NDM 2015
Neutrinos and Dark Matter in Nuclear Physics
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Spontaneuos Symmetry Breaking in Self-Induced Supernova Neutrino Flavor Conversions

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OUTLINE

Supernovae as neutrino sources

Self-induced SN ν oscillations: the collective behaviour of a dense ν gas

Spontaneous Symmetry Breaking effects of self-interacting neutrino gas

Open Issues and Conclusions

SUPERNOVA NEUTRINOS

Core collapse SN corresponds to the terminal phase of a massive star [$M \gtrsim 8\,M_{\odot}$] which becomes unstable at the end of its life. It collapses and ejects its outer mantle in a shock wave driven explosion.

- **ENERGY SCALES:** 99% of the released energy (~ 10^{53} erg) is emitted by ν and $\bar{\nu}$ of all flavors, with typical energies E ~ O(15 MeV).
- **TIME SCALES:** Neutrino emission lasts **~10 s**
- **EXPECTED:** **1-3 SN/century** in our galaxy (d ≈ $\mathcal{O}$(10) kpc).

(See talks by E.Endeve, T. Fischer .... for details on SN ν emission)

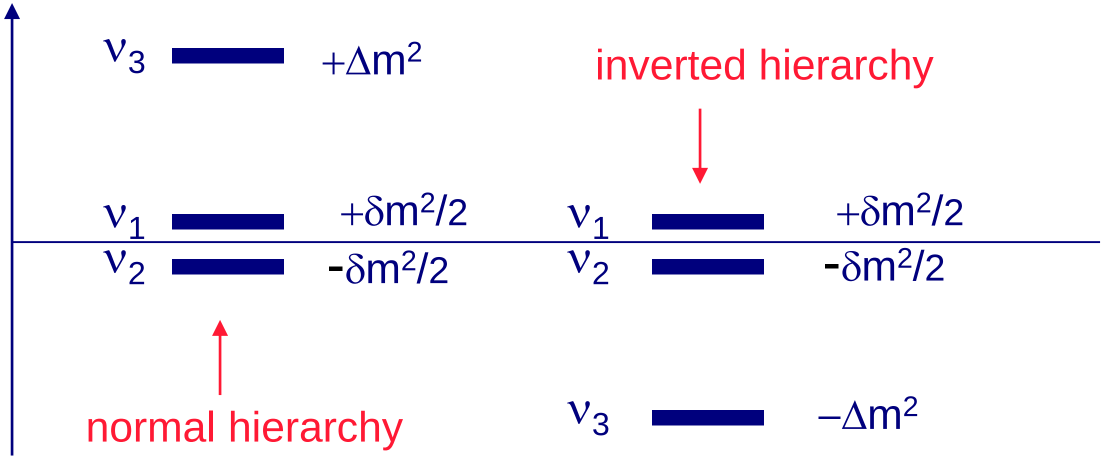
3ν FRAMEWORK

- **Mixing parameters:** $U = U(\theta_{12}, \theta_{13}, \theta_{23}, \delta)$ as for CKM matrix

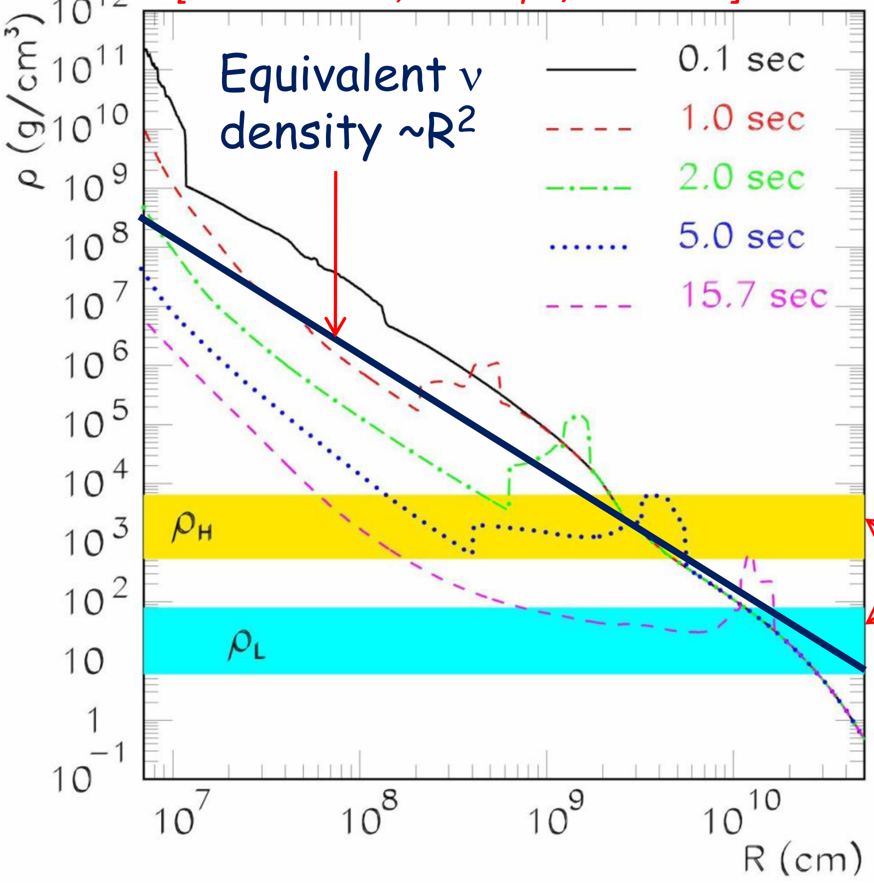
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & e^{-i\delta} s_{13} \\ & 1 & \\ -e^{-i\delta} s_{13} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$c_{12} = \cos\theta_{12}$, etc., δ CP phase

- **Mass-gap parameters:** $M^2 = \left(\underbrace{-\frac{\delta m^2}{2}, \ +\frac{\delta m^2}{2}}_{\text{“solar”}}, \ \underbrace{\pm \Delta m^2}_{\text{“atmospheric”}} \right)$



SNAPSHOT OF SN DENSITIES



[Tomas et al., astro-ph/0407132]

Equivalent ν density ~R²

- Matter bkg potential

$$\lambda = \sqrt{2} G_F N_e \quad \sim R^{-3}$$

- ν–ν interaction

$$\mu = \sqrt{2} G_F n_\nu \quad \sim R^{-2}$$

- Vacuum oscillation frequencies

$$\omega = \frac{\Delta m^2}{2E}$$

When μ>>λ, SN ν oscillations dominated by ν-ν interactions

Collective flavor transitions at low-radii [O (10^2 – 10^3 km)]

Two seminal papers in 2006 triggered a torrent of activities
Duan, Fuller, Qian, astro-ph/0511275, Duan et al. astro-ph/0606616

(See talks by B. Balantekin, G. McLaughlin, Y. Pehivan, C. Volpe)

