F-theory and all Things Rational:

a comprehensive study of U(1)s in F-theory and their Phenomenology

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Based on work in collaboration
with Craig Lawrie and Jin-Mann Wong 1504.05593
and with Sven Krippendorf and Jin-Mann Wong to appear

Goal

Determine universal, distinguishing characteristics of F-theory models, with distinct phenomenological signatures.

F-theory model building based on lots of examples: local and by now also global, with semi-realistic properties.

Challenge:
Combined package of realistic spectra, flavor, susy breaking, moduli stabilization, etc all into one framework, and genericity of such features.

Strategy:
Ask questions of universal nature: find characteristics that can be comprehensively understood and constrain the phenomenology

Setup

Constraining 4d $N = 1$ SUSY $SU(5)$ F-theory GUTs using additional symmetries: $U(1)$s and discrete.

1. Symmetries:
   What continuous and discrete symmetries are both geometrically consistent within F-theory and phenomenologically sound?
2. Anomalies:
   Spectra consistent with hypercharge flux (GUT breaking) induced anomalies
3. Flavor:
   Realistic quark sector Yukawa textures from distribution of matter, and using Froggatt-Nielsen type mechanism

String Theory Input: what are possible $U(1)$ symmetries in F-theory?

Summary

1. General characterization of global ways of realizing $U(1)$ symmetries and possible matter charges in F-theory [Lawrie, SSN, Wong]

⇒ Model-independent, superset of charges for GUTs

⇒ All charged matter and GUT-Singlet $U(1)$-charges

⇒ Classification of possible Higgsings for $U(1)$s to discrete symmetries

2. Phenomenological Implications:

Combined system of F-theory $U(1)$ charges, phenomenological consistency and anomaly cancellation has solutions with realistic flavor texture

[Krippendorf, SSN, Wong]

GUTs with extra $U(1)$s

- Toric Constructions with extra $U(1)$s.
  [Morrison, Park][Braun, Grimm, Keitel][Mayrhofer, Palti, Weigand][Cvetic, Klever, Piragua], [Morrison, Taylor]...
- All toric hypersurfaces: [Klever, Pena, Piragua, Oehlmann, Reuter]
- Multiple **10** matter loci:
  [Mayrhofer, Palti, Weigand], [Kuentzler, SSN], [Lawrie, Sacco],[Braun, Grimm, Keitel]
- Preliminary Pheno: [Krippendorf, Pena, Oehlmann, Ruehle]
- Systematic approach: Tate-like forms, however limited by ability to factor polynomials of UFD... [Kuentzler, SSN][Lawrie, Sacco]

Goal: Find general way to constrain $U(1)$s from first principles

[Lawrie, SSN, Wong]

Plan

I. Non-Abelian Gauge Groups in F-theory

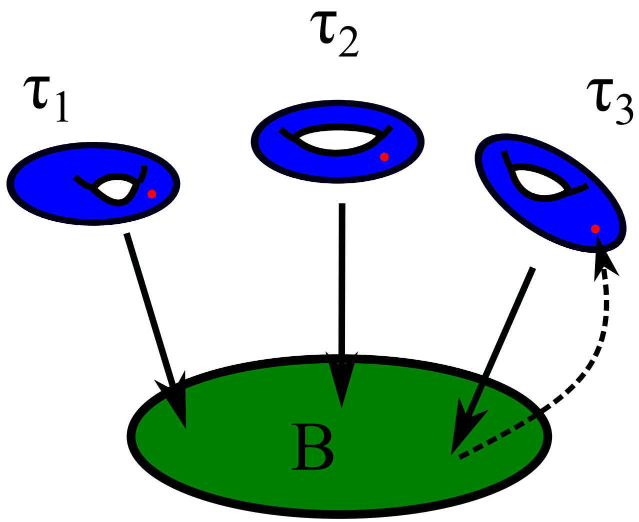
II. Systematics of $U(1)$s in F-theory

III. Phenomenology: Anomalies, PD, Flavor

I. Non-Abelian Gauge Groups in F-theory

F-theory and Elliptic Fibrations

4d vacua: Elliptically fibered Calabi-Yau, $\tau = C_0 + ie^{-\phi}$ axio-dilaton of IIB:



$\Rightarrow$ $\mathbb{E}_\tau$ fibers = Tori $\mathbb{C}/\mathbb{Z} \oplus \tau\mathbb{Z}$ with marked point O (elliptic curve, with O = origin) with complex structure τ

$\Rightarrow$ Exists "zero section" σ_0: $B \longrightarrow \mathbb{E}_\tau : b \mapsto O$

