Formal and interdisciplinary aspects of neutrino flavour conversion in astrophysical environments

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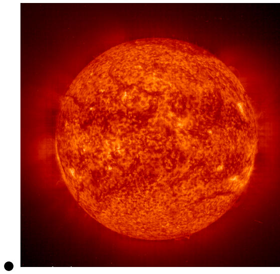
Outline

- Supernova neutrinos and ν flavour conversion
- Extended neutrino evolution equations
- Conclusions

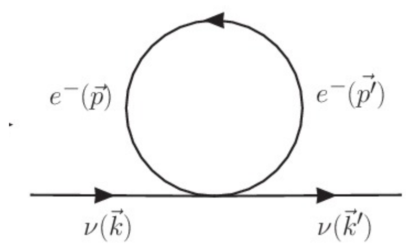
Solar neutrinos



The proton-proton (pp) fusion reaction chain produces 99% of solar energy transforming H into ^{4}He.

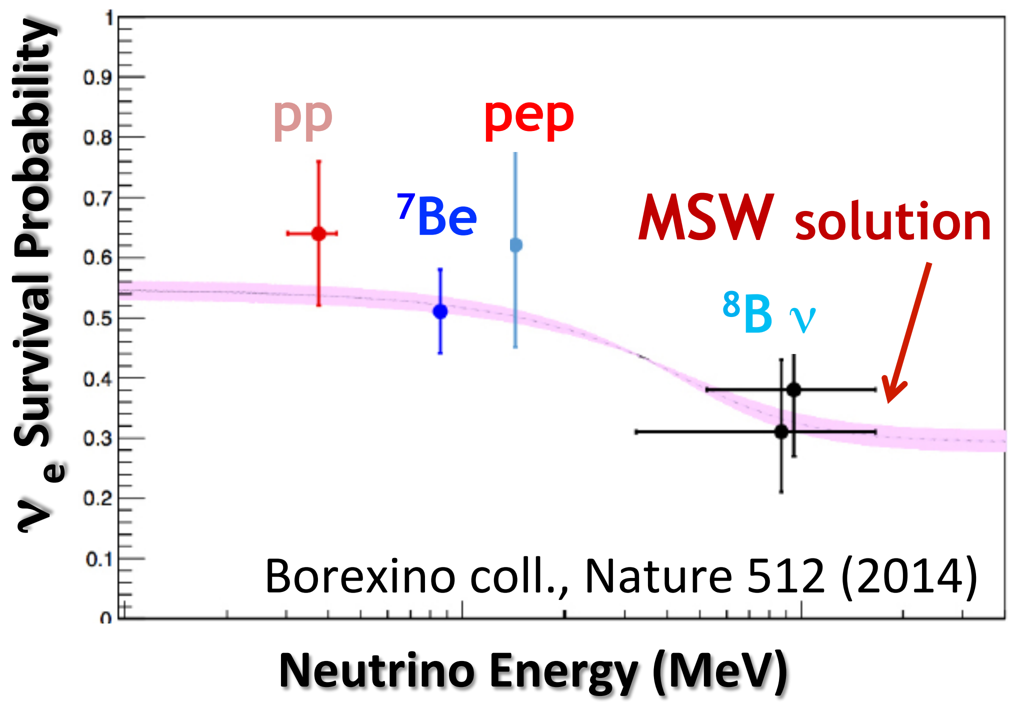
MSW effect : resonant flavour conversion due to ν-matter interaction

Wolfenstein PRD (1978)
Mikheev, Smirnov(1985)



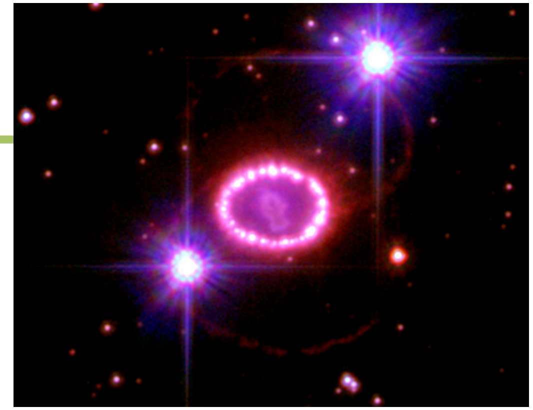
mean-field

$$\Gamma_{\nu_e}(\rho_e) = \sqrt{2} G_F \; \rho_e$$



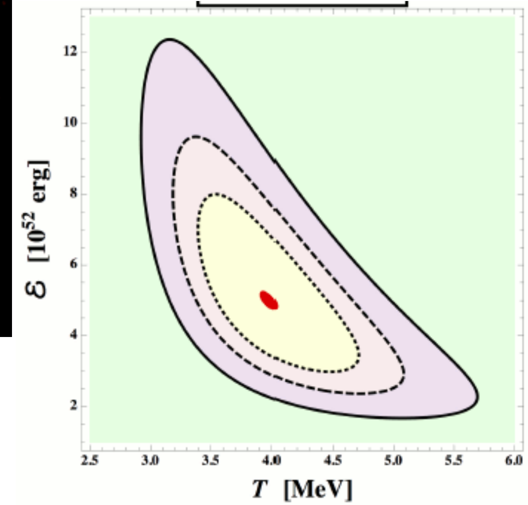
pp neutrinos - keystone fusion reaction - measured

Supernova ν



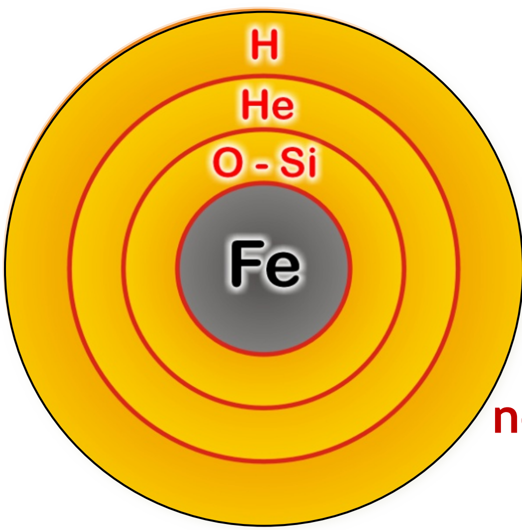
SN1987A (LMC)

