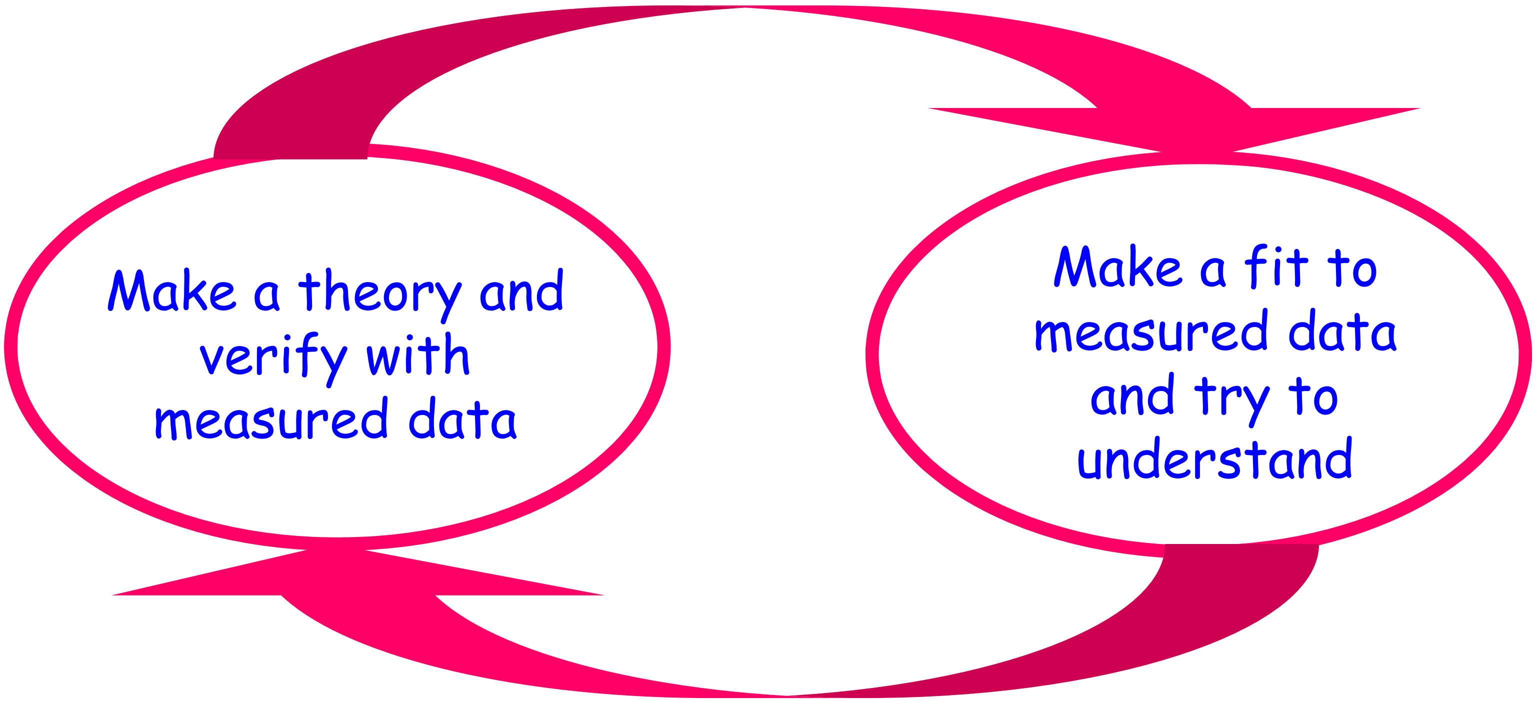
A New Local Field Quantity Describing the High-Gradient Limit of Accelerating Structures

December, 2008

Alexej Grudiev, Walter Wuensch

Motivation

To provide rf designers with a local field quantity which limits high-power/high-gradient performance in the presence of rf breakdowns.

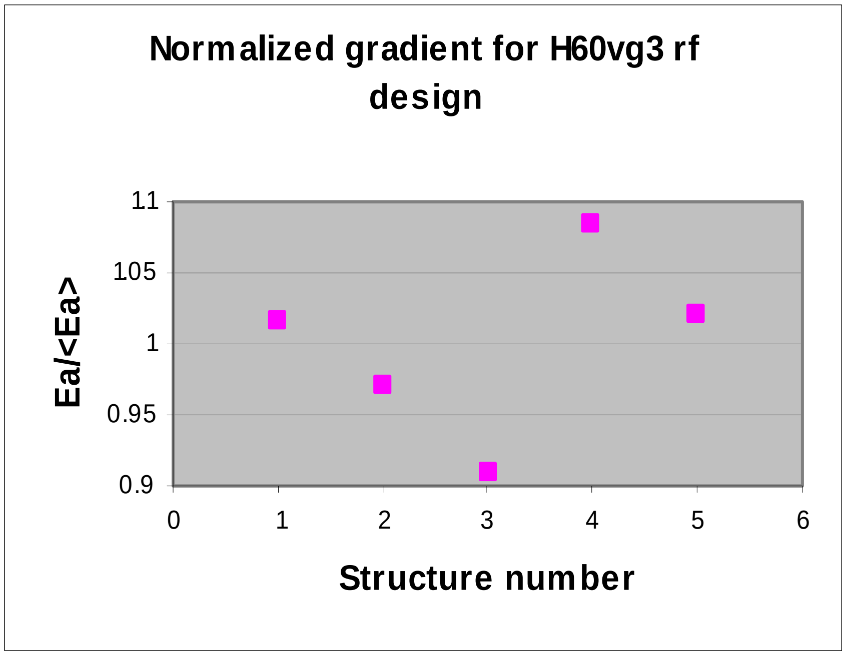


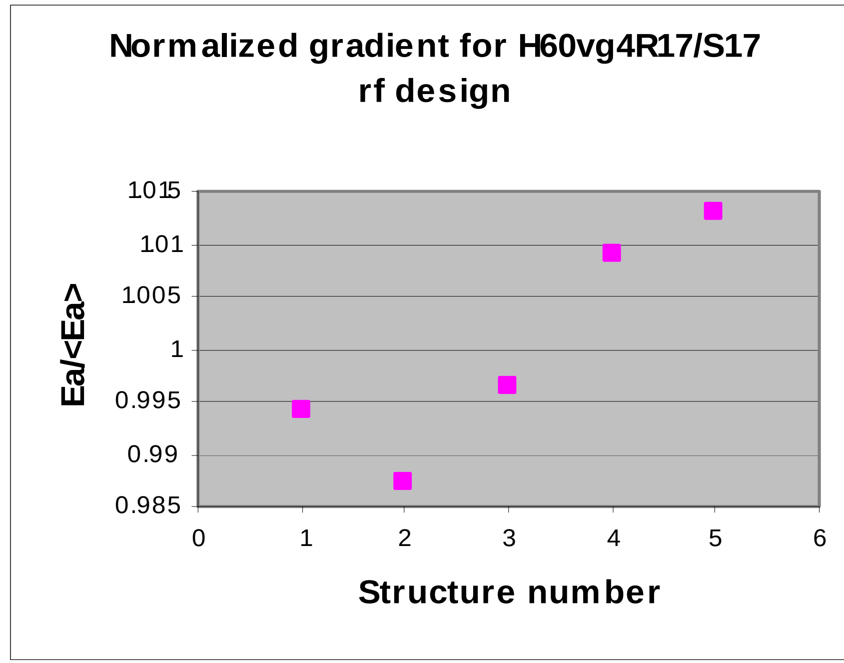
Introduction

The high-gradient performance depends on:

1. Geometry of the cavity: rf design
2. Surface of the cavity : anything else than rf design
   - Material
   - Heat treatment
   - Machining
   - Chemical treatment
3. Measurement technique and experimental setup

Variation of high-gradient performance of the same rf design.

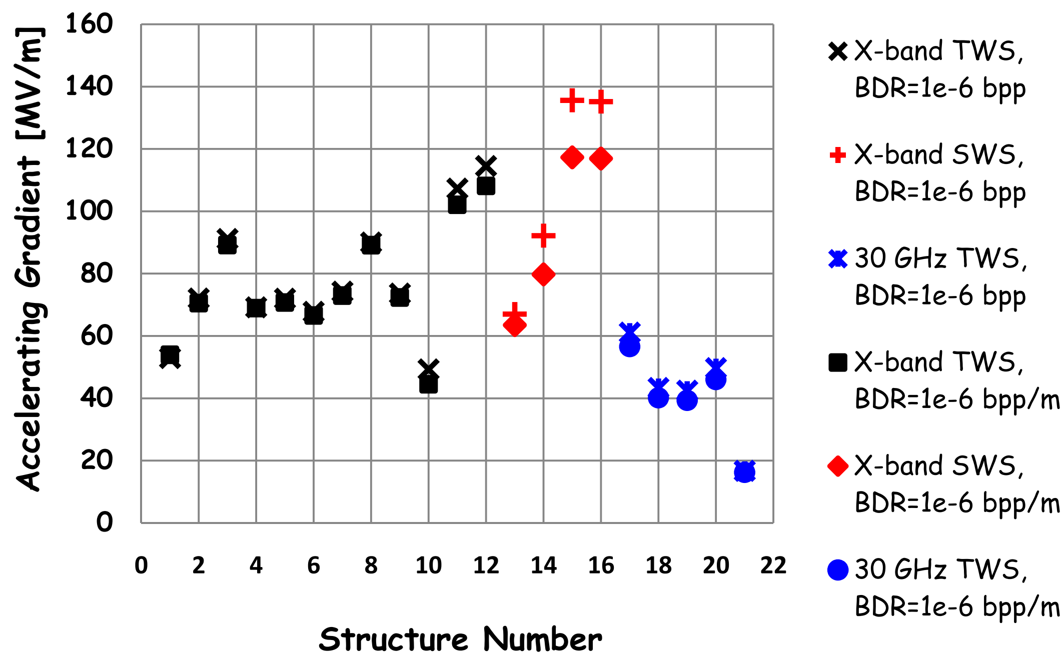




N.B. Variation of up to tens of percents can be expected from the difference in the surface state, statistics and measurement setup.