$\Rightarrow$ For such there is always a Weierstrass form with $O = [0, 1, 1]$

$$y^2 = x^3 + fxw^4 + gw^6 \qquad [w, x, y] \in \mathbb{P}(1, 2, 3)$$

4d gauge bosons from F-theory

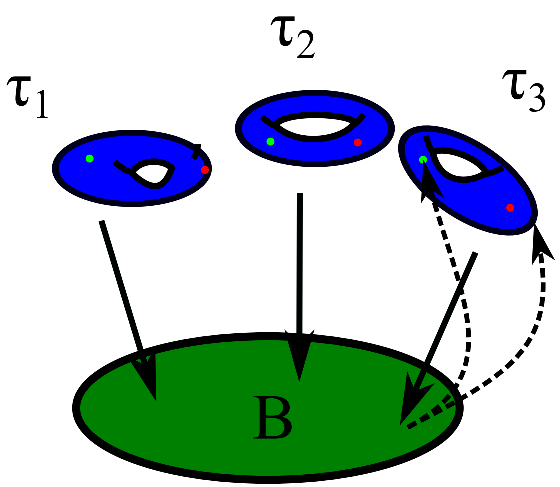
Reduce M-theory 3-form along (1, 1) forms $\omega^{(1,1)}$ in fiber:

$$C_3 = \omega^{(1,1)} \wedge A$$

$\Rightarrow$ abelian gauge potentials A. Two types

1. ω from special fibers (ADE like singularities) $\Rightarrow$ GUT gauge bosons
2. ω from rational sections $\Rightarrow$ $U(1)$s [Morrison, Vafa]

   Mathematically: maps from base to fiber: $\sigma: \quad B \longrightarrow \mathbb{E}_\tau$: $b \mapsto P$ with P a rational solution to $y^2 = x^3 + fxw^4 + gw^6$, $P \neq O$



$(1,1)$ Forms and Singular Fibers

[Kodaira]: $\exists$ "Singular fibers", which are $\mathbb{P}^1$s intersecting in affine ADE Dynkin diagram $\Rightarrow$ $\omega^{(1,1)}$ from volume form of $\mathbb{P}^1$

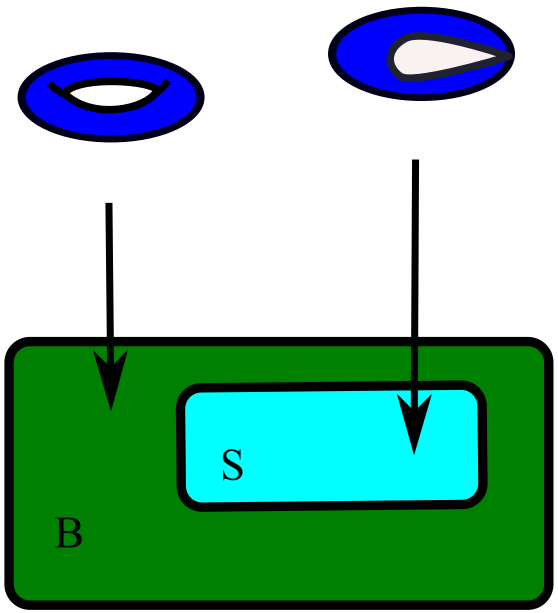
- Kodaira fibers from resolutions of singular fibrations
- Elliptic curve is $y^2 = x^3 + fxw^4 + gw^6$ singular if

$$\Delta = 4f^3 + 27g^2 = 0$$

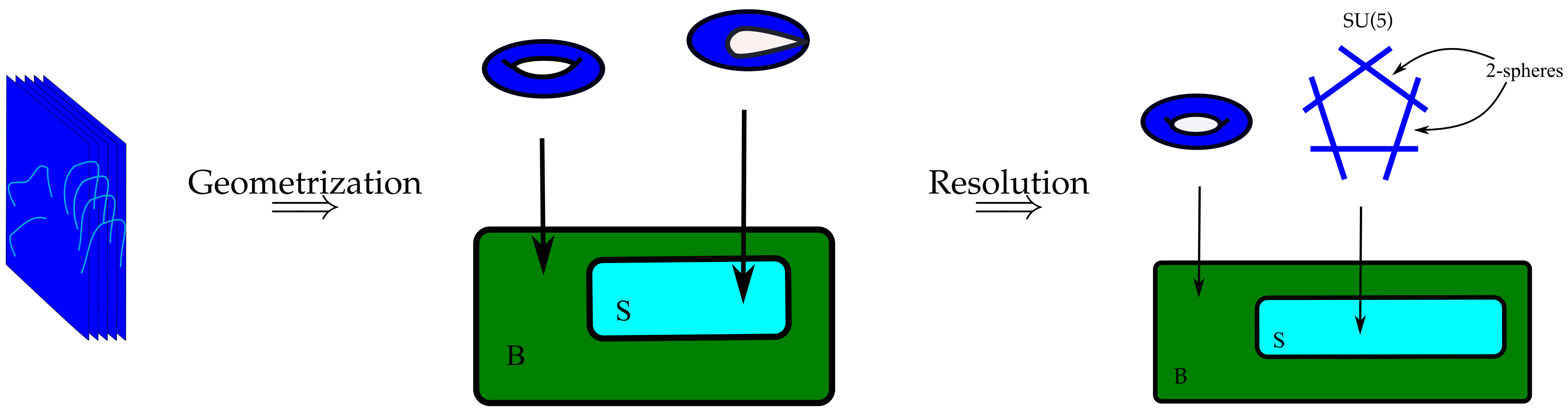
  Here Δ depends on base:

$$\Delta(z) = O(z^n) \quad \Leftrightarrow \quad z = 0 \text{ is surface } S \subset B$$

