Double beta decay matrix elements in IBM-2 with isospin restoration

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$0\nu\beta\beta$ and $2\nu\beta\beta$ nuclear matrix elements in the interacting boson model with isospin restoration

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We introduce a method for isospin restoration in the calculation of nuclear matrix elements (NMEs) for $0\nu\beta\beta$ and $2\nu\beta\beta$ decay within the framework of the microscopic interacting boson model (IBM-2). With this method, we calculate the NMEs for all processes of interest in $0\nu\beta^-\beta^-$ and $2\nu\beta^-\beta^-$ and in $0\nu\beta^+\beta^+$, $0\nu\mathrm{EC}\beta^+$, $R0\nu\mathrm{ECEC}$, $2\nu\beta^+\beta^+$, $2\nu\mathrm{EC}\beta^+$, and $2\nu\mathrm{ECEC}$. With this method, the Fermi matrix elements for $2\nu\beta\beta$ vanish, and those for $0\nu\beta\beta$ are considerably reduced.

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I. INTRODUCTION

The question of whether neutrinos are Majorana or Dirac particles, and of what are their masses and phases in the mixing matrix, remains one of the most important in physics today. A direct measurement of the average mass can be obtained from the observation of the neutrinoless double-β decay ($0\nu\beta\beta$)

$$ {}^{A}_{Z}X^{N} \rightarrow {}^{A}_{Z\pm2}Y_{N\mp2} + 2e^{\mp}, \tag{1}$$

two scenarios of which are shown in Fig. 1.

Several experiments are under way to detect this decay, and others are in the planning stage (for a review, see, e.g., Ref. [1]). The half-life for this decay can be written as

$$ \left[\tau^{0\nu}_{1/2}\right]^{-1} = G_{0\nu}|M_{0\nu}|^2|f(m_i,U_{ei})|^2, \tag{2}$$

where $G_{0\nu}$ is a phase-space factor, $M_{0\nu}$ is the nuclear matrix element, and $f(m_i,U_{ei})$ contains physics beyond the standard model through the masses m_i and the mixing matrix elements U_{ei} of neutrino species.

Concomitant with the neutrinoless modes, there is also the process allowed by the standard model, $2\nu\beta\beta$, depicted in Fig. 2. For this process, the half-life can be, to a good approximation, factorized in the form or approximately) into the product of a phase-space factor and a nuclear matrix element, which then are the crucial ingredients of any double-β decay calculation.

To extract physics beyond the standard model, contained in the function f in Eq. (2), we need an accurate calculation of both $G_{0\nu}$ and $M_{0\nu}$. These calculations will serve the purpose of extracting the neutrino mass $\langle m_\nu\rangle$ if $0\nu\beta\beta$ is observed and of guiding searches if $0\nu\beta\beta$ is not observed.

Recently we have started a systematic evaluation of phase-space factors (PSFs) and nuclear matrix elements (NMEs) for all processes of interest. The results for NMEs are presented in Refs. [3–7], and those for PSFs are presented in Refs. [2,7,8]. The calculations for the NMEs have been carried out within the framework of the microscopic interacting boson model (IBM-2).

Having completed the calculations in all nuclei of interest, we have now readdressed them with the purpose of providing as accurate as possible results. As shown in Table XV of Ref. [5], the Fermi matrix elements $M_F^{(2\nu)}$ for $2\nu\beta\beta$ decay in IBM-2 do not vanish in cases where protons and neutrons occupy the same major shell. Similarly, the Fermi matrix elements $M_F^{(0\nu)}$ for $0\nu\beta\beta$ decay are large when protons and neutrons are in the same major shell, as one can see from Table VII

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The IBM-2

- N_ν and N_π bosons are introduced for $2N_\nu$ valence protons and $2N_\pi$ valence neutrons, respectively.
- Bosons can be only in two states with positive parity

$$\begin{aligned} L = 0 &\implies s^\dagger_\nu, s^\dagger_\pi, \\ L = 2 &\implies d^\dagger_\nu, d^\dagger_\pi. \end{aligned}$$

- Bosons are allowed to interact with one and two body interactions.

