Rapid cavity prototyping using a Globalised Scattering Matrix Approach

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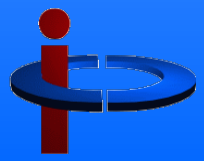


Dr Ian Shinton

The University of Manchester; Cockcroft Institute, Daresbury, UK

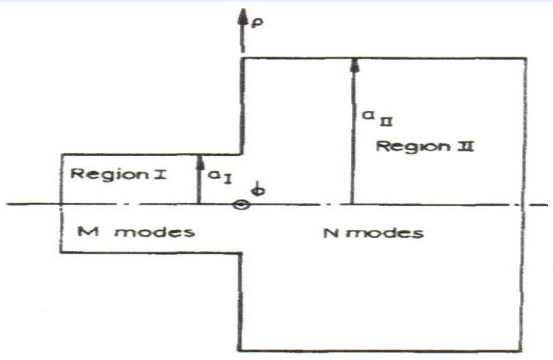
MANCHESTER 1824

INTRODUCTION

- Cavity design using traditional mesh based numerical means (such as the finite element or finite difference methods) requires numerous calculations and convergence studies in order to obtain accurate values and cavity optimisation is often not achieved.
- The mode matching method is a mature electromagnetic concept in which the analytical solutions of Maxwell's equations are given as a series expansion of modes. The method requires little in the way of computational resources and is exceedingly fast and accurate.
- Previous mode matching schemes, although extremely accurate and efficient, were limited to sharp transitions. Here we present a mode matching scheme which utilises a globalised scattering matrix approach that allows cavities with curved surfaces.
- The method allows rapid e.m. field calculations to be obtained for cavity prototyping and optimization.
- Combining GSM with mode matching allows extension to large scale simulations (of multi-module structures) is beyond the computational resources of a purely numerical approach such as FEM,FD…..

A QUICK SUMMARY OF GSM CASCADING

- Calculation involves cascading two RF junctions using S matrices to obtain the S matrix data of the combined structure
- The generalised scattering matrix technique is a well established mature RF technique.
- Can be used to describe any RF structure



$$\underline{\underline{S}}_{11} = \underline{\underline{S}}^a_{12}\left[\underline{\underline{I}} - \underline{\underline{S}}^b_{11}\ \underline{\underline{S}}^a_{22}\right]^{-1}\underline{\underline{S}}^b_{11}\underline{\underline{S}}^a_{21} + \underline{\underline{S}}^a_{11}$$

$$\underline{\underline{S}}_{12} = \underline{\underline{S}}^a_{12}\left[\underline{\underline{I}} - \underline{\underline{S}}^b_{11}\ \underline{\underline{S}}^a_{22}\right]^{-1}\underline{\underline{S}}^b_{12}$$

$$\underline{\underline{S}}_{21} = \underline{\underline{S}}^b_{21}\left[\underline{\underline{I}} - \underline{\underline{S}}^a_{22}\ \underline{\underline{S}}^b_{11}\right]^{-1}\underline{\underline{S}}^a_{21}$$

$$\underline{\underline{S}}_{22} = \underline{\underline{S}}^b_{21}\left[\underline{\underline{I}} - \underline{\underline{S}}^a_{22}\ \underline{\underline{S}}^b_{11}\right]^{-1}\underline{\underline{S}}^a_{22}\underline{\underline{S}}^b_{12} + \underline{\underline{S}}^b_{22}$$