- Physics:
  Syncs with 7-branes intuition in IIB, which sources F_9 and $\tau \sim \log(x - x_0)$ undergoes monodromy $SL_2\mathbb{Z}$



Gauge theory from Singular Fibers

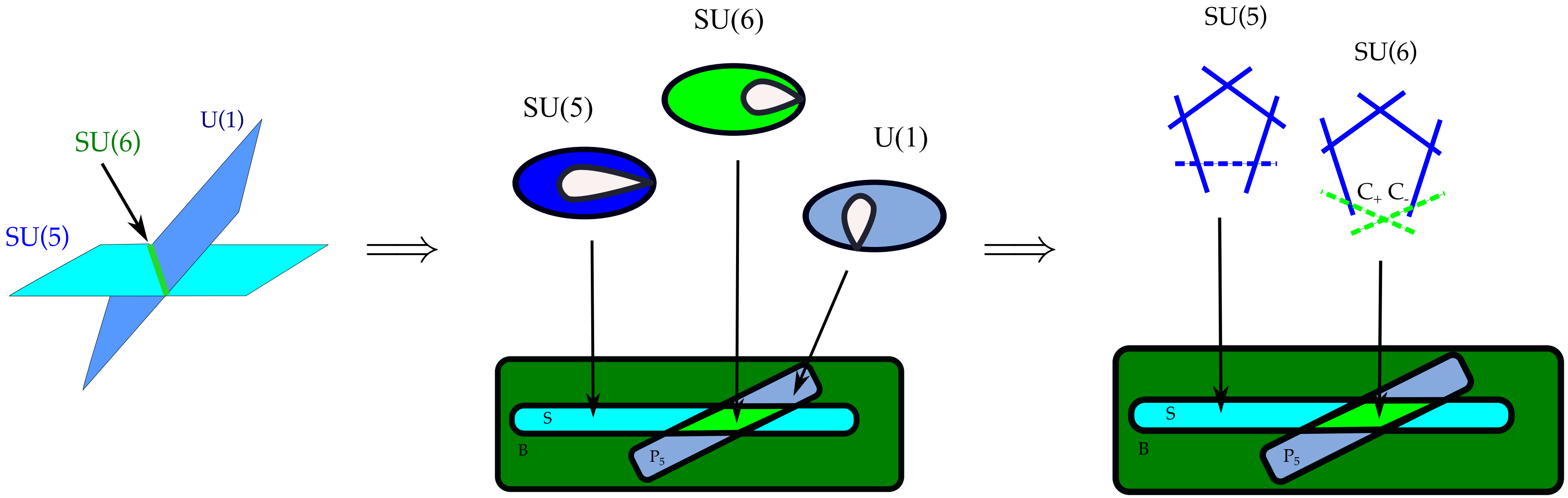


- Resolution of singularities:
  Trees of $\mathbb{P}^1$s, intersecting in Affine $SU(5)$ Dynkin diagram

  $\mathbb{P}^1 = S^2 =$ curves in resolved fiber $\overset{1:1}{\longleftrightarrow}$ simple roots of gauge group $SU(5)$

- M/F-theory:
  Gauge bosons from $C_3 = A_i \wedge \omega_i^{(1,1)}$ and wrapped M2

Matter

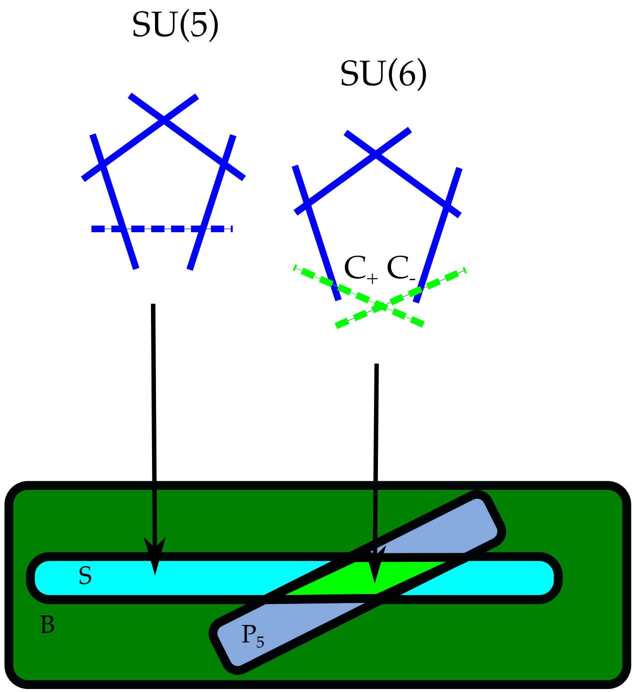


- Matter is localized along codimension 2 loci Σ: Singularity worsens

$$\Delta = P_{\mathbf{5}} z^5 + O(z^6)$$

- Matter determined by fiber type along codim 2:

$$z = P_5 = 0: \; SU(6) \to SU(5) \times U(1): \qquad \mathbf{35} \to \mathbf{24}_0 \oplus \mathbf{1}_0 \oplus \mathbf{5}_6 \oplus \overline{\mathbf{5}}_{-6}$$