The IBM-2 Hamiltonian

$$
\begin{aligned}
H &= \epsilon_{d_\pi} d_\pi^\dagger \cdot \tilde{d}_\pi + \epsilon_{d_\nu} d_\nu^\dagger \cdot \tilde{d}_\nu + \kappa Q_\pi \cdot Q_\nu + M + \omega_{\pi\pi} L_\pi \cdot L_\pi \\
&+ \frac{1}{2} \sum_{L=0,2,4} c_\pi^{(L)} \left(d_\pi^\dagger \times d_\pi^\dagger \right)^{(L)} \cdot \left(\tilde{d}_\pi \times \tilde{d}_\pi \right)^{(L)} + \omega_{\nu\nu} L_\nu \cdot L_\nu \\
&+ \frac{1}{2} \sum_{L=0,2,4} c_\nu^{(L)} \left(d_\nu^\dagger \times d_\nu^\dagger \right)^{(L)} \cdot \left(\tilde{d}_\nu \times \tilde{d}_\nu \right)^{(L)} + \omega_{\nu\pi} L_\nu \cdot L_\pi
\end{aligned}
$$

$$
Q_\rho = s_\rho^\dagger \tilde{d}_\rho + d_\rho^\dagger \tilde{s}_\rho + \chi_\rho \left(d_\rho^\dagger \times \tilde{d}_\rho \right)^{(2)}, \quad L_\rho = \sqrt{10} \left(d_\rho^\dagger \times \tilde{d}_\rho \right)^{(1)}, \quad \rho = \pi, \nu
$$

$$
M = \frac{1}{2} \xi_2 \left(d_\pi^\dagger s_\nu^\dagger - d_\nu^\dagger s_\pi^\dagger \right) \cdot \left(\tilde{d}_\pi s_\nu - \tilde{d}_\nu s_\pi \right) - \sum_{K=1,3} \xi_K \left(d_\nu^\dagger \times d_\nu^\dagger \right)^{(K)} \cdot \left(\tilde{d}_\nu \times \tilde{d}_\nu \right)^{(K)}
$$

IBM-2 and isospin

- For heavy nuclei the valence protons occupy orbits full of neutrons

$$T_{-} |\psi\rangle = 0 \Rightarrow T = |M_T|_{\mathrm{max}} = \frac{1}{2} |N - Z|$$

- For nuclei where protons and neutrons are in the same major shell, but with different character as particles or holes, isospin symmetry is violated only of order

$$\frac{1}{\Omega}$$

- For the rest of the cases (lighter nuclei) IBM-2 does not produce states with definite isospin

$$\Longrightarrow \text{IBM-3 and IBM-4 are required}$$

Source: J. P. Elliot, Prog. Part. Nucl. Phys. **25**, 325 (1990)

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Nuclear Matrix Elements

$$\left[\tau_{1/2}^{(0\nu)}\right]^{-1} = G_{0\nu}\left|M_{0\nu}\right|^2\left|f(m_i, U_{ei})\right|^2 \quad , \quad \left[\tau_{1/2}^{(2\nu)}\right]^{-1} = G_{2\nu}\left|m_e c^2 M_{2\nu}\right|^2 ,$$

$$M_{0\nu} = \left\langle F; J_F \left| -h_{0\nu}^F + h_{0\nu}^{GT} + h_{0\nu}^T \right| I; 0_1^+ \right\rangle$$

$$M_{2\nu} = \left\langle F; J_F \left| -h_{2\nu}^F + h_{2\nu}^{GT} \right| I; 0_1^+ \right\rangle$$

$$h_X^{F,GT,T} = -\frac{1}{4}\sum_{\alpha_\pi \alpha'_\pi}\sum_{\alpha_\nu \alpha'_\nu}\sum_J (-1)^J \, G_X^{F,GT,T}\left(\alpha_\pi \alpha'_\pi \alpha_\nu \alpha'_\nu; J\right)$$

$$\sqrt{1+(-1)^J \delta_{\alpha_\pi \alpha'_\pi}}\sqrt{1+(-1)^J \delta_{\alpha_\nu \alpha'_\nu}}$$

$$\left(\pi_{\alpha_\pi}^\dagger \times \pi_{\alpha'_\pi}^\dagger\right)^{(J)} \cdot \left(\tilde{\nu}_{\alpha_\nu} \times \tilde{\nu}_{\alpha'_\nu}\right)^{(J)}$$

$$\alpha_\rho = (n_\rho l_\rho j_\rho)\,,\ \rho = \nu, \pi,\ X = 0\nu, 0\nu_h, 2\nu$$

Source: J. Barea and F. Iachello, Phys. Rev. C **79**, 044301 (2009)

A couple of comments

- The closure approximation have been avoided in computing 2ν NMEs in IBM:

  N. Yoshida and F. Iachello,
  Prog. Theor. Exp. Phys. **2013** 043D01

- IBM is not restricted to just one major shell: the number of single particle levels can be extended to include spin-orbit partners in the formulation.