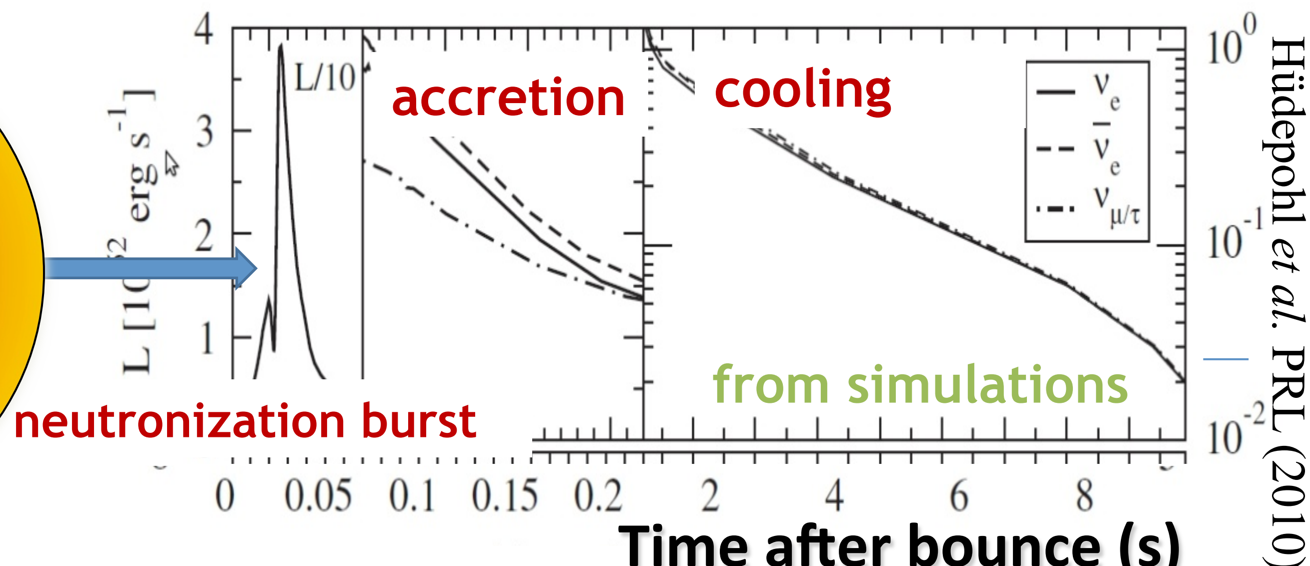
Vissani, JPG (2014)



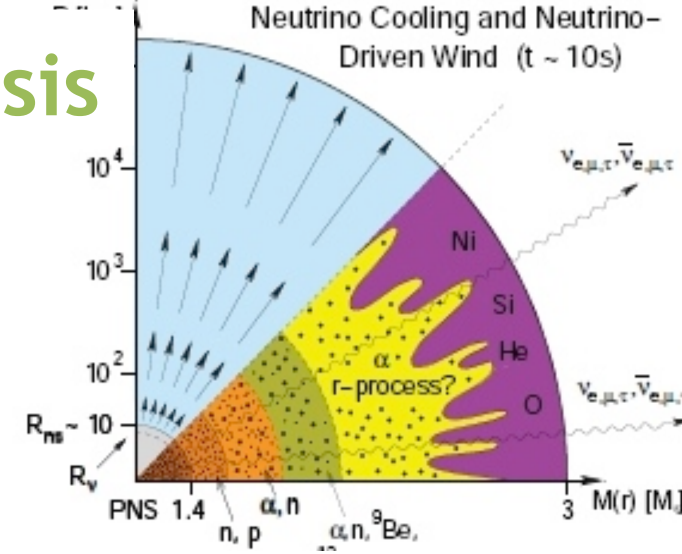
These comprise lighter massive stars (O-Ne-Mg), iron-core stars to collapsars (AD-BH).

10^{58} ν of about 10 MeV in 10 s from the gravitational collapse of massive stars.





SN explosions & nucleosynthesis



Supernova neutrinos linked to key issues :

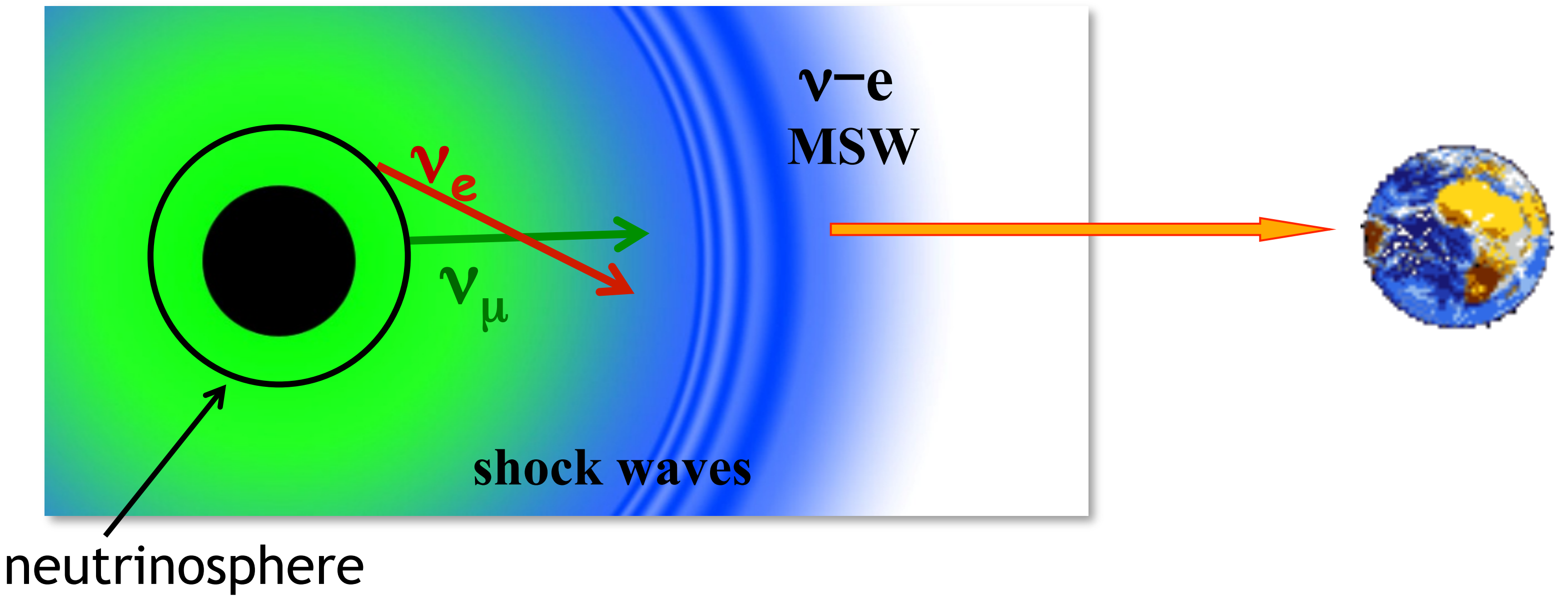
- ❑ **How do massive stars explode ?**
  Current simulations :
  multidimensional, realistic ν transport,
  convection and turbulence, hydrodynamical instabilities (SASI).

