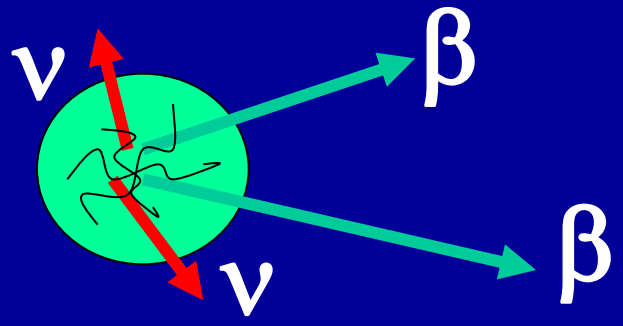
WESTFÄLISCHE
WILHELMS-UNIVERSTÄT
MÜNSTER

D. Frekers

Charge-exchange reactions and ββ-decay matrix elements

--GT transitions and beyond--



Outline

- **Charge-ex reactions (^{3}He,t) & (d,^{2}He)**
  - highlights & general features of $2\nu\beta\beta$ NME

    ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{136}Xe

    fragmentation – smallest/largest NME

  - solar neutrino SNU rates and (^{3}He,t) reaction

    ^{71}Ga(^{3}He,t)

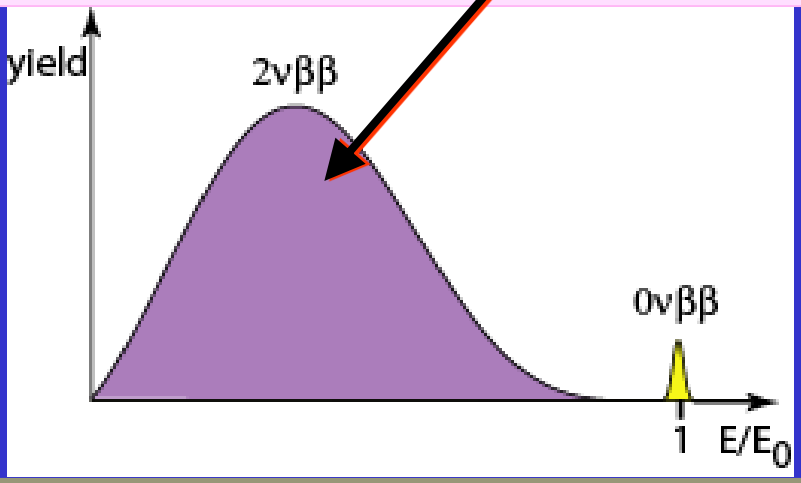
- **the $0\nu\beta\beta$ decay nuclear matrix elements**
  - 1st forbidden NME's and 2$^-$ states
  - the A=96 system

    the ^{96}Zr ($\beta-$) → ^{96}Nb Q-value

    and a direct test of $0\nu\beta\beta$ NME

Nucl. Matrix Elements

$2\nu\beta^-\beta^-$ decay



q-transfer like in ordinary β-decay

($q \sim 0.01$ fm^{-1} ~ 2 MeV/c)

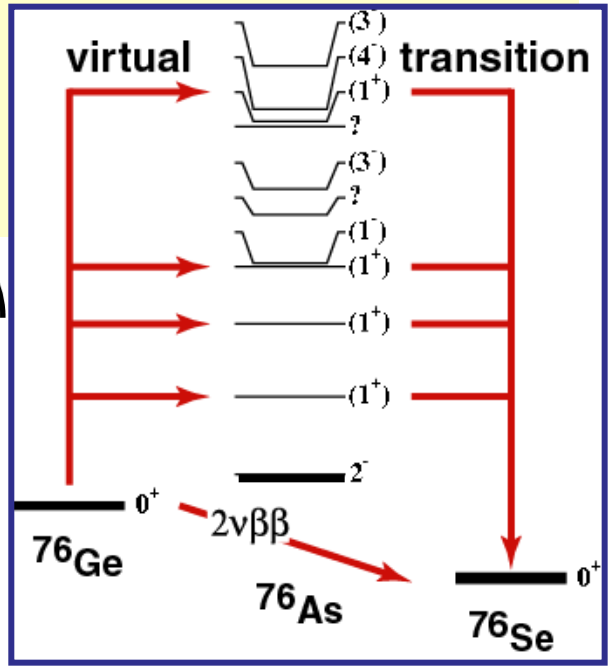
i.e. only allowed transitions possible

$$\Gamma^{2\nu}_{(\beta^-\beta^-)} = \frac{C}{8\pi^7}\left(\frac{G_F\, g_A}{\sqrt{2}}\cos(\Theta_C)\right)^4 \left|M^{(2\nu)}_{DGT}\right|^2 \mathcal{F}^2_{(-)}\, f(Q)$$

$$= G^{2\nu}(Q,Z) \qquad \left|M^{(2\nu)}_{DGT}\right|^2$$

$\propto Q^{11} \cdot Z^2$

$\exp \approx 10^{-3}$ MeV^{-2} extracted from half-life

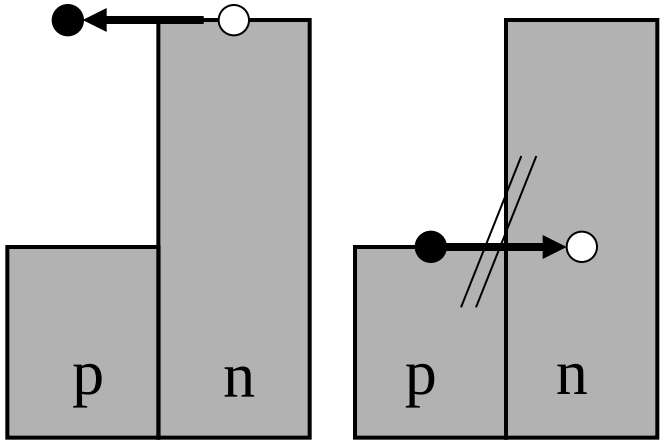


favorable:

1. high Q-value
2. large Z

unfavorable (but cannot be changed):

1. large neutron excess (Pauli-blocking)



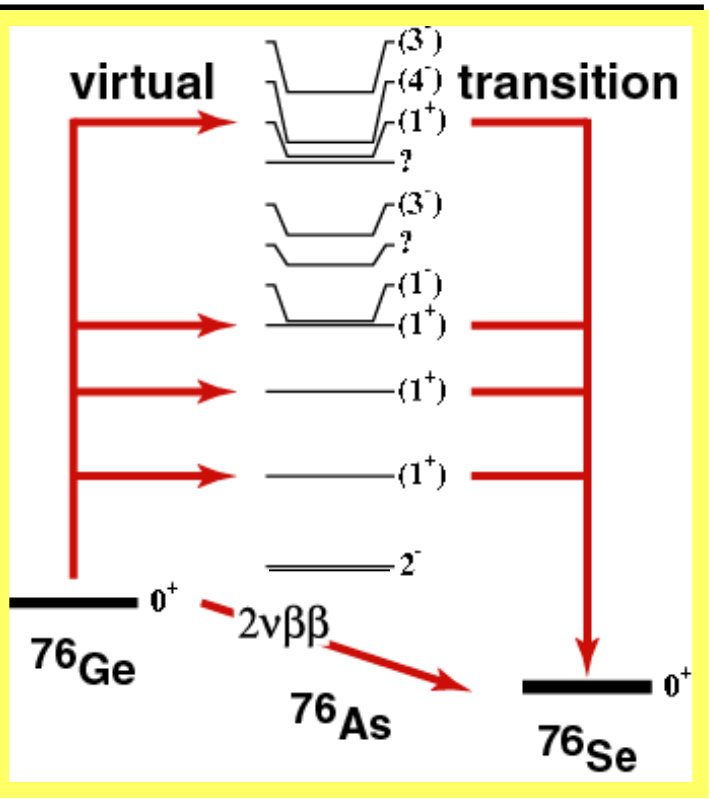
$$M_{DGT}^{(2\nu)} = \sum_m \frac{\left\langle 0_{g.s.}^{(f)} \left| \sum_k \sigma_k \tau_k^- \right| 1_m^+ \right\rangle \left\langle 1_m^+ \left| \sum_k \sigma_k \tau_k^- \right| 0_{g.s.}^{(i)} \right\rangle}{\frac{1}{2} Q_{\beta\beta}(0_{g.s.}^{(f)}) + E(1_m^+) - E_0}$$

$$= \sum_m \frac{M_m \; GT^+ \;\; M_m \; GT^-}{E_m}$$

to remember:

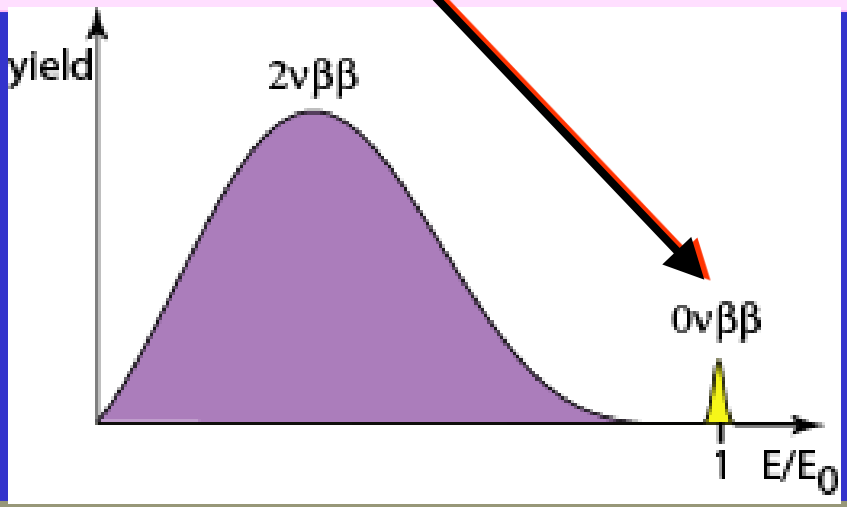
1. 2 sequential & „allowed" β^--decays of „Gamow-Teller" type
2. „1, 2, 3, ... forbidden" decays negligible
3. Fermi-transitions do no contribute (because of different isospin-multiplets)

Can be determined via charge-exchange reactions in the (n,p) and (p,n) direction (e.g. (d,^{2}He) or (^{3}He,t))



Nucl. Matrix Elements

$0\nu\beta^-\beta^-$ decay



neutrino is a virtual particle

$q \sim 0.5 fm^{-1}$ (~ 100 MeV/c)

(due to Heisenberg $\Delta q \cdot \Delta x \sim 1$)

degree of forbiddeness is lifted

$$\Gamma^{0\nu}_{(\beta^-\beta^-)} = G^{0\nu}(Q,Z)\, g_A^4 \left| M^{(0\nu)}_{DGT} - \left(\frac{g_V}{g_A}\right)^2 M^{(0\nu)}_{DF} \right|^2 \left| m_{\nu_e} \right|^2$$

!!

$\propto Q^5 \cdot Z^4$

theory $\approx$ 10 !!

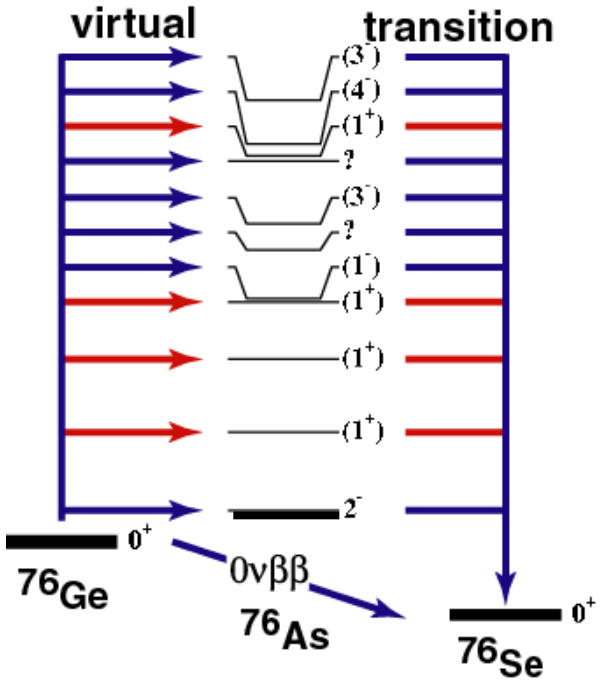
largely independent of (A,Z)
(except near magic nuclei)

mass of Majorana-ν !

to remember:

1. „higher-fold forbidden" transitions possible
2. Fermi-transitions important
3. „Pauli-blocking" largely lifted
4. large Q-value, high Z important

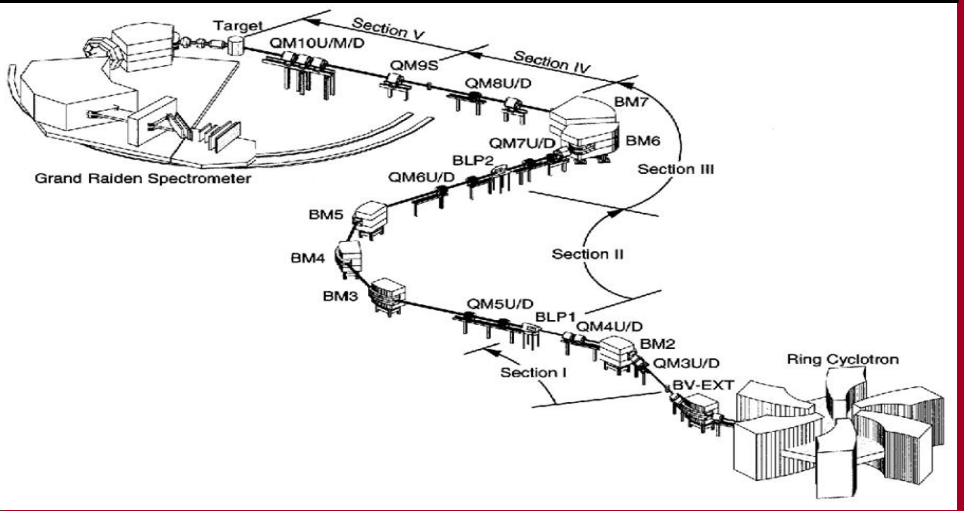
NOT (easily) accessible via charge-exchange reactions