Two body matrix elements I

$$G_X^{F,GT,T}\left(\alpha_\pi \alpha'_\pi \alpha_\nu \alpha'_\nu; J\right) = \left\langle \alpha_\pi \alpha'_\pi; JM \left| h_X^{F,GT,T} \right| \alpha_\nu \alpha'_\nu; JM \right\rangle$$

$$\begin{aligned} h_X^{F,GT,T} &\equiv h_X^{(s_1,s_2,\lambda)} \\ &= \frac{1}{2}\sum_{n,n'} \tau_n^+ \tau_{n'}^+ \left(\Sigma_n^{(s_1)} \times \Sigma_{n'}^{(s_2)}\right)^{(\lambda)} \cdot H_X(r_{nn'}) C^{(\lambda)}(\Omega_{nn'}), \end{aligned}$$

$$h^F \to h_X^{(0,0,0)}, \ \ h^{GT} \to h_X^{(1,1,0)}, \ \ h^T \to h_{0\nu}^{(1,1,2)}$$

$$\Sigma_n^{(0)} = 1, \ \ \Sigma_n^{(1)} = \vec{\sigma}_n, \ \ C^{(\lambda)}(\Omega) = \sqrt{4\pi/\left(2\lambda+1\right)}\, Y^{(\lambda)}(\Omega)$$

Two body matrix elements II

$$G_X^{F,GT,T}(\alpha_\pi \alpha'_\pi \alpha_\nu \alpha'_\nu; J; J) \equiv G_X^{(s_1,s_2,\lambda)}(\alpha_\pi \alpha'_\pi \alpha_\nu \alpha'_\nu; J; J) =$$

$$\sum_{k_1=|l_\pi - l_\nu|}^{l_\pi + l_\nu} \sum_{k_2=|l'_\pi - l'_\nu|}^{l'_\pi + l'_\nu} \sum_{k=k_{min}}^{k_{max}} i^{k_1 - k_2 + \lambda} \hat{k}_1^2 \hat{k}_2^2 \langle k_1 0\, k_2 0 \mid \lambda 0 \rangle$$
$$\times (-1)^{s_2 + k_1} \begin{Bmatrix} k_1 & s_1 & k \\ s_2 & k_2 & \lambda \end{Bmatrix} (-1)^{j'_\pi + j_\nu + J} \begin{Bmatrix} j_\pi & j'_\pi & J \\ j'_\nu & j_\nu & k \end{Bmatrix}$$
$$\times \hat{k} \hat{j}_\pi \hat{j}_\nu \begin{Bmatrix} \frac{1}{2} & l_\pi & j_\pi \\ \frac{1}{2} & l_\nu & j_\nu \\ s_1 & k_1 & k \end{Bmatrix} \hat{k} \hat{j}'_\pi \hat{j}'_\nu \begin{Bmatrix} \frac{1}{2} & l'_\pi & j'_\pi \\ \frac{1}{2} & l'_\nu & j'_\nu \\ s_2 & k_2 & k \end{Bmatrix}$$
$$\times \left\langle \tfrac{1}{2} \left\| \Sigma^{(s_1)} \right\| \tfrac{1}{2} \right\rangle (-1)^{-k_1} \hat{l}_\pi \langle l_\pi 0\, k_1 0 \mid l_\nu 0 \rangle$$
$$\times \left\langle \tfrac{1}{2} \left\| \Sigma^{(s_2)} \right\| \tfrac{1}{2} \right\rangle (-1)^{-k_2} \hat{l}'_\pi \langle l'_\pi 0\, k_2 0 \mid l'_\nu 0 \rangle$$
$$\times R_{X,k_1,k_2}^{(s_1,s_2,\lambda)}(n_\pi, l_\pi, n'_\pi, l'_\pi, n_\nu, l_\nu, n'_\nu, l'_\nu),$$

