

Photonic crystals, graphene, and new effects in Čerenkov radiation

Ido Kaminer



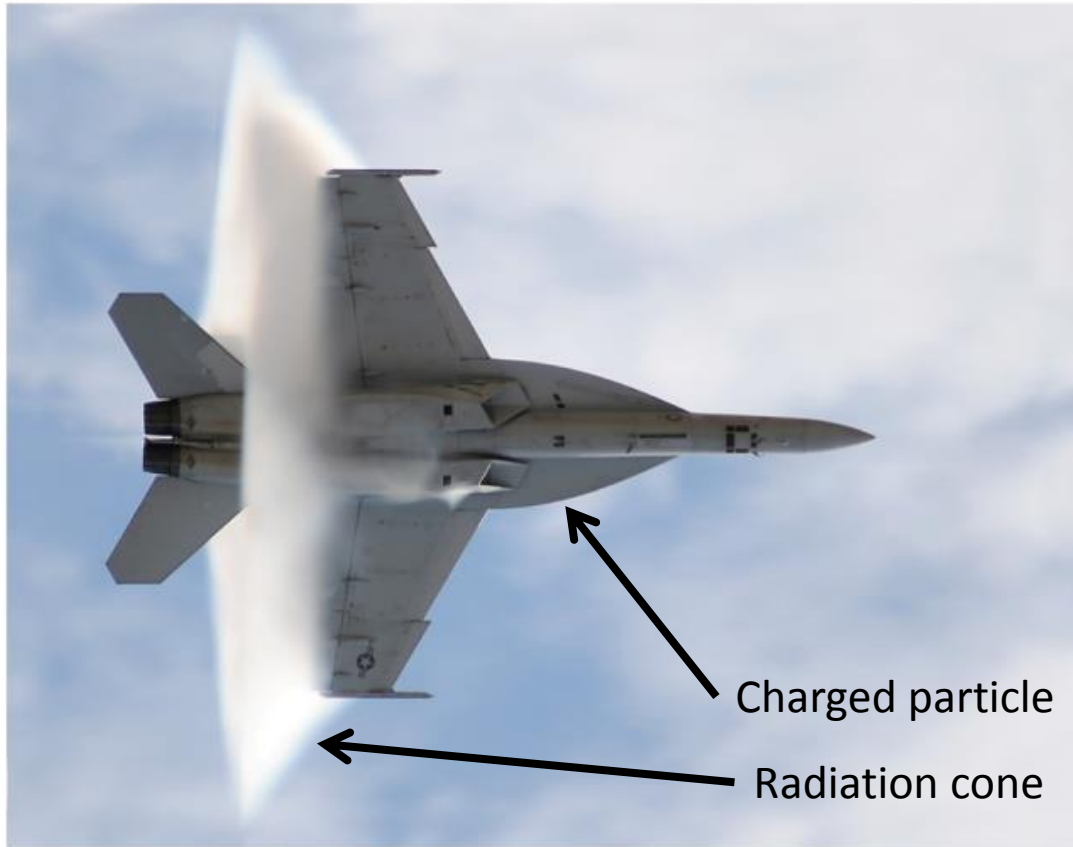
Postdoc with John D. Joannopoulos and Marin Soljačić, MIT
Ph.D. with Moti Segev, Technion



Marie Curie IOF
project BSICS

Čerenkov Radiation – Shock Wave of Light

$$v_{source} > v_{sound}$$



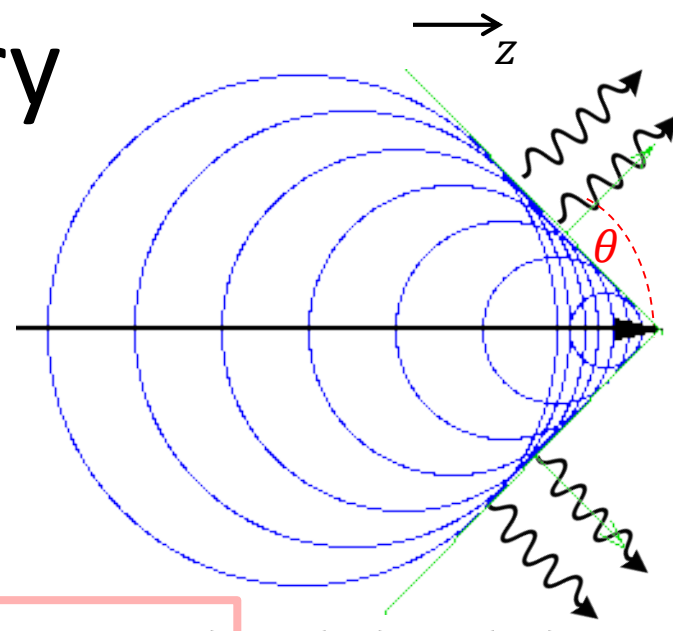
Čerenkov, **Dokl. Akad. Nauk SSSR** 2, 451 (1934)

Nobel Prize in Physics 1958

$$v_{particle} > v_{photon}$$



The Conventional Theory



$$\nabla^2 \vec{A} - \epsilon \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} = \mu_0 \vec{J} = \mu_0 q \vec{v} \delta(z - vt) \delta(x) \delta(y)$$

$$k_{\perp}^2 + k_z^2 = \epsilon \frac{\omega^2}{c^2}$$

$$k_z = \frac{\omega}{v} \leftarrow \mu_0 q \vec{v} e^{i\omega \frac{z}{v}} \delta(x) \delta(y)$$

$$k_{\perp} = \pm \sqrt{\epsilon \frac{\omega^2}{c^2} - \frac{\omega^2}{v^2}} = \pm \frac{\omega}{c} \sqrt{n^2 - \frac{1}{\beta^2}}$$

The Čerenkov threshold: $\beta > \frac{1}{n}$

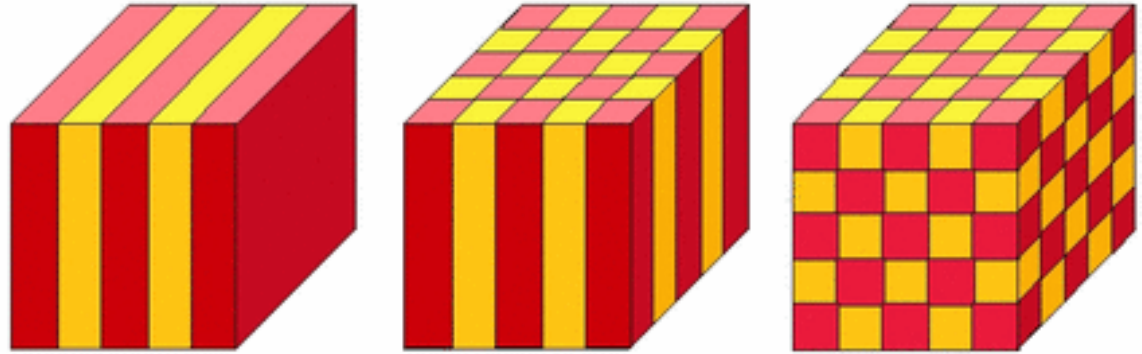
Tamm&Frank, **Dokl. Akad. Nauk SSSR** 14, 109 (1937)

Nobel Prize in Physics 1958

$$\text{The Čerenkov angle: } \cos(\theta) = \frac{1}{\beta n}$$

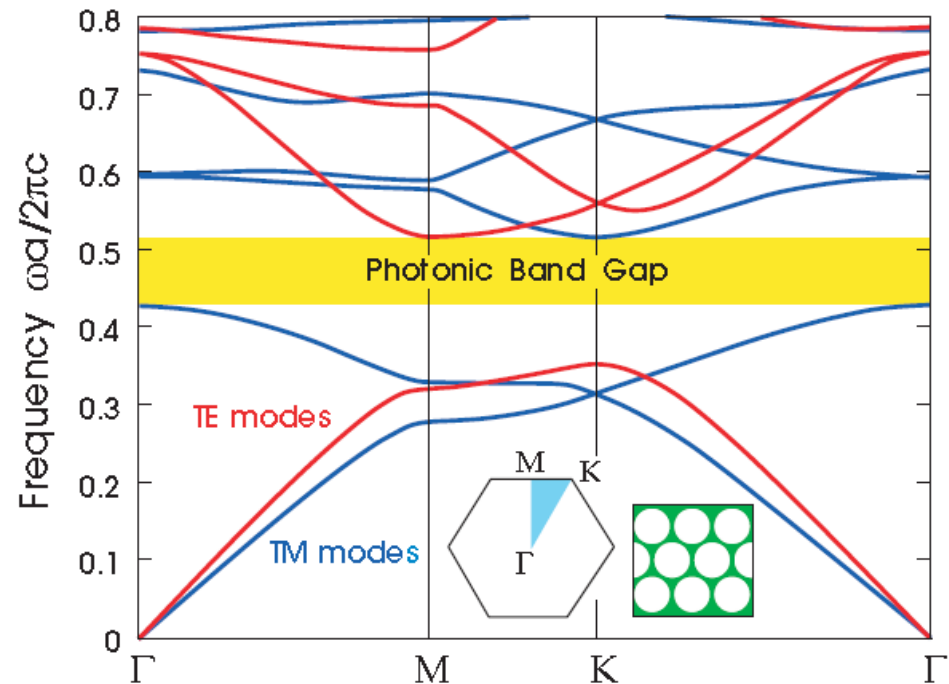
New materials and new types of matter

Photonic crystals



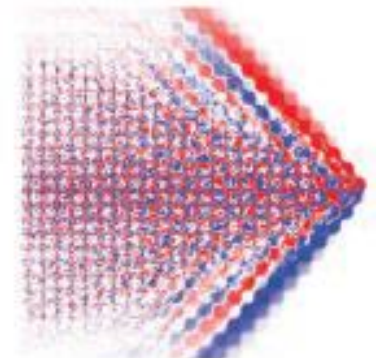
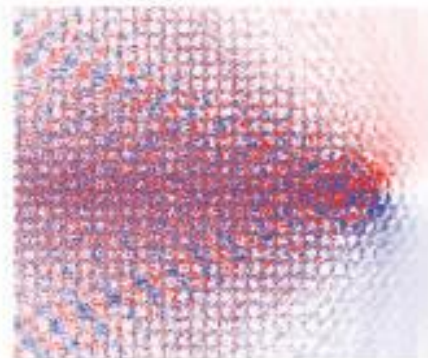
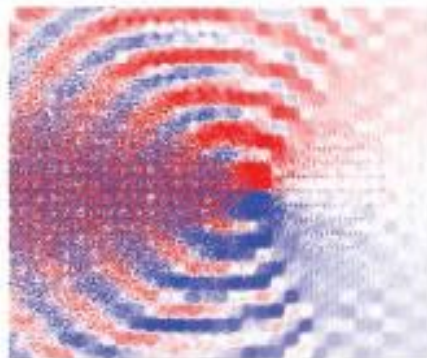
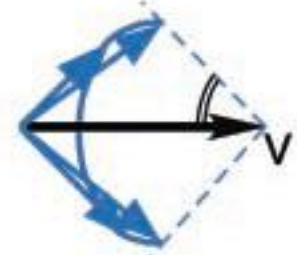
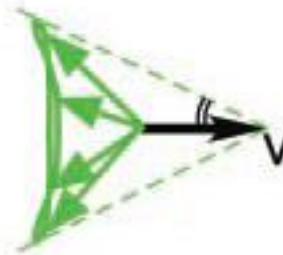
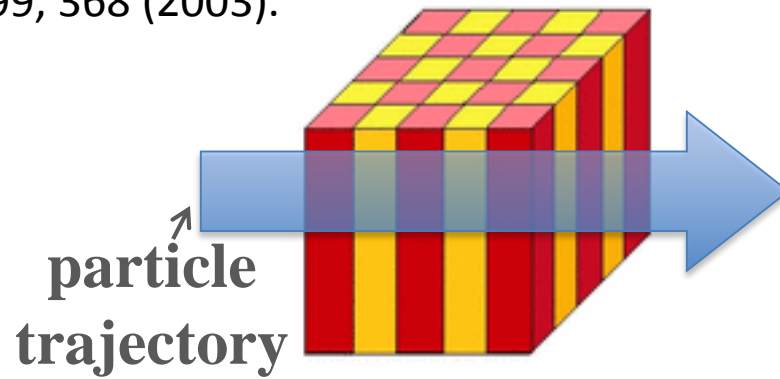
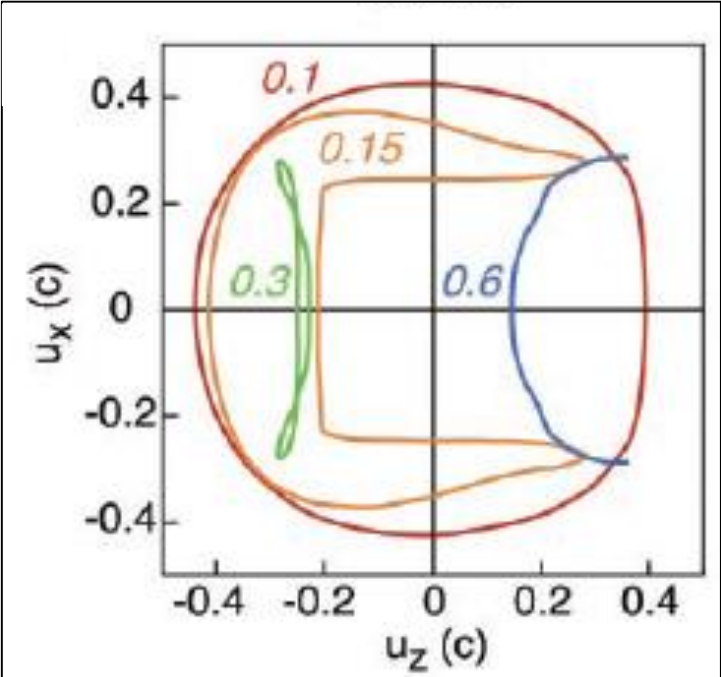
$$\cancel{k_x^2 + k_z^2 = \epsilon \frac{\omega^2}{c^2}}$$

