

# Predictions for Leptonic Dirac CP Violation

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After the measurement of  $\sin \theta_{13} = 0.15$  (Daya Bay, RENO, Double Chooz), one of the next most important goals of the future research in neutrino physics - determine the status of the CP symmetry in the lepton sector.

All compelling  $\nu$ -oscillation data is compatible with 3- $\nu$  mixing:

$$\nu_{l\text{L}}(x) = \sum_{j=1}^3 U_{lj} \nu_{j\text{L}}(x), \quad l = e, \mu, \tau.$$

B. Pontecorvo, 1957; 1958; 1967;  
Z. Maki, M. Nakagawa, S. Sakata, 1962;

$U$  is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix.

## The current "reference scheme": 3- $\nu$ mixing

$$|\nu_l\rangle = \sum_{j=1}^n U_{lj}^* |\nu_j\rangle, \quad \nu_j : m_j \neq 0; \quad l = e, \mu, \tau; \quad n = 3;$$

$$\nu_{l\text{L}}(x) = \sum_{j=1}^3 U_{lj} \nu_{j\text{L}}(x), \quad \nu_{j\text{L}}(x) : m_j \neq 0; \quad l = e, \mu, \tau.$$

The PMNS matrix  $U$  -  $3 \times 3$  unitary to a good approximation (at least:  $|U_{l,n}| \lesssim (\ll) 0.1$ ,  $l = e, \mu$ ,  $n = 4, 5, \dots$ ).

$\nu_j, m_j \neq 0$ : Dirac or Majorana particles.

**Data:** the 3  $\nu$ s are light:  $\nu_{1,2,3}, m_{1,2,3} \lesssim 1$  eV.

**3- $\nu$  mixing:** 3-flavour neutrino oscillations possible.

$\nu_\mu, E$ ; at distance  $L$ :  $P(\nu_\mu \rightarrow \nu_\tau) \neq 0$ ,  $P(\nu_\mu \rightarrow \nu_\mu) < 1$

$$P(\nu_l \rightarrow \nu_l) = P(\nu_l \rightarrow \nu_l; E, L; U, m_j^2 - m_k^2)$$

# Three Neutrino Mixing

$$\nu_{l\text{L}} = \sum_{j=1}^3 U_{lj} \nu_{j\text{L}} .$$

$U$  is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- $U - \textcolor{red}{n} \times \textcolor{red}{n}$  unitary:

$$\textcolor{red}{n} \quad \begin{matrix} 2 \\ 3 \\ 4 \end{matrix}$$

$$\text{mixing angles:} \quad \frac{1}{2}n(n-1) \quad \begin{matrix} 1 \\ 3 \\ 6 \end{matrix}$$

CP-violating phases:

- $\nu_j$  – Dirac:  $\frac{1}{2}(n-1)(n-2)$   $0$   $1$   $3$

- $\nu_j$  – Majorana:  $\frac{1}{2}\textcolor{red}{n}(\textcolor{red}{n}-1)$   $1$   $3$   $6$

$n = 3$ : 1 Dirac and

$\textcolor{blue}{2}$  additional CP-violating phases, Majorana phases

# PMNS Matrix: Standard Parametrization

$$U = V P, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin \theta_{ij}$ ,  $c_{ij} \equiv \cos \theta_{ij}$ ,  $\theta_{ij} \in [0, \frac{\pi}{2}]$ ,
- $\delta$  - Dirac CPV phase,  $\delta \in [0, 2\pi]$ ; CP inv.:  $\delta = 0, \pi, 2\pi$ ;
- $\alpha_{21}, \alpha_{31}$  - Majorana CPV phases; CP inv.:  $\alpha_{21(31)} = k(k')\pi$ ,  $k(k') = 0, 1, 2\dots$   
S.M. Bilenky, J. Hosek, S.T.P., 1980
- $\Delta m^2_0 \equiv \Delta m^2_{21} \cong 7.54 \times 10^{-5} \text{ eV}^2 > 0$ ,  $\sin^2 \theta_{12} \cong 0.308$ ,  $\cos 2\theta_{12} \gtrsim 0.28$  ( $3\sigma$ ),
- $|\Delta m^2_{31(32)}| \cong 2.47$  ( $2.42$ )  $\times 10^{-3} \text{ eV}^2$ ,  $\sin^2 \theta_{23} \cong 0.437$  ( $0.455$ ), NO (IO),
- $\theta_{13}$  - the CHOOZ angle:  $\sin^2 \theta_{13} = 0.0234$  (0.0240), Capozzi et al. NO (IO).  
F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)

- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.54 \times 10^{-5} \text{ eV}^2 > 0$ ,  $\sin^2 \theta_{12} \cong 0.308$ ,  $\cos 2\theta_{12} \gtrsim 0.28$  ( $3\sigma$ ),
- $|\Delta m_{31(32)}^2| \cong 2.47$  ( $2.42$ )  $\times 10^{-3} \text{ eV}^2$ ,  $\sin^2 \theta_{23} \cong 0.437$  ( $0.455$ ), NO (IO),
- $\theta_{13}$  - the CHOOZ angle:  $\sin^2 \theta_{13} = 0.0234$  ( $0.0240$ ), NH (IH).
- $1\sigma(\Delta m_{21}^2) = 2.6\%$ ,  $1\sigma(\sin^2 \theta_{12}) = 5.4\%$ ;
- $1\sigma(|\Delta m_{31(23)}^2|) = 2.6\%$ ,  $1\sigma(\sin^2 \theta_{23}) = 9.6\%$ ;
- $1\sigma(\sin^2 \theta_{13}) = 8.5\%$ ;
- $3\sigma(\Delta m_{21}^2) : (6.99 - 8.18) \times 10^{-5} \text{ eV}^2$ ;  $3\sigma(\sin^2 \theta_{12}) : (0.259 - 0.359)$ ;
- $3\sigma(|\Delta m_{31(23)}^2|) : 2.27(2.23) - 2.65(2.60) \times 10^{-3} \text{ eV}^2$ ;  
 $3\sigma(\sin^2 \theta_{23}) : 0.374(0.380) - 0.628(0.641)$ ;
- $3\sigma(\sin^2 \theta_{13}) : 0.0176(0.0178) - 0.0295(0.0298)$ .

F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)

- **Dirac phase**  $\delta$ :  $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ ,  $l \neq l'$ ;  $A_{CP}^{(l,l')} \propto J_{CP} \propto \sin \theta_{13} \sin \delta$

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

Current data:  $|J_{CP}| \lesssim 0.040 |\sin \delta|$  (can be relatively large!).

- **Majorana phases**  $\alpha_{21}, \alpha_{31}$ :

–  $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$  not sensitive;

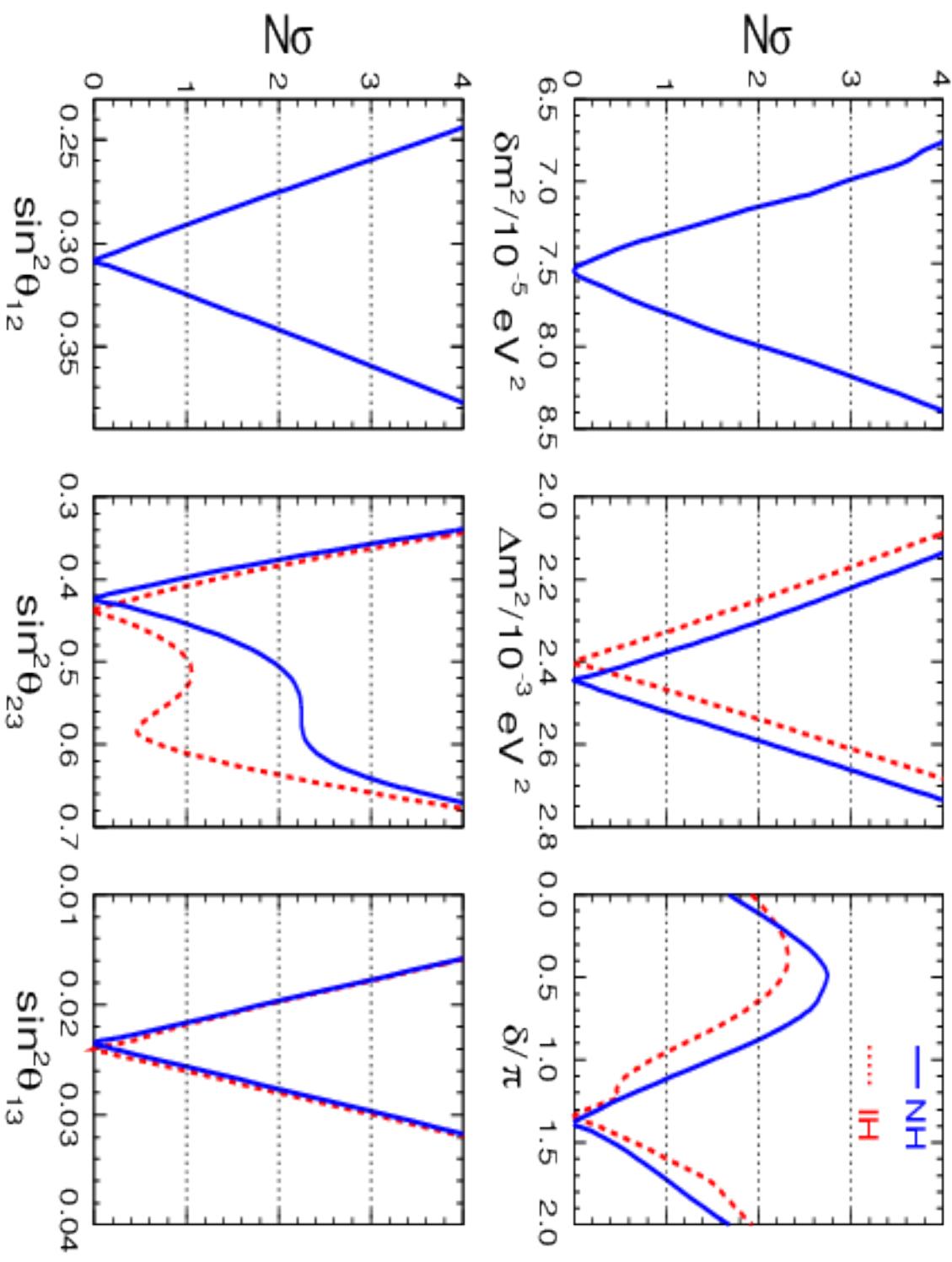
S.M. Bilenky, J. Hosek, S.T.P., 1980;  
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

- $|\langle m \rangle|$  in  $(\beta\beta)_{0\nu}$ -decay depends on  $\alpha_{21}, \alpha_{31}$ ;
- $\Gamma(\mu \rightarrow e + \gamma)$  etc. in SUSY theories depend on  $\alpha_{21,31}$ ;
- BAU, leptogenesis scenario:  $\delta, \alpha_{21,31}$  !