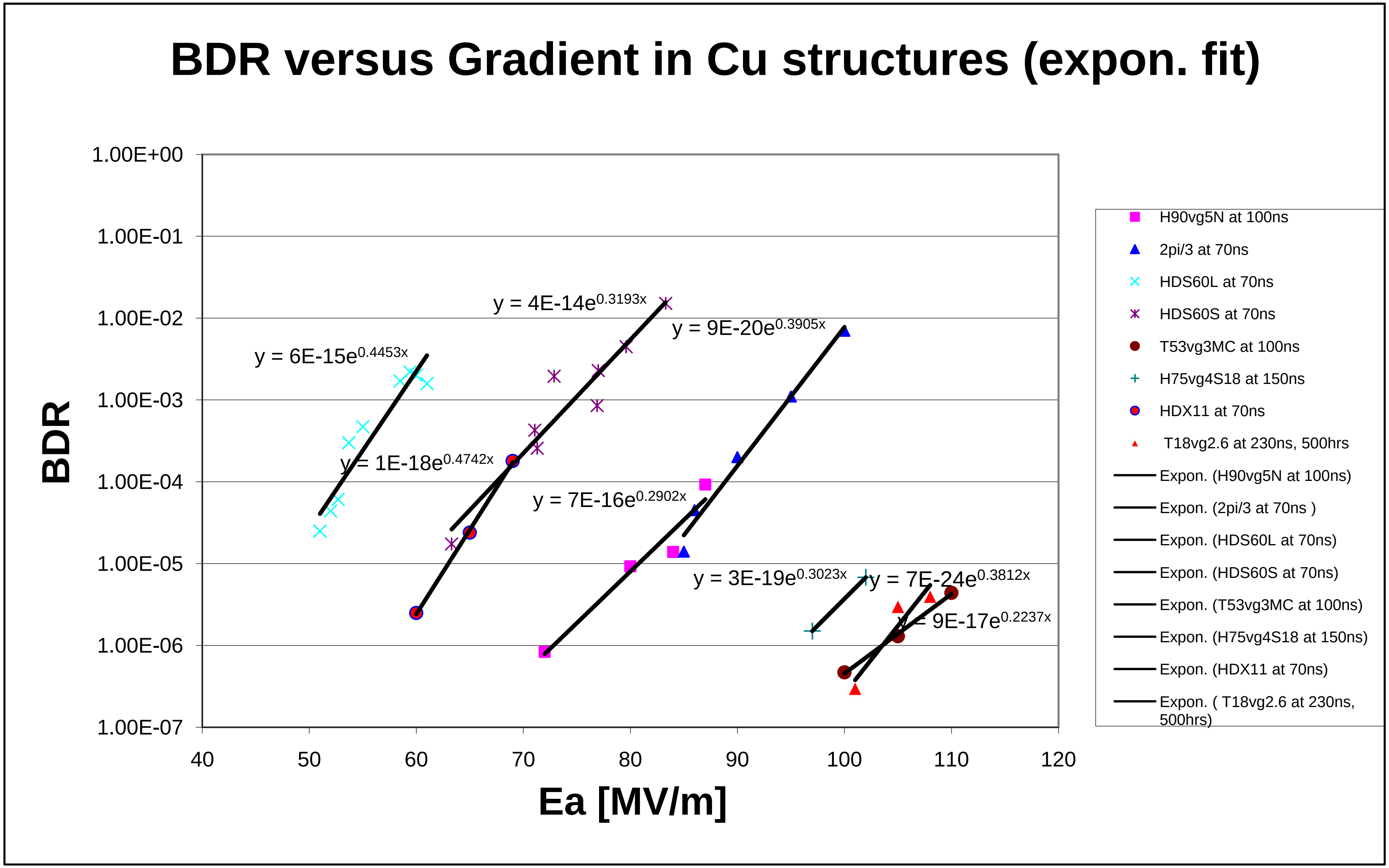
Experimental data @ 200ns, BDR=10^{-6}

N	Name	[GHz]	[º]	[%]	[m]
1	DDS1	11.424	120	11.7 – 3	1.8
2	T53vg5R	11.424	120	5.0 - 3.3	0.53
3	T53vg3MC	11.424	120	3.3 - 1.6	0.53
4	H90vg3	11.424	150	3.1 - 1.9	0.9
5	H60vg3	11.424	150	3 - 1.2	0.6
6	H60vg3S18 [	11.424	150	3.3 -1.2	0.6
7	H60vg3S17 [	11.424	150	3.6 -1.0	0.6
8	H75vg4S18	11.424	150	4.0 -1	0.75
9	H60vg4S17 [	11.424	150	4.5 -1	0.6
10	HDX11	11.424	60	5.1	0.05
11	CLIC-X-band	11.424	120	1.1	0.23
12	T18vg2.6	11.424	120	2.6 - 1.0	0.18
13	SW20a3.75	11.424	180	0	0.2
14	SW1a5.65t4.6	11.424	180	0	0.013
15	SW1a3.75t2.6	11.424	180	0	0.013
16	SW1a3.75t1.66	11.424	180	0	0.013
17	2π/3	29.985	120	4.7	0.1
18	π/2	29.985	90	7.4	0.1
19	HDS60	29.985	60	8.0 - 5.1	0.1
20	HDS60-Back	29.985	60	5.1 - 8.0	0.1
21	PETS9mm	29.985	120	39.8	0.4



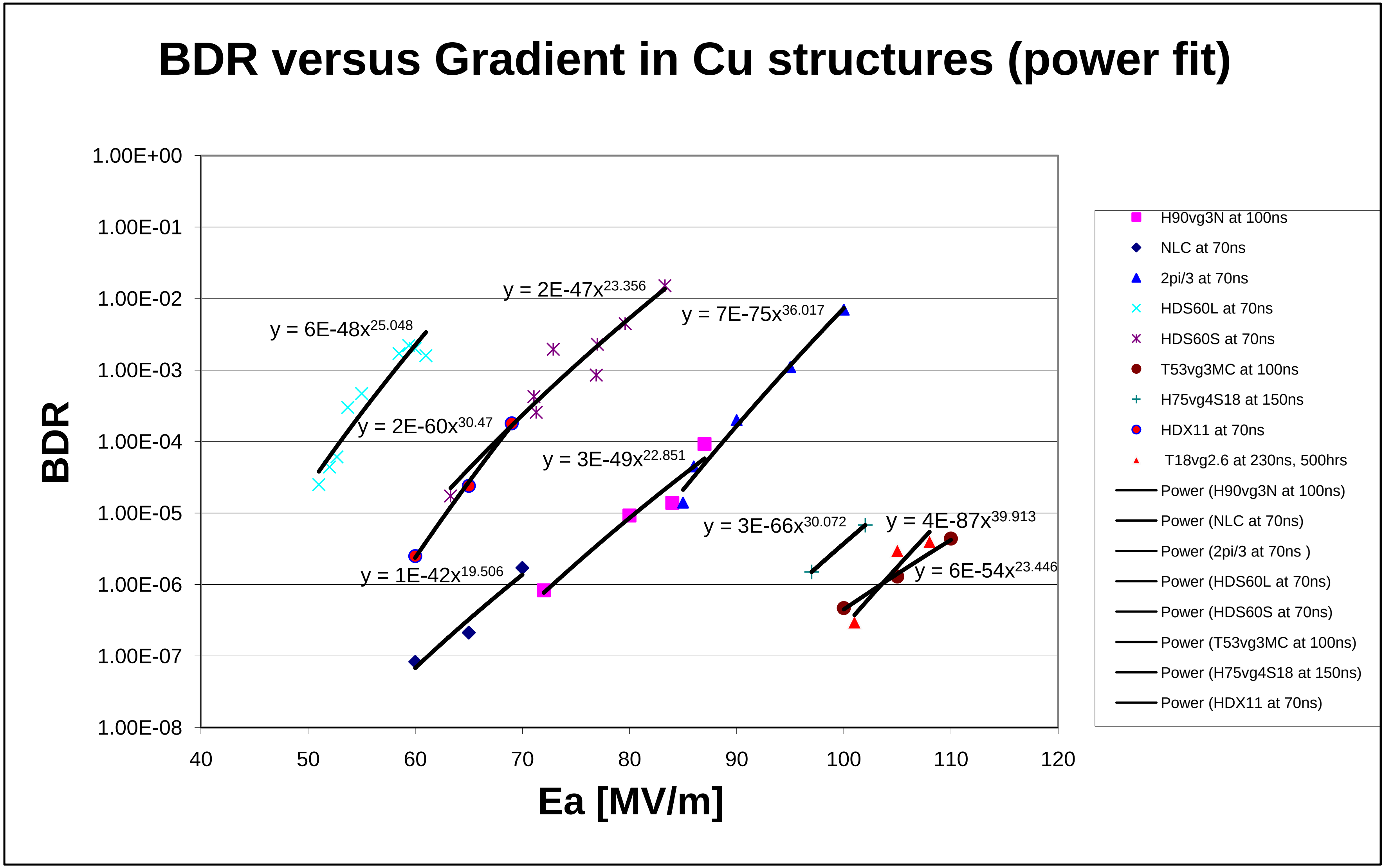
Accelerating gradient at 200ns, BDR=10^{-6}