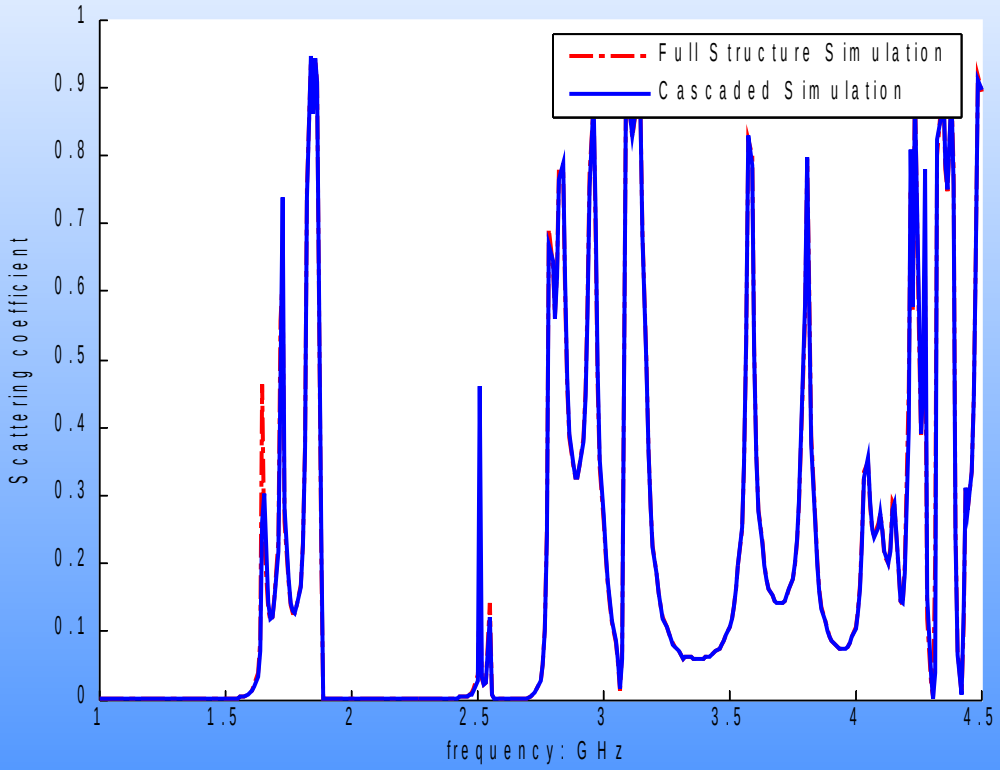
Advantages of Cascading

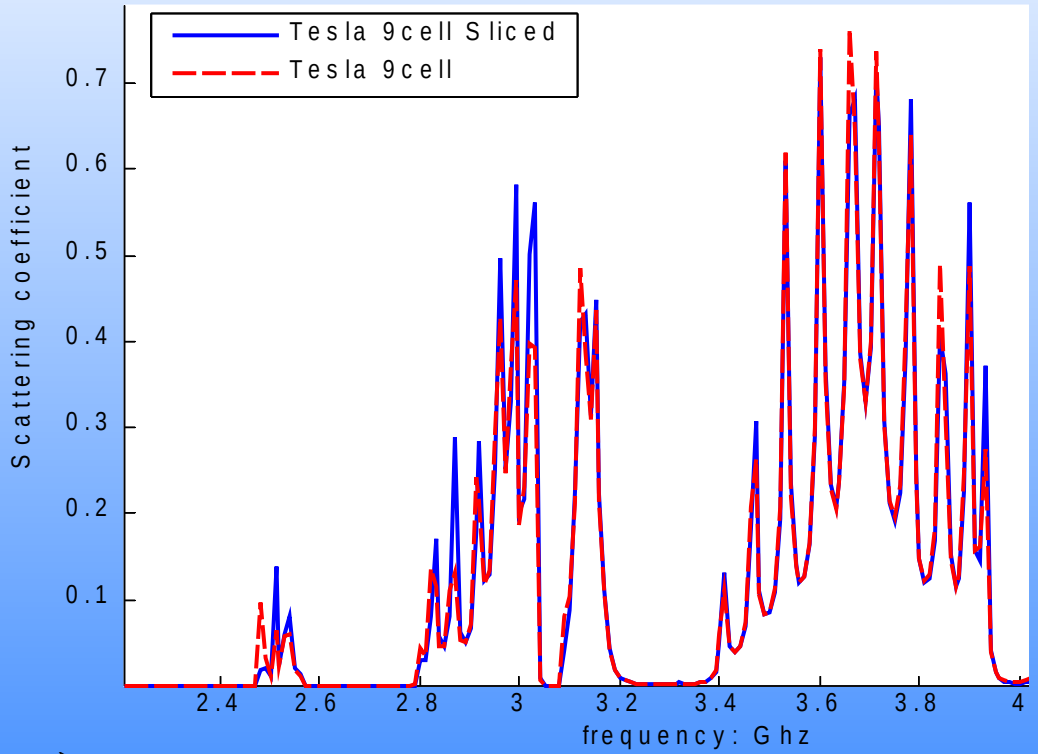
- Provided the correct physics of the problem have been considered the method is highly accurate
- The method requires little in the way of computational resources and is exceedingly fast (once the unit cell calculations have been made)
- Perturbations and cell miss-alignments can easily be implemented into the scheme without the necessity to remesh the entire structure as would be the case in a full numerical simulation.
- Large scale simulations (of multi-module structures) is beyond the computational resources of a purely numerical approach such as FEM,FD…..

A QUICK SUMMARY OF MODE MATCHING

- Mode matching is a mature RF technique, provides very accurate answers and requires very little in the way of computational resources or time.
- The mode matching technique relies upon splitting the structure into a series of sub-regions (typically WN or NW regions) in which an analytical solution of Maxwell's equations may be given as a series expansion over a set of orthogonal modes. The field solutions are then obtained by matching the field at the interfaces. In theory there are an infinite number of modes to be scattered into, however in practical numerical terms we truncate the series.
- The method is exact provided the following conditions are observed:
  - Since we are using a finite truncation of an infinite modal series, sufficient modes must be considered in terms of a convergence study.
  - A sufficient mode ratio should be employed on either side of the transition to enable the convergence criteria.
  - Gibbs phenomena will be present due to truncation, noticeable overshoots from this can be smoothed using a filter.

RESULTS OBTAINED USING GSM

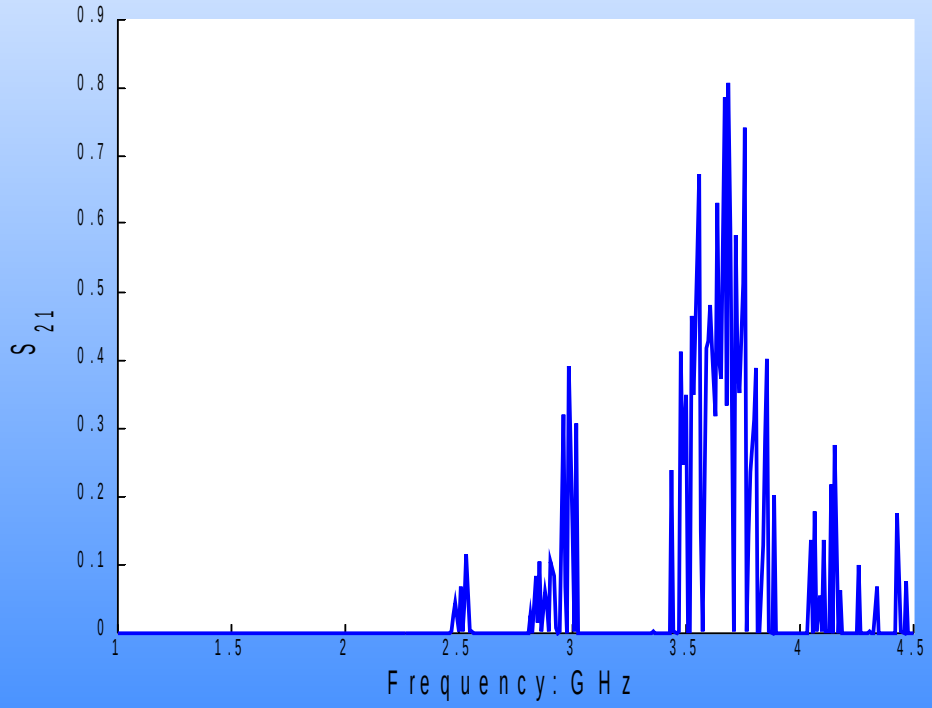


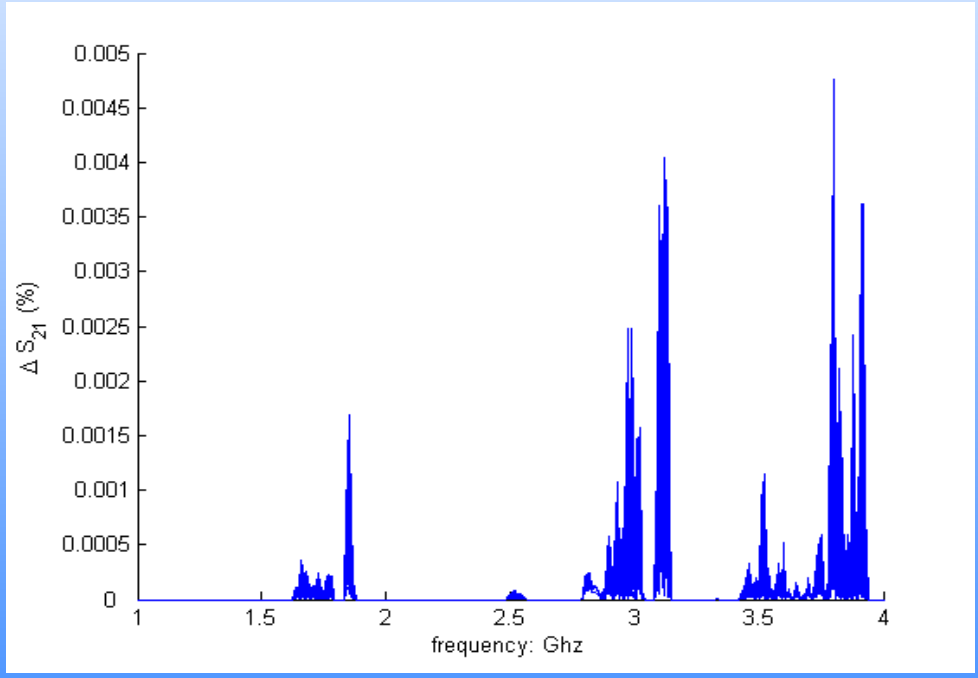


- Comparison between the cascaded and fully simulated structure for the dominant TE11 mode as a function of the TE11 mode for the S21 matrix for a structure constructed from 3 TESLA middle cells.
- The cascaded results in very good agreement with the fully simulated results with an average error of 0.014% but the amount of computational time saved through cascading is considerable; the total time required to calculate the fully simulated structure was 29hrs 25mins and 20sec, while the cascading calculation (using the referenced unit cell S matrix values) took a mere 1.5sec to run on a single processor machine.

