Structure of nuclei with strangeness and neutron star

Emiko Hiyama (RIKEN) and H. Togashi (RIKEN)
Nuclear chart with strangeness

Multi-strangeness system such as Neutron star

Double-$\Lambda$ Hypernuclei
$\Xi$ Hypernuclei

$\Lambda, \Sigma$ Hypernuclei

$\Lambda \sim 35$
$\Sigma 1$

Strangeness

$S=-2$
$S=-1$
$S=0$

Proton Number

Neutron Number
So far, it was considered that

Mass $\sim 1.5 \times$ solar mass

Almost: neutrons, protons, electrons, hyperons
In hypernuclear physics, it is one of important subject to understand the mass of neutron Star and structure of neutron star.
For this study, we need

(1) **Nucleon-Nucleon interaction**

(2) **Hyperon-Nucleon (Hyperon-Hyperon interaction)**
To determine the interactions in unified way:
one of the primary goals of the hypernuclear physics

In order to determine the interactions, two-body scattering experiment is most useful.

In fact, for this purpose, many nucleon-nucleon scattering experiments have been done so far.

We have much knowledge.
However, due to the difficulty of performing two-body hyperon-nucleon and hyperon-hyperon scattering experiments, we have small number of hyperon-nucleon scattering data and NO hyperon-hyperon data.

We have little knowledge about the interactions up to now.

Hyperon-nucleon and hyperon-hyperon potential models so far proposed. (ex. Nijmegen, Julich, Kyoto-Niigata) Large ambiguity.
Therefore, as a substitute for the 2-body limited $YN$ and non-existent $YY$ scattering data, the systematic investigation of the structure of light hypernuclei is essential.
It is important to produce a nucleus consisting of neutron + proton + hyperon and to study structure of this nucleus.

Most fundamental hypernuclei are composed of 3 and 4 particles. To study structure of these systems is to solve 3 and 4-body problems.
For the study of hyperon-nucleon and hyperon-
Hyperon interaction,
They are producing many hypernuclei using J-PARC, JLab,
Mainz and GSI facilities.
Strategy to determine Hyperon-Nucleon and Hyperon-Hyperon interactions from the studies of light Hypernuclear structure

- Hyperon-Nucleon interaction: Nijmegen model
- Hyperon-Hyperon interaction: Kyoto-Niigata

1. Use
2. Compare theoretical results with experimental data
3. Improvement

Accurate calculation of hypernuclear structure
Few-body, cluster model, shell model, ..... 

No direct information

Spectroscopy experiments
- High-resolution γ-ray spectroscopy experiment by Tamura and his collaborators
- Emulsion experiment by Nakazawa and his collaborators

EOS of Neutron star using a few-body method

My role
Our few-body calculational method

**Gaussian Expansion Method (GEM), since 1987**

- A variational method using Gaussian basis functions
- Take all the sets of Jacobi coordinates

Developed by Kyushu Univ. Group, Kamimura and his collaborators.

Review article:
E. Hiyama, M. Kamimura and Y. Kino,
Prog. Part. Nucl. Phys. 51 (2003), 223.

High-precision calculations of various 3- and 4-body systems:

- Exotic atoms / molecules
- Light hypernuclei
- 3- and 4-nucleon systems
- 3-quark systems
- Multi-cluster structure of light nuclei
Gaussian Expansion Method (GEM)

\[
H = -\frac{\hbar^2}{2\mu_{r_c}} \nabla^2_{r_c} - \frac{\hbar^2}{2\mu_{R}} \nabla^2_{R_c} + V^{(1)}(r_1) + V^{(2)}(r_2) + V^{(3)}(r_3).
\]

\[
[ H - E ] \Psi_{JM} = 0
\]

\[
\Psi_{JM} = \Phi^{(1)}_{JM}(r_1, R_1) + \Phi^{(2)}_{JM}(r_2, R_2) + \Phi^{(3)}_{JM}(r_3, R_3)
\]
Basis functions of each Jacobi coordinate

