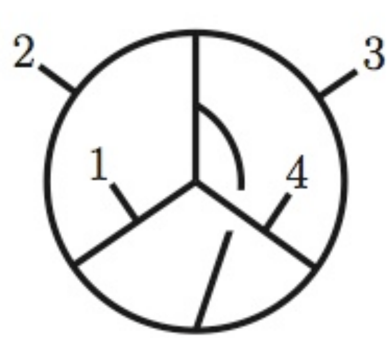
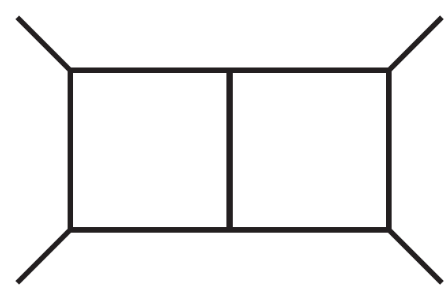
Quantum Inequivalence, Evanescent Operators and Gravity Divergences

CERN 2015

Zvi Bern, UCLA & CERN

ZB, Clifford Cheung, Huan-Hang Chi, Scott Davies, Lance Dixon, Josh Nohle (to appear)
ZB, Tristan Dennen, Scott Davies, Volodya Smirnov and Sasha Smirnov, arXiv:1309.2496
ZB, Tristan Dennen, Scott Davies, arXiv:1409.3089





Outline

1) Basic tools and ideas.

2) Review of ultraviolet properties of gravity and standard arguments, “enhanced cancellations”.

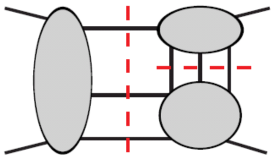
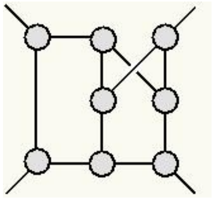
3) Conformal anomalies and quantum inequivalence under dualities.

4) Revisiting pure Einstein gravity at 2 loops. Surprising UV structure.

5) Status of supergravity divergences: Nontrivial examples of enhanced cancellations, meaning of divergences.

Our Basic Tools

We have powerful tools for computing amplitudes and for discovering new structures:

- **Unitarity method.**

  ZB, Dixon, Dunbar, Kosower
  ZB, Carrasco, Johansson , Kosower

- **Duality between color and kinematics. Gravity scattering amplitudes directly from gauge theory ones.**

  ZB, Carrasco and Johansson

- **Advanced loop integration technology.**

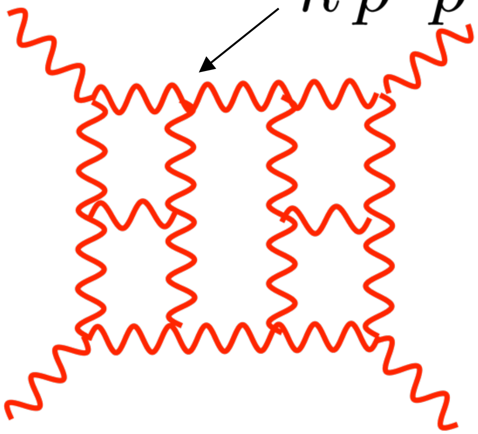
  Chetyrkin, Kataev and Tkachov; A.V. Smirnov; V. A. Smirnov, Vladimirov; Marcus, Sagnotti; Cazkon; etc

- **The above tools underlie everything we do in gravity.**
- **I won't talk about these but they underlie everything.**
- **Many other tools and advances that I won't discuss here.**

Argument for nonrenormalizability

$$\kappa = \sqrt{32\pi G_N}$$ ⟵ Dimensionful coupling

$\kappa\, p^\mu p^\nu$



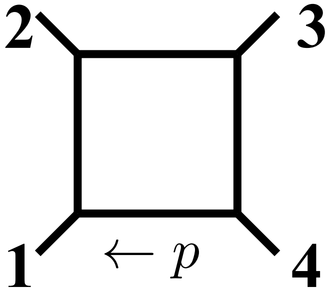
Gravity: $$\int \prod_{i=1}^{L} \frac{dp_i^D}{(2\pi)^D} \frac{(\kappa\, p_j^\mu p_j^\nu)\cdots}{\text{propagators}}$$

Gauge theory: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{(g\, p_j^\nu)\cdots}{\text{propagators}}$$

- **Extra powers of loop momenta in numerator means integrals are badly behaved in the UV.**
- **Much more sophisticated power counting in supersymmetric theories but this is basic idea.**
- **With more susy expect better UV properties.**

Enhanced UV Cancellations

Suppose diagrams in *all* possible covariant diagrammatic representations are UV divergent.



Pure gravity diagram necessarily is badly divergent

$$n_i \sim \prod_{i=1}^{4} p_\mu p_\nu \varepsilon_i^{\mu\nu}$$

Can't be moved to other diagrams

If sum over diagrams is UV finite by definition we have an "enhanced cancellation".

Pure Einstein gravity

$$\mathcal{L} = \sqrt{-g}R$$

Despite divergent diagrams, pure gravity is one loop finite

't Hooft and Veltman (1974)

One Loop Pure Gravity

Standard finiteness argument for 1 loop finiteness of pure gravity:

't Hooft and Veltman (1974)

$\cancel{R^2}$ $\quad$ $\cancel{R_{\mu\nu}^2}$

$\cancel{R_{\mu\nu\rho\sigma}^2}$

Counterterms vanish by equation of motion and can be eliminated by field redefinition.