BDR versus Gradient scaling



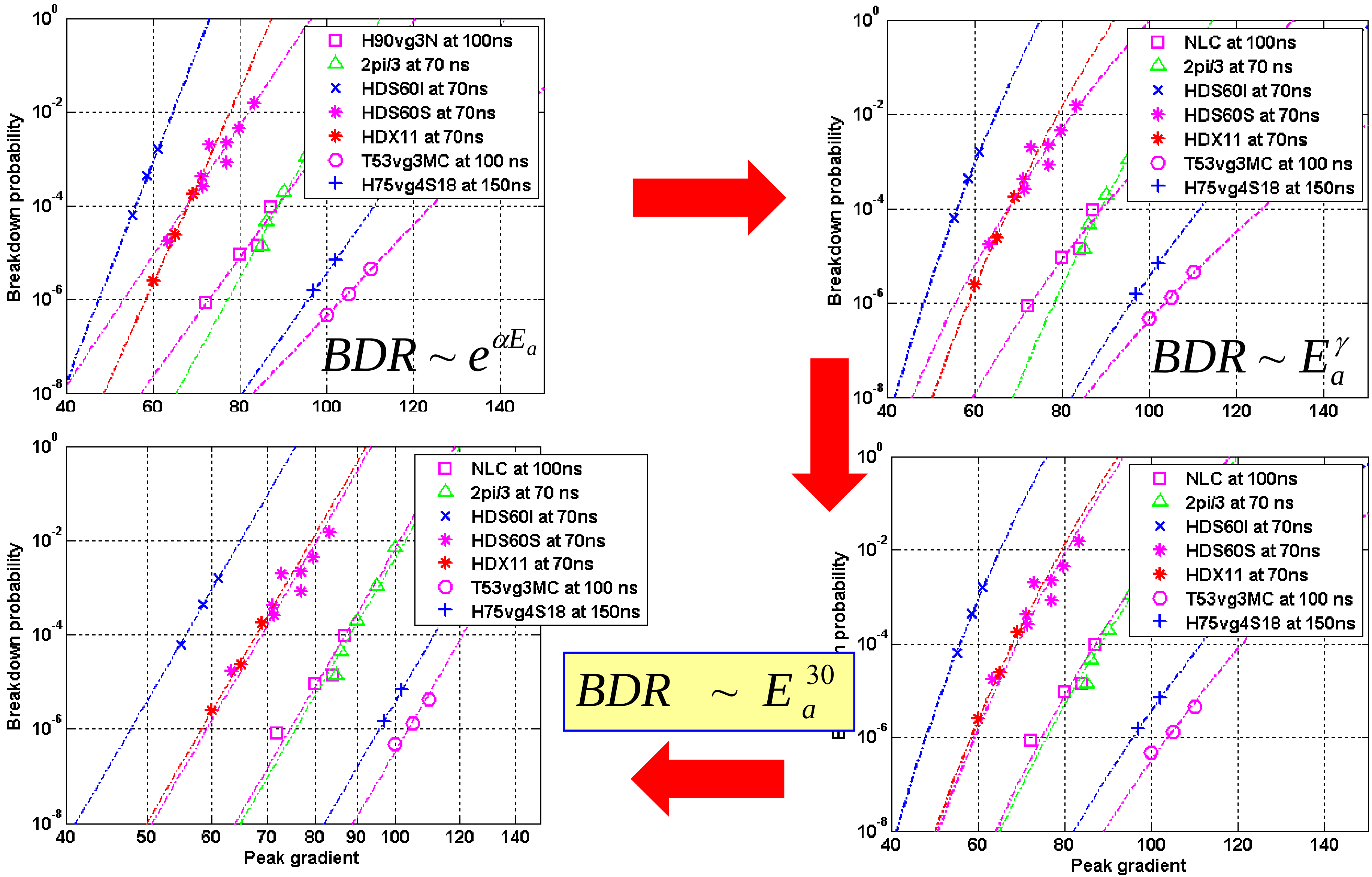
Exponential fit requires different slope depending on the gradient

