Solar Models and Neutrinos: The SSM and Beyond

Aldo Serenelli (ICE/CSIC-IEEC)

NDM’15 – 05/06/2015
**Plan**

**Part I**

* Standard solar models and the abundance problem
  
  what helioseismology and solar $\nu$s really tell us (also talk by Villante)

* A generalized approach to solar models (with dark energy channels)
  
  revisiting solar limits for axion-photon coupling and hidden photons

**Part II**

* Asymmetric dark matter in the Sun
  
  $q$ and $v_{\text{rel}}$ dependent interactions and a non-standard look at the solar abundance problem
SSM assumes

constant mass evolution – $1 \, M_\odot$
initially homogeneous
solar system age 4.57 Gyr

3 present-day constraints $\leftrightarrow$ 3 adjustable quantities

Solar radius $\rightarrow$ convection parameter – mixing length
Solar (photon) luminosity $\rightarrow$ initial helium
Metal to hydrogen surface abundance ($Z/X$) $\rightarrow$ initial metallicity
Probing the Sun: Helioseismology

Sound speed (& density) profile
Solar core structure (low degree modes)
Depth of convection/radiation transition boundary
Envelope helium abundance
**PROBING THE SUN: νS FROM PP-CHAIRS**

- **ppI**
- **ppII**
- **ppIII**

**Borexino: pp(10%) – pep(20%)**

- \( p + p \rightarrow ^2\text{H} + e^+ + \nu_e \) (99.76%)
- \( p + e^- + p \rightarrow ^2\text{H} + \nu_e \) (0.24%)

- \( ^2\text{H} + p \rightarrow ^3\text{He} + \gamma \) (84.6%)
- \( ^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + 2p \)
- \( ^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma \) (15.4%)
- \( ^3\text{He} + p \rightarrow ^4\text{He} + e^+ + \nu_e \) (2.5\times10^{-5}%)  

- \( ^7\text{Be} + e^- \rightarrow ^7\text{Li} + \nu_e \) (99.89%)
- \( ^7\text{Li} + p \rightarrow ^4\text{He} \)
- \( ^7\text{Be} + p \rightarrow ^8\text{B} + \gamma \) (0.11%)
- \( ^8\text{B} \rightarrow ^8\text{Be}^* + e^+ + \nu_e \)

**7Be: Borexino – 4.5%**

**8B: SNO, SuperK – 3%**
Dealing with convection

Hard to model with 1D models

Credit: N. Brummell

Magic et al. 2014
Solar Abundances: End Product

* 3D-RHD simulations of solar convection

* improved atomic data (e.g. line blends)

* NLTE line formation for certain elements

<table>
<thead>
<tr>
<th>Element</th>
<th>GS98</th>
<th>AGSS09+met</th>
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</thead>
<tbody>
<tr>
<td>C</td>
<td>8.52</td>
<td>8.43</td>
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<tr>
<td>N</td>
<td>7.92</td>
<td>7.83</td>
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<tr>
<td>O</td>
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<td>Ne</td>
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<td>Ar</td>
<td>6.40</td>
<td>6.40</td>
</tr>
<tr>
<td>Fe</td>
<td>7.50</td>
<td>7.45</td>
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</tbody>
</table>

Z/X 0.0229 0.0178

Differences of

CNO(Ne)~30-40%
refractories~10%
relative difference in sound speed
relative difference in sound speed

\[
\frac{c_{\text{sun}} - c_{\text{mod}}}{c_{\text{sun}}}
\]

<table>
<thead>
<tr>
<th></th>
<th>GS98</th>
<th>AGSS09</th>
<th>Helios.</th>
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</thead>
<tbody>
<tr>
<td>(Z/X_\odot)</td>
<td>0.0229</td>
<td>0.0178</td>
<td>—</td>
</tr>
<tr>
<td>(R_{\odot}/R_\odot)</td>
<td>0.712</td>
<td>0.723</td>
<td>0.713 ± 0.001</td>
</tr>
<tr>
<td>(Y_S)</td>
<td>0.2429</td>
<td>0.2319</td>
<td>0.2485 ± 0.0034</td>
</tr>
<tr>
<td>(\langle \delta c/c \rangle)</td>
<td>0.0009</td>
<td>0.0037</td>
<td>—</td>
</tr>
<tr>
<td>(\langle \delta \rho/\rho \rangle)</td>
<td>0.011</td>
<td>0.040</td>
<td>—</td>
</tr>
</tbody>
</table>
Frequency ratios sensitive to solar core

\[ \int_{0}^{R} \frac{dc}{dr} \frac{dr}{r} \]
Borexino ($^7$Be) – SNO & SuperK ($^8$B)