In *D* = 4 asymptotically flat space Gauss-Bonnet theorem eliminates Riemann square term.

$$\int d^4x \sqrt{-g}(R^2 - 4R_{\mu\nu} + R_{\mu\nu\rho\sigma}^2) = 32\pi^2\chi$$

Pure gravity divergence with nontrivial topology:

$$\mathcal{L}^{\mathrm{GB}} = \frac{1}{(4\pi)^2}\frac{53}{90\epsilon}(R^2 - 4R_{\mu\nu} + R_{\mu\nu\rho\sigma}^2)$$

Capper and Duff (1974)
Tsao (1977); Critchley (1978)
Gibbons, Hawking, Perry (1978)
Goroff and Sagnotti (1986)
Bornsen and van de Ven (2009)

Euler characteristic vanishes in flat space. 't Hooft and Veltman (1974)

Dimensional regularization makes it subtle. Capper and Kimber (1980)

The Conformal Anomaly

Capper and Duff (1974); Tsao (1977); Critchley (1978); Gibbons, Hawking, Perry (1978); Critchly (1978); Duff and van Nieuwenhuizen (1980); Siegel (1980); Grisaru, Nielsen, Siegel, Zanon (1984); Goroff and Sagnotti (1986); Bornsen and van de Ven (2009); Etc.

$$T^{\mu}{}_{\mu} = -\frac{1}{(4\pi)^2}\frac{2}{360}\Big(\underbrace{4\cdot 53}_{\text{graviton}} + \underbrace{1}_{\text{scalar}} + \underbrace{91}_{\text{2 form}} - \underbrace{180}_{\text{3 form}}\Big)\underbrace{(R^2 - 4R_{\mu\nu} + R^2_{\mu\nu\rho\sigma})}_{\text{Gauss-Bonnet}}$$

Referred to as conformal, trace or Weyl anomaly.

The Gauss-Bonnet counterterm exactly corresponds to trace anomaly

$$\mathcal{L}^{\rm GB} = \frac{1}{(4\pi)^2}\frac{1}{360\epsilon}\Big(4\cdot 53 + 1 + 91 - 180\Big)\sqrt{-g}(R^2 - 4R_{\mu\nu} + R^2_{\mu\nu\rho\sigma})$$

$$D = 4 - 2\epsilon$$

At first sight this result looks wrong: In $D = 4$ a three form has zero degrees of freedom. Also two form is dual to (pseudo)scalar.

Quantum Inequivalence

$$T^{\mu}{}_{\mu} = -\frac{1}{(4\pi)^2}\frac{2}{360}\Big(4\cdot 53 + 1 + 91 - 180\Big)(R^2 - 4R_{\mu\nu} + R^2_{\mu\nu\rho\sigma})$$

graviton scalar 2 form 3 form Gauss-Bonnet

two form dual to scalar

$$\partial_\mu \phi = \varepsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma}$$

three form not dynamical

$$\Lambda = \varepsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho\sigma}$$

- **Quantum inequivalence under duality transformations.** Duff and van Nieuwenhuizen (1980)
- **Quantum equivalence under duality. Gauge dependence.** Seigel (1980)
- **Quantum equivalence of UV (ignoring trace anomaly).** Fradkin and Tseytlin (1984)
- **Quantum equivalence of at 1 loop effective action (with repeat of Siegel's argument for higher loops)** Grisaru, Nielsen, Siegel, Zanon (1984)

What is physical significance?

One loop really isn't good enough because anyway evanescent

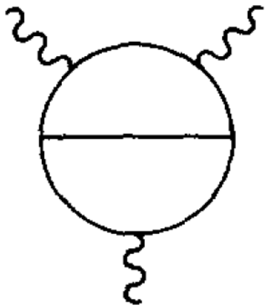
Two Loop Pure gravity

Pure gravity is "well understood":
By two loops there is a valid *R*³ counterterm and corresponding divergence.

Goroff and Sagnotti (1986); Van de Ven (1992)

Using standard MS-bar prescriptions Goroff and Sagnotti showed Einstein gravity diverges at 2 loops.

$$\mathcal{L}^{R^3} = \frac{209}{2880}\frac{1}{(4\pi)^4}\frac{1}{2\epsilon}\sqrt{-g}\,R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}$$



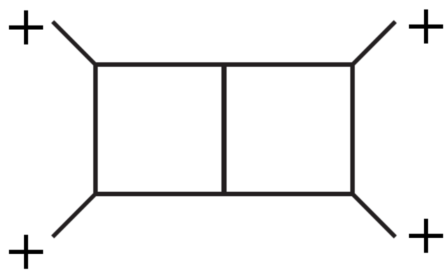
$D = 4 - 2\epsilon$

- **The Goroff and Sagnotti result is definitely correct in all details.**
- **There does not seem to be anything weird going on here.**

However, the goal of this talk is to show you that UV in gravity subtle and weird.

Two Loop Amplitudes

ZB, Cheung, Chi, Davies, Dixon and Nohle (to appear)

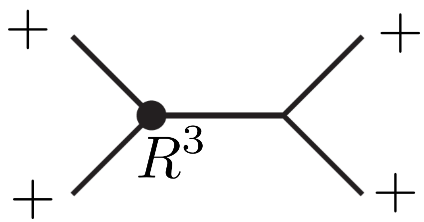


Initial Questions:

1. **Does the conformal anomaly feed into the two-loop divergence?**

2. **Are there physical effects in asymptotically flat space from the conformal anomaly?**

Two Loop Identical Helicity Amplitude

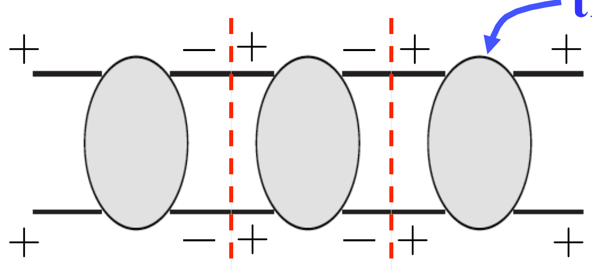


Pure gravity identical helicity amplitude sensitive to Goroff and Sagnotti divergence.

$$A^{R^3} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4} \frac{209}{24} \frac{1}{(4\pi)^4} \frac{1}{\epsilon}\, stu\, \mathcal{T}^2 \qquad \mathcal{T} = \frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

Curious feature:

tree amplitude vanishes



- **Unitarity cut vanishes for four-dimensional loop momenta.**
- **Nonvanishing because of ϵ-dimensional loop momenta.**

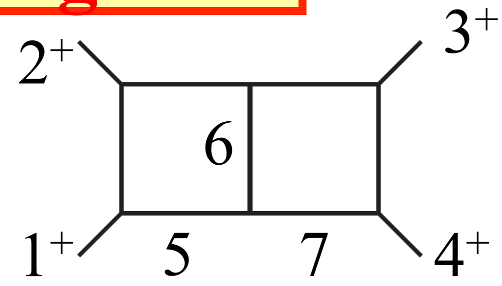
A surprise:

Divergence is *not* generic but is tied to anomalous behavior!