Geometry:

$\mathbb{P}^1$ associated to root α splits into "weights" of $\bar{\mathbf{5}}$

$$\mathbb{P}^1_\alpha \quad \longrightarrow \quad C_+ + C_-$$

M/F-theory picture:

Wrapped M2-branes give matter transforming in representation of $SU(5)$

$\Rightarrow$ Classification of posssible codim 2 fibers?

Coulomb phases and Resolutions

M-theory on resolved $\tilde{Y}_{3/4} \Rightarrow$ Coulomb branch of $3/5d$, $N = 2$ theory

- Vector multiplet for gauge group G: $\mathbb{V} = (\phi, A)$
- $\langle \phi \rangle \in \mathrm{CSA}(G) \Rightarrow G \to U(1)^{\mathrm{rank}(G)}$

$\Rightarrow$ Coulomb branch $\cong \mathbb{R}^{\mathrm{rank}(G)}/W_G$ = Weyl chamber

Including matter:
N chiral multiplets Q in representation $\mathbf{R}$ of G introduces substructure in Weyl chamber.

[deBoer, Hori, Oz][Aharony, Hanany, Intriligator, Seiberg, Strassler]
[Diaconescu, Gukov][Grimm, Hayashi]

Coulomb Phases with Matter

Substructure in Weyl chamber:

Q in rep $\mathbf{R}$ with weight λ

$$\mathcal{L} \supset |\langle \phi, \lambda \rangle|^2 |Q|^2$$

$\Rightarrow$ new walls (where additional massless states arise)

$$\langle \phi, \lambda \rangle = 0$$

- Lie algebra $\mathfrak{g}$, and positive roots $\mathbf{\Phi}^+$.
- Weyl chamber:

$$\mathcal{C}^* = \{\phi \in \mathfrak{h}\,, \quad \langle \phi, \alpha \rangle > 0\,, \quad \text{for all} \quad \alpha \in \mathbf{\Phi}^+\} \subset \mathfrak{h} = \text{CSA}$$

- Representation $\mathbf{R}$ with weights λ_I, $I = 1, \cdots r = \dim R$, with

$$\begin{aligned} \widetilde{\mathfrak{g}} \quad &\longrightarrow \quad \mathfrak{g} \oplus \mathfrak{u}(1) \\ \text{Adj}(\widetilde{\mathfrak{g}}) \quad &\longrightarrow \quad \text{Adj}(\mathfrak{g}) \oplus \text{Adj}(\mathfrak{u}(1)) \oplus \mathbf{R}_+ \oplus \overline{\mathbf{R}}_- \,, \end{aligned}$$

$\Rightarrow$ Phases for $\mathbf{R}$ are subwedges with definite sign of $\langle \phi, \lambda \rangle$:

$$\Phi_{\epsilon_1 \cdots \epsilon_r} = \langle \phi \in \mathcal{C}^* : \quad \text{sign}(\langle \phi, \lambda_I \rangle) = \epsilon_I = \pm 1\,, \quad I = 1, \cdots, r \rangle_{\mathbb{Z}^+}$$

Phases are 1:1 with elements of

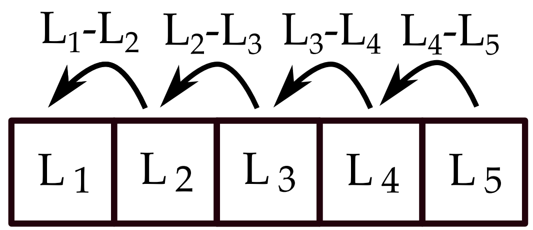
$$\{\mathbf{\Phi}_{\epsilon_1 \cdots \epsilon_r}\} \quad \overset{1:1}{\longleftrightarrow} \quad \frac{W_{\tilde{\mathfrak{g}}}}{W_{\mathfrak{g}}}$$

Main example: $\mathfrak{su}(5)$

$\mathfrak{g} = \mathfrak{su}(5)$, $\mathbf{R} = \mathbf{5}$ or $\mathbf{10}$.