Metamaterials



Čerenkov radiation in photonic crystal

Luo, Ibanescu, Johnson, and Joannopoulos, **Science** 299, 368 (2003).



Phase matching condition $\omega = \vec{k} \cdot \vec{v}$

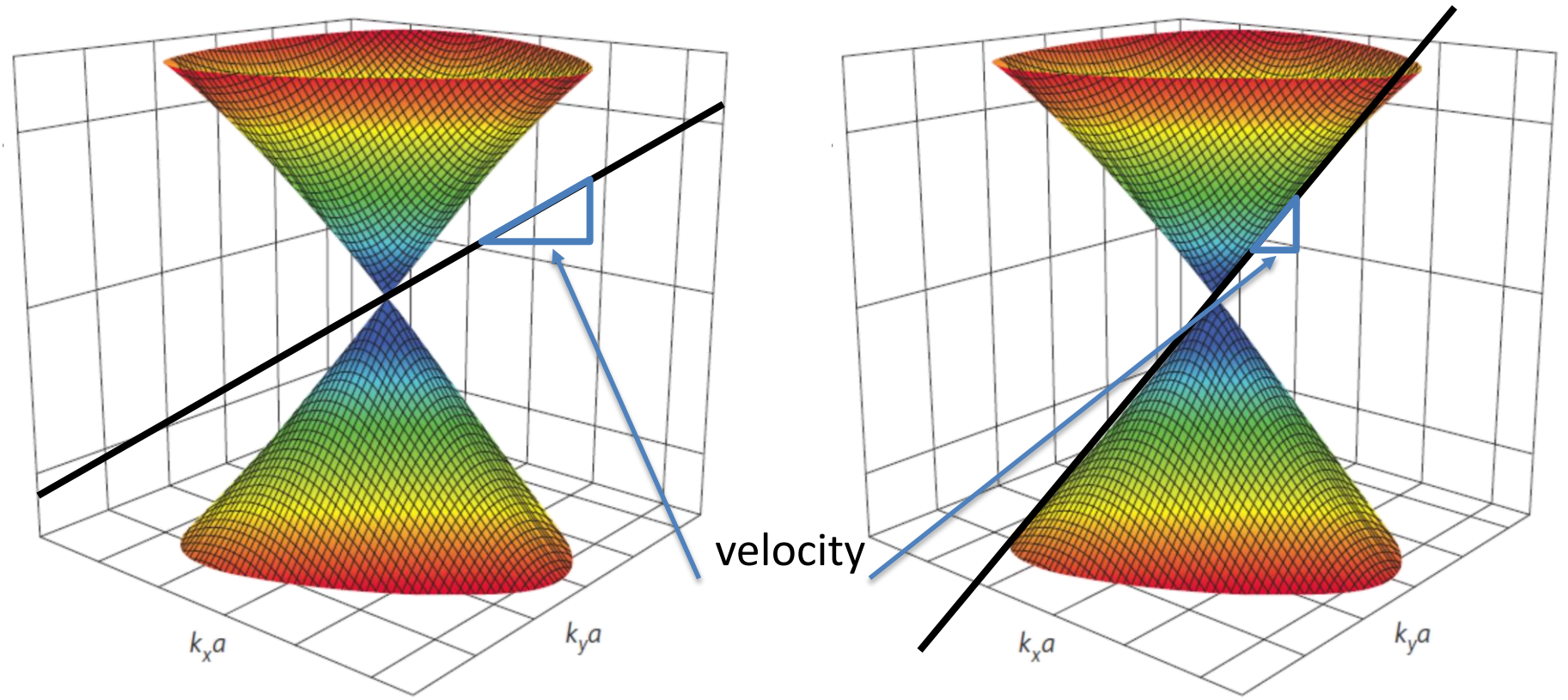
$$k_z = \frac{\omega}{v} \leftarrow \mu_0 q \vec{v} e^{i\omega \frac{z}{v}} \delta(x) \delta(y)$$

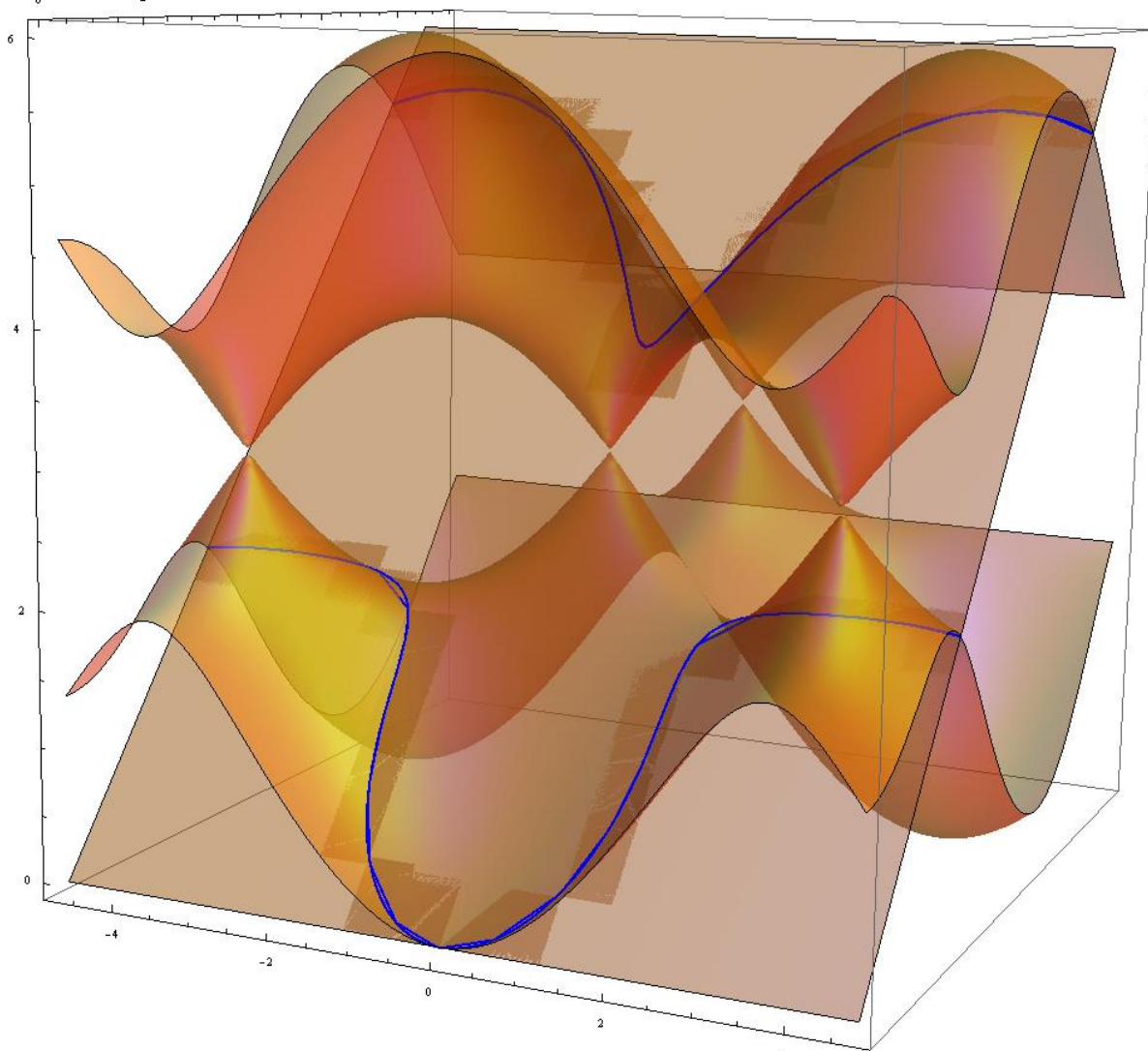
$\omega(\vec{k}) = \text{photonic bandstructure}$

Can be found numerically by MPB (MIT Photonic Bands)

Johnson and Joannopoulos, "Block-iterative frequency-domain methods for Maxwell's equations in a planewave basis," **Optics Express** 8, 173 (2001)

The case of a homogeneous medium - conventional Čerenkov effect:

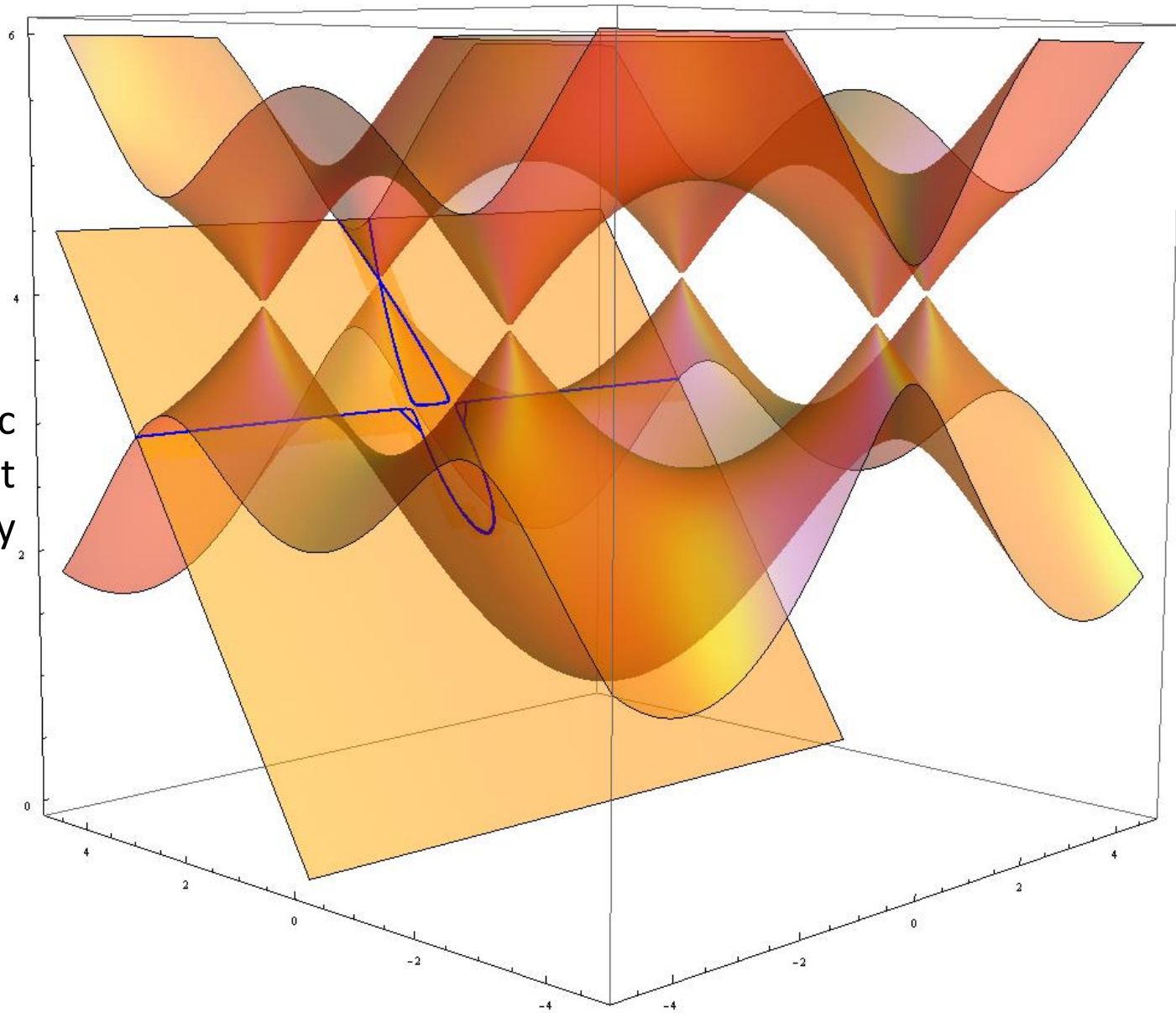




Different frequencies are generally emitted to different directions.
No longer a single Čerenkov angle

Enhancements from the properties of photonic crystals

A supercollimation effect in the Čerenkov radiation: directional monochromatic radiation

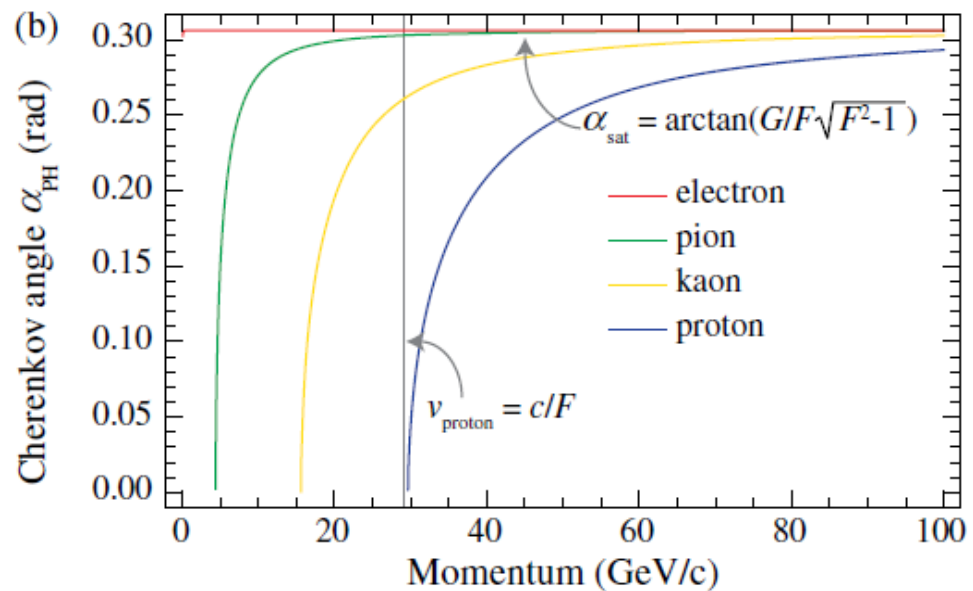
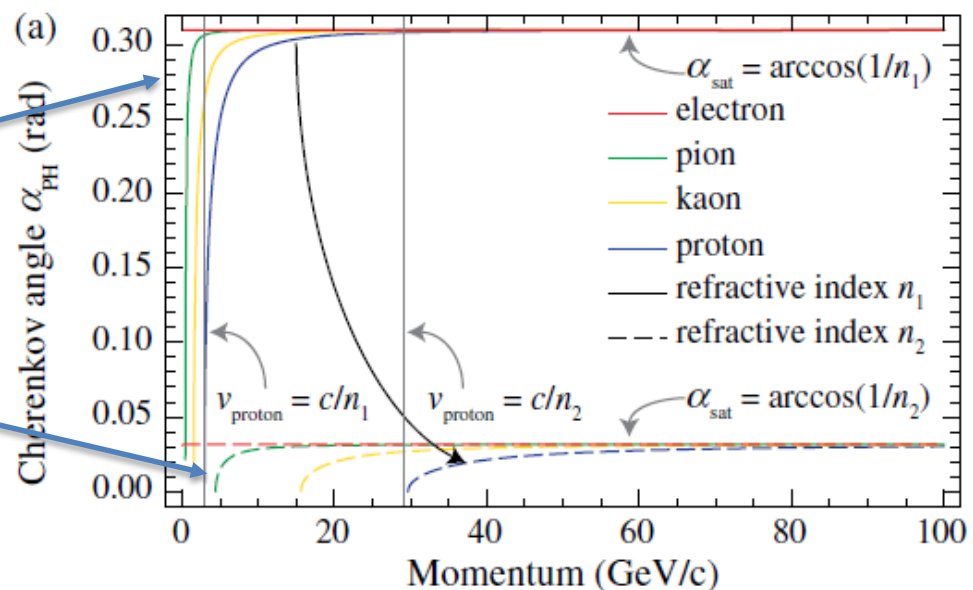
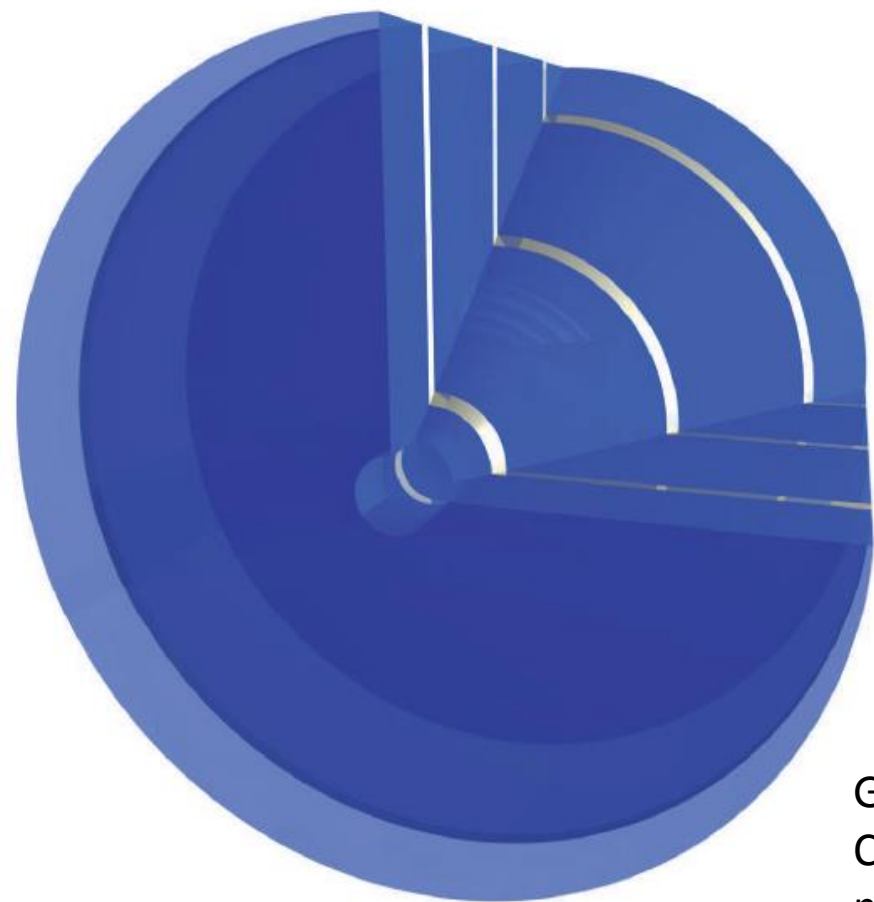


Extremely enhanced
coupling of a charged
particle at one specific
velocity to radiation at
one specific frequency

An artificial strongly anisotropic medium

large opening angles of a silica aerogel radiator ($n=1.05$)

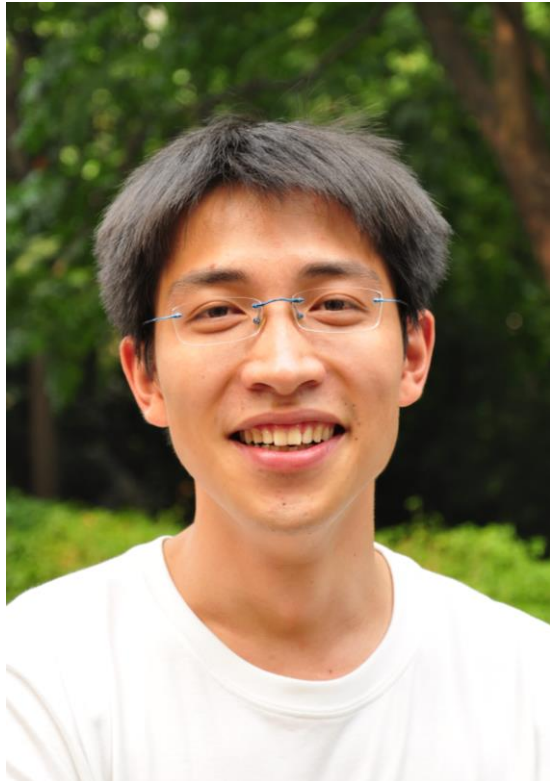
resolution of a CF_4 radiator ($n=1.0005$)



Ginis, Danckaert, Veretennicoff, & Tassin, "Controlling Cherenkov radiation with transformation-optical metamaterials." **PRL** 113, 167402 (2014).

