Spontaneous Symmetry Breaking in Self-Induced Supernova Neutrino Flavor Conversions

Alessandro MIRIZZI
University of BARI & Sez. INFN Bari, Italy
OUTLINE

Supernovae as neutrino sources

Self-induced SN $\nu$ oscillations: the collective behaviour of a dense $\nu$ gas

Spontaneous Symmetry Breaking effects of self-interacting neutrino gas

Open Issues and Conclusions
Core collapse SN corresponds to the terminal phase of a massive star \([M \gtrsim 8 \, M_\odot]\) which becomes unstable at the end of its life. It collapses and ejects its outer mantle in a shock wave driven explosion.

- **ENERGY SCALES**: 99\% of the released energy (~ \(10^{53}\) erg) is emitted by \(\nu\) and \(\bar{\nu}\) of all flavors, with typical energies \(E \sim O(15\, \text{MeV})\).

- **TIME SCALES**: Neutrino emission lasts \(~10\, s\)

- **EXPECTED**: 1–3 SN/century in our galaxy \((d \approx O(10)\, \text{kpc})\).

(See talks by E.Endeve, T. Fischer .... for details on SN \(\nu\) emission)
Mixing parameters: \( U = U (\theta_{12}, \theta_{13}, \theta_{23}, \delta) \) as for CKM matrix

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
1 & & \\
\ c_{23} & s_{23} & \\
- s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & e^{-i\delta} s_{13} & 1 \\
e^{-i\delta} s_{13} & c_{13} & &
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} \\
-s_{12} & c_{12} & 1
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

\( c_{12} = \cos \theta_{12} \), etc., \( \delta \) CP phase

Mass-gap parameters: \( M^2 = \left\{ -\frac{\delta m^2}{2}, +\frac{\delta m^2}{2}, \pm \Delta m^2 \right\} \)

```
+\Delta m^2
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\( \nu_3 \)

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+\delta m^2/2
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\( \nu_1 \)

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-\delta m^2/2
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\( \nu_2 \)

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-\Delta m^2
```
\( \nu_3 \)

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SNAPSHOT OF SN DENSITIES

- Matter bkg potential
  \[ \lambda = \sqrt{2}G_F N_e \sim R^{-3} \]

- \(\nu-\bar{\nu}\) interaction
  \[ \mu = \sqrt{2}G_F n_\nu \sim R^{-2} \]

- Vacuum oscillation frequencies
  \[ \omega = \frac{\Delta m^2}{2E} \]

When \(\mu \gg \lambda\), SN \(\nu\) oscillations dominated by \(\nu-\bar{\nu}\) interactions

Collective flavor transitions at low-radii \([O(10^2 - 10^3 \text{ km})]\)

[Two seminal papers in 2006 triggered a torrent of activities
Duan, Fuller, Qian, astro-ph/0511275, Duan et al. astro-ph/0606616]

(See talks by B. Balantekin, G. McLaughlin, Y. Pehivan, C. Volpe)
Conclusions

Simultaneous $\nu$ and $\bar{\nu}$ flavor conversion possible by bipolar collective oscillation mode at few 10 to few 100 km above neutrino sphere

Depending on primary neutrino flux spectra, may
- Modify energy transfer to shock wave
- Modify neutrino-driven nucleosynthesis
- Modify observable signatures of SN neutrino oscillations

Collective Oscillations of Supernova Neutrinos

Georg Raffelt, Max-Planck-Institut für Physik, München
Toy Supernova in “Single-Angle” Approximation

- Assume 80% anti-neutrinos
- Vacuum oscillation frequency \( \omega = 0.3 \text{ km}^{-1} \)
- Neutrino-neutrino interaction energy at \( \nu \) sphere (\( r = 10 \text{ km} \)) \( \mu = 0.3 \times 10^5 \text{ km}^{-1} \)
- Falls off approximately as \( r^{-4} \) (geometric flux dilution and \( \nu \)s become more co-linear)

Decline of oscillation amplitude explained in pendulum analogy by increasing moment of inertia
(Hannestad, Raffelt, Sigl & Wong astro-ph/0608695)
Two seminal papers in 2006 triggered a torrent of activities
Duan, Fuller, Qian, astro-ph/0511275, Duan et al. astro-ph/0606616