Dependence on core temperature

No current probe directly sensitive to composition (next talk by Villante)
CN(O) neutrinos store additional (linear) dependence

Next talk by F. Villante
**Robust Inferences from SSMs?**

Phenomenological approach
- let input parameters vary and find SSM that best fits data
- changes in input parameters of order $1-\sigma$ & adjustment in composition
- excellent phenomenological description of solar structure

Even better than the real thing!!

*Villante, Serenelli et al. 2014*
* Solar limits on axion-photon coupling (well-studied case – for comparison)

\[ \epsilon_{a\gamma} \propto g_{a\gamma}^2 T^7 F(\kappa^2) \sim g_{a\gamma}^2 T^6 \]

* Solar limits on hidden (dark) photons

\[ \epsilon_{hp} = \frac{\chi^2 m^2}{e^{\omega_P/T} - 1} \frac{\omega_P^3}{4\pi} \frac{1}{\rho} \sim \chi^2 m^2 T \]

No explicit composition dependence -- > relative changes are the same regardless composition

Robust limits as long as good T-\( \rho \) solar profiles

Vinyoles et al. arxiv:1501.01639
Extending the Method for Particle Studies

* Sound speed variations

AGSS09 – low Z

GS98 – high Z
Extending the Method for Particle Studies

* Variation of neutrino fluxes

- Axions
- Hidden photons

Graphs showing the variation of axion and hidden photon fluxes with parameters.
Our approach: input parameters absorb variations to get best possible fit – e.g. increase metal abundances: freely or constrained by spectroscopy, S11, L⊙, etc.

Limits derived are based on forcing solar models (+ dark channel) to fit solar data as best as possible --- > limits then derived from irreducible residuals

Full solution: composition is free and pulls computed to minimize $\chi^2$ for fixed $g_{10}$

GS98 – high Z
AGSS09 – low Z

![Graphs showing Oxygen and Iron abundances for GS98 and AGSS09 models.](image)
Final upper limit – $g_{10} < 4$ @ 3-σ C.L.
2 times better than previous solar limit

$\chi_m < 2$ @ 3-σ C.L.
improves previous limit by factor 2

previous solar limit

previous solar limit
Comments on Solar Constraints

Effective limit in dark channels
$L_{hp} < 2\% \, L_{\odot}$ — $L_{\alpha' < 3\% \, L_{\odot}}$

using pp $\nu$ flux offers a model independent test – but needs measurement ~ 1%
CURRENT LIMITS

Hadronic axions

Hidden photons

KSVZ axion

CAST

IAXO

HB

CMB

SHIPS

XENON10

Sun–L (global)

Sun–L (ν)

Sun–T (ν)

Coulomb

10^{-13}

10^{-12}

10^{-11}

10^{-10}

10^{-9}

10^{-8}

10^{-7}

10^{-6}

10^{-5}

10^{-4}

10^{-3}

10^{-2}

10^{-1}

10^{0}

10^{1}

10^{2}

m [eV]

10^{-12}

10^{-11}

10^{-10}

10^{-9}

10^{-8}

10^{-7}

10^{-6}

10^{-5}

10^{-4}

10^{-3}

10^{-2}

10^{-1}

10^{0}

10^{1}

10^{2}

m [eV]

10^{-3}

10^{-2}

10^{-1}

10^{0}

10^{1}

10^{2}

g_{\alpha\gamma} [GeV^{-1}]

10^{-11}

10^{-10}

10^{-9}

10^{-8}

10^{-7}

10^{-6}

10^{-5}

10^{-4}

10^{-3}

10^{-2}

10^{-1}

10^{0}

10^{1}

10^{2}

m_\alpha [eV]

