# Variational study of the equation of state for hyperonic neutron stars

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## Outline

1 : Introduction

2 : Variational method for hyperonic nuclear matter

3 : Application to neutron stars

4 : Summary

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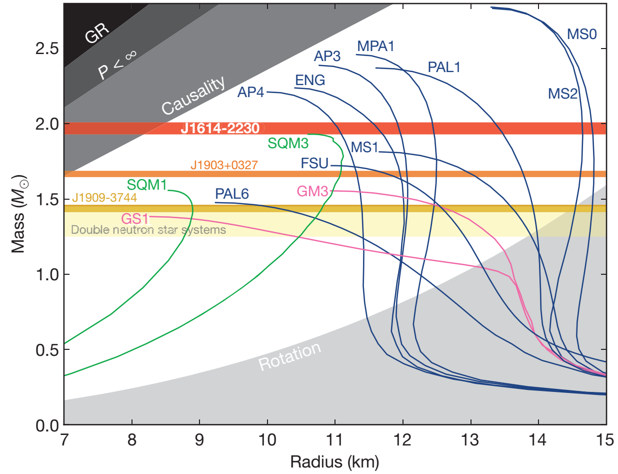
# 1: Introduction

***Hyperon interactions play important roles in the study of neutron stars.***

**Hyperons (Λ, Σ, Ξ ) are expected to appear in the core of neutron stars.**

### *HYPERON PUZZLE*

- Nuclear equation of state (EOS) becomes softer due to the hyperon mixing.
- Maximum mass of neutron star tends to be lower than the observational data.




P. B. Demorest et al., NATURE 467 (2010)

**The hyperon mixing in neutron stars has been studied with various nuclear theories.**

- **Relativistic mean field theory** (C. Ishizuka et al., J. Phys. G 35 (2008) 085201)
- **Relativistic Hartree-Fock theory** (T. Miyatsu, M. Cheoun, PRC 88 (2013) 01802)
- **Brueckner-Hatree-Fock theory** (H. Schulze, T. Rijken, PRC 84 (2011) 035801)
- **Variational many-body theory** (D. Lonardoni et al., PRL 114 (2015) 092301)

# Variational Many-Body Theory

- **Akmal, Pandharipande and Ravenhall (APR)** (PRC 58 (1998) 1804)

  Method : *Fermi Hypernetted Chain method* Potential : AV18 + UIX

  **Symmetric Nuclear Matter and Pure Neutron Matter**

- **D. Lonardoni, A. Lovato, S. Gandolfi, and F. Pederiva** (PRL 114 (2015) 092301)

  Method : *Auxiliary field diffusion Monte Carlo* Potential : AV8' + UIX + $\Lambda N$ + $\Lambda NN$

  **Hyperonic Nuclear Matter (composed only of neutrons and Λ hyperons)**

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**We have constructed the nuclear EOS for core-collapse supernovae with the variational method.**

Collaboration with M. Takano (Waseda University) , K. Sumiyoshi (Numazu College of Tech.), Y. Takehara, S. Yamamuro, K. Nakazato, H. Suzuki (Tokyo Univ. of Science)

Method : *Simple cluster variational method* Potential : AV18 + UIX

**Cold and Hot Asymmetric Nuclear Matter for arbitrary particle fractions**

(NPA902 (2013) 53, PTEP 2014 (2014) 023D05)

⇨ ***It is relatively easy to extend our simple variational method to calculate the EOS of hyperonic nuclear matter.***

# Aim of This Study

**To construct the EOS of nuclear matter**
**containing Λ and $\Sigma^-$ hyperons**
**starting from <u>bare baryon interactions</u>**
**by <u>the cluster variational method</u>**

- Our cluster variational method for asymmetric nuclear matter is extended to calculate energies of **hyperonic nuclear matter**.

- We employ **reliable hyperon interactions** so as to reproduce the experimental data on Λ hypernuclei.

(Reported by E. Hiyama)

# 2. Variational method for hyperonic nuclear matter

**Two-body Hamiltonian**

$$H_2 = -\sum_i \left[ m_i c^2 + \frac{\hbar^2}{2m_i} \nabla_i^2 \right] + \sum_{i<j} V_{ij}$$