- The generalised cascaded S21 matrix for a 9-cell TESLA structure comparison between an unperturbed structure and the RMS of a randomly perturbed structure for the TE11 mode scattered into the TE11 mode. Here the middle cells in the structure were randomly perturbed with the RMS taken over 20 different simulations (representing 20 possible fabricated cavities).
- The inclusion of realistic defects for an RMS calculation in traditionally used numerical techniques is prohibitively time consuming as it will require re-meshing of the problem domain. A GSM calculation circumvents this issue by only altering the specific regions (where the defects are located) in its calculation to include these effects directly.

LARGE SCALE SIMULATIONS USING GSM





➢GSM result for the dominant TE11 mode as a function of the TE11 mode for the S21 matrix for the FLASH accelerator – 6 TESLA cryomodules – 432 cells.

➢Large scale simulations beyond the means of traditionally employed numerical techniques such as the finite element method can be achieved using GSM.

GSM compilation error across a hypothetical symmetric 100m structure – the **error obtained from cascading from any cell is less that 0.005%**

FULL 3D ELECTROMAGNETIC FIELD SIMULATIONS USING MODEMATCHING GSM

- Extending the currently developed mode matching scheme, full 3D electromagnetic fields can be calculated for symmetrical structures. These fields are the product extending the previously developed concepts, in which we still assume that a dominant propagating mode is launched into the structure – here we utilise the analytical solution for sharp transitions.
- The electromagnetic field in any structure can be subdivided into one of three different regions (using a series of WN or NW regions). Since the S matrices are directly proportional to the modal amplitudes then by applying the mode matching procedure we obtain the fields as a sum over all the modes.
- The method is exact provided the following conditions are observed
  - Since we are using a finite truncation of an infinite modal series, sufficient modes must be considered in terms of a convergence study.
  - A sufficient mode ratio should be employed on either side of the transition to enable the convergence criteria.
  - Gibbs phenomena will be present due to truncation, noticeable overshoots from this can be smoothed using a filter.

Region 1 electric field

$$E_{\parallel} = e_{n1}e^{ik_{n1}(z-z1)} - \sum_{n1=1}^{N} S_{11}(n1,1)e_{n1}e^{ik_{n1}(z-z1)}$$

$$E_{\perp} = e_{n1}e^{ik_{n1}(z-z1)} - \sum_{n1=1}^{N} S_{11}(n1,1)e_{n1}e^{ik_{n1}(z-z1)}Y_{n1}$$

Region 2 electric field

$$E_{\perp} = \sum_{n2=1}^{N} \left(T_z S_{21}^0 + T_{-z} S_{11}^0\right)\bar{e}_{n2}$$

$$E_{\parallel} = \sum_{n2=1}^{N} \left(T_z S_{21}^0 - T_{-z} S_{11}^0\right)Y_{n2}\bar{e}_{n2}$$

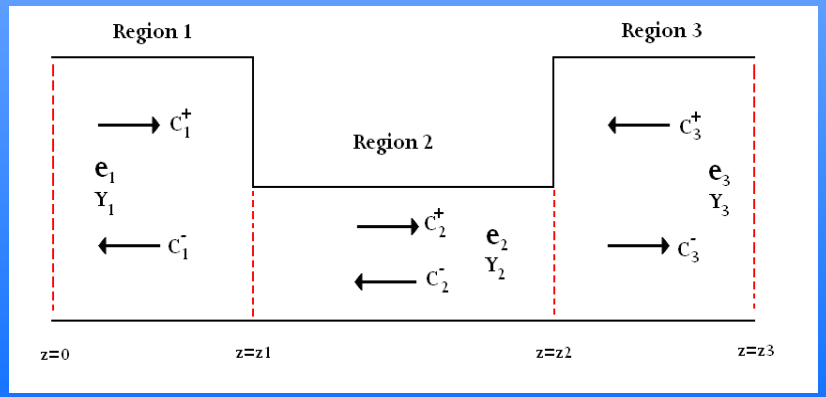
$$S_{11}^0 = T_{-z}\left[U - T_g S_{11}^{II} T_g S_{22}^{I}\right]^{-1} T_g S_{11}^{II} T_g S_{21}^{I}$$

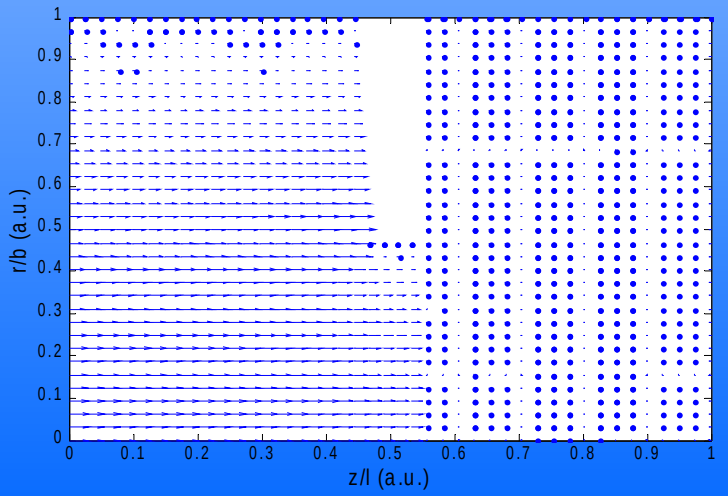
$$S_{21}^0 = T_z\left[U - S_{22}^{I} T_g S_{11}^{II} T_g\right]^{-1} S_{21}^{I}$$

Region 3 electric field

$$E_{\parallel} = \sum_{n3=1}^{N} S_{21}(n3,1)e_{n3}e^{-ik_{n3}(z-z2)}$$

$$E_{\perp} = \sum_{n3=1}^{N} S_{21}(n3,1)e_{n3}e^{-ik_{n3}(z-z2)}Y_{n3}$$

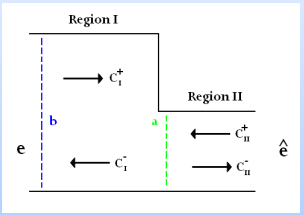




ANALYTICALLY OBTAINING S MATRIX OF A WN JUNCTION

➢It is possible to analytically derive the S matrix for a sharp transition – this allows very fast calculations to be made and we are no longer tied down to using commercial mesh based (FEM, FDM etc)software.