Neutrinos and Dark Matter in Nuclear Physics

NDM06, 3-9 September 2006, Paris

Conclusions

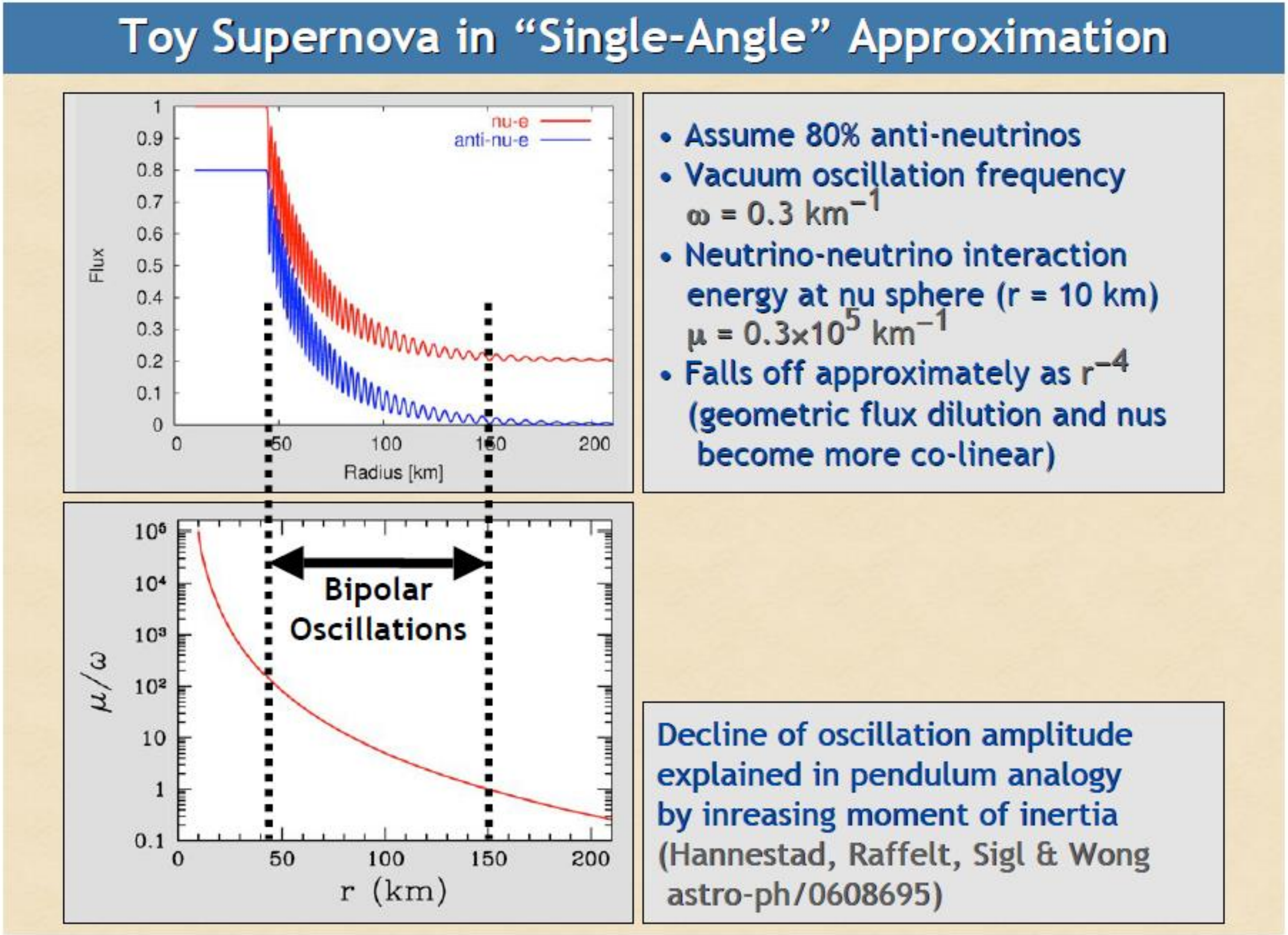
Simultaneous ν and $\bar{\nu}$ flavor conversion possible by bipolar collective oscillation mode at few 10 to few 100 km above neutrino sphere

Depending on primary neutrino flux spectra, may
- Modify energy transfer to shock wave
- Modify neutrino-driven nucleosynthesis
- Modify observable signatures of SN neutrino oscillations

Collective Oscillations of Supernova Neutrinos

Georg Raffelt Max-Planck-Institut für Physik München

From G. Raffelt talk at NDM 2006



Toy Supernova in “Single-Angle” Approximation

- Assume 80% anti-neutrinos
- Vacuum oscillation frequency $\omega = 0.3\ \mathrm{km}^{-1}$
- Neutrino-neutrino interaction energy at nu sphere (r = 10 km) $\mu = 0.3\times10^5\ \mathrm{km}^{-1}$
- Falls off approximately as r^{-4} (geometric flux dilution and nus become more co-linear)

Bipolar Oscillations

Decline of oscillation amplitude explained in pendulum analogy by inreasing moment of inertia (Hannestad, Raffelt, Sigl & Wong astro-ph/0608695)

Collective Supernova Nu Oscillations since 2006

Two seminal papers in 2006 triggered a torrent of activities
Duan, Fuller, Qian, astro-ph/0511275, Duan et al. astro-ph/0606616

Balantekin, Gava & Volpe [0710.3112]. Balantekin & Pehlivan [astro-ph/0607527]. Blennow, Mirizzi & Serpico [0810.2297]. Cherry, Fuller, Carlson, Duan & Qian [1006.2175, 1108.4064]. Cherry, Wu, Fuller, Carlson, Duan & Qian [1109.5195]. Cherry, Carlson, Friedland, Fuller & Vlasenko [1203.1607]. Chakraborty, Choubey, Dasgupta & Kar [0805.3131]. Chakraborty, Fischer, Mirizzi, Saviano, Tomàs [1104.4031, 1105.1130]. Choubey, Dasgupta, Dighe & Mirizzi [1008.0308]. Dasgupta & Dighe [0712.3798]. Dasgupta, Dighe & Mirizzi [0802.1481]. Dasgupta, Dighe, Raffelt & Smirnov [0904.3542]. Dasgupta, Dighe, Mirizzi & Raffelt [0801.1660, 0805.3300]. Dasgupta, Mirizzi, Tamborra & Tomàs [1002.2943]. Dasgupta, Raffelt & Tamborra [1001.5396]. Dasgupta, O'Connor & Ott [1106.1167]. Duan, Fuller, Carlson & Qian [astro-ph/0608050, 0703776, 0707.0290, 0710.1271]. Duan, Fuller & Qian [0706.4293, 0801.1363, 0808.2046, 1001.2799]. Duan, Fuller & Carlson [0803.3650]. Duan & Kneller [0904.0974]. Duan & Friedland [1006.2359]. Duan, Friedland, McLaughlin & Surman [1012.0532]. Esteban-Pretel, Mirizzi, Pastor, Tomàs, Raffelt, Serpico & Sigl [0807.0659]. Esteban-Pretel, Pastor, Tomàs, Raffelt & Sigl [0706.2498, 0712.1137]. Fogli, Lisi, Marrone & Mirizzi [0707.1998]. Fogli, Lisi, Marrone & Tamborra [0812.3031]. Friedland [1001.0996]. Gava & Jean-Louis [0907.3947]. Gava & Volpe [0807.3418]. Galais, Kneller & Volpe [1102.1471]. Galais & Volpe [1103.5302]. Gava, Kneller, Volpe & McLaughlin [0902.0317]. Hannestad, Raffelt, Sigl & Wong [astro-ph/0608695]. Wei Liao [0904.0075, 0904.2855]. Lunardini, Müller & Janka [0712.3000]. Mirizzi, Pozzorini, Raffelt & Serpico [0907.3674]. Mirizzi & Serpico [1111.4483]. Mirizzi & Tomàs [1012.1339]. Pehlivan, Balantekin, Kajino & Yoshida [1105.1182]. Pejcha, Dasgupta & Thompson [1106.5718]. Raffelt [0810.1407, 1103.2891]. Raffelt & Sigl [hep-ph/0701182]. Raffelt & Smirnov [0705.1830, 0709.4641]. Raffelt & Tamborra [1006.0002]. Sawyer [hep-ph/0408265, 0503013, 0803.4319, 1011.4585]. Sarikas, Raffelt, Hüdepohl & Janka [1109.3601]. Sarikas, Tamborra, Raffelt, Hüdepohl & Janka [1204.0971]. Saviano, Chakraborty, Fischer, Mirizzi [1203.1484]. Wu & Qian [1105.2068]………………………

DENSITY MATRIX FOR THE NEUTRINO ENSEMBLE

Diagonal elements related to flavor content

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{e\mu} & \rho_{e\tau} \\ \rho^{*}_{e\mu} & \rho_{\mu\mu} & \rho_{\mu\tau} \\ \rho_{e\tau} & \rho^{*}_{\mu\tau} & \rho_{\tau\tau} \end{pmatrix}$$

Off-diagonal elements responsible for flavor conversions

- $$\rho_{\alpha\alpha} = \frac{F_{\nu_\alpha}(E,r)}{F(E,r)}$$

- In 2ν scenario. Decompose density matrix over Pauli matrices to get the "**polarization**" (Bloch) vector **P**. Survival probability Pee =1/2(1+$\mathbf{P_z}$) . $\mathbf{P_z}$ = -1 -> Pee =0 ; $\mathbf{P_z}$ = 0 -> Pee =1/2 (flavor decoherence)