**Two-body potential**

$$V_{ij} = \sum_{N,Y} [V_{ij}^{NN} + V_{ij}^{YN} + V_{ij}^{YY}]$$

**Three-body Hamiltonian**

$$H_3 = \sum_{i<j<k} V_{ijk}$$

**Three-nucleon potential : UIX**

*NN* potential: **AV18 two-body potential** (PRC 51 (1995) 38)

$$V_{ij}^{NN} = \sum_{p=+}^{-} \sum_{s=0}^{1} [V_{\mathrm{C}ps}(r_{ij}) + sV_{\mathrm{T}p}(r_{ij}) S_{\mathrm{T}ij} + sV_{\mathrm{SO}p}(r_{ij})(\boldsymbol{L}_{ij} \cdot \boldsymbol{s}) + V_{\mathrm{qL}ps}(r_{ij}) \left|\boldsymbol{L}_{ij}\right|^2 + sV_{\mathrm{qSO}p}(r_{ij})(\boldsymbol{L}_{ij} \cdot \boldsymbol{s})^2] P_{psij}^{\mu=NN}$$

*YN* and *YY* potential: Central three-range Gaussian potential

$$V_{ij}^{\mu} = \sum_{p=+}^{-} \sum_{s=0}^{1} V_{\mathrm{C}ps}^{\mu}(r_{ij}) P_{psij}^{\mu}$$

$P_{psij}^{\mu}$ : Projection operator

Parity: $p = (+, -)$ Total spin: $s = (1, 0)$

Baryon pair: $\mu = (NN, \Lambda\mathrm{n}, \Lambda\mathrm{p}, \Sigma^-\mathrm{n}, \Sigma^-\mathrm{p}, \Lambda\Lambda)$

# *YN* and *YY* potentials

## Λ*N* interaction (E. Hiyama et al., PRC 74 (2006) 054312)

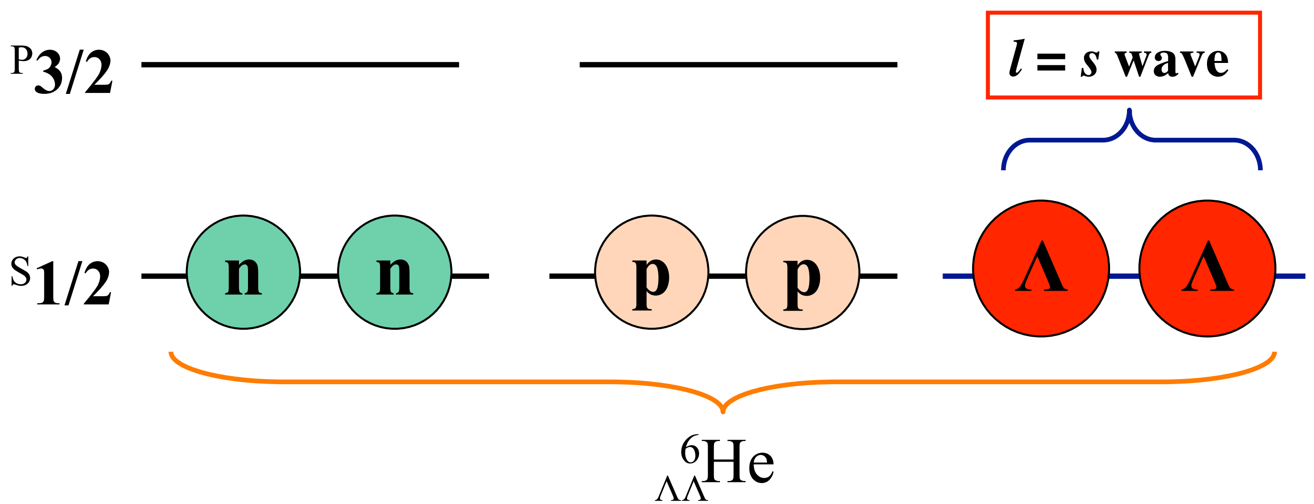
- The *ab initio* variational calculations for Λ hypernuclei reproduce their experimental eigenvalues.

## $\Sigma^{-}N$ interaction

- Based on the latest version of the **Nijmegen model ESC08**.