Simple roots

$$\alpha_i = L_i - L_{i+1}$$

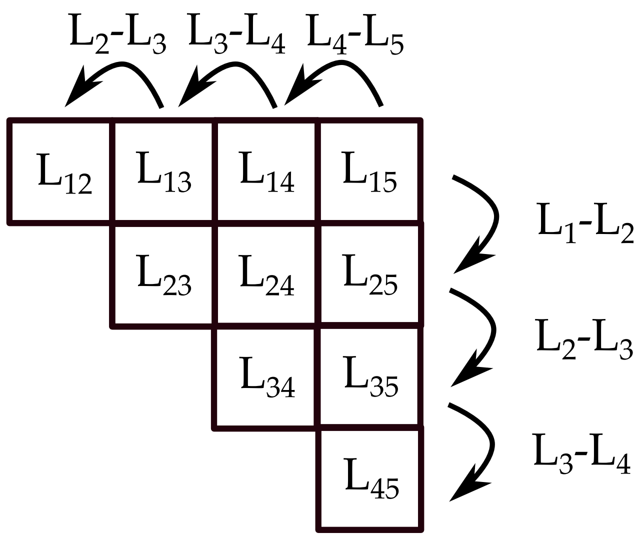


Fundamental weights:

$$\mathbf{5}: \qquad \{L_1, \cdots, L_5\}$$

Anti-symmetric representation:

$$\mathbf{10}: \qquad \{L_i + L_j,\ i < j\}$$

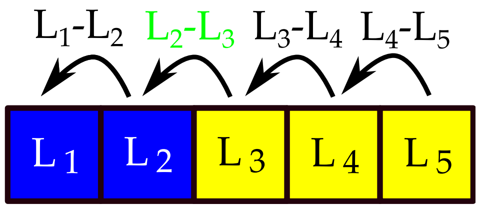


Box Graphs

Then a phase defines a sign/coloring to each weight:

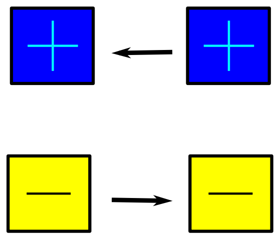
$\Phi_{\epsilon_1 \cdots \epsilon_r}$ $\Rightarrow$ $\pm$/coloring (blue or yellow) of representation graph $\equiv$ Box graph

$\Phi_{++---} = \langle L_1, L_2, -L_3, -L_4, -L_5 \rangle_{\mathbb{Z}^+}$ is realized in terms of:

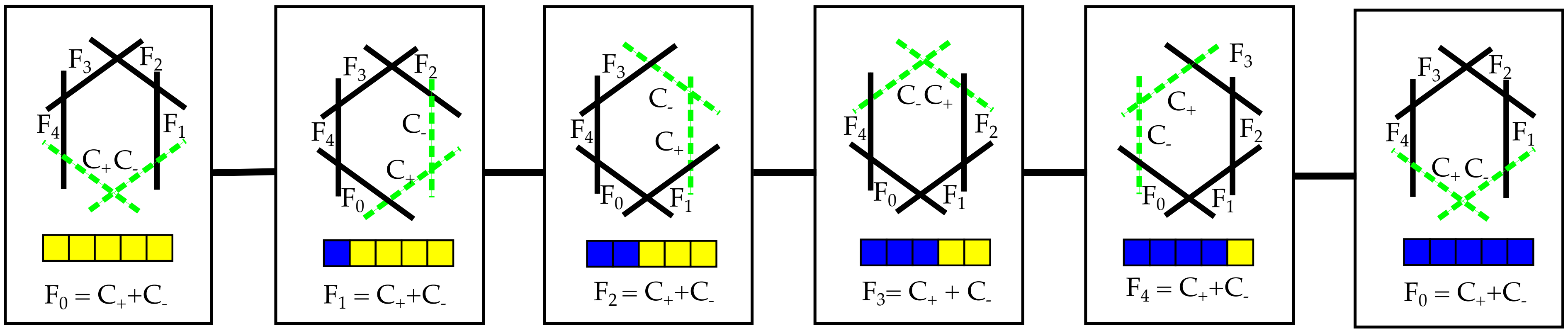


More importantly: Conversely a sign decoration of a rep graph defines phase if it satisfies flow rules:

5:



Codim 2 Fibers/Phases: $\mathfrak{su}(5) \oplus \mathfrak{u}(1) \rightarrow \mathfrak{su}(6)$



Classification of Singular Fibers

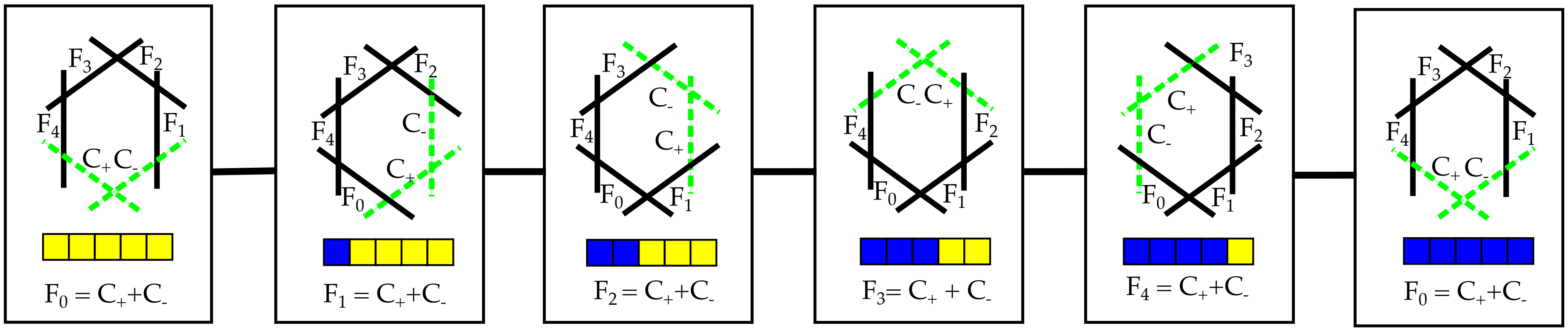
- Codim 1: Classic Algebraic Geometry [Kodaira][Néron]: Lie algebra $\mathfrak{g}$