BDR versus Gradient scaling

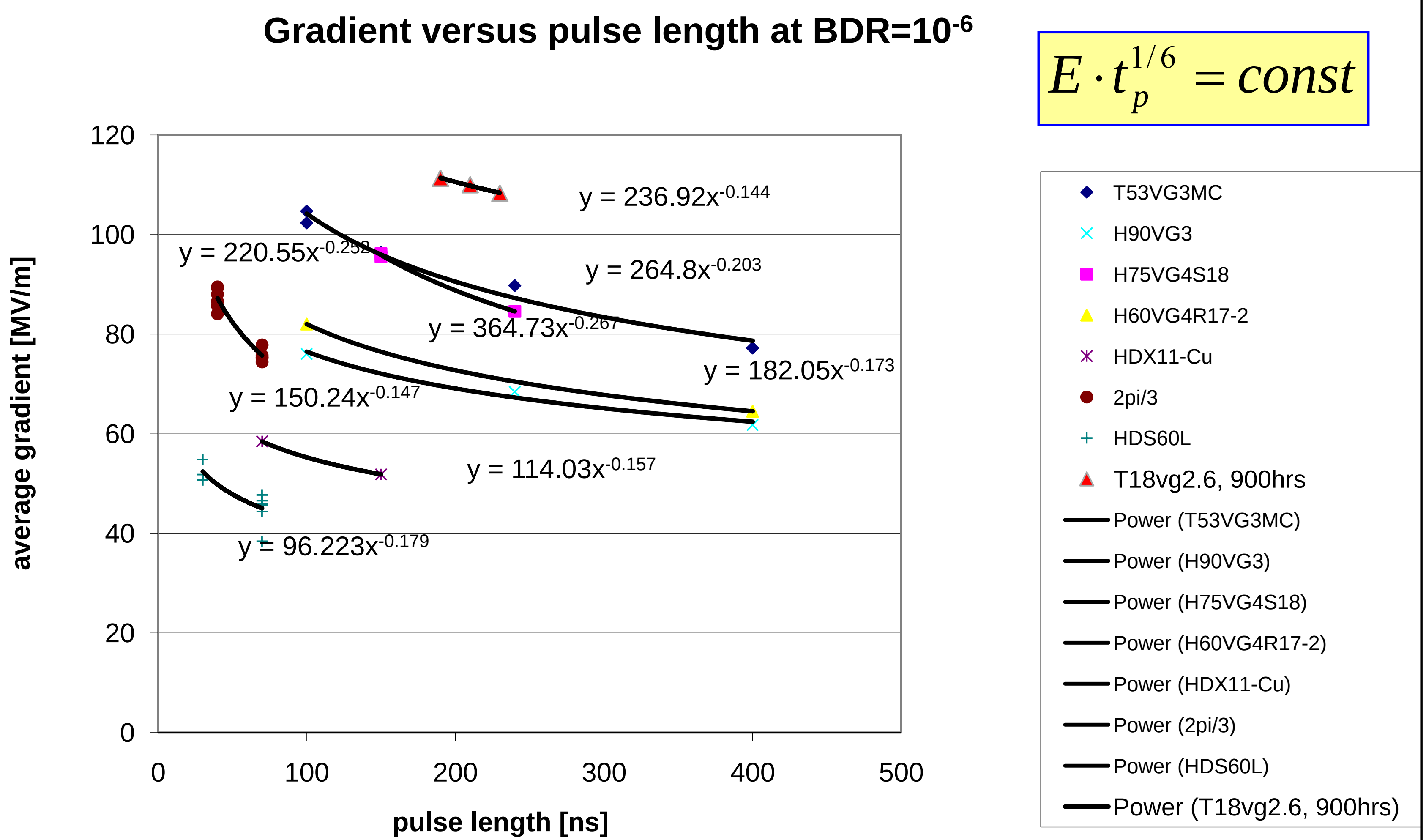


Power fit can be done with the same power for all gradients

BDR versus Gradient scaling



Gradient versus pulse length scaling



$$E \cdot t_p^{1/6} = const$$

N.B. This is very well known scaling law being confirmed again and again

Summary on gradient scaling

For a fixed pulse length

$$BDR \sim E_a^{30}$$

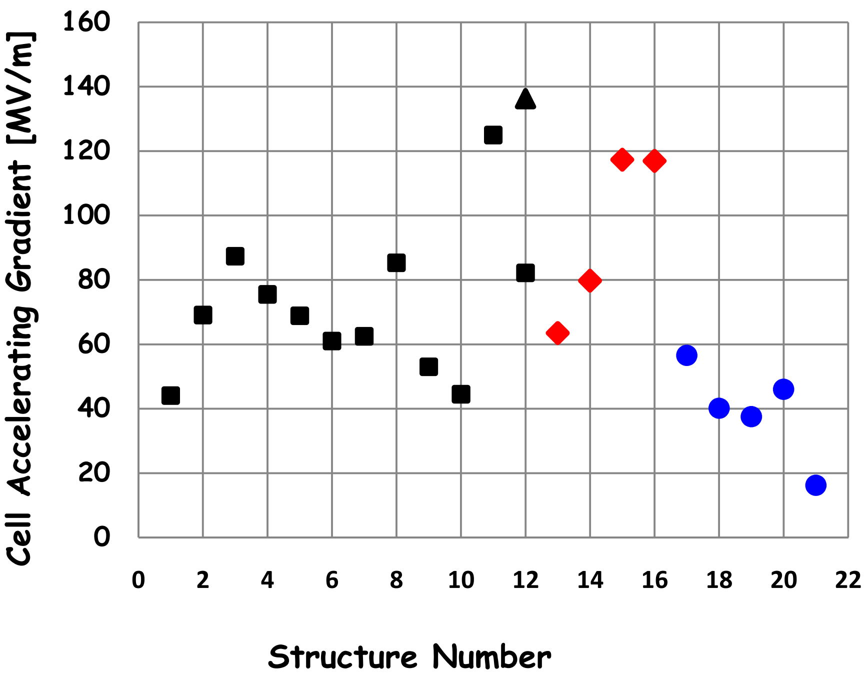
For a fixed BDR

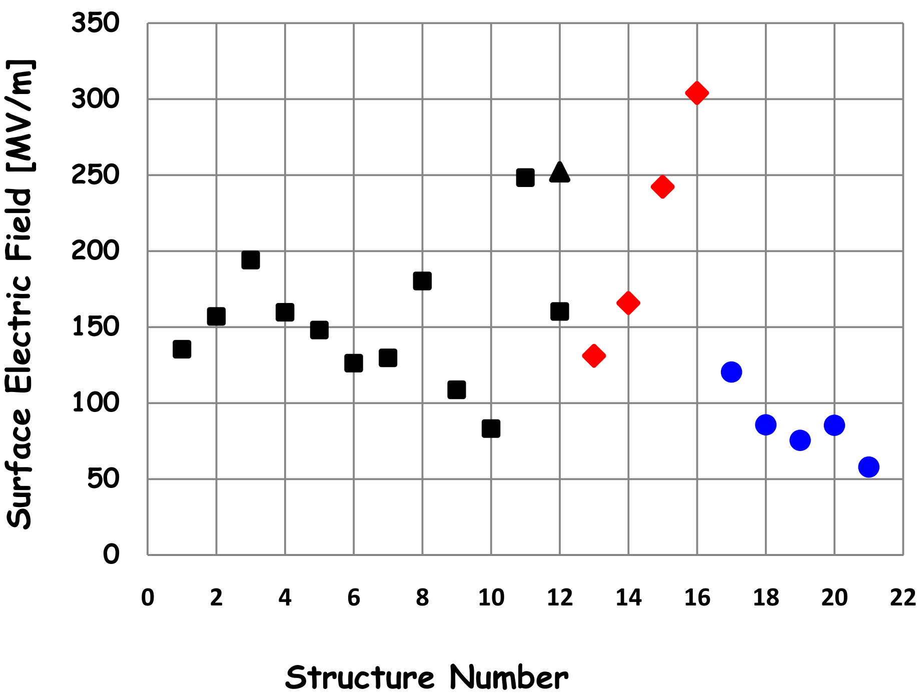
$$E_a \cdot t_p^{1/6} = const$$

$$\frac{E_a^{30} \cdot t_p^{5}}{BDR} = const$$

- In **a** Cu structure, ultimate gradient E_a can be scaled to certain BDR and pulse length using above power law. It has been used in the following analysis of the data.
- The aim of this analysis is to find a field quantity X which is geometry independent and can be scaled among **all** Cu structures.

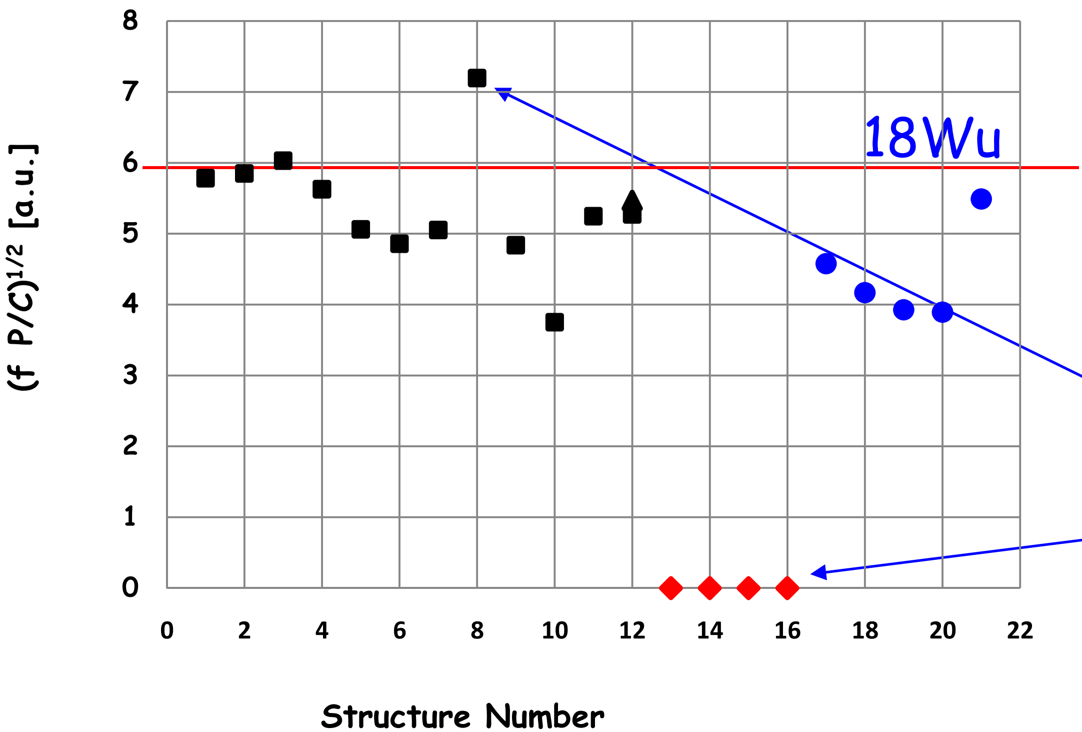
Cell gradients





Cell Accelerating and surface gradients @ 200ns, BDR=10^{-6} bpp/m

Power over circumference



Much better agreement

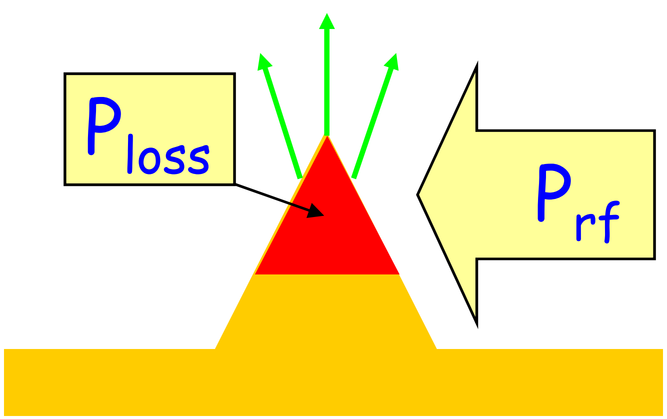
but

1. This is not a local field quantity.
2. H75vg4S18 does not really fit.
3. Does not describe standing wave structures.
4. Needs frequency scaling

Power over circumference @ 200ns, BDR=10^{-6} bpp/m

Qualitative picture

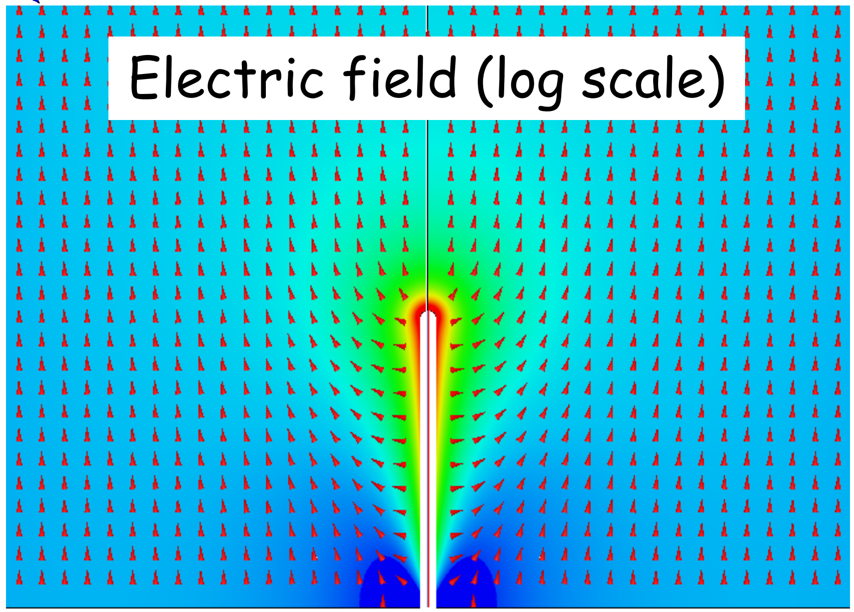
- Field emission currents J_{FN} heat a (potential) breakdown site up to a temperature rise ΔT on each pulse.
- After a number of pulses the site got modified so that J_{FN} increases so that ΔT increases above a certain threshold.
- Breakdown takes place.