## ΛΛ interaction (E. Hiyama et al., PRC 66 (2002) 024007)

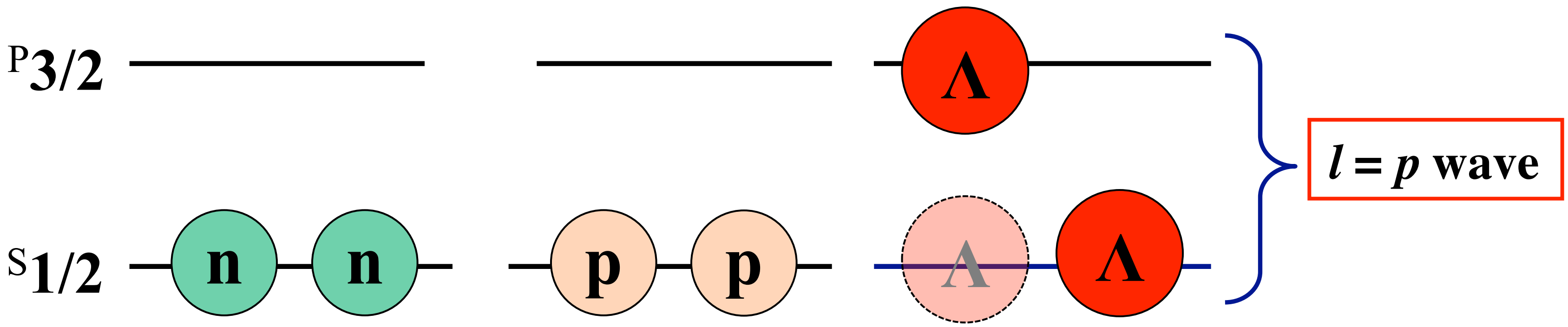
- The even-state part of the ΛΛ interaction is constructed so as to reproduce **the experimental ΛΛ binding energy given by the NAGARA event.**



# *YN* and *YY* potentials

**The odd-state part of the ΛΛ interaction is also necessary to calculate the energy of hyperonic nuclear matter.**

## ΛΛ interaction (odd-state)




- Two Λs in the experimentally known double Λ hypernuclei are in the *s*-orbit.

# *YN* and *YY* potentials

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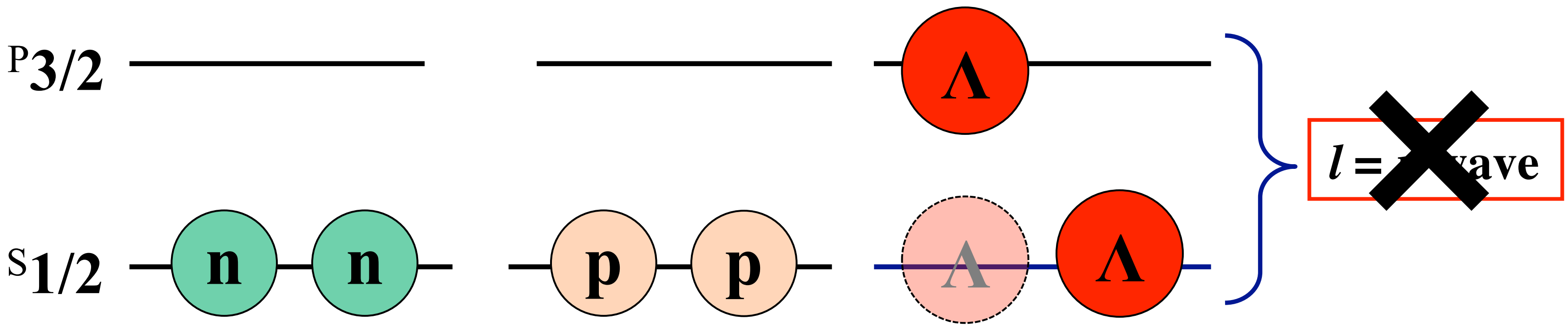



- Two Λs in the experimentally known double Λ hypernuclei are in the *s*-orbit.

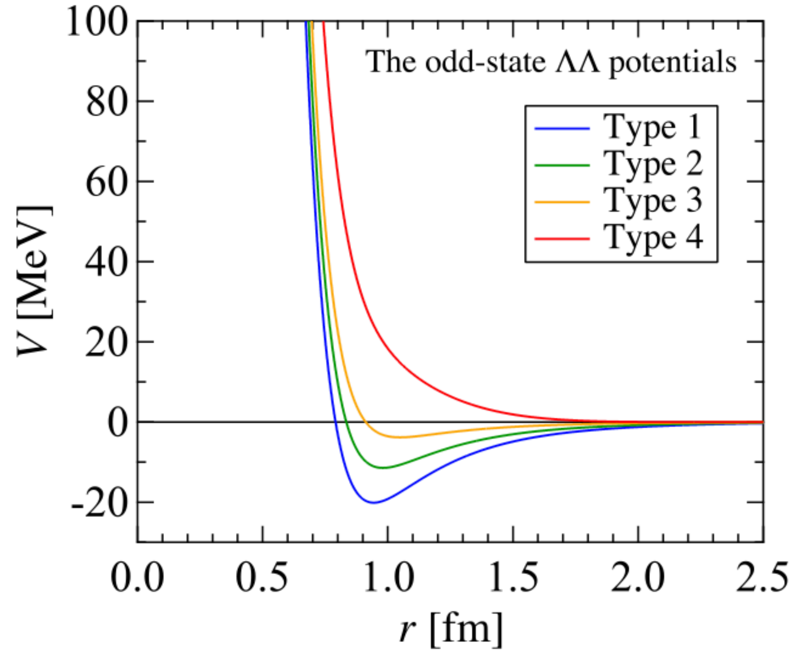
*The experimental data on hypernuclei give no information on* ***the odd-state part of the ΛΛ interactions****.*

# The odd-state part of the ΛΛ interaction

We prepare **four different models** for the odd-state part of the ΛΛ interaction.