EQUATIONS OF MOTION FOR A DENSE NEUTRINO GAS

(Sigl & Raffelt, 1992)

$$\partial_t \varrho_{\mathbf{p},\mathbf{x}} + \mathbf{v}_{\mathbf{p}} \cdot \nabla_{\mathbf{x}} \varrho_{\mathbf{p},\mathbf{x}} + \dot{\mathbf{p}} \cdot \nabla_{\mathbf{p}} \varrho_{\mathbf{p},\mathbf{x}} \quad \longleftarrow \text{ Liouville operator}$$

$$= -i[\Omega_{\mathbf{p},\mathbf{x}}, \varrho_{\mathbf{p},\mathbf{x}}] \ ,$$

Hamiltonian $\Omega_{p,x} = \Omega_{vac} + \Omega_{matt} + \Omega_{\nu\nu}$

$\partial_t \varrho_{\mathbf{p},\mathbf{x}} \longrightarrow$ Explicit time evolution

$\mathbf{v}_{\mathbf{p}} \cdot \nabla_{\mathbf{x}} \varrho_{\mathbf{p},\mathbf{x}} \longrightarrow$ Drift term due to space inhomogeneities

$\dot{\mathbf{p}} \cdot \nabla_{\mathbf{p}} \varrho_{\mathbf{p},\mathbf{x}} \longrightarrow$ Force term acting on neutrinos (negligible)

7-dimensional problem. Never solved in its complete form. Symmetries have been used to reduce the complexity of the problem.

SPACE/TIME HOMOGENEITIY

- Space Homogeneity:

$$\partial_t \rho_{p,x} + v_p \cdot \cancel{\nabla_x \rho_{p,x}} = -i[\Omega_{p,x}, \rho_{p,x}]$$

Pure temporal evolution (Neutrinos in Early Universe)

- Time Homogeneity:

$$\cancel{\partial_t \rho_{p,x}} + v_p \cdot \nabla_x \rho_{p,x} = -i[\Omega_{p,x}, \rho_{p,x}]$$

Stationary space evolution (SN neutrinos)

MULTI-ANGLE (M.A.) EOMs FOR SN NEUTRINOS

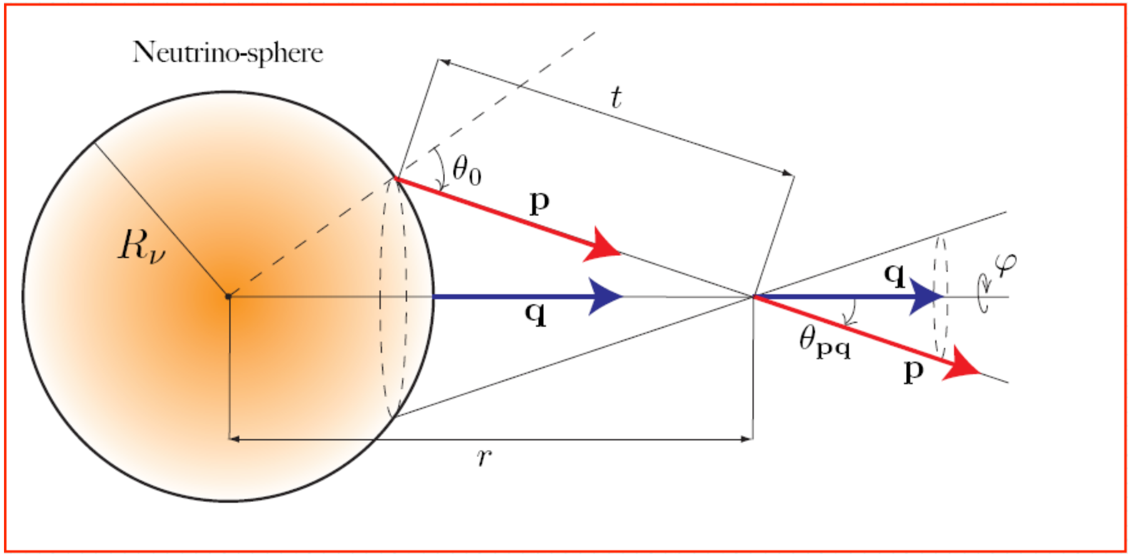
Evolution in space for ν's streaming from a SN core in quasi-stationary situation

$$i\,\vec{\mathrm{v}}_{\mathrm{p}}\cdot\vec{\nabla}_x\rho_{p,x}=\left[H(\omega,\lambda,\rho_{p',x}),\rho_{p,x}\right]$$

Liouville operator for free streaming ν

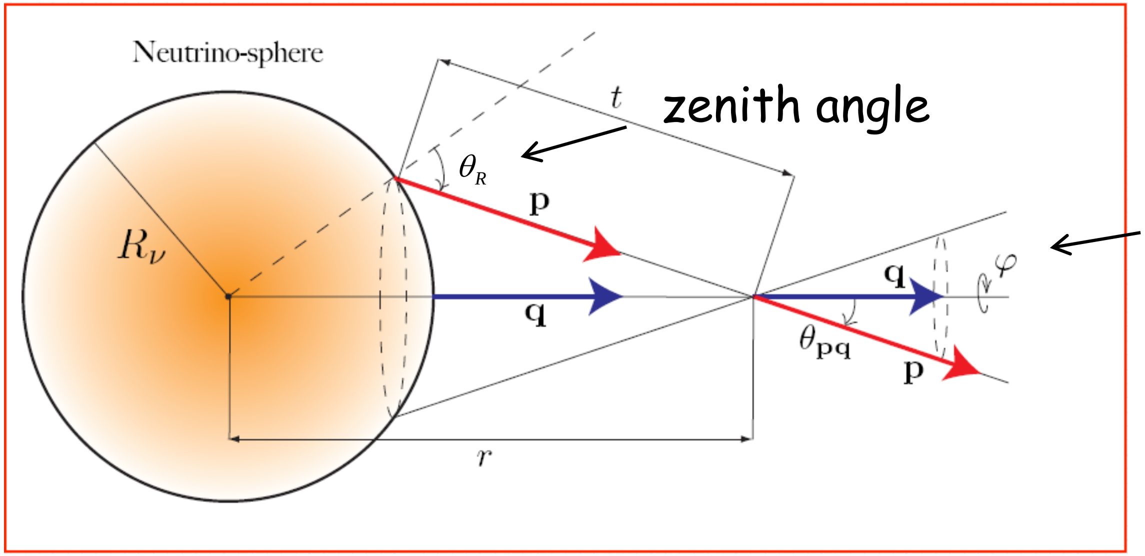
MULTI-ANGLE ν–ν HAMILTONIAN

$$H_{\nu\nu}=\sqrt{2}G_F\int d\vec{q}\left(1-\vec{\mathrm{v}}_{\mathrm{p}}\cdot\vec{\mathrm{v}}_q\right)\left(\rho_{q,x}-\bar{\rho}_{q,x}\right)$$



BULB MODEL

[see, e.g., Duan et al., astro-ph/0606616] ⟶ First large-scale multi-angle simulations



zenith angle

Azimuthal angle

- Neutrinos are emitted uniformly and (half)-isotropically from the surface of a sphere (ν-sphere), like in a blackbody.