This scenario can explain:

- Dependence of the breakdown rate on the gradient (Fatigue)
- Pulse length dependence of the gradient (1D÷3D heat flow from a point-like source)

EM fields around a tip of β=30



Unperturbed rf power flow:

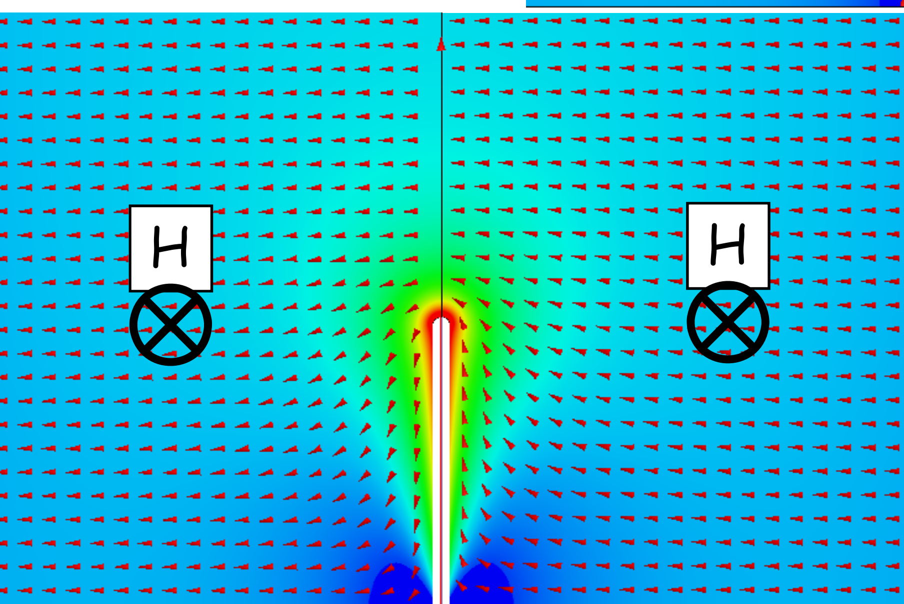
$S = E \times H$

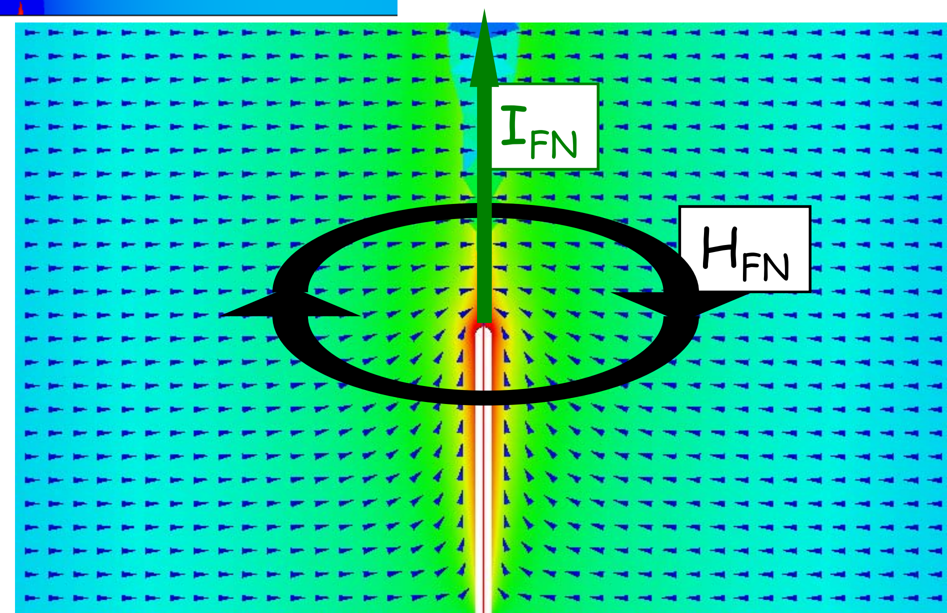
$H = \text{const}$

Field emission power flow:

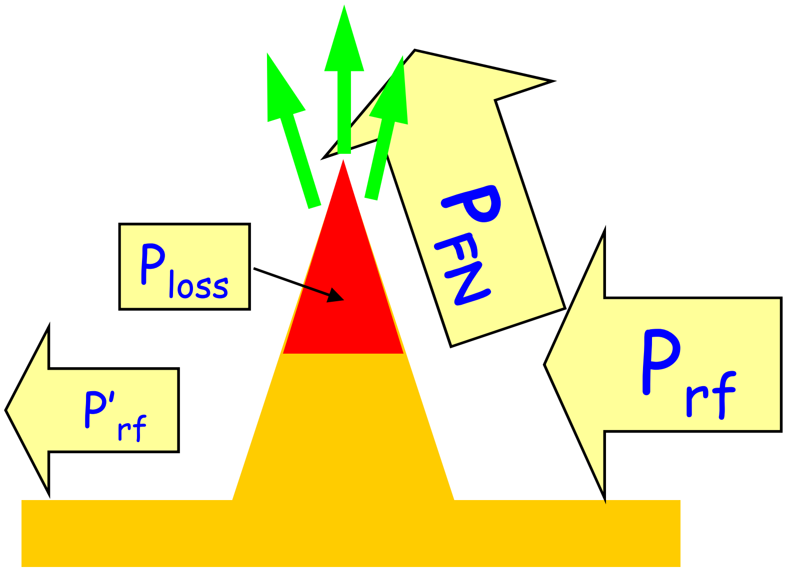
$S_{FN} = E \times H_{FN}$

$H_{FN} = I_{FN}/2\pi r$





Field emission and rf power flow



$$\Delta T \sim P_{loss} << P_{FN} \leq P_{rf}$$

$$P_{loss} = \int_V J_{FN}^2 \rho \, dv$$

$$P_{FN} = \oint_S E \times H_{FN} \, ds \sim E \cdot I_{FN}$$

$$P_{rf} = \oint_S E \times H \, ds$$

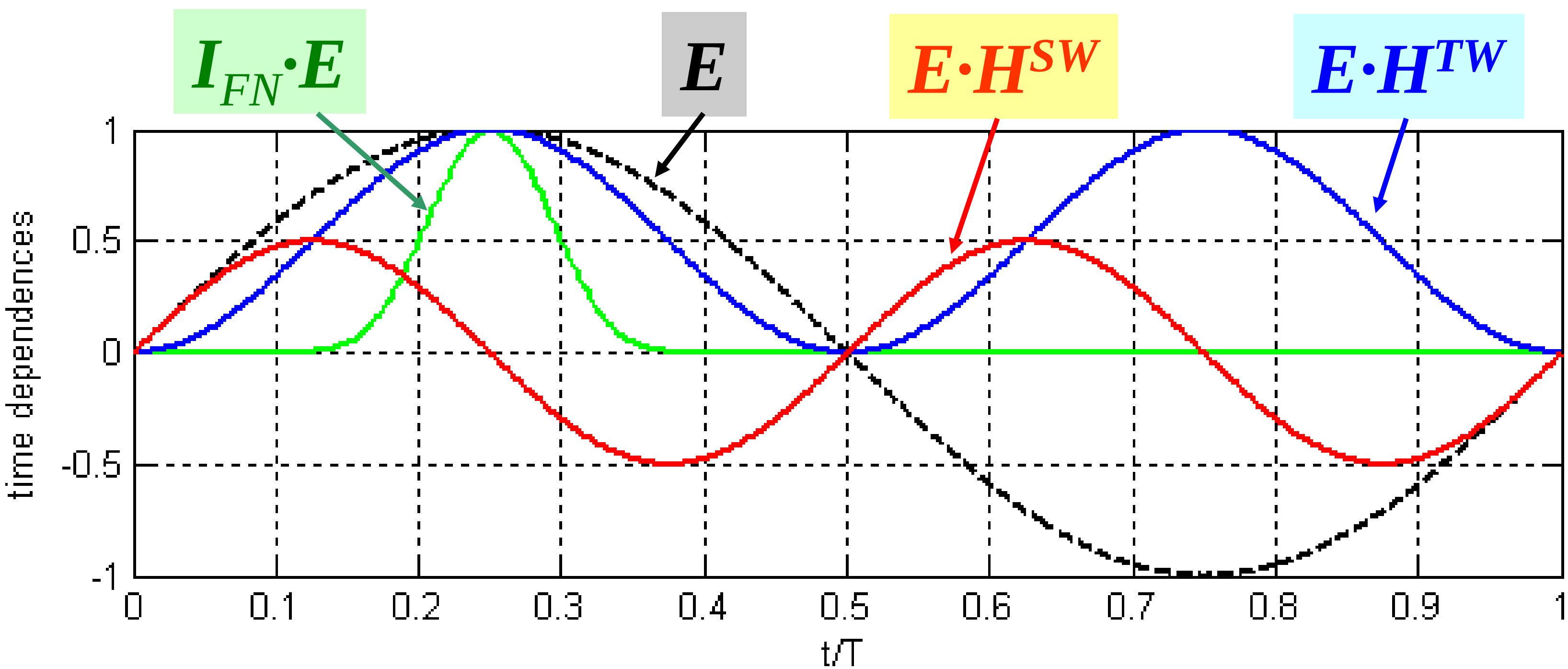
There are two regimes depending on the level of rf power flow