10^{-12}

10^{-11}

10^{-10}

10^{-9}

10^{-8}

10^{-7}

10^{-6}

10^{-5}

10^{-4}

10^{-3}

10^{-2}

10^{-1}

10^{0}

10^{1}

10^{2}

g_{\alpha\gamma} [GeV^{-1}]

10^{-11}

10^{-10}

10^{-9}

10^{-8}

10^{-7}

10^{-6}

10^{-5}

10^{-4}

10^{-3}

10^{-2}

10^{-1}

10^{0}

10^{1}

10^{2}

m_\alpha [eV]

10^{-12}

10^{-11}

10^{-10}

10^{-9}

10^{-8}

10^{-7}

10^{-6}

10^{-5}

10^{-4}

10^{-3}

10^{-2}

10^{-1}

10^{0}

10^{1}

10^{2}

g_{\alpha\gamma} [GeV^{-1}]

10^{-11}

10^{-10}

10^{-9}

10^{-8}

10^{-7}

10^{-6}

10^{-5}

10^{-4}

10^{-3}

10^{-2}

10^{-1}

10^{0}

10^{1}

10^{2}

m_\alpha [eV]

10^{-12}

10^{-11}

10^{-10}

10^{-9}

10^{-8}

10^{-7}

10^{-6}

10^{-5}

10^{-4}

10^{-3}

10^{-2}

10^{-1}

10^{0}

10^{1}

10^{2}

m_\alpha [eV]
DM-nucleon scattering allows DM collisions with nuclei in the Sun

→ gravitational capture and settling the to solar core
→ nuclear scattering inside the Sun
→ additional energy transport (abundance problem)

Vincent et al. – arxiv:1411.6626 / 1504.04378

SECOND PART --- ADM & SOLAR ABUNDANCE PROBLEM

helioseismology  solar neutrinos
Capture & Transport

Enhancement/suppression factors

\[ \sigma = \sigma_0 \left( \frac{q}{q_0} \right)^{2n} \]

\[ \sigma = \sigma_0 \left( \frac{v_{\text{rel}}}{v_0} \right)^{2n} \]

\[ \sigma_0 = 10^{-35} \text{ cm}^2 \]

Solid = SI, Dashed = SD
Impact on Observables: SD, SI - constant

$^{7}\text{Be} \& ^{8}\text{B} \text{ neutrinos} + \text{convective radius} + \text{surface helium}$

Frequency separation ratios $r_{02}, r_{13}$
Impact on Observables: $\text{SI}, v^2, q^2$

$^7\text{Be} \& ^8\text{B}$ neutrinos + convective radius + surface helium
Frequency separation ratios $r_{02}, r_{13}$

Notice $\chi^2$
**Best Model – $q^2$**

$q^2$ coupling
$q_0 = 40$ MeV
$m_\chi = 3$ GeV
$\sigma_0 = 10^{-37}$ cm$^2$

**Sound speed for best $q^2$ SI and SD models**

- Modelling error
- Helioseismology error
- Standard Solar Model
- Spin-Dependent ADM
- Spin-Independent ADM
- Momentum-dependent ADM
**Best Model – \( q^2 \)**

**Frequency separation ratios – zooming into the solar core**

![Graph showing frequency separation ratios for different models](image)

- BiSON data
- Standard Solar Model
- Spin-Dependent ADM
- Spin-Independent ADM
- Momentum-dependent ADM
* Energy extracted from core $M < 0.2 \, M_\odot$

* Deposited at intermediate range

* Core change in $T$-gradient $\rightarrow$ sound speed, frequencies, $\nu$-fluxes

* Smaller $T$-grad. change at $R_{CZ}$ $\rightarrow$ deeper convection
## BEST MODEL – $q^2$