- Physical conditions depend only on the the distance r from the center of the star (azimuthal symmetry)

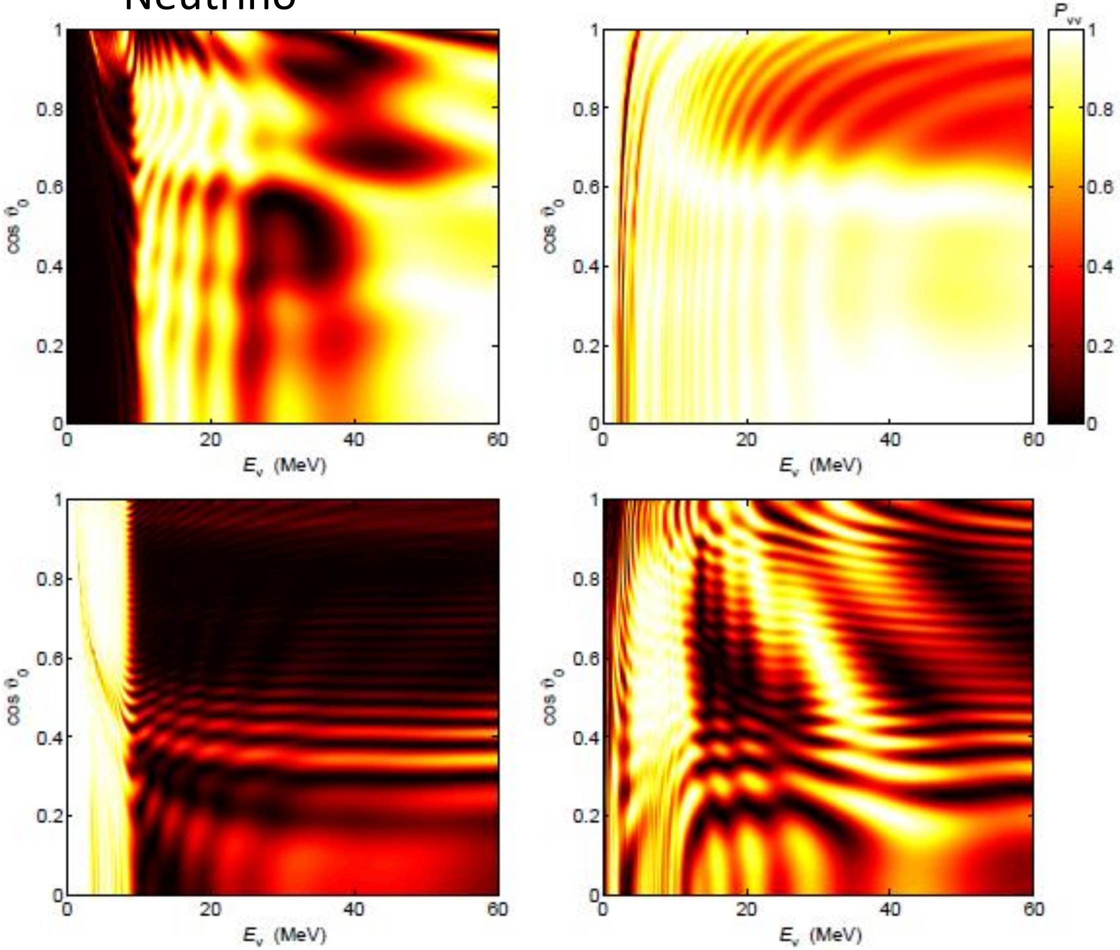
- Only multi-zenith-angle (MZA) effects in terms of $u = \sin^2 \theta_R$

- Project evolution along radial direction (ODE problem) $\vec{\mathrm{v}}_p \cdot \vec{\nabla}_x \rightarrow \mathrm{v}_\mathrm{r} d_r$

MULTI-ANGLE LARGE SCALE SIMULATIONS

First multi-angle simulations in 2006 by Duan, Fuller, Qian (2006). Major breakthrough!

Survival probability of ν_e vs E and emission angle cos θ



Neutrino

Anti-neutrino

Normal hierarchy

Inverted hierarchy

Significant angular dependence on the Pee

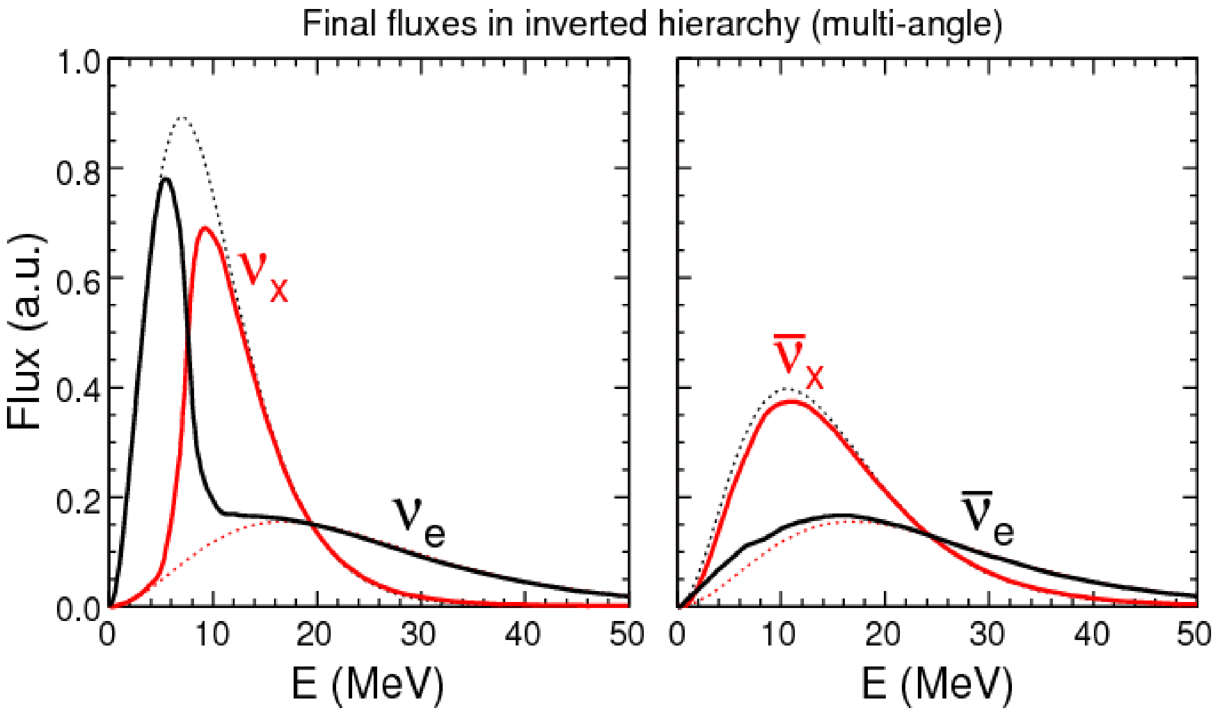
Convergence required > 10^3 angular bins ⟶ Large scale numerical simulations

MULTI-ANGLE SIMULATIONS BY DIFFERENT GROUPS

- *Duan, Fuller, Carlson & Qian, astro-ph/0606616, 0608050*
- *Fogli, Lisi, Marrone & A.M., 0707.1998, Fogli, Lisi, Marrone, A.M & Tamborra, 0808.0807;*
- *Esteban-Pretel, Pastor, Tomas, Raffelt & Sigl, 0706.2498*
- *Duan & Friedland, 1006.2359*
- *A.M. & Tomas, 1012.1339*
- *Cherry, Fuller, Carlson, Duan, Qian, 1006.2175*

SELF-INDUCED SPECTRAL SPLITS

[Fogli, Lisi, Marrone, A.M. , arXiV: 0707.1998 [hep-ph], Duan, Carlson, Fuller, Qian, astro-ph/0703776, Raffelt and Smirnov, 0705.1830 [hep-ph], Dasgupta, Dighe, Raffelt & Smirnov, arXiv:0904.3542 [hep-ph], Duan & Friedland, arXiv: 1006.2359, A.M. & Tomas, arXiv:1012.1339, Choubey, Dasgupta, Dighe, A.M., 1008.0308....]