Bardeen & Cangemi pointed out nonvanishing of identical helicity is connected to anomaly in self-dual symmetries.

Full Two-Loop Integrand

Using spinor helicity very compact:



$$N_D = \frac{1}{2}D(D-3)$$

$$t_0 = 16((\vec{\mu}_5\cdot\vec{\mu}_7)^2 - \mu_5^2\mu_7^2)s_{12}$$

$$t_1 = \mu_5^2\mu_7^2 s_{12} + \mu_5^2(\vec{\mu}_5+\vec{\mu}_7)^2 s_{12} + \mu_7^2(\vec{\mu}_5+\vec{\mu}_7)^2 s_{12} - 4\vec{\mu}_5\cdot\vec{\mu}_7(\mu_5^2+\mu_7^2)s_{67}$$

$$t_2 = -\frac{\mu_5^2\mu_7^2}{s_{12}}s_{68}s_{67}$$

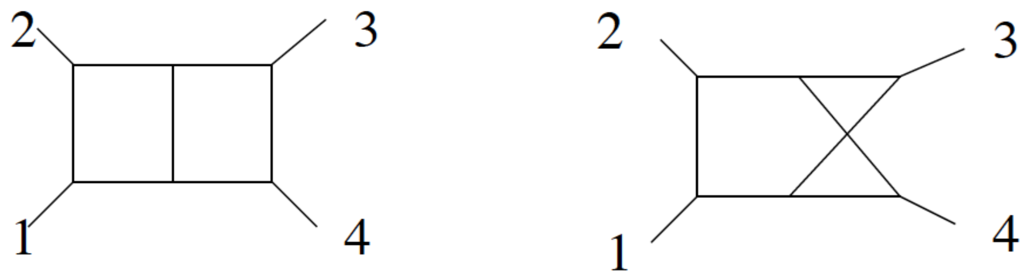
$$p_i = p_i^{(4)} + \mu_i$$

$$\begin{aligned} n =& +N_D^2 t_2^2 + D N_D t_2 t_1 + N_D\Big(\Big((\mu_5^2+\mu_7^2)((\vec{\mu}_5+\vec{\mu}_7)^2 s_{12} - 4\vec{\mu}_5\cdot\vec{\mu}_7 s_{67})\Big)^2 \\ &+ \mu_5^4\mu_7^4(s_{12}^2 + 16 s_{68}s_{67}) - 2\mu_5^2\mu_7^2\vec{\mu}_5\cdot\vec{\mu}_7(\vec{\mu}_5+\vec{\mu}_7)^2 s_{12}^2 + 4\mu_5^2\mu_7^2\vec{\mu}_5\cdot\vec{\mu}_7(\mu_5^2+\mu_7^2)s_{67}s_{12} \\ &+ 4((\vec{\mu}_5\cdot\vec{\mu}_7)^2 - \mu_5^2\mu_7^2)(\mu_5^4+\mu_7^4)s_{67}s_{68}\Big) \\ &+ 3D\Big(-8\mu_5^4\mu_7^4 s_{68}s_{67} - 6\mu_5^2\mu_7^2(\vec{\mu}_5\cdot\vec{\mu}_7)(\mu_5^2+\mu_7^2)s_{67}s_{12} + 3s_{12}^2(\vec{\mu}_5\cdot\vec{\mu}_7)\mu_5^2\mu_7^2(\vec{\mu}_5+\vec{\mu}_7)^2 \\ &+ s_{12}^2\mu_5^2\mu_7^2(\mu_5^2-\mu_7^2)^2 - 4((\vec{\mu}_5\cdot\vec{\mu}_7)^2 - \mu_5^2\mu_7^2)\Big(-s_{12}(\mu_5^2+\mu_7^2)((\vec{\mu}_5+\vec{\mu}_7)^2 s_{12} - 4(\vec{\mu}_5\cdot\vec{\mu}_7)s_{67}) \\ &+ 4\mu_5^2\mu_7^2 s_{67}s_{68}\Big)\Big) + 144((\vec{\mu}_5\cdot\vec{\mu}_7)^2 - \mu_5^2\mu_7^2)^2 s_{12}^2 \end{aligned}$$

- **Integrand vanishes for *D* = 4 loop momenta.**
- **Nonplanar similar.**
- **Upon integration ultraviolet divergent, but certainly not generic.**

Two Loop Bare Divergence

ZB, Cheung, Chi, Davies, Dixon and Nohle (to appear)



$$209 = 11 \cdot 19$$

$$3431 = 47 \cdot 73$$

Integrating we obtain:

$$\mathcal{M}_4^{(2)}\Big|_{\rm bare} = -\left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4} \frac{3431}{5400\epsilon} stu\mathcal{T}^2$$

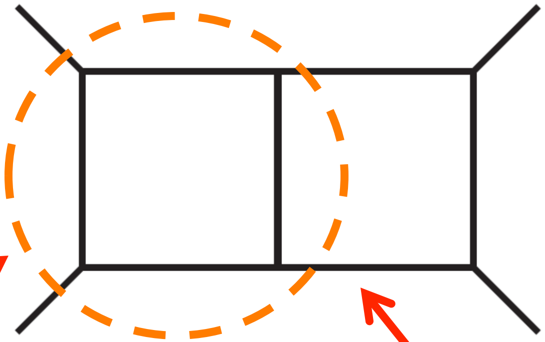
Not the same as the Goroff and Sagnotti result

However, Goroff and Sagnotti subtracted subdivergences integral by integral.

Subdivergences? What subdivergences?
There are no one-loop divergences. Right?

Subdivergences?

The integrand has subdivergences



representative diagram
sum over all diagrams

Gauss-Bonnet subdivergence

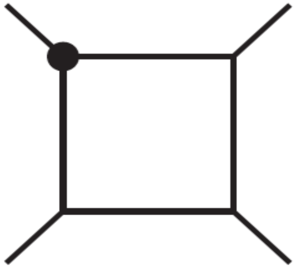
$D = 4$, **no subdivergences**
$D \neq 4$, **subdivergences!**

A very strange phenomenon: no one loop divergences,
Yet there are one-loop subdivergences!