Type 1 : *The most attractive*
Type 2 : *Less attractive*
Type 3 : *Slightly repulsive*
Type 4 : *The most repulsive*

The repulsion strength of Type 4 is comparable to that of the odd-state repulsion of Λ*N* interaction.




**The repulsive effect increases monotonically from Type 1 to Type 4**.

- We investigate the effects of **the odd-state part of bare ΛΛ interactions** on **the structure of neutron stars**.

# Expectation Value of $H_2$

**Jastrow wave function**

$$\Psi = \mathrm{Sym}\left[\prod_{i<j} f_{ij}\right] \Phi_{\mathrm{F}}$$

Sym [ ]: Symmetrizer

$\Phi_{\mathrm{F}}$: The Fermi-gas wave function

**$f_{ij}$ : Two-body correlation function**

$$f_{ij} = \sum_{p=+}^{-} \sum_{\mu} \sum_{s=0}^{1} \left[ \underbrace{f^{\mu}_{\mathrm{C}ps}(r_{ij})}_{\textbf{Central}} + \underbrace{s f^{\mu}_{\mathrm{T}p}(r_{ij})}_{\textbf{Tensor}} S_{\mathrm{T}ij} + \underbrace{s f^{\mu}_{\mathrm{SO}p}(r_{ij})}_{\textbf{Spin-orbit}} (\boldsymbol{L}_{ij} \cdot \boldsymbol{s}) \right] P^{\mu}_{psij}$$

$\boldsymbol{E_2}$ is the expectation value of $H_2$ with the Jastrow wave function in *the two-body cluster approximation*.

$$E_2(n_{\mathrm{p}}, n_{\mathrm{n}}, n_{\Lambda}, n_{\Sigma^-}) = \frac{\langle H_2 \rangle_2}{A} [f^{\mu}_{\mathrm{C}ps}, f^{\mu}_{\mathrm{T}p}, f^{\mu}_{\mathrm{SO}p}]$$

$n_{\mathrm{p}}$ : Proton number density

$n_{\mathrm{n}}$ : Neutron number density

$n_{\Lambda}$ : $\Lambda$ number density

$n_{\Sigma^-}$ : $\Sigma^-$ number density

$E_2$ is minimized with respect to $f_{\mathrm{C}ps}{}^{\mu}(r)$, $f_{\mathrm{T}p}{}^{\mu}(r)$ and $f_{\mathrm{SO}p}{}^{\mu}(r)$ by solving the Euler-Lagrange Equations with the appropriate constraints.