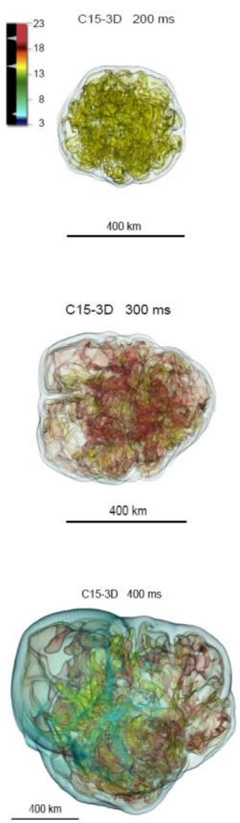
Swap of the original SN ν spectra in inverted mass hierarchy

Strong dependence of collective oscillations on mass hierarchy and on the energy ("splits")

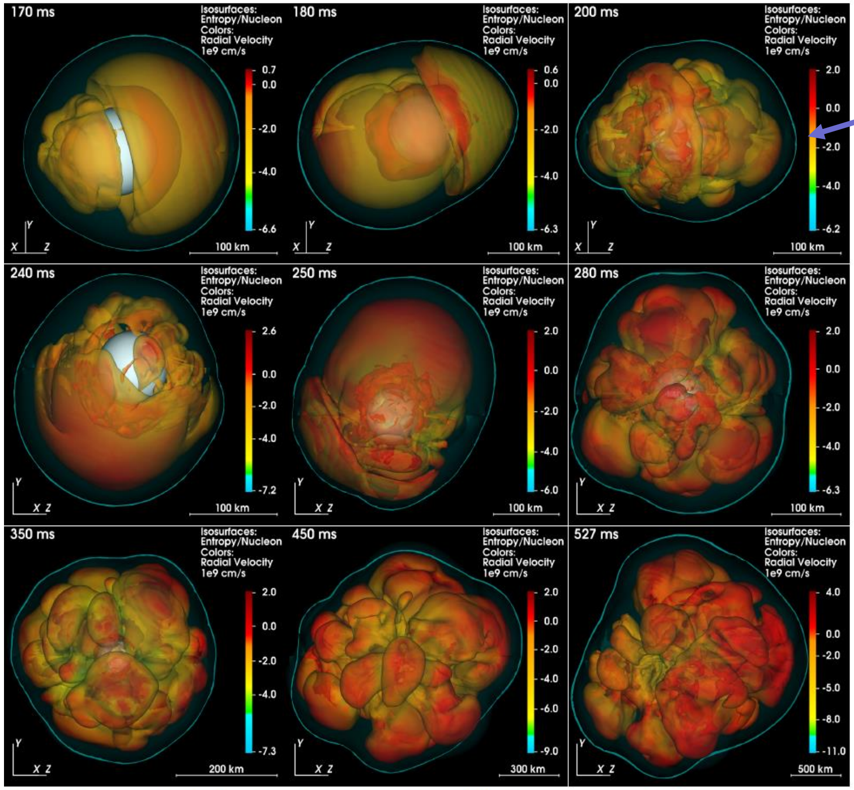
Splits possible in both normal and inverted hierarchy, for ν & $\bar{\nu}$!!

FIRST EXPLODING 3D SIMULATIONS

See talk by E.Endeve



[Lentz et al., 1504.07631]



[Melson, Janka et al., 1504.07631]

Shock-front

- 3D SN simulations show strong anisotropies and asphericities in the matter profile and in the neutrino emission.
- Real SN environment very far from the idealized bulb model.
- How deviations from the bulb model would affect the self-induced effects?

MULTI-AZIMUTHAL-ANGLE (MAA) INSTABILITY

- Self-induced flavor conversions are associated to an instability in the flavor space *[Sawyer,0803.4319; Banerjee, Dighe & Raffelt, 1107.2308]*

- Instability required to get started (exponential growth of the off-diagonal density matrix part)

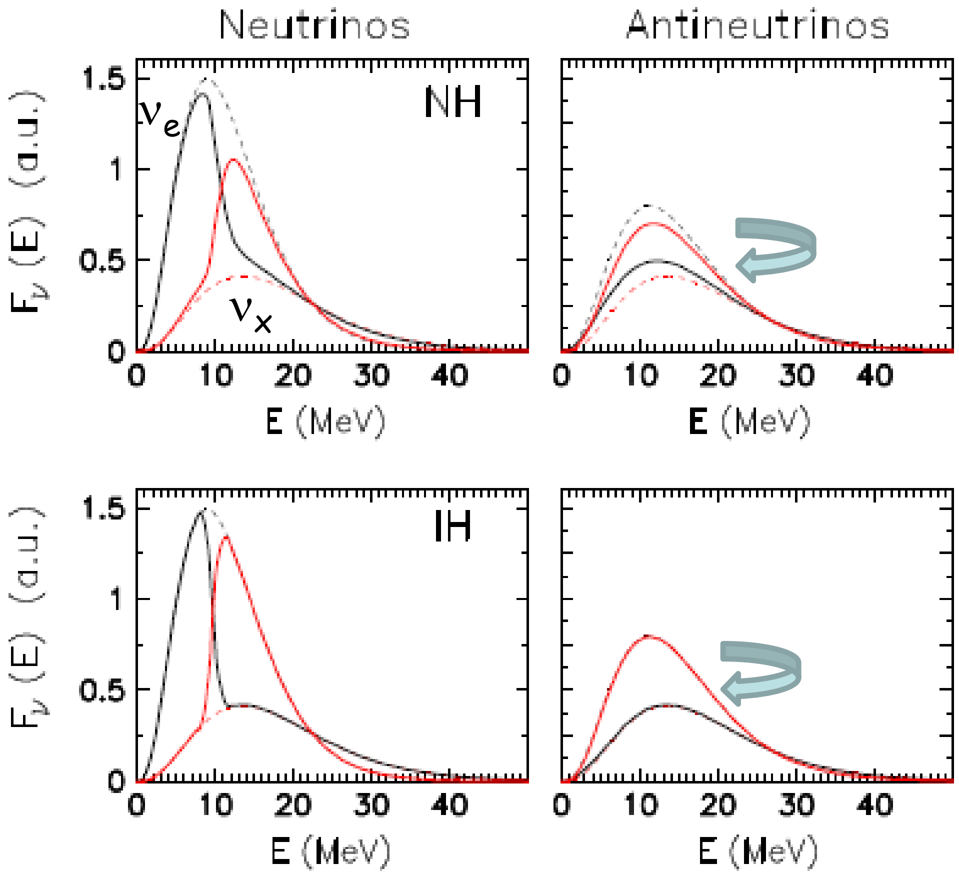
- The onset of the conversions can be found through a stability analysis of the linearized EoMs.

In *[Raffelt, Sarikas, Seixas, 1305.7140]* a stability analysis of the EoMs has been performed including the azimuthal angle ϕ of the ν propagation and without enforcing axial symmetry.
Also starting with an intial axial symmetric ν emission.....
.....A new multi-azimuthal-angle (MAA) instability has been found!!

- In the unstable case, numerical simulations are mandatory.

SPECTRAL SPLITS FOR SN NEUTRINO FLUXES

[Chakraborty & A.M., 1308.5255]



- Spectral swaps and splits in both NH & IH !!
- In the axial symmetric case, only IH unstable

SPONTANEOUS SYMMETRY BREAKING IN SELF-INDUCED OSCILLATIONS

- Symmetries have been used to reduce the complexity of the SN ν flavor evolution.

- However, the discovery of the MAA instability suggests self-interacting ν can lead to a spontaneous symmetry breaking (SSB) of the symmetry inherent to the initial conditions.

- Small deviations from the space/time symmetries of the bulb model have to be expected. Can these act as seed for new instabilities?

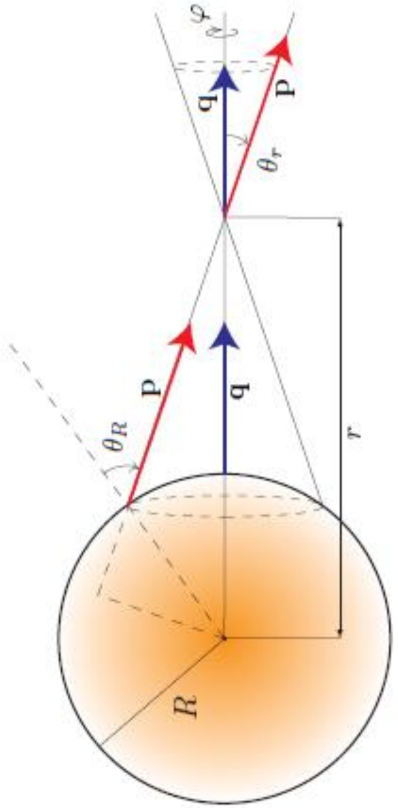
FIRST INVESTIGATIONS WITH TOY MODELS

- With a simple toy model in *[Mangano, A.M. & Saviano, 1403.1892]* it has been shown that self-interacting ν can break translational symmetries in space and time.