To match the G&S result we need to subtract subdivergences.
We use counterterm method.

Gauss-Bonnet

Gauss-Bonnet insertion:



$$\mathcal{L}^{\mathrm{GB}} = \frac{1}{(4\pi)^2}\frac{53}{90\epsilon}(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma})$$

- **total derivative in *D* = 4**
- **evanescent operator**

Gauss-Bonnet counterterm inserted into one loop

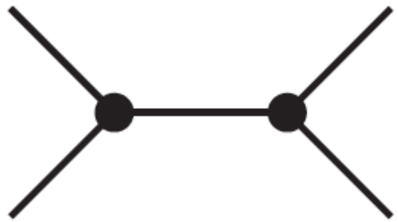
$$\mathcal{M}_4^{(1)\mathrm{GB}}\Big|_{\mathrm{div.}} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\frac{689}{675\epsilon} stu\mathcal{T}^2 \qquad \mathcal{T} \equiv \frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

Even though *no* one-loop divergence there is a one-loop subdivergence!

- **GB evanescent term contributes at two loops even in flat space!**
- **It plays an important role in UV structure.**

Double Gauss-Bonnet Counterterm

double Gauss-Bonnet counterterm:



$$D = 4 - 2\epsilon$$

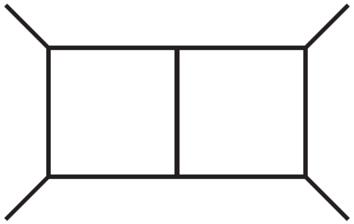
$$\mathcal{M}_4^{(0)\mathrm{GB}^2}\Big|_{\mathrm{div.}} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\frac{5618}{675\epsilon} stu\mathcal{T}^2$$

$$\mathcal{T} \equiv \frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

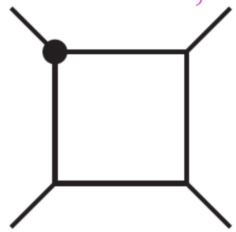
UV divergence

Two Loop Identical Helicity Amplitude

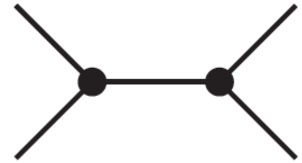
ZB, Cheung, Chi, Davies, Dixon and Nohle (to appear)



bare



single GB counterterm



double GB counterterm

$$\mathcal{T} \equiv \frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

Add the pieces:

$$\mathcal{M}_4^{(2)}\Big|_{\text{bare}} = -\left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\frac{3431}{5400\epsilon} stu\mathcal{T}^2$$

$$\mathcal{M}_4^{(1)\text{GB}}\Big|_{\text{div.}} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\frac{689}{675\epsilon} stu\mathcal{T}^2$$

$$\mathcal{M}_4^{(0)\text{GB}^2}\Big|_{\text{div.}} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\frac{5618}{675\epsilon} stu\mathcal{T}^2$$

$$A^{R^3} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\frac{209}{24}\frac{1}{(4\pi)^4}\frac{1}{\epsilon}\, stu\,\mathcal{T}^2$$

Goroff and Sagnotti divergence reproduced

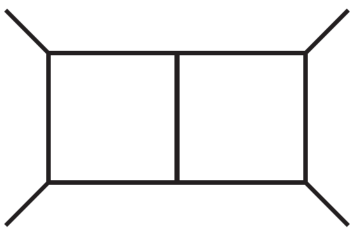
This is pure Einstein gravity. Demonstrates directly the central role the conformal anomaly and GB term play in divergence.

Three Forms

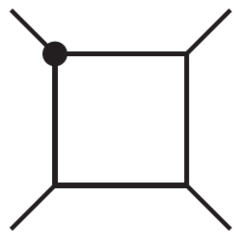
What does the divergence mean?

Let's add a three form:

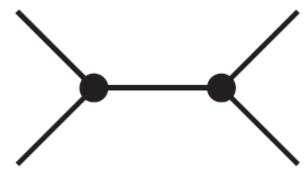
- **On the one hand no degrees of freedom in *D* = 4, so no change in divergence expected.**
- **On the other hand the conformal anomaly and Gauss-Bonnet term is effected, so we do expect divergence to change.**



bare



single GB counterterm



double GB counterterm:

Pure gravity $$M^{(2)}\Big|_{\text{div.}} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4} \mathcal{T}^2 \frac{1}{\epsilon} \frac{209}{24}$$

Gravity+3 form $$M^{(2)}\Big|_{\text{div.}} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4} \mathcal{T}^2 \frac{1}{\epsilon} \frac{29}{24}$$

Four Theories and Dualities

ZB, Cheung, Chi, Davies, Dixon and Nohle (to appear)

$$\mathcal{L}_g = \frac{2}{\kappa^2}\sqrt{-g}R$$

$$\mathcal{L}_{gd} = \sqrt{-g}\Big(\frac{2}{\kappa^2}R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi\Big)$$

$$\mathcal{L}_{ga} = \sqrt{-g}\Big(\frac{2}{\kappa^2}R + \frac{1}{6}H_{\mu\nu\rho}H^{\mu\nu\rho}\Big)$$

Focus on these two

$$\mathcal{L}_{gda} = \sqrt{-g}\Big(\frac{2}{\kappa^2}R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{6}e^{-2\kappa\phi/\sqrt{D-2}}H_{\mu\nu\rho}H^{\mu\nu\rho}\Big)$$

The *gd* and *ga* theories are equivalent by duality

$$\partial_\mu\phi = \varepsilon_{\mu\nu\rho\sigma}H^{\nu\rho\sigma}$$

Divergences Differ Under Dualities

ZB, Cheung, Chi, Davies, Dixon and Nohle (to appear)

Coefficient of $\frac{1}{\epsilon}$

Note conformal anomaly

theory	g	gd	ga
bare divergence	$-\frac{3431}{5400}$	$-\frac{793}{1200}$	$\frac{2027}{1200}$
GB c.t. in 1 loop	$\frac{4\cdot 53}{360}\cdot\frac{2\cdot 13}{15}$	$\frac{4\cdot 53+1}{360}\cdot\frac{2\cdot(13-1)}{15}$	$\frac{4\cdot 53+91}{360}\cdot\frac{2\cdot(13-91)}{15}$
GB^2 c.t. in tree	$24\left(\frac{4\cdot 53}{360}\right)^2$	$24\left(\frac{4\cdot 53+1}{360}\right)^2$	$24\left(\frac{4\cdot 53+91}{360}\right)^2$
RHH c.t. in 1 loop	0	0	$\frac{1}{4}\cdot 20$
total	$\frac{209}{24}$	$\frac{139}{16}$	$\frac{239}{16}$

G&S

Same under duality but divergences differ

These results for UV divergences suggest that theories are quantum mechanically *in*equivalent as proposed by Duff and van Nieuwenhuizen. But wait: what about finite parts?

