Neutrino Emissivities from Deuteron-Breakup and Formation in Supernovae


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Introduction

Light elements in supernova (SN) $^2$H, $^3$H, $^3$He ... etc.

Snapshot of supernova profile (150 ms after core bounce)

Mass fraction abundance of light elements based on nuclear statistical equilibrium

Sumiyoshi, Röpke, PRC 77, 055804 (2008)

Q: Can light elements in SN influence SN evolution?
Yes, it does!


Neutrino absorption by deuteron can boost shockwave drastically in some cases
Neutrino emission from light elements in SN

ν-emission previously considered (A≤2)

* $p + e^- \rightarrow n + \nu_e$
* $n + e^+ \rightarrow p + \bar{\nu}_e$
* $n + n \rightarrow p + n + e^- + \bar{\nu}_e$
* $p + p \rightarrow p + n + e^+ + \nu_e$
* $N + N \rightarrow N + N + \nu + \bar{\nu}$
Neutrino emission from light elements in SN

ν-emission previously considered (A≤2)

* \( p + e^- \to n + \nu_e \)
* \( n + e^+ \to p + \bar{\nu}_e \)
* \( n + n \to p + n + e^- + \bar{\nu}_e \)
* \( p + p \to p + n + e^+ + \nu_e \)
* \( N + N \to N + N + \nu + \bar{\nu} \)

New agents considered here

\( d + e^- \to n + n + \nu_e \)
\( d + e^+ \to p + p + \bar{\nu}_e \)
\( n + n \to d + e^- + \bar{\nu}_e \)
\( p + p \to d + e^+ + \nu_e \)
\( p + n \to d + \nu + \bar{\nu} \)
Neutrino emission from light elements in SN

**ν-emission previously considered (A≤2)**

<table>
<thead>
<tr>
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<td>* $p + e^- \rightarrow n + \nu_e$</td>
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<td>* $p + p \rightarrow p + n + e^+ + \nu_e$</td>
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Questions to be addressed:

- How much $\nu$ are emitted by deuteron processes compared with the conventional ones?
- Regarding SN evolution, what is implied by the new $\nu$-emission from light elements?

Contents of this talk: description of model ➔ numerical results ➔ conclusion
Calculational method

Well-established method for electroweak processes in few-nucleon systems

\[ \langle \psi_f | H_{\text{ew}} | \psi_i \rangle \]

\( |\psi\rangle \): solution of Schröding eq. with high-precision \( NN \) (+ \( NNN \)) potential

AV18, Nijmegen, Bonn, chiral etc.
Weak Interaction Hamiltonian

\[ H^{CC}_W = \frac{G'_F V_{ud}}{\sqrt{2}} \int d\mathbf{x} [J^{CC}_\lambda(\mathbf{x}) L^\lambda(\mathbf{x}) + \text{h. c.}] \quad \text{for CC} \]

\[ H^{NC}_W = \frac{G'_F}{\sqrt{2}} \int d\mathbf{x} [J^{NC}_\lambda(\mathbf{x}) L^\lambda(\mathbf{x}) + \text{h. c.}] \quad \text{for NC} \]

\[ L^\lambda(\mathbf{x}) = \bar{\psi}_l(\mathbf{x}) \gamma^\lambda (1 - \gamma^5) \psi_\nu(\mathbf{x}) \]
Nuclear current

\[
J^C_C(x) = V^\pm(x) + A^\pm(x)
\]

\[
J^{NC}_\lambda(x) = V^3_\lambda - 2\sin^2\theta_W (V^3_\lambda + V^s_\lambda) + A^3_\lambda
\]

\(V(A)\): Vector (Axial) current

\(V^s\): Isoscalar vector current

\(\theta_W\): Weinberg Angle \(\sin^2\theta_W = 0.23\)

\(J_\lambda = \) (one-body current) + (two-body exchange current)
Impulse approximation (IA) current

\[
< p' | V_\lambda(0) | p > = \bar{u}(p') \left[ f_V \gamma_\lambda + i \frac{f_M}{2M_N} \sigma_{\lambda \rho} q^\rho \right] u(p)
\]

\[
< p' | A_\lambda(0) | p > = \bar{u}(p') \left[ f_A \gamma_\lambda \gamma^5 + f_P \gamma^5 q_\lambda \right] u(p)
\]

\[ q_\lambda \equiv p'_\lambda - p_\lambda \]

\[ f_M : \text{CVC} \quad f_P : \text{PCAC} \]

\[
f_A(q_\mu^2) = -g_A \left( 1 - \frac{q_\mu^2}{1.04 \text{[GeV}^2]\right)}^{-2}, \quad g_A = 1.2670 \pm 0.0030 \text{ (PDG)}
\]
Exchange axial-vector current