**DENSITY MATRIX FOR THE NEUTRINO ENSEMBLE**

Diagonal elements related to flavor content

\[ \rho = \begin{pmatrix}
\rho_{ee} & \rho_{e\mu} & \rho_{e\tau} \\
\rho^{*}_{e\mu} & \rho_{\mu\mu} & \rho_{\mu\tau} \\
\rho^{*}_{e\tau} & \rho^{*}_{\mu\tau} & \rho_{\tau\tau}
\end{pmatrix} \]

Off-diagonal elements responsible for flavor conversions

- \( \rho_{\alpha\alpha} = \frac{F_{\nu\alpha}(E,r)}{F(E,r)} \)

In 2ν scenario, Decompose density matrix over Pauli matrices to get the "polarization" (Bloch) vector \( \mathbf{P} \). Survival probability \( P_{ee} = \frac{1}{2}(1+P_z) \). \( P_z = -1 \rightarrow P_{ee} = 0 \); \( P_z = 0 \rightarrow P_{ee} = 1/2 \) (flavor decoherence)
EQUATIONS OF MOTION FOR A DENSE NEUTRINO GAS

(Sigl & Raffelt, 1992)

\[
\begin{align*}
\partial_t \rho_{p,x} + v_p \cdot \nabla_x \rho_{p,x} + \dot{p} \cdot \nabla_p \rho_{p,x} &= -i[\Omega_{p,x}, \rho_{p,x}] , \\
= -i[\Omega_{p,x}, \rho_{p,x}] , & \quad \text{Liouville operator}
\end{align*}
\]

Hamiltonian \( \Omega_{p,x} = \Omega_{\text{vac}} + \Omega_{\text{mat}} + \Omega_{\text{vv}} \)

\[
\partial_t \rho_{p,x} \quad \text{Explicit time evolution}
\]

\[
v_p \cdot \nabla_x \rho_{p,x} \quad \text{Drift term due to space inhomogeneities}
\]

\[
\dot{p} \cdot \nabla_p \rho_{p,x} \quad \text{Force term acting on neutrinos (negligible)}
\]

7-dimensional problem. Never solved in its complete form. Symmetries have been used to reduce the complexity of the problem.

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**SPACE/TIME HOMOGENEITY**

- **Space Homogeneity:**
  \[ \partial_t \rho_{p,x} + v_p \cdot \nabla_{x} \rho_{p,x} = -i[\Omega_{p,x}, \rho_{p,x}] \]

  **Pure temporal evolution (Neutrinos in Early Universe)**

- **Time Homogeneity:**
  \[ \partial_t \rho_{p,x} + v_p \cdot \nabla_{x} \rho_{p,x} = -i[\Omega_{p,x}, \rho_{p,x}] \]

  **Stationary space evolution (SN neutrinos)**

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MULTI-ANGLE (M.A.) EOMs FOR SN NEUTRINOS

Evolution in space for $\nu$'s streaming from a SN core in quasi-stationary situation

$$i \vec{\nabla}_p \cdot \vec{\nabla}_x \rho_{p,x} = \left[ H(\omega, \lambda, \rho_{p',x}), \rho_{p,x} \right]$$

Liouville operator for free streaming $\nu$

MULTI-ANGLE $\nu-\nu$ HAMILTONIAN

$$H_{\nu\nu} = \sqrt{2} G_F \int d\vec{q} \left( 1 - \vec{v}_p \cdot \vec{v}_q \right) \left( \rho_{q,x} - \bar{\rho}_{q,x} \right)$$
Neutrinos are emitted uniformly and (half)-isotropically from the surface of a sphere (\(\nu\)-sphere), like in a blackbody.

Physical conditions depend only on the the distance \(r\) from the center of the star (azimuthal symmetry)

Only multi-zenith-angle (MZA) effects in terms of \(u = \sin^2 \theta_R\)

Project evolution along radial direction (ODE problem) \(\mathbf{v}_p \cdot \nabla_x \rightarrow v_r d_r\)
MULTI-ANGLE LARGE SCALE SIMULATIONS

First multi-angle simulations in 2006 by Duan, Fuller, Qian (2006). Major breakthrough!