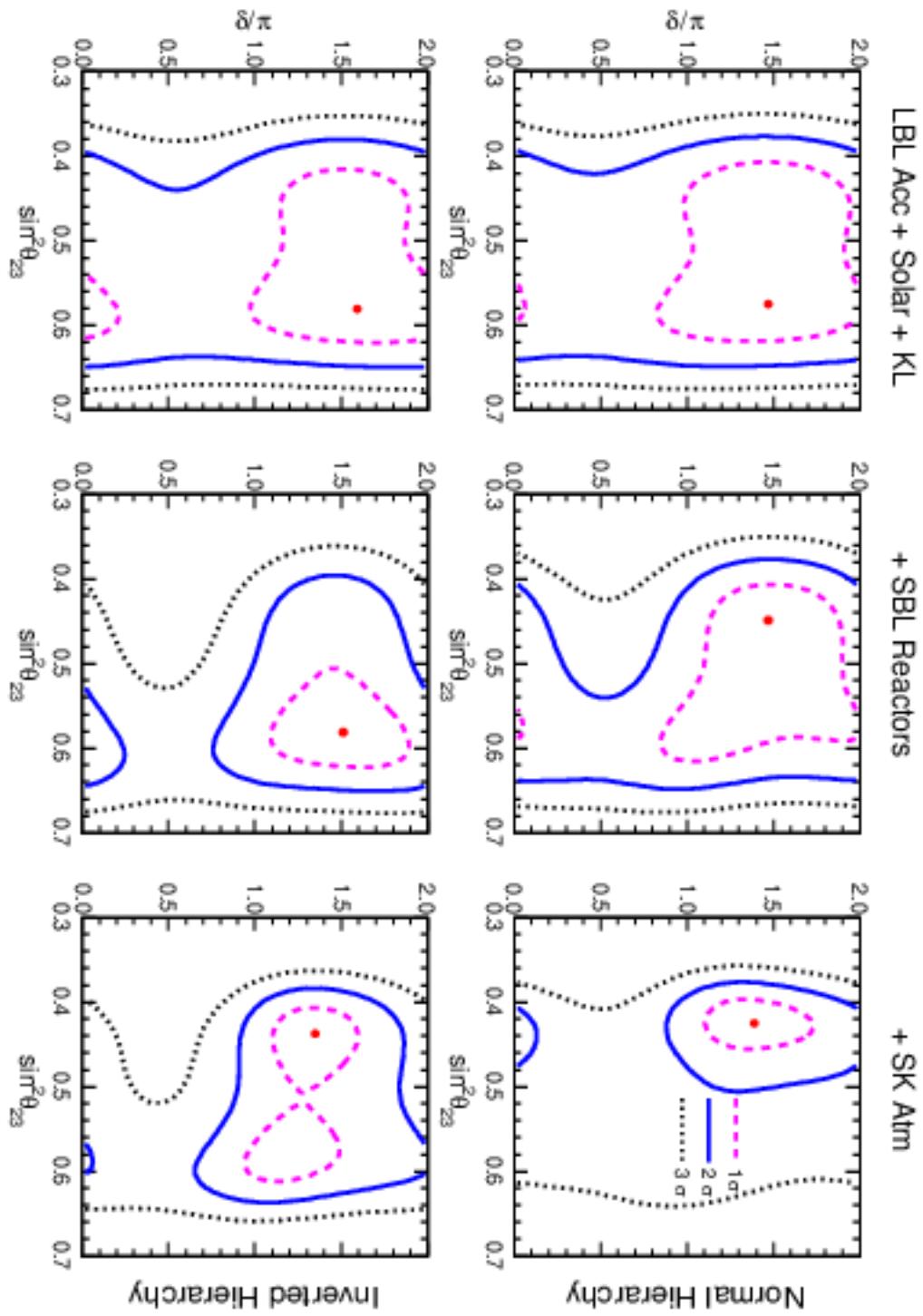
The next most important steps are:

- determination of the status of the CP symmetry in the lepton sector;
- determination of the nature - Dirac or Majorana, of massive neutrinos.
- determination of the neutrino mass hierarchy;
- determination of the absolute neutrino mass scale (or  $\min(m_j)$ ).

# LBL Acc + Solar + KL + SBL Reactors + SK Atm



F. Capozzi, E. Lisi et al., arXiv:1312.2878



F. Capozzi, E. Lisi et al., arXiv:1312.2878

## Large $\sin \theta_{13} \cong 0.15$ (Daya Bay, RENO) + $\delta = 3\pi/2$ – far-reaching implications:

- For the searches for CP violation in  $\nu$ -oscillations; for the b.f.v. one has  $J_{CP} \cong -0.035$ ;
- Important implications also for the “flavoured” leptonogenesis scenario of generation of the baryon asymmetry of the Universe (BAU).

If all CPV, necessary for the generation of BAU is due to  $\delta$ , a necessary condition for reproducing the observed BAU is

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.09$$

S. Pascoli, S.T.P., A. Riotto, 2006.

# Understanding the Pattern of Neutrino Mixing. Predictions for the CPV Phase $\delta$ .

## Neutrino Mixing: New Symmetry?

- $\theta_{12} = \theta_\odot \cong \frac{\pi}{5.4}$ ,    $\theta_{23} = \theta_{\text{atm}} \cong \frac{\pi}{4}(\text{?})$ ,    $\theta_{13} \cong \frac{\pi}{20}$

$$U_{\text{PMNS}} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}(\text{?}) \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}(\text{?}) \end{pmatrix};$$

Very different from the CKM-matrix!

- $\theta_{12} \cong \pi/4 - 0.20$ ,    $\theta_{13} \cong 0 + \pi/20$ ,    $\theta_{23} \cong \pi/4 \mp 0.10$ .
- $U_{\text{PMNS}}$  due to new approximate symmetry?

## A Natural Possibility (vast literature):

$$U = U_{\text{lep}}^\dagger(\theta_{ij}^\ell, \delta^\ell) \ Q(\psi, \omega) U_{\text{TBM}, \text{BM}, \text{LC}, \dots} \ \bar{P}(\xi_1, \xi_2),$$

with

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\text{BM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \pm\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \pm\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \mp\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $U_{\text{lep}}^\dagger(\theta_{ij}^\ell, \delta^\ell)$  - from diagonalization of the  $l^-$  mass matrix;
- $U_{\text{TBM}, \text{BM}, \text{LC}, \dots}$   $\bar{P}(\xi_1, \xi_2)$  - from diagonalization of the  $\nu$  mass matrix;
- $Q(\psi, \omega)$ , - from diagonalization of the  $l^-$  and/or  $\nu$  mass matrices.

$U_{\text{LC}}, U_{\text{GRAM}}, U_{\text{GRBM}}, U_{\text{HGM}}$ :

$$U_{\text{LC}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{\sqrt{2}}{c_{23}^\nu} & \frac{c_{23}^\nu}{\sqrt{2}} & s_{23}^\nu \\ \frac{s_{23}^\nu}{\sqrt{2}} & -\frac{s_{23}^\nu}{\sqrt{2}} & c_{23}^\nu \end{pmatrix}; \quad \mu - \tau \text{ symmetry: } \theta_{23}^\nu = \mp\pi/4;$$

$$U_{\text{GR}} = \begin{pmatrix} c_{12}^\nu & s_{12}^\nu & 0 \\ -\frac{s_{12}^\nu}{\sqrt{2}} & \frac{c_{12}^\nu}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \\ -\frac{s_{12}^\nu}{\sqrt{2}} & \frac{c_{12}^\nu}{\sqrt{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\text{HGM}} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{pmatrix}, \quad \theta_{12}^\nu = \pi/6.$$

$U_{\text{GRAM}}$ :  $\sin^2 \theta_{12}^\nu = (2+r)^{-1} \cong 0.276$ ,  $r = (1 + \sqrt{5})/2$   
 $(\mathbf{GR}$ :  $r/1$ ;  $a/b = a + b/a$ ,  $a > b$ )

$U_{\text{GRBM}}$ :  $\sin^2 \theta_{12}^\nu = (3-r)/4 \cong 0.345$ .

- $U_{\text{TBM}}:$   $s_{12}^2 = 1/3$ ,  $s_{23}^2 = 1/2$ ,  $s_{13}^2 = 0$ ;  $s_{13}^2 = 0$  must be corrected; if  $\theta_{23} \neq \pi/4$ ,  $s_{23}^2 = 0.5$  must be corrected .
- $U_{\text{BM}}:$   $s_{12}^2 = 1/2$ ,  $s_{23}^2 = 1/2$ ,  $s_{13}^2 = 0$ ,  $s_{12}^2 = 1/2$  and possibly  $s_{23}^2 = 1/2$  must be corrected.

$U_{\text{TBM(BM)}}$ : Groups  $A_4$ ,  $S_4$ ,  $T'$ , ... (vast literature)

(Reviews: G. Altarelli, F. Feruglio, arXiv:1002.0211; M. Tanimoto et al., arXiv:1003.3552; S. King and Ch. Luhn, arXiv:1301.1340)

- $U_{\text{GRA}}$ : Group  $A_5, \dots$ ;  $s_{13}^2 = 0$  and possibly  $s_{12}^2 = 0.276$  and  $s_{23}^2 = 1/2$  must be corrected.
- $U_{\text{LC}}$ : alternatively  $U(1)$ ,  $L' = L_e - L_\mu - L_\tau$   
L. Everett, A. Stuart, arXiv:0812.1057; ...
- $U_{\text{LC}}$ :  $s_{12}^2 = 1/2$ ,  $s_{13}^2 = 0$ ,  $s_{23}^2$  - free parameter;  
 $s_{13}^2 = 0$  and  $s_{12}^2 = 1/2$  must be corrected.

- $U_{\text{GRB}}$ :  $s_{13}^2 = 0$  and possibly  $s_{12}^2 = 0.345$  and  $s_{23}^2 = 1/2$  must be corrected.

- $U_{\text{HG}}$ :  $s_{13}^2 = 0$ ,  $s_{12}^2 = 0.25$  and possibly  $s_{23}^2 = 1/2$  must be corrected.

**None of the symmetries leading to  $U_{\text{TBM}}$ ,  $U_{\text{BM}}$  or other approximate forms of  $U_{\text{PMNS}}$  can be exact.**

**Which is the correct approximate symmetry, i.e., approximate form of  $U_{\text{PMNS}}$  (if any)?**

In the cases of  $U_{\nu}$  given by  $U_{\text{TBM}}$ ,  $U_{\text{BM}}$ , etc. the requisite corrections of some of the mixing angles are small and can be considered as perturbations to the corresponding symmetry values.

Depending on the symmetry leading to  $U_{\text{TBM}, \text{BM}}$ , etc. and on the form of  $U_{\text{lep}}$ , one obtains different experimentally testable predictions for the sum of the neutrino masses, the neutrino mass spectrum, the nature (Dirac or Majorana) of  $\nu_j$  and the CP violating phases in the neutrino mixing matrix. Future data will help us understand whether there is some new fundamental symmetry behind the observed patterns of neutrino mixing and  $\Delta m_{ij}^2$ .

**What is the minimal  $U_{\text{lep}}$  providing the requisite corrections to  $U_{\text{TBM}, \text{BM}, \text{LC}, \text{GRM}, \text{HGM}}$ ?**

## Predictions for $\delta$

Assume:

$$\bullet U_{PMNS} = U_{\text{lept}}^\dagger(\theta_{ij}^\ell, \delta^\ell) Q(\psi, \omega) U_{\text{TBM, BM, GR, HG}} \bar{P}(\xi_1, \xi_2),$$

- $U_{\text{lept}}^\dagger$  - minimal, such that
  - i)  $\sin \theta_{13} \cong 0.16$ ; BM:  $\sin^2 \theta_{12} \cong 0.31$ ;
  - ii)  $\sin^2 \theta_{23}$  can deviate significantly (by more than  $\sin^2 \theta_{13}$ ) from 0.5 (b.f.v. = 0.40-0.45).