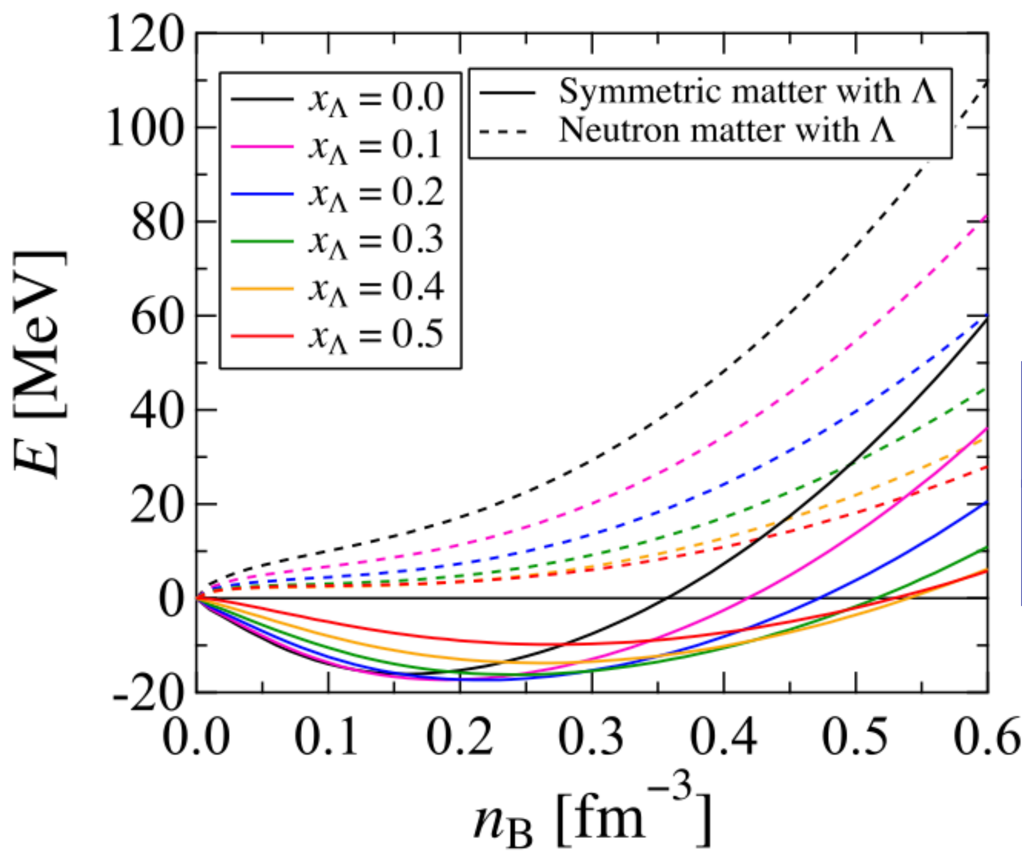
# Energy of Hyperonic Nuclear Matter

**Energy per baryon** $\boxed{E(n_p, n_n, n_\Lambda, n_{\Sigma^-}) = E_2(n_p, n_n, n_\Lambda, n_{\Sigma^-}) + E_3^N}$

Three-nucleon energy $E_3^N$

Based on the expectation value of $H_3$ with the Fermi-gas wave function

(NPA902 (2013) 53)

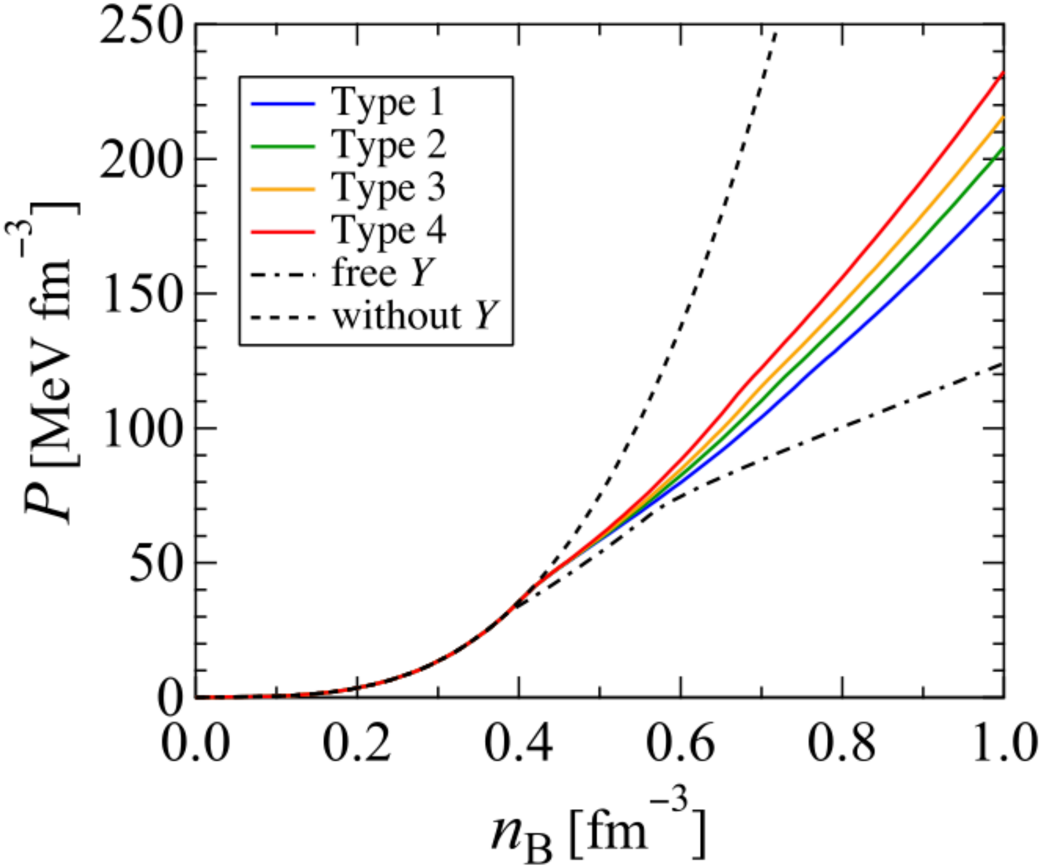



Energy of hyperonic nuclear matter (Type 1)

| $n_0$[fm$^{-3}$] | $E_0$[MeV] | $K$ [MeV] | $E_{sym}$[MeV] |
|---|---|---|---|
| 0.16 | −16.1 | 245 | 30.0 |