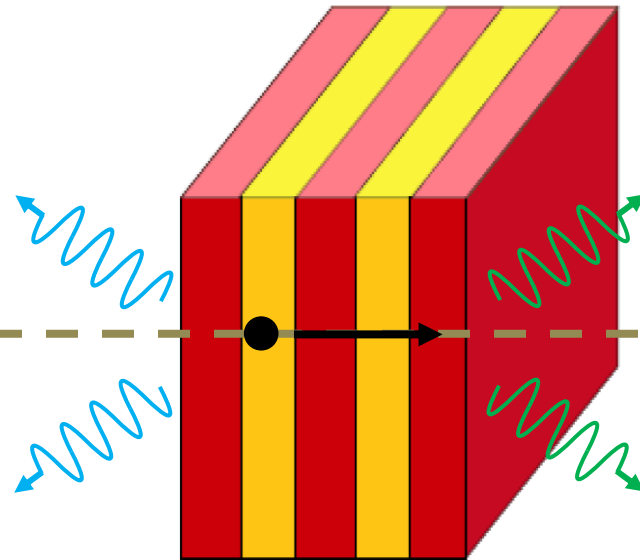
Our current efforts

1D Photonic Crystals for Radiation Detectors



Xiao Lin

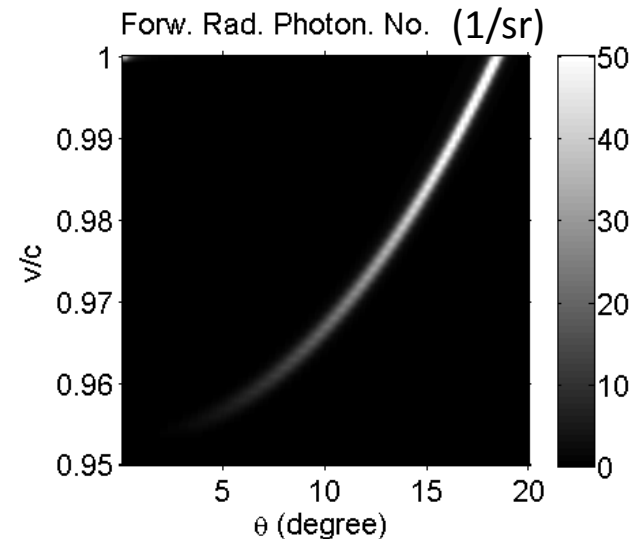
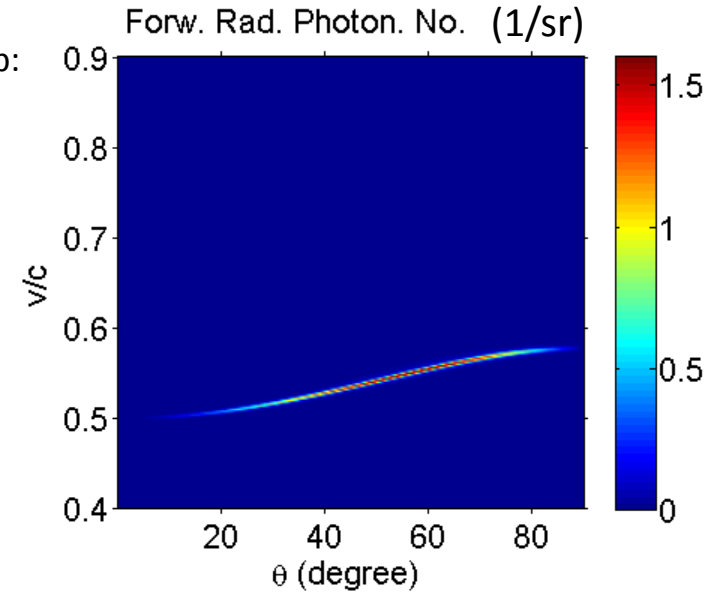
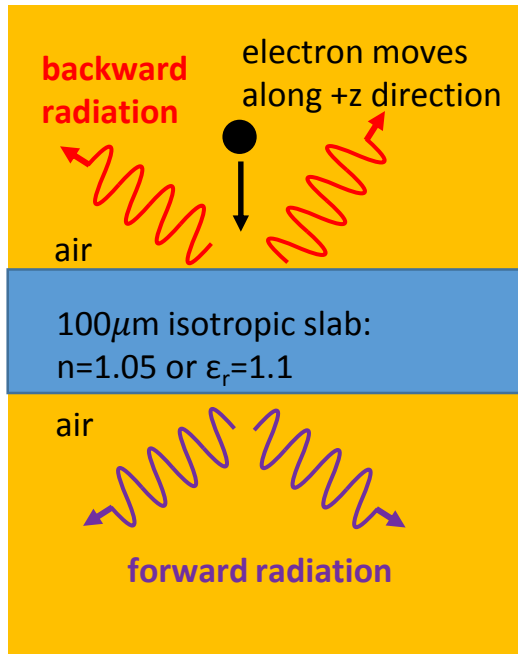
MIT & Zhejiang University



Dr. Sajan Easo
LHCb & STFC Rutherford

Homogeneous natural materials

20 μm isotropic slab:
 $n=2$ or $\epsilon_r=4$

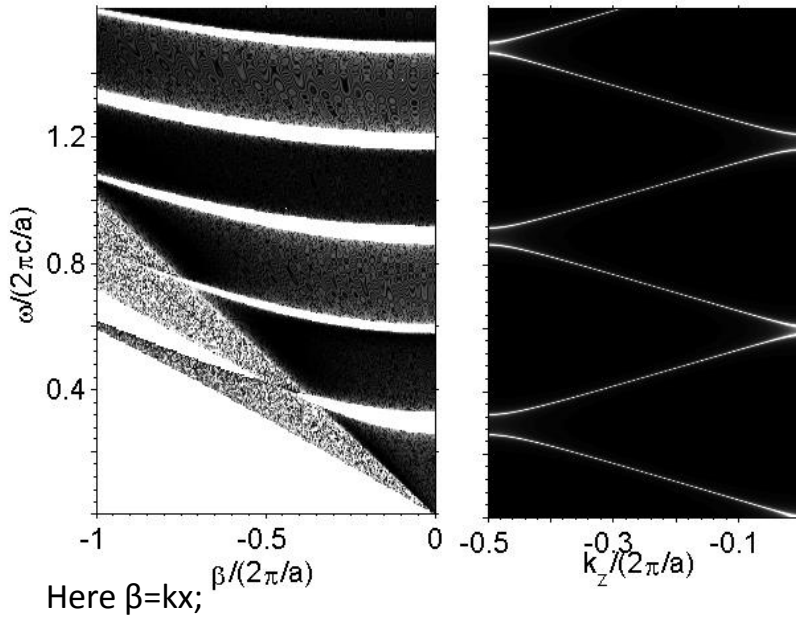
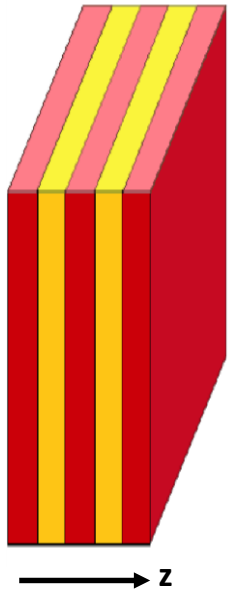


wavelength range: 200nm to 600nm
silica aerogel ($n=1.05$); CF4 ($N=1.0005$)

(No existing natural material with low index and high quality)

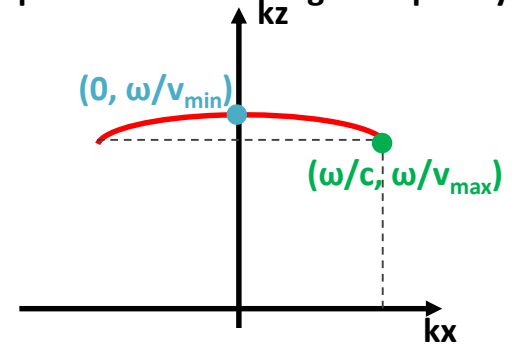
Can we do better with 1D photonic crystals?

Aim: a flat iso-frequency contour, supercollimation

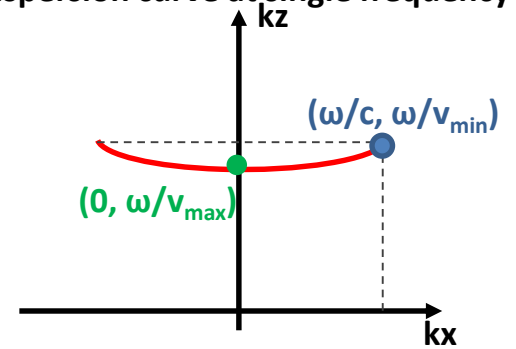


Goal

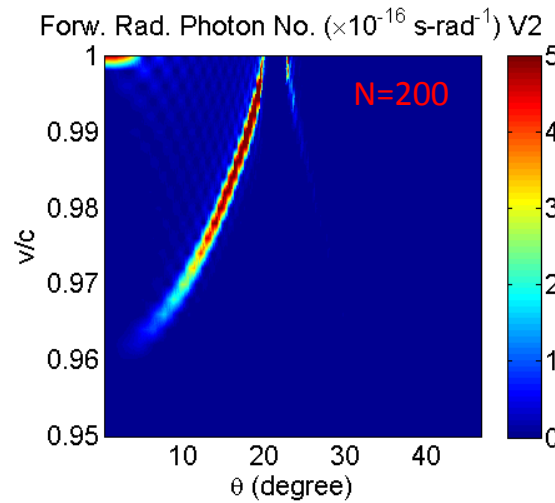
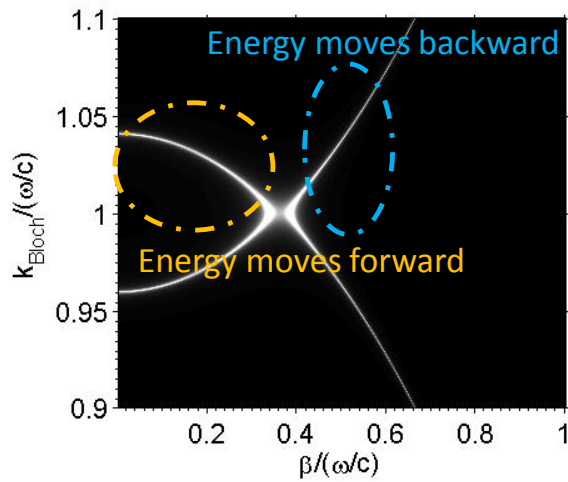
Dispersion curve at single frequency



Dispersion curve at single frequency



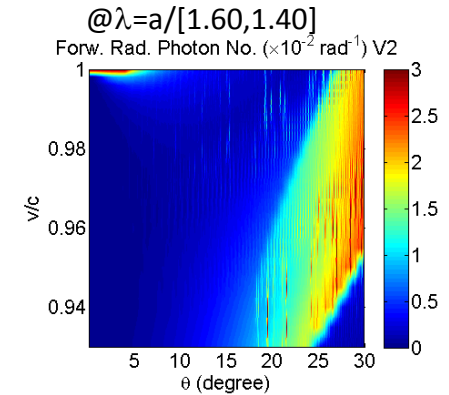
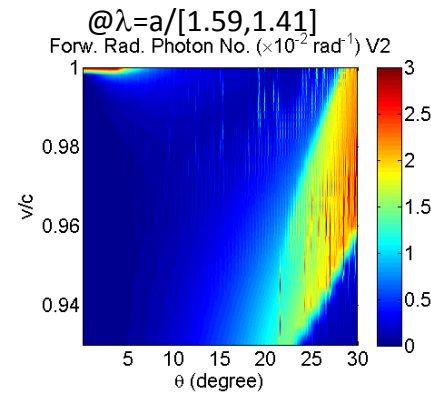
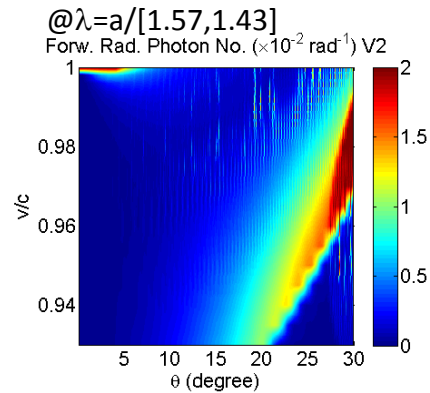
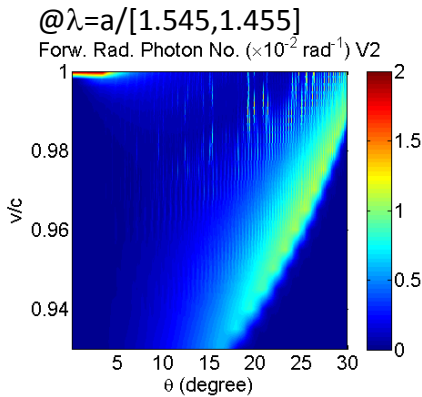
$$(k_z = \omega/v)$$



Structure: air |(4|2)_N4|air; a=600nm;
d₁ = 300nm; d₂ = 300nm;

forward CR
 $\lambda = a/1.5 = 400\text{nm}$

Chromatic aberrations are significant



Analogous to the broadening coming from the material dispersion in the current detectors.
The periodic structure of the 1D PhC makes the whole system very sensitive to frequency.

One possible solution: increase N, i.e. increase the thickness of the periodic system.

