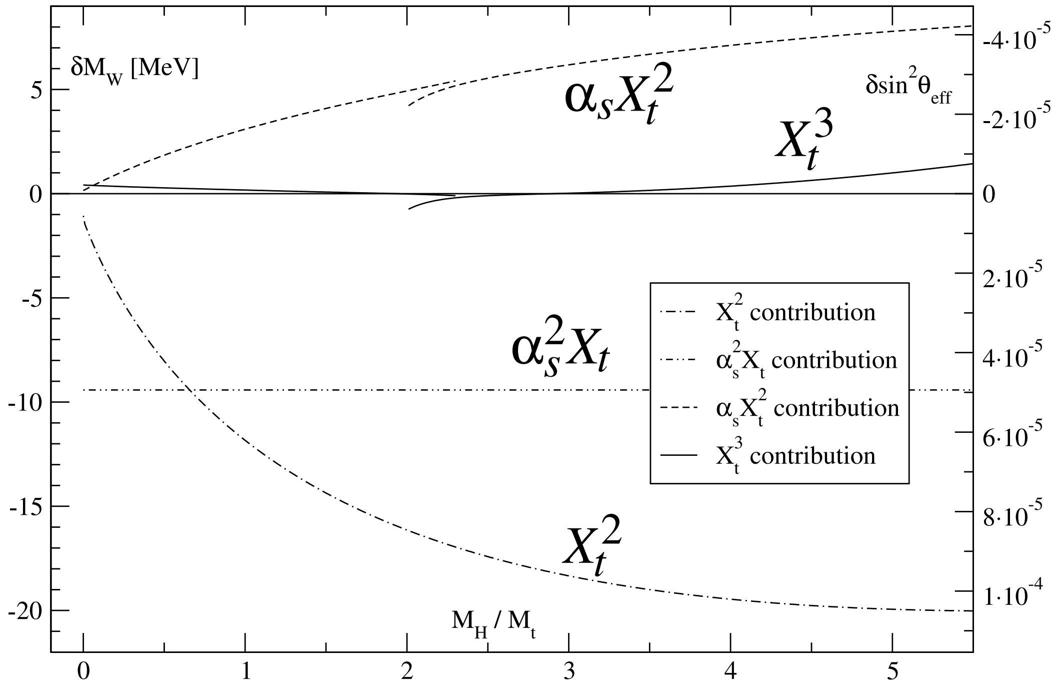
$$\delta M_W \approx M_W \frac{1}{2}\frac{\cos^2\theta_w}{\cos^2\theta_w - \sin^2\theta_w}\delta\rho \approx 5.7 \times 10^4\,\delta\rho \text{ [MeV]}$$

$$\delta\rho_t = 3X_t\left(1 - 2.8599\left(\frac{\alpha_s}{\pi}\right) - 14.594\left(\frac{\alpha_s}{\pi}\right)^2 - 93.1\left(\frac{\alpha_s}{\pi}\right)^3\right)$$

$\downarrow$ $\qquad$ $\downarrow$

$\delta M_W = 9.5$ MeV $\qquad$ $\delta M_W = 2.1$ MeV

α_s^3: 4 loop (Chetyrkin, JK, Maierhöfer, Sturm; Boughezal, Czakon, 2006) mixed QCD $\star$ electroweak



three loop $(X_t \equiv G_F m_t^2)$

X_t^3 (purely weak) $\Rightarrow$ 200eV

$\alpha_s X_t^2$ (mixed) $\Rightarrow$ 2.5MeV

$\alpha_s^2 X_t$ (QCD three loop) $\Rightarrow -9.5$MeV

$\alpha_s^3 X_t$ (QCD four loop) $\Rightarrow$ 2.1MeV

the future

individual uncalculated higher orders below 0.5 MeV, examples:

$\alpha_s^2 X_t^2$ presumably feasible (4 loop tadpoles), $\alpha_s^4 X_t$ 5 loop tadpoles?