# 3. Application to neutron stars

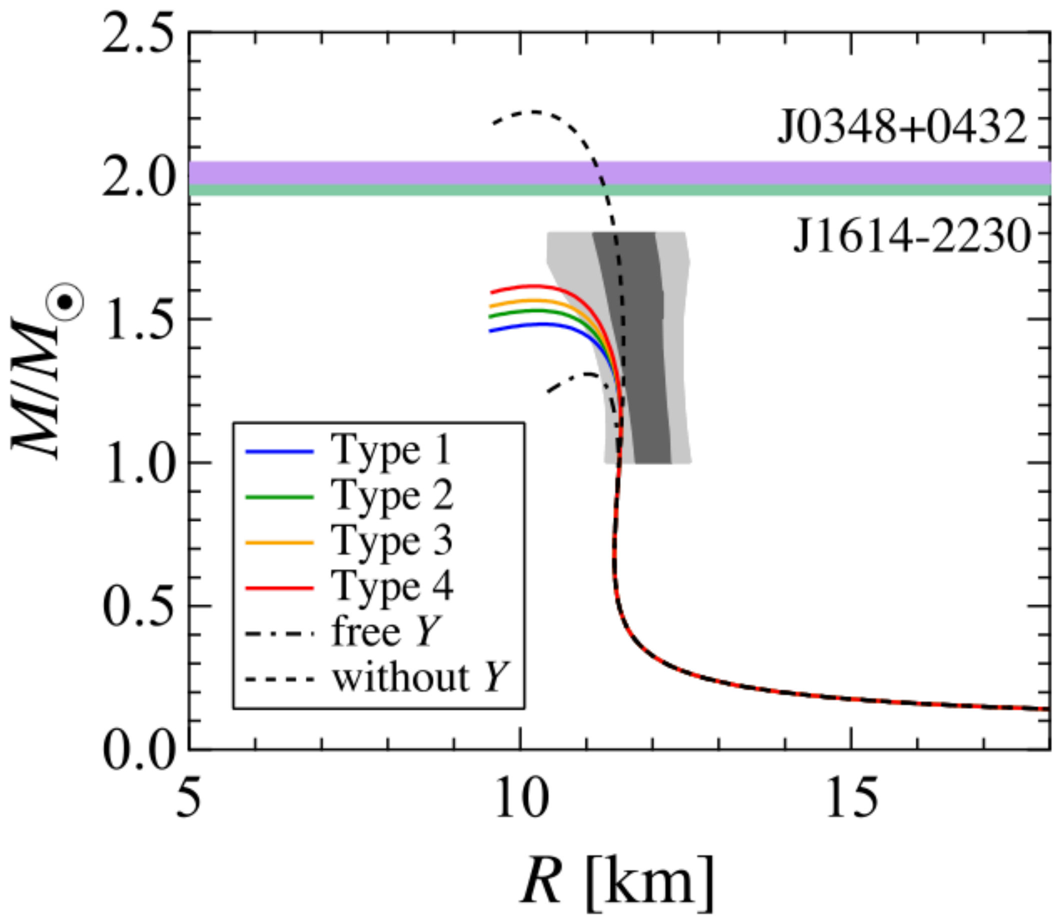
We investigate the effects of
**the odd-state part of bare ΛΛ interactions**
on **the structure of neutron stars**.




Pressure of neutron star matter

Type 1 : *The most attractive*
Type 2 : *Less attractive*
Type 3 : *Slightly repulsive*
Type 4 : *The most repulsive*

# Gravitational Mass of Neutron Stars




Mass-radius relations of neutron stars

**Maximum mass of neutron stars**

| | |
|---|---|
| Type 1 | 1.48 $M_\odot$ |
| Type 2 | 1.53 $M_\odot$ |
| Type 3 | 1.57 $M_\odot$ |
| Type 4 | 1.62 $M_\odot$ |
| free $Y$ | 1.31 $M_\odot$ |
| without $Y$ | 2.22 $M_\odot$ |