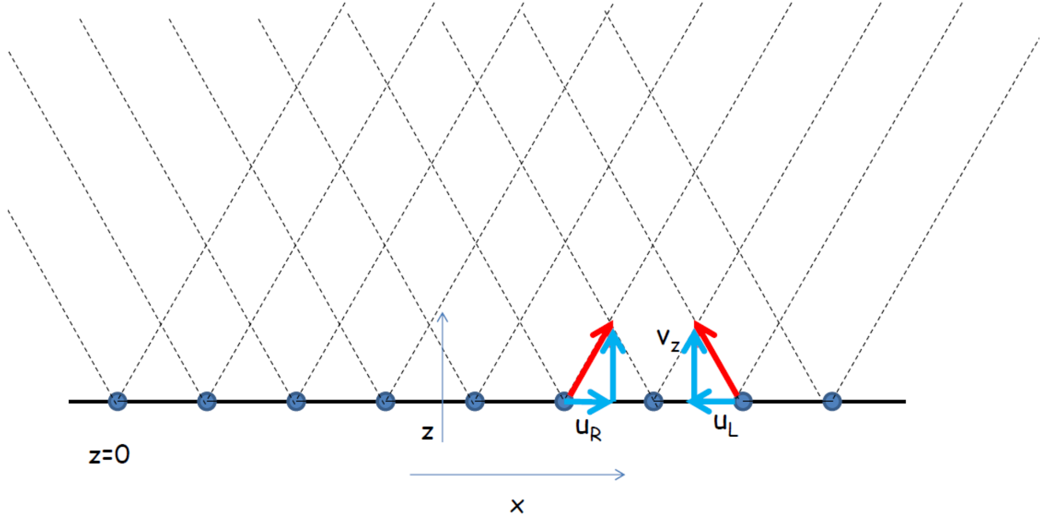
- By a stability analysis in *[Duan & Shalgar, 1412.7097]* is has been found that self-interacting ν can break the spatial symmetries of a 2D model.

2D MODEL FOR SELF-INTERACTING ν

FROM BULB MODEL → PLANAR MODEL



[Duan & Shalgar, 1412.7097]



Nu evolving in the plane (x,z) emitted from an infinite boundary at z=0, in only two directions (L and R). Excess of ν_e over $\overline{\nu_e}$.

SPHERICAL ↔ TRANSLATIONAL

AXIAL ↔ L-R

BROKEN SYMMETRIES ?

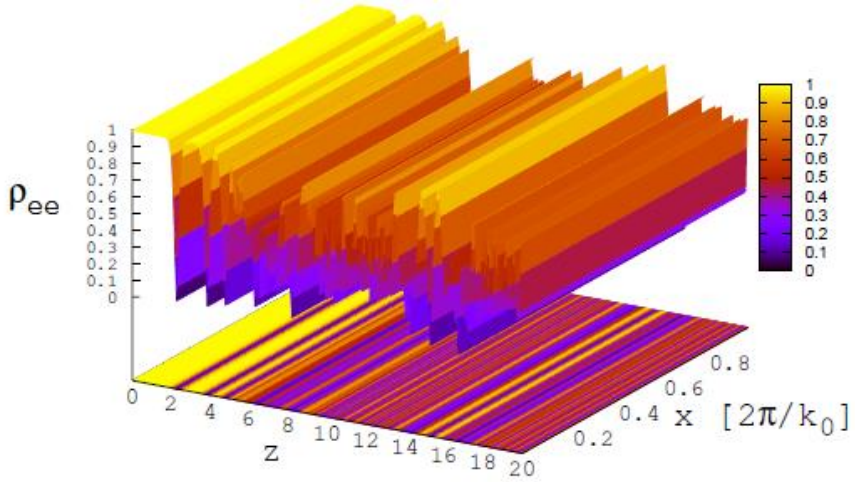
SSB IN 2D MODEL

Perturbing with seeds the L-R and the translational symmetries

~~L-R~~

TRANSLATIONAL

Both NH and IH are unstable. In L-R symmetric case only IH unstable.
(Analogous of MAA instability of bulb model)
[Raffelt & de Sousa Seixas, 1307.7625]



NH

1D problem along z direction

~~L-R~~

~~TRANSLATIONAL~~

2D Flavor evolution in (x,z) plane
[Duan & Shalgar, 1412.7097]

EoM FOR THE 2D MODEL

$$\begin{aligned}\hat{\mathbf{v}}_L\cdot\nabla_{\mathbf{x}}\mathsf{P}_L(x,z) &= [+\omega\mathsf{B}+\mu\mathsf{D}_R(x,z)]\times\mathsf{P}_L(x,z)\\ \hat{\mathbf{v}}_L\cdot\nabla_{\mathbf{x}}\overline{\mathsf{P}}_L(x,z) &= [-\omega\mathsf{B}+\mu\mathsf{D}_R(x,z)]\times\overline{\mathsf{P}}_L(x,z)\end{aligned}$$

(analogous for the R mode. L ←→ R symmetry)

$$\rho_p=\frac{1}{2}(1+P\cdot\sigma)$$ Two-flavor polarization vectors

$$\omega=\frac{\Delta m^2}{2E}$$ Vacuum oscillation frequency

$$B\cdot\hat{e}_3=-\cos\theta$$ Mass eigenstate direction in flavor space

$$\mu=\sqrt{2}G_F[F^0_{\bar{\nu}_e}-F^0_{\bar{\nu}_x}](1-\hat{\mathbf{v}}_L\cdot\hat{\mathbf{v}}_R)$$ ν–ν potential

$$D_R=P_R-\overline{P}_R$$

SOLVING THE PROBLEM IN FOURIER SPACE

[Mangano, A.M. & Saviano, 1403.1892; A.M. , Mangano & Saviano, 1503.03485]

The partial differential equation can be transformed into a tower of ordinary differential equations for the Fourier modes

$$\mathsf{P}_{L(R),k}(z) = \int_{-\infty}^{+\infty} dx \;\; \mathsf{P}_{L(R)}(x,z) e^{-ikx}$$

We assume a monochromatic perturbation (with wave-number $k_0 = 2\pi/\Lambda_0$) in the translational symm. along x at z=0

$$P^3_{L,R}(x,0) = \langle P^3_{L,R}(x,0)\rangle + \epsilon \cos(k_0 x) \quad \text{with} \quad \varepsilon << 1$$

Non-linear interaction →

$$\boxed{\begin{aligned} v_z \frac{d}{dz}\mathsf{P}_{L,n}(z) &= -i u_L k_n \mathsf{P}_{L,n} + \omega \mathsf{B} \times \mathsf{P}_{L,n} \\ &+ \mu \sum_{j=-\infty}^{+\infty} \mathsf{D}_{R,n-j} \times \mathsf{P}_{L,j} \; . \end{aligned}}$$

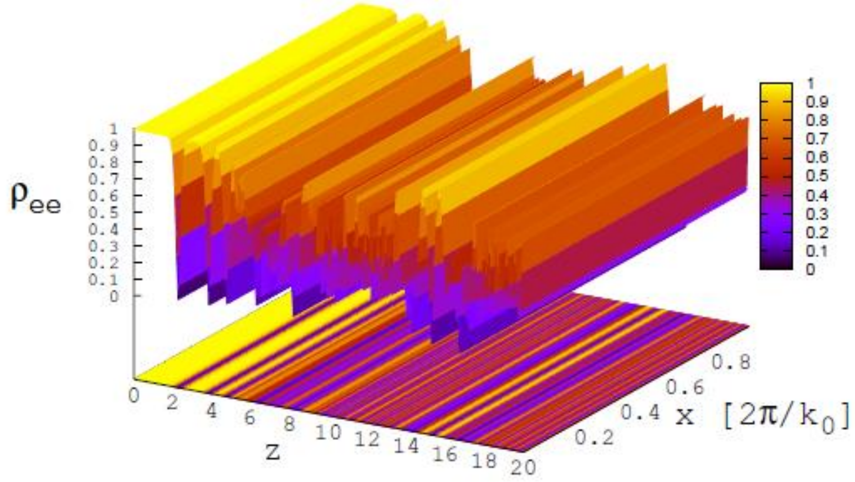
$$k_n = n k_0$$

Solution in real space by inverse Fourier transform

$$P(x,z) = \int_{-\infty}^{+\infty} dk P_k(z) e^{ikx}$$

2D FLAVOR EVOLUTION IN THE PLANE

[A.M. , Mangano & Saviano, 1503.03485]