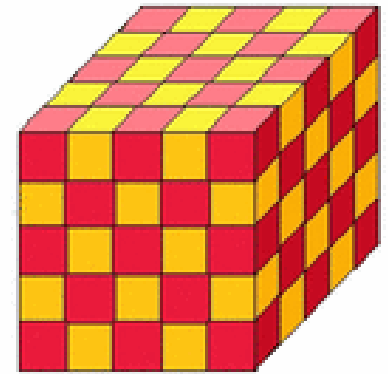
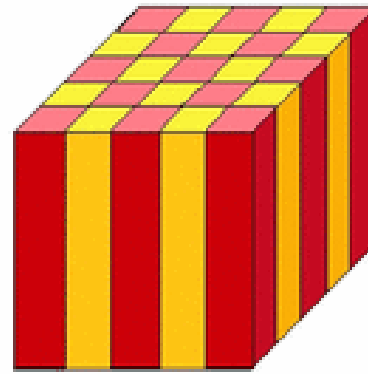
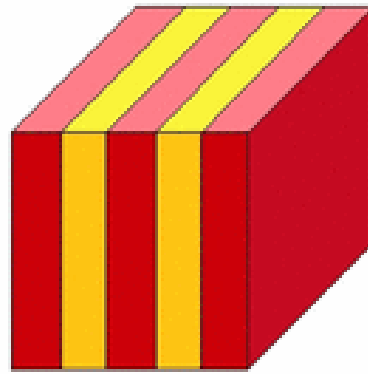
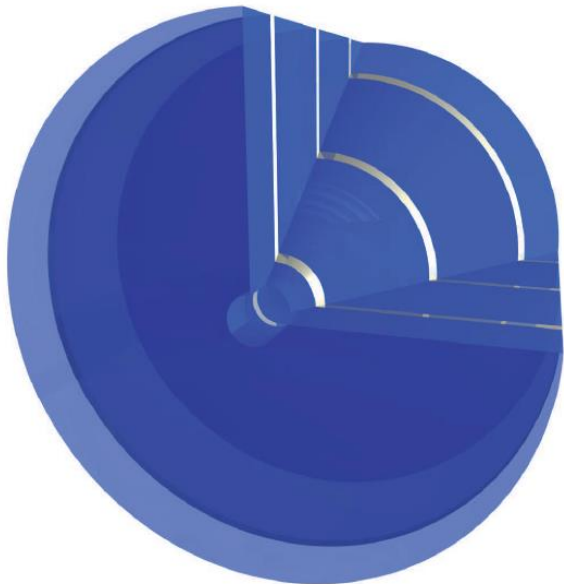
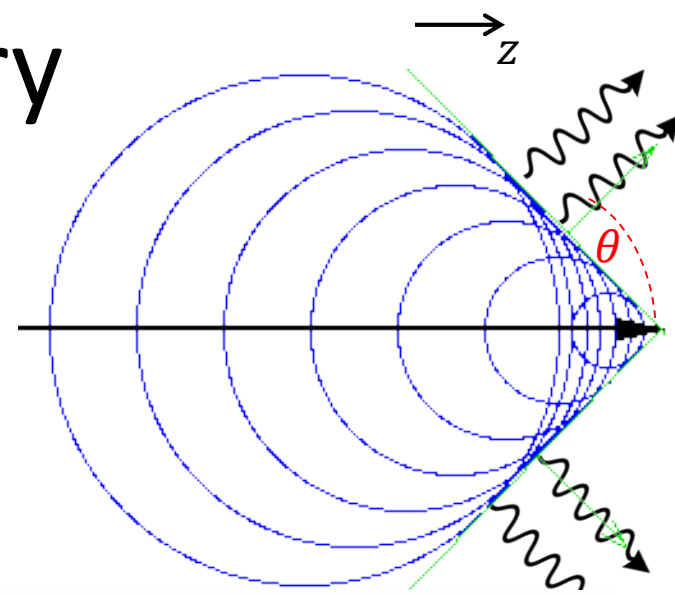
Looking for good ideas for how to circumvent this challenge

The Conventional Theory

$$v_{\text{source}} > v_{\text{sound wave}}$$

What about the repulsion?

Where is the conservation of energy?



$$= \pm \frac{\omega}{c} \sqrt{n^2 - \frac{1}{\beta^2}}$$

The Čerenkov threshold: $\beta > \frac{1}{n}$

Tamm&Frank, **Dokl. Akad. Nauk SSSR** 14, 109 (1937)

Nobel Prize in Physics 1958

$$\text{The Čerenkov angle: } \cos(\theta) = \frac{1}{\beta n}$$

This may be written

$$\cos \theta = 1/n\beta + (\Lambda/\lambda)(n^2 - 1)/2n^2,$$

where $\beta = u/c$ and Λ is the de Broglie wave-length of the electron before the emission of the photon.

The first term on the right is the expression for $\cos \theta$ derived classically by Frank and Tamm.

The second, in which h is a factor, illustrates the rule that a quantum equation becomes identical with the corresponding classical one when h is replaced by zero. Or, to say much the same thing in another way, this term is a natural one to find as the difference between a quantum calculation and a classical one, since the classical calculation of course ignores any effect associated with the wave-length of the electron.

This term will be very small since, as was mentioned earlier, the wave-length of the electron is very much smaller than that of the photon. Hence the classical approximation is a very good one, and the direction of emission of the light does not appreciably depend upon its wave-length except as the wave-length determines the refractive index.

Cox, *Phys. Rev.* 66, 106 (1944)

Quantum Corrections

$$\cos(\theta_{\check{C}R}) = \frac{1}{\beta n} + \frac{\hbar\omega}{\beta\gamma mc^2} \frac{n^2 - 1}{2n}$$

$$\cos(\theta) = \frac{1}{\beta n} \quad \text{Conventional Čerenkov angle}$$

Later papers, e.g,

Jauch&Watson, *Phys. Rev.* 74, 1485 (1948)

Neamtan, *Phys. Rev.* 92, 1362 (1953)

Derived the Čerenkov Effect

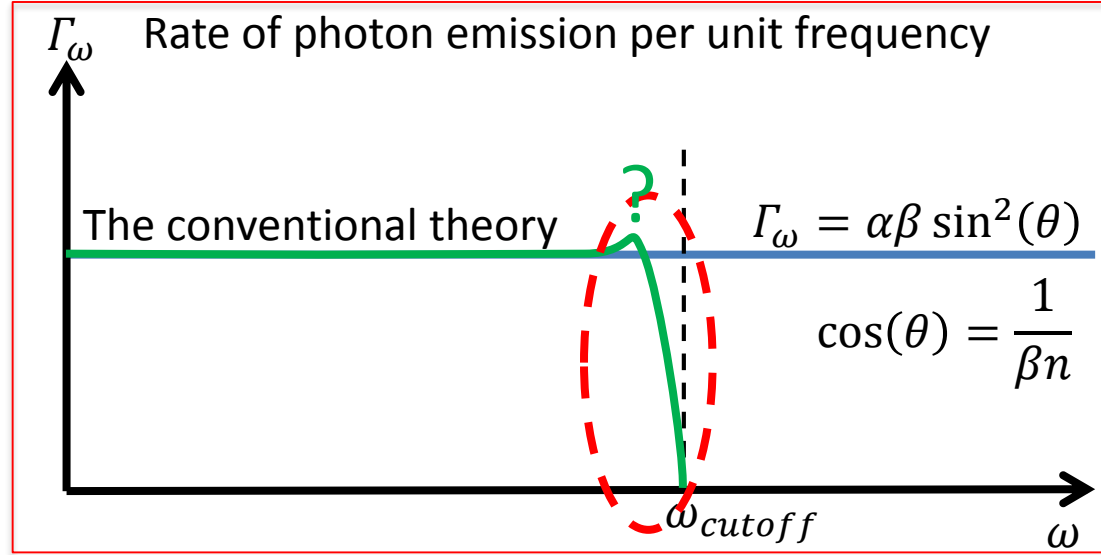
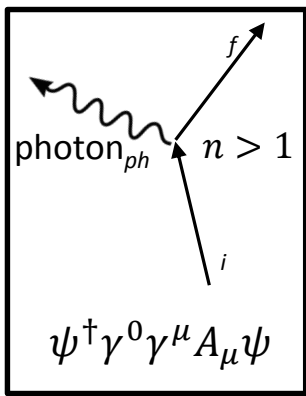
from the Dirac Hamiltonian, *always*

reconfirming the conventional result

$$\omega_{cutoff} = \frac{2mc^2}{\hbar} \frac{\beta n - 1}{(n^2 - 1)\sqrt{1 - \beta^2}}$$

$\omega_{Compton}$

No Čerenkov radiation for $\omega > \omega_{cutoff}$

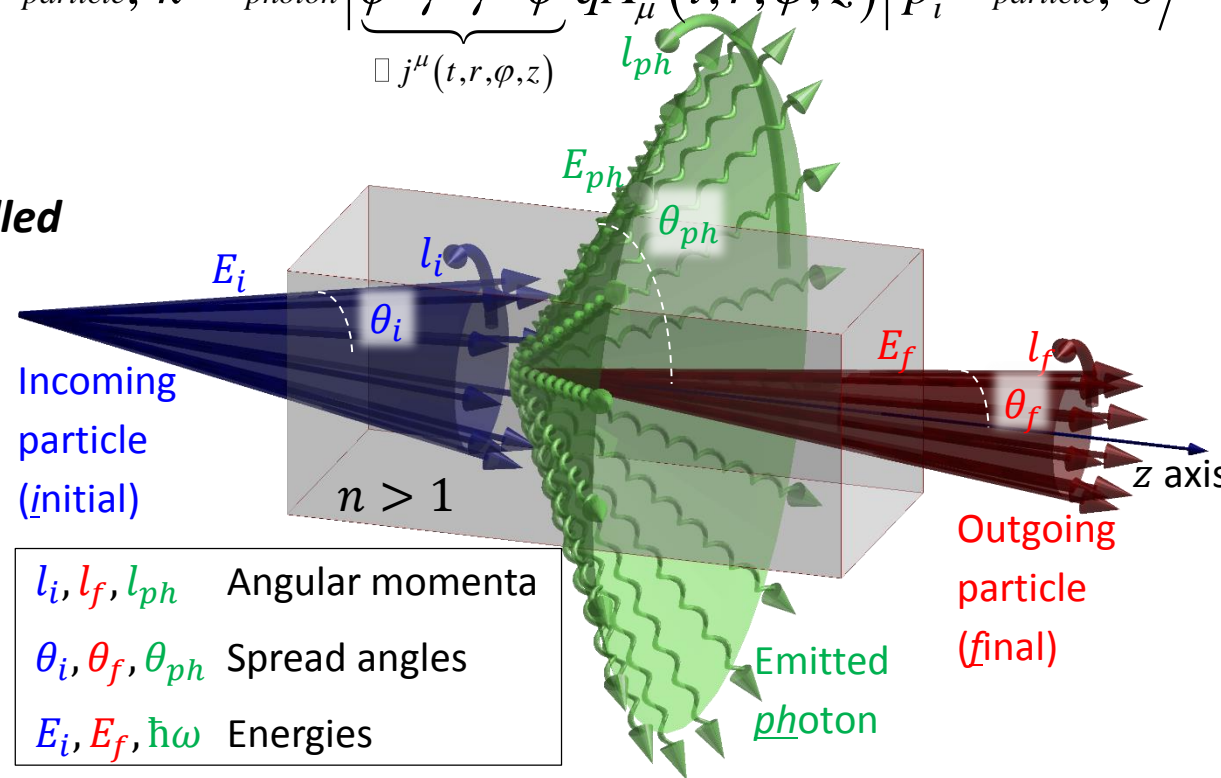


$$M_{p_i^{cyl} \rightarrow p_f^{cyl} + k^{cyl}}^{density} (t, r, \varphi, z) = \left\langle p_f^{cyl} \text{ particle}, k^{cyl} \text{ photon} \left| \underbrace{\psi^\dagger \gamma^0 \gamma^\mu \psi}_{\square j^\mu(t, r, \varphi, z)} q A_\mu (t, r, \varphi, z) \right| p_i^{cyl} \text{ particle}, 0 \right\rangle$$

The Čerenkov angle splits in two!