dominant contribution from $m_t(pole) \Rightarrow \bar{m}_t$

crucial input: $\boxed{m_t}$ also for stability of the universe

$\delta M_W \approx 6 \times 10^{-3} \delta m_t$

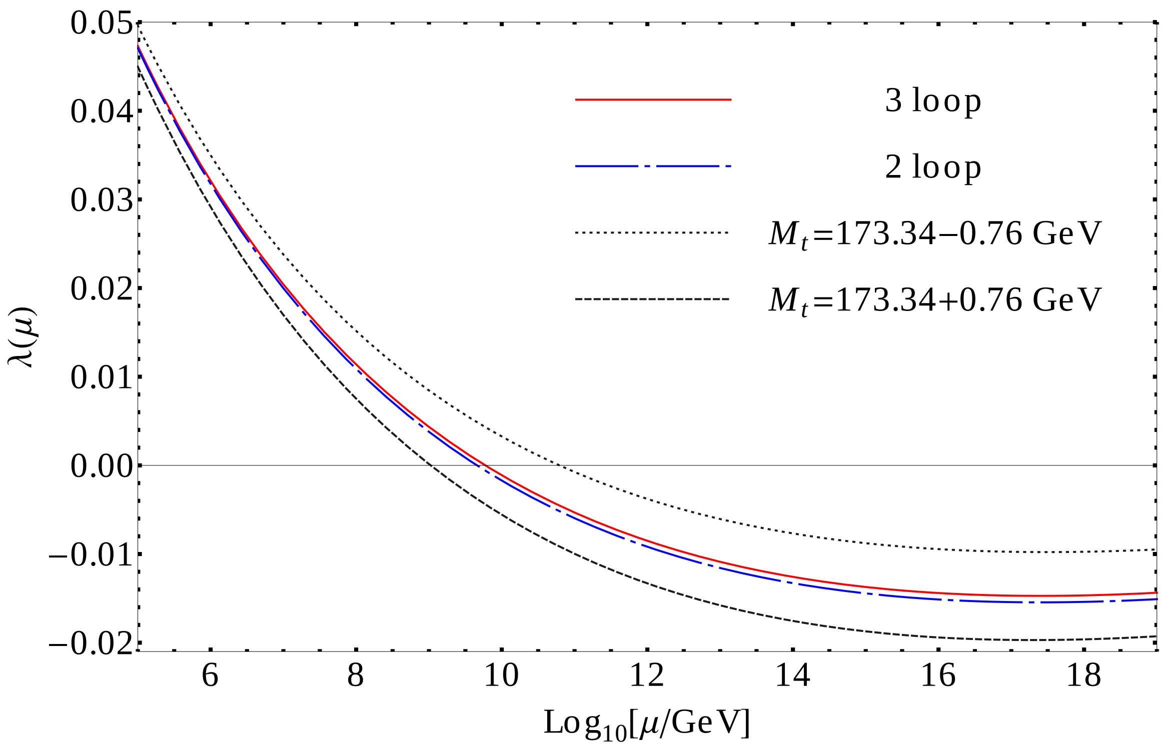
$\delta m_t = 1$ GeV

$\Rightarrow \delta M_W \approx 6$ MeV (status)

conversely:

TLEP: $\delta M_W = 0.5$ MeV

requires $\delta m_t = 100$ MeV

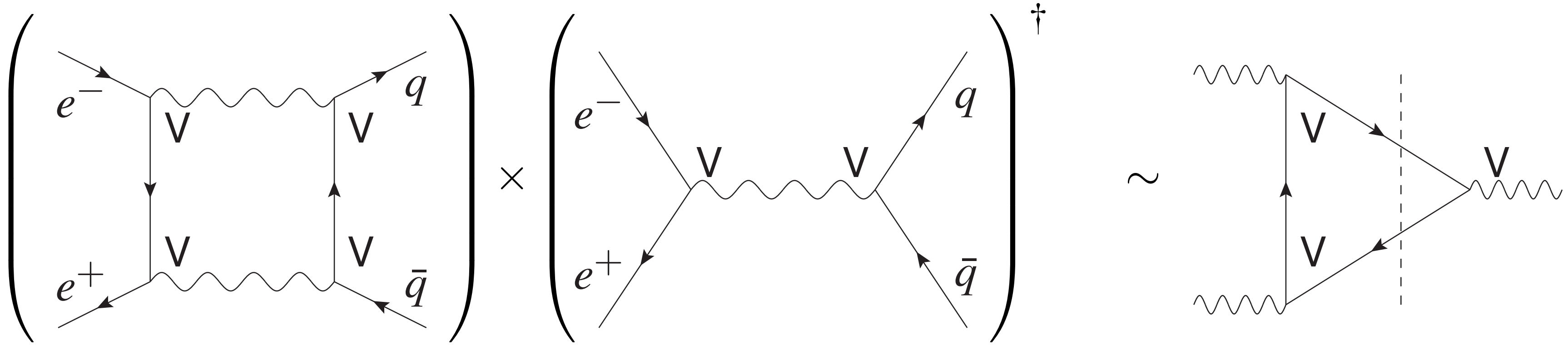


(Zoller)