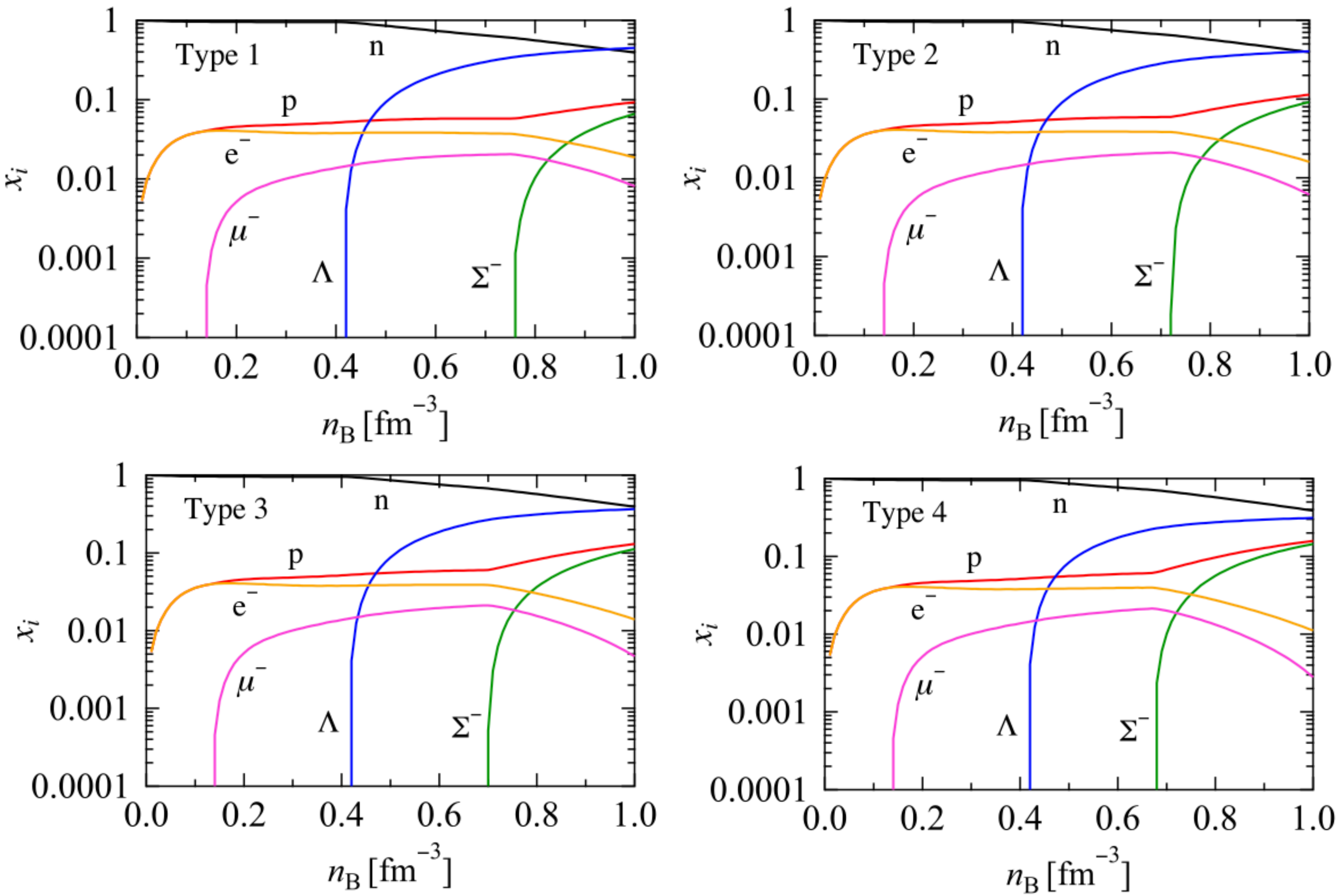
*The maximum mass increases by about 9%.*

*(1.48 $M_\odot \rightarrow$ 1.62 $M_\odot$)*

J0348+0432: Science 340 (2013) 1233232 J1614-2230: Nature 467 (2010) 1081

Shaded region is the observationally suggested region by Steiner et al. (Astrophys. J. 722 (2010) 33)