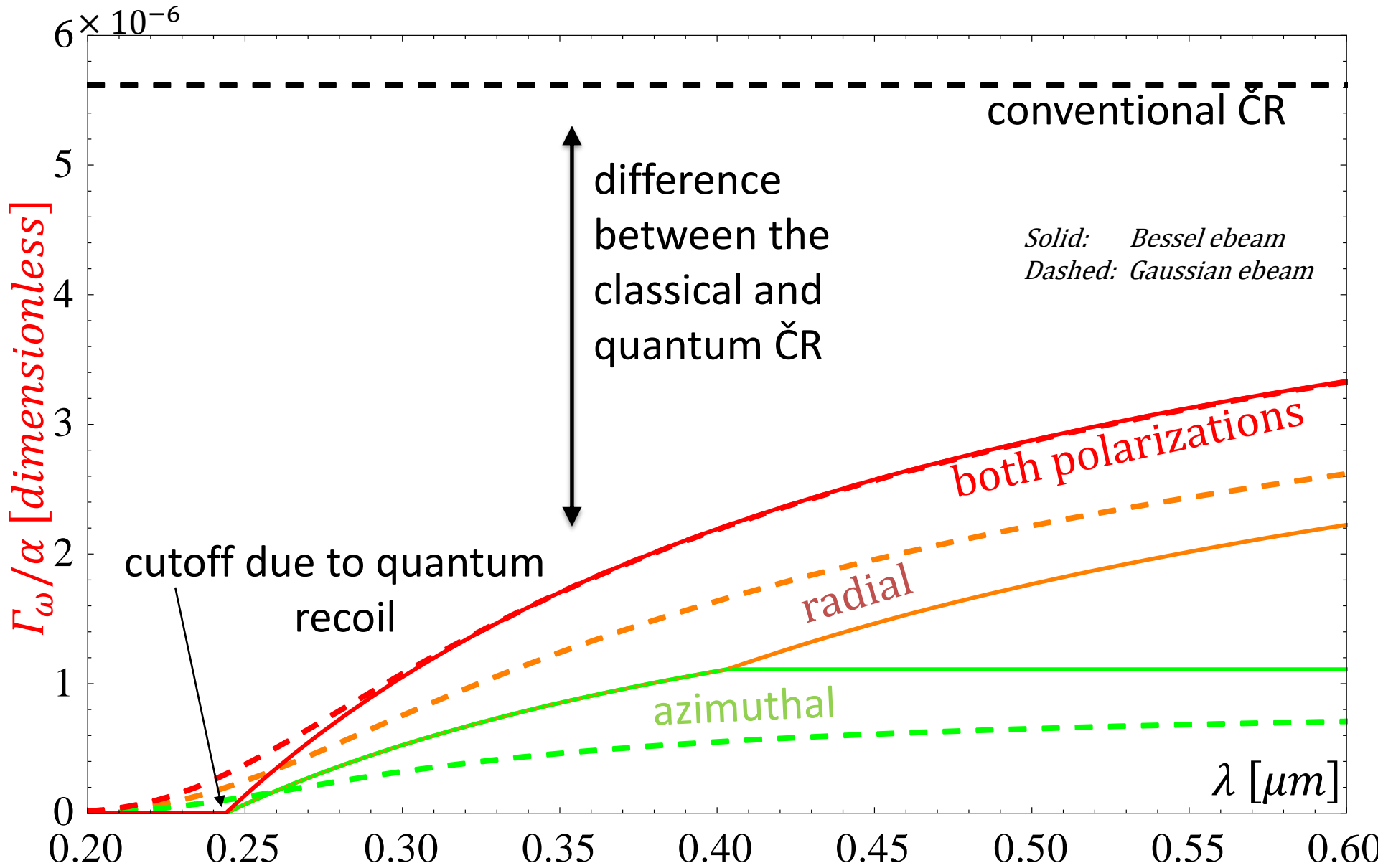
Preferred emission angles controlled by OAM, spin, and polarization

Entanglement between the charged particle and the emitted photon

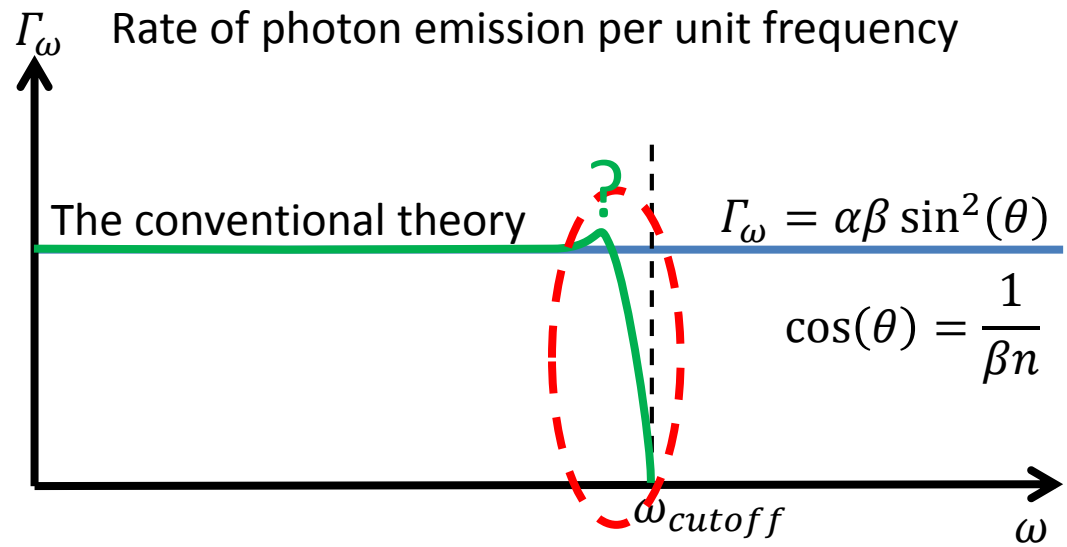
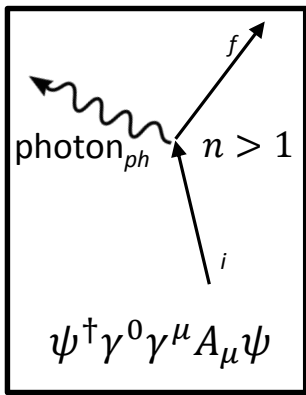


- l_i, l_f, l_{ph} Angular momenta
- $\theta_i, \theta_f, \theta_{ph}$ Spread angles
- $E_i, E_f, \hbar\omega$ Energies

Kaminer et al., "Quantum Čerenkov Radiation: Spectral Cutoffs and the Role of Spin and Orbital Angular Momentum." **PRX** 6, 011006 (2016).



Kaminer et al., "Quantum Čerenkov Radiation: Spectral Cutoffs and the Role of Spin and Orbital Angular Momentum." **PRX** 6, 011006 (2016).



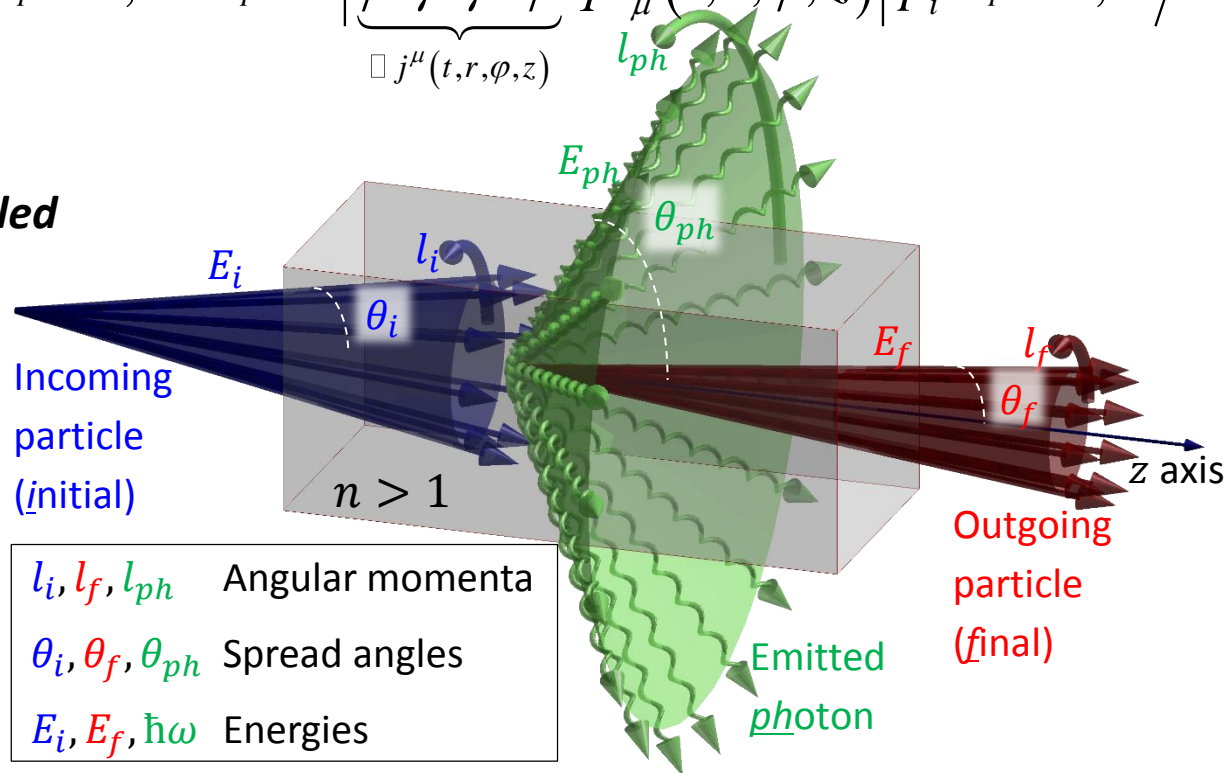
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Kaminer et al., "Quantum Čerenkov Radiation: Spectral Cutoffs and the Role of Spin and Orbital Angular Momentum." **PRX** 6, 011006 (2016).



l_i, l_f, l_{ph}	Angular momenta
$\theta_i, \theta_f, \theta_{ph}$	Spread angles
$E_i, E_f, \hbar\omega$	Energies

Spectral signature of the particle OAM

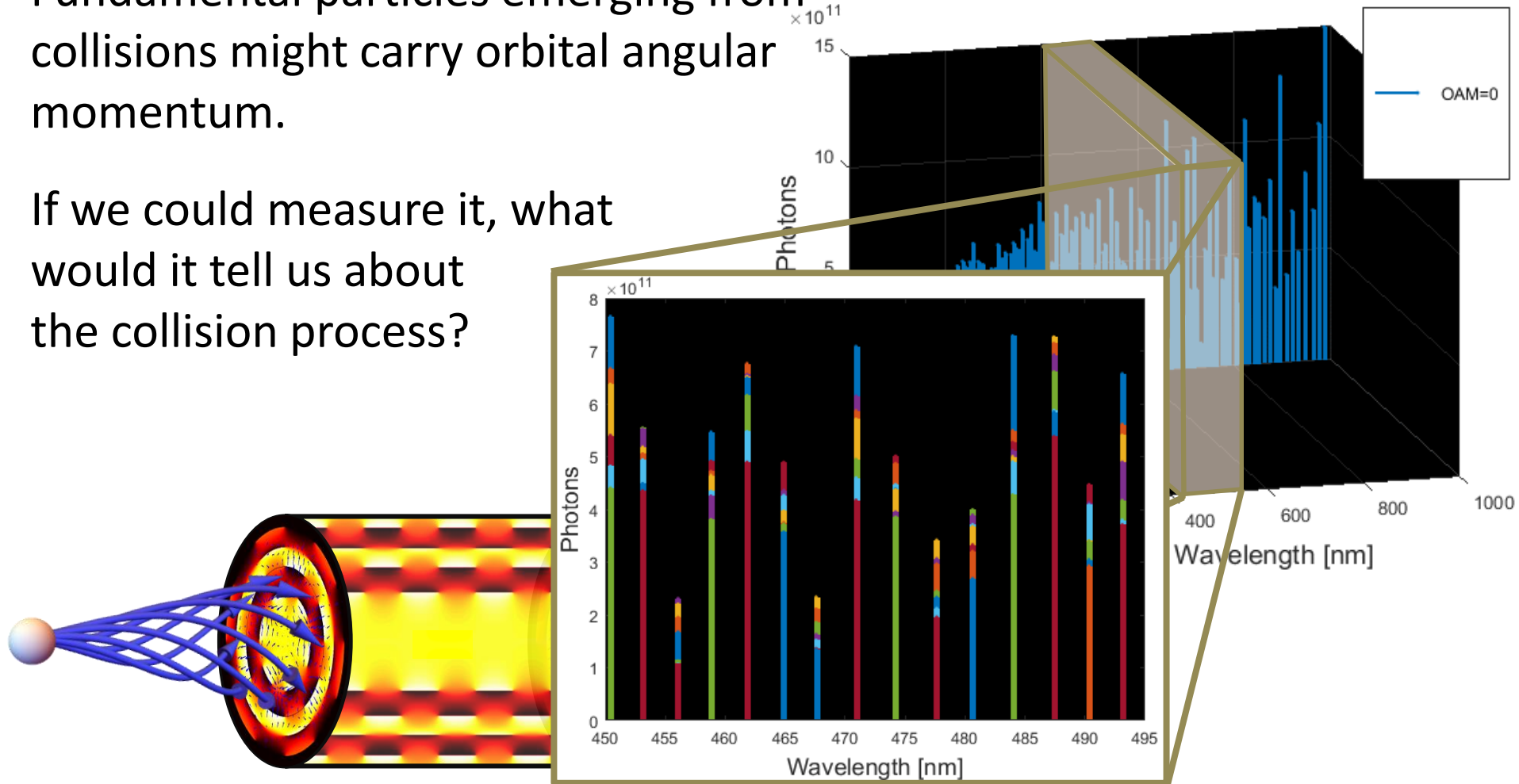
Fundamental particles emerging from collisions might carry orbital angular momentum.