The Scattering Amplitudes

Pure Gravity:

$$\mathcal{M}^{(2)}(1^+,2^+,3^+,4^+) = \mathcal{N}\Big\{\frac{1}{\epsilon}\frac{209}{24}stu + \frac{117617}{21600}stu + \frac{1}{10}stu\log(-s) - \frac{1}{60}s^3\log(-s) + \frac{1}{120}(s^2+t^2+u^2)\,s\log(-s)^2 + \text{perms}\Big\}$$

Gravity + 3 Form:

$$\mathcal{M}^{(2)}(1^+,2^+,3^+,4^+) = \mathcal{N}\Big\{\frac{1}{\epsilon}\frac{29}{24}stu + \frac{411617}{21600}stu + \frac{1}{10}stu\log(-s) - \frac{1}{60}s^3\log(-s) + \frac{1}{120}(s^2+t^2+u^2)\,s\log(-s)^2 + \text{perms}\Big\}$$

Divergences are different but logarithms are identical!
No physical effect! The 3 form is a Cheshire Cat field.

Using a scheme with or without the 3 form changes the divergence but not the physics. Similar results comparing *gd* to *ga* theories

The Scattering Amplitudes

If actual divergences depend on UV completion then what is meaningful?

Renormalization scale: $\log(M^2)$

In all theories we have studied. Independent of duality transformations for two loop four graviton amplitude we find

$$M^{(2)}\Big|_{\text{div.}} = -\left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\, \mathcal{T}^2\, \frac{N_s}{4} \log(M^2)$$

N_s **is number of states in the theory.**

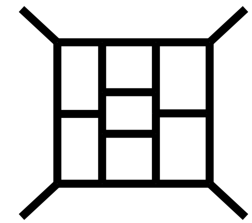
A much more sensible measure of the divergence properties. Also note the simplicity of the number!

Status of Supergravity Divergences

Current Status of $N = 8$ Divergences

Consensus is that in $N = 8$ supergravity trouble starts at 5 loops and by 7 loops we have valid UV counterterm in $D = 4$ under all known symmetries (suggesting divergences).

Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Björnsson ; Bossard , Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger



For $N = 8$ sugra in $D = 4$:

- **All counterterms ruled out until 7 loops.**
- **D^8R^4 counterterm available at 7 loops under all known symmetries. Oddly, it is not a full superspace integral.**
- **All earlier calculations explained.**

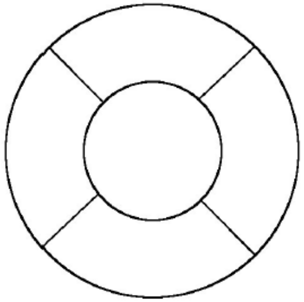
Bossard, Howe, Stelle and Vanhove

Based on this, a reasonable person would conclude that $N = 8$ supergravity almost certainly diverges at 7 loops in $D = 4$.

Predictions of Ultraviolet Cancellations

Björnsson and Green developed a first quantized formulation of Berkovits' pure-spinor formalism.

Key point: *all* supersymmetry cancellations are exposed.



They identify contributions that are poorly behaved.

Poor UV behavior, unless new types of cancellations between diagrams exist that are "not consequences of supersymmetry in any conventional sense":

Bjornsson and Green

- **$N = 8$ sugra should diverge at 7 loops in $D = 4$.**
- **$N = 8$ sugra should diverge at 5 loops in $D = 24/5$.**

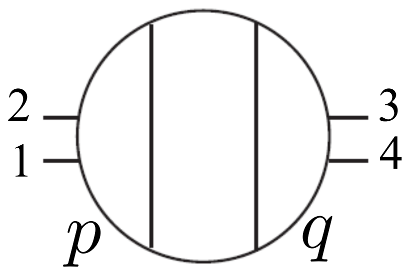
Maximal Cut Power Counting

ZB, Davies, Dennen

Maximal cuts of diagrams poorly behaved:

already log divergent

$\mathcal{N}=4$ sugra



$\mathcal{N}=4$ sugra: pure YM x $\mathcal{N}=4$ sYM

$$n_i \sim s^3 t A_4^{\rm tree} \, (p\cdot q)^2 \, \varepsilon_1 \cdot p \, \varepsilon_2 \cdot p \, \varepsilon_3 \cdot q \, \varepsilon_4 \cdot q + \ldots$$

This diagram is log divergent

$\mathcal{N}=8$ sugra should diverge at 7 loops in $D=4$. **Bet with David Gross**

$\mathcal{N}=8$ sugra should diverge at 5 loops in $D=24/5$ **Bet with Kelly Stelle**

$\mathcal{N}=4$ sugra should diverge at 3 loops in $D=4$

$\mathcal{N}=5$ sugra should diverge at 4 loops in $D=4$

Unfortunately no bets

This result equivalent to Björnsson and Green's approach: Identify poorly behaved terms and count.

All other groups that looked at the question of symmetries agree. Looked like a safe bet that these divergences are present.

Examples of Enhanced Cancellations

A safe bet can be wrong: phenomenon of enhanced cancellations not taken into account.