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<tr>
<th></th>
<th>SSM</th>
<th>SD</th>
<th>SI</th>
<th>$q^2$ SI</th>
<th>Obs. a</th>
<th>$\sigma_{obs}$</th>
<th>$\sigma_{model}$</th>
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<tr>
<td>$\phi_{\nu}^8\mathrm{B}$ b</td>
<td>4.95</td>
<td>4.39</td>
<td>4.58</td>
<td>3.78</td>
<td>5.00</td>
<td>3%</td>
<td>14%</td>
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<tr>
<td>$\phi_{\nu}^7\mathrm{Be}$ c</td>
<td>4.71</td>
<td>4.58</td>
<td>4.62</td>
<td>4.29</td>
<td>4.82</td>
<td>5%</td>
<td>7%</td>
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<td>$R_{CZ}/R_{\odot}$</td>
<td>0.722</td>
<td>0.721</td>
<td>0.721</td>
<td>0.718</td>
<td>0.713</td>
<td>0.001</td>
<td>0.004</td>
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<td>$Y_s$</td>
<td>0.2356</td>
<td>0.2351</td>
<td>0.2353</td>
<td>0.2327</td>
<td>0.2485</td>
<td>0.0034</td>
<td>0.0035</td>
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<tr>
<td>$\chi^2_{8\mathrm{B}}$</td>
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<td>0.9</td>
<td>0.9</td>
<td>4.9</td>
<td></td>
<td></td>
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<tr>
<td>$\chi^2_{7\mathrm{Be}}$</td>
<td>0.1</td>
<td>0.4</td>
<td>0.4</td>
<td>1.9</td>
<td></td>
<td></td>
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<tr>
<td>$\chi^2_{R_{CZ}}$</td>
<td>4.8</td>
<td>3.8</td>
<td>3.8</td>
<td>1.5</td>
<td></td>
<td></td>
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<tr>
<td>$\chi^2_{Y_s}$</td>
<td>7.0</td>
<td>7.5</td>
<td>7.3</td>
<td>10.5</td>
<td></td>
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<tr>
<td>$\chi^2_{r_{02}}$</td>
<td>156.6</td>
<td>95.3</td>
<td>105.2</td>
<td>5.6</td>
<td></td>
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<tr>
<td>$\chi^2_{r_{13}}$</td>
<td>119.3</td>
<td>50.7</td>
<td>67.2</td>
<td>3.1</td>
<td></td>
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<tr>
<td>$\chi^2_{total}$</td>
<td>287.8</td>
<td>158.5</td>
<td>185.2</td>
<td>27.5</td>
<td></td>
<td></td>
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<tr>
<td>$p$</td>
<td>$&lt;10^{-10}$</td>
<td>$&lt;10^{-10}$</td>
<td>$&lt;10^{-10}$</td>
<td>0.845</td>
<td></td>
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**SUMMARY**

First part

* Recap on standard solar models and solar abundance problem
* relevance of CNO
* SSM with correct T-$\rho$ structure from data -- > phenomenological description of solar interior
* Extension to axion and hidden photons -- > new solar limits lower by x2

Second part

* Momentum exchange $q^2$ ADM models -- > agreement in solar data and models ($\sigma_0=10^{-37}$ cm$^2$, $m_\chi = 3$ GeV)
* Preferred mass and x-section range not excluded by direct experiment
* Caveat: evaporation not accounted for (will do)
EXTRA SLIDES
Helioseismology

Low degree modes; $l=0, 1, 2, 3$ – frequency separation ratios

$$
\begin{align*}
    r_{02} &= \frac{\nu_{n,0} - \nu_{n-1,2}}{\nu_{n,1} - \nu_{n-1,1}} \\
    r_{13} &= \frac{\nu_{n,1} - \nu_{n-1,3}}{\nu_{n+1,0} - \nu_{n,0}} \\
\end{align*}
\right)

\propto \int_0^R \frac{dc}{dr} \frac{dr}{r}

Frequency ratios: probing solar core
Extending the Method for Particle Studies

* Variation of neutrino fluxes

\[
\frac{\Phi(8B)}{\Phi_{SSM}(8B)} = \left( \frac{L_x + L_\odot}{L_\odot} \right)^\alpha
\]

\[\alpha = 4.4 \text{ (ax)}\]

\[\alpha = 5.7 \text{ (hp)}\]
Solar limits on axion-photon coupling

\[ \mathcal{L}_{a\gamma} = g_{a\gamma} B \cdot E_a \]
\[ g_{a\gamma} = g_{10} \times 10^{-10} \text{ GeV}^{-1} \]