- ❑ **What are the sites where heavy elements are made ?**
  Candidate sites for heavy elements nucleosynthesis :
  supernovae, AD-BH, neutron star mergers.

  Neutrinos determine if the site is neutron-rich
  (r-process elements) or proton-rich (νp-process).

see e.g. Focus issue «Nucleosynthesis and the role of neutrinos»,
J.Phys. G 41 (2014)

Neutrinos flavour conversion in supernovae



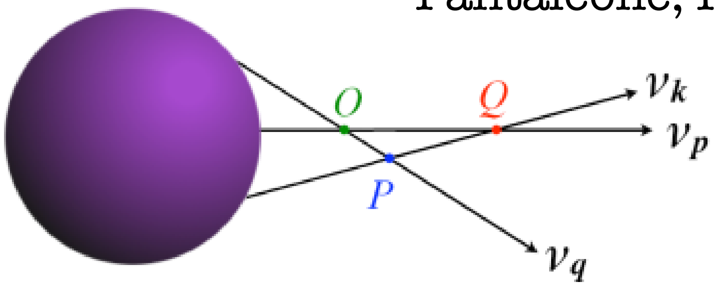
Numerous aspects investigated, different from the Sun.
In particular :

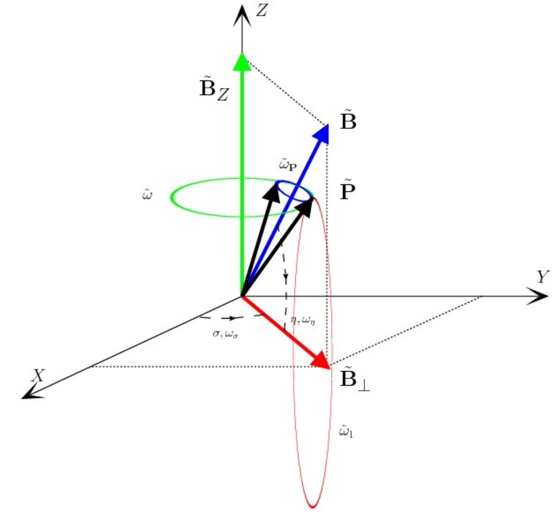
- ❑ **the ν interaction with ν** and **with matter** (MSW effect).
- ❑ dynamical aspects - **shock waves and turbulence**.

Important progress, key open questions

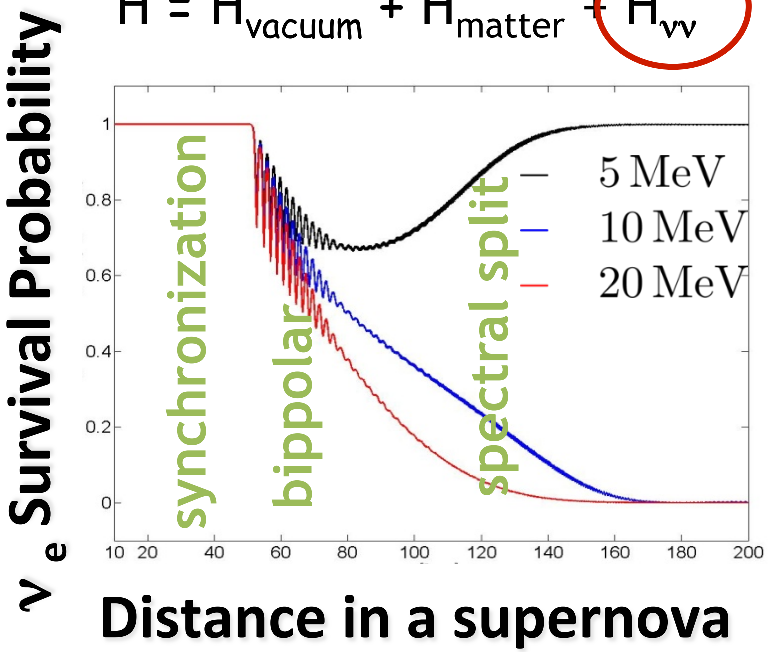
The νν interaction effects

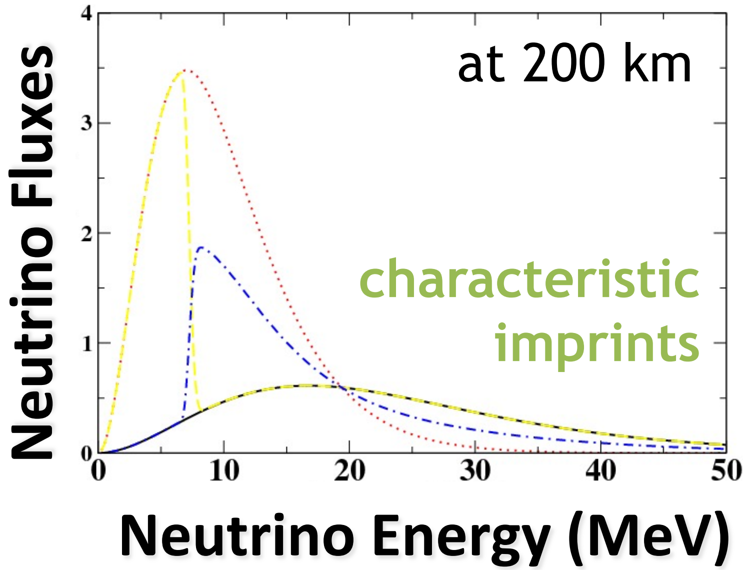
Pantaleone, PLB 287 (1992)