Three examples in supergravity:

1) $\mathcal{N} = 4$ **supergravity in** $D = 4$ **at 3 loops.**

2) $\mathcal{N} = 5$ **supergravity in** $D = 4$ **at 4 loops.**

3) **Half-maximal supergravity in** $D = 5$ **at 2 loops.**

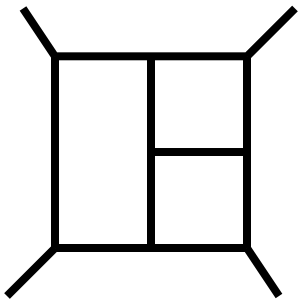
At present no known standard symmetry explanation for any of these (though last one explained via color kinematics duality)

Three-Loop $N = 4$ Supergravity Construction

Use duality between color and kinematics ZB, Carrasco, Johansson

$N = 4$ sugra : ($N = 4$ sYM) x ($N = 0$ YM)

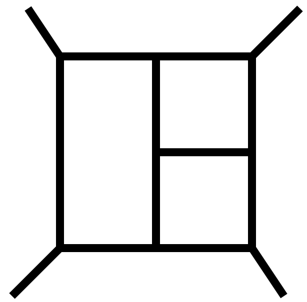
$N = 4$ sYM



$$l \cdot k\, s^2 t A_4^{\text{tree}}$$

BCJ representation

pure YM



$$(\varepsilon \cdot l)^4\, l^4$$

Feynman representation

$c_i \rightarrow n_i$

$N = 4$ sugra diagrams linearly divergent

$$\int (d^D l)^3 \frac{k^7 l^9}{l^{20}}$$

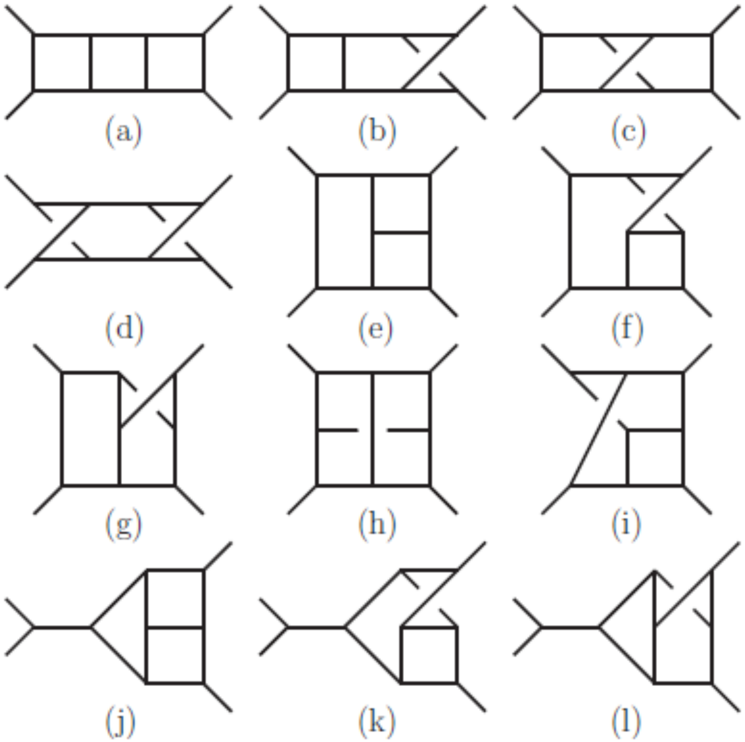
ZB, Davies, Dennen, Huang

- **Ultraviolet divergences are obtained by series expanding small external momentum (or large loop momentum).**
- **Introduce mass regulator for IR divergences.**
- **In general, subdivergences must be subtracted.**

Vladimirov; Marcus and Sagnotti

$N = 4$ Supergravity Enhanced Cancellation

ZB, Davies, Dennen, Huang



Graph	$(\text{divergence})/(\langle 12\rangle^2[34]^2 st A^{\text{tree}}(\frac{\kappa}{2})^8)$
(a)-(d)	0
(e)	$\frac{263}{768}\frac{1}{\epsilon^3}+\frac{205}{27648}\frac{1}{\epsilon^2}+\left(-\frac{5551}{768}\zeta_3+\frac{326317}{110592}\right)\frac{1}{\epsilon}$
(f)	$-\frac{175}{2304}\frac{1}{\epsilon^3}-\frac{1}{4}\frac{1}{\epsilon^2}+\left(\frac{593}{288}\zeta_3-\frac{217571}{165888}\right)\frac{1}{\epsilon}$
(g)	$-\frac{11}{36}\frac{1}{\epsilon^3}+\frac{2057}{6912}\frac{1}{\epsilon^2}+\left(\frac{10769}{2304}\zeta_3-\frac{226201}{165888}\right)\frac{1}{\epsilon}$
(h)	$-\frac{3}{32}\frac{1}{\epsilon^3}-\frac{41}{1536}\frac{1}{\epsilon^2}+\left(\frac{3227}{2304}\zeta_3-\frac{3329}{18432}\right)\frac{1}{\epsilon}$
(i)	$\frac{17}{128}\frac{1}{\epsilon^3}-\frac{29}{1024}\frac{1}{\epsilon^2}+\left(-\frac{2087}{2304}\zeta_3-\frac{10495}{110592}\right)\frac{1}{\epsilon}$
(j)	$-\frac{15}{32}\frac{1}{\epsilon^3}+\frac{9}{64}\frac{1}{\epsilon^2}+\left(\frac{101}{12}\zeta_3-\frac{3227}{1152}\right)\frac{1}{\epsilon}$
(k)	$\frac{5}{64}\frac{1}{\epsilon^3}+\frac{89}{1152}\frac{1}{\epsilon^2}+\left(-\frac{377}{144}\zeta_3+\frac{287}{432}\right)\frac{1}{\epsilon}$
(l)	$\frac{25}{64}\frac{1}{\epsilon^3}-\frac{251}{1152}\frac{1}{\epsilon^2}+\left(-\frac{835}{144}\zeta_3+\frac{7385}{3456}\right)\frac{1}{\epsilon}$

Spinor helicity used to clean up table, but calculation for all states

***All* three-loop divergences and subdivergences cancel completely!**

3-loop 4-point $N = 4$ sugra UV finite contrary to predictions

A pity we did not bet on this theory

Tourkine and Vanhove understand this result by extrapolating from two-loop heterotic string amplitudes.

Explanations?

Key Question:

Is there an ordinary symmetry explanation for this?
Or is something extraordinary happening?

Bossard, Howe and Stelle (2013) showed that 3 loop finiteness of N =4 sugra can be explained by ordinary superspace + duality symmetries, *assuming* a 16 supercharge off-shell superspace exists.

If true, there is a perfectly good "ordinary" symmetry explanation.