Schlattl et al. 1999 – \( g_{10} < 10 \)
\( \text{Sound speed at } R = 0.1 \ R_\odot \) – equivalent to \( L_a < 0.2 \ L_\odot \)

Gondolo & Raffelt 2009 – \( g_{10} < 7 \)
\( ^8\text{B flux} < 1.5 \ ^8\text{B}_{SSM} \ (3-\sigma) \) – equivalent to \( L_a < 0.1 \ L_\odot \)

Maeda & Shibahashi 2013 – \( g_{10} < 2.5 \)
\( ^8\text{B flux constrained by sound speed (1-}\sigma) \)
seismic (not evolutionary models – neglect basic physics)

Vinyoles et al. 2015 – \( g_{10} < 4 \ (3-\sigma) \)
seismic + neutrino data
extend the method used to construct best-fit SSM
Testing the Method: Axions-Photon Coupling

Variations in sound speed without variations in composition and pulls

AGSS09 – low Z
GS98 – high Z
Extending the Method for Particle Studies

* Sound speed variations

Relative changes are similar regardless of solar composition
Testing the Method: Axions-Photon Coupling

Variations in other quantities without variations in composition and pulls

Changes due to axions and “zero point” of SSM to be accounted for by composition and systematics (pulls)
Testing the Method: Axions-Photon Coupling
Testing the Method: Axions-Photon Coupling

- **opa**
- **age**
- **lumi**
- **diffu**
- **s11**
- **s33**
- **s34**
- **sbe7**
- **s17**
- **shep**
- **sn14**

σmas
Recent Stellar Limits for $g_{\alpha}$

Ayala et al. 2014 – $g_{10} < 0.66$
- R parameter – HB/RGB stars – no syst. study of stellar uncertainties
- He-core burning is a tricky business in stellar evolution

Friedland et al. 2013 – $g_{10} < 0.8$
- blue loop of Cepheids – stellar calculations are plainly wrong!!
Energy losses dominated by the L-channel

\[ \epsilon_{hp} = \frac{\chi^2 m^2}{e^{\omega_P/T} - 1} \frac{\omega_P^3}{4\pi} \frac{1}{\rho} \sim \chi^2 m^2 T \]

No explicit composition dependence

Weak T-dependence --> broad production region
More relevant as T decreases (nucl. energy higher T-dependence)
Variations in sound speed over whole radiative interior

**Axions**
- $g_{10} = 0$
- $g_{10} = 2$
- $g_{10} = 4$
- $g_{10} = 6$
- $g_{10} = 8$

**Hidden Photons**
- $\chi_m = 0$
- $\chi_m = 1$
- $\chi_m = 2$
- $\chi_m = 3$
- $\chi_m = 4$
Hidden Photons

Depth of convective envelope more sensitive to hidden photons than axions
Hidden Photons

Our approach: solar model absorbs these variations without influencing boundaries derived for particle properties – e.g. increase metal abundances: freely or constrained by spectroscopy

Limits derived are based on forcing solar models (+ dark channel) to fit solar data as best as possible --- > limits then derived from irreducible residuals
$\chi m < 2$ @ 3-$\sigma$ C.L. -- improves previous limit by factor 2
**Comments on Solar Constraints**

Effective limit in dark channels

\[ L_{hp} < 2\% \ L_\odot \quad \Rightarrow \quad L_{\alpha \gamma} < 3\% \ L_\odot \]

\[ \frac{\Phi(8B)}{\Phi_{SSM}(8B)} = \left( \frac{L_x + L_\odot}{L_\odot} \right)^\alpha \]

Relations are not universal depend on the type of particle

\[ \alpha = 4.4 \ (ax) \]
\[ \alpha = 5.7 \ (hp) \]
Physical Motivation

Standard models – dominant term constant in DM-quarks interactions

\[ \chi \bar{\chi} Q \bar{Q} \rightarrow \sigma_{SI} \]
\[ \chi \gamma_\mu \gamma_5 \bar{\chi} Q \gamma^\mu \gamma_5 \bar{Q} \rightarrow \sigma_{SD} \]

Going beyond: non-zero particle radius, parity violation coupling, etc...