Fit $AN\Delta$ coupling to tritium $\beta$-decay rate with rigorous three-body calculation

Predicted rates for muon captures on deuteron and $^3$He are consistent with data

Marcucci et al., PRC 83 (2011)

$\nu_e + d \rightarrow e^- + p + p$ , $\nu + d \rightarrow \nu + p + n$ for SNO expt.  SN et al. PRC 63 (2000) ; NPA707 (2002)
Emissivity ($Q$)

\[ e^- + d \rightarrow \nu_e + n + n \]

\[
Q = \frac{(2\pi)^4}{S} \int \frac{dp_{N_1}}{(2\pi)^3} \frac{dp_{N_2}}{(2\pi)^3} \frac{dp_d}{(2\pi)^3} \frac{dp_{\nu}}{(2\pi)^3} \frac{dp_{e^-}}{(2\pi)^3} \delta^4(p_f - p_i) \\
\times E_{\nu} \sum_{\text{spin}} |\langle \psi_f | H_{\text{ew}} | \psi_i \rangle|^2 f_d f_e (1 - f_{N_1}) (1 - f_{N_2})
\]

\[
f_k(p_k) = \frac{1}{\exp\left((e_k(p_k) - \mu_k)/k_B T\right) \pm 1}
\]

+ : fermion
- : boson

$S$ : symmetry factor for identical particles
Numerical results
Emissivities presented are for:

- **Surface** region of proto-neutron star ($r > 10$ km)
- **Inner** region of proto-neutron star ($r < 10$ km)

Deuteron can be largely modified, or even doesn’t exist

⇒ “deuteron” as two-nucleon correlation in matter

supernova profile (150 ms after core bounce)

Sumiyoshi, Röpke, PRC 77, 055804 (2008)
Emissivities presented are for:

- **Surface** region of proto-neutron star \( (r > 10 \text{ km}) \)

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supernova profile (150 ms after core bounce)

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$\nu_e$-emissivity \hspace{1cm} (r > 10 \text{ km})$

electron capture \hspace{1cm} NN fusion

\[Q(e^- p) > Q(e^- d) > Q(\text{NN} \rightarrow d)\]
Change of $\nu_e$ emissivity due to deuteron

Mass fraction

$Q(N+d) / Q(N)$
Change of $\nu_e$ emissivity due to deuteron

Deuterons exit at the cost of the proton abundance plus $\sigma(e^-p) > \sigma(e^-d)$

Effectively reduced $\nu_e$ emissivity $\Rightarrow$ less $\nu$-flux, $\nu$-heating

Careful estimate of light element abundance & emissivity needed
Whenever $NN$ brem is important, $NN \rightarrow d$ (correlation) can be also important

$\Rightarrow$ Possible important role in proto-neutron star cooling
$\nu$-emission in SN: \[ e^\pm + d \rightarrow \nu_e(\bar{\nu}_e) + N + N \quad N + N \rightarrow d + l + \bar{l} \]

New agents other than direct & modified Urca, $NN$ bremsstrahlung

Emissivities $\leftrightarrow$ $NN$ wave functions based on high-precision $NN$ potential
+ 1 & 2-body nuclear weak currents (tested by data)

Electron captures $\Rightarrow$ effectively reduced $\nu_e$ emissivity

$\Rightarrow$ light element abundance & emissivity are important

$NN$ fusions $\Rightarrow$ play a role comparable to $NN$ bremsstrahlung & modified Urca

$\Rightarrow np \rightarrow d\nu\bar{\nu}$ can be important for $\nu_\mu$ emissivity
Backups
$\nu_\mu$-emissivity

Whenever $NN$ bremsstrahlung is important, $np \rightarrow d$ can be also important

Possible important role in proto-neutron star cooling
Medium effect on deuteron

Deuteron wave function with Pauli-blocking at $r = 11.7$ km, $\rho = 6 \times 10^{13}\text{g/cm}^3$, $T=25$ MeV