1. If the rf power flow dominates, the electric field remains unperturbed by the field emission currents and heating is limited by the rf power flow (We are in this regime)
2. If power flow associated with field emission current P_{FN} dominates, the electric field is reduced due to "beam loading" thus limiting field emission and heating

Field emission and power flow

$$E \times H = E_0 \cdot H_0^{TW} \sin^2 \omega t + E_0 \cdot H_0^{SW} \sin \omega t \cos \omega t$$

$$I_{FN} \cdot E = A E_0^3 \sin^3 \omega t \cdot \exp\left(\frac{-62\, GV/m}{\beta E_0 \sin \omega t} \right)$$



Field emission and rf power coupling

What matters for the breakdown is the amount of rf power coupled to the field emission power flow.

$$P_{coup} = \int_0^{T/4} P_{rf} \cdot P_{FN} dt \Bigg/ \left(\int_0^{T/4} P_{FN} dt \cdot \int_0^{T/4} P_{rf} dt \right)$$
$$= C^{TW} E_0 H_0^{TW} + C^{SW} E_0 H_0^{SW}$$

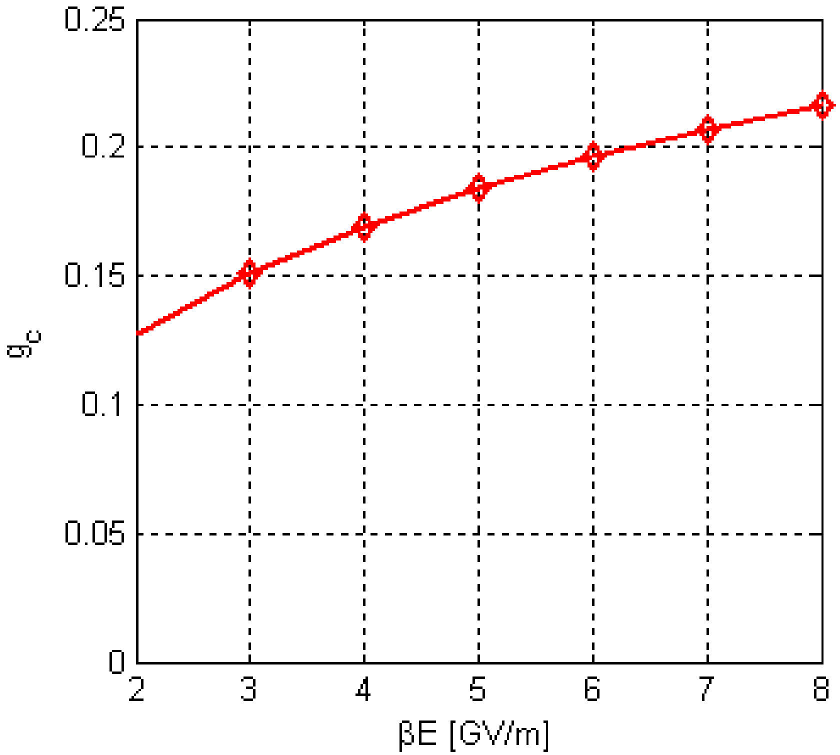
Assuming that all breakdown sites have the same geometrical parameters the breakdown limit can be expressed in terms of modified Poynting vector S_c.

$$S_c = E_0 H_0^{TW} + \frac{C^{SW}}{C^{TW}} E_0 H_0^{SW} = \mathrm{Re}\{\mathbf{S}\} + g_c \cdot \mathrm{Im}\{\mathbf{S}\}$$

Field emission and rf power coupling

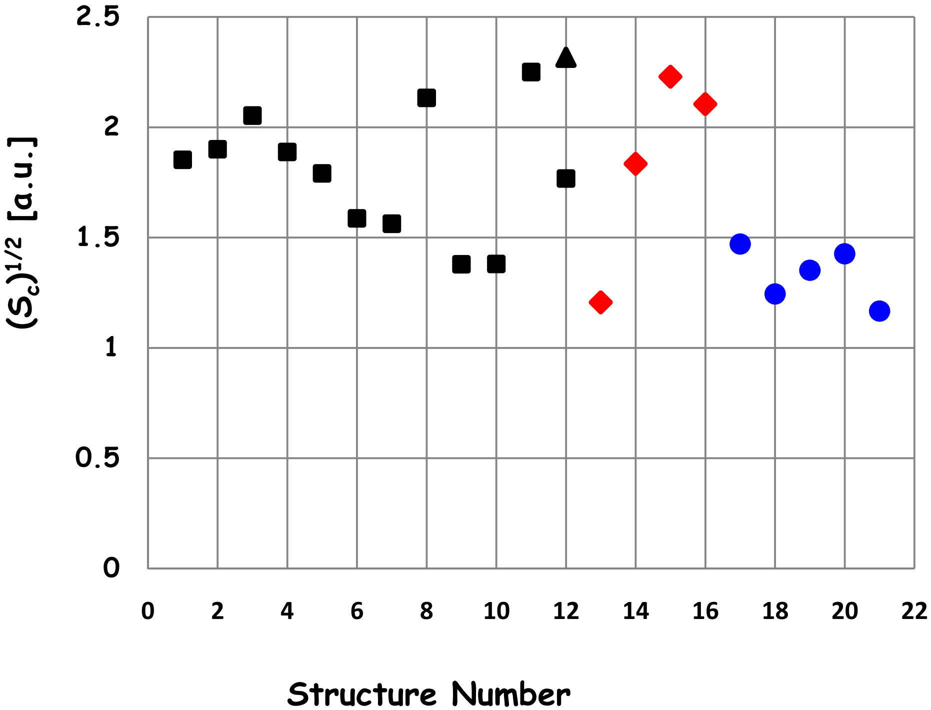
Constant g_c depends only on the value of the local surface electric field βE_0

$$g_c = \frac{\displaystyle\int_0^{\pi/2} \sin^4 x \cos x \cdot \exp\left(\frac{-62\,GV/m}{\beta E_0 \sin x}\right) dx}{\displaystyle\int_0^{\pi/2} \sin^5 x \cdot \exp\left(\frac{-62\,GV/m}{\beta E_0 \sin x}\right) dx}$$



g_c is in the range: from 0.15 to 0.2

New rf breakdown constraint S_c



$$S_c = \mathrm{Re}\{\mathbf{S}\} + \mathrm{Im}\{\mathbf{S}\}/6$$

$S_c = 4 \div 5$ [MW/mm^2]