\( (c = 1 - 3) \)

\[ \psi_{JM} = \Phi_{JM}^{(1)}(r_1, R_1) + \Phi_{JM}^{(2)}(r_2, R_2) + \Phi_{JM}^{(3)}(r_3, R_3) \]

\[ \phi_{nl}^{(c)}(r_c) \ Y_{lm}(\widehat{r}_c), \quad \psi_{NL}^{(c)}(R_c) \ Y_{LM}(\widehat{R}_c) \]

\[ \Phi_{JM}^{(c)}(r_c, R_c) = \sum_{nl, NL} C_{NL, lm} \phi_{nl}^{(c)}(r_c) \ \psi_{NL}^{(c)}(R_c) \ [Y_{L}(\widehat{r}_c) \ \otimes \ Y_{L}(\widehat{R}_c)]_{JM} \]

Determined by diagonalizing \( H \)
Radial part: Gaussian function

$$\phi_{nl}(r) = r^l e^{-(r/r_n)^2}$$

$$\psi_{NL}(R) = R^L e^{-(R/R_N)^2}$$

Gaussian ranges in geometric progression

$$r_n = r_1 a^{n-1} \quad (n = 1 - n_{\text{max}})$$

$$R_N = R_1 A^{N-1} \quad (N = 1 - N_{\text{max}})$$

Both the short-range correlations and the exponentially-damped tail are simultaneously reproduced accurately.
Next, by solving eigenstate problem, we get eigenenergy $E$ and unknown coefficients $C_n$.

\[
\begin{pmatrix}
(H_{in}) - E(N_{in})
\end{pmatrix}
\begin{pmatrix}
C_n
\end{pmatrix} = 0
\]
The merit of this method:

1. To calculate the energy of bound state very accurately
2. To calculate the wavefunction very precisely

successful example
Benchmark-test calculation to solve the 4-nucleon bound state

7 different groups (18 co-authors)

1. Faddeev-Yakubovski (Kamada et al.)
2. Gaussian Expansion Method (Kamimura and Hiyama)
3. Stochastic variational (Varga et al.)
4. Hyperspherical variational (Viviani et al.)
5. Green Function Variational Monte Carlo (Carlson at al.)
6. Non-Core shell model (Navratil et al.)
7. Effective Interaction Hypershperical HarmonicsEIHH (Barnea et al.)

Good agreement among these 7 different methods in the binding energy, r.m.s. radius and two-body correlation function
Good agreement among 7 different methods
In the binding energy, r.m.s. radius and wavefunction density

TABLE I. The expectation values $\langle T \rangle$ and $\langle V \rangle$ of kinetic and potential energies, the binding energies $E_b$ in MeV, and the radius in fm.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\langle T \rangle$</th>
<th>$\langle V \rangle$</th>
<th>$E_b$</th>
<th>$\sqrt{\langle r^2 \rangle}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FY</td>
<td>102.39(5)</td>
<td>-128.33(10)</td>
<td>-25.94(5)</td>
<td>1.485(3)</td>
</tr>
<tr>
<td>SVM</td>
<td>102.30</td>
<td>-128.20</td>
<td>-25.90</td>
<td>1.482</td>
</tr>
<tr>
<td>HH</td>
<td>102.35</td>
<td>-128.27</td>
<td>-25.92</td>
<td>1.486</td>
</tr>
<tr>
<td>GFMC</td>
<td>102.44(1.0)</td>
<td>-128.34</td>
<td>-25.90(1)</td>
<td>1.483</td>
</tr>
<tr>
<td>NCSM</td>
<td>103.35(1.0)</td>
<td>-129.45</td>
<td>-25.80(20)</td>
<td>1.485</td>
</tr>
<tr>
<td>E1HH</td>
<td>100.8(9)</td>
<td>-126.7(9)</td>
<td>-25.944(10)</td>
<td>1.486</td>
</tr>
</tbody>
</table>

very different techniques and the complexity of the nuclear force chosen. Except for NCSM and E1HH, the expectation values of $T$ and $V$ also agree within three digits. The NCSM results are, however, still within 1% and E1HH within 1.5% of the others, but note that the E1HH results for $T$ and $V$ are