Does this superspace exist in D = 5 or D = 4?

Prediction of superspace: If you add N = 4 vector multiplets, amplitude should develop no *new* 2, 3 loop divergences.

Bossard, Howe and Stelle (2013)

Subsequent explicit calculation proves new divergences at 2, 3 loops.
Conclusion: currently no viable standard-symmetry understanding.

ZB, Davies, Dennen (2013)

Four-loop $\mathcal{N} = 4$ Supergravity Divergences

ZB, Davies, Dennen, Smirnov, Smirnov

To make a deeper probe we calculated four-loop divergence in $\mathcal{N} = 4$ supergravity.

Industrial strength software needed: FIRE5 and C++

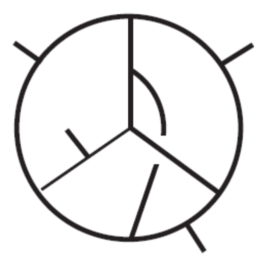
$\mathcal{N} = 4$ sugra: ($\mathcal{N} = 4$ sYM) x ($\mathcal{N} = 0$ YM)

$\mathcal{N} = 4$ sYM

$$(l \cdot k)^2 s^2 t A_4^{\text{tree}}$$

BCJ representation

pure YM



$$(\varepsilon \cdot l)^4 l^6$$

Feynman representation

$\mathcal{N} = 4$ sugra diagrams quadratically divergent

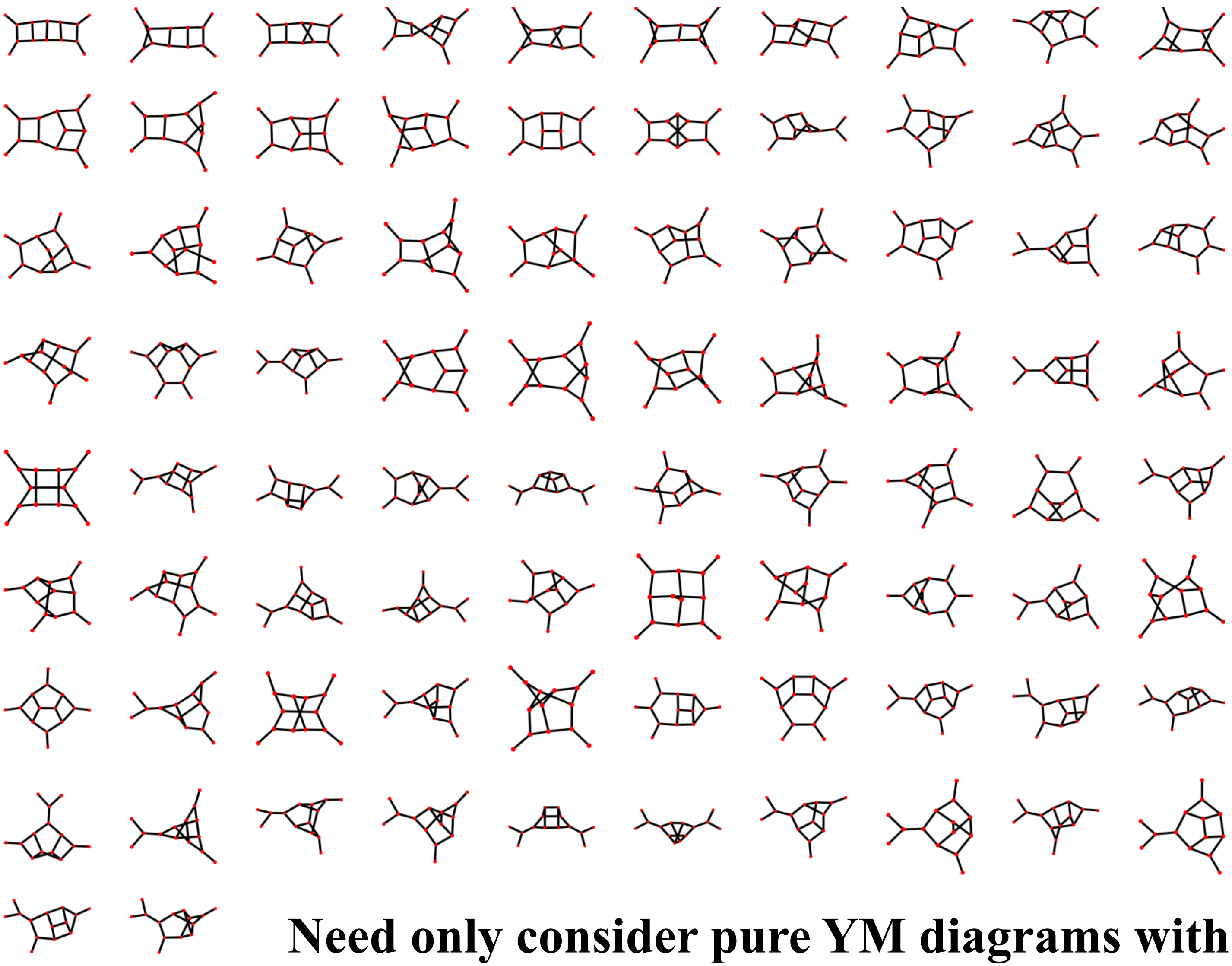
$$\int (d^D l)^4 \frac{k^8 l^{12}}{(l^2)^{13}}$$

$D^2 R^4$ counterterm

82 nonvanishing diagram types using $\mathcal{N} = 4$ sYM BCJ form.

82 nonvanishing numerators in BCJ representation

ZB, Carrasco, Dixon, Johansson, Roiban ($\mathcal{N} = 4$ sYM)



Need only consider pure YM diagrams with color factors that match these.