➢ Herein we will consider the case for a propagating monopole mode being launched form the starting port of a Wide Narrow transition. The S matrix at the transition is obtained by mode matching the fields in terms of the modal amplitudes.



$$\bar{a}_{nm} = \frac{2a^2 X_n J_0(X_n a/b)}{b^2 J_1(X_n)\left((X_n a/b)^2 - X_m^2\right)}$$

$$S_{21} = 2\left(U + \hat{Z}\bar{a}^t Y \bar{a}\right)^{-1} \hat{Z}\bar{a}^t Y$$

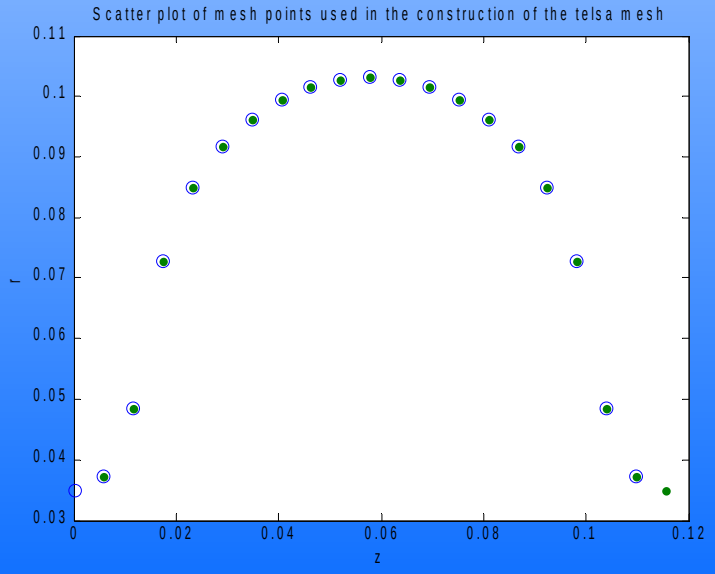
$$S_{11} = \bar{a} S_{21} - U$$

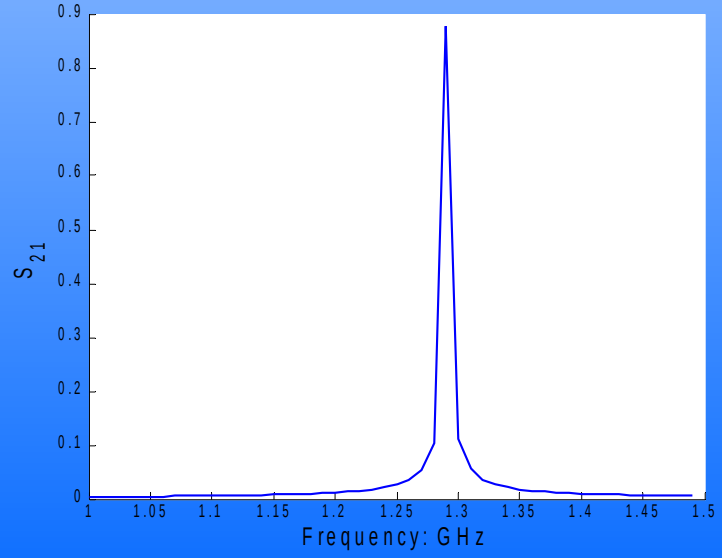
$$S_{22} = 2\left(U + \hat{Z}\bar{a}^t Y \bar{a}\right)^{-1} - U$$

$$S_{12} = \bar{a}(U + S_{22})$$

➢The GSM technique is very fast and allows an analytical calculation to be made for the S matrix of a symmetrical cavity – removing the need for numerical means (such as HFSS). By combining a series of WN and NW junctions we are able to simulate the curved cavities.

➢This use of the technique has not appeared in the literature before, as it has been applied herein.



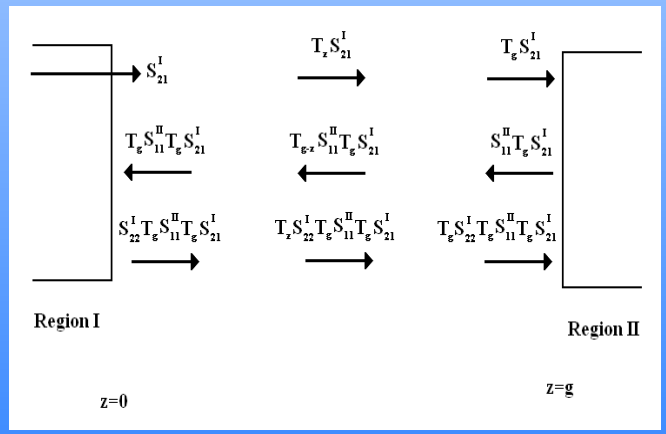


Presented here are: (Centre) the S_{21} matrix for the dominant propagating monopole mode scattered into all other modes for a TESLA structure which has been calculated analytically using GSM and a series of WN and NW transitions
(Far left) : The series of WN and NW sections used to calculate GSM result.

USEFUL CAVITY DESIGN TOOLS FROM A GSM MODEMATCHING SCHEME

- Recently at LINAC08 X.Li presented a reworked TESLA design which is an intermediate design between the KEK Ichiro and the Cornel re-entrant cavity design which improved the performance of the cavity by 10% - effectively what he is presenting is the design that should have been used for the TESLA cell i.e. an optimised design.
- Typically engineers use FEM or FD codes to design cavities, this requires a lot of computational time as dispersion curves, kick factors and R/Q quantises must be determined and “tweaked”.
- A spin off from the GSM mode matching scheme is the ability to calculate an entire dispersion curve very quickly – in one go! rather than having to tediously run may calculations. This method also allows cavity design optimization to be obtained by concurrently running many designs and observing the outcome.
- We obtain a direct dispersion relation by consider the S matrix of a cell in an infinite periodic structure (obtained from GSM). We assume that there is a propagating mode being launched from one port to the other. In equation the equation below “a” represent the incident waves and “b” the reflected waves on each port, the subscripts refer to the ports.

$$\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$$



- As this cell is in an infinite periodic structure:

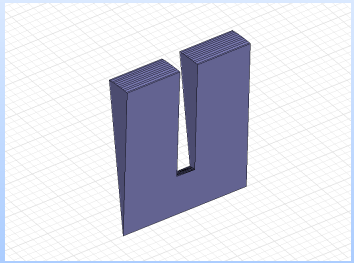
$$b_1 = e^{-i\Phi} a_2$$

$$b_2 = e^{-i\Phi} a_1$$

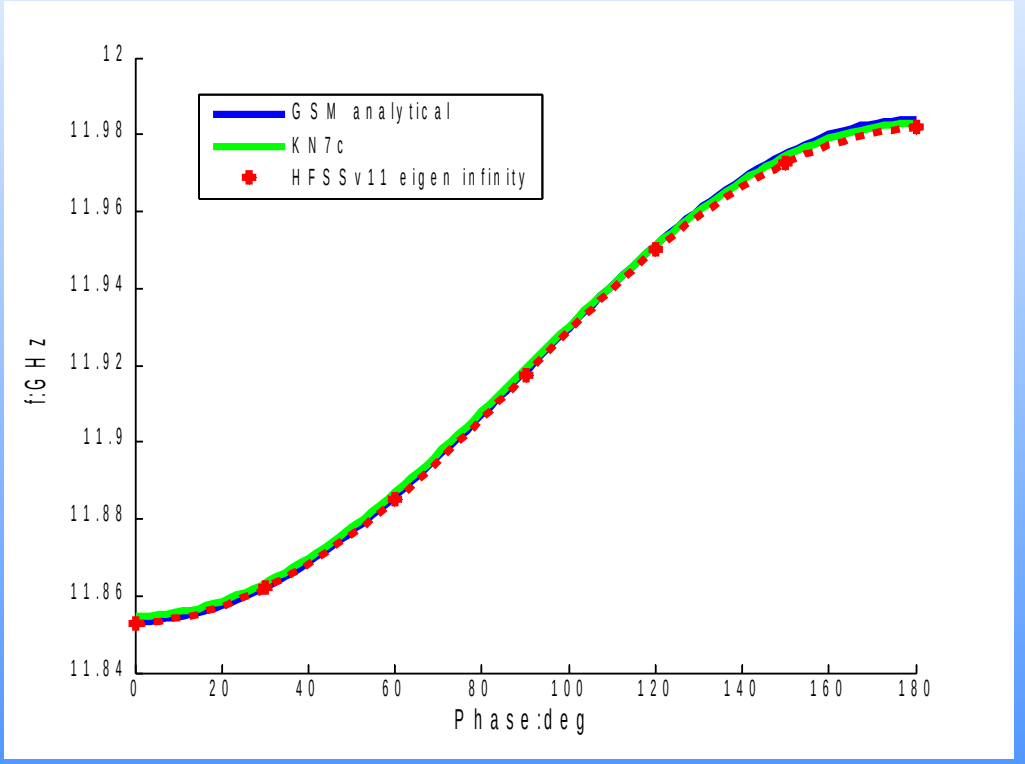
- The above equations represent an eigen mode system which after some rearranging we obtain the following dispersion relationship

$$\cos\Phi = \frac{1 + S_{21}(1,1)^2 - S_{11}(1,1)^2}{2S_{21}(1,1)}$$

COMPARISION OF THE ANALYTICAL MODEMATCHING GSM DISPERSION CURVE WITH OTHER METHODS AND CODES FOR A WNW TRANSITION



- Here we are comparing analytical dispersion curve obtained from using KN7c, The analytical S matrix calculation and the eigen mode solution from HFSSv11 using linear elements.
- Note eigen mode values have been extrapolated to infinite mesh values

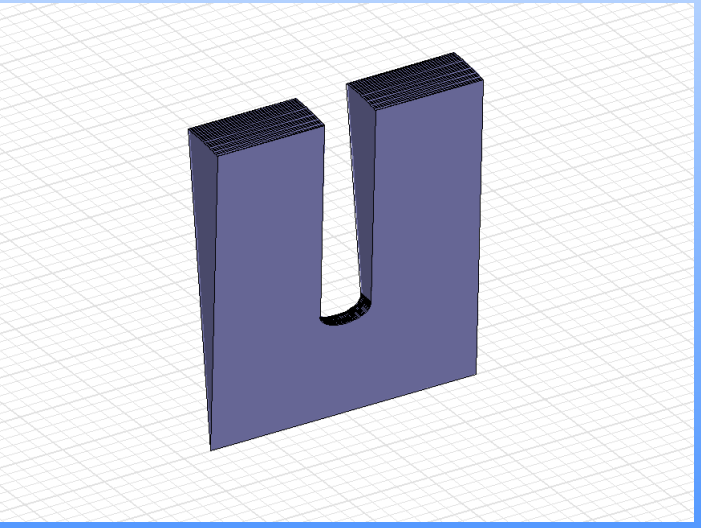


HFSSv11 eigen fpi: GHz	KN7c fpi:GHz	S matrix fpi: GHz	Max range error: MHz
11.9818	11.9828	11.984	2.2

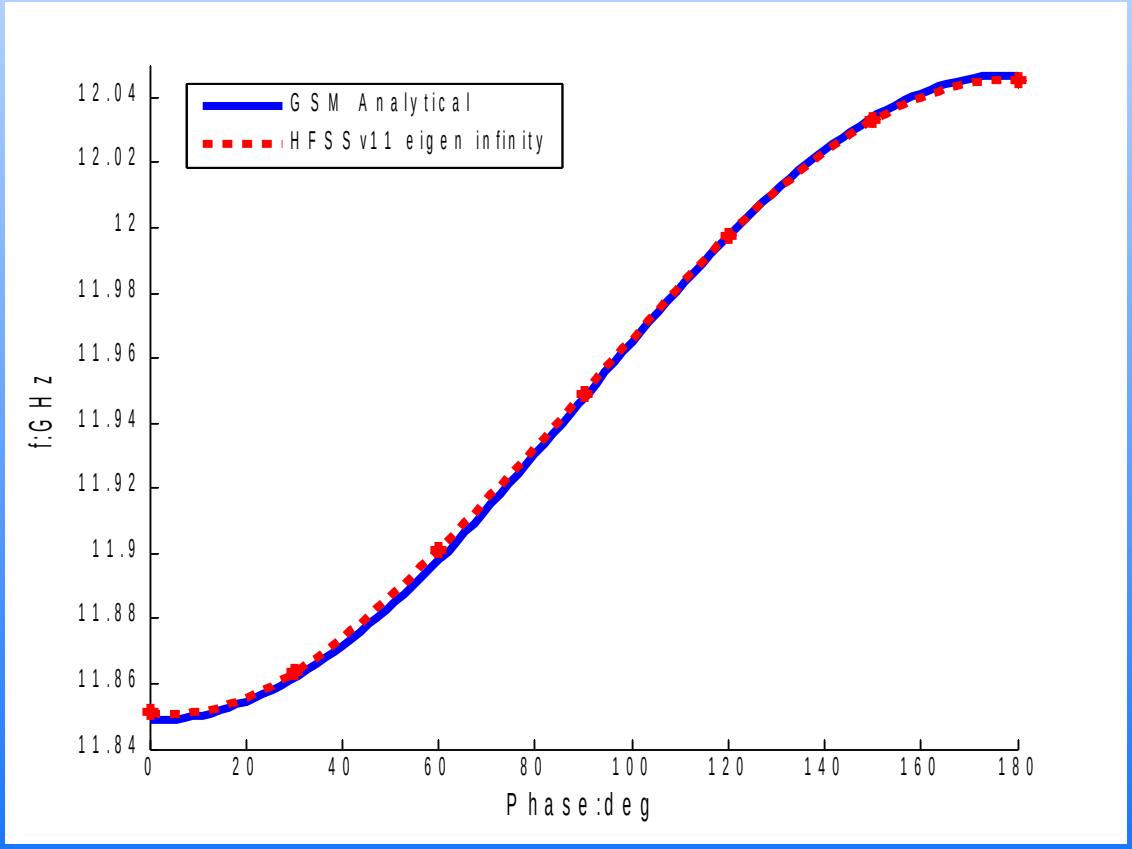
HFSSv11 eigen f0: GHz	KN7c f0:GHz	S matrix f0: GHz	Max range error: MHz
11.853	11.8548	11.853	1.8

CAVITY PROTOTYPING USING GSM – EXTENDED TO CURVED SURFACES

- Initial GSM prototyping dispersion curves were limited to sharp transitions (WNW transitions)
- The technique has now been extended to cavities with curved surfaces
- The idea relies upon approximating any curved surface by using a series of NW or WN transitions
- This use of the technique has not appeared in the literature before.