If we could measure it, what would it tell us about the collision process?

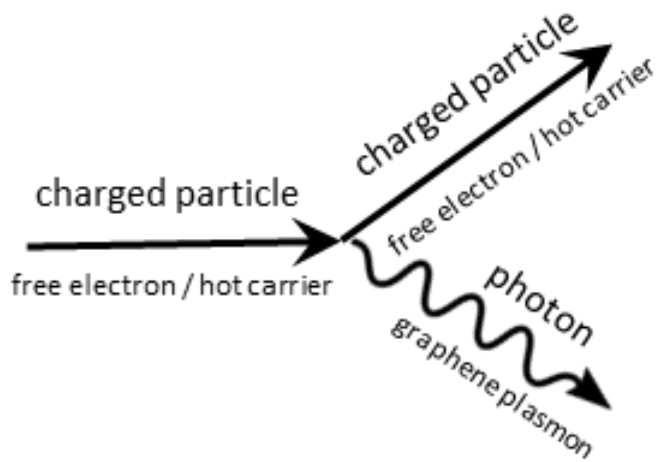
Spectral signature of the particle OAM

Fundamental particles emerging from collisions might carry orbital angular momentum.

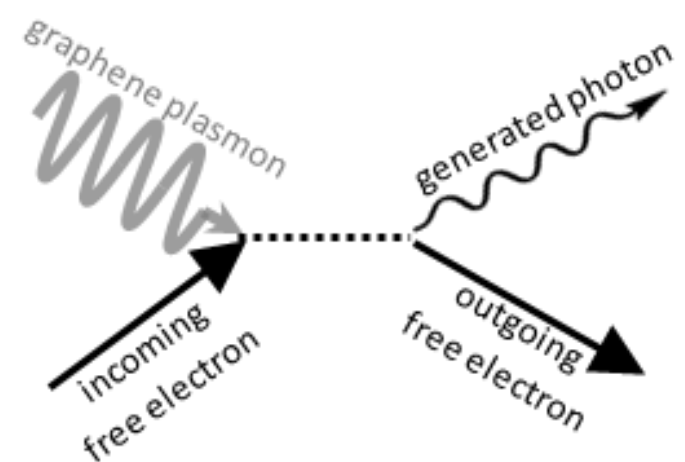
If we could measure it, what would it tell us about the collision process?



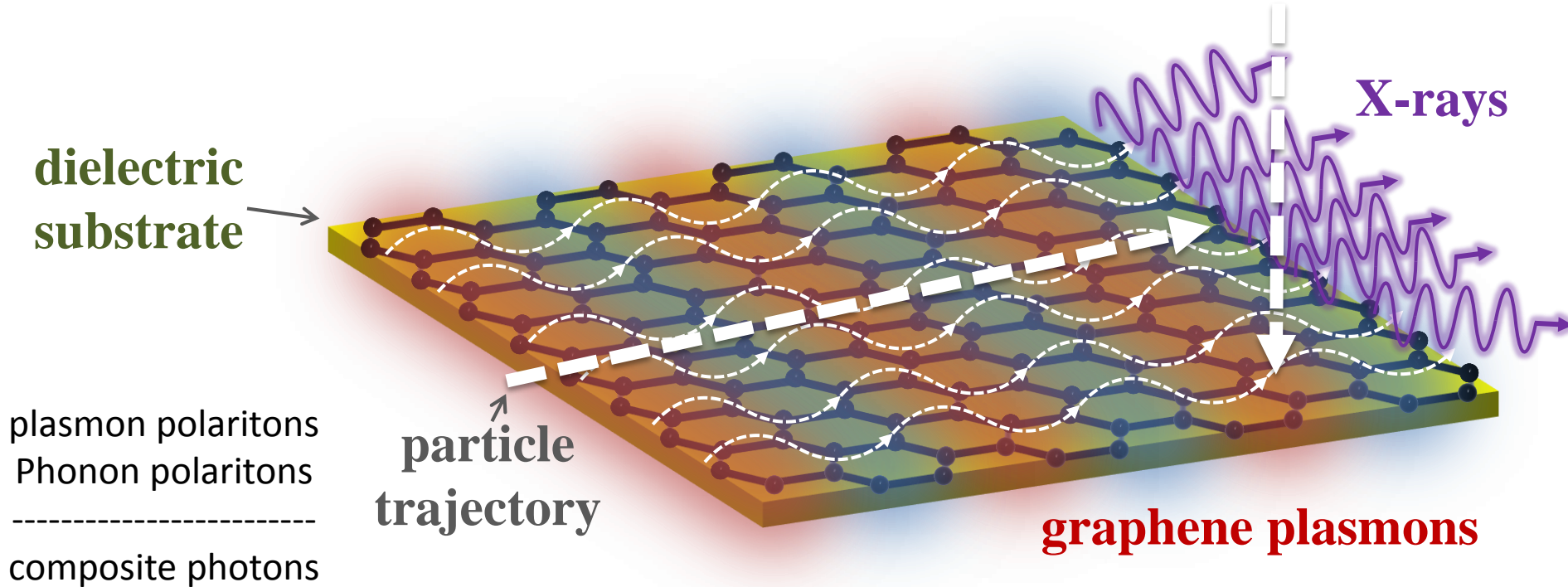
Shapira, Mutzafi, Harari, Kammer, Alon and Segev, "Čerenkov Radiation from Particles Carrying Orbital Angular Momentum in a Cylindrical Waveguide". *in preparation*



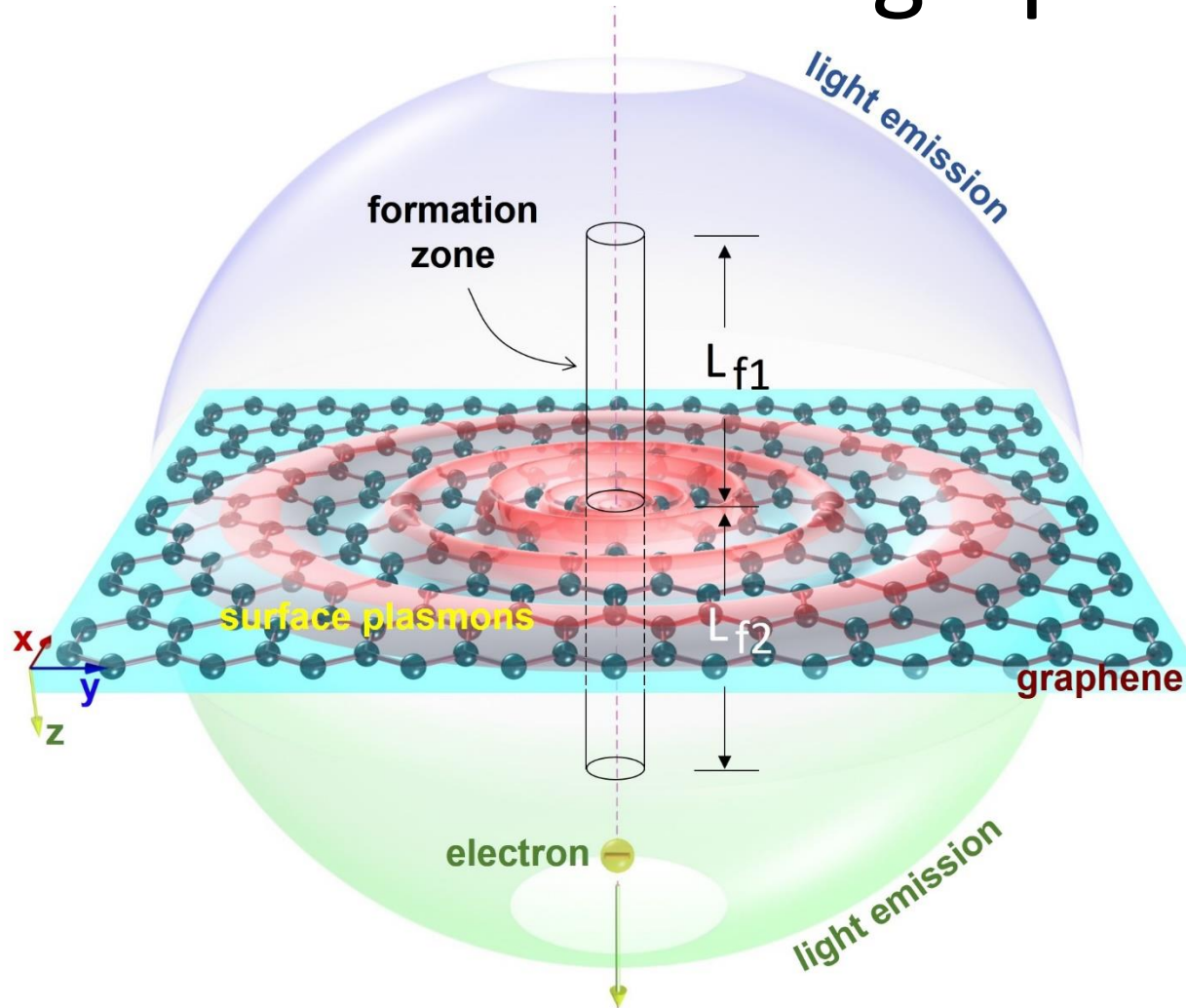
Kaminer, Katan, Buljan, Shen, Ilic, López, Wong, Joannopoulos, Soljačić
(Nature Comm., 7, 11880, (2016))



Wong*, Kaminer*, Ilic, Joannopoulos, Soljačić
(Nature Photon. 10, 46 (2016))



Transition radiation - graphene



X. Lin, I. Kaminer, X. Shi, F. Gao, Z. Yang, Z. Gao, H. Buljan, J. Joannopoulos, M. Soljačić, H. Chen, B. Zhang, "Splashing-like Dynamics of 2D Plasmons Launched by Swift Electrons" ([arXiv:1507.08369](https://arxiv.org/abs/1507.08369)).