$$H = H_{vacuum} + H_{matter} + H_{\nu\nu}$$





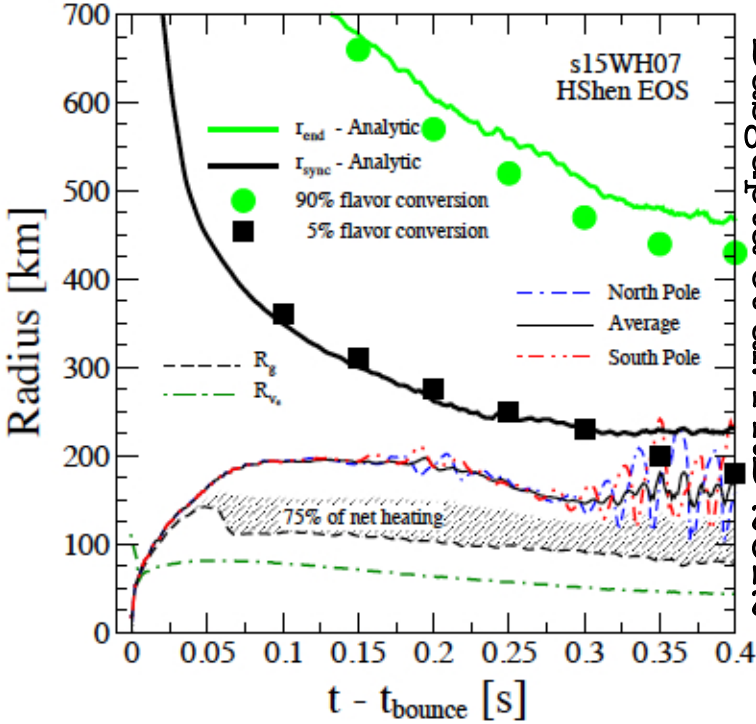
collective stable and unstable ν modes in flavor space

see e.g. Duan, Fuller, Qian, Ann. Rev.. 60 (2010)

key open questions

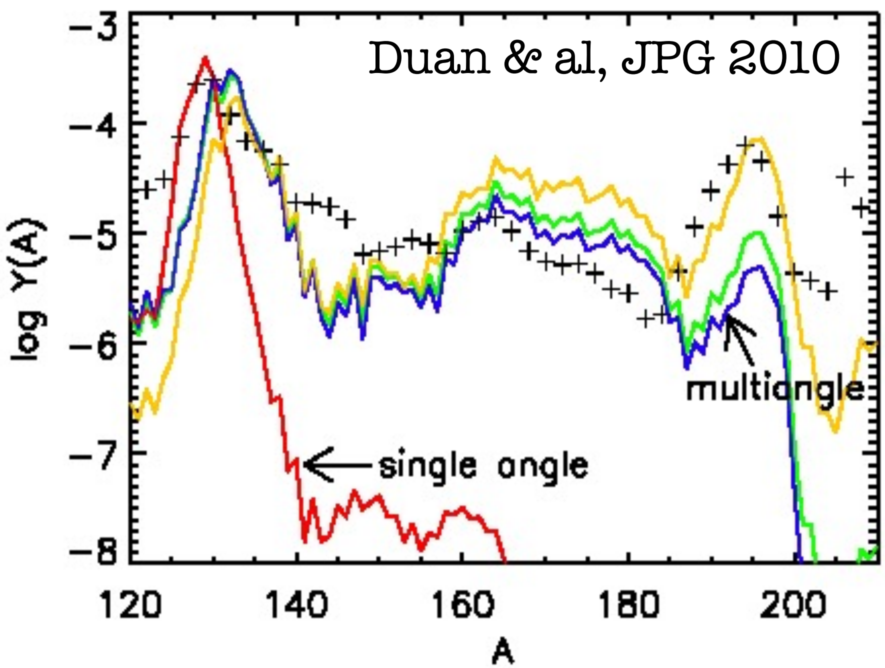
Further work needed to finally assess the impact of flavour conversion

- ❑ on the shock wave



Dasgupta et al. PRD 2012

- ❑ on nucleosynthesis (r-process, vp process)