FIG. 1. Correlation functions in the different calculational schemes: E1HH (dashed-dotted curves), FY, CRCGV, SVM, HH, and NCSM (overlapping curves).
Strategy to determine Hyperon-Nucleon and Hyperon-Hyperon interactions from the studies of light Hypernuclear structure

- **Hyperon-Nucleon interaction**: Nijmegen model
- **Hyperon-Hyperon interaction**: Kyoto-Niigata

1. Use
2. No direct information
3. Improvement

Accurate calculation of hypernuclear structure
- Few-body, cluster model, shell model, ..... 

My role using a few-body method

Spectroscopy experiments
- High-resolution γ-ray spectroscopy experiment by Tamura and his collaborators
- Emulsion experiment by Nakazawa and his collaborators

Compare theoretical results with experimental data

EOS of Neutron star
**ΛN interaction**  (effectively including ΛN -ΣN coupling)

\[
V_{ΛN} = V_0 + \sigma_Λ \cdot \sigma_N V_{σσ} + \mathbf{L} \cdot (s_Λ + s_N) V_{SLS} + \mathbf{L} \cdot (s_Λ - s_N) V_{ALS} + S_{12} V_{tensor} + \cdots
\]

Almost determined since 1998

One of the important issue

\[
\mathbf{L} \cdot (s_Λ + s_N) V_{SLS} \quad ----- \quad SLS \ (Symmetric \ LS)
\]

\[
\mathbf{L} \cdot (s_Λ - s_N) V_{ALS} \quad ----- \quad ALS \ (Anti \ symmetric \ LS)
\]

Two kinds of spin-orbit force

1) **Meson theory based**: Nijmegen model D, F and soft core ’97a-f

2) **Quark model based**: Kyoto-Niigata group (FSS)
3- and 4-body calculations:
E. Hiyama, M. Kamimura, T. Motoba, T. Yamada and Y. Yamamoto

**YN LS force**
1) **Meson theory**: Nijmegen Model D, F, soft core'97 a – f.
2) **Qurak model**: Kyoto-Niigata, FSS
ΛN LS force and $^9$Be and $^{13}$C

$^9$Be

\begin{align*}
3/2^+ & \quad \downarrow \quad 80 \quad \uparrow \quad 3/2^+ \quad 35 \\
5/2^+ & \quad \downarrow \quad 200 \quad \uparrow \quad 5/2^+ \\
1) \text{Meson} & \quad \text{Exp.} \quad 43 \pm 5 \text{ keV}
\end{align*}

$^{13}$C

\begin{align*}
1/2^- & \quad \downarrow \quad 360 \quad \uparrow \quad 1/2^- \quad 150 \\
3/2^- & \quad \downarrow \quad 960 \quad \uparrow \quad 3/2^- \quad 200 \\
1) \text{Meson} & \quad \text{Exp.} \quad 152 \pm 54 \pm 36 \text{ keV} \\
2) \text{Quark} & \quad \text{BNL-E929}
\end{align*}

Nijmegen model D,F
Soft core '97a-f

H. Akikawa et al.
H. Tamura et al.

BNL-E930

S. Ajimura et al.
LS splitting in $^{9}$Be $\Lambda$

the improved YN interaction based on meson theory, extended soft core potential 06 (ESC06) by Th. A Rijken

\[
\begin{align*}
3/2^+ & \quad 5/2^+ \\
39 \text{ keV} & \\
\text{ESC06} & \\
\text{(small)} & \\
\text{SLS + ALS} & \\
\text{Hiyama (2007)} & \\
\text{Good agreement} & \\
\text{BNL-E930} & \\
\text{Exp.} & 43 \pm 5 \text{ keV} \\
\end{align*}
\]