Radial Integrals

$$R^{(s_1,s_2,\lambda)}_{X,k_1,k_2}(n_\pi, l_\pi, n'_\pi, l'_\pi, n_\nu, l_\nu, n'_\nu, l'_\nu) =$$

$$\int_0^\infty h^{(s_1,s_2,\lambda)}_X(p)\, p^2 dp \quad \times \quad \int_0^\infty R_{n_\pi l_\pi}(r_1)\, R_{n_\nu l_\nu}(r_1)\, j_{k_1}(pr_1)\, r_1^2 dr_1$$

$$\times \quad \int_0^\infty R_{n'_\pi l'_\pi}(r_2)\, R_{n'_\nu l'_\nu}(r_2)\, j_{k_2}(pr_2)\, r_2^2 dr_2$$

$$h^{(s_1,s_2,\lambda)}_X(p) = v_X(p)\, \underbrace{\tilde{h}^{(s_1,s_2,\lambda)}(p)}_{\substack{+\text{FNS}\\ +\text{SRC}\\ +\text{HOC}}};$$

$$v_X(p) = \begin{cases} \frac{\delta(p)}{p^2} & \text{for } 2\nu \\ \frac{2}{\pi}\frac{1}{p\left(p+\tilde{A}\right)} & \text{for light } 0\nu \\ \frac{2}{\pi}\frac{1}{m_e m_p} & \text{for heavy } 0\nu \end{cases}$$

The transition operator in IBM-2

$$h_X^{F,GT,T} = h_{X,ss}^{F,GT,T}\; s_\pi^\dagger \cdot \tilde{s}_\nu + h_{X,dd}^{F,GT,T}\; d_\pi^\dagger \cdot \tilde{d}_\nu$$

Expansion coefficients

$$\begin{aligned} h_{X,ss}^{F,GT,T} &= -\sum_{j_\pi}\sum_{j_\nu} G_X^{F,GT,T}\left(\alpha_\pi\alpha'_\pi\alpha_\nu\alpha'_\nu; J=0\right) A_{j_\pi}^{(01)}\, \tilde{A}_{j_\nu}^{(01)} \\ h_{X,dd}^{F,GT,T} &= -\frac{1}{2}\sum_{j_\pi j'_\pi}\sum_{j_\nu j'_\nu}\sqrt{1+\delta_{j_\pi j'_\pi}}\sqrt{1+\delta_{j_\nu j'_\nu}} \\ &\times\; G_X^{F,GT,T}\left(\alpha_\pi\alpha'_\pi\alpha_\nu\alpha'_\nu; J=2\right) B_{j_\pi j'_\pi}^{(01)}\, \tilde{B}_{j_\nu j'_\nu}^{(01)} \end{aligned}$$

Isospin correction of the transition operator

Monopole term removed

$$R^{(s_1,s_2,\lambda)}_{X,k_1,k_2} \rightarrow R^{(s_1,s_2,\lambda)}_{X,k_1,k_2} - \delta_{k_1 0}\delta_{k_2 0}\delta_{k 0}\delta_{\lambda 0}\delta_{\alpha_\pi \alpha_\nu}\delta_{\alpha'_\pi \alpha'_\nu} R^{(s_1,s_2,0)}_{X,0,0}$$

Consequences:

$M^F_{2\nu} \sim 0$ and $M^F_{0\nu}$ are strongly reduced

$M^{GT}_{2\nu}$ and $M^{GT,T}_{0\nu}$ does not change

J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 87, 014315 (2015)