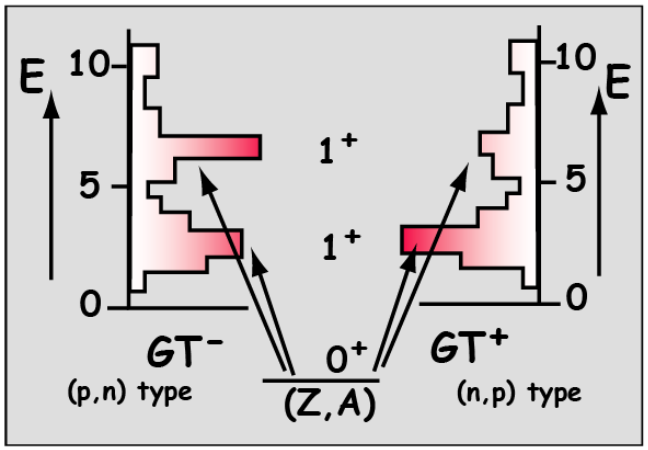


Charge-exchange reactions

Grand Raiden Magnetic Spectrometer

$\Delta E/E \sim 5 \times 10^{-5}$ ~ 25 keV at 420 MeV (^{3}He)





Q: what is the connection between „weak στ operator" and the hadronic reaction

A: dominance of the $V_{\sigma\tau}$ effective interaction at medium energies

$$M(GT) = \langle 1^+ || \sigma\tau^{\pm} || 0_{g.s.}^{i} \rangle$$

$$B(GT) = \frac{1}{2J_i+1} \, | \, M(GT) \, |^2$$

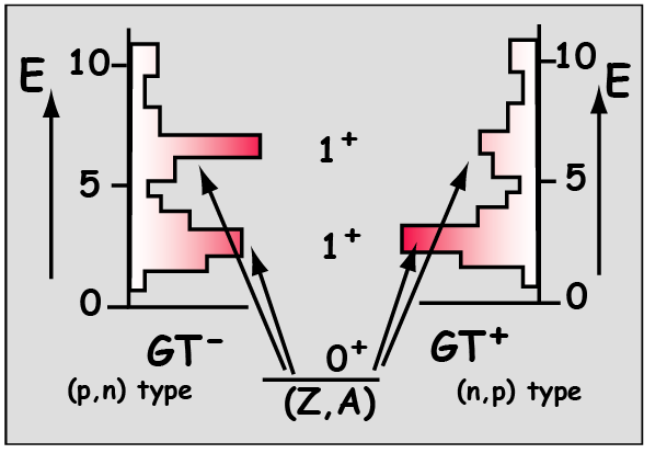
hadronic probes: (n,p), (d,^{2}He), (t,^{3}He)

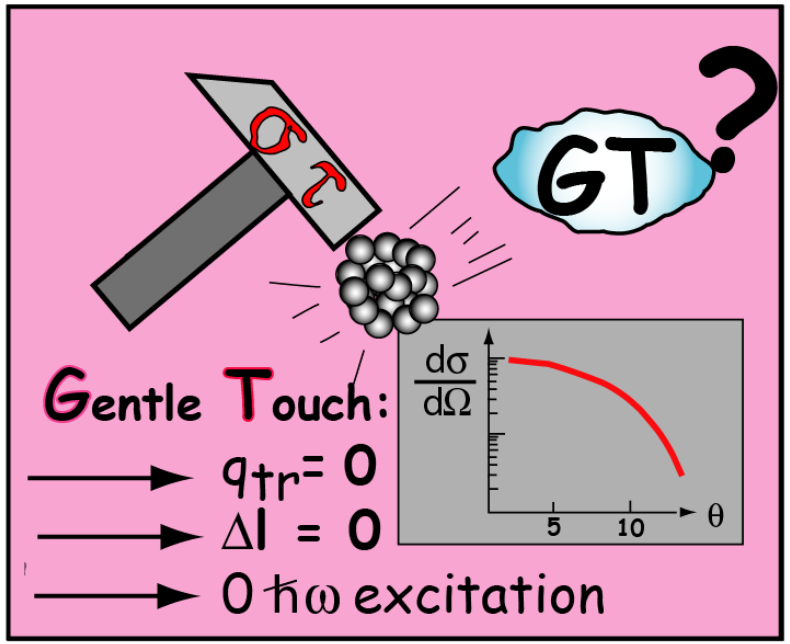
or (p,n), (^{3}He,t)

$$\left[\frac{d\sigma}{d\Omega}\right] = \left[\frac{\mu}{\pi\hbar}\right]^2 \frac{k_f}{k_i} \, N_d \, |V_{\sigma\tau}|^2 \, |\langle f \, | \, \sigma\tau \, | \, i \rangle|^2$$

$q = 0$!!

largest at 100 - 200 MeV/A





$$M(GT) = \langle 1^+ \| \sigma\tau^{\pm} \| 0_g^i{}_s. \rangle$$

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hadronic probes: (n,p), (d,^{2}He), (t,^{3}He)

or (p,n), (^{3}He,t)

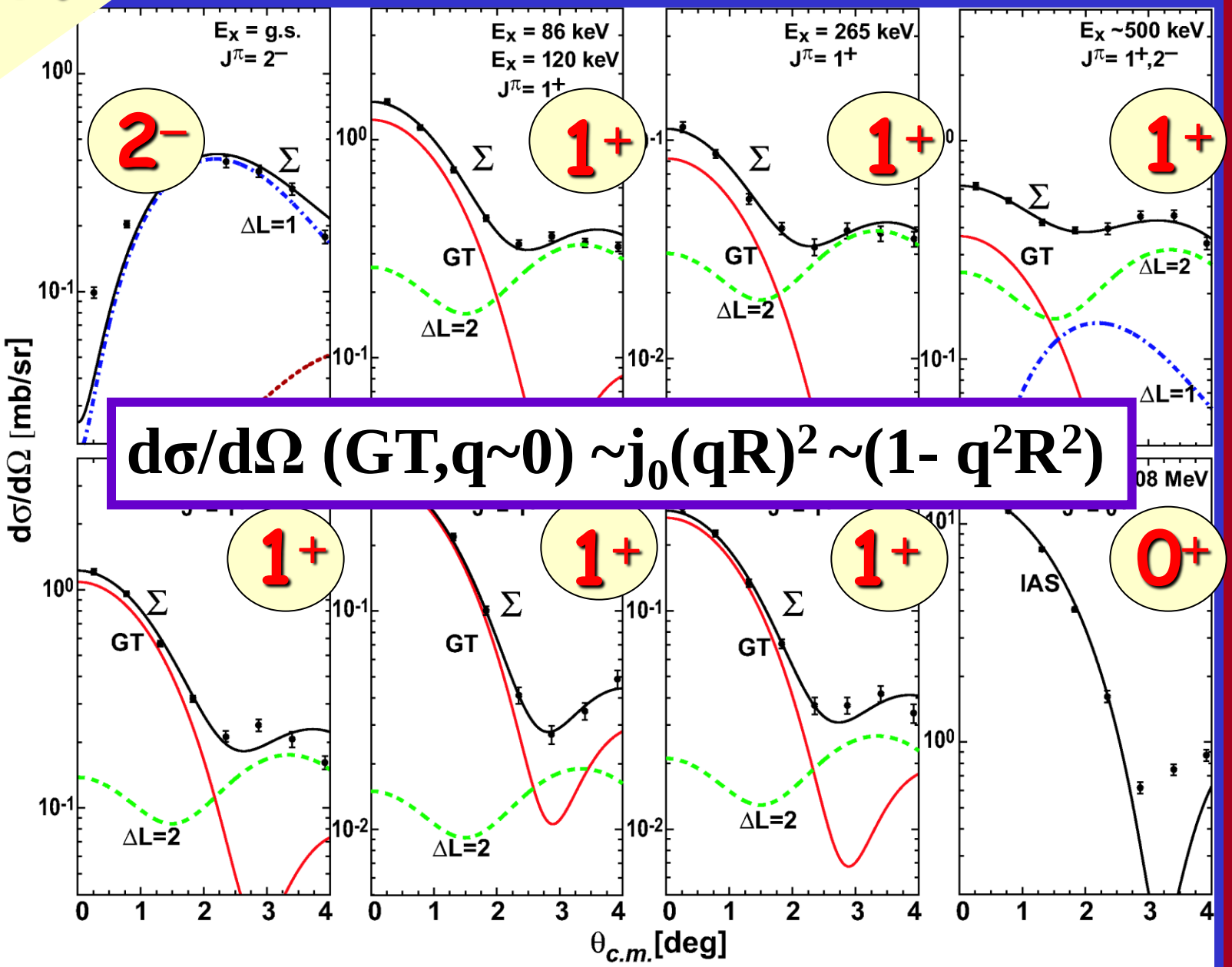
$$\left[\frac{d\sigma}{d\Omega}\right] = \left[\frac{\mu}{\pi\hbar}\right]^2 \frac{k_f}{k_i} N_d \, |V_{\sigma\tau}|^2 \, |\langle f \, | \, \sigma\tau \, | \, i \rangle|^2$$

$q = 0$!!