Singular Fiber Codim 1	$\longleftrightarrow$	(Decorated) affine Dynkin diagram of $\mathfrak{g}$

- Codim 2: **R**= representation of $\mathfrak{g}$ [Hayashi, Lawrie, Morrison, SSN]

Singular Fiber Codim 2	$\longleftrightarrow$	Box Graph = Decorated rep graph of **R**

Tool: Coulomb phases of $3d$ $N = 2$ susy gauge theories.



NB: known also now for other matter and higher rank

II. Abelian Gauge Groups in F-theory

Mordell-Weil group and $U(1)$s

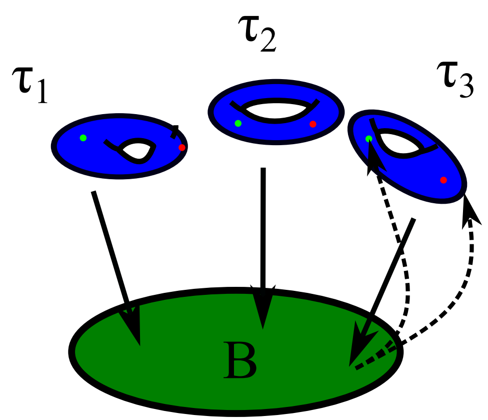
$U(1)$s arise from additional $(1,1)$-forms in fibration

$$C_3 = A \wedge \omega^{(1,1)}$$

(1,1)-forms in elliptic fibration:

- Kodaira singular fiber ($\Rightarrow$ GUT gauge bosons)
- Rational sections of fibration (″rational solutions to the elliptic curve equation″ or ″marked points″)

$U(1)$s $\leftrightarrow$ rational sections



Math fun facts:

- Elliptic curves have group laws: can add points on curves $p \boxplus q$
- The rational points on an elliptic curve form a free abelian group

$$\text{Mordell-Weil group} \cong \mathbb{Z}^n \oplus \Gamma$$

- Rational points:

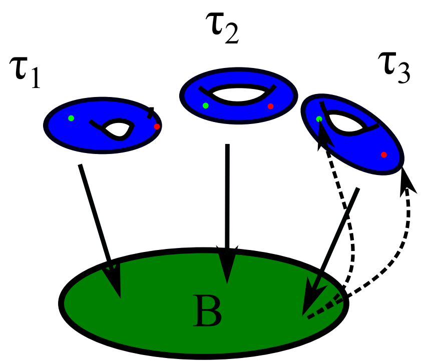
$$y^2 = x^3 + fxw^4 + gw^6 \qquad \sigma_0 : \;\; w = 0,\; x = y = 1$$

$\Rightarrow$ Recall: Weierstrass generically has only one marked point "origin"

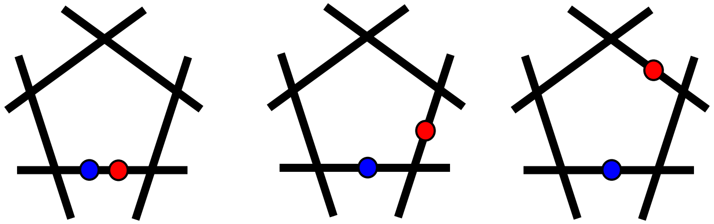
$$y\left(y + bx^2\right) = wP(x,y,w) \qquad \begin{cases} \sigma_0 : & w = 0,\; y = 0 \\ \sigma_1 : & w = 0,\; y = -bx^2 \end{cases}$$

$\Rightarrow$ σ_0 is the origin and σ_1 generates Mordell-Weil=$\mathbb{Z}$

Elliptic fibrations with rational sections



Codim 1: $SU(5)$ singular fiber with σ_0 and σ_1 intersecting one of the $\mathbb{P}^1$s:



Codim 2:

- $\mathbb{P}^1 \to C^+ + C^-$ with $C^\pm$ weights of matter representation.
- $U(1)$ charge: σ_1 intersected with $C^\pm$
- Question: what can σ_0 and σ_1 do in codim 2?
  $\Rightarrow$ Universal characterization of $U(1)$s in F-theory

Strategy

[Lawrie, SSN, Wong]

Fibers in codim 2 (Box graphs)