~~L-R~~

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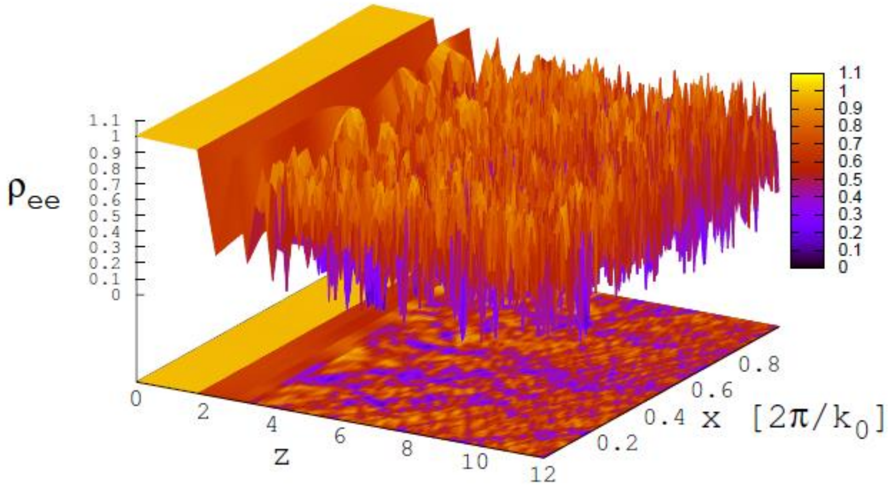
Evolution uniform in the x direction.

Coherent behavior along x direction.



~~L-R~~

~~TRANSLATIONAL~~ $k_0 = 0.2\sqrt{2\omega\mu}$

Large variations in the x direction at smaller and smaller scales.

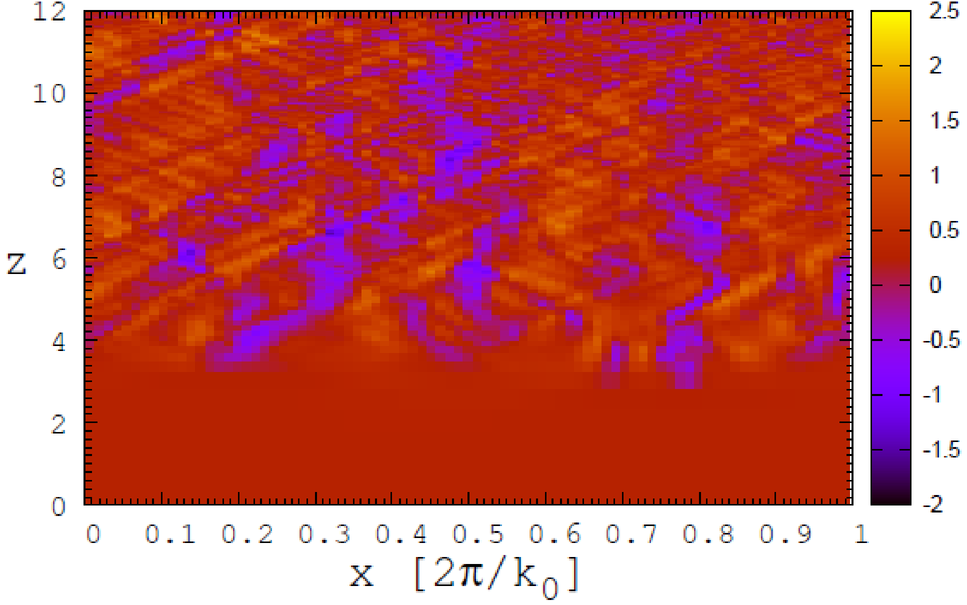
Planes of common phase broken.

Coherent behavior of oscillation lost.

LEPTON NUMBER

[A.M. , Mangano & Saviano, 1503.03485]

Map of L_0



Lepton current $L^{\mu} = (L_0, L)$

$$\mathrm{L}_0 = \mathrm{D}_L \cdot \mathrm{B} + \mathrm{D}_R \cdot \mathrm{B} \ ,$$

$$\mathbf{L} = \hat{\mathbf{v}}_L(\mathrm{D}_L \cdot \mathrm{B}) + \hat{\mathbf{v}}_R(\mathrm{D}_R \cdot \mathrm{B})$$

Continuity equation

$$\partial_t \mathrm{L}_0 + \nabla_{\mathbf{x}} \cdot \mathbf{L} = \nabla_{\mathbf{x}} \cdot \mathbf{L} = 0$$

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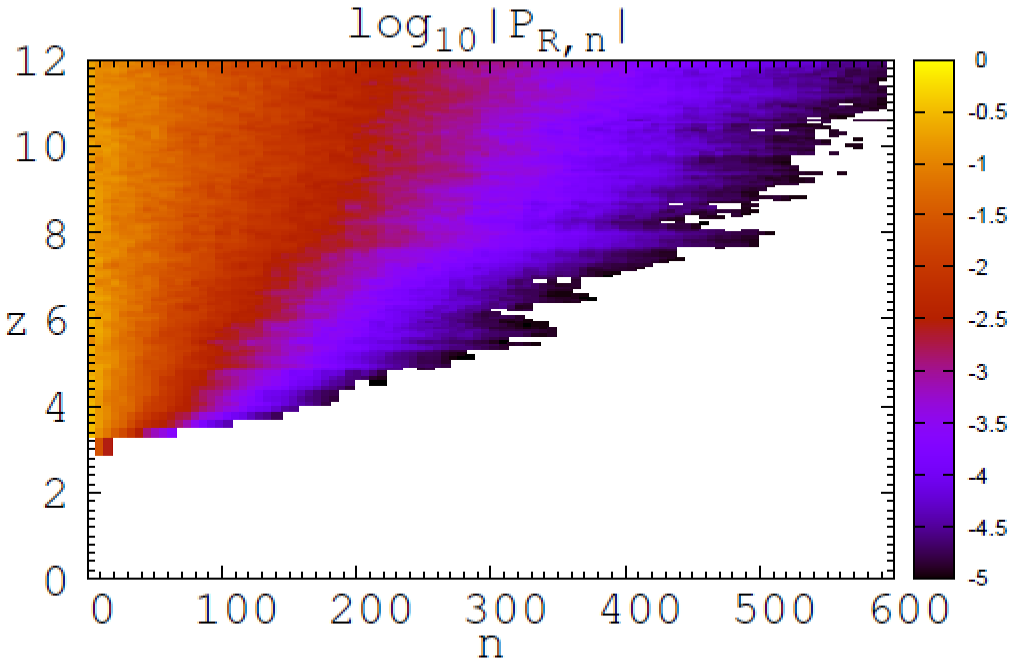
L_0 shows a non-trivial domain structure with different net lepton number flux

GROWTH OF FOURIER MODES

[A.M. , Mangano & Saviano, 1503.03485]



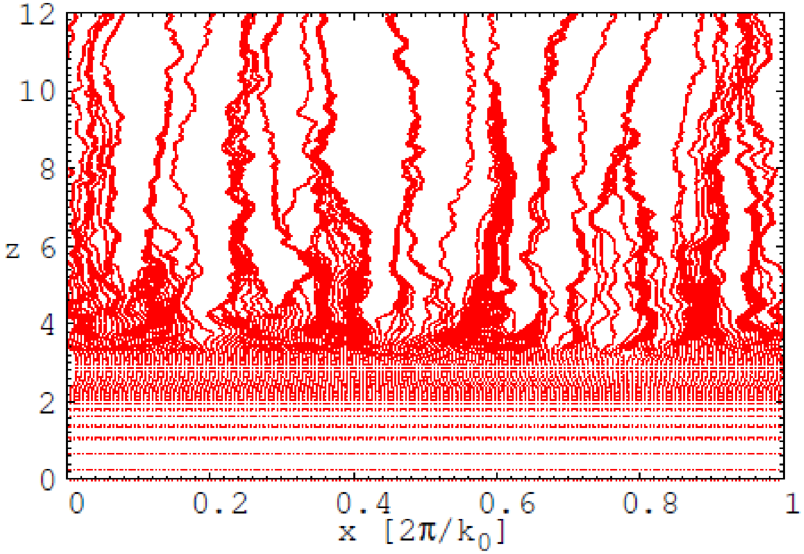
~~TRANSLATIONAL~~ ⇨

Growth of n>0 modes in Fourier space. Cascade process. Flavor wave diffuses to higher harmonics (smaller scales)