largest at 100 - 200 MeV/A

examples

76Ge-76As

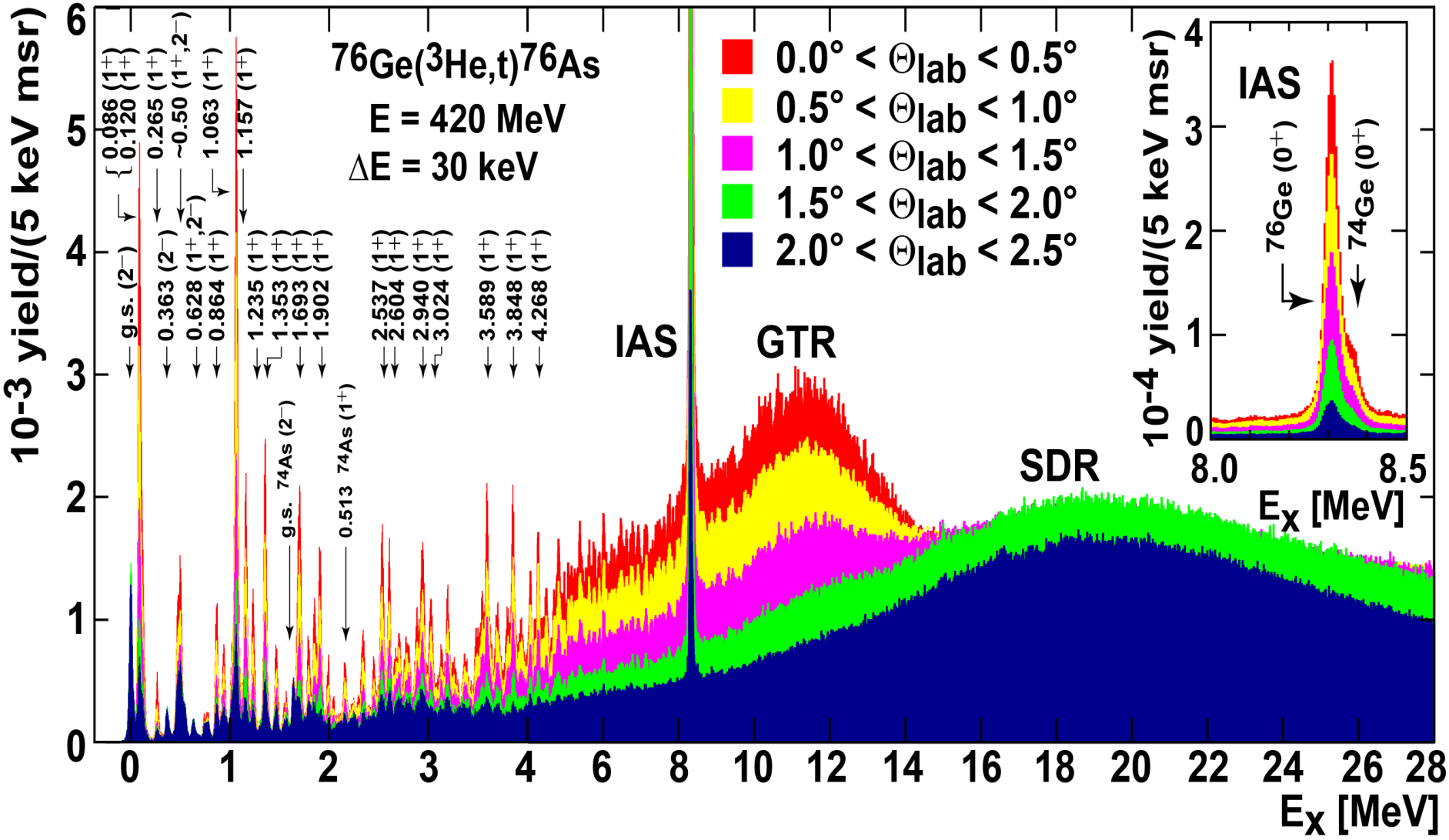


$$d\sigma/d\Omega\ (GT,q\sim 0) \sim j_0(qR)^2 \sim (1- q^2R^2)$$

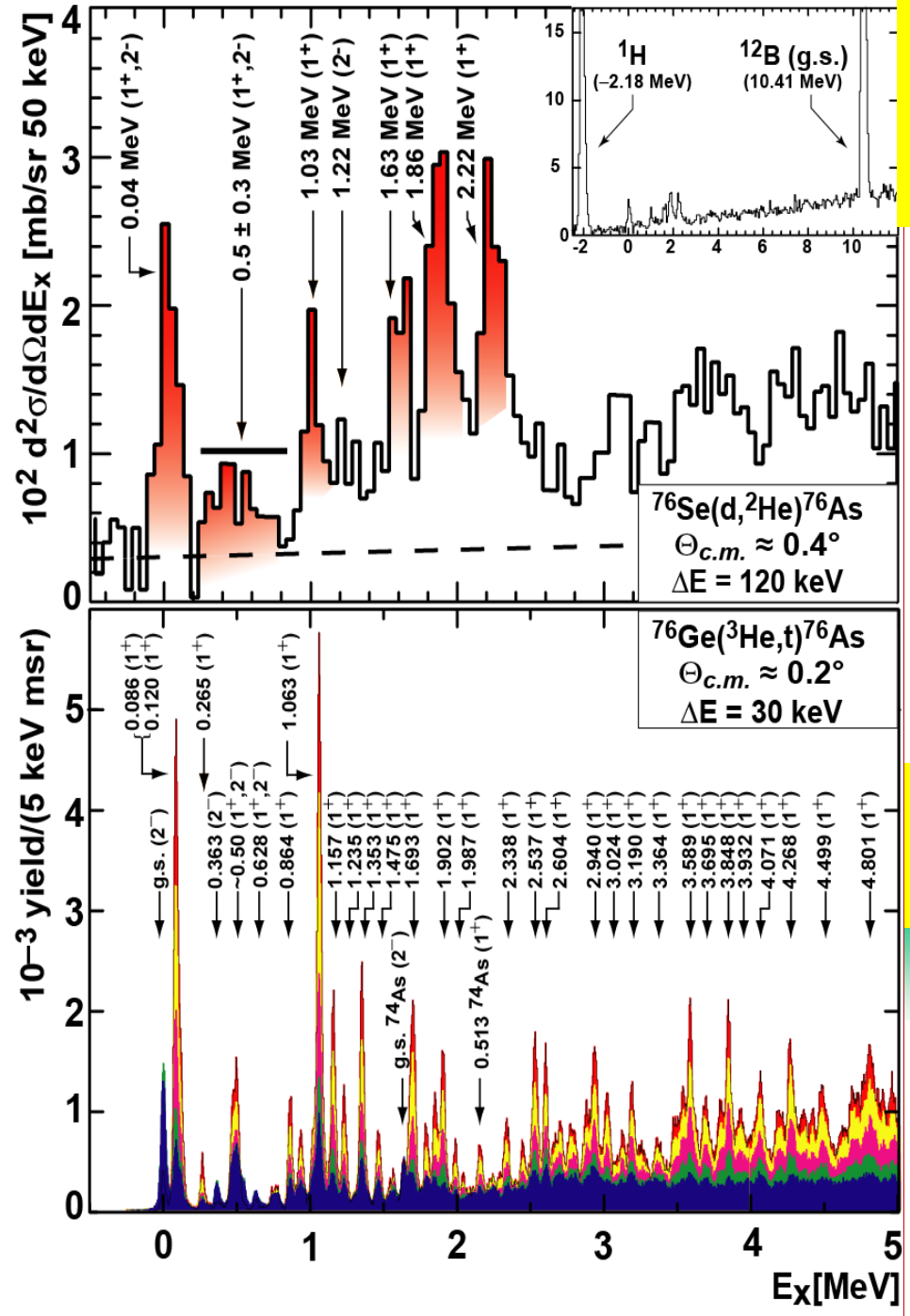
^{76}Ge

N-Z=10

Resolution is the key !!!

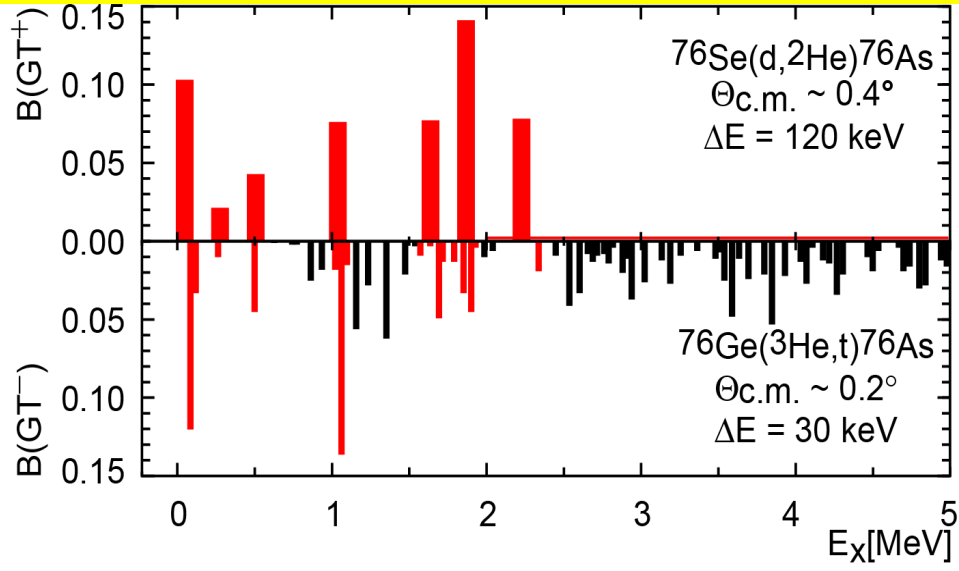


almost 70 !! resolved single states up to 5 MeV identified as GT 1+ transitions !!!



~ 70 !! single states up to 5 MeV !!!

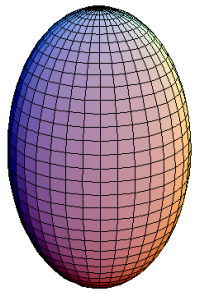
???? anti-correlation ????

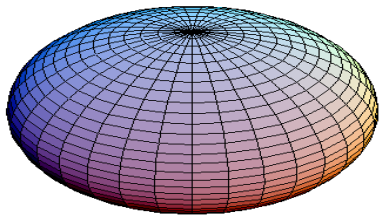


is the anti-correlation a property of deformation ??

^{76}Ge

^{76}Se



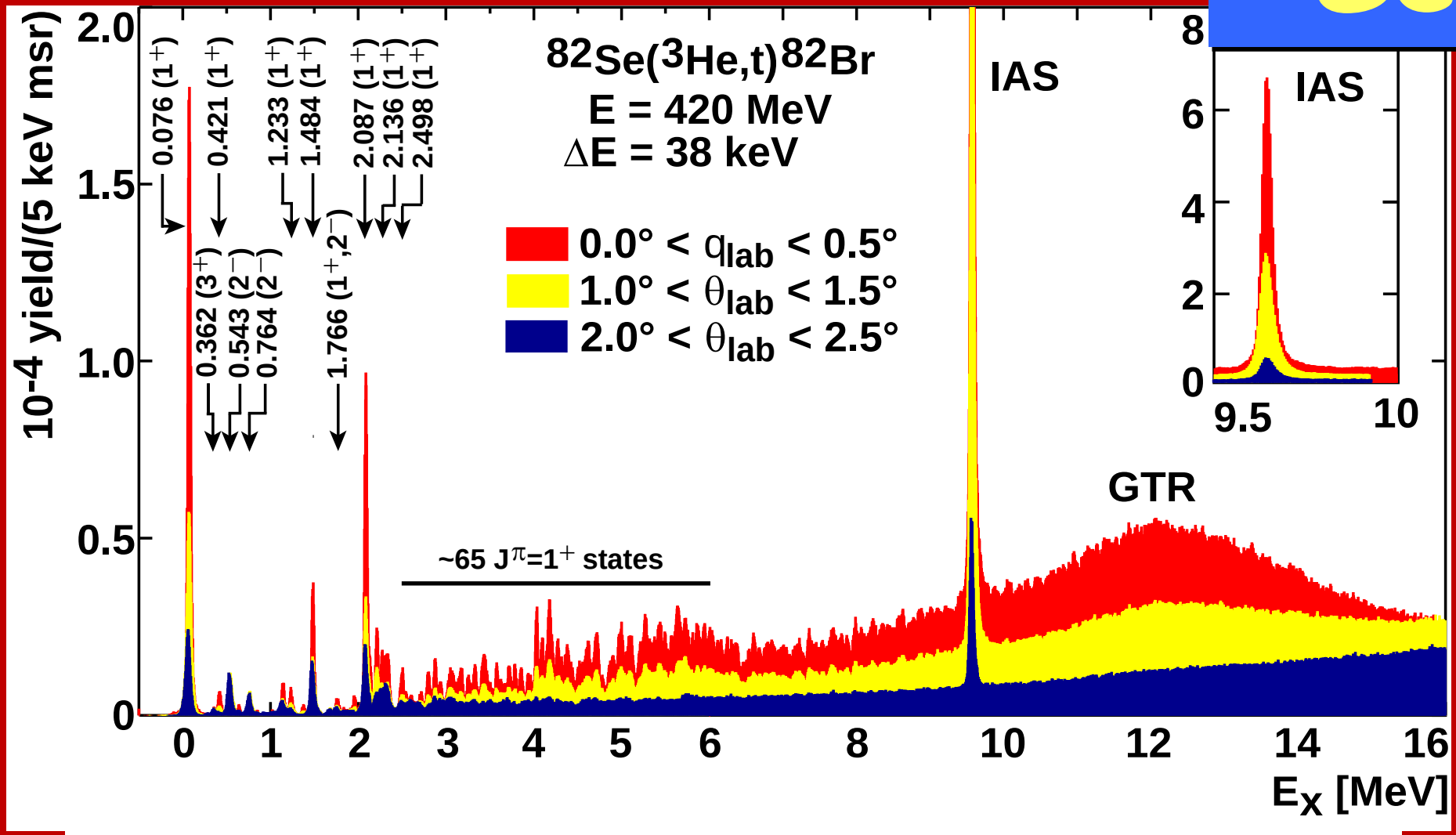


^{82}Se

N-Z=14

Resolution is the key !!!

^{82}Se

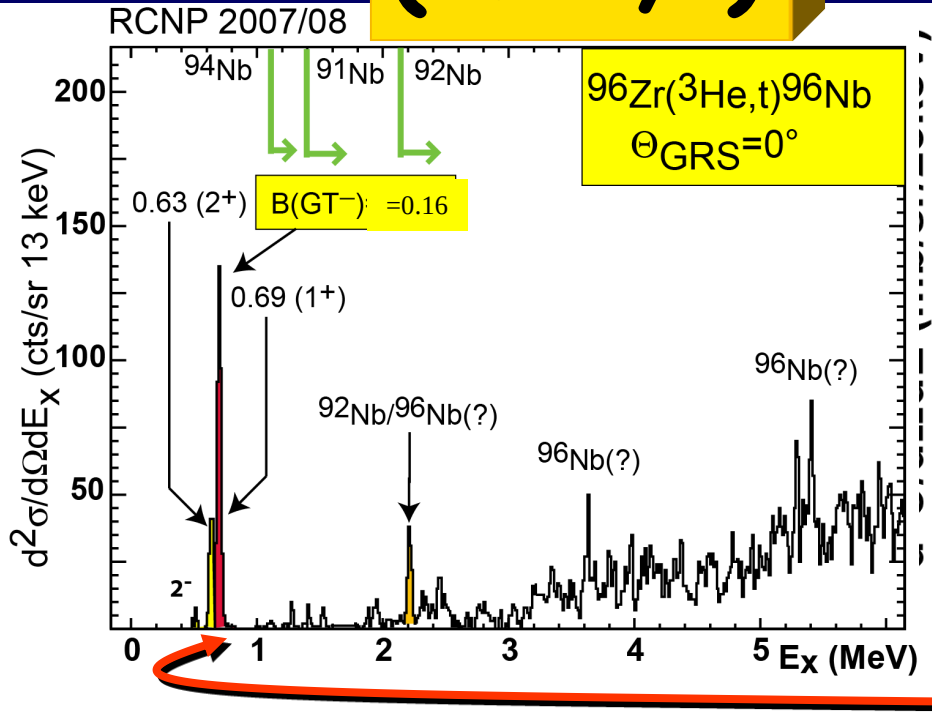


3 isolated GT transition below 2 MeV-fragmentation recedes to GT resonance

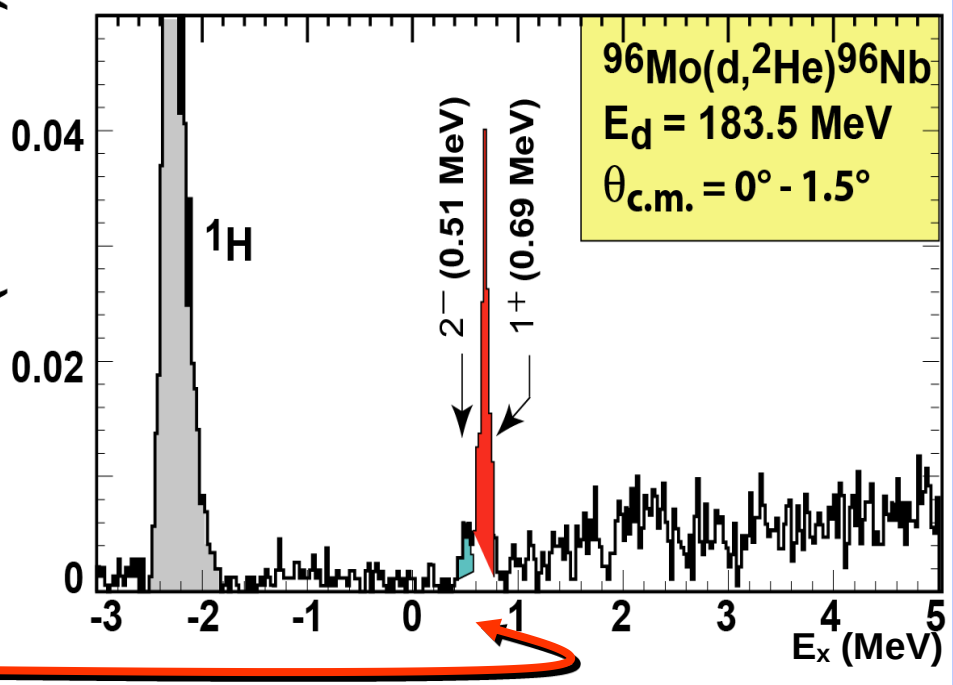
^{96}Zr

N-Z=16

(^{3}He,t)