- Significant decrease of B.E. at $P_d=0$
- B.E. recovered free B.E. at $P_d=1$ GeV

Reduction rate : $1 - Q(\text{in-medium})/Q(\text{free})$

<table>
<thead>
<tr>
<th>$r$ (km)</th>
<th>11.7</th>
<th>19.7</th>
<th>40.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduction rate (%)</td>
<td>15</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>
Most recent applications of the model to weak processes

★ *pp-fusion* \((p + p \rightarrow d + e^+ + \nu_e)\) for solar model, Schiavilla et al. PRC 58 (1998)

★ *Muon capture* \((\mu^- + d \rightarrow n + n + \nu_\mu, \mu^- + ^3He \rightarrow ^3H + \nu_\mu)\), Marcucci et al., PRC 83 (2011)

<table>
<thead>
<tr>
<th></th>
<th>[1]</th>
<th>[2]</th>
<th>Theory</th>
<th>MuSun@PSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Gamma(\mu^- + d) [s^{-1}])</td>
<td>409 ± 40</td>
<td>470 ± 29</td>
<td>393</td>
<td>???</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>(\Gamma(\mu^- + ^3He) [s^{-1}])</td>
<td>1496 ± 4</td>
</tr>
</tbody>
</table>

|----------------|---------------------------------|

★ *vd-reactions* \((\nu_e + d \rightarrow e^- + p + p, \nu^- + d \rightarrow \nu + p + n)\) for SNO experiment

SN et al. PRC 63 (2000); NPA707 (2002)

⇒ evidence of \(\nu\)-oscillation, solar \(\nu\) problem resolved
Emissivity \((Q)\)

\[ \begin{align*} 
N_1 + N_2 & \rightarrow N'_1 + N'_2 + \nu + \bar{\nu} \\
Q & = \frac{(2\pi)^4}{S} \int \frac{dp_{N_1}}{(2\pi)^3} \frac{dp_{N_2}}{(2\pi)^3} \frac{dp_{N'_1}}{(2\pi)^3} \frac{dp_{N'_2}}{(2\pi)^3} \frac{dp_{\nu}}{(2\pi)^3} \frac{dp_{\bar{\nu}}}{(2\pi)^3} \delta^{(4)}(p_f - p_i) \\
& \times (E_{\nu} + E_{\bar{\nu}}) \sum_{\text{spin}} \left| \langle \psi_f | H_{\text{ew}} | \psi_i \rangle \right|^2 F_{N_1} F_{N_2} (1 - F_{N'_1})(1 - F_{N'_2}) \\
F_N & = \frac{1}{1 + \exp[(\varepsilon_N - \mu_N)/kT]} 
\end{align*} \]
Emissivity \((Q)\)

\[ N_1 + N_2 \rightarrow d + \nu + \bar{\nu} \]

\[ Q = \frac{(2\pi)^4}{S} \int \frac{dp_{N_1}}{(2\pi)^3} \frac{dp_{N_2}}{(2\pi)^3} \frac{dp_d}{(2\pi)^3} \frac{dp_{\nu}}{(2\pi)^3} \frac{dp_{\bar{\nu}}}{(2\pi)^3} \delta^{(4)}(p_f - p_i) \]

\[ \times (E_\nu + E_{\bar{\nu}}) \sum_{\text{spin}} |\langle \psi_f | H_{\text{ew}} | \psi_i \rangle|^2 F_{N_1} F_{N_2} \]

11 dimensional integral !!

Approximation necessary to evaluate \(Q\)
Emissivity \((Q)\)

\[
Q \propto \int dp_N \int dp_{N_2} \int dp_\nu \int dp_{\bar{\nu}} \omega \delta(E_f - E_i) \delta^{(3)}(\vec{p}_f - \vec{p}_i) |M|^2 F_{N_1}(\vec{p}_1) F_{N_2}(\vec{p}_2)
\]

\[
p^2 dp d\Omega_p dP
\]

\[
\delta \left[ \left( \frac{D}{2} + \frac{p_t^2}{2M_d} \frac{\omega}{\omega} \right) \left( \frac{p^2}{4M_N} \frac{\omega}{M_N} \right) \right]
\]

Approximation!