H. Akikawa et al.  
H. Tamura et al.  
Hypernuclear $\gamma$-ray data since 1998 (figure by H. Tamura)

\[ V_{\Lambda N} = V_0 + \sigma_\Lambda \cdot \sigma_N V_{0-0} + L \cdot (s_\Lambda + s_N) V_{SLS} + L \cdot (s_\Lambda - s_N) V_{ALS} + S_{12} V_{\text{tensor}} + \cdots \]

- Millener (p-shell model),  
- Hiyama (few-body)
Section 3

S=-2 hypernuclei and

YY interaction
It is conjectured that extreme limit, which includes many \(\Lambda\)s in nuclear matter, is the core of a neutron star.

In this meaning, the sector of \(S=-2\) nuclei, double \(\Lambda\) hypernuclei and \(\Xi\) hypernuclei is just the entrance to the multi-strangeness world.

However, we have hardly any knowledge of the YY interaction because there exist no YY scattering data.

Then, in order to understand the YY interaction, it is crucial to study the structure of double \(\Lambda\) hypernuclei and \(\Xi\) hypernuclei.

So far, we have discussed about single \(\Lambda\) hypernuclei.
Before 2000

Only three double $\Lambda$ hypernuclei

Ambiguity for identifying these double $\Lambda$ hypernuclei

There was NO observed double $\Lambda$ hypernuclei without ambiguity.
In 2001, the epoch-making data has been reported by the KEK-E373 experiment.

**Observation of $^{6}_{\Lambda\Lambda}\text{He}$**

Uniquely identified without ambiguity for the first time.

\[ \alpha + \Lambda + \Lambda \]

7.25 ± 0.1 MeV

\( ^{6}_{\Lambda\Lambda}\text{He} \)

\( ^{6}_{\Lambda\Lambda}\text{He} \rightarrow ^{5}\text{He} + p + \pi^- \)

\( \Xi^- + ^{12}\text{C} \rightarrow ^{6}_{\Lambda\Lambda}\text{He} + ^{4}\text{He} + t \)

H. Takahashi et al., PRL 87, 212502-1 (2001)
Strategy of how to determine YY interactions from the study of light hypernuclear structure

1. Use Nijmegen model D
2. Compare between the theoretical result and the experimental data of the binding energy of $^6\text{He}$
3. Suggest reducing the strength of spin-independent force by half
4. Prediction of new double $\Lambda$ hypernuclei

Accurate structure calculation

Spectroscopic experiments
Emulsion experiment (KEK-E373) by Nakazawa and his collaborators

Calculation of EOS of neutron star
KEK-E373 experiment analysis is still in progress.

**E07**

Approved proposal at J-PARC

“Systematic Study of double strangness systems at J-PARC”

by Nakazawa and his collaborators

It is difficult to determine:

1. spin-parity
2. whether the observed state is the ground state or an excited state

Emulsion experiment

\[ \Lambda \Lambda \text{ interaction to reproduce the observed binding energy of } {}^6\text{He} \]

My theoretical contribution using few-body calculation

Theoretical calculation

the identification of the state
Successful example to determine spin-parity of double $\Lambda$ hypernucleus --- Demachi-Yanagi event for $^{10}_{\Lambda\Lambda}$Be

Observation of $^{10}\Lambda\Lambda$Be --- KEK-E373 experiment

$^{8}\text{Be} + \Lambda + \Lambda$ to $^{10}\Lambda\Lambda$Be

12.33 $^{+0.35}_{-0.21}$ MeV

$^{10}_{\Lambda\Lambda}$Be

$^{10}_{\Lambda\Lambda}$Be: ground state or excited state?

Demachi-Yanagi event
In this way, we succeeded in interpreting the spin-parity by comparing the experimental data and our theoretical calculation.

E. Hiyama, M. Kamimura, T. Motoba, T. Yamada and Y. Yamamoto
Phys. Rev. 66 (2002), 024007
Therefore, the 4-body calculation has predictive power. Hoping to observe new double $\Lambda$ hypernuclei in future experiments, I predicted level structures of these double $\Lambda$ hypernuclei within the framework of the $\alpha+x+\Lambda+\Lambda$ 4-body model.