(d,^{2}He)



B(GT$^-$) = 0.16

B(GT$^+$) = 0.3

Fascination: With only 1 state:

$$T_{1/2}^{calc.}(2\nu\beta\beta) = (2.1 \pm 0.4) \cdot 10^{19}\ \text{years}$$

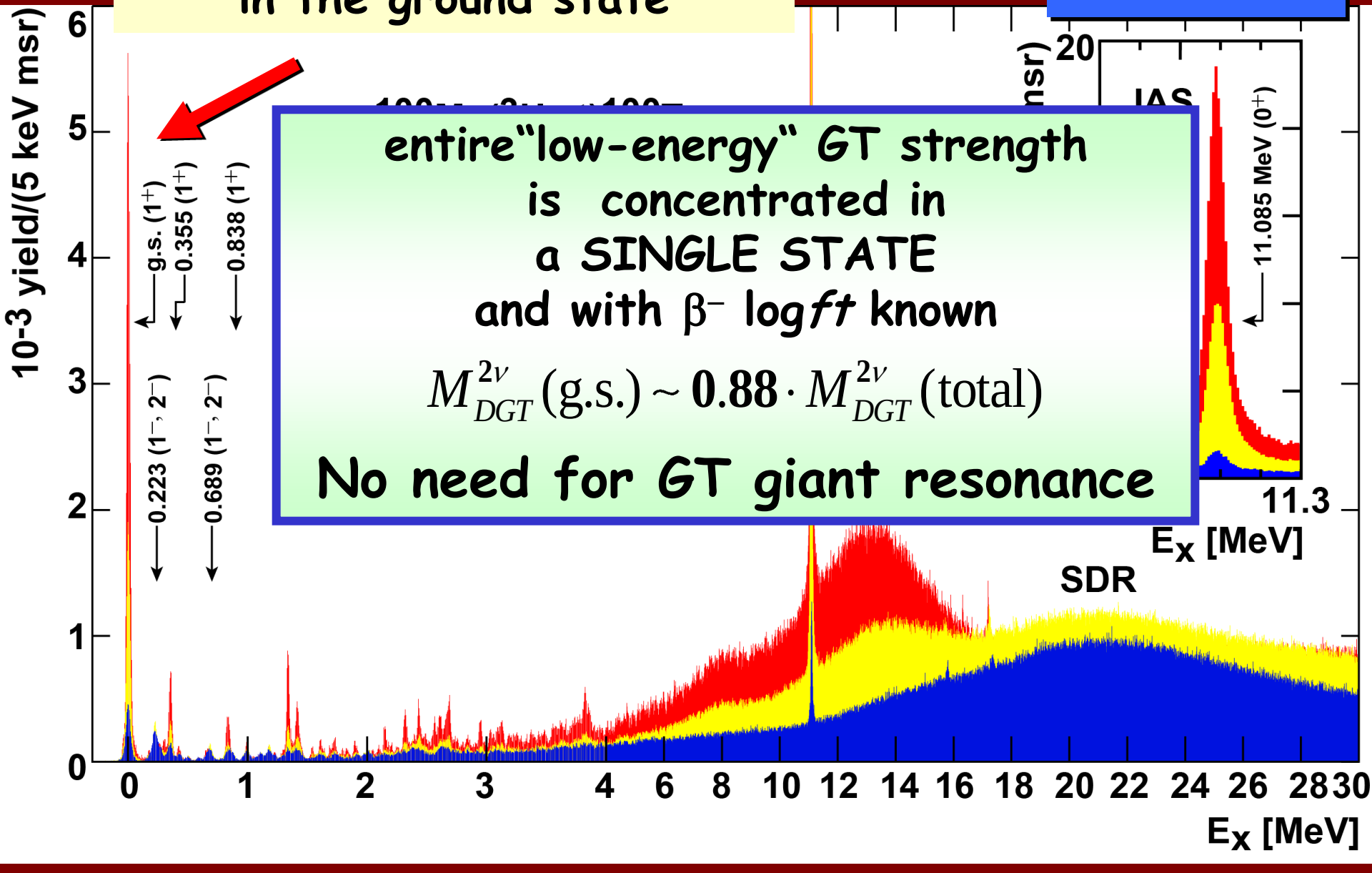
$$T_{1/2}^{exp.}(2\nu\beta\beta = (2.3 \pm 0.2) \cdot 10^{19}\ \text{years}\ \ \text{(NEMO3-result)}$$

^{100}Mo

N-Z=16

HERE: almost the entire GT strength is concentrated in the ground state

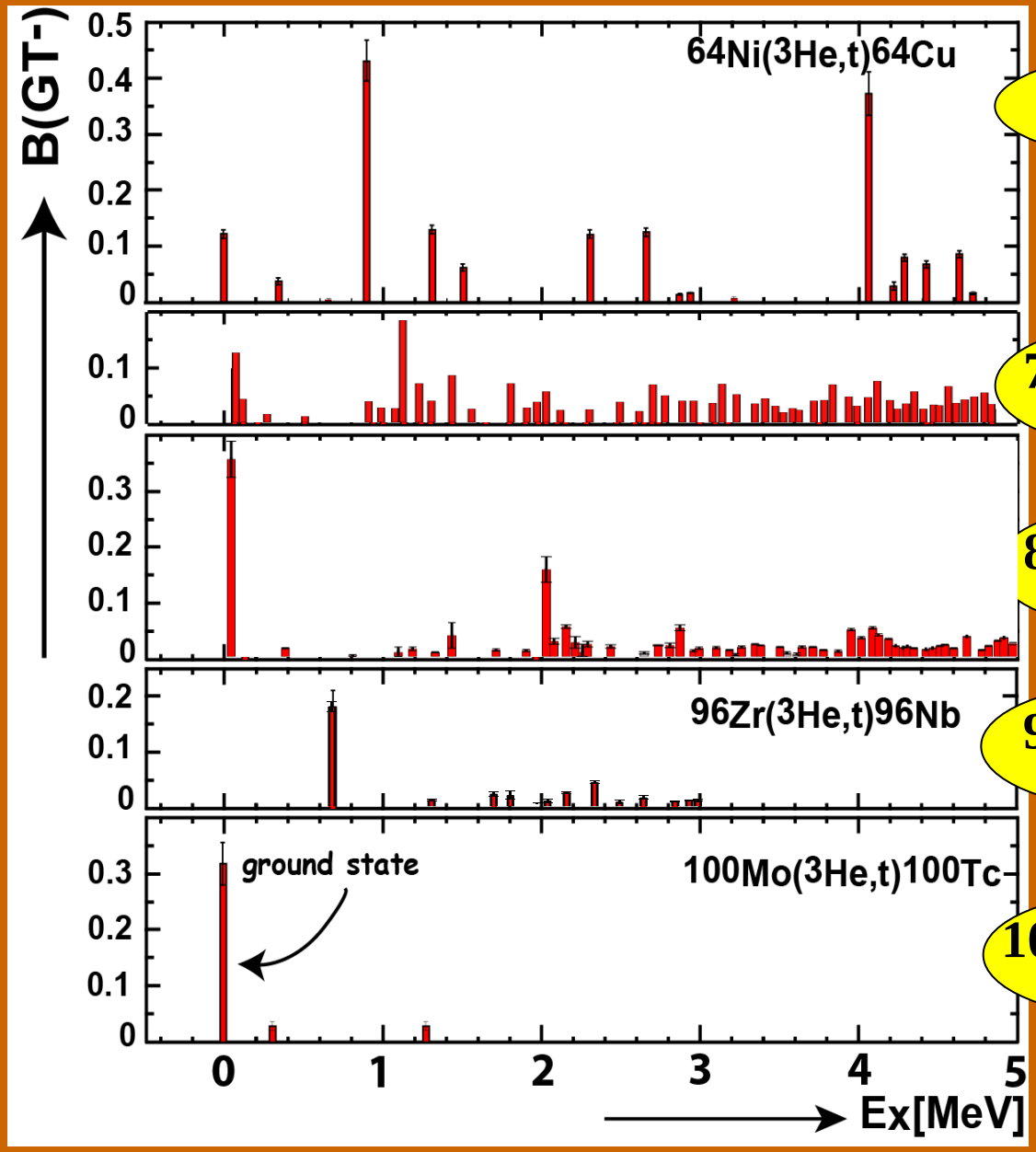
^{100}Mo



entire "low-energy" GT strength is concentrated in a SINGLE STATE and with β^- log*ft* known

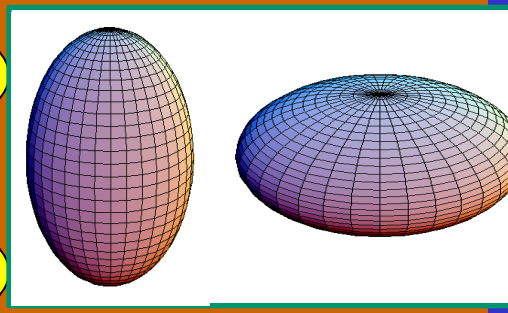
$$M^{2\nu}_{DGT}(\text{g.s.}) \sim \mathbf{0.88} \cdot M^{2\nu}_{DGT}(\text{total})$$

No need for GT giant resonance



^{64}Zn(εε, εβ+)

^{76}Ge(β⁻β⁻)



^{82}Se(β⁻β⁻)

^{96}Zr(β⁻β⁻)

^{100}Mo(β⁻β⁻)