The simplest case ( $SU(5) \times T'$ , ...)

$U_{\text{lep}} \cong O_{12}^\ell(\theta_{12}^\ell)$ ; now  $Q = \text{diag}(e^{i\varphi}, \mathbf{1}, \mathbf{1})$ ;

$\sin^\ell \theta_{13}$ ,  $\sin^\ell \theta_{23}$  - negligibly small ( $SU(5) \times T'$ , ...).

$U_{\text{BM}}(\text{LC})$ :  $\sin^2 \theta_{12} = \frac{1}{2} + \sin 2\theta_{12}' \sin \theta_{13} \cos \delta$ ,

$\delta$  is the Dirac CPV phase;

$U_{\text{BM}}$ : requires  $\cos \delta \cong \mp 1$  as  $\sin 2\theta_{12}' = \pm 1$ .

$U_{\text{TBM}}$ :  $\sin^2 \theta_{12} = \frac{1}{3} \mp 2 \frac{\sqrt{2}}{3} \sin \theta_{13} \cos \delta$ .

Problem for  $U_{\text{TBM,BM,GRA(B),HG}}$  if  $\sin^2 \theta_{23} \cong 0.44 - 0.45$ :

$$\sin^2 \theta_{23} = \frac{1 - 2 \sin^2 \theta_{13}}{2(1 - \sin^2 \theta_{13})} \cong 0.5(1 - \sin^2 \theta_{13}).$$

Larger correction to  $\sin^2 \theta'_{23} = 0.5$  needed.

**Minimal case:**  $U_{\text{lep}} \cong U_{\text{lep}}(\theta_{12}^\ell, \theta_{23}^\ell)$ ,  
 $Q = \text{diag}(1, e^{-i\psi}, e^{-i\omega})$ .

Two possibilities.

"Standard" Ordering:  
 $U_{\text{lep}}(\theta_{12}^\ell, \theta_{23}^\ell) = O_{23}^T(\theta_{23}^\ell) O_{12}^T(\theta_{12}^\ell)$  (**GUTs typically**);  
in many theories - a consequence of  $m_e^2 \ll m_\mu^2 \ll m_\tau^2$ .

"Inverse" Ordering:

$$U_{\text{lep}}(\theta_{12}^\ell, \theta_{23}^\ell) = O_{12}^T(\theta_{12}^\ell) O_{23}^T(\theta_{23}^\ell)$$

## Standard Ordering

$$U = O_{12}(\theta_{12}^\ell)O_{23}(\theta_{23}^\ell)\text{diag}(1, e^{-i\psi}, e^{-i\omega})O_{23}(\theta_{23}^\nu)O_{12}(\theta_{12}^\nu)\bar{P},$$
$$\bar{P} = \text{diag}(1, e^{i\xi_1}, e^{i\xi_2}).$$

Can be shown to be equivalent to:

$$U = O_{12}(\theta_{12}^\ell)\text{diag}(1, e^{i\phi}, 1)O_{23}(\hat{\theta}_{23})O_{12}(\theta_{12}^\nu)\bar{P}(\xi_1, \xi_2 + \beta)$$

$$\hat{\theta}_{23} = \hat{\theta}_{23}(\theta_{23}^\ell, \psi - \omega, \theta_{23}^\nu), \quad \phi = \phi(\theta_{23}^\ell, \psi, \omega, \theta_{23}^\nu).$$

$$\theta_{12}^\nu = \pi/4 \text{ (BIM, LC)}, \text{ or } \sin^{-1}(1/\sqrt{3}) \text{ (TBM)}, \text{ etc.}$$

Thus,  $\theta_{12}, \theta_{23}, \theta_{13}, \delta$  - functions of  $\theta_{12}^\ell, \phi, \hat{\theta}_{23}$ .

$\phi$  serves as a "source" for  $\delta$ .

Expect  $\delta = \delta(\theta_{12}, \theta_{23}, \theta_{13})(!)$

For arbitrary fixed  $\theta_{12}^\nu$ :

$$\cos \delta = \frac{\tan \theta_{23}}{\sin 2\theta_{12} \sin \theta_{13}} \left[ \cos 2\theta_{12}^\nu + (\sin^2 \theta_{12} - \cos^2 \theta_{12}^\nu) (1 - \cot^2 \theta_{23} \sin^2 \theta_{13}) \right].$$

This results is exact. For TBM and BM cases derived first in:

S.T.P., arXiv:1405.6006

D. Marzocca, S.T.P., A. Romanino, M.C. Sevilla, arXiv:1302.0423

In a complete self-consistent theory corrections are possible:  $\theta_{13}^e \neq 0$  (see further), RG running effects (negligible for  $m_0 \lesssim 0.01$  eV), etc.

Comparing the imaginary and real parts of  $U_{e1}^* U_{\mu 3}^* U_{e3} U_{\mu 1}$  in the two parametrisations:

$$\sin \delta = - \frac{\sin 2\theta_{12}^\nu}{\sin 2\theta_{12}} \sin \phi,$$

$$\cos \delta = \frac{\sin 2\theta_{12}^\nu}{\sin 2\theta_{12}} \cos \phi \left( \frac{2 \sin^2 \theta_{23}}{\sin^2 \theta_{23} \cos^2 \theta_{13} + \sin^2 \theta_{13}} - 1 \right) \\ + \frac{\cos 2\theta_{12}^\nu}{\sin 2\theta_{12}} \frac{\sin 2\theta_{23} \sin \theta_{13}}{\sin^2 \theta_{23} \cos^2 \theta_{13} + \sin^2 \theta_{13}}.$$

S.T.P., arXiv:1405.6006

The relations are exact.

$$\sin \theta_{13} = |U_{e3}| = \sin \theta_{12}^\ell \sin \hat{\theta}_{23},$$

$$\begin{aligned}\sin^2 \theta_{23} &= \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2} = \sin^2 \hat{\theta}_{23} \frac{\cos^2 \theta_{12}^\ell}{1 - \sin^2 \theta_{12}^\ell \sin^2 \hat{\theta}_{23}}, \\ \sin^2 \theta_{12} &= \frac{|U_{e2}|^2}{1 - |U_{e3}|^2} = \frac{|\sin \theta_{12}^\nu \cos \theta_{12}^\ell + e^{i\phi} \cos \theta_{12}^\nu \cos \hat{\theta}_{23} \sin \theta_{12}^\ell|^2}{1 - \sin^2 \theta_{12}^\ell \sin^2 \hat{\theta}_{23}},\end{aligned}$$

$$\sin^2 \theta_{23} = \frac{\sin^2 \hat{\theta}_{23} - \sin^2 \theta_{13}}{1 - \sin^2 \theta_{13}}, \quad \hat{\theta}_{23} \cong \theta_{23}.$$

From first two eqs.:

$$\theta_{12}^\ell = \theta_{12}^\ell(\theta_{13}, \theta_{23}), \quad \hat{\theta}_{23} = \hat{\theta}_{23}(\theta_{13}, \theta_{23});$$

substitute in the 3rd:  $\cos \phi = \cos \phi(\theta_{12}, \theta_{23}, \theta_{13}, \theta_{12}^\nu)$ .

$$\sin^2 \hat{\theta}_{23} = \frac{1}{2} \left( 1 - 2 \sin \theta_{23}^\ell \cos \theta_{23}^\ell \cos(\omega - \psi) \right),$$

$$\phi = \arg \left( e^{-i\psi} \cos \theta_{23}^\ell + e^{-i\omega} \sin \theta_{23}^\ell \right),$$

$$\gamma = \arg \left( -e^{-i\psi} \cos \theta_{23}^\ell + e^{-i\omega} \sin \theta_{23}^\ell \right),$$

$$\bar{P} = \text{diag}(1, e^{i\xi_1}, e^{i(\xi_2 + \beta)}), \quad \beta = \gamma - \phi,$$

$$U = O_{12}(\theta_{12}^\ell) \text{diag}(1, e^{i\phi}, 1) O_{23}(\hat{\theta}_{23}) O_{12}(\theta_{12}'^\nu) \bar{P}.$$

In all cases TBM, BM (LC), GRA, GRB, HG:

- New sum rules relating  $\theta_{12}, \theta_{13}, \theta_{23}$  and  $\delta$ ;
- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13})$ .
- TBM case:  $\delta \cong 3\pi/2$  or  $\pi/2$ ;  
b.f.v. of  $\theta_{ij}$ :  $\delta \cong 263.5^\circ$  or  $96.5^\circ$ .
- GRA case, b.f.v. of  $\theta_{ij}$ :  $\delta \cong 286.8^\circ$  or  $73.2^\circ$ .
- GRB case, b.f.v. of  $\theta_{ij}$ :  $\delta \cong 258.5^\circ$  or  $101.5^\circ$ .
- HG case, b.f.v. of  $\theta_{ij}$ :  $\delta \cong 298.4^\circ$  or  $61.6^\circ$ .
- BM, LC cases:  $\delta \cong \pi$  ( $\sin^2 \theta_{12} = 0.32$ , b.f.v. of  $\theta_{13,23}$ )

- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13})$ .
- TBM case:  $\delta \cong 3\pi/2$  or  $\pi/2$ ; b.f.v. of  $\theta_{ij}$ :  
 $\delta \cong 263.5^\circ$  or  $96.5^\circ$ ,  $\cos \delta = -0.114$ ,  $J_{CP} \cong \mp 0.034$ .
- GRAM case, b.f.v. of  $\theta_{ij}$ :  $\delta \cong 286.8^\circ$  or  $73.2^\circ$ ;  
 $\cos \delta = 0.289$ ,  $J_{CP} \cong \mp 0.0327$ .
- GRBM case, b.f.v. of  $\theta_{ij}$ :  $\delta \cong 258.5^\circ$  or  $101.5^\circ$ ;  
 $\cos \delta = -0.200$ ,  $J_{CP} \cong 0.0333$ .
- HGM case, b.f.v. of  $\theta_{ij}$ :  $\delta \cong 298.4^\circ$  or  $61.6^\circ$ ;  
 $\cos \delta = 0.476$ ,  $J_{CP} \cong \mp 0.0299$ .
- BM, LC cases:  $\delta \cong \pi$ ,  $\cos \delta \cong -0.978$ ,  $J_{CP} \cong \mp 0.008$

The results shown - for NO neutrino mass spectrum; the results are practically the same for IO spectrum. (Best fit values of  $\theta_{ij}$ : F. Capozzi et al., arXiv:1312.2878v1.)

By measuring  $\cos \delta$  one can distinguish between different symmetry forms of  $\tilde{U}_\nu$ !

Relatively high precision measurement of  $\delta$  will be performed at the future planned neutrino oscillation experiments, see, e.g., A. de Gouvea *et al.*, arXiv:1310.4340, P. Coloma *et al.*, arXiv:1203.5651.