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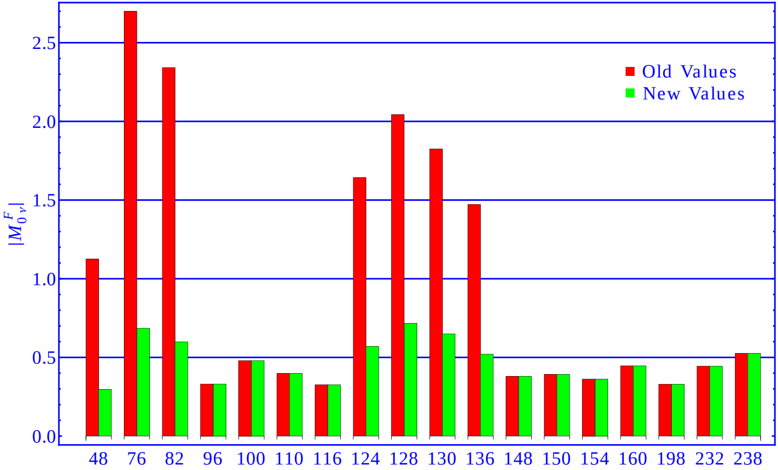
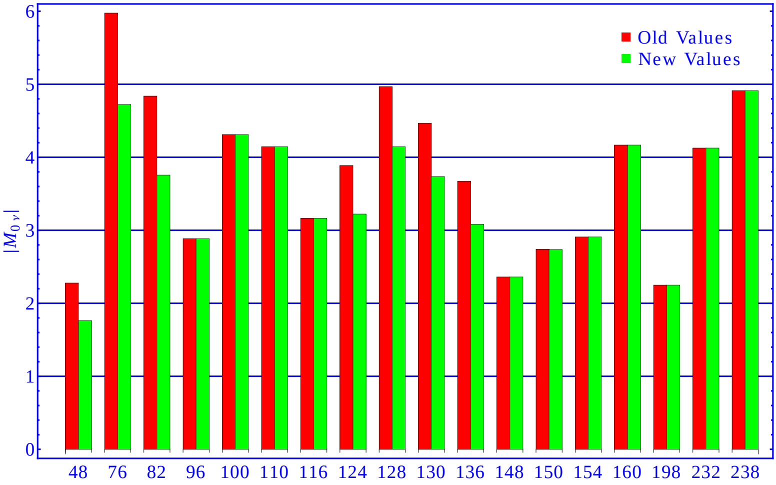
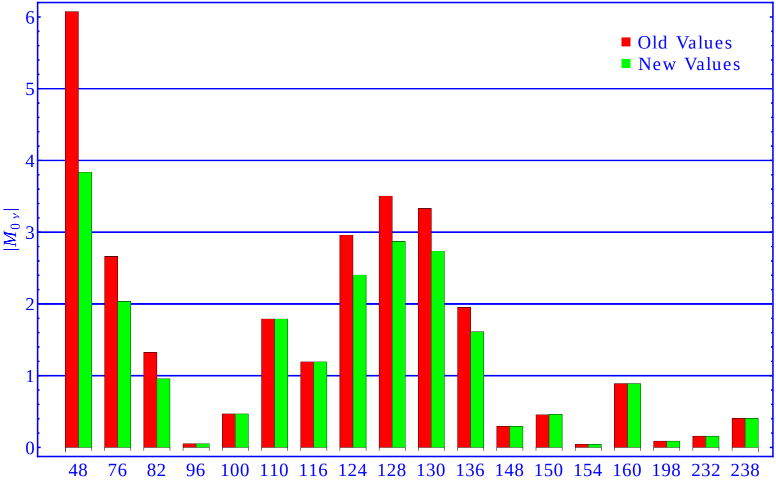
Fermi NMEs for $\beta^-\beta^-$ (0ν) to the ground state



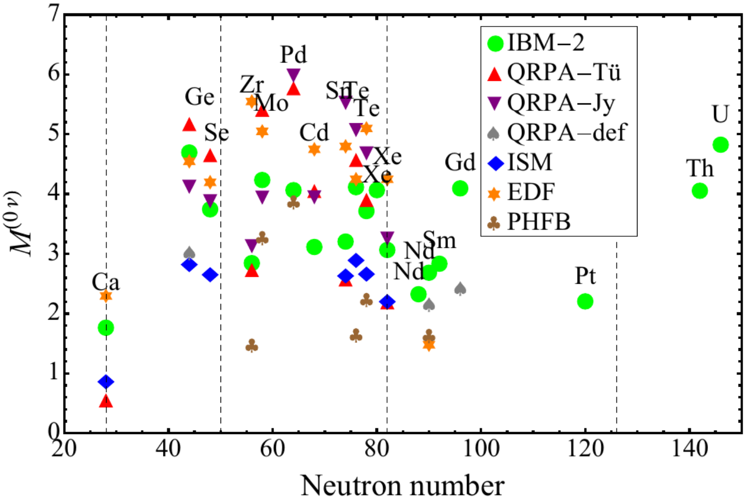
NMEs for $\beta^-\beta^-$ (0ν) to the ground state