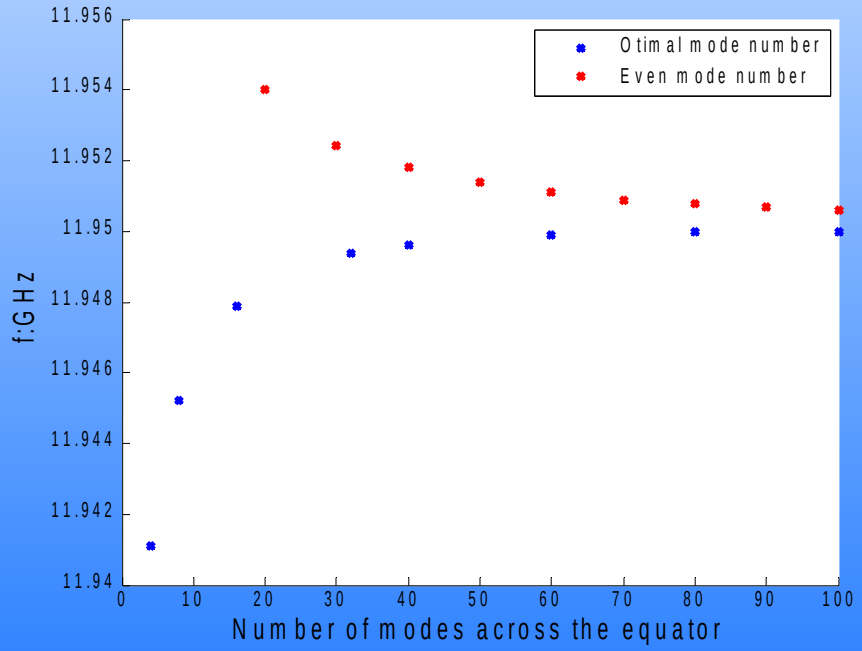


- Here we see the technique applied to one of the CLIC ZC designs of Vasim. Using 0.001GHz steps 30 modes per port and 50 segments to approximate the iris.
- The technique can be applied to any symmetric structure for rapid cavity prototyping

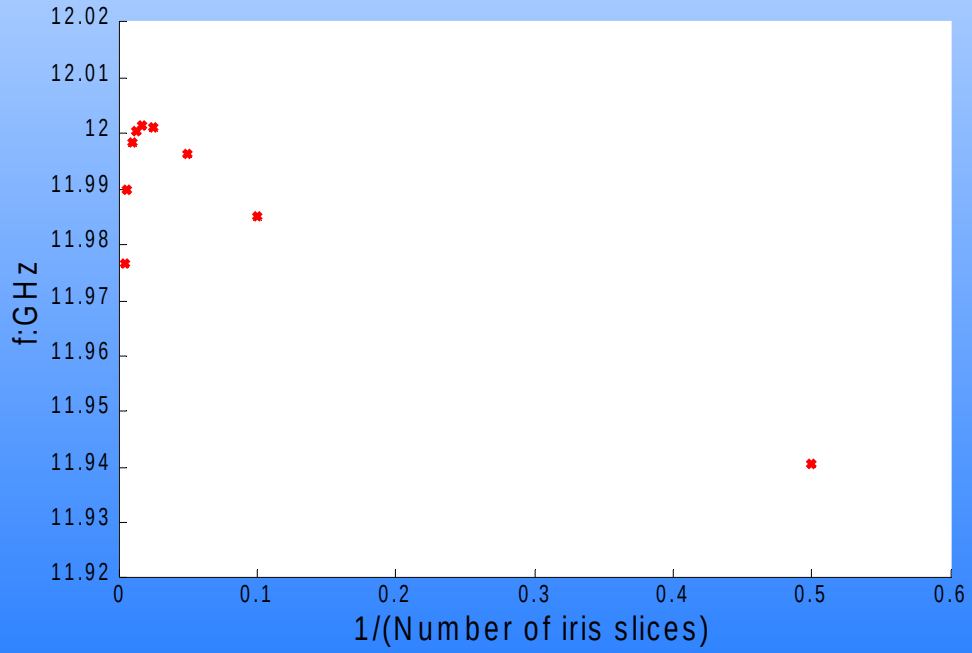


CONVERGENCE STUDIES

- Convergence studies I have run concur with other mode matching work that an optimised mode ratio leads to rapid convergence – basic rule of thumb is the ratio of equator to iris radii gives the optimal mode ratio.
- Effectively one could simply use many modes 100's of modes for example – the method using this sledge hammer approach is still faster than any numerical mesh based method (FEM etc); however an optimised mode ratio allows super fast convergence.
- The convergence study using a number of thin slices to approximate curved surfaces works unless too small a slice is used to approximate the curvature – towards a nano meter scale the physics will break down – as we will end up simulating a straight pipe!!! – it is interesting to note that the convergence plot resembles what you would see for a Numerical simulation in which you reach machine error. So provided one does not use ridiculously small transitions the physics of the problem holds and a solution at infinity is obtained.



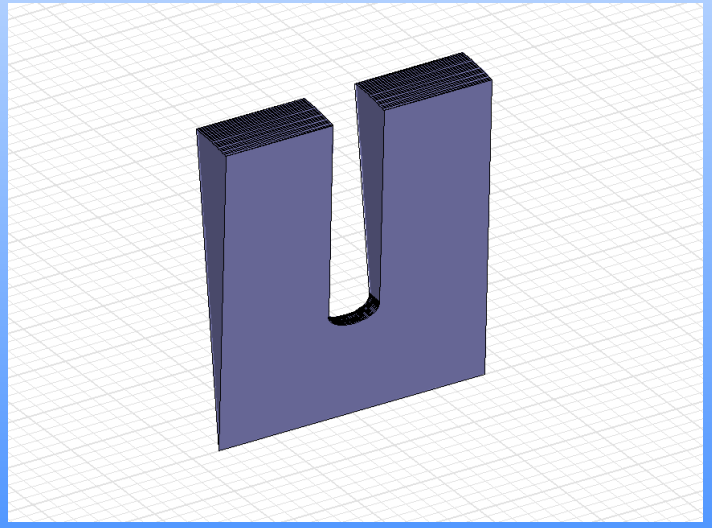
Convergence study of a WNW transition – here we see that using the optimised mode ratio infinite convergence is reach just after 60 modes