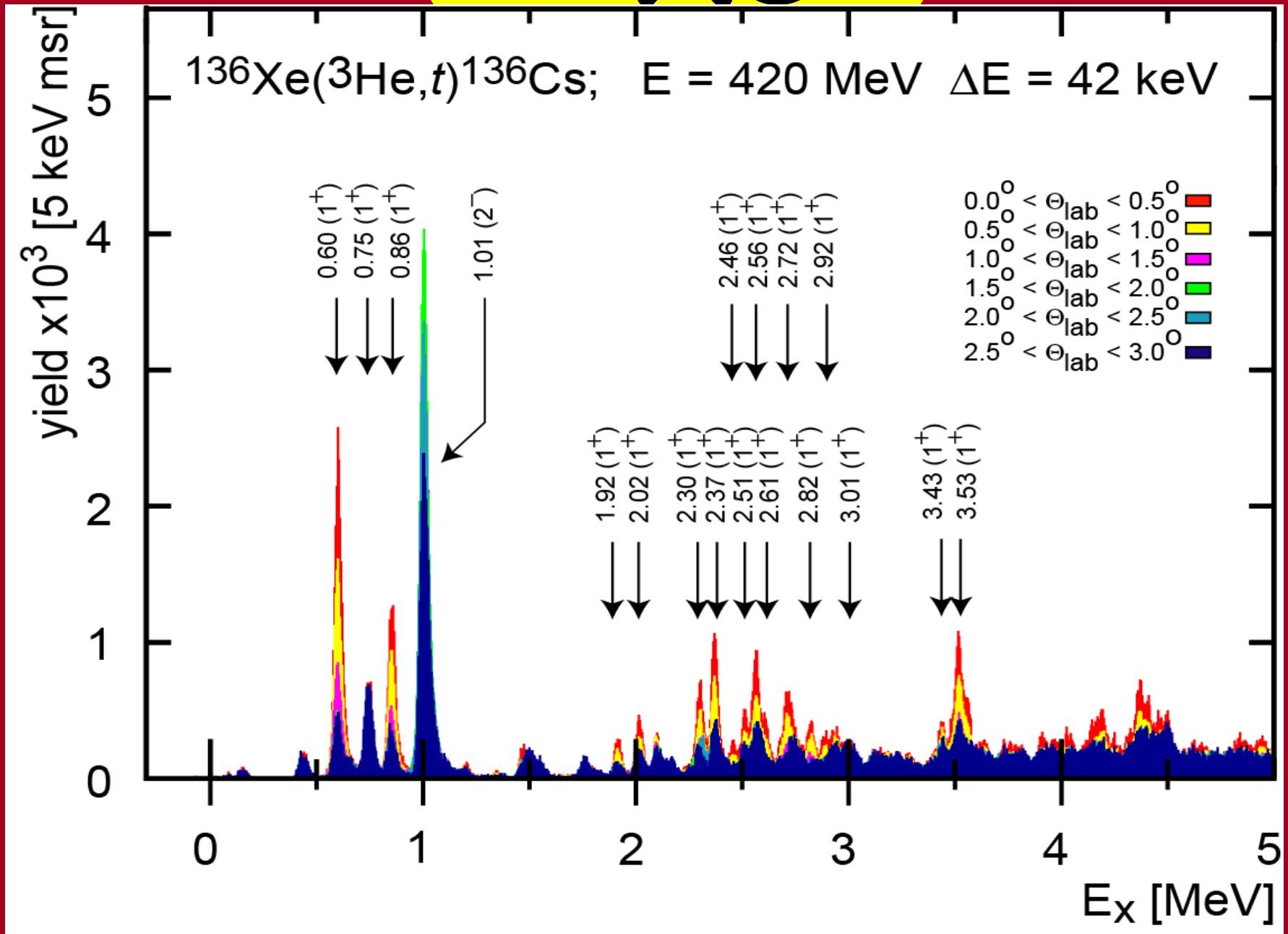
reduced fragmentation of GT strength

^{136}Xe

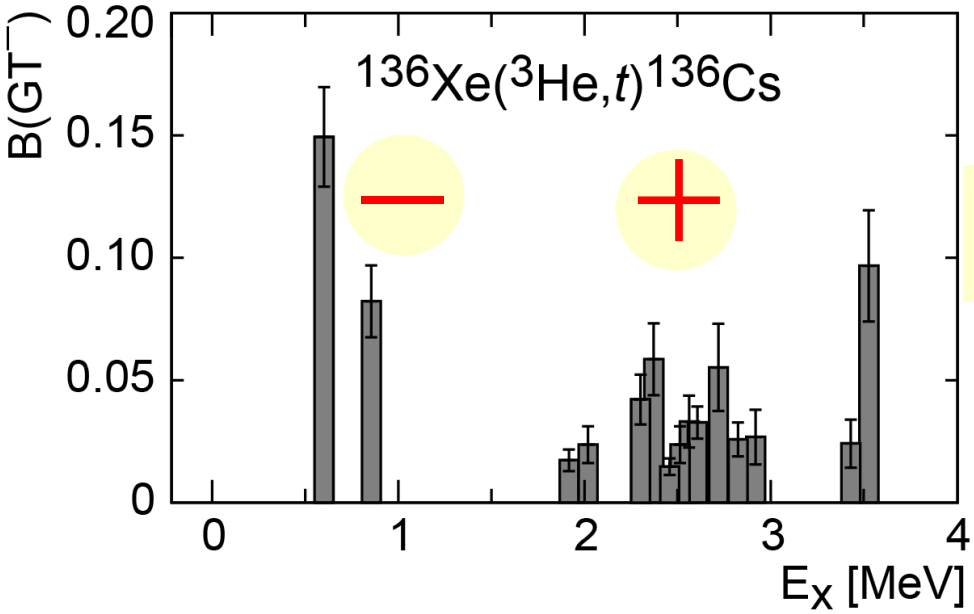
N-Z=28

question: why so stable !!!

^{136}Xe

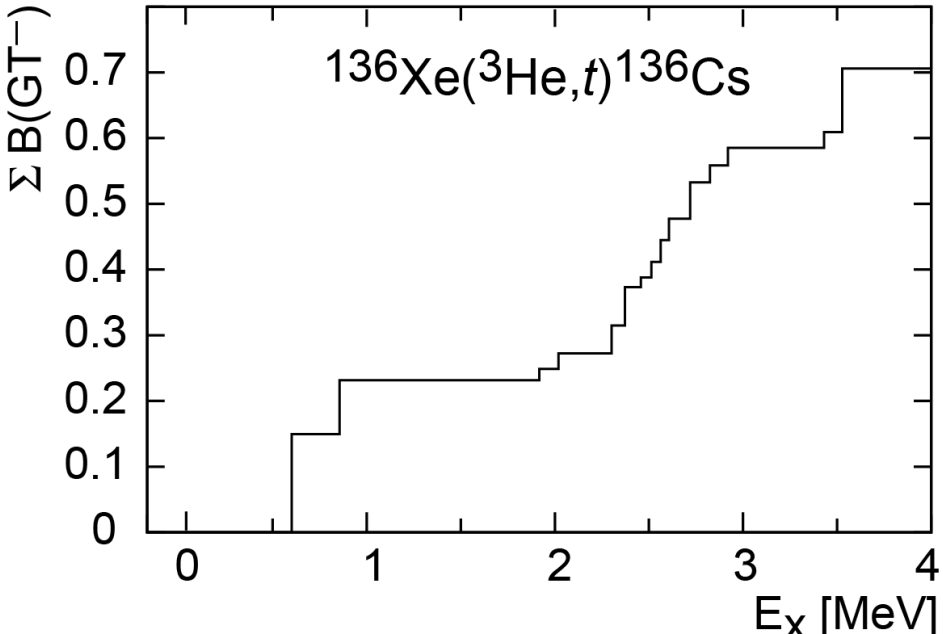

136Xe(3He,t)136Cs; E = 420 MeV ΔE = 42 keV
yield x10^3 [5 keV msr]
Ex [MeV]
0.60 (1+)
0.75 (1+)
0.86 (1+)
1.01 (2−)
1.92 (1+)
2.02 (1+)
2.30 (1+)
2.37 (1+)
2.51 (1+)
2.61 (1+)
2.82 (1+)
3.01 (1+)
2.46 (1+)
2.56 (1+)
2.72 (1+)
2.92 (1+)
3.43 (1+)
3.53 (1+)
0.0° < Θlab < 0.5°
0.5° < Θlab < 1.0°
1.0° < Θlab < 1.5°
1.5° < Θlab < 2.0°
2.0° < Θlab < 2.5°
2.5° < Θlab < 3.0°

What's the size of the NME?



$$T_{1/2}^{2\nu} = 2.2 \cdot 10^{21}\,\mathrm{yr}$$

$$M_{\mathrm{DGT}}^{(2\nu)} \sim 0.019\ \mathrm{MeV}^{-1}$$



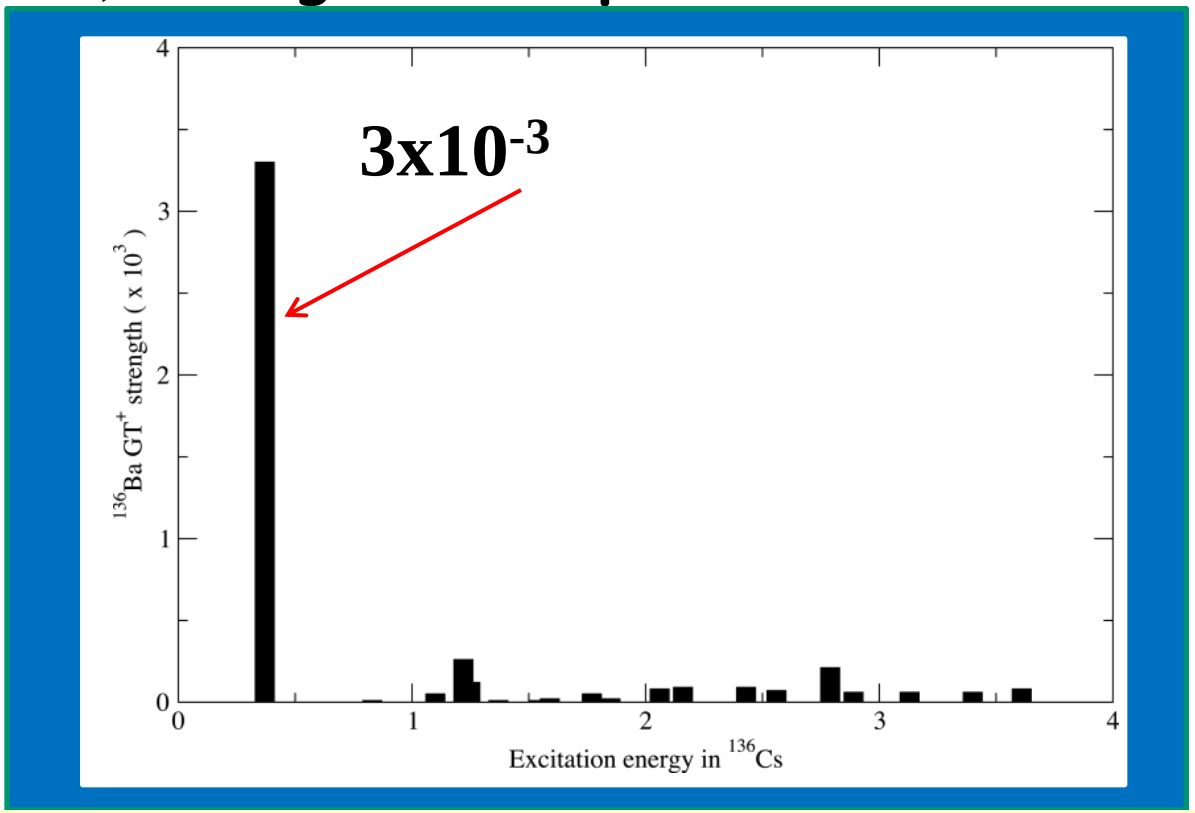
all signs positive —>

$$B_m\ GT^+ \approx 10^{-2} \cdot B_m\ GT^-$$

$$B_m\ GT^+ \approx 10^{-3}\ !!!!$$

A. Poves (simultaneous to our publication):
NO CANCELLATION !!
there is no B(GT$^+$) strength, except for lowest 1$^+$ state

Recall:
^{136}Xe is almost doubly magic!!



Shell model provides conclusive explanation for the deemed „pathologically“ long half-life of ^{136}Xe.
Expt‘l test: ^{136}Ba(d,^{2}He)^{136}Cs

Lessons from charge-exchange reactions

1. **Deformation plays an important role for $2\nu\beta\beta$ decay – and likely also for $0\nu\beta\beta$ decay. It reduces the NME !!**

2. **The simple shell structure of ^{100}Mo and ^{96}Zr causes a particularly large NME for both systems.**

3. **In all cases only the „low-energy“ part of the GT distrib‘n is relevant for the 2ν decay. The GT giant resonance does not contribute at all (already conjectured by Ericson et al. PLB 328, 1994)**

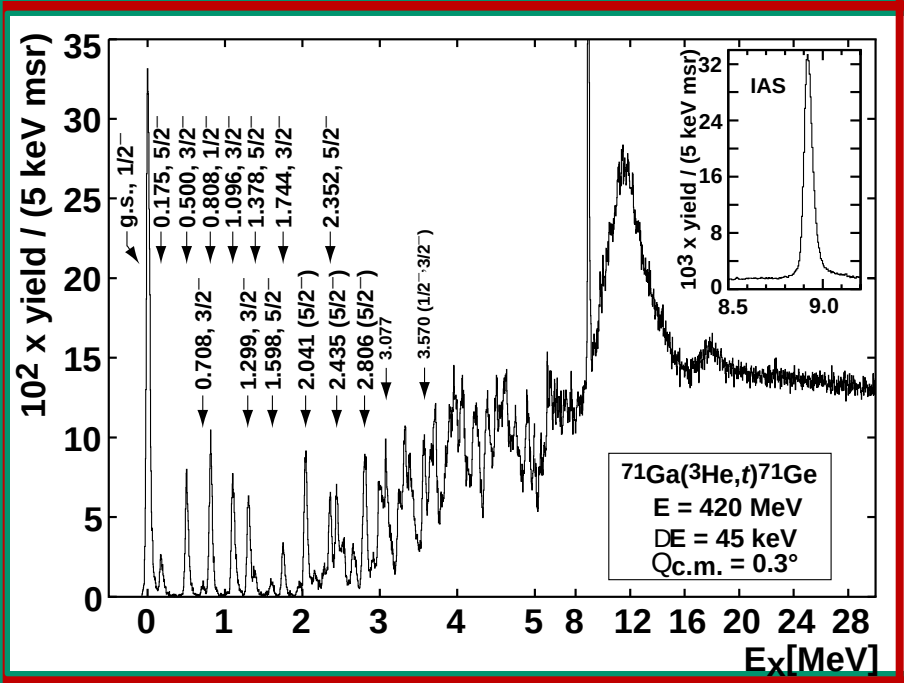
4. **the half-life of ^{136}Xe is long because:**
   the GT+ „leg“ of the $2\nu\beta\beta$ NME is almost completely Pauli-blocked
   (→ reduced exp‘l „background“ for $0\nu\beta\beta$ decay)

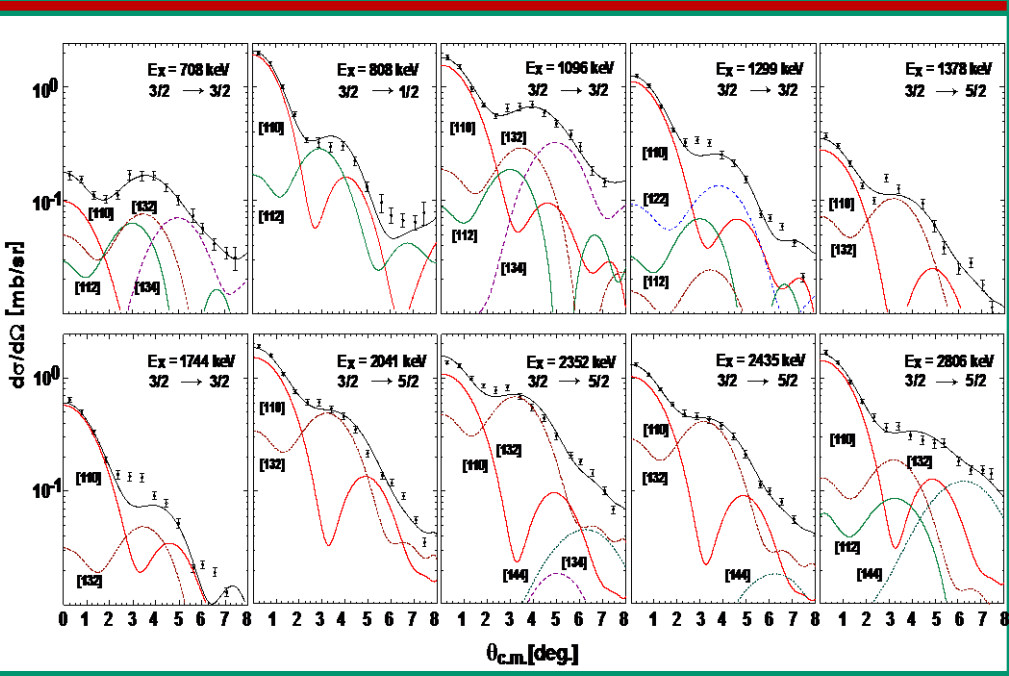
5. **The GT distribution mirrors the g.s. nuclear wave function**