↘

Constraining all possible U(1) charges

↗

General properties of sections

Constraining rational sections in codim 2: CY3 and CY4

[Lawrie, SSN, Wong]

- Compatibility codim 1 and codim 2: $\sigma \cdot F = 1$ etc.
- New effect: sections can contain $\mathbb{P}^1$s of fiber $\Rightarrow$ "wrapping"

1. Constraints on normal bundle of rational curves C: If $C \subset \sigma \subset Y$, and σ and Y smooth, with σ divisor:

$$0 \to N_{C/\sigma} \to N_{C/Y} \to N_{\sigma/Y}|_C \to 0$$

2. Connecting normal bundle to charge:

$$\sigma \cdot_Y C = -2 - \deg N_{C/\sigma}$$

3. Know $N_{C/Y}$ from codim 2 fibers/box graphs
$\Rightarrow$ determine all possible embeddings of $N_{C/\sigma}$

Key assumption: σ is smooth.

Determining intersections and charges

Y= smooth Calabi-Yau three-fold (similar analysis for CY four-fold in paper), $\sigma \subset Y$ a non-singular divisor, $C \subset \sigma$ a rational curve.

(i) Let $(C)^2_\sigma = \deg(N_{C/\sigma}) = k$. If $k \geq -1$ the short exact sequence of normal bundles splits and

$$N_{C/Y} = \mathcal{O}(k) \oplus \mathcal{O}(-2-k)\,.$$

(ii) Let $N_{C/Y} = \mathcal{O}(-1) \oplus \mathcal{O}(-1)$. If σ smooth, $C \subset \sigma$, then

$$N_{C/D} = \mathcal{O}(k)\,, \qquad k \leq -1$$

and there exists a non-trivial embedding

$$\mathcal{O}(k) \hookrightarrow N_{C/Y} = \mathcal{O}(-1) \oplus \mathcal{O}(-1)$$

and

$$\sigma \cdot_Y C = -2 - k \geq -1$$

(iii) Let $N_{C/Y} = \mathcal{O} \oplus \mathcal{O}(-2)$. If σ smooth, $C \subset \sigma$, then

$$N_{C/\sigma} = \mathcal{O}(k)\,, \qquad k = 0 \quad \text{or} \quad k \leq -2$$

and there exists a non-trivial embedding

$$\mathcal{O}(k) \hookrightarrow N_{C/Y} = \mathcal{O} \oplus \mathcal{O}(-2)$$

and

$$\sigma \cdot_Y C = -2 - k = \begin{cases} -2 & k = 0 \\ \geq 0 & k \leq -2 \end{cases} .$$

Codim 2 Fibers: $SU(5) \longrightarrow SU(6)$

Example: $F_1 \to C^+ + C^-$. Then from box graph analysis $C^\pm$ are $(-1,-1)$ curves:

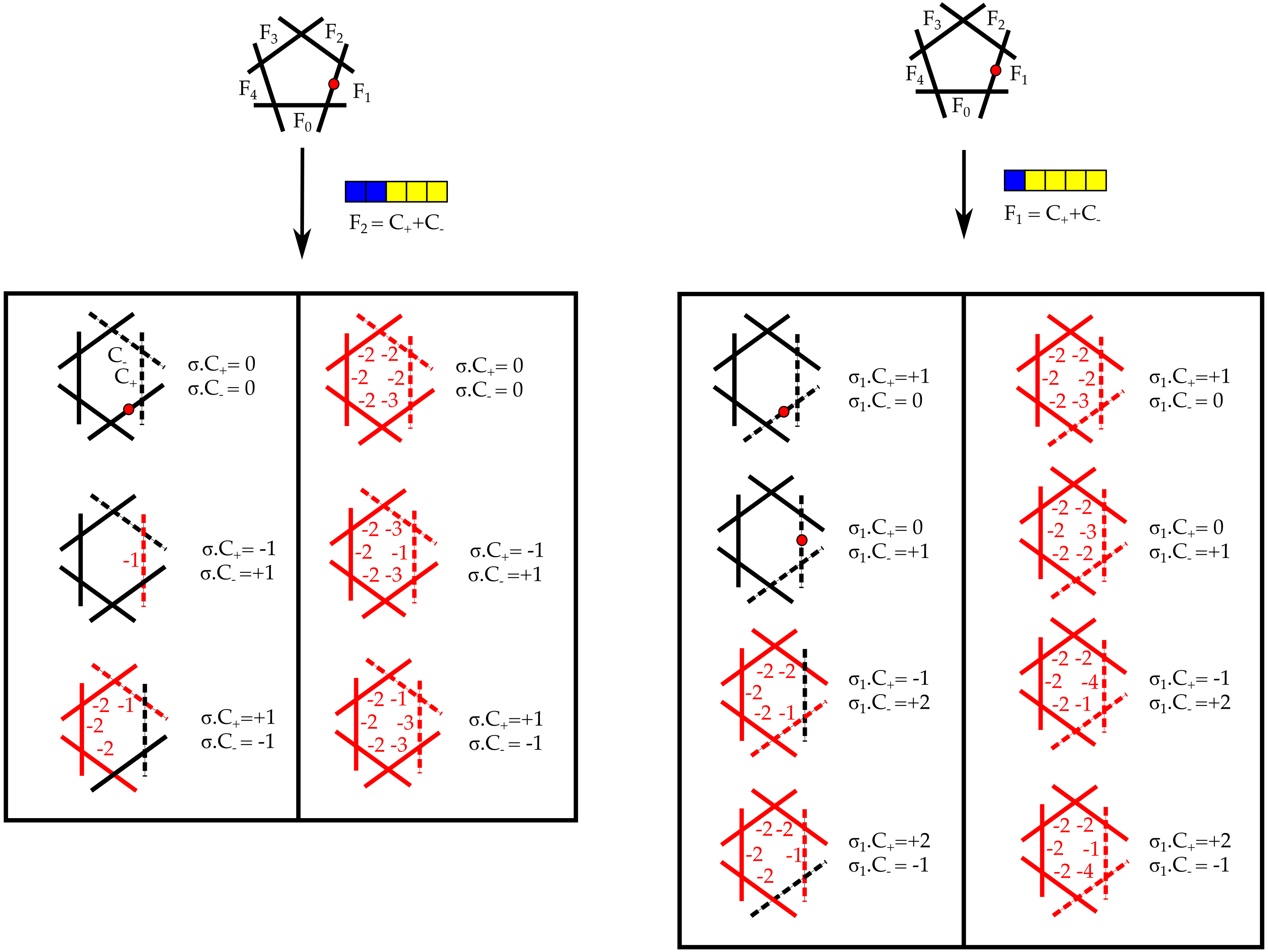
$$N_{C^\pm/Y} = \mathcal{O}(-1) \oplus \mathcal{O}(-1)$$