The 4 loop Divergence of $N = 4$ Supergravity

ZB, Davies, Dennen, Smirnov, Smirnov

Pure $N = 4$ supergravity is divergent at 4 loops with divergence

Result is for Siegel dimensional reduction.

dim. reg. UV pole

$$\boxed{\mathcal{M}^{\text{4-loop}}\Big|_{\text{div.}} = \frac{1}{(4\pi)^8}\frac{1}{\epsilon}\left(\frac{\kappa}{2}\right)^{10}\frac{1}{144}(1-264\zeta_3)\,\mathcal{T}}$$

$$\mathcal{T} = st A^{\text{tree}}_{\mathcal{N}=4}\left(\mathcal{O}_1 - 28\mathcal{O}_2 - 6\mathcal{O}_3\right)$$

$D = 4 - 2\epsilon$

$$\mathcal{O}_1 = \sum_{\mathcal{S}_4}(D_\alpha F_{1\mu\nu})(D^\alpha F_2^{\mu\nu})\,F_{3\rho\sigma}F_4^{\rho\sigma}$$

$$\mathcal{O}_2 = \sum_{\mathcal{S}_4}(D_\alpha F_{1\mu\nu})(D^\alpha F_2^{\nu\sigma})\,F_{3\sigma\rho}F_4^{\rho\mu}$$

$$\mathcal{O}_3 = \sum_{\mathcal{S}_4}(D_\alpha F_{1\mu\nu})(D_\beta F_2^{\mu\nu})\,F_{3\sigma}{}^{\alpha}F_4^{\sigma\beta}$$

$$s = (k_1+k_2)^2$$
$$t = (k_2+k_3)^2$$

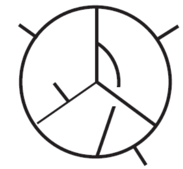
$$F_j^{\mu\nu} \equiv i(k_j^\mu \varepsilon_j^\nu - k_j^\nu \varepsilon_j^\mu)\,,$$
$$D^\alpha F_j^{\mu\nu} \equiv -k_j^\alpha(k_j^\mu \varepsilon_j^\nu - k_j^\nu \varepsilon_j^\mu)$$

Valid for all nonvanishing 4-point amplitudes of pure $N = 4$ sugra

Meaning of $N = 4$ Divergence?

- All subdivergences cancelled as confirmed by extensive checks.
- $\mathrm{Log}(M^2)$ coefficient matches the UV divergence.
- Nevertheless, some peculiar properties connected to anomalies.

Some Peculiar Properties



Linear combinations to expose $D = 4$ helicity structure

Refers to helicities of pure YM component

$$\mathcal{O}^{--++} = \mathcal{O}_1 - 4\mathcal{O}_2$$
$$\mathcal{O}^{-+++} = \mathcal{O}_1 - 4\mathcal{O}_3$$
$$\mathcal{O}^{++++} = \mathcal{O}_2$$

$$\mathcal{O}^{--++} = 4s^2 t \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle}\,,$$
$$\mathcal{O}^{-+++} = -12 s^2 t^2 \frac{[2\,4]^2}{[1\,2]\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,[4\,1]}\,,$$
$$\mathcal{O}^{++++} = 3st(s+t)\frac{[1\,2]\,[3\,4]}{\langle 1\,2\rangle\,\langle 3\,4\rangle}\,,$$

The latter two configurations would vanish if the *U*(1) symmetry were not anomalous.

See Carrasco, Kallosh, Tseytlin and Roiban

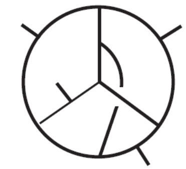
All three independent configurations have similar divergence!

Very peculiar because the nonanomalous sector should have a very different analytic structure. Not related by any supersymmetry Ward identities.

For anomalous sectors:

- **Might expect UV divergence to be suppressed by ϵ .**
- **Strange that the only know examples of divergences in pure (super)gravity are not generic, but are wound up with anomalies.**

Relation to $U(1)$ Anomaly



unitarity cut

Anomalous sector feeds poor UV behavior into non-anomalous sector

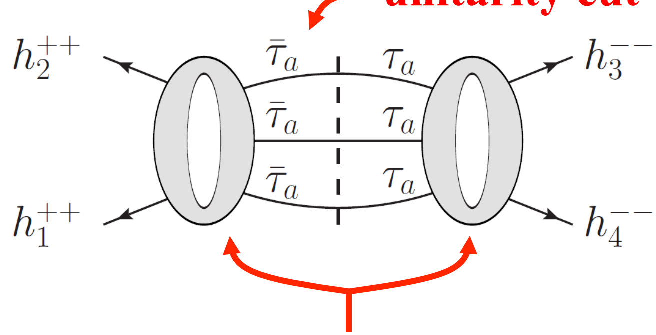


Figure from arXiv:1303.6219
Carrasco, Kallosh, Tseytlin and Roiban

Anomalous 1-loop amplitudes

- **As pointed out by Carrasco, Kallosh Roiban,Tseytlin the anomalous amplitudes are poorly behaved and contribute to a 4-loop UV divergence (unless somehow canceled).**
- **Via anomaly it is easy to understand why all three sectors can have similar divergence structure.**
- **The dependence of the divergence on vector multiplets matches anomaly.**

$$\mathcal{M}^{4\text{-loop}}_{n_V}\Big|_{\text{div.}} = \frac{1}{(4\pi)^8}\left(\frac{\kappa}{2}\right)^{10} \boxed{\frac{n_V+2}{2304}} \left[\frac{6(n_V+2)n_V}{\epsilon^2} + \frac{(n_V+2)(3n_V+4) - 96(22-n_V)\zeta_3}{\epsilon}\right]\mathcal{T}$$

anomaly has exactly this factor

n_V is number vector multiplets

Bottom line: The divergence looks specific to $N = 4$ sugra and likely due to an anomaly. Won't be present in $N \geq 5$ sugra.

If anything, this suggests $N = 8$ sugra UV finite at 8 loops.

Summary

- Reproduced result from Goroff and Sagnotti on 2 loop divergence of pure gravity. Gauss-Bonnet counterterm nontrivial.
- Pure gravity divergence has anomaly-like 0/0 behavior, connected to Bardeen & Cangemi's observations.
- UV divergences depend on field representation! Value of divergence physically meaningless. Log(M^2) better to look at.
- Quantum equivalence under duality. Logs identical.
- Four examples of enhanced UV cancellations. Only one-loop pure gravity explained. Others remain a challenge.
- $N = 4$ supergravity diverges at 4 loops, but a peculiar structure suggesting similar divergences won't happen in $N \geq 5$ sugra.

The UV properties of gravity theories rich and interesting, and full of subtleties and interesting twists.
We can expect many more surprises as we probe gravity theories using modern perturbative tools.