NMEs for $\beta^-\beta^-$ (0ν) to the first excited state



Comparison with other approaches



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Conclusions

- We have cancelled 2ν Fermi NMEs in those cases where isospin was broken and the 0ν Fermi NMEs was strongly reduced.
- The Gamow-Teller and Tensor NMEs remain with the same values.
- The same effects are present in the NMEs for $0\nu\beta^+\beta^+$ and the other channels: $\{0,2\}\,\nu EC\beta$, $R0\nu ECEC$ and $2\nu ECEC$.
- These effects are less pronounced in the NME for heavy neutrino exchange.

THANK YOU FOR YOUR ATTENTION

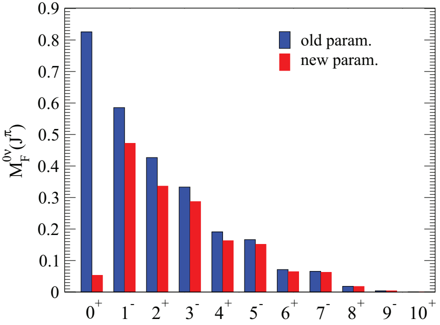


FIG. 4. (Color online) Multipole decomposition of the matrix element $M_F^{0\nu}$. The results with the old and new parametrizations are compared. Note the dominant effect for the 0^+ multipole and the relatively small effects for the other multipoles. This is the case of ^{76}Ge.

Source: F. Šimkovic *et al*, Phys. Rev. **C** 87, 045501 (2013)

Short Range Correlations

In momentum space

$$h^{F,GT}(p) = \frac{2}{\pi}\int_0^\infty j_0(pr)H^{F,GT}(r)r^2dr$$
$$h^{T}(p) = \frac{2}{\pi}\int_0^\infty j_2(pr)H^{T}(r)r^2dr$$

Jastrow function

$$H^{F,GT,T}(r) \to H^{F,GT,T}(r)f^2(r)$$

$$f(r) = 1 - Ce^{-Ar^2}\left(1 - Br^2\right)$$

SRC Parametrizations

Name	A (fm^{-2})	B (fm^{-2})	C
Miller-Spencer	1.10	0.68	1.00
Argonne	1.59	1.45	0.92
CD-Bonn	1.52	1.88	0.46

Source: F. Šimkovic *et al*, Phys. Rev. **C** 79, 055501 (2009)

HOC terms I

$$
\begin{aligned}
h &= \sum_{n,n'} \tau_n^\dagger \tau_{n'}^\dagger \left[-h^F(p) + h^{GT}(p) \vec{\sigma}_n \cdot \vec{\sigma}_{n'} + h^T(p) S^p_{nn'} \right] \\
S^p_{nn'} &= 3 \left[(\vec{\sigma}_n \cdot \hat{p}) (\vec{\sigma}_{n'} \cdot \hat{p}) \right] - \vec{\sigma}_n \cdot \vec{\sigma}_{n'} \\
h^{F,GT,T}(p) &= v(p) \tilde{h}^{F,GT,T}(p); \;\; v(p) = \begin{cases} \frac{2}{\pi} \frac{1}{p(p+\tilde{A})} & \text{for light } \nu \\ \frac{2}{\pi} \frac{1}{m_e m_p} & \text{for heavy } \nu \end{cases}
\end{aligned}
$$