For  $\theta_{13}^e \neq 0$ ,  $|\sin \theta_{13}^e| \ll 1$ :

$$\cos \delta(\theta_{13}^e) = \cos \delta - \Delta(\cos \delta),$$

$\cos \delta$  - from the exact sum rule,

$\Delta(\cos \delta)$  - correction due to  $|\sin \theta_{13}^e| \neq 0$ ,

$$\Delta(\cos \delta) \cong \frac{\sin \theta_{13}^e}{\sin \theta_{13}} \frac{\cos \kappa}{\sin \hat{\theta}_{23}} \tan \theta'_{12} \cot \theta_{12} \tan \theta_{23},$$

$$\kappa = \arg(c_{23}^e e^{-i\omega} - s_{23}^e e^{-i\psi}).$$

For  $|\sin \theta_{13}^e| \lesssim 10^{-3}$ , the correction  $|\Delta(\cos \delta)|$  to the exact sum rule result for  $\cos \delta$  does not exceed 11% (4.9%) in the case of the TBM (GRB) form and is even smaller for the BM, GRA and HG forms of  $\tilde{U}_\nu$ . In what follows we concentrate on the case of negligibly small  $\sin \theta_{13}^e \cong 0$ .

"Leading order" (in  $\sin \theta_{13}$ ,  $\theta_{23}^\ell$ , with  $\theta_{13}^\ell$  assumed to be  $\theta_{13}^\ell \ll \theta_{13}, \theta_{23}^\ell$ ) sum rule:

$$\theta_{12} \approx \theta_{12}^\nu + \theta_{13} \cos \delta.$$

S.Antusch, S. King, 2005

Can be obtained from

$$\sin \theta_{12} \approx \sin \theta_{12}^\nu + \frac{\sin 2\theta_{12}^\nu}{2 \sin \theta_{12}^\nu} \sin \theta_{13} \cos \delta,$$

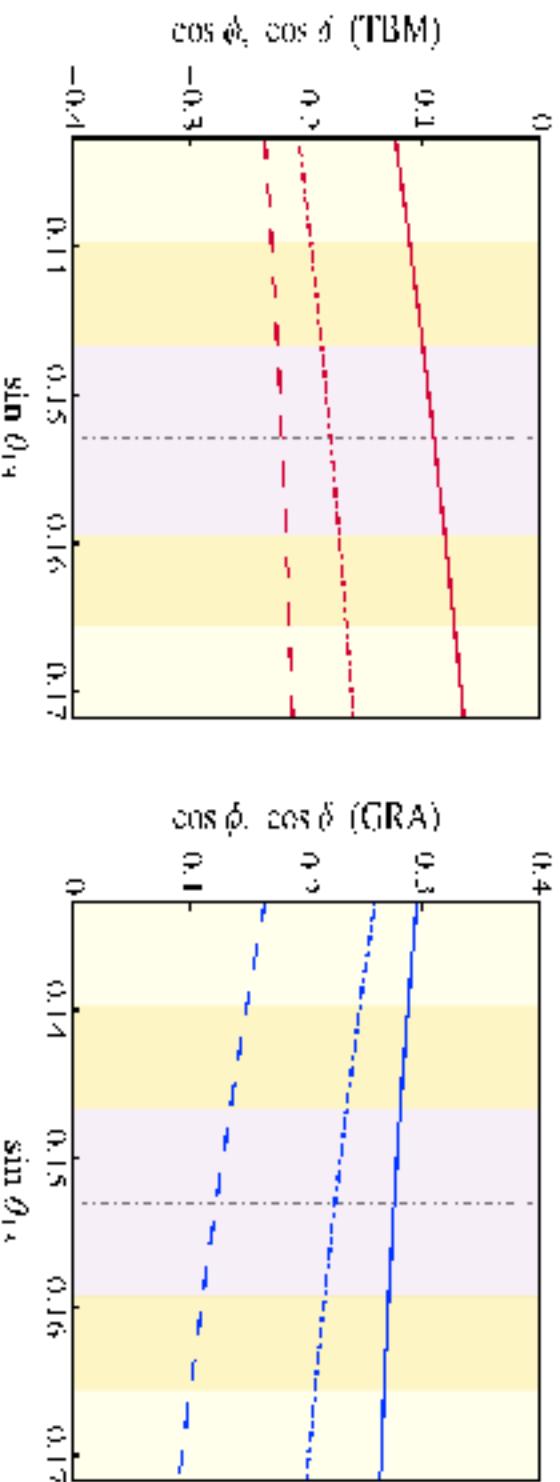
From the exact sum rules we get in "leading order":

$$\sin^2 \theta_{12} \cong \sin^2 \theta_{12}^\nu + \sin 2\theta_{12} \sin \theta_{13} \cos \delta,$$

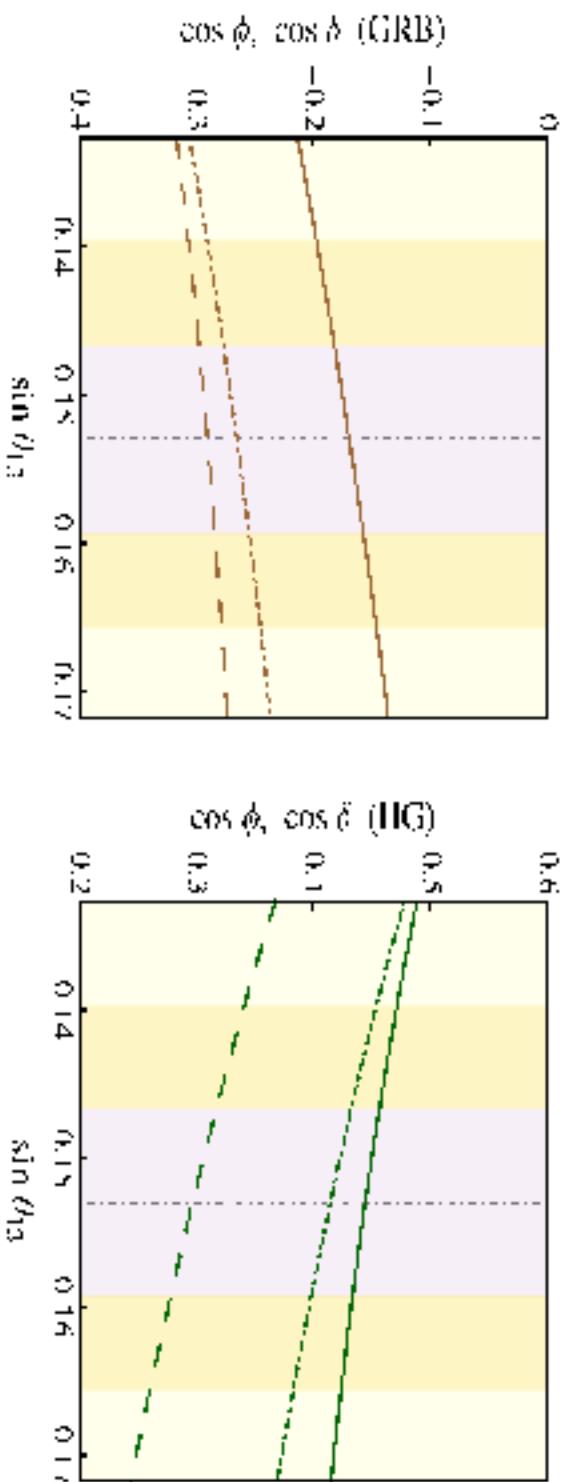
$$\sin^2 \theta_{12} \cong \sin^2 \theta_{12}^\nu + \sin 2\theta_{12}^\nu \sin \theta_{13} \cos \phi.$$

To "leading order",  $\cos \delta = \cos \phi$ .

---



$$\sin^2 \theta_{12} = 0.308, \sin^2 \theta_{23} = 0.437 \text{ (b.f.v.)}$$



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

$\sin^2 \theta_{13} = 0.0234$ ,  $\sin^2 \theta_{23} = 0.437$  (b.f.v.):

	TBM	GRA	GRB	HG
$\sin^2 \theta_{12}$	0.308			
$(\cos \delta)_E$	-0.0906	0.275	-0.169	0.445
$(\cos \delta)_L^{LO}$	-0.179	0.225	-0.265	0.415
$(\cos \delta)_E / (\cos \delta)_L^{LO}$	0.506	1.22	0.636	1.07
$(\cos \phi)_E$	-0.221	0.123	-0.29	0.297
$(\cos \delta)_E / (\cos \phi)_E$	0.41	2.24	0.581	1.50
$(\cos \phi)_E / (\cos \phi)_L^{LO}$	1.23	0.547	1.10	0.716

For TBM, GRA, GRB, HG symmetry forms of  $\tilde{U}_\nu$ :

$$|\theta_{12} - \theta_{12}'| \sim \sin^2 \theta_{13}.$$

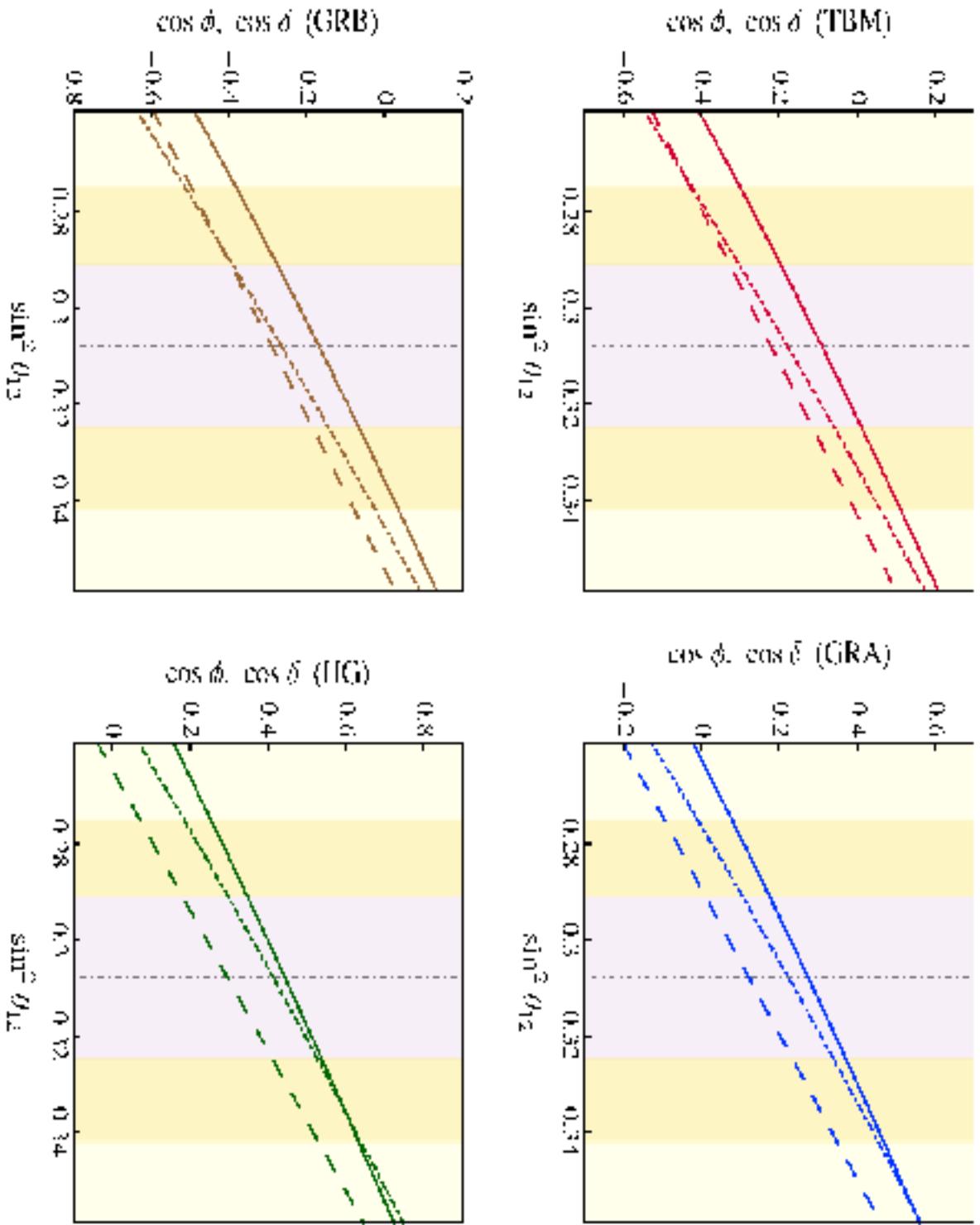
Thus, the next-to-leading order terms important.