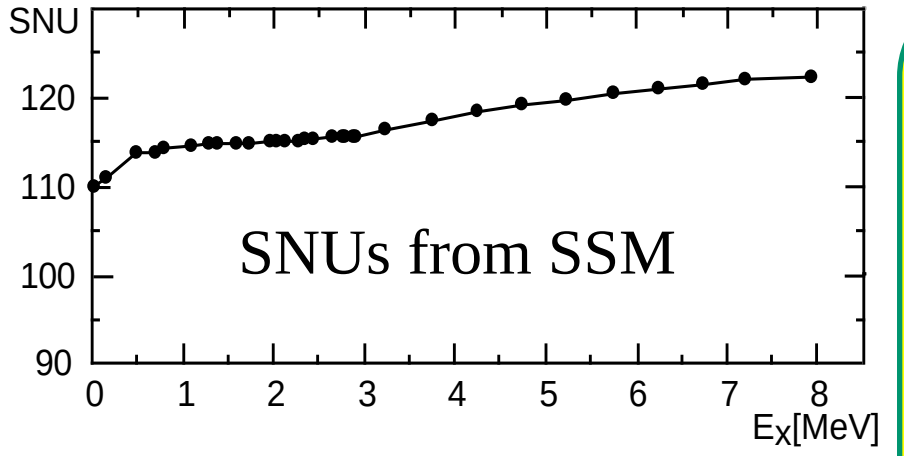
a side remark on (^{3}He,t) tool because of its power & beauty

^{71}Ga($\nu_\odot$,e^-) SNUs from
^{71}Ga(^{3}He,t)^{71}Ge charge-ex reaction

^{71}Ga($\nu_\odot$,e$^-$) SNUs from (^{3}He,*t*) charge-exchange reaction







^{71}Ga($\nu_\odot$,e$^-$)

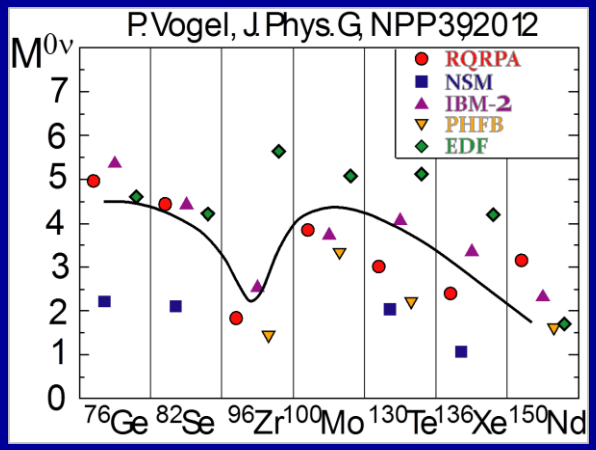
$$R = 122.4 \pm 3.4(\text{stat}) \pm 1.1(\text{sys})$$

stat. err. mostly due to CNO ν's

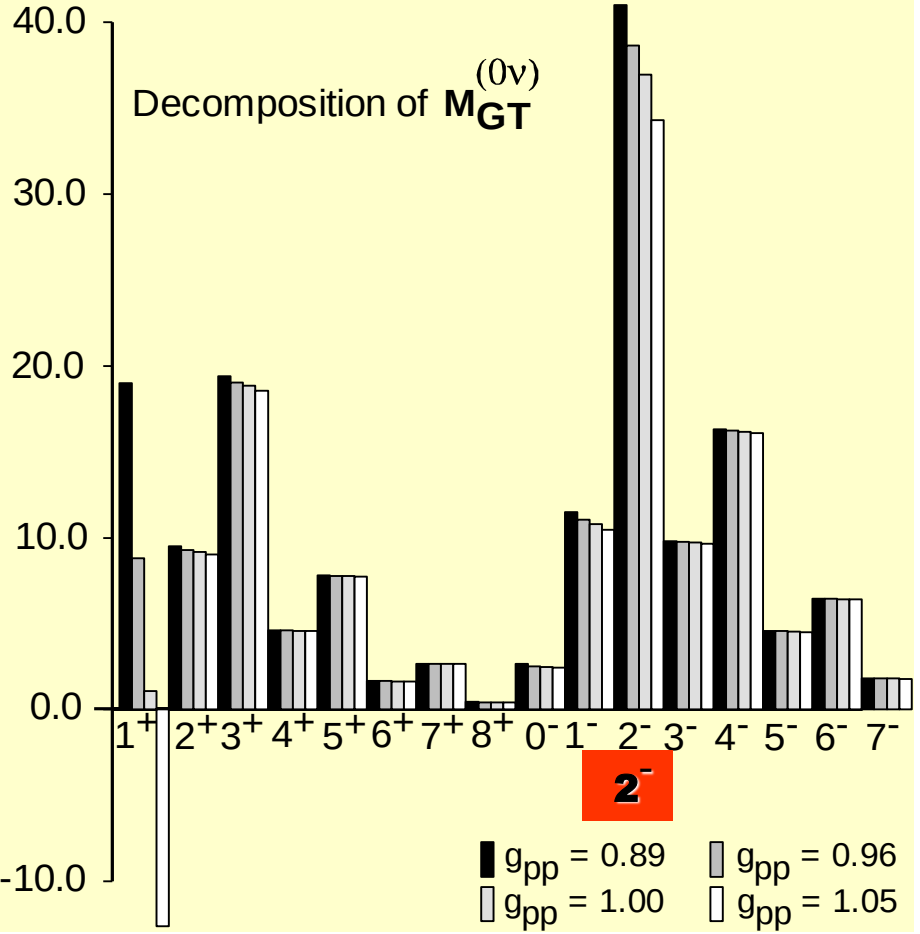
prev'ly: 132 ± 18

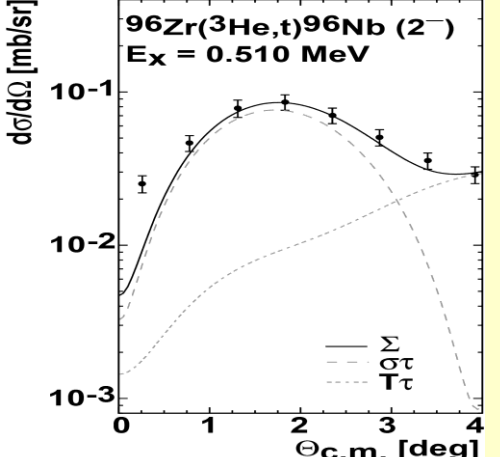
DF et al, PRC91,2015

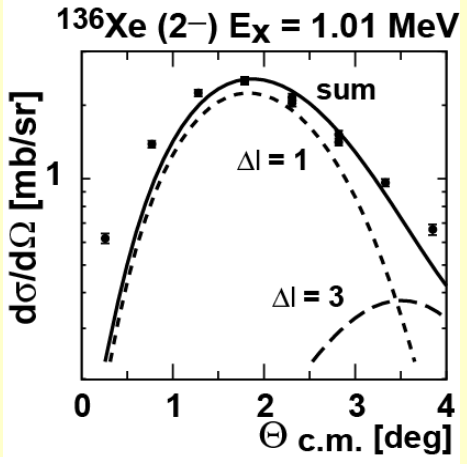
experiments towards the 0νββ NMEs

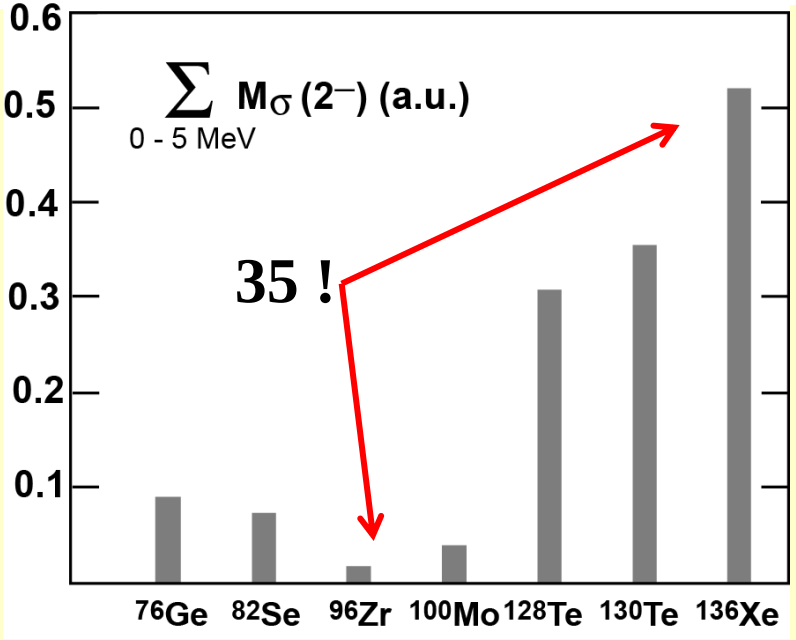


2^- states and occupation/vacancy numbers via charge-ex reactions









relative 2^- strength to ~ 5 MeV

Theory:

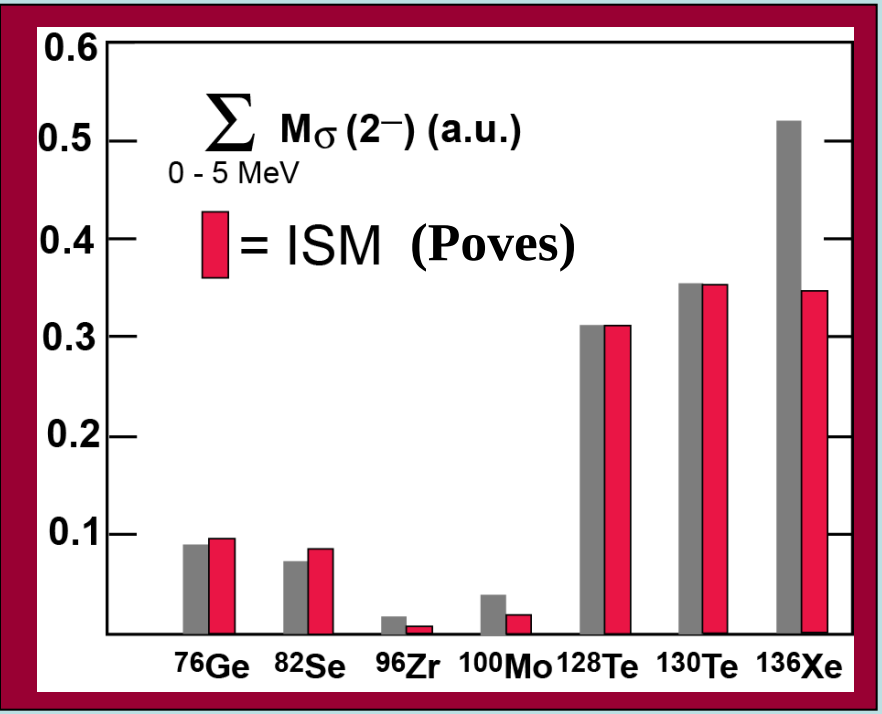
The 2^- strength makes up ~ 20-30% of the 0νββ ME!!

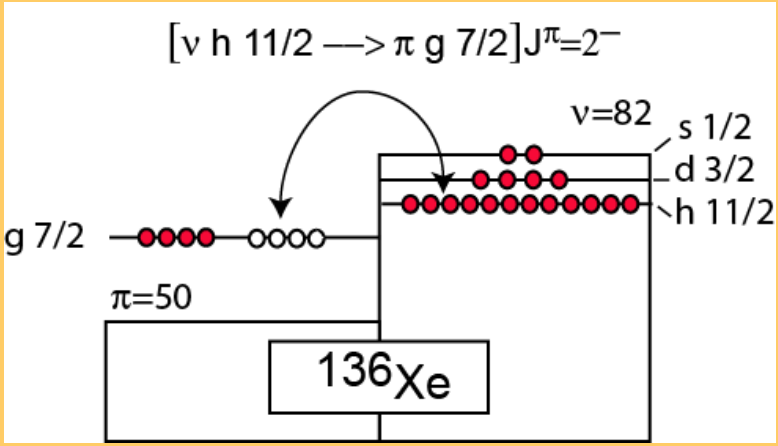
J. Suhonen, Phys. Lett B607, 87 (2005)

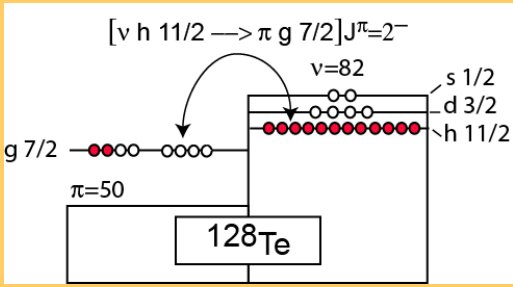
Expmt:

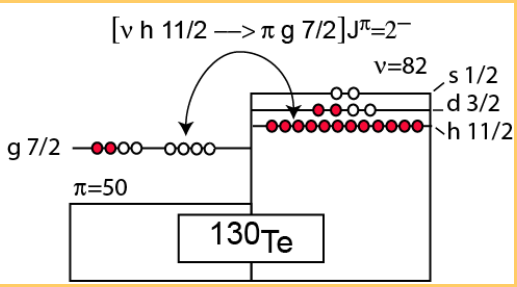
^{136}Xe exhibits largest 2^- strength

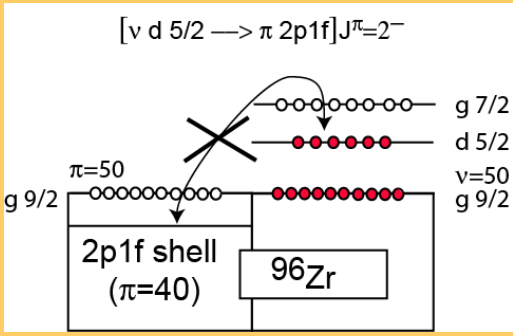
0νββ ME enhanced?!?!