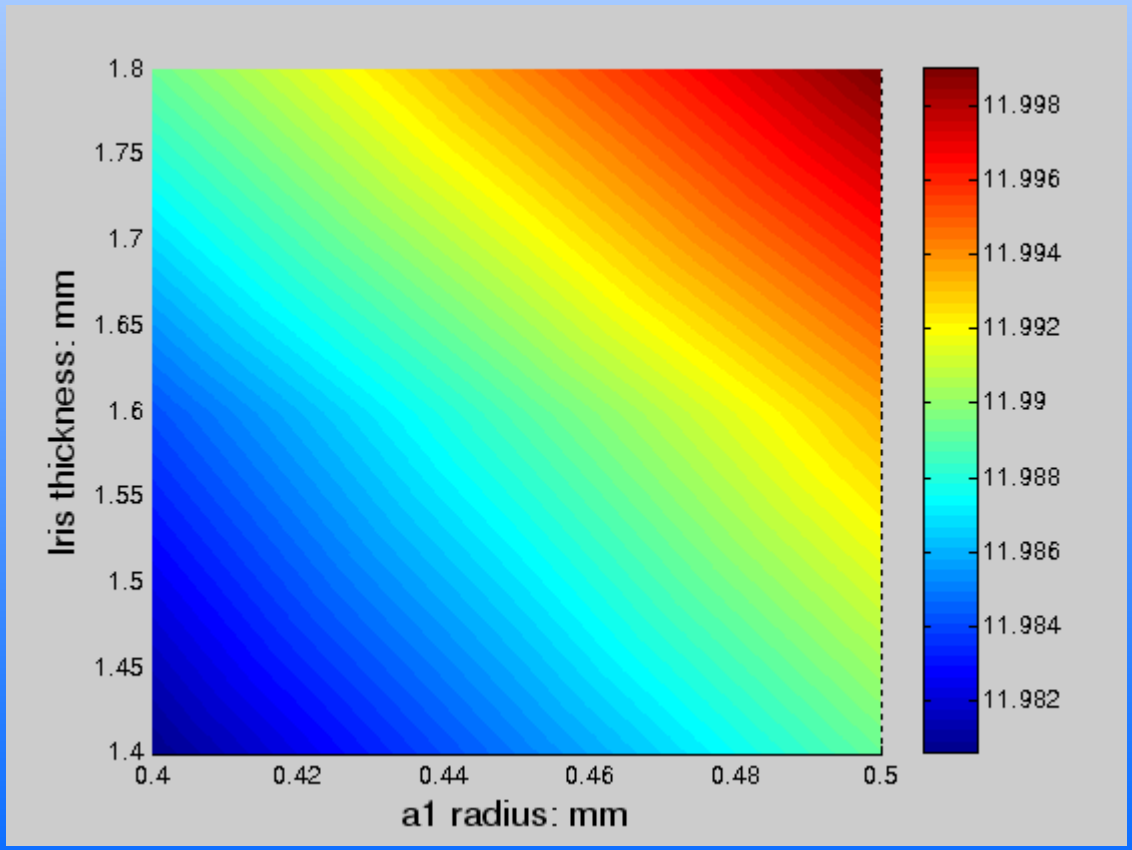
Convergence study of a CLIC ZC cell1 using GSM mode matching technique where the curved surface is approximated by a series of WN and NW transitions– here we see that if too small a transition is used (less than a nano meter) then the physics breaks down - since we end up simulating a pipe rather than a transition.

CAVITY PROTOTYPING (CURVED SURFACES) USING GSM – OPTIMISATION

- Basically because the method is so fast – I can simply run many simulations over a large parameter sweep to locate a desired optimised design point (Ok so it is not a true optimisation procedure i.e. "simplex" method etc – however large parameter sweeps using ABCI for cavities/collimators etc are a common practice in RF cavity design).
- The technique quickly allows one to look/optimise a cavity to the desired parameters – **any electromagnetic quantity**

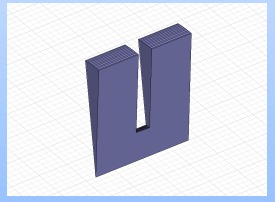


- Here we see the technique applied to one of the CLIC ZC designs of V.Khan.
- Here we see a slice of the results of the variation of iris thickness and a1 vertical iris ellipse radius for a particular equator radius b=9.881mm at 120deg phase – the technique allows a full parameter sweep. The colour axis if the frequency in GHz.

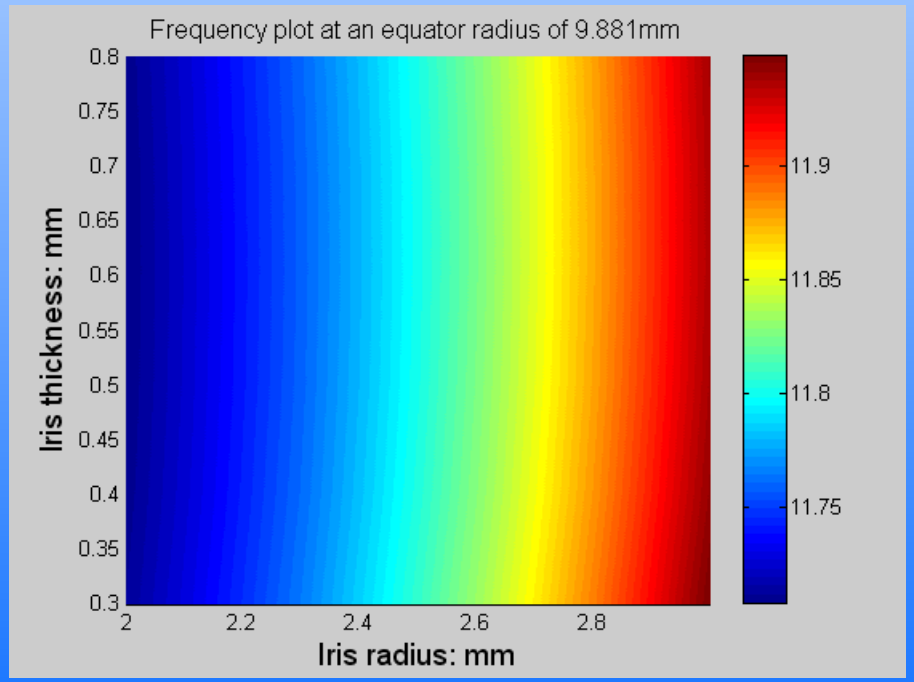


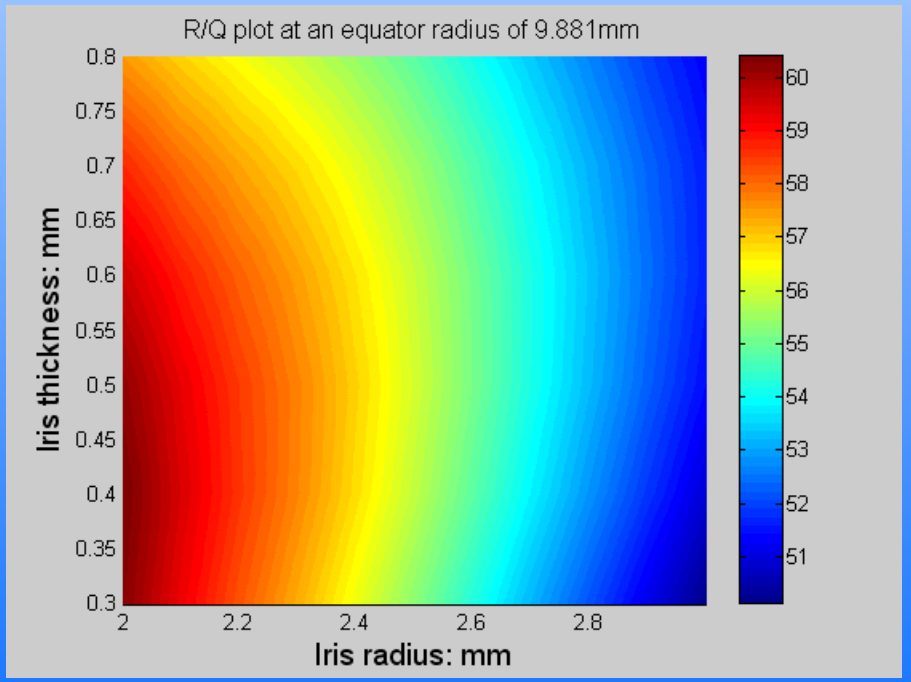
R/Q, KLOSS AND KICK FACTOR CALCULATIONS - OPTIMISATIONS