For the BM form:  $|\theta_{12} - \theta_{12}'| \sim \sin \theta_{13}$ , LO is O.K.

The “leading order” sum rule (S. Antusch + S. King, 2005) provides reasonably accurate predictions for  $\cos\delta$  in the BM and HG cases, and largely incorrect predictions for  $\cos\delta$  in the cases of TBM, **GRA** and **GRB** symmetry forms of  $\tilde{U}_\nu$ :  
 $(\cos\delta)_E / (\cos\delta)_\text{LO} = 0.506, 1.22 \text{ and } 0.636.$

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$$\sin^2 \theta_{13} = 0.0234, \sin^2 \theta_{23} = 0.437 \text{ (b.f.v.)}$$



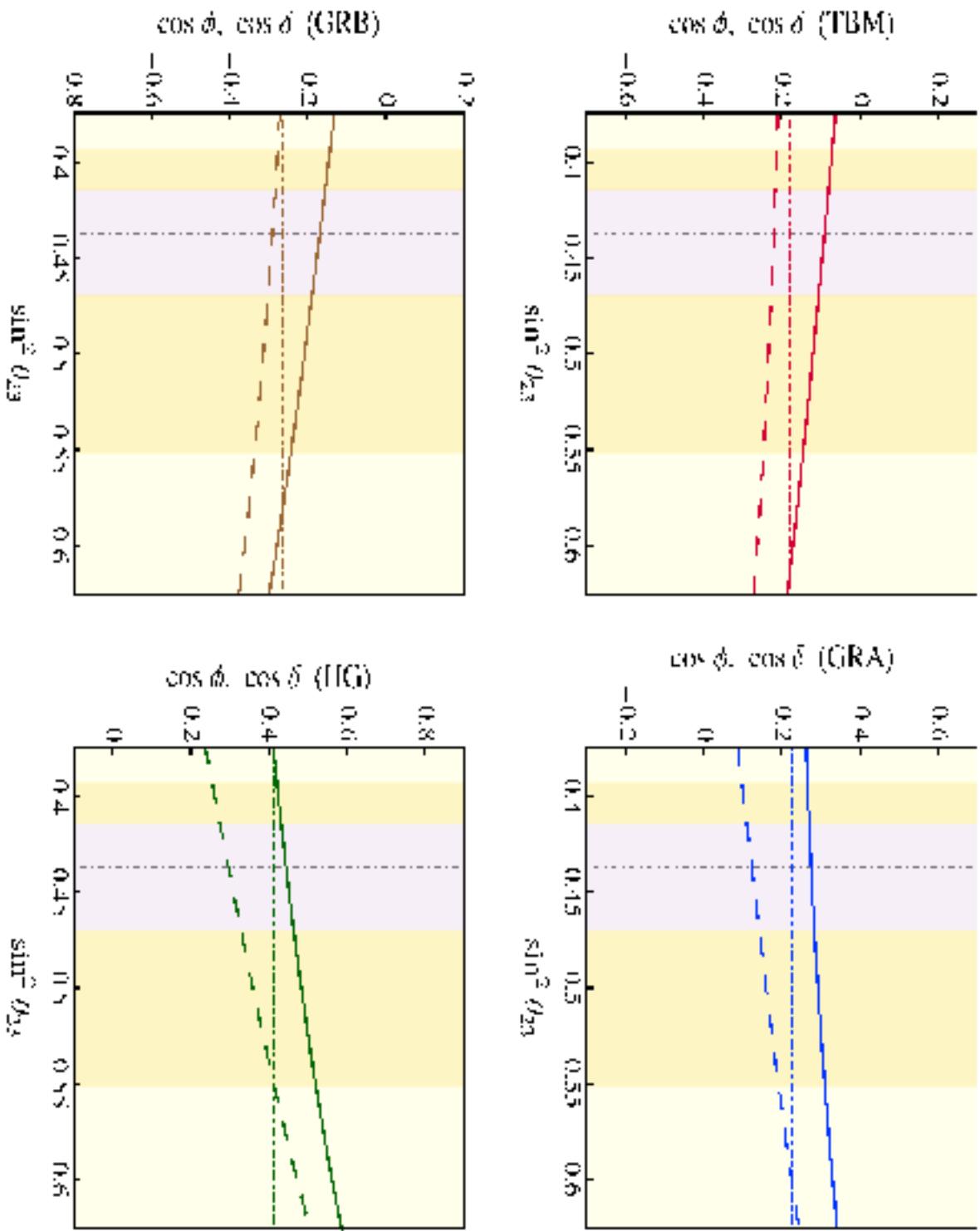
I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

$$\sin^2 \theta_{13} = 0.0234, \sin^2 \theta_{23} = 0.437 \text{ (b.f.v.)}$$

	TBM	GRA	GRB	HG
$(\cos \delta)_E$	<b>-0.408</b>	<b>-0.0223</b>	<b>-0.490</b>	<b>0.156</b>
$(\cos \delta)_{LO}$	-0.548	-0.129	-0.637	0.0673
$(\cos \delta)_E / (\cos \delta)_{LO}$	0.744	0.172	0.769	2.32
$(\cos \phi)_E$	-0.529	-0.202	-0.596	-0.0386
$(\cos \delta)_E / (\cos \phi)_E$	0.771	0.110	0.822	-4.05
$(\cos \phi)_E / (\cos \phi)_{LO}$	0.966	1.57	0.935	-0.573

	TBM	GRA	GRB	HG
$\sin^2 \theta_{12} = 0.359$	<b>0.210</b>	<b>0.562</b>	<b>0.135</b>	<b>0.725</b>
$(\cos \delta)_E$	0.175	0.564	0.092	0.749
$(\cos \delta)_{LO}$	1.20	0.996	1.46	0.969
$(\cos \delta)_E / (\cos \delta)_{LO}$	0.100	0.461	0.0279	0.647
$(\cos \phi)_E$	2.09	1.22	4.83	1.12
$(\cos \phi)_E / (\cos \phi)_{LO}$	0.573	0.817	0.303	0.864

$$\sin^2 \theta_{13} = 0.0234, \sin^2 \theta_{12} = 0.308 \text{ (b.f.v.)}$$



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

$\sin^2 \theta_{13} = 0.0234$ ,  $\sin^2 \theta_{12} = 0.308$  (b.f.v.):

	TBM	GRA	GRB	HG
$\sin^2 \theta_{23} = 0.374$	<b>-0.0618</b>	<b>0.262</b>	<b>-0.131</b>	<b>0.412</b>
$(\cos \delta)_E$				
$(\cos \delta)_L$	-0.179	0.225	-0.265	0.415
$(\cos \delta)_E / (\cos \delta)_L$	0.345	1.17	0.494	0.993
$(\cos \phi)_E$	-0.211	0.0866	-0.271	0.237
$(\cos \delta)_E / (\cos \phi)_E$	0.293	3.03	0.483	1.74
$(\cos \phi)_E / (\cos \phi)_L$	1.18	0.385	1.02	0.572
<hr/>				
$\sin^2 \theta_{23} = 0.626$	TBM	GRA	GRB	HG
$(\cos \delta)_E$	<b>-0.186</b>	<b>0.343</b>	<b>-0.299</b>	<b>0.588</b>
$(\cos \delta)_L$	-0.179	0.225	-0.265	0.415
$(\cos \delta)_E / (\cos \delta)_L$	1.04	1.52	1.13	1.42
$(\cos \phi)_E$	-0.272	0.244	-0.376	0.506
$(\cos \delta)_E / (\cos \phi)_E$	0.684	1.41	0.794	1.16
$(\cos \phi)_E / (\cos \phi)_L$	1.52	1.09	1.42	1.22

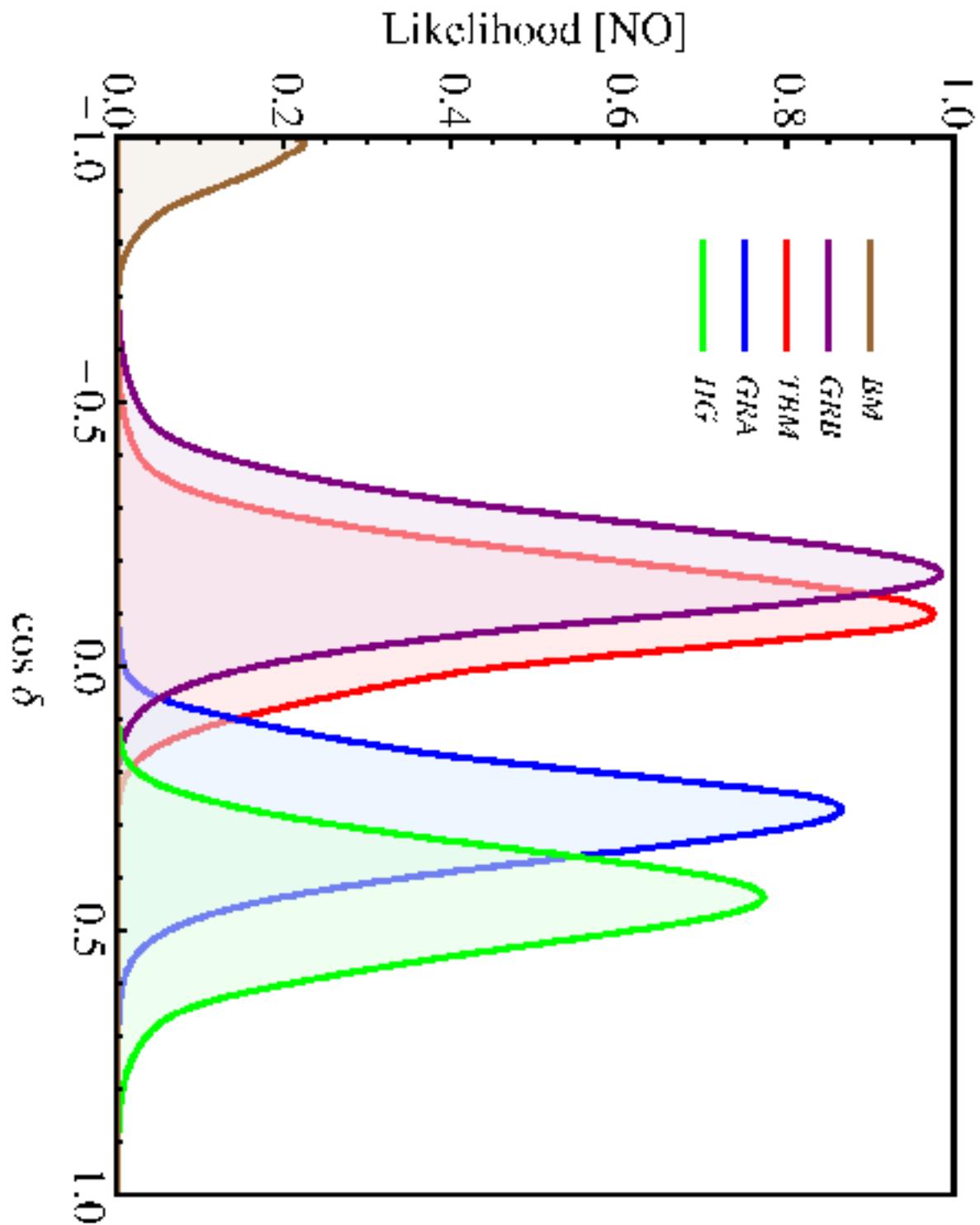
**Statistical analysis, likelihood method;  
input “data”:  $\sin^2 \theta_{13}$ ,  $\sin^2 \theta_{12}$ ,  $\sin^2 \theta_{23}$ ,  $\delta$   
from F. Capozzi et al., arXiv:1312.2878v2 (May 5,  
2014).**