- Neutrino
- Anti-neutrino

Survival probability of $v_e$ vs $E$ and emission angle $\cos \theta$

Convergence required $> 10^3$ angular bins $\rightarrow$ Large scale numerical simulations

Significant angular dependence on the Pee

Normal hierarchy

Inverted hierarchy
MULTI-ANGLE SIMULATIONS BY DIFFERENT GROUPS

- Duan, Fuller, Carlson & Qian, astro-ph/0606616, 0608050

- Fogli, Lisi, Marrone & A.M., 0707.1998, Fogli, Lisi, Marrone, A.M & Tamborra, 0808.0807;

- Esteban-Pretel, Pastor, Tomas, Raffelt & Sigl, 0706.2498

- Duan & Friedland, 1006.2359

- A.M. & Tomas, 1012.1339

- Cherry, Fuller, Carlson, Duan, Qian, 1006.2175
SELF-INDUCED SPECTRAL SPLITS


Strong dependence of collective oscillations on mass hierarchy and on the energy ("splits")

Splits possible in both normal and inverted hierarchy, for \( \nu \) & \( \bar{\nu} \)!!

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3D SN simulations show strong anisotropies and asphericities in the matter profile and in the neutrino emission.

Real SN environment very far from the idealized bulb model.

How deviations from the bulb model would affect the self-induced effects?
Self-induced flavor conversions are associated to an instability in the flavor space. [Sawyer, 0803.4319; Banerjee, Dighe & Raffelt, 1107.2308]

Instability required to get started (exponential growth of the off-diagonal density matrix part).

The onset of the conversions can be found through a stability analysis of the linearized EoMs.

In [Raffelt, Sarikas, Seixas, 1305.7140] a stability analysis of the EoMs has been performed including the azimuthal angle $\phi$ of the $\nu$ propagation and without enforcing axial symmetry. Also starting with an initial axial symmetric $\nu$ emission.....

......A new multi-azimuthal-angle (MAA) instability has been found!!

In the unstable case, numerical simulations are mandatory.

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Spectral swaps and splits in both NH & IH !!

In the axial symmetric case, only IH unstable
**SPONTANEOUS SYMMETRY BREAKING IN SELF-INDUCED OSCILLATIONS**

- **Symmetries** have been used to reduce the complexity of the SN $\nu$ flavor evolution.

- However, the discovery of the MAA instability suggests self-interacting $\nu$ can lead to a spontaneous symmetry breaking (SSB) of the symmetry inherent to the initial conditions.

- Small deviations from the space/time symmetries of the bulb model have to be expected. Can these act as seed for new instabilities?

**FIRST INVESTIGATIONS WITH TOY MODELS**

- With a simple toy model in [Mangano, A.M. & Saviano, 1403.1892] it has been shown that self-interacting $\nu$ can break translational symmetries in space and time.

- By a stability analysis in [Duan & Shalgar, 1412.7097] is has been found that self-interacting $\nu$ can break the spatial symmetries of a 2D model.
2D MODEL FOR SELF-INTERACTING $\nu$

FROM BULB MODEL $\rightarrow$ PLANAR MODEL

Nu evolving in the plane $(x,z)$ emitted from an infinite boundary at $z=0$, in only two directions (L and R). Excess of $\nu_e$ over $\bar{\nu}_e$.

[\textit{Duan & Shalgar, 1412.7097}]

SPHERICAL $\rightarrow$ TRANSLATIONAL

AXIAL $\rightarrow$ L-R

BROKEN SYMMETRIES?
Perturbing with seeds the L-R and the translational symmetries

Both NH and IH are unstable. In L-R symmetric case only IH unstable. (Analogous of MAA instability of bulb model) 
[Raffelt & de Sousa Seixas, 1307.7625]

1D problem along z direction

2D Flavor evolution in (x,z) plane 
[Duan & Shalgar, 1412.7097]
EoM FOR THE 2D MODEL

\[
\hat{v}_L \cdot \nabla_x P_L(x, z) = [+\omega B + \mu D_R(x, z)] \times P_L(x, z)
\]
\[
\hat{v}_L \cdot \nabla_x \overline{P}_L(x, z) = [-\omega B + \mu D_R(x, z)] \times \overline{P}_L(x, z)
\]

(analogous for the R mode. L \leftrightarrow R symmetry)

\[
\rho_p = \frac{1}{2}(1 + P \cdot \sigma)
\]

Two-flavor polarization vectors

\[
\omega = \frac{\Delta m^2}{2E}
\]

Vacuum oscillation frequency

\[
B \cdot \hat{e}_3 = -\cos \theta
\]

Mass eigenstate direction in flavor space

\[
\mu = \sqrt{2}G_F[F_{\nu_e}^0 - F_{\nu_x}^0](1 - \hat{v}_L \cdot \hat{v}_R)
\]