4) Perspectives for $e^+e^- \to$ Hadrons above Z

pure QED

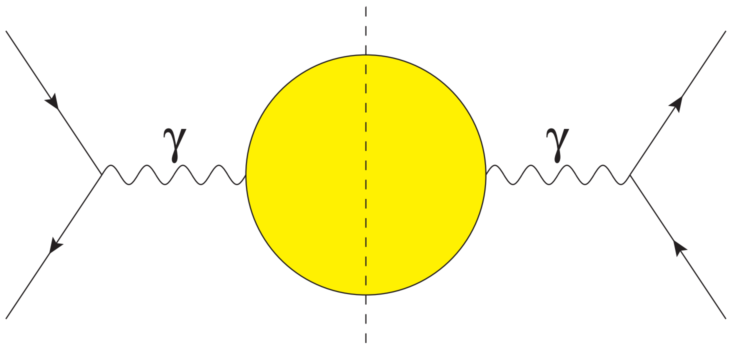
$\Rightarrow$ absence of



(Yang Theorem)

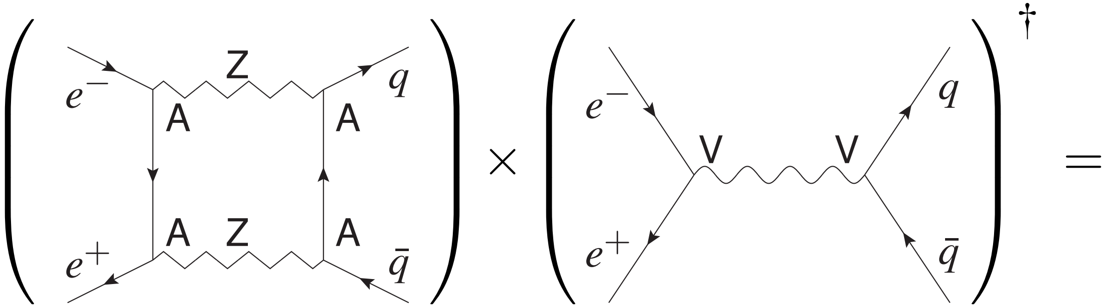
corrections of $O(\alpha^2)$!

$\sigma\left(e^+e^- \to \text{had}\right) \sim$



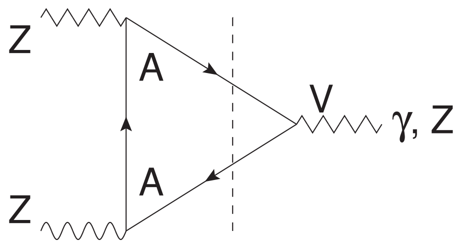
$\left(1 + O(\alpha^2)\right)$

electroweak theory



$$\sum_q \left((g_A^e)^2 g_V^e \right) \left((g_A^q)^2 g_V^q \right) = \left((g_A^e)^2 g_V^e \right) \left((g_A^u)^2 \sum_q g_V^q \right) \neq 0$$