$$L(\cos \delta) \propto \exp \left( -\frac{\chi^2(\cos \delta)}{2} \right)$$

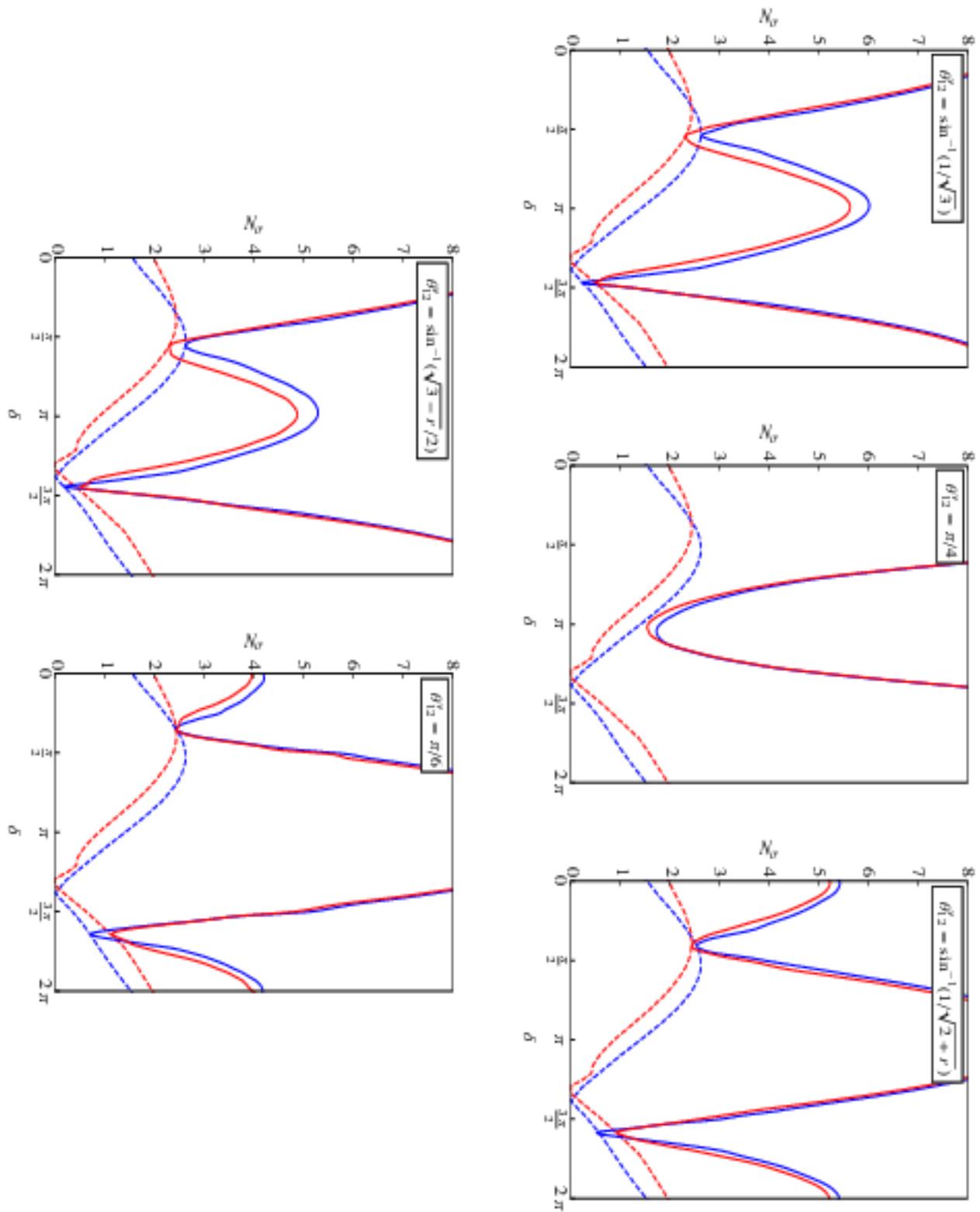
**$n\sigma$  confidence level interval of values of  $\cos \delta$ :**

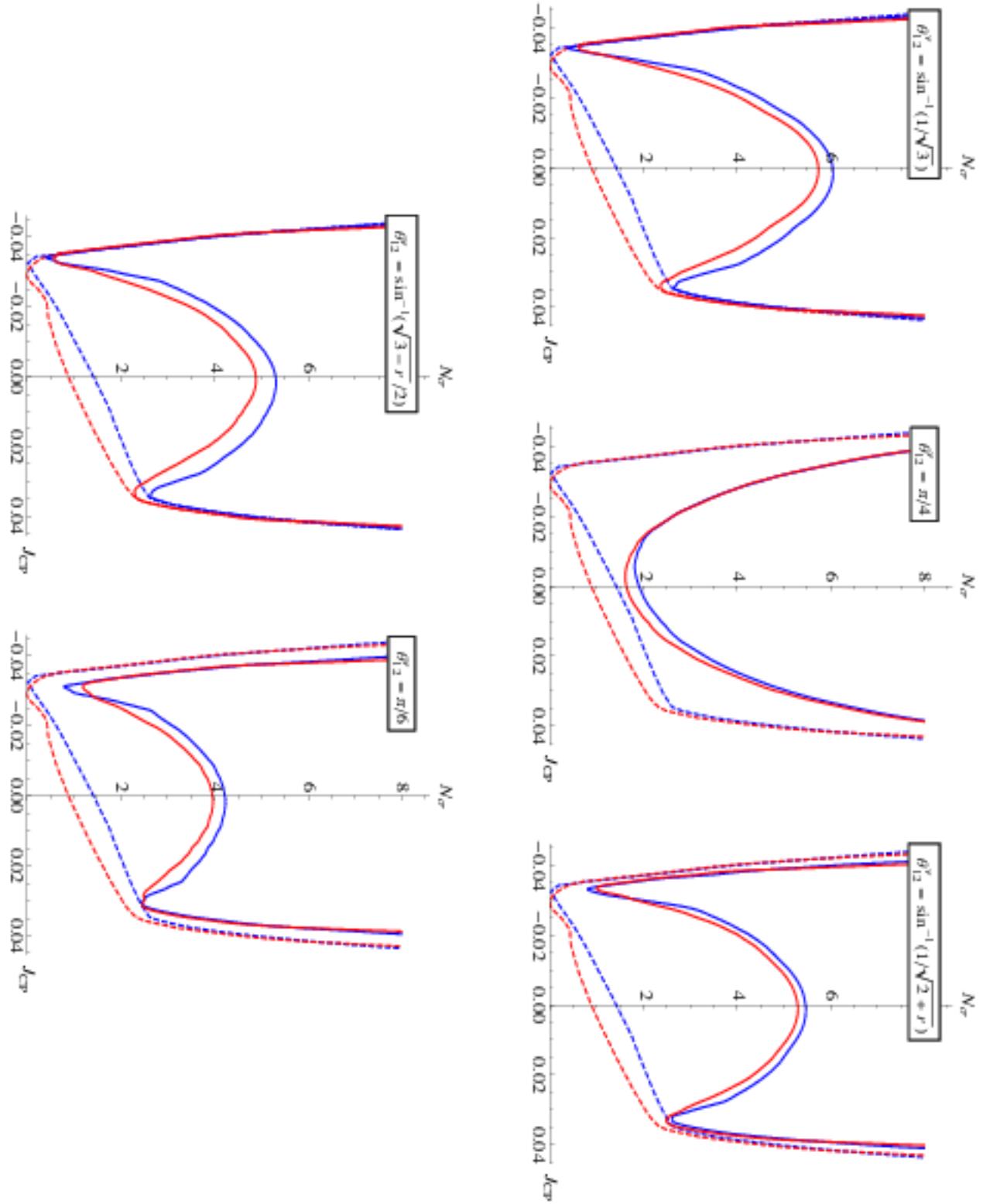
$$L(\cos \delta) \geq L(\chi^2_{\min}) \cdot L(\chi^2 = n^2)$$