# Composition of Neutron Star Matter



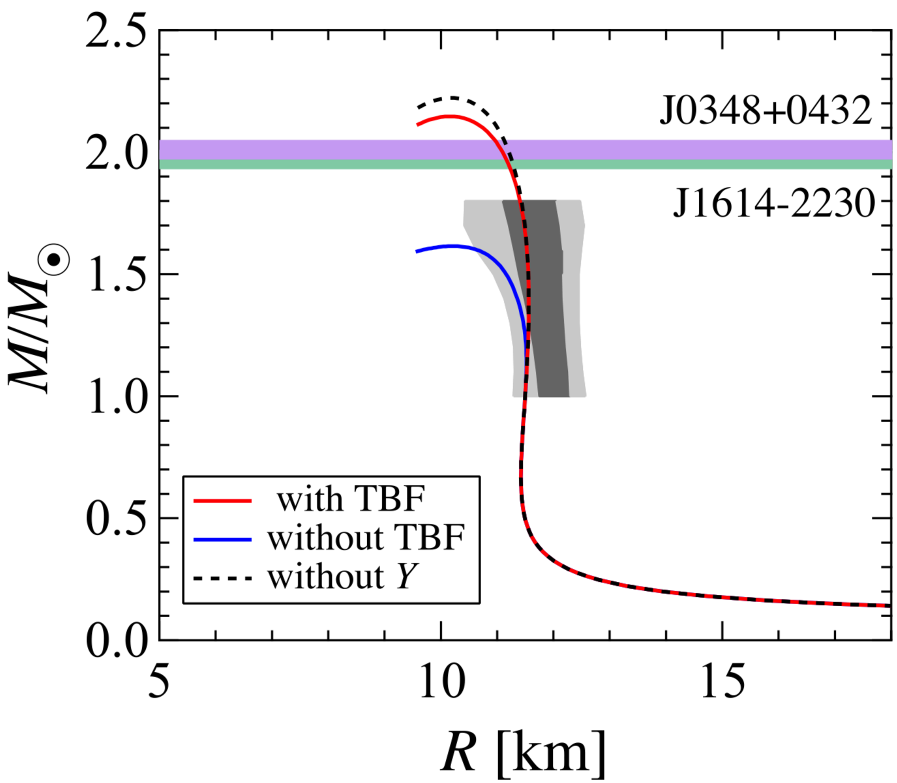

Onset density of Λ is insensitive to the odd-state part of the ΛΛ interaction.

Onset density of $\Sigma^-$ is strongly depends on the odd-state part of the ΛΛ interaction.

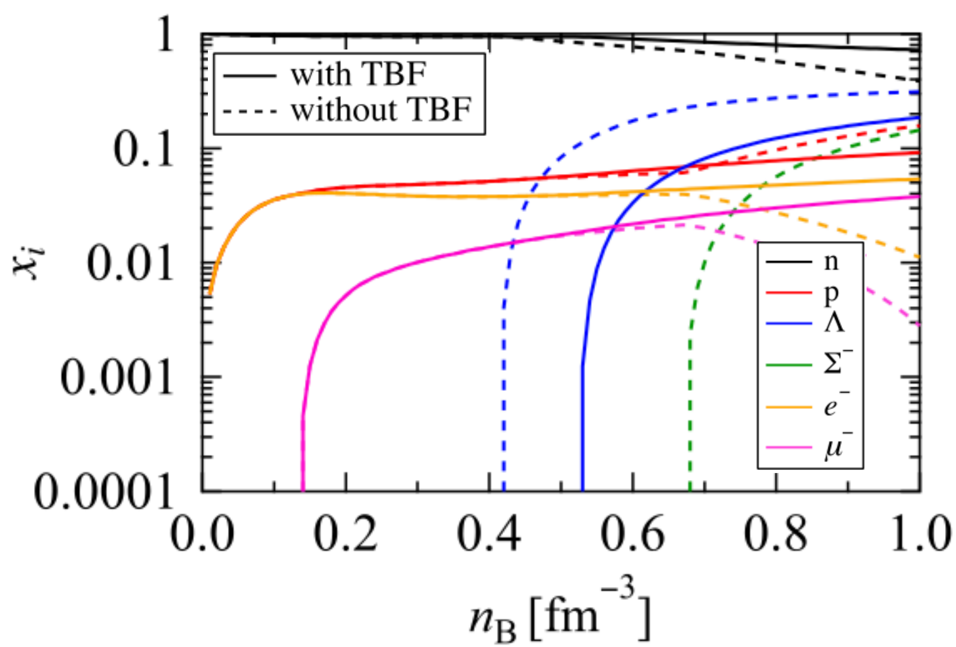
# Neutron Star Matter with Three-Baryon Force

We take into account a density dependent two-body effective potential **as a phenomenological three-baryon repulsive force (TBF)**.

Y. Yamamoto et al., PRC 90 (2014) 045805




Mass-radius relations of neutron stars (Type 4)




Composition of neutron star matter (Type 4)

# 4. Summary

**We construct the EOS of nuclear matter containing Λ and $\Sigma^-$ hyperons by <u>the cluster variational method</u>.**

**We investigate the effects of the odd-state ΛΛ interactions on the structure of neutron stars.**

- The repulsion in the odd-state ΛΛ interaction raises the maximum mass of neutron stars by about 9%. ($1.48\ M_\odot \rightarrow 1.62\ M_\odot$)
- The onset density of $\Sigma^-$ strongly depends on the odd-state ΛΛ interaction.
- Maximum mass of neutron stars with TBF is consistent with the observational data.

## Future Plans

- Taking into account mixing of other hyperons ($\Sigma^0$, $\Sigma^+$, $\Xi^0$, $\Xi^-$)
- Calculation of triple-Λ hypernuclei
- Employing more sophisticated baryon interactions (e.g. Nijmegen)