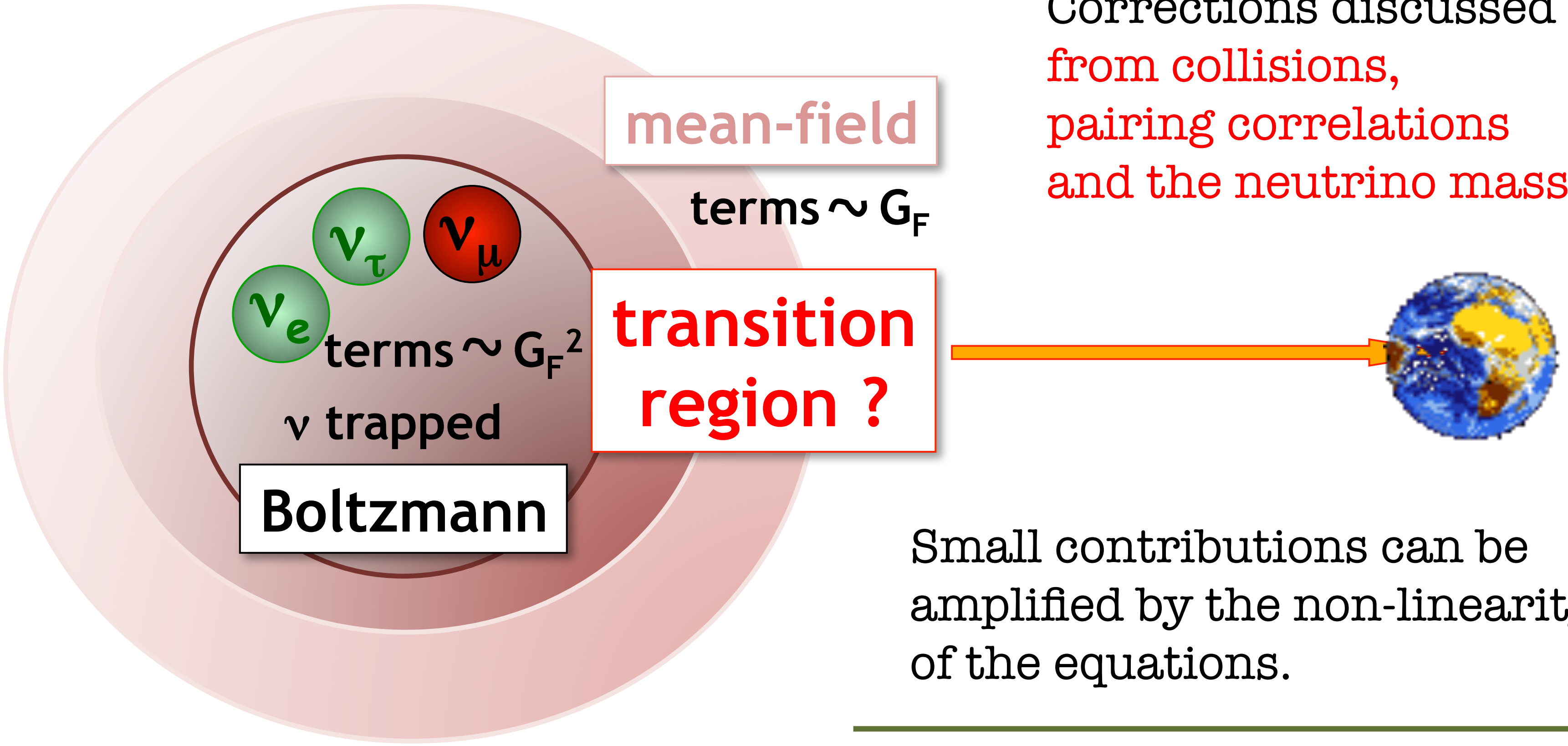
- ❑ role of decoherence, or of symmetry breaking

  see e.g. Chakraborty, et al. PRL 107 (2011), Raffelt et al. PRL111(2013)

Theoretical description of ν evolution

Evolution equations are based on :

- ❑ mean-field approximation
- ❑ extended mean-field approximation
- ❑ Boltzmann approximation («molecular» chaos assumption)



Corrections discussed :
from collisions,
pairing correlations
and the neutrino mass

Small contributions can be amplified by the non-linearity of the equations.

Theoretical description of ν evolution

Different approaches used to derive evolution equations :

- density matrix formalism, perturbative expansion of interaction terms

  Dolgov, Sov. J. (1981), Stodolsky, Sigl, Raffelt, PRL 51993),
  Sigl, Raffelt, Nucl. Phys. B406 (1993), McKellar, Thomson PRD49 (1994),...

- Born-Bogoliubov-Green-Kirkwood-Yvon (BBGKY) hierarchy

  Volpe, Väänänen, Espinoza, PRD87 (2013)
  – ν-antiν pairing correlations, mass contributions.

- neutrino (iso)spins and precession equations

  see e.g. Balantekin, Pehlivan, Kajino, PRD84(2011), PRD90(2014).

- path integral approach

  Balantekin, Pehlivan, JPG 34 (2007)

- Green's functions, Closed-Time-Path and 2PI effective action.

  Yamada, PRD 62 (2000),
  Vlasenko, Fuller, Cirigliano, PRD89 (2014) – spin coherence
  see also Herranen et al, JHEP 1012 (2010), Fidler et al JHEP 1202 (2012)

Volpe, to be submitted to IJMPE.

Extended ν equations for astrophysical media

Serreau and Volpe, PRD 90 (2014), arXiv:1409.3591

General mean-field equations to describe neutrino propagation in an inhomogeneous medium, for massive Dirac (or Majorana) neutrinos.

The most general mean-field Hamiltonian takes the bilinear form :

$$H_{\text{eff}}(t) = \int d^3x\, \bar{\psi}_i(t,\vec{x})\Gamma_{ij}(t,\vec{x})\psi_j(t,\vec{x}),$$

All possible two-point correlators are :

NORMAL DENSITIES

$$\rho_{ij}(t,\vec{q},h,\vec{q}',h') = \langle a_j^\dagger(t,\vec{q}',h')a_i(t,\vec{q},h)\rangle,$$

$$\bar{\rho}_{ij}(t,\vec{q},h,\vec{q}',h') = \langle b_i^\dagger(t,\vec{q},h)b_j(t,\vec{q}',h')\rangle,$$

They usually present off-diagonal terms in flavour due to the mixings. They can also have helicity structure in presence of magnetic fields or of neutrino mass corrections.

ABNORMAL DENSITIES (PAIRING)

$$\kappa_{ij}(t,\vec{q},h,\vec{q}',h') = \langle b_j(t,\vec{q}',h')a_i(t,\vec{q},h)\rangle,$$

$$\kappa_{ij}^\dagger(t,\vec{q},h,\vec{q}',h') = \langle a_j^\dagger(t,\vec{q}',h')b_i^\dagger(t,\vec{q},h)\rangle,$$

The Ehrenfest theorem is used :

$$i\dot{\rho}_{ij}(t,\vec{q},h,\vec{q}',h') = \langle[a_j^\dagger(t,\vec{q}',h')a_i(t,\vec{q},h), H_{\text{eff}}(t)]\rangle.$$