at 200ns, BDR=1e-6 bpp/m

$S_c^{15} t_p^{5}/BDR = const$

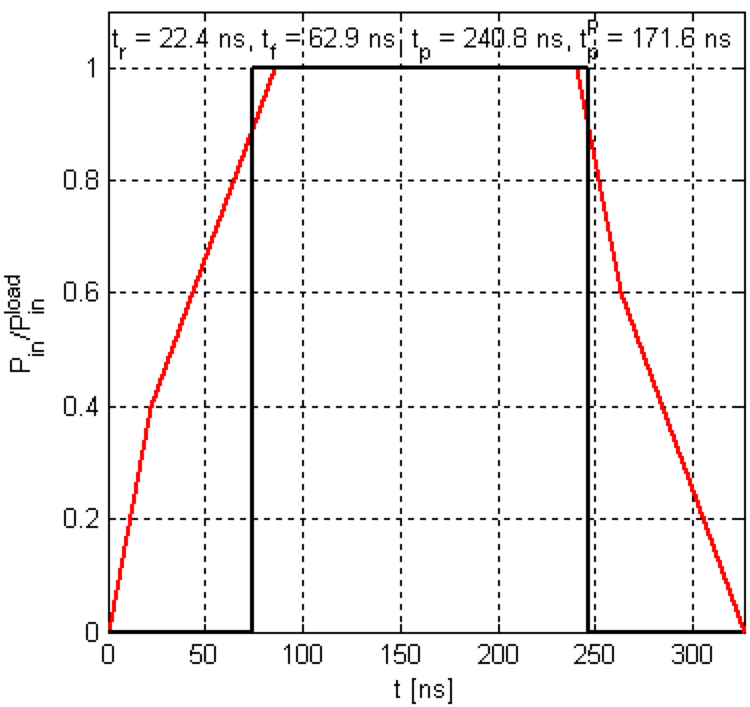
S_c in CLIC_G

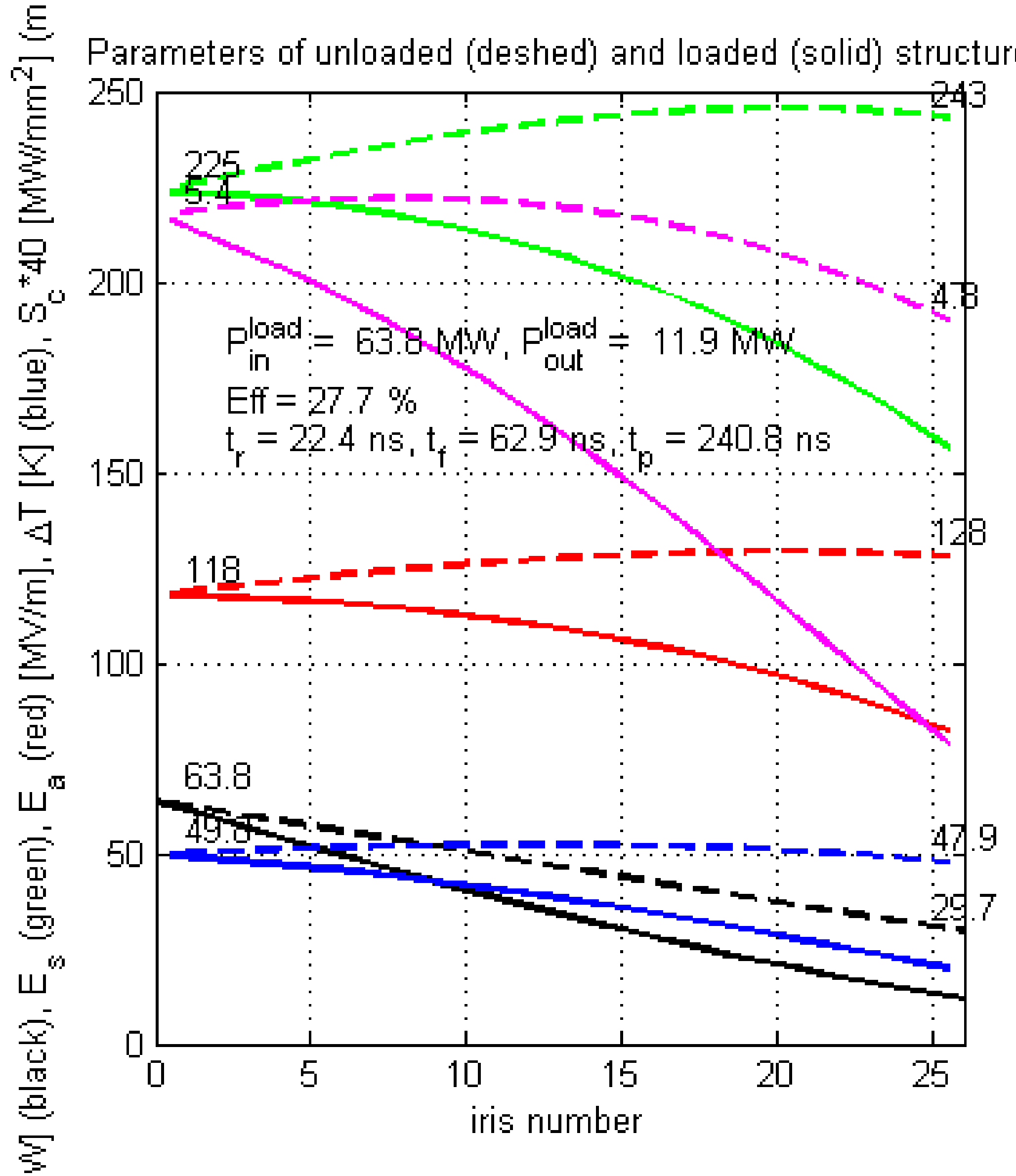
S_c reaches 5.55 for nominal parameters.

Scaling it to 200ns gives:

5.55*(171.6/200)^1/3 = 5.3

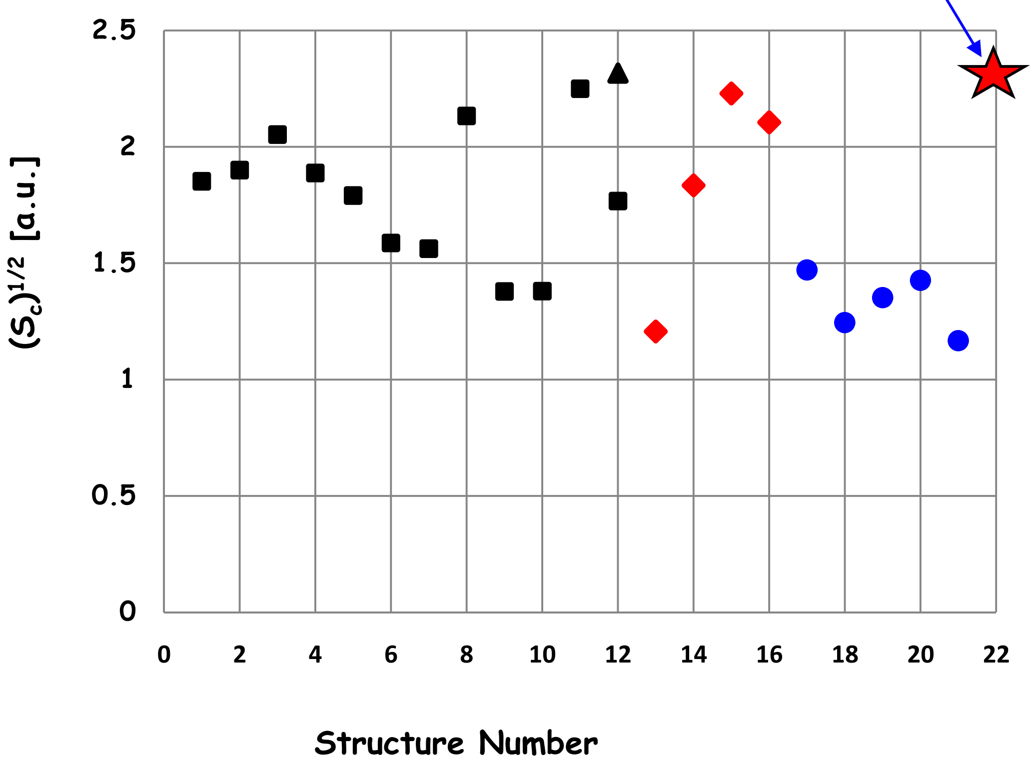
To be compared with the measured data.

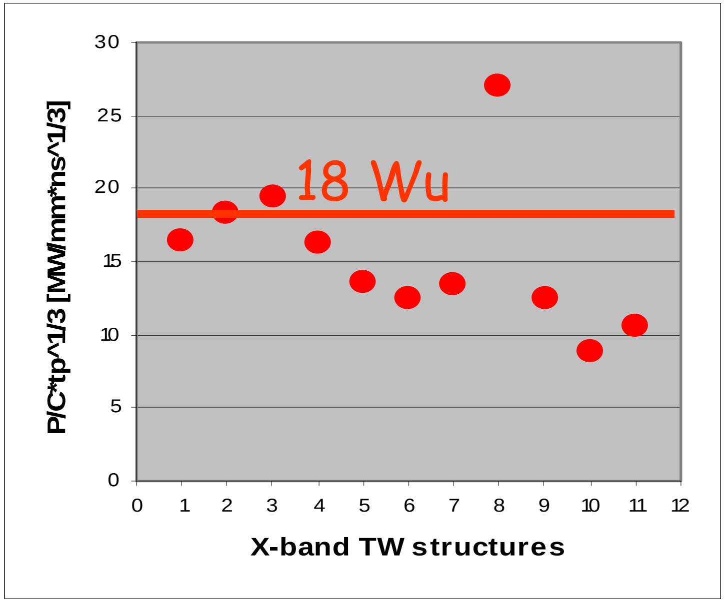


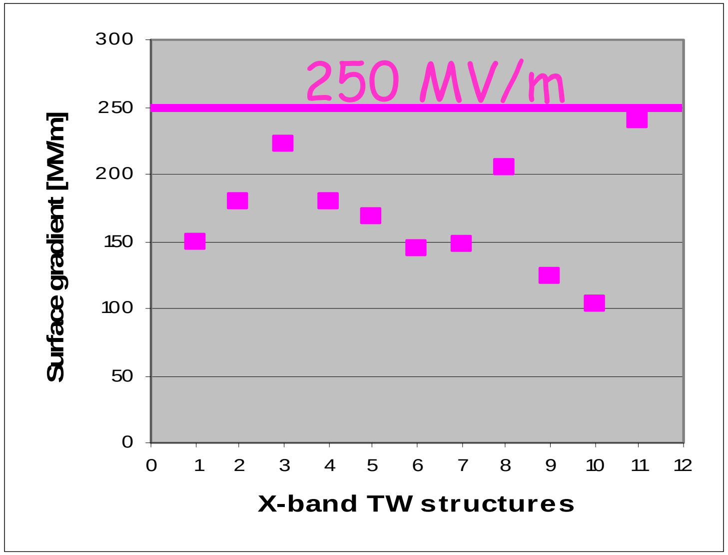


S_c in CLIC_G

S_c values in CLIC_G for the nominal parameters is very challenging



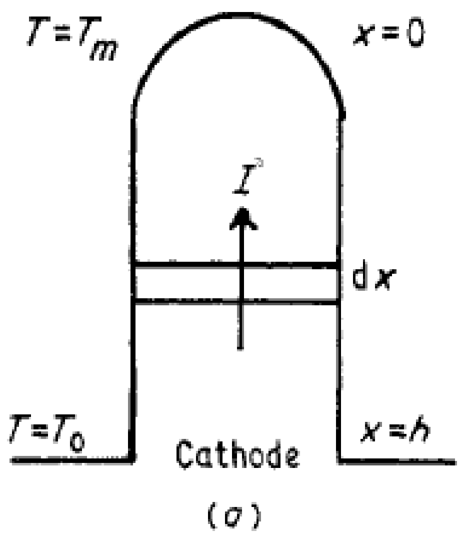




Analytical estimates for a cylindrical tip

For a cylindrical protrusion heat conduction is described by:

$$C_V \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial x^2} + J^2 \rho$$