E. Hiyama, M. Kamimura, T. Motoba, T. Yamada and Y. Yamamoto

I have been looking forward to having new data in this mass-number region.
Strategy of how to determine YN and YY interactions from the study of light hypernuclear structure

1. Use Nijmegen model D

2. Compare between the theoretical result and the experimental data of the binding energy of $^6\text{He}$

3. Suggest reducing the strength of spin-independent force by half

4. Prediction of new double $\Lambda$ hypernuclei

Accurate structure calculation

Spectroscopic experiments
- Emulsion experiment (KEK-E373) by Nakazawa and his collaborators

Calculation of EOS of neutron star
ΛN interaction determined by the study of structure of Λ hypernuclei
ΛΛ interaction determined by the study of structure of double Λ hypernuclei

NN interaction: AV18 realistic interaction +UIX three-body force

To calculate EOS, we employ the cluster variational method which was developed by M. Takano and H. Togashi et al.


The detail will be reported by H. Togashi in this session:
‘Variational study of the equation of state for hyperonic neutron stars’
17:30～
Calculated by H. Togashi

$M/M_\odot$ vs. $R [\text{km}]$

- Blue line: Two-body $YN$ and $YY$
- Red line: Three-body $YNN + YYN + YYY$

J0348+0432
J1614-2230

$1.62 \, M_\odot$
hypernuclei
For the study of $\Xi N$ interaction, it is important to study the structure of $\Xi$ hypernuclei. So far there was no observed $\Xi$ hypernucleus.
This year, we observed Ξ hypernucleus for the first time. From this evidence, we found that Ξ-N interaction is attractive. In order to obtain further information on ΞN interaction, it is important to predict theoretically what kinds of Ξ hypernuclei will exist as bound states.
For this purpose, recently, I studied these Ξ hypernuclei.

\[(d+Ξ^-)\]  \[(t +Ξ^-)\]  \[(^5\text{Li}+Ξ^-)\]

\[(^6\text{He}+Ξ^-)\]  \[(^9\text{Be}+Ξ^-)\]  \[(^{11}\text{B}+Ξ^-)\]

ΞN interaction to reproduce the experimental data of \(^{12}\text{C}(K^-,K^+)\) reaction

Spectroscopy of $\Xi$ hypernuclei at J-PARC

($pn+\Xi$) $(pnn+\Xi)$ ($^6\text{He}+\Xi$) ($^6\text{He}+\Xi$) ($^9\text{Be}e+\Xi$) ($^{11}\text{B}+\Xi$)

$^2_d+\Xi$ 1/2 $^+$$\quad an+\Xi$ 1 $^-$ $(\alpha+\Xi)+nn$ 1/2 $^+$ $(^8\text{Be}+\Xi)+n$ 11B+Xi

t +Xi 2 $^-$ 1 $^-$ 2 $^-$

MeV

0
-2
-5
-10
-20
Approved proposal at J-PARC

E05 “Spectroscopic study of $\Xi$-Hypernucleus, $^{12}$Be, via the $^{12}$C($K^-,K^+$) Reaction” by Nagae and his collaborators

Day-1 experiment

$^{11}$B

$^{12}$C

$K^-$

$p$

$\Xi^-$

$K^+$

$\Xi$ hypernucleus

This observation will give further information about $\Xi N$ interaction.
Spectroscopy of $\Xi$ hypernuclei at J-PARC

\[ (p + \Xi^+) + \Xi^{-} \quad (p + \Xi^{-}) + n \quad (\alpha + \Xi^{-}) + n \quad (\alpha + \Xi^{-}) + n \quad (\alpha + \Xi^{-}) + n \quad (\alpha + \Xi^{-}) + n \]
Concluding remark

Multi-strangeness system such as Neutron star
Thank you!