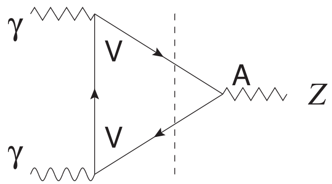
leading contribution $O(\alpha)$



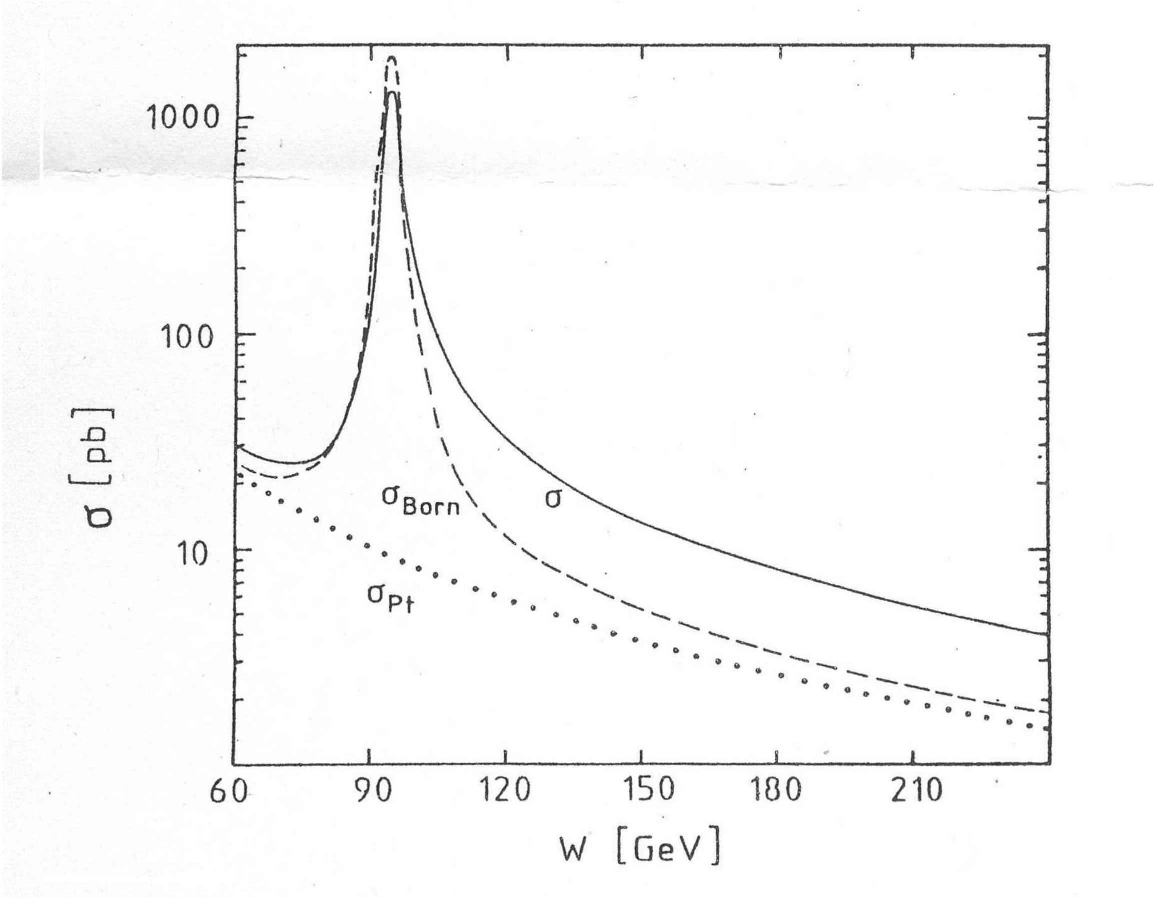
$$\sum_q \left((g_A^q)^2 g_V^q \right) = \left((g_A^u)^2 \sum_q g_V^q \right) \neq 0$$