INHOMOGENEOUS BACKGROUND, MASSIVE NEUTRINO CASE

Implementing the neutrino Dirac fields :

$$\phi(\vec{x}) = \sum_h \int \frac{d^3\vec{p}}{(2\pi)^3 2E_p}[a(\vec{p},h)u_{\vec{p},h}e^{i\vec{p}\cdot\vec{x}} + b^\dagger(\vec{p},h)v_{\vec{p},h}e^{-i\vec{p}\cdot\vec{x}}]$$

the general mean-field Hamiltonian has the quadratic form :

$$H_{\text{eff}}(t) = a^\dagger(t)\cdot\Gamma^{\nu\nu}(t)\cdot a(t) + b(t)\cdot\Gamma^{\bar{\nu}\bar{\nu}}(t)\cdot b^\dagger(t) + a^\dagger(t)\cdot\Gamma^{\nu\bar{\nu}}(t)\cdot b^\dagger(t) + b(t)\cdot\Gamma^{\bar{\nu}\nu}(t)\cdot a(t),$$

note that : $[A\cdot B]_{ij}(\vec{q},h,\vec{q}\,',h') \equiv \int A_{ik}(\vec{q},h,\vec{p},s)B_{kj}(\vec{p},s,\vec{q}\,',h')$.

the kernels : $\Gamma^{\nu\nu}_{ij}(t,\vec{q},h,\vec{q}\,',h') = \bar{u}_i(\vec{q},h)\tilde{\Gamma}_{ij}(t,\vec{q}-\vec{q}\,')u_j(\vec{q}\,',h')$,

$$i\dot{\rho}(t) = \Gamma^{\nu\nu}(t)\cdot\rho(\quad i\dot{\rho} = [h(\rho),\rho]$$

The extended equation for the density matrix and ν-antiν correlators :

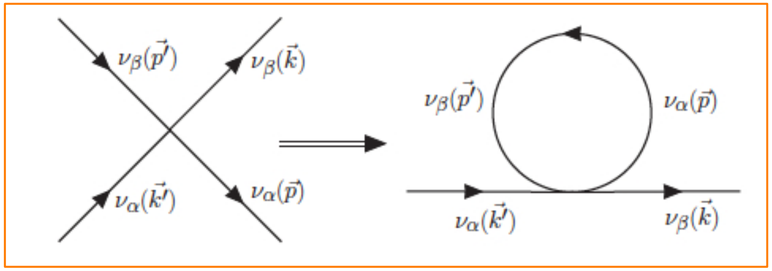
$$\mathcal{R}(t) = \begin{pmatrix} \rho(t) & \kappa(t) \\ \kappa^\dagger(t) & 1 - \bar{\rho}(t) \end{pmatrix} \quad i\dot{\mathcal{R}}(t) = [\mathcal{H}(t),\mathcal{R}(t)]\,. \quad \mathcal{H}(t) = \begin{pmatrix} \Gamma^{\nu\nu}(t) & \Gamma^{\nu\bar{\nu}}(t) \\ \Gamma^{\bar{\nu}\nu}(t) & \Gamma^{\bar{\nu}\bar{\nu}}(t) \end{pmatrix}$$

HOMOGENEOUS BACKGROUND, RELATIVISTIC CASE

Taking e.g. the neutral-current low energy SM Hamiltonian

$$H^{\rm self}_{\rm int} = \frac{G_F}{4\sqrt{2}} \sum_{\alpha,\beta} \int d^3x\, j^\mu_\alpha(t,\vec{x}) j_{\beta,\mu}(t,\vec{x}), \quad j^\mu_\alpha(t,\vec{x}) = \bar{\psi}_\alpha(t,\vec{x})\gamma^\mu(1-\gamma_5)\psi_\alpha(t,\vec{x})$$

the corresponding mean-field kernel is



$$\Gamma^{\rm self}_{ij}(t,\vec{x}) = \frac{G_F}{\sqrt{2}}\gamma_\mu(1-\gamma_5)\,\frac{1}{2}\langle\bar{\psi}_j(t,\vec{x})\gamma^\mu(1-\gamma_5)\psi_i(t,\vec{x})\rangle.$$

and the homogeneity condition : $\rho^{\nu_\beta,\nu_\alpha}_{\vec{p}'h',\vec{p}h} = (2\pi)^3 2E_p \delta_{hh'}\delta^3(\vec{p}-\vec{p}')\rho^{\nu_\beta,\nu_\alpha}_{\vec{p}}$

CONTRIBUTIONS FROM PAIRING CORRELATIONS

The extended Hamiltonian with mixings, ν interactions with matter and ν :

$$\mathcal{H}(t,\vec{q}) = \begin{pmatrix} S(t,q) - \hat{q}\cdot\vec{V}(t) & -\hat{\epsilon}_q^*\cdot\vec{V}(t) \\ -\hat{\epsilon}_q\cdot\vec{V}(t) & \bar{S}(t,q) + \hat{q}\cdot\vec{V}(t) \end{pmatrix}.$$

The off-diagonal term introduces neutrino-antineutrino mixing,
if medium anisotropic and pairing correlations present.

The quantities in H are *(trace terms taken out to simplify expressions)* :

$$S(t,q) = h^0(q) + h^{\mathrm{mat}}(t) + \sqrt{2}G_F \int_{\vec{p}} \ell(t,\vec{p}) \qquad \ell(t,\vec{q}) = \rho(t,\vec{q}) - \bar{\rho}(t,-\vec{q}).$$

$$\vec{V}(t) = -\sqrt{2}G_F \int_{\vec{p}} \left\{\hat{p}\,\ell(t,\vec{p}) + \hat{\epsilon}_p\kappa(t,\vec{p}) + \hat{\epsilon}_p^*\kappa^\dagger(t,\vec{p})\right\}.$$