- R/Q, kloss and kick factor calculations have also now been added to the code – these have also be incorporated into the cavity optimisation program.
- The results I have obtained are in accordance with other programs i.e. see table below, in which the results are taken at b=9.881mm, a=2.991mm, t=0.8mm, phase=120deg



HFSS R/Q	GSM R/Q	Percentage difference
51.76	51.18	1.12



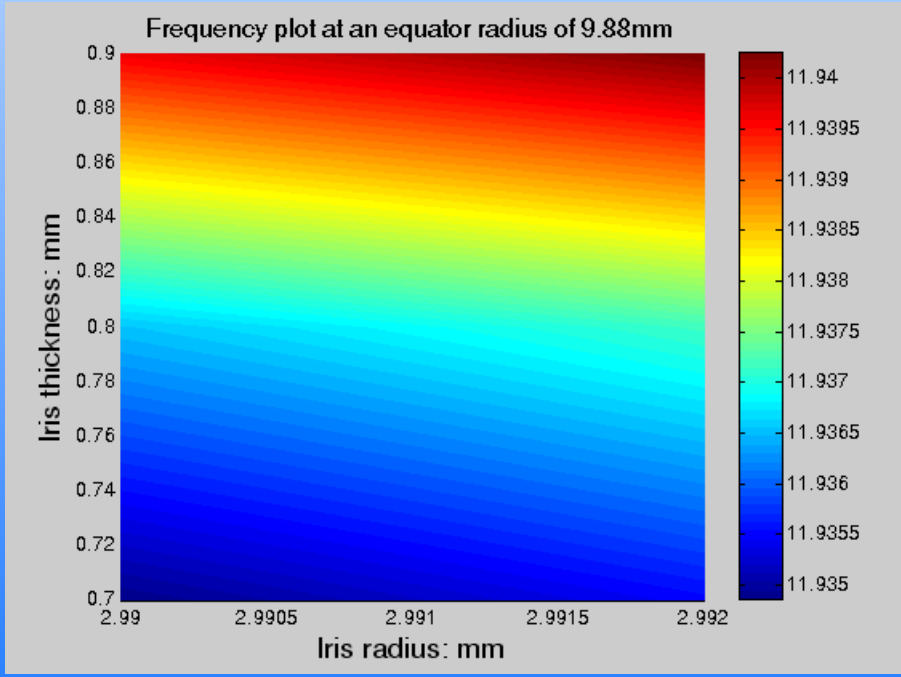


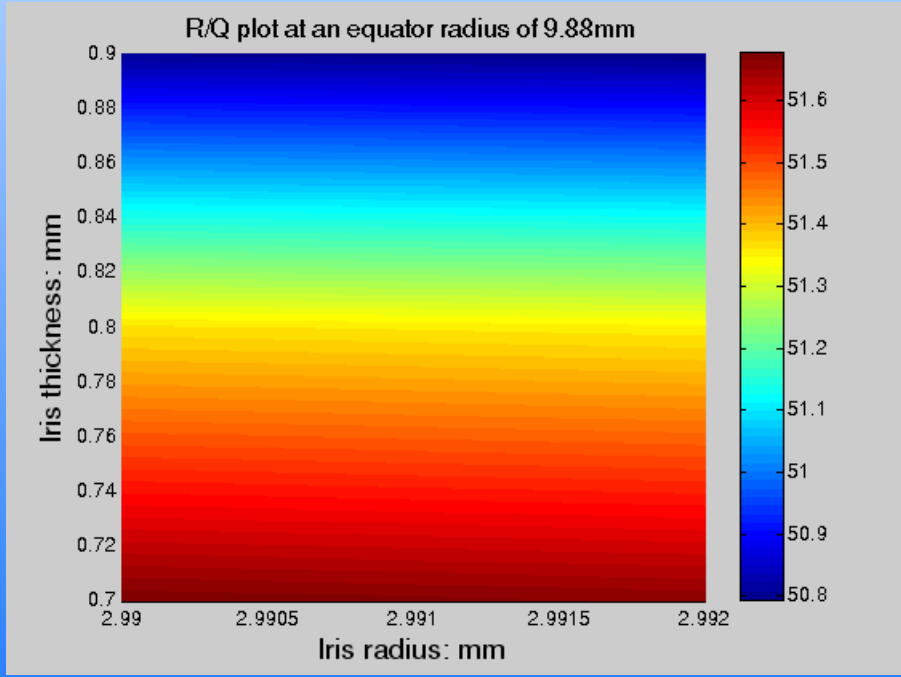
Here we see a slice of the results for a WNW transition for the variation of iris thickness and iris radius for a particular equator radius b=9.88mm at 120deg phase – the technique allows a full parameter sweep. The colour axes are the frequency in GHz and the R/Q in Ohms respectively

R/Q, KLOSS AND KICK FACTOR CALCULATIONS – OPTIMISATIONS ZOOMED IN RESULTS

➢Since the results are taken over then entire range of the dispersion curve in what can be referred to “N” dimensional space (where there are N dimensions that the user is investigating) – it is quite easy to narrow in on an optimal point/quantity.

➢As all the results within the sweep range that has been used to generate the dispersion curve are automatically calculated – any phase/frequency/….quantity that has been investigated will be present in the “N” dimensional there is no need to run separate parameter runs – as would be the case using traditional numerical eigen mode software.





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FUTURE ASPECTS

- Combining GSM with mode matching allows extension to large scale simulations (of multi-module structures) that is beyond the computational resources of a purely numerical approach such as FEM,FD….. (this is also an advantage of using a GSM method to obtain the amplitudes over a standard mode matching scheme).
- Couplers and other complicated structures can be added to the scheme by utilising the CSC technique (U.Van Rien, H.W.Glock, K.Rothemund) or GSM (V.A.Dolgashev).
- HOM such as dipoles will be included.
- Losses can be incorporated by altering the boundary conditions on the conductor walls and resolving Maxwell's equations.

SUMMARY

- A mode matching scheme has been combined with a GSM technique, which as a spin-off can be used for rapid cavity prototyping
- Cavities with curved surfaces can be simulated with this method – previous mode matching methods have been limited to sharp transitions.
- The scheme requires little in the way of computational resources or time and is highly accurate.
- The method allows a full parameter sweep of a cavity to be undertaken in a single go – allowing "N" parameter optimisation in a single calculation – one does not have to run separate simulations as would be the case using an eigen mode method….
- Rapid cavity optimisation over "N" parameters can be undertaken.