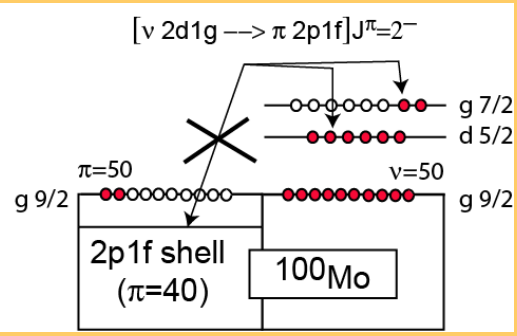

0.6
0.5
0.4
0.3
0.2
0.1
$\sum$ Mσ (2⁻) (a.u.)
0 - 5 MeV
= ISM (Poves)
76Ge
82Se
96Zr
100Mo
128Te
130Te
136Xe

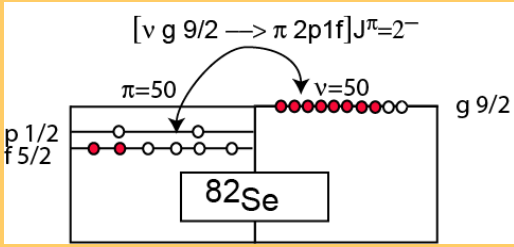

[ν h 11/2 —> π g 7/2]J^π=2⁻
ν=82
s 1/2
d 3/2
h 11/2
g 7/2
π=50
136Xe

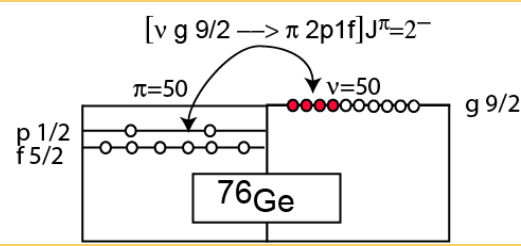

[ν h 11/2 —> π g 7/2]J^π=2⁻
ν=82
s 1/2
d 3/2
h 11/2
g 7/2
π=50
128Te

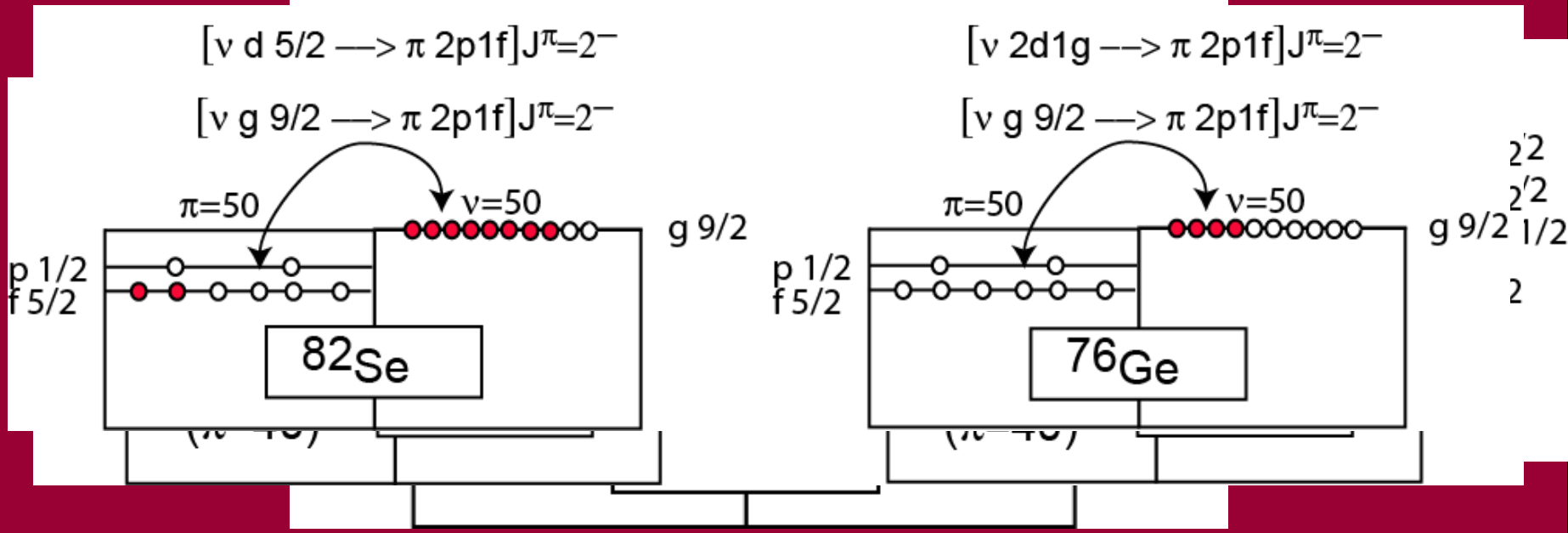

[ν h 11/2 —> π g 7/2]J^π=2⁻
ν=82
s 1/2
d 3/2
h 11/2
g 7/2
π=50
130Te

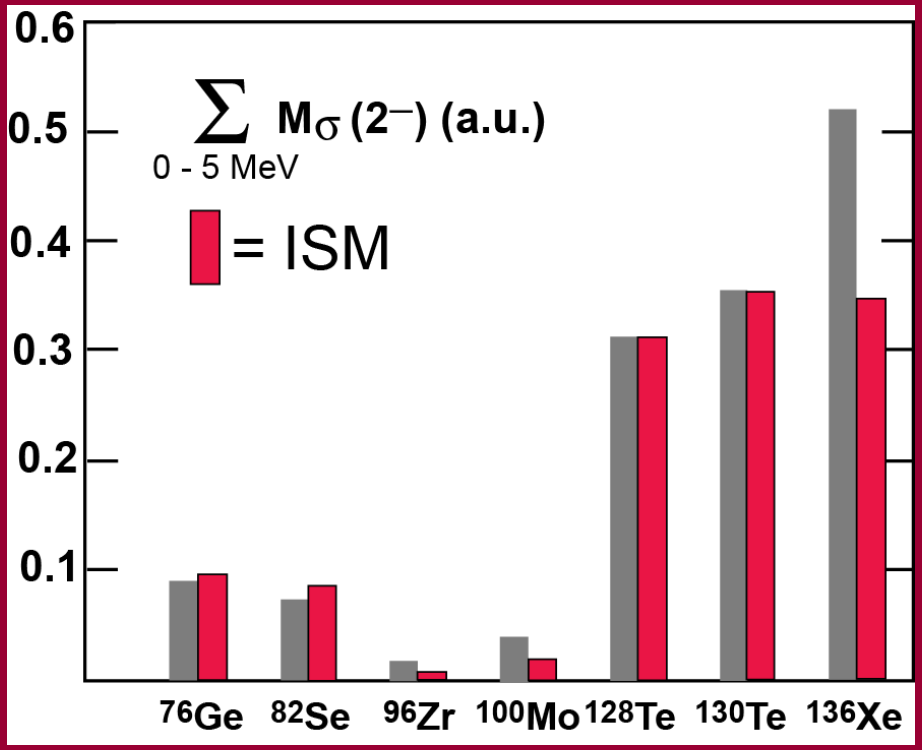

[ν d 5/2 —> π 2p1f]J^π=2⁻
g 7/2
d 5/2
π=50
ν=50
g 9/2
g 9/2
2p1f shell
(π=40)
96Zr


[ν 2d1g —> π 2p1f]J^π=2⁻
g 7/2
d 5/2
π=50
ν=50
g 9/2
g 9/2
2p1f shell
(π=40)
100Mo

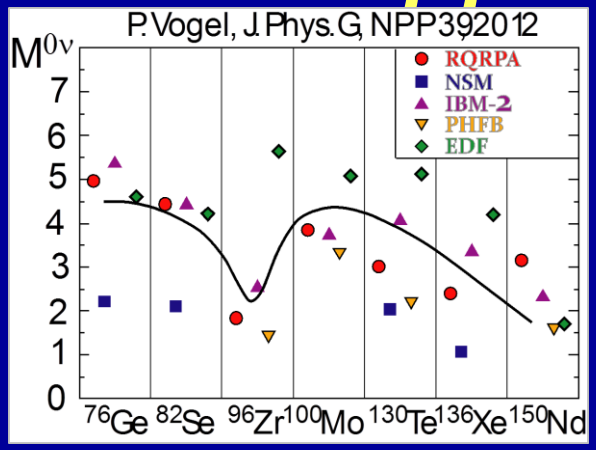

[ν g 9/2 —> π 2p1f]J^π=2⁻
π=50
ν=50
g 9/2
p 1/2
f 5/2
82Se


[ν g 9/2 —> π 2p1f]J^π=2⁻
π=50
ν=50
g 9/2
p 1/2
f 5/2
76Ge


[ν d 5/2 —> π 2p1f]J^π=2^−
[ν g 9/2 —> π 2p1f]J^π=2^−
π=50
ν=50
g 9/2
p 1/2
f 5/2
82Se
[ν 2d1g —> π 2p1f]J^π=2^−
[ν g 9/2 —> π 2p1f]J^π=2^−
π=50
ν=50
g 9/2
p 1/2
f 5/2
76Ge


0.6
0.5
0.4
0.3
0.2
0.1
Σ Mσ (2⁻) (a.u.)
0 - 5 MeV
= ISM
76Ge
82Se
96Zr
100Mo
128Te
130Te
136Xe

experiments towards the 0νββ NMEs



the A=96 system in ββ decay

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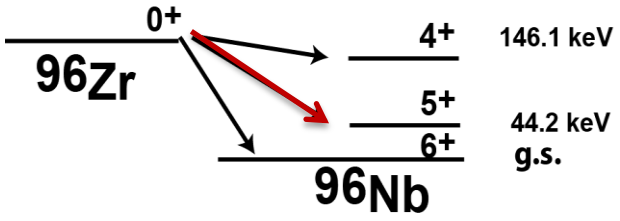
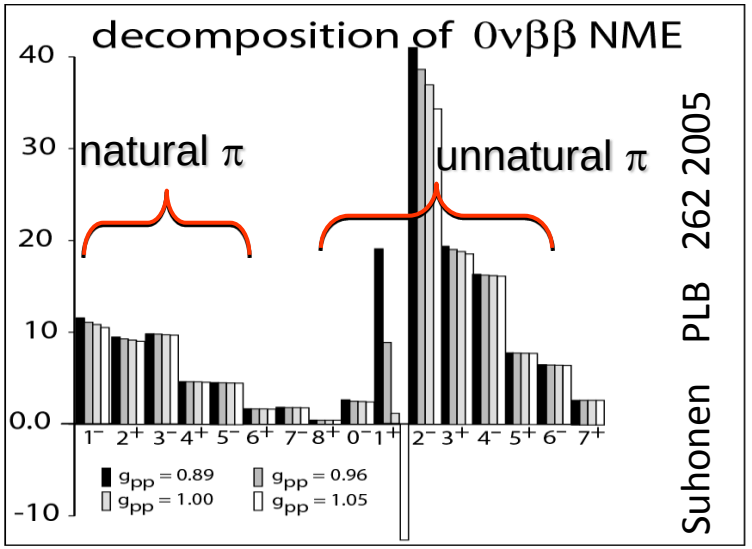
Motivation



- measure Q-value for ^{96}Zr ⟶ ^{96}Nb **single β-decay** by accurate **^{96}Nb** mass measuremt

- provide information about the ^{96}Zr **4-fold forbidden β-decay NME** and confront with theory
- confront with same theories aimed at calculating **0νββ-decay NME for the same nucleus!!**



Competition between β & ββ decay in ^{96}Zr

two conflicting half-lives:

NEMO-3: $T_{1/2}^{2\nu\beta\beta} = (2.3 \pm 0.2) \times 10^{19}$ y

geo-chem: $T_{1/2} = (0.94 \pm 0.32) \times 10^{19}$ y (1)

can this difference be reconciled ?
yes, if single β competes with ββ decay

$$\left(T_{1/2}\right)^{-1} = \left(T_{1/2}^{2\nu\beta\beta}\right)^{-1} + \left(T_{1/2}^{\beta}\right)^{-1}$$

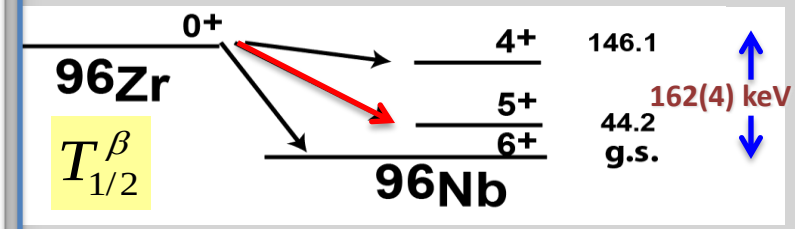
expected $T_{1/2}^{\beta} = (1.6 \pm 0.9) \times 10^{19}$ y

experiment $T_{1/2}^{\beta} > 2.6 \times 10^{19}$ y (2)