If medium is isotropic and ν-ν pairing correlations are negligible, one recovers the « usual » mean-field equations :

$$i\dot{\rho} = [h(\rho),\rho] \qquad h = h^0 + h^{mat} \qquad h_{\nu\nu} = \sqrt{2}G_F\int_{\vec{p}}(1-\hat{q}\cdot\hat{p})\ell(t,\vec{p}).$$

likely to be small

NEUTRINO MASS CONTRIBUTIONS

Serreau and Volpe, PRD 90 (2014), arXiv:1409.3591

With the same procedure extended equations with neutrino mass terms can be derived. For Majorana neutrinos one has :

$$\rho_M(t,\vec{q}) \to \begin{pmatrix} \rho_M(t,\vec{q}) & \zeta_M(t,\vec{q}) \\ \zeta_M^\dagger(t,\vec{q}) & \bar{\rho}_M^T(t,-\vec{q}) \end{pmatrix} \qquad \zeta_{ij}(t,\vec{q}) = \langle a_j^\dagger(t,\vec{q},+)a_i(t,\vec{q},-)\rangle$$

$$\Gamma_M^{\nu\nu}(t,\vec{q}) \to \begin{pmatrix} H_M(t,\vec{q}) & \Phi_M(t,\vec{q}) \\ \Phi_M^\dagger(t,\vec{q}) & -\bar{H}_M^T(t,-\vec{q}) \end{pmatrix}$$

$$H_M(t,\vec{q}) = S(t,q) - \hat{q}\cdot\vec{V}(t) - \hat{q}\cdot\vec{V}_m(t),$$
$$\bar{H}_M(t,\vec{q}) = \bar{S}(t,q) + \hat{q}\cdot\vec{V}(t) + \hat{q}\cdot\vec{V}_m(t),$$
$$\Phi_M(t,\vec{q}) = e^{i\phi_q}\hat{\epsilon}_q^*\cdot\left[\vec{V}(t)\frac{m}{2q} + \frac{m}{2q}\vec{V}^T(t)\right].$$

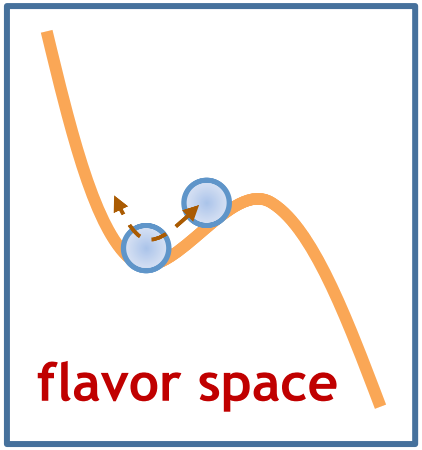
The off-diagonal term introduces neutrino-antineutrino mixing, if medium anisotropic. This is referred to as helicity coherence.

Vlasenko, Fuller, Cirigliano, PRD89 (2014) – spin coherence

First calculation shows it might have an impact under appropriate conditions.

Vlasenko, Fuller, Cirigliano, arXiv:1406.6724

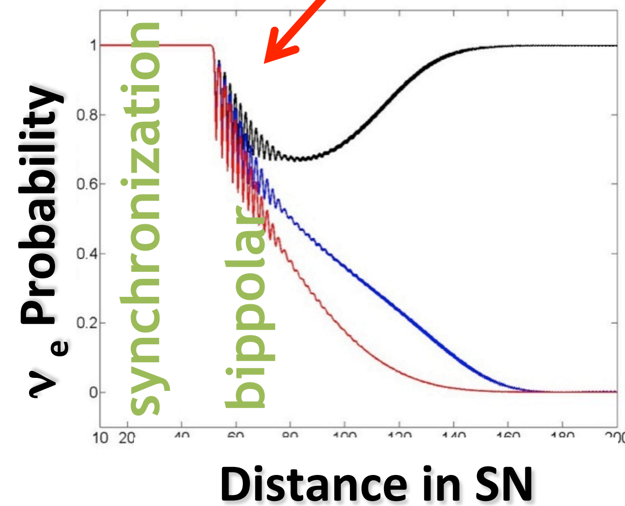
Flavor collective modes and instabilities



Small amplitude motion

Collective modes and instabilities can be studied with the linearization.

Banerjee et al., PRD84 (2011)



Stability matrix

$$\begin{pmatrix} A & B \\ \bar{B} & \bar{A} \end{pmatrix} \begin{pmatrix} \rho' \\ \bar{\rho}' \end{pmatrix} = \omega \begin{pmatrix} \rho' \\ \bar{\rho}' \end{pmatrix}$$

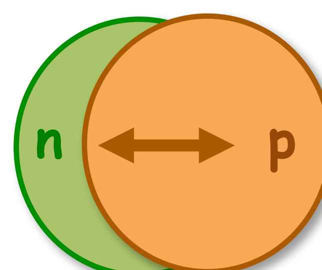
S eigenvalues :

-> real : stable collective

-> imaginary : instabilities

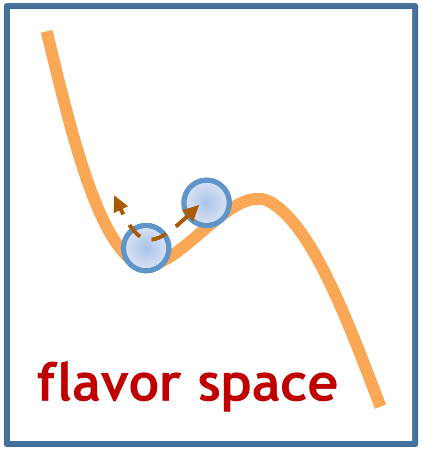
connection to collective modes in other many-body systems (nuclei, clusters, ...)

Väänänen and Volpe, PRD88 (2013)



nuclear resonances

Flavor collective modes and instabilities



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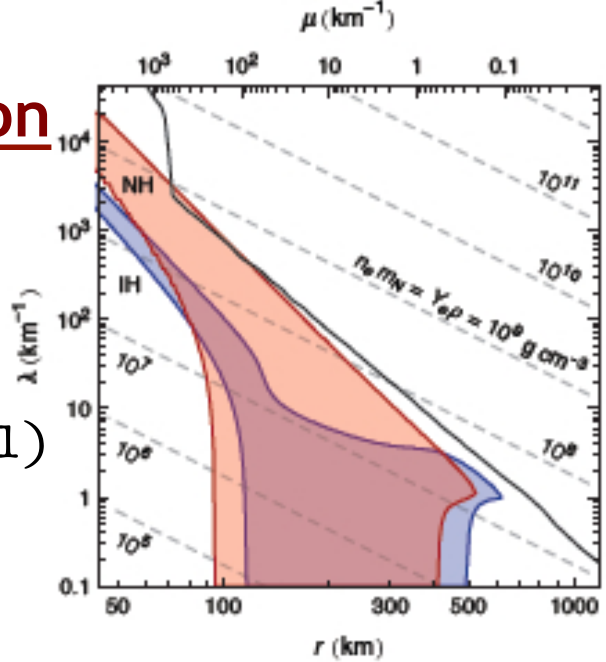