HOC terms II

HOC term	$\tilde{h}(p)$
$\tilde{h}^{F}_{VV}$	$g_A^2 \frac{g_V^2/g_A^2}{\left(1+p^2/M_V^2\right)^4}$
$\tilde{h}^{GT}_{AA}$	$g_A^2 \frac{1}{\left(1+p^2/M_A^2\right)^4}$
$\tilde{h}^{GT}_{AP}$	$g_A^2 \left[-\frac{2}{3}\frac{1}{\left(1+p^2/M_A^2\right)^4}\frac{p^2}{p^2+m_\pi^2}\left(1-\frac{m_\pi^2}{M_A^2}\right)\right]^2$
$\tilde{h}^{GT}_{PP}$	$g_A^2 \left[\frac{1}{\sqrt{3}}\frac{1}{\left(1+p^2/M_A^2\right)^2}\frac{p^2}{p^2+m_\pi^2}\left(1-\frac{m_\pi^2}{M_A^2}\right)\right]$
$\tilde{h}^{GT}_{MM}$	$g_A^2 \left[\frac{2}{3}\frac{g_V^2}{g_A^2}\frac{1}{\left(1+p^2/M_V^2\right)^4}\frac{\kappa_\beta^2 p^2}{4m_p^2}\right]$
$\tilde{h}^{T}_{AP}$	$-h^{GT}_{AP}$
$\tilde{h}^{T}_{PP}$	$-h^{GT}_{PP}$
$\tilde{h}^{T}_{MM}$	$\frac{1}{2}h^{GT}_{MM}$

Closure Approximation

Example: $M_{GT}^{(2\nu)}$

$$\sum_N \frac{\langle F\,|\tau^+\vec{\sigma}|\,N\rangle\,\langle N\,|\tau^+\vec{\sigma}|\,I\rangle}{\frac{1}{2}Q_{\beta\beta}+m_e c^2+E_N-E_I} \longrightarrow \frac{\langle F\,|\tau^+\tau^+\vec{\sigma}\cdot\vec{\sigma}|\,I\rangle}{\frac{1}{2}Q_{\beta\beta}+m_e c^2+\langle E_N\rangle-E_I}$$

Boson expansion

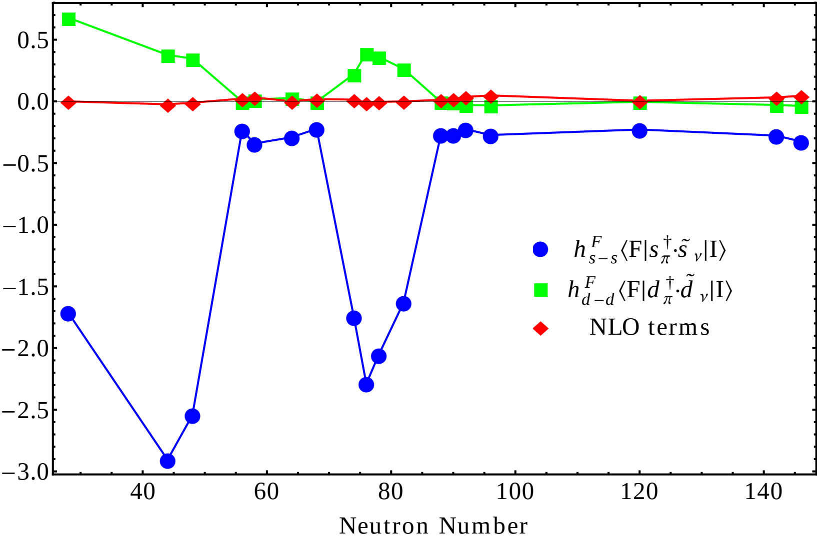
Boson expansion of the coupled pairs operators

$$
\begin{aligned}
\left(\pi^\dagger_{j_\pi} \times \pi^\dagger_{j_\pi}\right)^{(0)} &\longmapsto A^{(01)}_{j_\pi} s^\dagger_\pi + A^{(11)}_{j_\pi} s^\dagger_\pi \left(d^\dagger_\pi \tilde{d}_\pi\right)^{(0)} + \ldots \\
\left(\pi^\dagger_{j_\pi} \times \pi^\dagger_{j'_\pi}\right)^{(2)} &\longmapsto B^{(01)}_{j_\pi j'_\pi} d^\dagger_\pi \\
&+ \quad B^{(11)}_{j_\pi j'_\pi} s^\dagger_\pi \left(s^\dagger_\pi \tilde{d}_\pi\right)^{(2)} + B^{(12)}_{j_\pi j'_\pi} s^\dagger_\pi \left(d^\dagger_\pi \tilde{d}_\pi\right)^{(2)} \\
&+ \quad \ldots \\
\left(\tilde{\nu}_{j_\nu} \times \tilde{\nu}_{j_\nu}\right)^{(0)} &\longmapsto \tilde{A}^{(01)}_{j_\nu} \tilde{s}_\nu + \tilde{A}^{(11)}_{j_\nu} \tilde{s}_\nu \left(d^\dagger_\nu \tilde{d}_\nu\right)^{(0)} + \ldots \\
\left(\tilde{\nu}_{j_\nu} \times \tilde{\nu}_{j'_\nu}\right)^{(2)} &\longmapsto \tilde{B}^{(01)}_{j_\nu j'_\nu} \tilde{d}_\nu \\
&+ \quad \tilde{B}^{(11)}_{j_\nu j'_\nu} \left(d^\dagger_\nu \tilde{s}_\nu\right)^{(2)} \tilde{s}_\nu + \tilde{B}^{(12)}_{j_\nu j'_\nu} \left(d^\dagger_\nu \tilde{d}_\nu\right)^{(2)} \tilde{s}_\nu \\
&+ \quad \ldots
\end{aligned}
$$