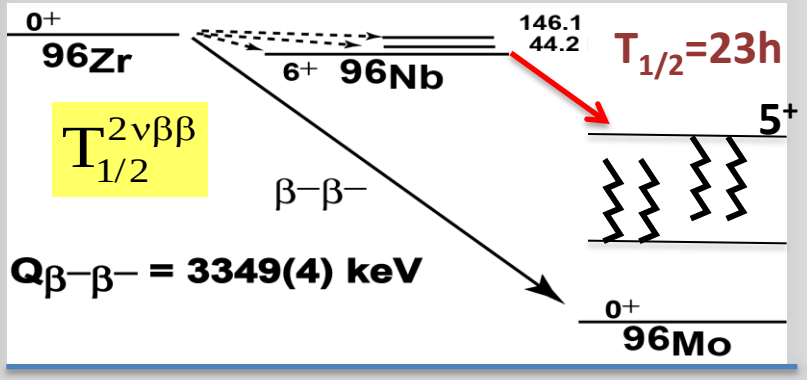
pred. (QRPA) $T_{1/2}^{\beta} = 24 \times 10^{19}$ y (3)

BUT

$$(T_{1/2}^{\beta})^{-1} \propto o(Q^{13}) \left\langle M_{\beta}^{4u} \right\rangle^2$$



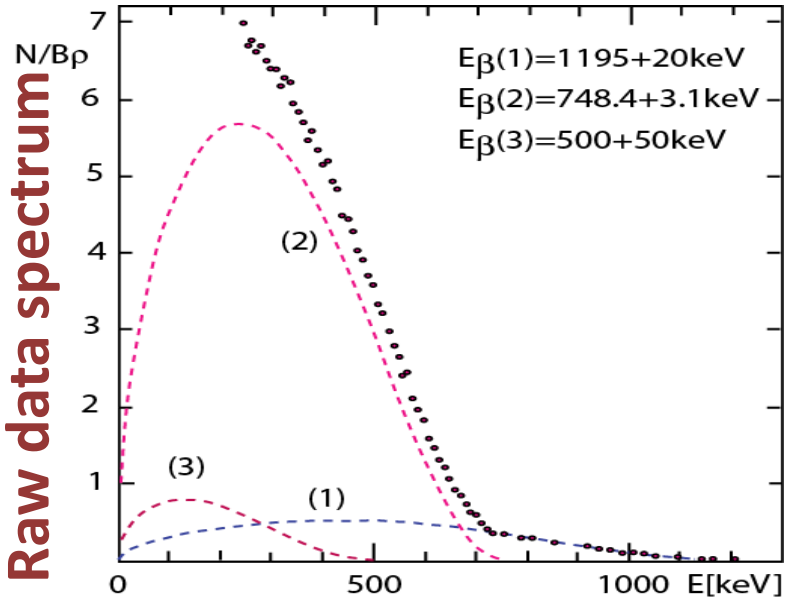
$0^+ \rightarrow 6^+$ 6-fold non-unique (unobservably long)
$0^+ \rightarrow 5^+$ 4-fold unique (possible)
$0^+ \rightarrow 4^+$ 4-fold non-unique (no phase space)

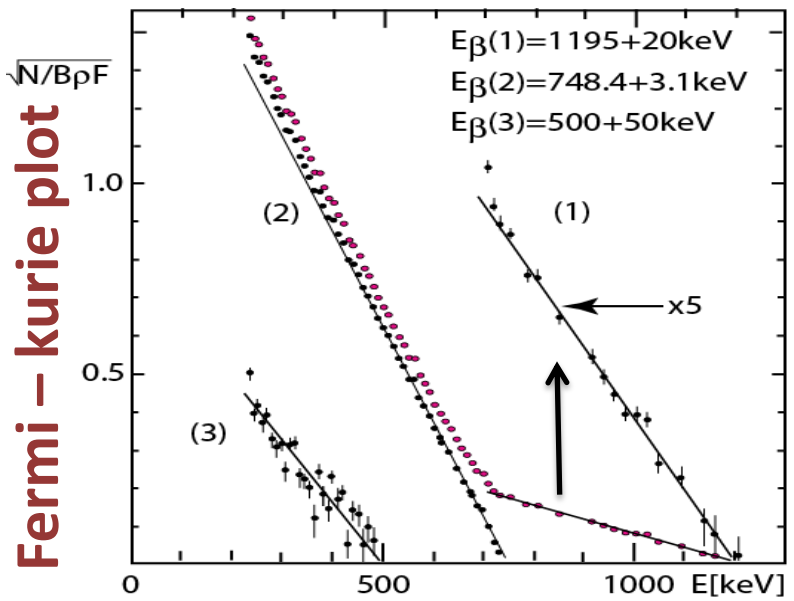


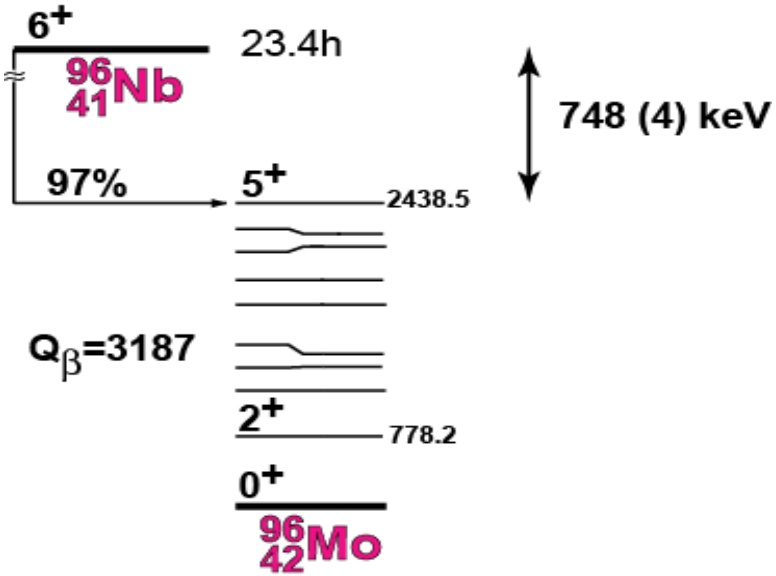
Q-value → M_{β}^{4u} → $(T_{1/2}^{0\nu\beta\beta})^{-1} \propto Q^5 \left| M_{\beta\beta}^{0\nu} \right|^2 \left\langle m_{\beta\beta} \right\rangle^2$

(1) *Wieser, PRC64,2001,* (2) *Barabash, JPG-NPP22, 1996* (3) *Heiskanen, JPG3,2007*

The only ^{96}Nb mass: deduced from β^- decay end point energy {Antman et al. NPA 110 1968}







the 1195 keV is NOT a ^{xx}Nb long-lived isomer!!

the 500 keV transition does NOT exist.
→ fit depends on 2 transitions, which don't exist!!

→ Q-value is in doubt !!

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$T_{1/2}$ of ^{96}Zr decay

Geochemical method (zircon sample, i.e. $Zr(SiO_4)_4$

accumulation of decay daughter over geological time

- **Pros**: can provide longer $T_{1/2}$
- **Cons**: - measure only total decay
  - systematics from ^{95}Mo (n,γ) reaction
    n-sources: ^{238}U sf , cosmics
    BUT: zircon contains Gd (i.e. ^{157}Gd(n,γ))
  - needs chemical isobar separation (A=96)

age of zircon sample: 1.822(±0.003) ×10^9 y
dated by U-Pb technique

origin: Capel Australia

advantage: Mo is embedded in zircon lattice with infinite retention time

geo. $T_{1/2}$ calculation

- # of ^{96}Mo excess atoms in sample
- # of ^{96}Zr atoms in sample
- age of the sample

measurable limit

$$T^{\beta}_{1/2} \gtrsim 15 \times 10^{19}\ \text{y}$$

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EXOTIC decay

Decay limit comes for free in geochem. analysis

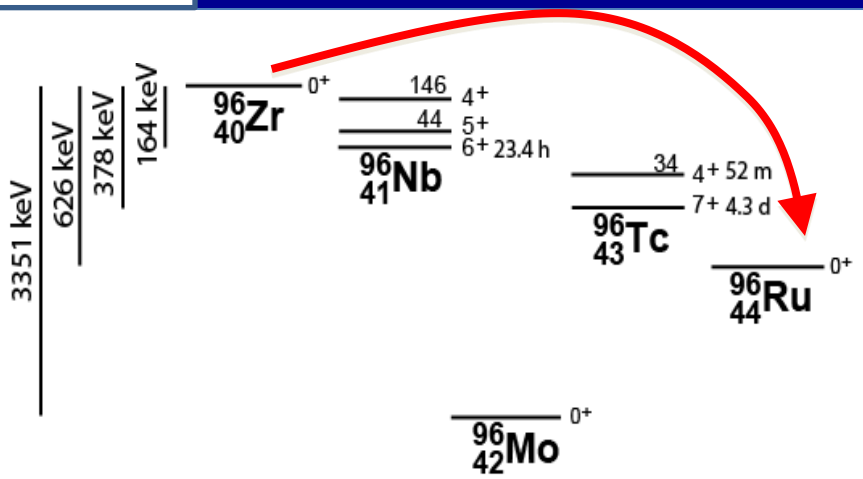
$$^{96}\mathrm{Zr} \xrightarrow{0\nu 4\beta} {}^{96}\mathrm{Ru}$$

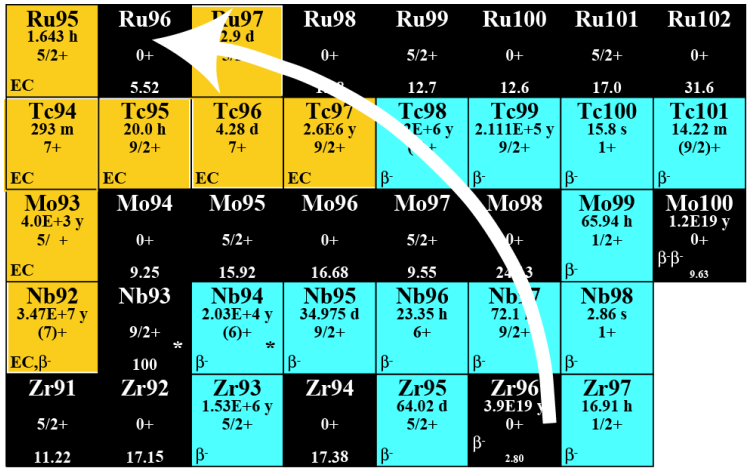
$$(A, Z) \longrightarrow (A, Z+4) + 4\,e$$

$$(T_{1/2}^{0\nu 4\beta})^{-1} \propto Q^{11} \left| M_{0\nu 4\beta} \right|^2$$

Ref. : J. Heeck EPL103, 32001 (2013)

0ν4β decay





The physics specialties:

- theor. estimated $T_{1/2} = 10^{55\pm20}$ y
- the 4β decay is possible for Dirac & Majorana ν
- the 0ν4β decay has the largest phase space
- all intermediate nuclei are lower in energy than parent (unique!!)

Advantages of geochem. analysis:

- the 4β daughter ^{96}Ru is stable
- ^{96}Ru is p-process nucleus
  → low natural abundance (5.5%)
- no contribution from n-capture
- no contribution from β-decay of fission products.

expected geochem. limit: $T_{1/2} = 10^{22}$ y

Conclusions

1. **Charge-ex can be used BEYOND GT extraction**
   1. **2^- states**
   2. **occupation numbers**

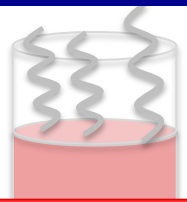
2. **Q-value for the ^{96}Zr-^{96}Nb 4-fold forbidden β decay is found to be in doubt**

3. **precise Q-value sets the ground for a new measurement of the ^{96}Zr-^{96}Nb half-life**
   a) **either by direct counting**
   b) **or through an improved geochemical method**

the result provides a test for the $0\nu\beta\beta$ NME calculations for the same ^{96}Zr nucleus

Isobar separation method (U of Calgary)

+



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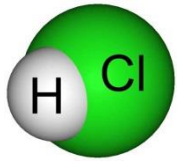
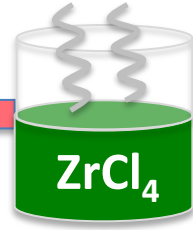




re-dissolved in acid

with 4 M HCl

ion-exchange chemistry

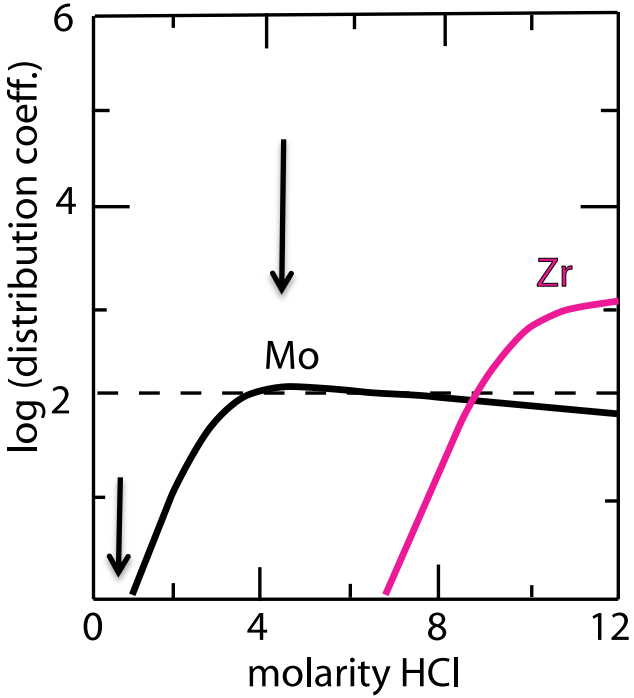
Zr & Mo

separate Zr from Mo

Zr

purified Mo>10 ng

plasma mass spectrometer



$(\Delta^{96}Mo/^{95}Mo)=0.01\%$

measurable limit
$T^{\beta}_{1/2} \lesssim 15 \times 10^{19}$ y