$\Rightarrow$ interference of order α

similarly for



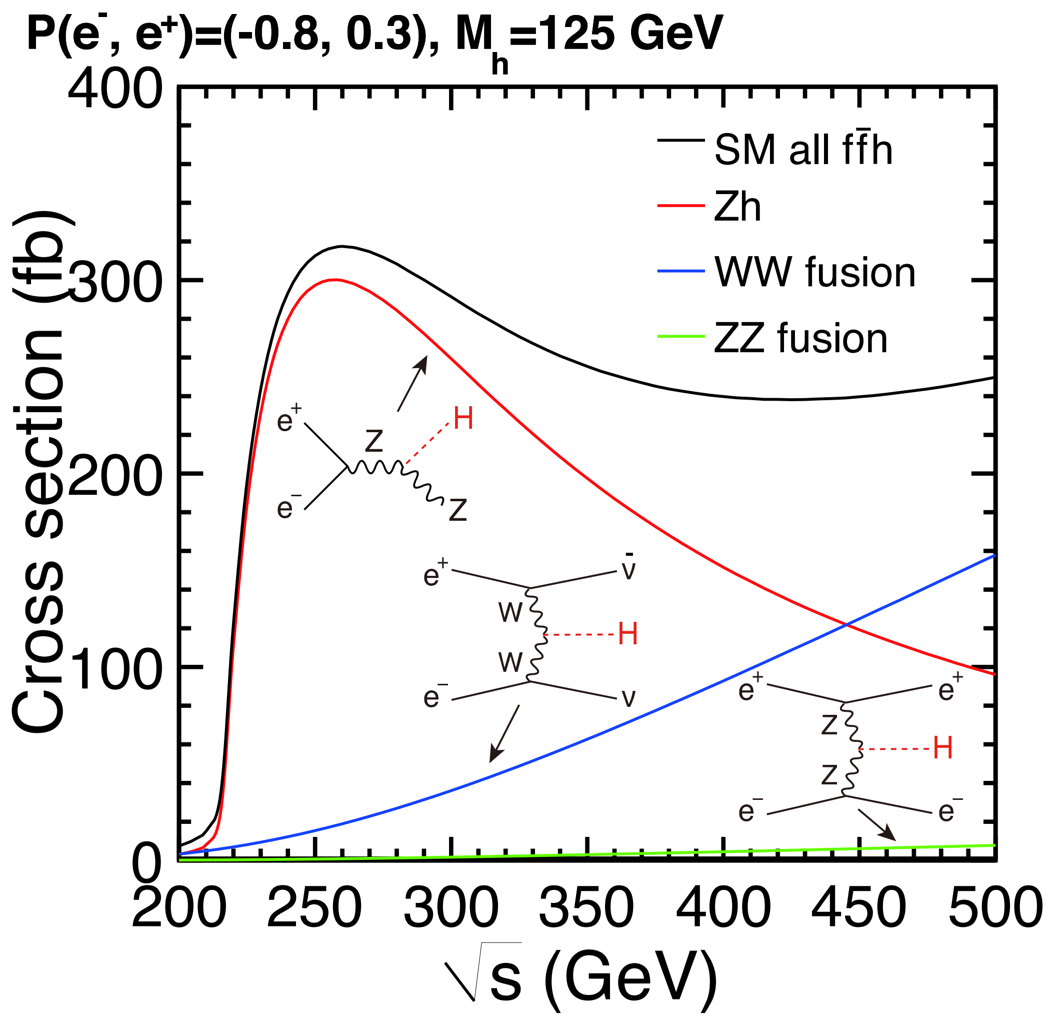
Important radiative tail from Z



precise predictions $\left(< \frac{1}{2}\%\right)$ difficult:

large radiative tail from the Z (factor 3 compared to Born cross section)

5) Perspectives for $e^+e^- \to Z + H(\to$ hadrons$)$



Cross sections for the three major Higgs production processes as a function of center of mass energy (from arXiv:1306.6352) example: $H \to b\bar{b}$ dominant decay mode, all branching ratios are affected!

TLEP: $\sigma_{HZ} \times Br(H \to b\bar{b})$: aim 0.2%

Higgs WG, arXiv:1307.1347 (Table 1) assumes $\alpha_s = 0.119 \pm 0.002,\ m_b|_{pole} = 4.49 \pm 0.06$ GeV:

$\frac{\delta\Gamma(H\to b\bar{b})}{\Gamma(H\to b\bar{b})} = \pm 2.3\%|_{\alpha_s} \pm 3.2\%|_{m_b} \pm 2.0\%|_{th} \quad \Rightarrow\ 7.5\%$

Our estimate: $\Gamma(H \to b\bar{b}) = \frac{G_F M_H}{4\sqrt{2}\pi} m_b^2(M_H)\, R^S(s = M_H^2, \mu^2 = M_H^2)$

$$
\begin{aligned}
R^S(M_H) &= 1 + 5.667\left(\frac{\alpha_s}{\pi}\right) + 29.147\left(\frac{\alpha_s}{\pi}\right)^2 + 41.758\left(\frac{\alpha_s}{\pi}\right)^3 - 825.7\left(\frac{\alpha_s}{\pi}\right)^4 \\
&= 1 + 0.1948 + 0.03444 + 0.0017 - 0.0012 \\
&= 1.2298 \qquad \text{(Chetyrkin, Baikov, JK, 2006)}
\end{aligned}
$$

for $\alpha_s(M_Z) = 0.118,\ \alpha_s(M_H) = 0.108$

Theory uncertainty ($M_H/3 < \mu < 3M_H$) : $5‰$ (four loop) reduced to $1.5‰$ (five loop) present parametric uncertainties:

$m_b(10\text{GeV}) = 3610 - \frac{\alpha_s - 0.1189}{0.002} 12 \pm 11$ MeV (Karlsruhe, arXiv:0907.2110)

$$\left(\begin{array}{ll} \text{Bodenstein+Dominguez:} & 3623(9)\ \text{MeV} \\ \text{HPQCD} & 3617(25)\ \text{MeV} \end{array} \right)$$