Transition operator in IBM-2

$$
\begin{aligned}
h^{F,GT,T} \quad \longmapsto \quad & h^{F,GT,T}_{s-s}\; s^\dagger_\pi \cdot \tilde{s}_\nu + h^{F,GT,T}_{d-d}\; d^\dagger_\pi \cdot \tilde{d}_\nu \\
+ \quad & h^{F,GT,T}_{d-dss}\; d^\dagger_\pi \cdot d^\dagger_\nu \tilde{s}_\nu \tilde{s}_\nu + h^{F,GT,T}_{d-dds}\; d^\dagger_\pi \cdot \left(d^\dagger_\nu \tilde{d}_\nu\right)^{(2)} \tilde{s}_\nu \\
+ \quad & h^{F,GT,T}_{ssd-d}\; s^\dagger_\pi s^\dagger_\pi \tilde{d}_\pi \cdot \tilde{d}_\nu + h^{F,GT,T}_{sdd-d}\; s^\dagger_\pi \left(d^\dagger_\pi \tilde{d}_\pi\right)^{(2)} \cdot \tilde{d}_\nu \\
+ \quad & h^{F,GT,T}_{ssd-dss}\; s^\dagger_\pi s^\dagger_\pi \tilde{d}_\pi \cdot d^\dagger_\nu \tilde{s}_\nu \tilde{s}_\nu \\
+ \quad & h^{F,GT,T}_{ssd-dds}\; s^\dagger_\pi s^\dagger_\pi \tilde{d}_\pi \cdot \left(d^\dagger_\nu \tilde{d}_\nu\right)^{(2)} \tilde{s}_\nu \\
+ \quad & h^{F,GT,T}_{sdd-dss}\; s^\dagger_\pi \left(d^\dagger_\pi \tilde{d}_\pi\right)^{(2)} \cdot d^\dagger_\nu \tilde{s}_\nu \tilde{s}_\nu \\
+ \quad & h^{F,GT,T}_{sdd-dds}\; s^\dagger_\pi \left(d^\dagger_\pi \tilde{d}_\pi\right)^{(2)} \cdot \left(d^\dagger_\nu \tilde{d}_\nu\right)^{(2)} \tilde{s}_\nu \\
+ \quad & \ldots
\end{aligned}
$$

Different orders in the boson expansion of the Fermi NME



S and D pairs

Calculation of α_j and $\beta_{jj'}$

$$S^\dagger = \sum_j \alpha_j \sqrt{\frac{\Omega_j}{2}} \left(c_j^\dagger \times c_j^\dagger \right)^{(0)}, \quad D^\dagger = \sum_{j \leq j'} \beta_{jj'} \frac{1}{\sqrt{1+\delta_{jj'}}} \left(c_j^\dagger \times c_{j'}^\dagger \right)^{(2)}$$

$$H_{SDI} = \sum_j \varepsilon_j + A_T V_{SDI}$$

- Single particle energies ε_j taken from spectra of nuclei with one nucleon of valence
- A_T fitted to reproduce the $2_1^+ - 0_{gs}^+$ in nuclei with 2 nucleons of valence.
- α_j and $\beta_{jj'}$ are extracted from the lowest 0^+ and 2^+ states obtained diagonalizing H_{SDI}.

S. Pittel, P.D. Duval, B.R. Barret, Ann. Phys. **144**, 168 (1982).