I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056





I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

**TBM, GRA, GRB, HG:**  $J = 0$  excluded at  $5\sigma$ ,  $4\sigma$ ,  
 $4\sigma$ ,  $3\sigma$  confidence level.

**At  $3\sigma$ :**  $0.020 \leq |J_{CP}| \leq 0.039$ .

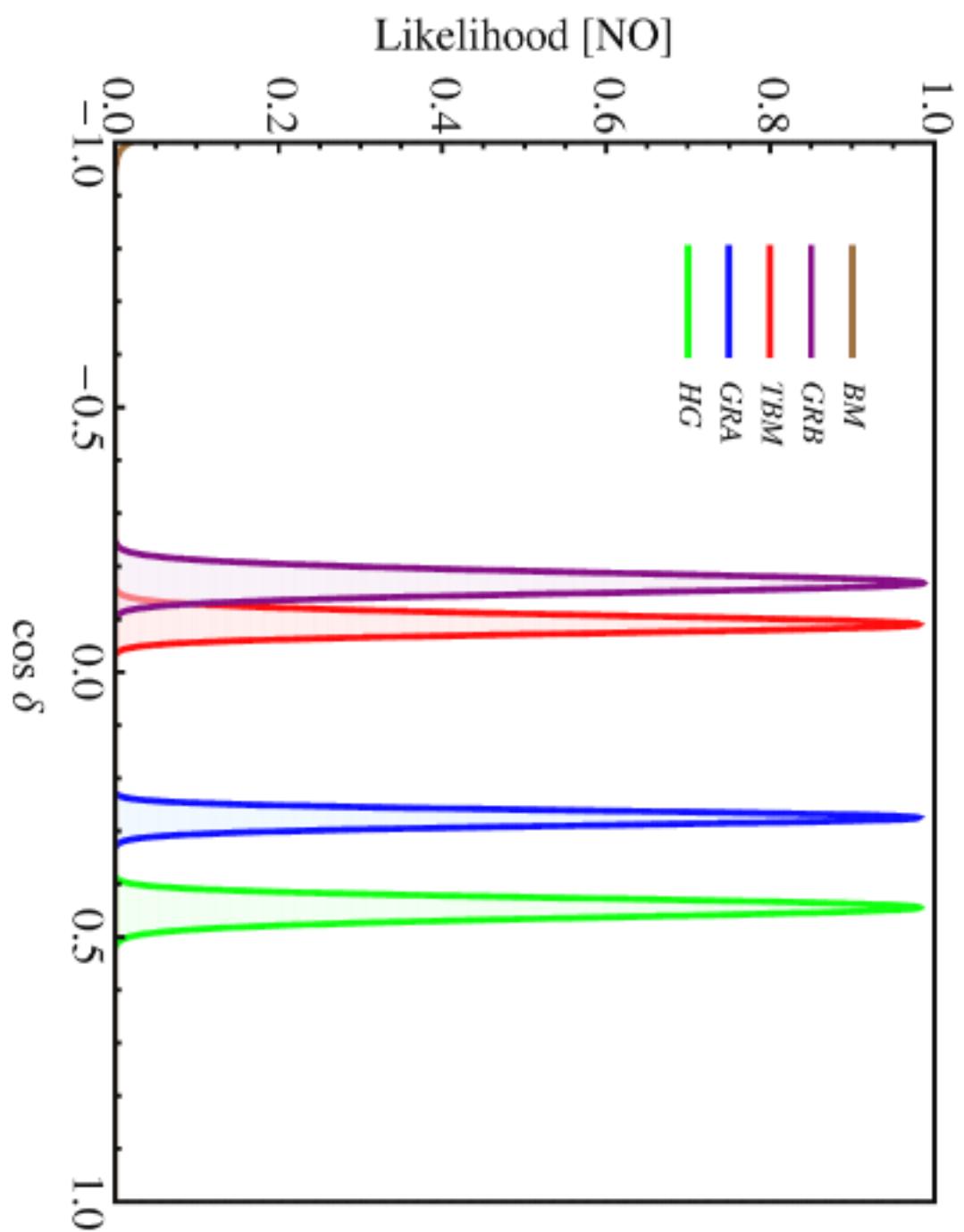
**BM (LC), b.f.v.:**  $J_{CP} = 0$ ;  
at  $3\sigma$ :  $-0.026$  ( $-0.025$ )  $\leq J_{CP} \leq 0.021$  ( $0.023$ ) for NO  
**(IO) neutrino mass spectrum.**

## Prospective precision:

$\delta(\sin^2 \theta_{12}) = 0.7\%$  (**JUNO**),

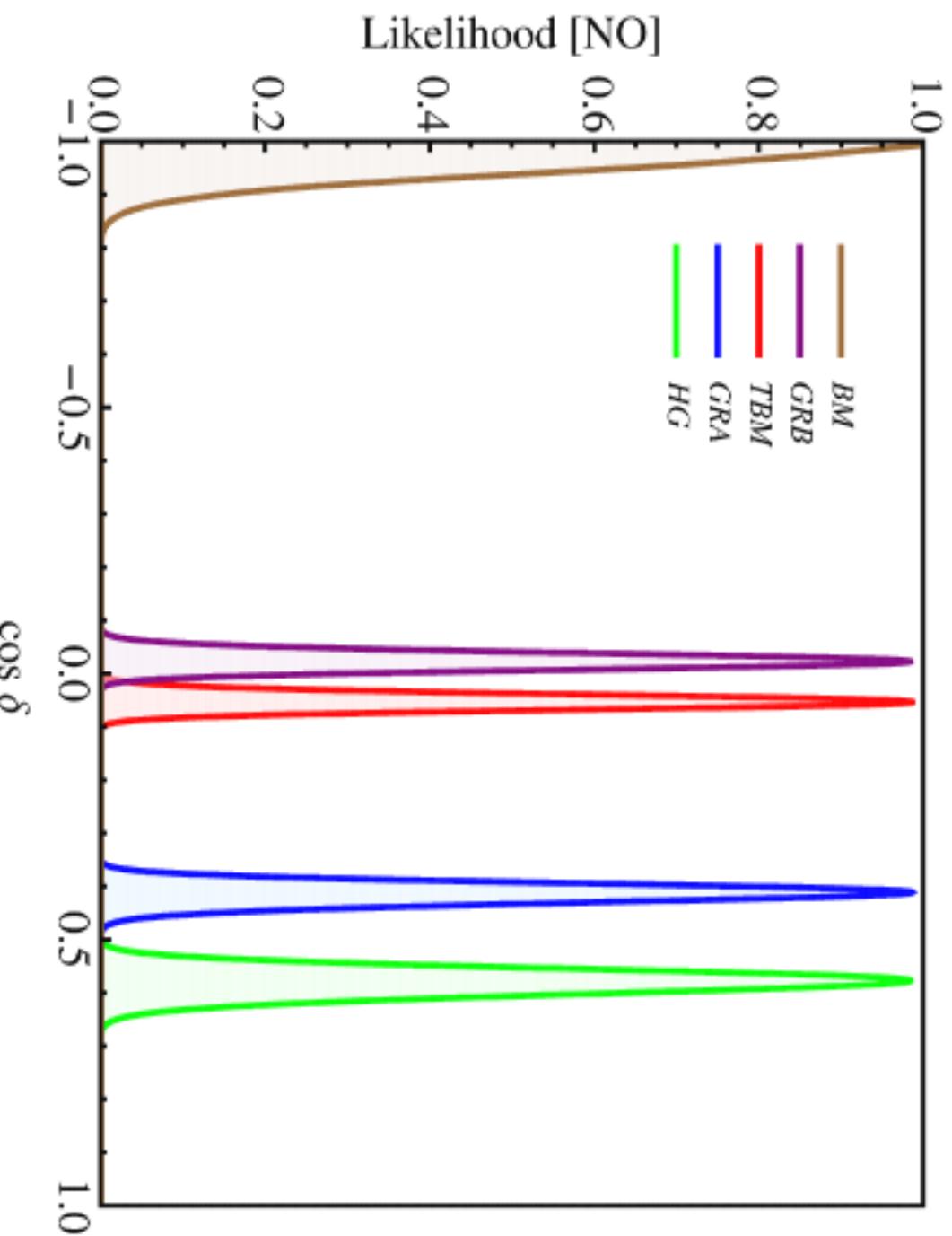
$\delta(\sin^2 \theta_{13}) = 3\%$  (**Daya Bay**),

$\delta(\sin^2 \theta_{23}) = 5\%$  (**T2K, NO $\nu$ A combined**).



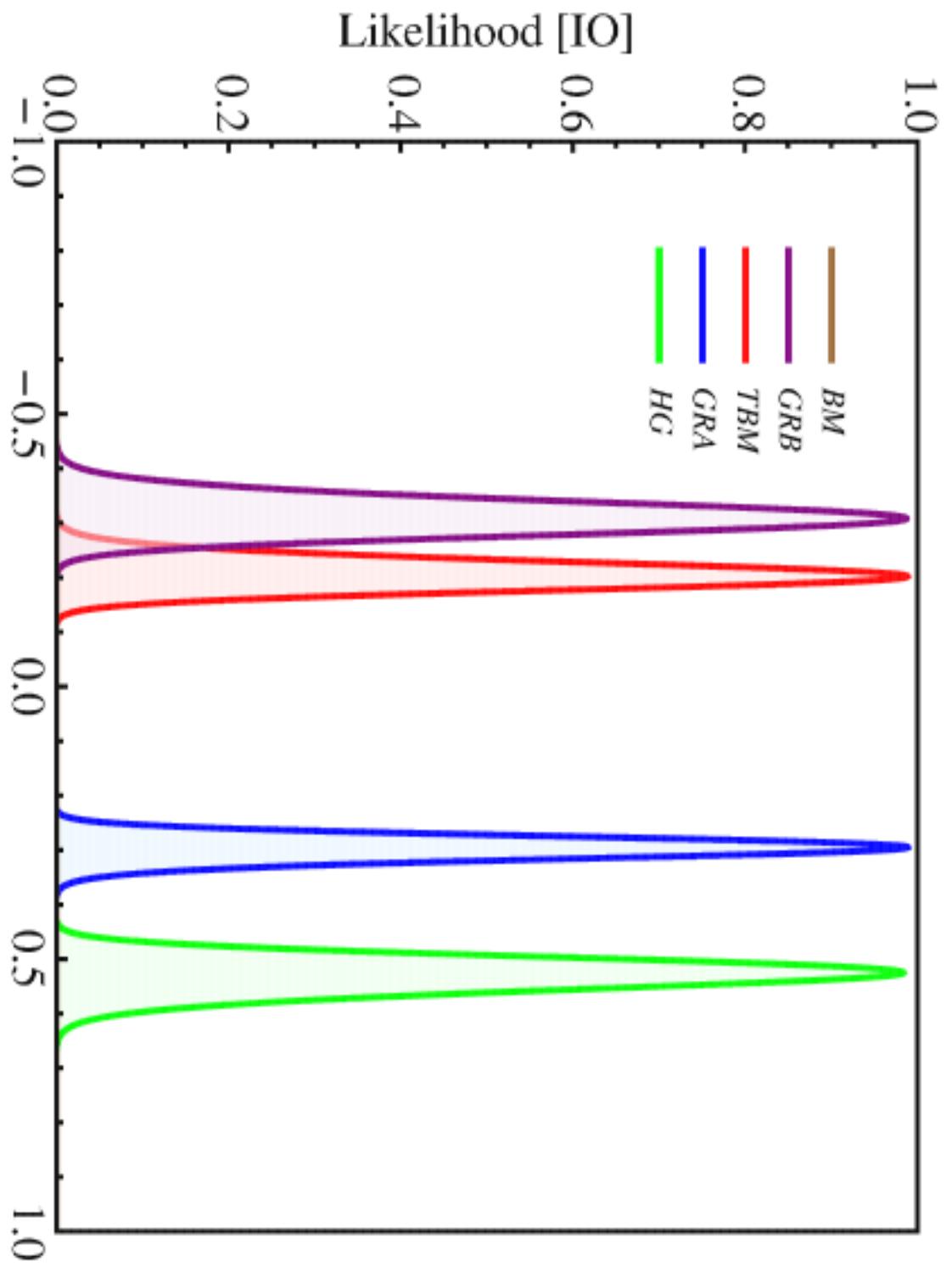
b.f.v. of  $\sin^2 \theta_{ij}$  (Capozzi et al., 2014) + the prospective precision used.

I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



The same, but for  $\sin^2 \theta_{12} = 0.33$  (the BM prediction dependence on  $\sin^2 \theta_{12}$ ).

I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

$\sin^2 \theta_{23} = 0.557$  (b.f.v.: C. Gonzales-Garcia et al., 2014, IO case).