(α_s uncertainties are presently dominant, assuming $\delta = 0.002$, they influence m_b-determination; running to M_H; R^S)

running from 10 GeV to M_H depends on

anomalous mass dimension, β-function and α_s

$m_b(M_H) = 2759 \pm 8|_{m_b} \pm 27|_{\alpha_s}$ MeV

γ_4 (five loop): Baikov + Chetyrkin, 2012

β_4 under construction

$$\frac{\delta m_b^2(M_H)}{m_b^2(M_H)} = -1.4 \times 10^{-4} \left(\frac{\beta_4}{\beta_0} = 0\right) \quad | \quad -4.3 \times 10^{-4} \left(\frac{\beta_4}{\beta_0} = 100\right) \quad | \quad -7.3 \times 10^{-4} \left(\frac{\beta_4}{\beta_0} = 200\right)$$

to be compared with $\delta\Gamma(H \to b\bar{b})/\Gamma(H \to b\bar{b}) = 2.0 \times 10^{-4}$ (FCC-ee) perspectives: (assume $\delta\alpha_s = \delta\alpha_s(\text{now})/10 = 2\times 10^{-4}$)

$\delta m_b(10\text{GeV})/m_b \sim 10^{-3}$ conceivable (dominated by $\delta\Gamma(\Upsilon \to e^+e^-)$)

$\Rightarrow \frac{\delta\Gamma_{H\to b\bar{b}}}{\Gamma_{H\to b\bar{b}}} = \pm 2\times 10^{-3}|_{m_b} \pm 1.3\times 10^{-3}|_{\alpha_s,\text{running}} \pm 1\times 10^{-3}|_{\text{theory}}$

similarly: $\Gamma_{H\to c\bar{c}}$

$$\begin{aligned} \delta m_c(3\ \text{GeV})/m_c(3\ \text{GeV}) &= 13\ \text{MeV}/986\ \text{MeV} \quad \text{(now)} \\ &= 5\ \text{MeV}/986\ \text{MeV} \quad \text{(conceivable)} \end{aligned}$$

$$\begin{aligned} m_c(M_H) &= (609 \pm 8|_{m_c} \pm 9|_{\alpha_s})\ \text{MeV} \quad \text{(now)} \\ & \qquad \pm 3\ \text{MeV} \quad \text{(conceivable)} \\ \Rightarrow \frac{\delta\Gamma_{H\to c\bar{c}}}{\Gamma_{H\to c\bar{c}}} &= \pm 5.5\times 10^{-2} \quad \text{(now)} \\ &= \pm 1\times 10^{-2} \quad \text{(conceivable)} \end{aligned}$$

Starting from order α_s^3 the separation of $H \to gg$ and $H \to b\bar{b}$

is no longer unambiguously possible. (Chetyrkin, Steinhauser, 1997)

$H \to gg$ to $O(\alpha_s^5)$ (hep-ph/0604194; Baikov, Chetyrkin)

(separation of gg, $b\bar{b}$, $c\bar{c}$ difficult in $O(\alpha_s^4)$ and higher)

$\Gamma(H \to gg) = K \cdot \Gamma_{\text{Born}}(H \to gg)$

and

$$
\begin{aligned}
K = 1 \quad &+ \quad 17.9167\, a_s' + (156.81 - 5.7083 \ln \frac{M_t^2}{M_H^2})(a_s')^2 \\
&+ \quad (467.68 - 122.44 \ln \frac{M_t^2}{M_H^2} + 10.94 \ln^2 \frac{M_t^2}{M_H^2})\ (a_s')^3 .
\end{aligned}
$$

take $M_t = 175$ GeV, $M_H = 120$ GeV and $a_s' = \alpha_s^{(5)}(M_H)/\pi = 0.0363$:

$$
\begin{aligned}
K &= 1 + 17.9167\, a_s' + 152.5 (a_s')^2 + 381.5 (a_s')^3 \\
&= 1 + 0.65038 + 0.20095 + 0.01825 .
\end{aligned}
$$

Claim: experimental precision of $\sigma(HZ)$ BR $(H \to gg) = 1.4\%$

$\sim$ approximately equal to last calculated correction

SUMMARY

- QCD corrections are crucial for precise predictions at Fcc-ee (or a linear collider).

- Z decays are crucial for a precise determination of α_s.

- Important implications of α_s for $M_W^2 = f\,(G_F, M_Z, \ldots)$.

- Large radiative corrections for $e^+e^- \to$ hadrons above the Z;

  precise measurements of $e^+e^- \to$ hadrons not yet established.

- attractive perspectives for $e^+e^- \to Z + H$, in particular for $\Gamma(H \to$ hadrons).