\[
\begin{align*}
(\bar{\chi} \gamma_5 \chi)(\bar{Q} Q) \\
(\bar{\chi} \chi)(\bar{Q} \gamma_5 Q) \\
(\bar{\chi} \gamma_5 \chi)(\bar{Q} \gamma_5 Q) \\
(\bar{\chi} \gamma_\mu \gamma_5 \chi)(\bar{Q} \gamma^\mu Q)
\end{align*}
\]

can lead to dependence on the transferred momentum

\[ \sigma_{\chi q} \propto q^n \]
DM-nucleon interaction with $q$ or $v_{\text{rel}}$ dependences

$$
\sigma = \sigma_0 \left( \frac{q}{q_0} \right)^{2n} \quad \sigma = \sigma_0 \left( \frac{v_{\text{rel}}}{v_0} \right)^{2n}
$$

$$
\sigma_{N,i} = \frac{m_{\text{nuc}}^2 (m_\chi + m_p)^2}{m_p^2 (m_\chi + m_{\text{nuc}})^2} \sigma_{\text{SI}} A_i^2 + \sigma_{\text{SD}} \frac{4(J_i + 1)}{3J_i} \left| \langle S_{p,i} \rangle + \langle S_{n,i} \rangle \right|^2
$$

SI – $A^2$ dependence $\rightarrow$ enhanced by metals sensitive to solar composition can be dominant

SD– couples mostly to H
Energy Transport by ADM

Dark matter number density

\[ n_{X,\text{LTE}}(r) = n_{X,\text{LTE}}(0) \left[ \frac{T(r)}{T(0)} \right]^{3/2} \exp \left[ -\int_0^r \frac{k_B\alpha(r')}{k_B T(r')} \frac{dT(r')}{dr'} + m_X \frac{d\phi(r')}{dr'} \right] \]

Dark matter conductive luminosity

\[ L_{X,\text{LTE}}(r) = 4\pi r^2 \zeta^2 n(r) \kappa(r) n_{X,\text{LTE}}(r) l_X(r) \left[ \frac{k_B T(r)}{m_X} \right]^{1/2} k_B \frac{dT(r)}{dr} \]

Energy injection rate

\[ \epsilon_{X,\text{LTE}}(r) = \frac{1}{4\pi r^2 \rho(r)} \frac{dL_{X,\text{LTE}}(r)}{dr} \]

Two limiting behavior: LTE & Isothermal
Intermediate: Knudsen regime \( \chi \sim r_\chi \) --> Boltzmann eq.
**Energy Transport by ADM**

**Dark matter number density**

\[ n_{\chi,\text{LTE}}(r) = n_{\chi,\text{LTE}}(0) \left[ \frac{T(r)}{T(0)} \right]^{3/2} \exp \left[ - \int_0^r \frac{k_B \alpha(r')}{k_B T(r')} \frac{dT(r')}{dr'} + m_{\chi} \frac{d\phi(r')}{dr'} \right] \]

**Dark matter conductive luminosity**

\[ L_{\chi,\text{LTE}}(r) = 4\pi r^2 \zeta^2 n(r) \kappa(r) n_{\chi,\text{LTE}}(r) l_{\chi}(r) \left[ \frac{k_B T(r)}{m_{\chi}} \right]^{1/2} k_B \frac{dT(r)}{dr} \]

**Energy injection rate**

\[ \epsilon_{\chi,\text{LTE}}(r) = \frac{1}{4\pi r^2 \rho(r)} \frac{dL_{\chi,\text{LTE}}(r)}{dr} \]

Two limiting behavior: LTE & Isothermal

Intermediate: Knudsen regime \( \frac{\chi}{r} \sim r_{\chi} \) --> Boltzmann eq.
Impact on Observables: SI, $v^{-2}$, $q^{-2}$

$^7$Be & $^8$B neutrinos + convective radius + surface helium
Frequency separation ratios $r_{02}$, $r_{13}$
**Impact on Observables: SI, $v^4$, $q^4$**

$^7\text{Be}$ & $^8\text{B}$ neutrinos + convective radius + surface helium

Frequency separation ratios $r_{02}$, $r_{13}$