ANALOGY WITH A TURBULENT FLUID

[A.M. , Mangano & Saviano, 1503.03485]

ν_e streamlines



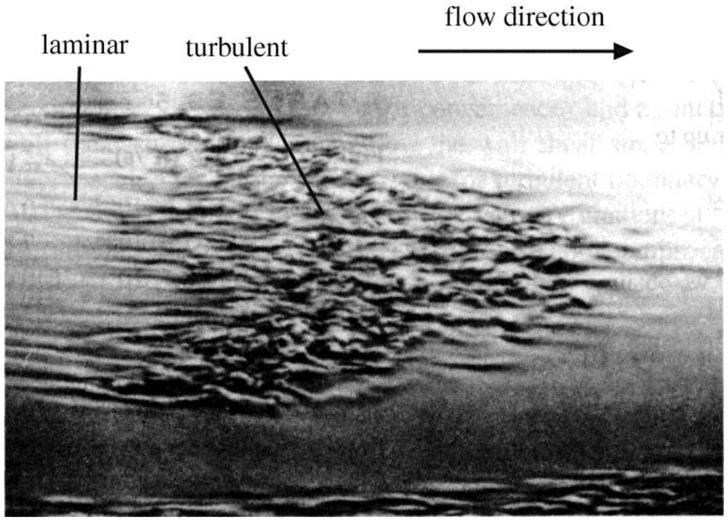
- ν_e average velocity

$$\langle \hat{\mathbf{v}}_e \rangle_{\mathbf{x}} = \frac{\varrho_{ee,L}\hat{\mathbf{v}}_L + \varrho_{ee,R}\hat{\mathbf{v}}_R}{\varrho_{ee,L} + \varrho_{ee,R}}$$

- streamlines of the ν_e flux

$$\frac{d\mathbf{x}}{ds} = \frac{\langle \hat{\mathbf{v}}_e \rangle_{\mathbf{x}}}{|\langle \hat{\mathbf{v}}_e \rangle_{\mathbf{x}}|} = \hat{\mathbf{F}}_{e,\mathbf{x}}$$

fluid streamlines

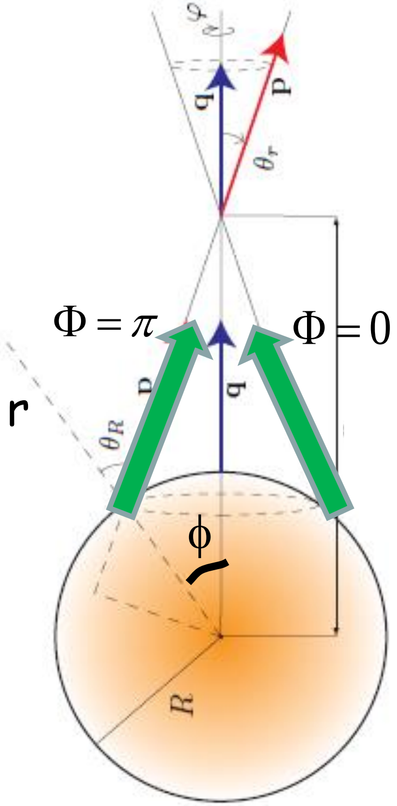


Analogy:
transition btw the coherent → incoherent behavior of the ν oscillations

transition btw the laminar → turbulent behaviour of a fluid.
(Non-linear Navier-Stokes equations)

2D TOY SUPERNOVA

[A.M. in preparation]



Neutrinos emitted in a plane from a ring with two azimuthal angles $\Phi = 0, \pi$ and with zenith angles u $= \sin^2\theta_R$ in [0;1]

$$v\cdot\nabla_x P(r,\phi) = v_r \frac{\partial}{\partial r}P(r,\phi) + \frac{v_\phi}{r}\frac{\partial}{\partial\phi}P(r,\phi)$$

$$v_r = \cos\Theta_r = \sqrt{1-\frac{R^2}{r^2}\sin^2\vartheta_R}$$

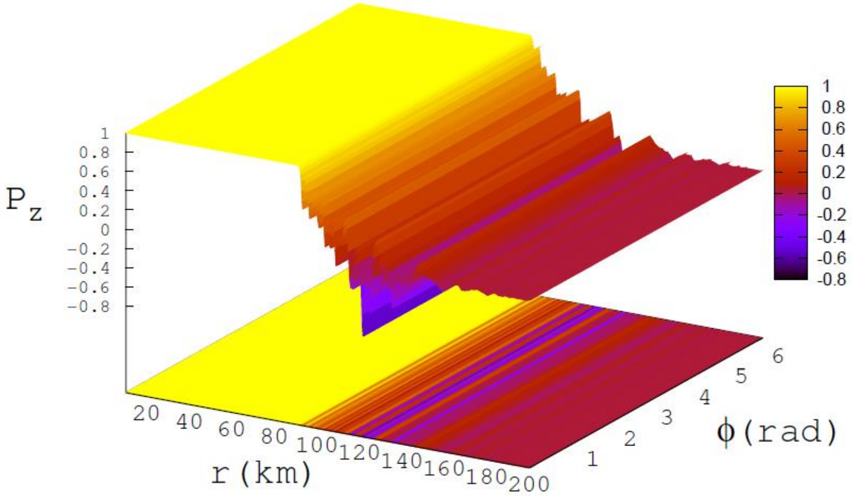
$$v_\phi = \sin\Theta_r\cos\Phi = \frac{R}{r}\sin\vartheta_R\cos\Phi$$

One can apply to this problem the same technique based on FT.

2D FLAVOR EVOLUTION IN A TOY SN

[A.M. in preparation]

NH case

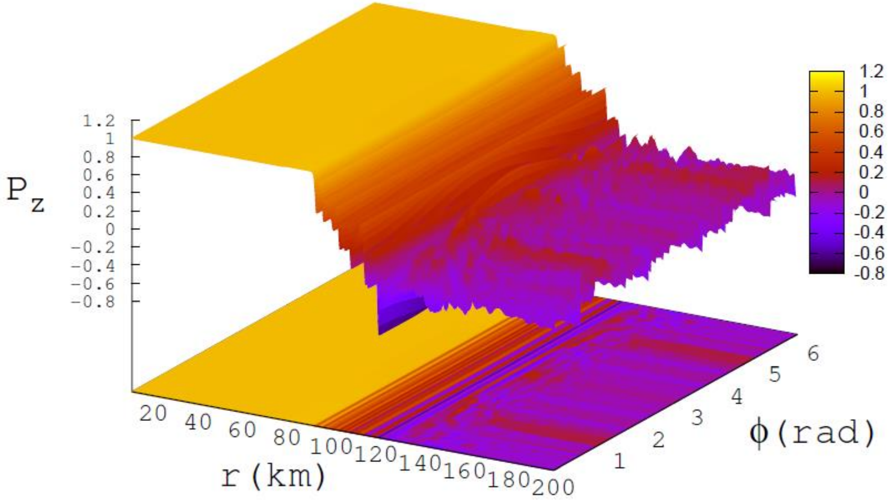


~~AXIAL~~

SPHERICAL

MAA instability in NH

Evolution uniform in the r direction.



~~AXIAL~~

~~SPHERICAL~~

Significant variations in the ϕ direction.

Planes of common phase broken.

Coherent behavior of oscillation lost.

OPEN ISSUES AND CONCLUSIONS

- Self-interacting neutrinos spontaneously break spatial symmetries (axial symmetry, translational symmetry,...)
- Self-induced flavor evolution of SN neutrinos obtained in the spherically symmetric bulb model should be critically reconsidered!
- Going beyond the bulb model would add additional layers of complications to this vexed problem
- Studies with simple toy models just begun

Ten years after the first studies on self-induced effects in SNe we are still far from a complete description of this flavor dynamics.....

LOT OF FUN/WORK WAITING FOR THE NEXT GALACTIC SN EXPLOSION !