Williams & Williams, J. Appl. Rhys. D, 5 (1972) 280

Let's get approximate solution it in two steps:

1. Solve it in steady-state (i.e. left hand side is zero) for a threshold current density required to reach melting temperature T_m
2. Solve time dependent equation in linear approximation to get the threshold time required to reach melting temperature

Analytical estimates for a cylindrical tip

Case B: Resistivity is temperature-dependent: $\rho = \rho_0 \cdot T / T_0$ (Bloch-Grüneisen)

Step 1:

$$K \frac{\partial^2 T}{\partial x^2} + J^2 \rho = 0; \quad T\big|_{x=h} = T_0; \; T\big|_{x=0} = T_m; \; \frac{\partial T}{\partial x}\bigg|_{x=0} = 0$$

$$T = T_m \cos \sqrt{\frac{J^2 \rho_0}{K T_0}} x; \quad J_m^{\rho 1} = \sqrt{\frac{K T_0}{h^2 \rho_0}} \arccos \frac{T_0}{T_m}$$

Step 2:

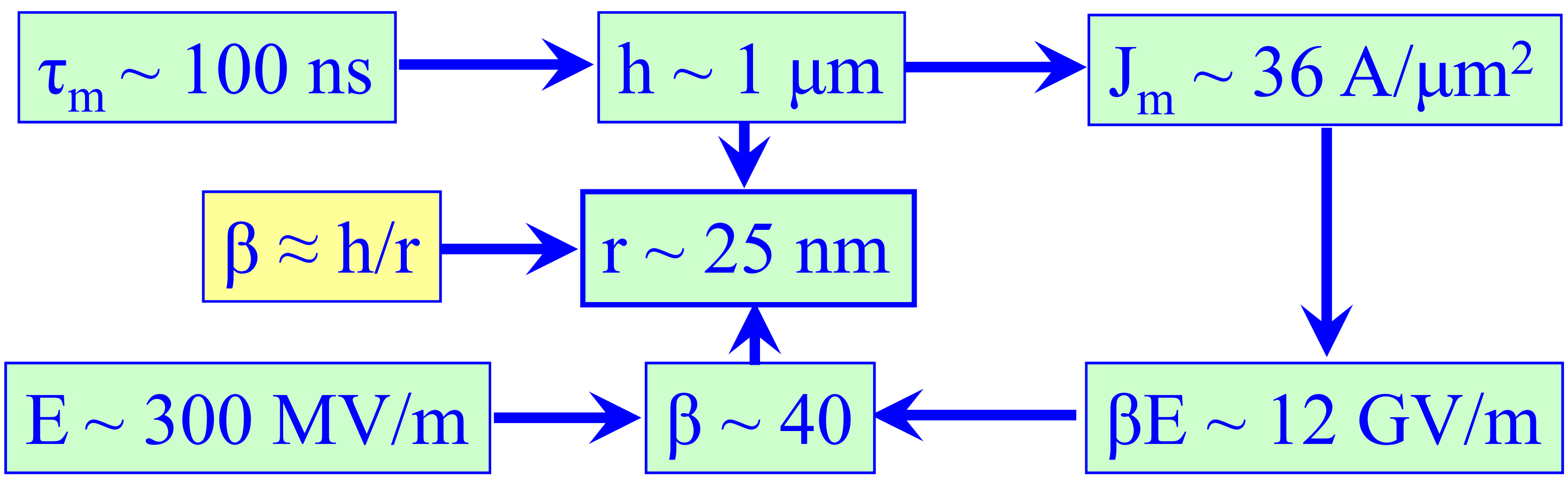
$$C_V \frac{\partial T}{\partial t} = J^2 \rho; \quad T\big|_{t=0} = T_0; \quad T\big|_{t=t_m} = T_m$$

$$T = T_0 \exp \frac{J^2 \rho_0}{C_V T_0} t; \quad \tau_m^{\rho 1} = \frac{C_V T_0}{J^2 \rho_0} \ln \frac{T_m}{T_0} = \frac{C_V}{K} h^2 \ln \frac{T_m}{T_0} \bigg/ \arccos^2 \frac{T_0}{T_m}$$

Analytical estimates for a cylindrical tip

Fundamental constants for copper	
Thermal conductivity: K [W/m·K]	400
Volumetric heat capacity: C_V [MJ/m^3·K]	3.45
Resistivity@300K: ρ_0 [nΩ·m]	17
Melting temperature: T_m [K]	1358

Some numbers for Case B: $\rho = \rho_0 \cdot T/T_0$



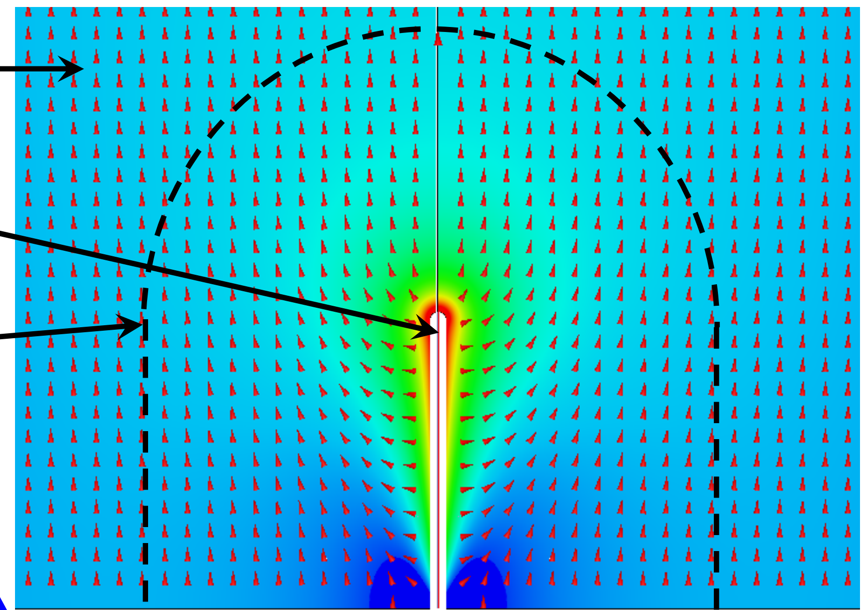
Some numbers for Case 2: $\rho = \rho_0 \cdot T/T_0$ (Continue)

$\beta \sim 40$

$E \sim 300$ MV/m

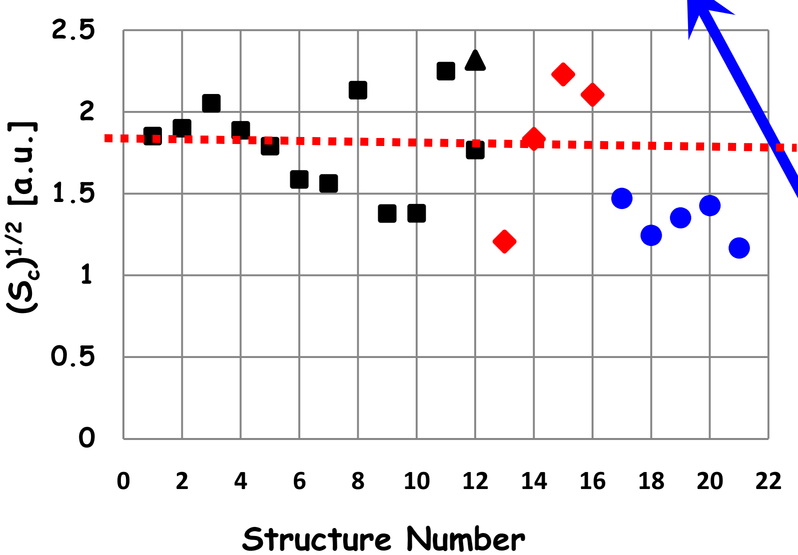
$J_m \sim 36$ A/µm²

$S_{FN} \sim 3.4$ W/µm²



$r \sim 25$ nm

$h \sim 1$ µm



$$S_{FN}\Big|_h = E\,H_{FN} = E\frac{J\pi r^2}{2\pi h} = E\frac{J r}{2\beta}$$

Conclusions on the new rf constraint

- All (?) available results of the high gradient rf tests has been collected and analyzed
- A model of the breakdown trigger has been developed based on the pulsed heating of the potential breakdown site by the field emission currents
- A new field quantity, modified Poynting vector: S_c, has been derived which takes into account both active and reactive power flow
- This new field quantity describes both travelling wave and standing wave accelerating structure experimental results rather well.
- The value of S_c achieved in the experiments agrees well with analytical estimate

Acknowledgements

- Sergio Calatroni
- Chris Adolphsen
- Steffen Doebert
- …