In I. Giradi et al., arXiv:1410.8056, we have investigated also the dependence of the predictions for  $(\cos \delta)_E$ ,  $(\cos \delta)_{LO}$ ,  $(\cos \delta)_E / (\cos \delta)_{LO}$ ,  $(\cos \phi)_E$ , etc. on  $\sin^2 \theta_{13}$  and  $\sin^2 \theta_{12}$  when the latter are varied in their respective  $3\sigma$  allowed intervals in the case of  $\theta_{23}^\ell \cong 0$ . In this case:

$$\sin^2 \theta_{23} = \frac{1 - 2 \sin^2 \theta_{13}}{2(1 - \sin^2 \theta_{13})} \cong 0.5(1 - \sin^2 \theta_{13}).$$

The predictions for  $(\cos \delta)_E$ ,  $(\cos \delta)_{LO}$  and  $(\cos \delta)_E / (\cos \delta)_{LO}$  are very similar to those shown for non-negligible  $\theta_{23}^\ell$ ; in particular, also in this case the “leading order” sum rule provides largely incorrect predictions for  $\cos \delta$  for the TBM, GRA and GRB forms of  $\tilde{U}_\nu$ .

The predictions obtained for  $\cos\delta$  are valid in a large class of theoretical models of (lepton) flavour based on discrete symmetries.

J. Gehrlein *et al.*, “An  $SU(5) \times A_5$  Golden Ratio Flavour Model”, arXiv:1410.2095;

I. Girardi *et al.*, “Generalised Geometrical CP Violation in a  $T'$  Lepton Flavour Model”, arXiv:1312.1966, JHEP 1402 (2014) 050.

$T'$  model of lepton flavour:  $U_{\text{TBM}}$ ,  $\delta \cong 3\pi/2$  or  $\pi/2$ .

L. Girardi, A. Meroni, STP, M. Spinrath, arXiv:1312.1966

- Light neutrino masses: type I seesaw mechanism.
- $\nu_j$  - Majorana particles.
- Diagonalisation of  $M_\nu$ :  $U_{\text{TBM}} \Phi$ ,  $\Phi = \text{diag}(1, 1, 1(i))$
- $U_{\text{TBM}}$  "corrected" by  
 $U_{\text{lep}}^\dagger Q = R_{12}(\theta_{12}^\ell) R_{23}(\theta_{23}^\ell) Q$ ,  $Q = \text{diag}(1, e^{i\phi}, 1)$

$T'$  model of lepton flavour:  $U_{\text{TBM}}$ ,  $\delta \cong 3\pi/2$  or  $\pi/2$ .

- $T'$ : double covering of  $A_4$  (tetrahedral symmetry group).

•  $T'$ : **1**, **1'**, **1''**; **2**, **2'**, **2''**; **3**.

- $T'$  model:  $\psi_{eL}(x), \psi_{\mu L}(x), \psi_{\tau L}(x)$  - triplet of  $T'$ ;  $e_R(x), \mu_R(x)$  - a doublet,  $\tau_R(x)$  - a singlet, of  $T'$ ;  $\nu_{eR}(x), \nu_{\mu R}(x), \nu_{\tau R}(x)$  - a triplet of  $T'$ ; the Higgs doublets  $H_u(x)$ ,  $H_d(x)$  - singlets of  $T'$ .
- The discrete symmetries of the model are  $T' \times H_{\text{CP}} \times Z_8 \times Z_4^2 \times Z_3^2 \times Z_2$ , the  $Z_n$  factors being the shaping symmetries of the superpotential required to forbid unwanted operators.

## Predictions of the $T'$ Model

- $m_{1,2,3}$  determined by 2 real parameters +  $\Phi^2$ :

NO spectrum A :  $(m_1, m_2, m_3) = (4.43, 9.75, 48.73) \cdot 10^{-3} \text{ eV}$ ,

NO spectrum B :  $(m_1, m_2, m_3) = (5.87, 10.48, 48.88) \cdot 10^{-3} \text{ eV}$ ,

IO spectrum :  $(m_1, m_2, m_3) = (51.53, 52.26, 17.34) \cdot 10^{-3} \text{ eV}$ ,

$$\text{NO A : } \sum_{j=1}^3 m_j = 6.29 \times 10^{-2} \text{ eV},$$

$$\text{NO B : } \sum_{j=1}^3 m_j = 6.52 \times 10^{-2} \text{ eV},$$

$$\text{IO : } \sum_{j=1}^3 m_j = 12.11 \times 10^{-2} \text{ eV},$$

- $\theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{21}, \alpha_{31}$  determined by 3 real parameters.

Given the values of  $\theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{21}, \alpha_{31}$  are predicted:

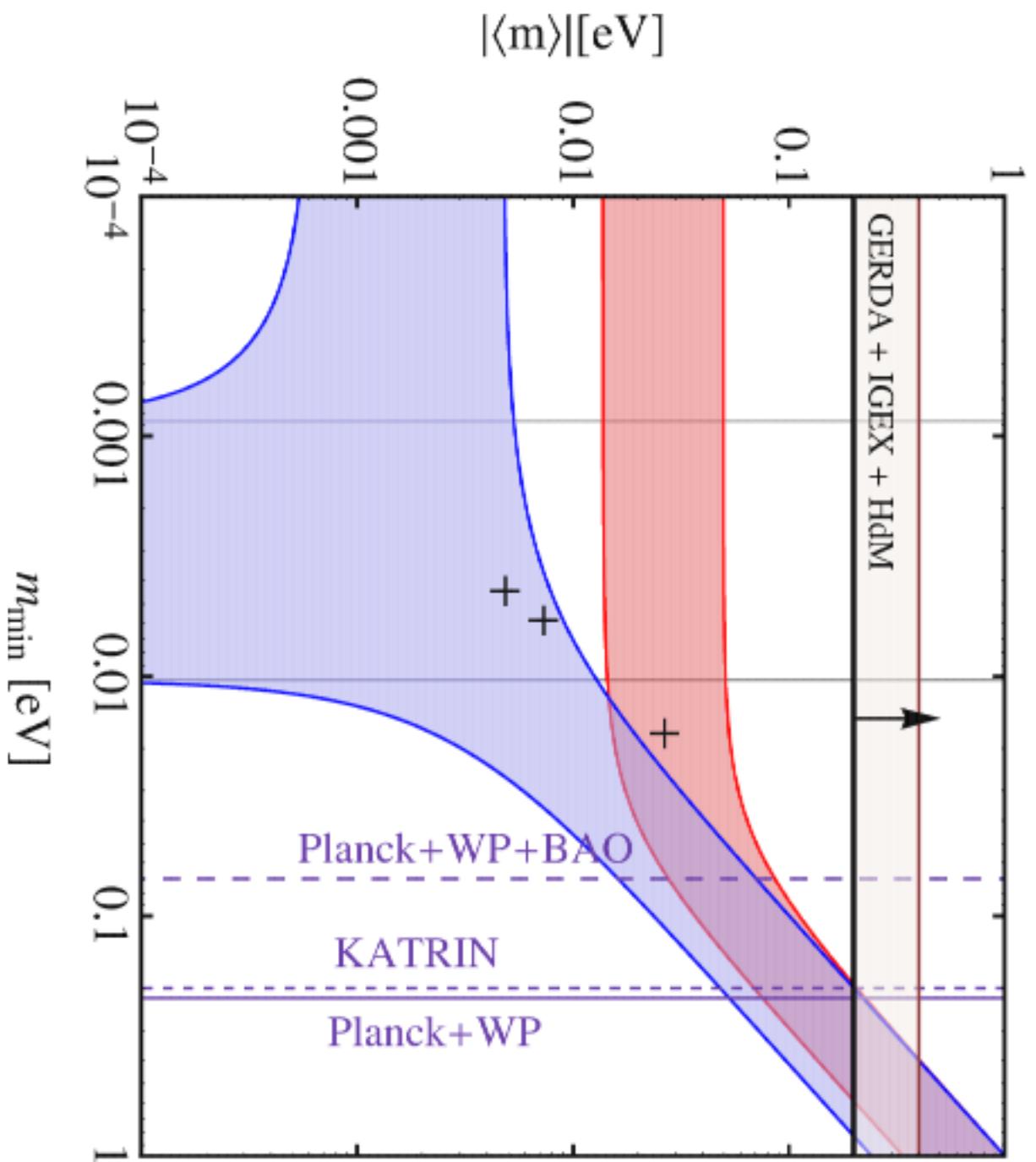
$$\delta \cong 3\pi/2 (266^\circ) \text{ (or } \pi/2 (94^\circ)\text{)};$$

$$\text{NO A: } \alpha_{21} \cong +47.0^\circ \text{ (or } -47.0^\circ\text{)} (+2\pi),$$

$$\alpha_{31} \cong -23.8^\circ \text{ (or } +23.8^\circ\text{)} (+2\pi).$$

The model is falsifiable.

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## Conclusions.

- We have considered a simple scheme for obtaining  $\sin^2 \theta_{13} \cong 0.16$  and  $\sin^2 \theta_{23} \cong 0.44$  from TBM, BM (LC), GRA, GRB and HG and the charged lepton corrections;  $U_{\text{lep}} \equiv U_{\text{lep}}(\theta_{12}^\ell, \theta_{23}^\ell)$  required.
- The results depend strongly on the ordering of the 1-2 and 2-3 rotations in  $U_{\text{lep}}$ .
- Interesting results in the case of “Standard Ordering”,  $O_{23}(\theta_{23}^\ell)O_{12}(\theta_{12}^\ell)$ :
  - i) new exact “sum rules”:  $\delta = \delta(\theta_{12}, \theta_{23}, \theta_{13}; \theta_{12}^\nu)$ ;
  - ii) BM (LC):  $\delta \cong \pi$ ;
  - iii) TBM:  $\delta \cong 3\pi/2$  (hints from data),  $J_{CP} \neq 0$  at  $5\sigma$ , b.f.v.  $J_{CP} = -0.034$ ;
  - iv) GRA, GRB, HG:  $J_{CP} \neq 0$  at  $\sim 4\sigma$ , b.f.v.  $J_{CP} = (-0.30) - (-0.33)$ ;  
TBM, GRA, GRB, HG:  $|J_{CP}| \gtrsim 0.02$  at  $3\sigma$ ;
- The measurement of  $\cos \delta$  can allow to distinguish between different symmetry forms of  $U_\nu$ :  $\cos \delta \cong (-1); (-0.091); 0.275; (-0.169); 0.445$  for the BM; TBM; GRA; GRB; HG forms (b.f.v. of  $\theta_{ij}$  used).
- The predictions for  $\cos \delta$  in the TBM, GRA, GRB and HG cases exhibit strong dependence on  $\sin^2 \theta_{12}$  varied in its  $3\sigma$  allowed range; depend also on  $\sin^2 \theta_{23}$ ;
- “Leading order” sum rule (King + Antusch, 2005):  $\delta = \delta(\theta_{12}, \theta_{12}^\nu, \theta_{13})$ ; gives typically incorrect predictions for  $\cos \delta$  in the TBM, GRA, GRB and HD cases; can be reasonably accurate for some of the forms for specific values of  $\theta_{ij}$ .
- The predictions for  $\cos \delta$  and  $J_{CP}$  will be tested in LBL neutrino oscillation experiments.

The measurement of the Dirac phase in the PMNS mixing matrix, together with an improvement of the precision on the mixing angles  $\theta_{12}$ ,  $\theta_{13}$  and  $\theta_{23}$ , can provide unique information about the possible existence of new fundamental symmetry in the lepton sector. These measurements could also provide an indication about the structure of the matrix  $\tilde{U}^e$  originating from the charged lepton sector, and thus about the charged lepton mass matrix.