Čerenkov Radiation

Čerenkov (*Dokl. Akad. Nauk SSSR* 2, 451, 1934)
Nobel Prize in Physics 1958

$$v_{particle} > v_{photon}$$

$$\beta c > \frac{c}{n}$$

The Čerenkov threshold: $\beta > \frac{1}{n}$

$$\cos(\theta) = \frac{1}{\beta n} \quad \text{Conventional Čerenkov angle}$$

Most of optics: $n \sim 1.5 - 2$

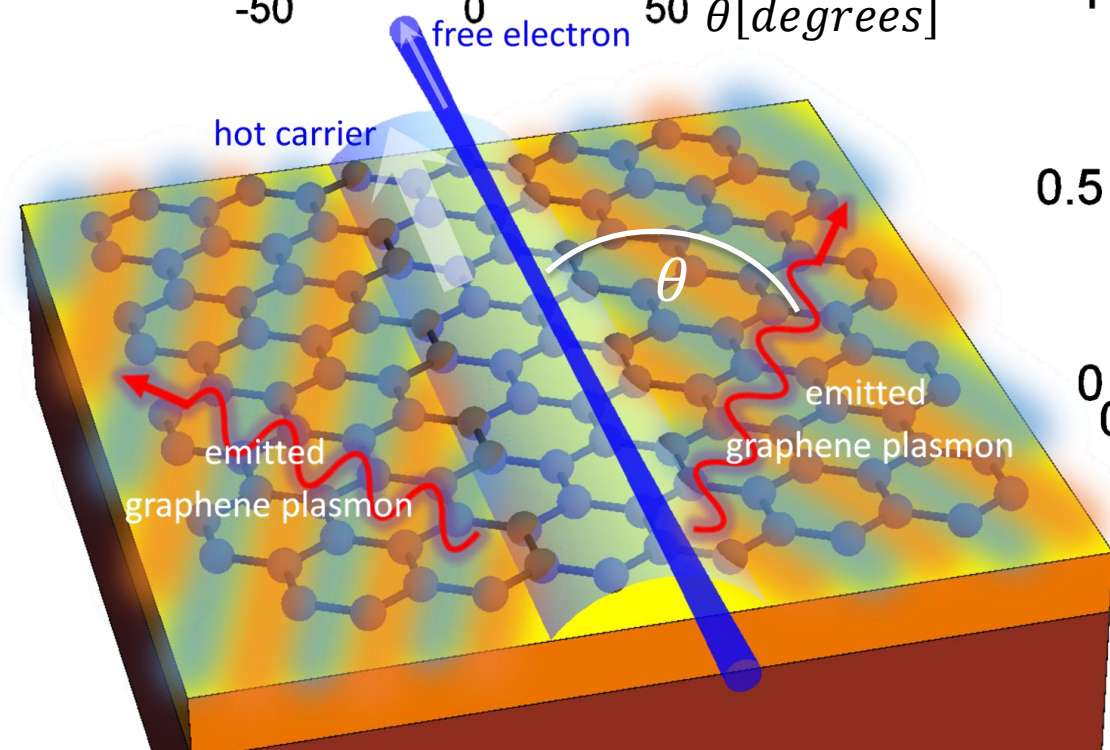
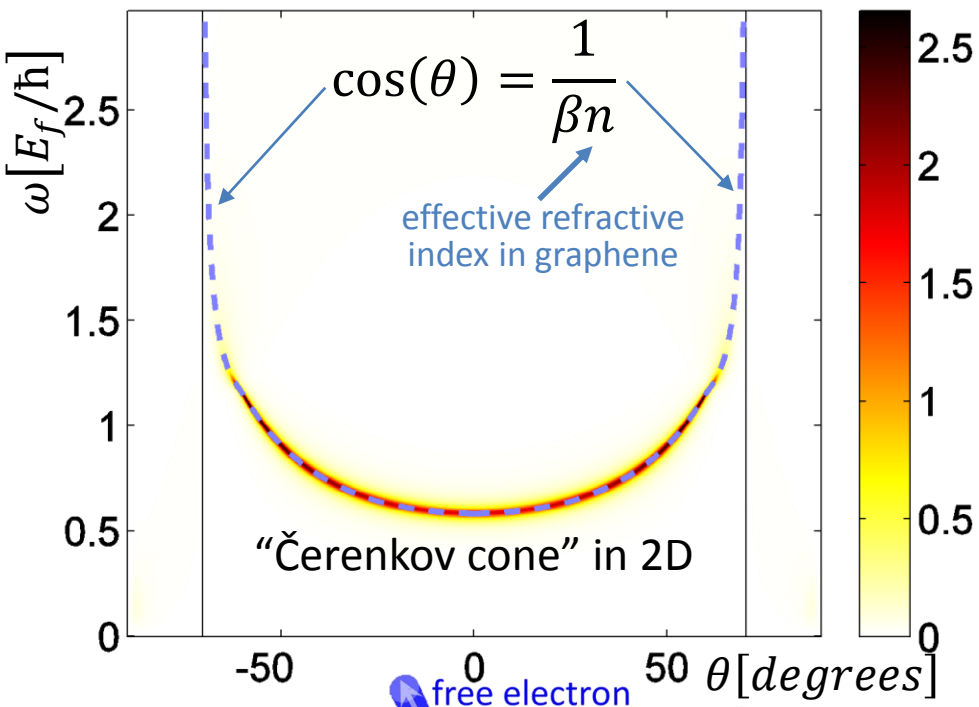
→ particle has to be relativistic

Taking this concept to graphene:

- Charge particles with non-relativistic velocities can emit Čerenkov radiation
 - Even the Fermi velocity can cross the threshold
 - Čerenkov effect from charge particles flowing inside graphene (hot carriers)
- electron's (Fermi) velocity: $c/300$ \sim plasmons (phase) velocity: $\sim c/300$
- The quantum correction becomes significant [Kaminer, et al. *PRX*, (2016)]
 - lowering the velocity threshold in graphene

effective refractive index in graphene

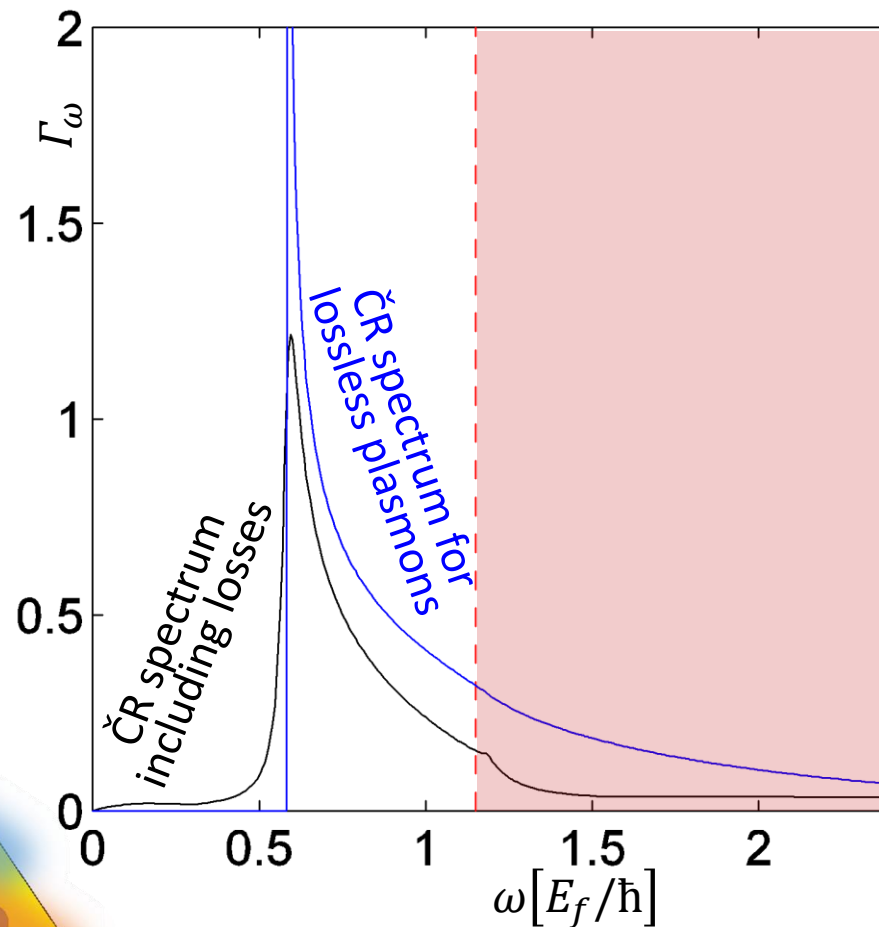
I. Kaminer, Y. T. Katan, H. Buljan, Y. Shen, O. Ilic, J. J. López, L. J. Wong, J. D. Joannopoulos, M. Soljačić (*Nature Comm.*, 7, 11880, (2016))



$$\beta c = v = 3v_f$$

$$n_s = 3 \times 10^{13} \text{ cm}^{-2} (E_f = 0.639 \text{ eV})$$

$$\Gamma_\omega \sim 1$$

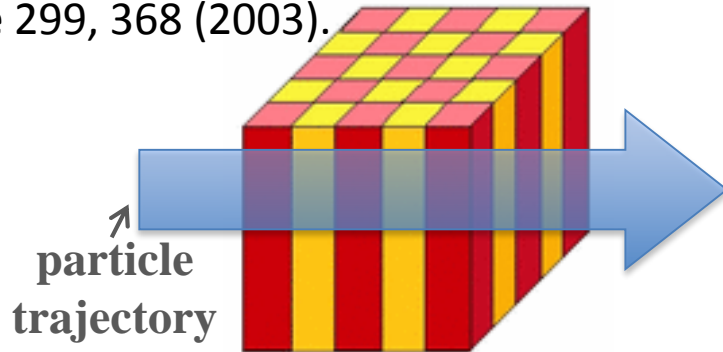


I. Kaminer, Y. T. Katan, H. Buljan, Y. Shen, O. Ilic, J. J. López, L. J. Wong, J. D. Joannopoulos, M. Soljačić
(Nature Comm., 7, 11880, (2016))

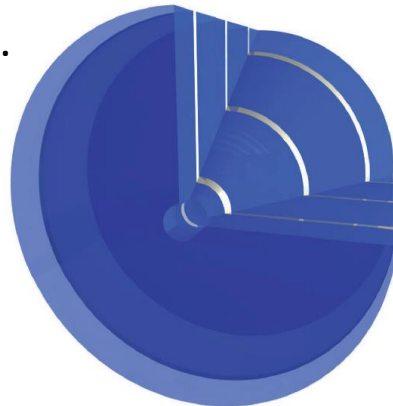
Summary

Čerenkov Radiation in Photonic Crystals

Luo, Ibanescu, Johnson, and Joannopoulos, "Čerenkov radiation in photonic crystal." *Science* 299, 368 (2003).

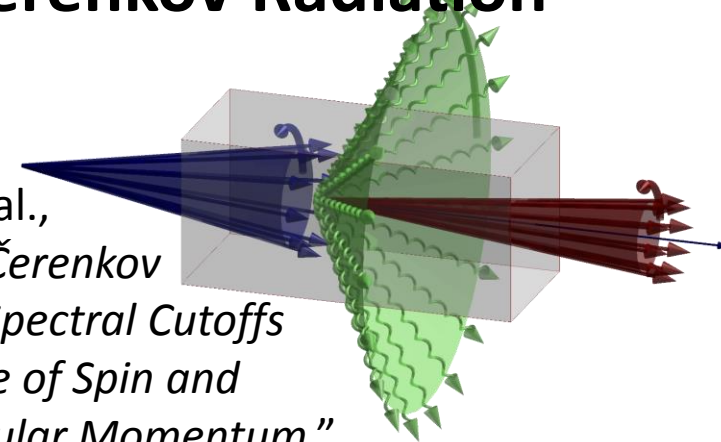


Ginis, Danckaert, Veretennicoff, & Tassin, "Controlling Čerenkov radiation with transformation-optical metamaterials." *PRL* 113, 167402 (2014).



New Effects in Čerenkov Radiation

Kaminer et al., "Quantum Čerenkov Radiation: Spectral Cutoffs and the Role of Spin and Orbital Angular Momentum." *PRX* 6, 011006 (2016).



Kaminer, et al., "Quantum Čerenkov Effect from Hot Carriers in Graphene: An Efficient Plasmonic Source." *Nature Comm.*, 7, 11880, (2016)