\[
Q \propto \int dp_\nu \int dp_{\bar{\nu}} \omega p^2 \left[ \int d\Omega_p |M|^2 \right] \left[ \int dP F_{N_1}(\vec{P}/2 + \vec{p}) F_{N_2}(\vec{P}/2 - \vec{p}) \right]
\]

\[
8\pi^2 \frac{p^2}{p_\nu p_{\bar{\nu}}} \int dp_\nu \int dp_{\bar{\nu}} \int d\cos \theta_{\nu\bar{\nu}}
\]

3 dimensional integral
Previous common approximation to evaluate $Q_{\text{NN-brem}}$

- One-pion-exchange potential, Born approximation
  Low-energy theorem

- Nuclear matrix element $\Rightarrow$ long wave length limit
  $\Rightarrow$ constant

Neither of them are adopted in this work
Previous common approximation to evaluate $Q_{\text{NN-brem}}$

- One-pion-exchange potential, Born approximation
- Low-energy theorem
  
- Neglect momentum transfer ($\vec{p}_\nu + \vec{p}_{\bar{\nu}}$)
  
  ➔ also angular correlation between $\nu$ and $\bar{\nu}$

- Nuclear matrix element ➔ long wave length limit
  
  ➔ constant
Emissivites from direct Urca, $e^+e^-$ annihilation, $NN$ brems ⇐ compilation I

Emissivites from election captures on $d$ & $NN$ fusion ⇐ compilation II

• Compilation I : Shen EoS, $N$, $^4$He, a heavy nucleus

• Compilation II : light elements abundance from Sumiyoshi & Röpke (2008)

Both have the same density, temperature, electron fraction
Exchange vector current

★ Current conservation for one-pion-exchange potential
★ $VΝΔ$ coupling is fitted to $np \rightarrow dγ$ data
Comparison with $np \rightarrow d\gamma$ data

$n+p \rightarrow d+\gamma$

Exchange currents contribute about 10%
Exchange axial charge

Kubodera, Delorme, Rho, PRL 40 (1978)

\[
\begin{array}{c}
\uparrow \\
\pi \\
\downarrow \\
\end{array}
\]

Soft pion theorem + PCAC
Why tritium $\beta$ decay?

$\nu d$: Gamow-Teller ($^3S_1 \rightarrow ^1S_0$) $\Rightarrow$ $A_{EXC}$ is main correction

$^3H$: Fermi ($^1S_0 \rightarrow ^1S_0$) & Gamow-Teller

$\rho(x)$ [fm$^{-1}$]

$\rho_{vd} \approx \rho_{^3H}$ = const.

Schiavilla et al., PRC58,1263(1998)
$E_{\nu}$-dependence of energy transfer cross section

$\nu d$ CC

* solid: $T_{\nu} = 5\text{MeV}$
* dashed: $T_{\nu} = 10\text{MeV}$

\[
\sigma \omega (E_{\nu}) \times f(T, E_{\nu}) \times 10^{-42} \text{cm}^2 \text{MeV}
\]

\[
\begin{array}{c}
0 & 20 & 40 & 60 & 80 & 100 & 120 & 140 \\
\end{array}
\]

$E_{\nu} [\text{MeV}]$

* Main contribution is from $E_{\nu} = 20 (60) \text{MeV}$ for $T_{\nu} = 5 (10) \text{MeV}$

* High energy tail of $\sigma \omega \times f$ is appreciable
\[ Q(e^+ n) > Q(e^+ d) > Q(NN \rightarrow d) \]
\( \nu_\mu \)-emissivity

Whenever NN brem is important, \( np \rightarrow d \) can be also important

Possible important role in proto-neutron star cooling
Neutrino spectrum

\[ Q[\text{erg/cm}^3/\text{sec}/\text{MeV}] \]

\[ E_\nu [\text{MeV}] \]

\[ r_c = 20 \text{Km} \]
\[ r_c = 50 \text{Km (x 5e5)} \]

\[ nn \rightarrow d\bar{e}\bar{\nu}_e \]

\[ \bar{e} d \rightarrow nn \nu_e \]
Supernova profile

Sumiyoshi, Röpke, PRC 77, 055804 (2008)
Meson exchange current effect on $Q$

Large effect on NN fusion!
Why so large meson exchange current effect?

Higher NN kinetic energy causes large exchange current effect

Axial exchange current & higher partial waves are important; uncertainty