Raffelt et al, PRL(2013)

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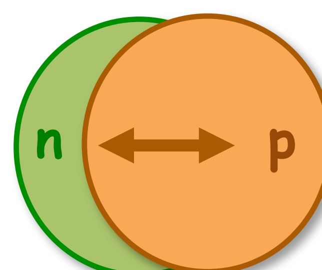


nuclear resonances

Conclusions and perspectives

- Steady progress in our understanding of ν flavour conversion in dense media. Important questions still needs to be investigated.

- Extended equations provided to investigate the transition region. Instabilities searched.
  More sophisticated modelling required with symmetry breaking.

- Established connections between a gas of SN ν and other many body systems – nuclei, condensed matter.


Thank you

Born-Bogoliubov-Green-Kirkwood-Yvon (BBGKY) hierarchy

The s-reduced density matrix : $\rho_{1\ldots s} = \langle\psi(t)|a_s^\dagger \ldots a_1^\dagger a_1 \ldots a_s|\psi(t)\rangle$

one-body density

$$\rho_1 = \langle\psi(t)|a_1^\dagger a_1|\psi(t)\rangle$$

two-body density

$$\rho_{12} = \langle\psi(t)|a_2^\dagger a_1^\dagger a_1 a_2|\psi(t)\rangle$$

Solving exactly the many-body problem is equivalent to

$$\mathrm{i}\dot{\rho}_1 = [t_1, \rho_1] + \mathrm{Tr}_{(2)}\left\{[v_{12}, \rho_{12}]\right\}$$

$$\mathrm{i}\dot{\rho}_{12} = [t_1 + t_2 + v_{12}, \rho_{12}] + \mathrm{Tr}_{(3)}\left\{[v_{13} + v_{23}, \rho_{123}]\right\}$$

$$\vdots$$

$$\mathrm{i}\dot{\rho}_{1\cdots n} = \left[\sum_{i=1}^{n} t_i + \sum_{j>i=1}^{n} v_{ij}, \rho_{1\cdots n}\right] + \sum_{i=1}^{n} \mathrm{Tr}_{(n+1)}\left\{\left[v_{i(n+1)}, \rho_{1\cdots(n+1)}\right]\right\}$$

a hierarchy of equations for s-reduced density matrices

Volpe, Väänänen, Espinoza, PRD87 (2013)
– BBGKY hierarchy, ν-antiν pairing correlations, mass cont.

A novel perspective to ν conversion

Volpe, Väänänen, Espinoza, PRD87 (2013), arXiv: 1302.2374

BBGKY for neutrinos :
- ❖ a system of particles and anti-particles
- ❖ particles with mixings

occupation number op.

ν

$$\rho_\nu = \begin{pmatrix} \langle a^\dagger_{\nu_\alpha,i} a_{\nu_\alpha,i}\rangle & \langle a^\dagger_{\nu_\beta,j} a_{\nu_\alpha,i}\rangle \\ \langle a^\dagger_{\nu_\alpha,i} a_{\nu_\beta,j}\rangle & \langle a^\dagger_{\nu_\beta,j} a_{\nu_\beta,j}\rangle \end{pmatrix}$$

anti-ν

$$\bar{\rho}_\nu = \begin{pmatrix} \langle b^\dagger_{\nu_\alpha,i} b_{\nu_\alpha,i}\rangle & \langle b^\dagger_{\nu_\beta,j} b_{\nu_\alpha,i}\rangle \\ \langle b^\dagger_{\nu_\alpha,i} b_{\nu_\beta,j}\rangle & \langle b^\dagger_{\nu_\beta,j} b_{\nu_\beta,j}\rangle \end{pmatrix}$$

decoherence or mixing terms

The BBGKY is a rigorous theoretical framework :
- ✓ to go from the N-body to the 1-body description
- ✓ that is very general, equivalent the Green's function formalism (equal-time limit)

UNIFIED APPROACH for ASTROPHYSICAL and COSMOLOGICAL APPLICATIONS

that allows to go beyond current approximations

The first BBGKY equation

$$\rho_1 = \langle \psi(t) | a_1^\dagger a_1 | \psi(t) \rangle \qquad \rho_{12} = \langle \psi(t) | a_2^\dagger a_1^\dagger a_1 a_2 | \psi(t) \rangle$$

one-body density   two-body density

The two-body density matrix can be written as :

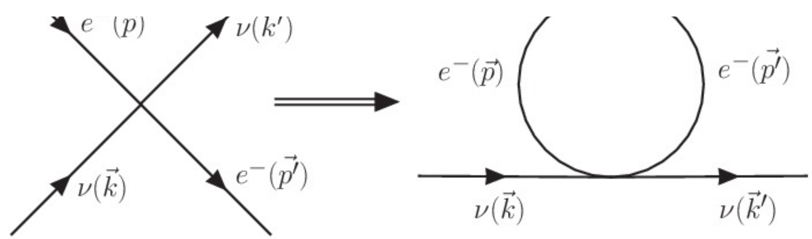
$$\rho_{12} = \rho_1 \rho_2 + c_{12}$$

two-body correlation function

The first BBGKY equations gives for the mean-field evolution equations

$$i\dot{\rho}_1 = [t_1, \rho_1] + \mathrm{Tr}_{(2)} \{[v_{12}, \rho_{12}]\}$$

~~$$i\dot{\rho}_{12} = [t_1 + t_2 + v_{12}, \rho_{12}] + \mathrm{Tr}_{(3)} \{[v_{13} + v_{23}, \rho_{123}]\}$$~~



$$\Gamma_{1,ij}(\rho) = \sum_{mn} v_{(im,jn)} \rho_{2,nm}$$

MEAN-FIELD

the mean-field approximation