\nu-\nu potential

\[
D_R = P_R - \overline{P}_R
\]

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The partial differential equation can be transformed into a tower of ordinary differential equations for the Fourier modes

\[ P_{L(R),k}(z) = \int_{-\infty}^{+\infty} dx \ P_{L(R)}(x,z)e^{-ikx} \]

We assume a monochromatic perturbation (with wave-number \( k_0 = 2\pi/\lambda_0 \)) in the translational symm. along \( x \) at \( z=0 \)

\[ P^3_{L,R}(x,0) = \langle P^3_{L,R}(x,0) \rangle + \epsilon \cos(k_0 x) \quad \text{with} \quad \epsilon << 1 \]

Non-linear interaction

Solution in real space by inverse Fourier transform

\[ P(x,z) = \int_{-\infty}^{+\infty} dk P_k(z)e^{ikx} \]
2D FLAVOR EVOLUTION IN THE PLANE

Evolution uniform in the x direction.

Coherent behavior along x direction.

Large variations in the x direction at smaller and smaller scales.

Planes of common phase broken.

Coherent behavior of oscillation lost.

[A.M., Mangano & Saviano, 1503.03485]
LEPTON NUMBER

[A.M. Mangano & Saviano, 1503.03485]

Map of $L_0$

Lepton current $L^\mu = (L_0, L)$

$L_0 = D_L \cdot B + D_R \cdot B$

$L = \hat{v}_L(D_L \cdot B) + \hat{v}_R(D_R \cdot B)$

Continuity equation

$\partial_t L_0 + \nabla_x \cdot L = \nabla_x \cdot L = 0$

TRANSLATIONAL

$L_0$ shows a non-trivial domain structure with different net lepton number flux

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GROWTH OF FOURIER MODES

[A.M., Mangano & Saviano, 1503.03485]

Growth of $n > 0$ modes in Fourier space. Cascade process. Flavor wave diffuses to higher harmonics (smaller scales).

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**ANALOGY WITH A TURBULENT FLUID**

- \( v_e \) average velocity
  \[
  \langle \hat{v}_e \rangle_x = \frac{\rho_{ee,L} \hat{V}_L + \rho_{ee,R} \hat{V}_R}{\rho_{ee,L} + \rho_{ee,R}}
  \]
- streamlines of the \( v_e \) flux
  \[
  \frac{dx}{ds} = \frac{\langle \hat{v}_e \rangle_x}{|\langle \hat{v}_e \rangle_x|} = \hat{F}_{e,x}
  \]

---

**Analogy:**
- transition btw the coherent \( \rightarrow \) incoherent behavior of the \( v \) oscillations
- transition btw the laminar \( \rightarrow \) turbulent behaviour of a fluid.

(Non-linear Navier-Stokes equations)
Neutrinos emitted in a plane from a ring with two azimuthal angles $\Phi = 0, \pi$ and with zenith angles $u = \sin^2 \theta_R$ in $[0; 1]$

$$v \cdot \nabla_x P(r, \phi) = v_r \frac{\partial}{\partial r} P(r, \phi) + \frac{v_\phi}{r} \frac{\partial}{\partial \phi} P(r, \phi)$$

$$v_r = \cos \Theta_r = \sqrt{1 - \frac{R^2}{r^2} \sin^2 \theta_R}$$

$$v_\phi = \sin \Theta_r \cos \Phi = \frac{R}{r} \sin \theta_R \cos \Phi$$

One can apply to this problem the same technique based on FT.
2D FLAVOR EVOLUTION IN A TOY SN

[A.M. in preparation]

NH case

Evolution uniform in the r direction.

MAA instability in NH

Significant variations in the φ direction.

Planes of common phase broken.

Coherent behavior of oscillation lost.

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OPEN ISSUES AND CONCLUSIONS

- Self-interacting neutrinos spontaneously break spatial symmetries (axial symmetry, translational symmetry, ...)

- Self-induced flavor evolution of SN neutrinos obtained in the spherically symmetric bulb model should be critically reconsidered!

- Going beyond the bulb model would add additional layers of complications to this vexed problem

- Studies with simple toy models just begun

Ten years after the first studies on self-induced effects in SNe we are still far from a complete description of this flavor dynamics.....

LOT OF FUN/WORK WAITING FOR THE NEXT GALACTIC SN EXPLOSION!