In particular $\sigma =$ smooth rational section, $C^+ \subset \sigma$

$$\Rightarrow \quad N_{C^+/\sigma} = \mathcal{O}(k)\,, \qquad k \leq -1$$

$$\Rightarrow \quad \sigma \cdot_Y C^+ = -2 - k \geq -1$$

Codim 2 Fibers: $SU(5) \longrightarrow SU(6)$

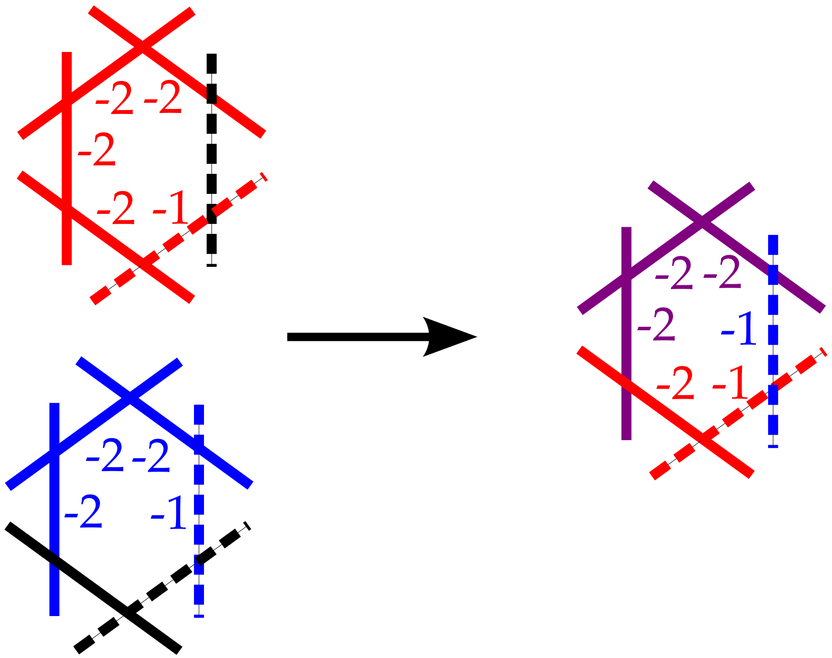


$U(1)$ charges

The $U(1)$ charge obtained from intersecting with (Shioda)

$$S = 5\sigma_1 - 5\sigma_0 + S_f \,,$$

S_f ensures that roots of $SU(5)$ remain uncharged under $U(1)$.



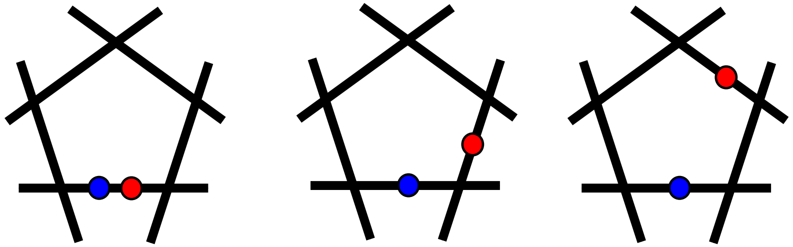
$S.C_{+}$=-14

$S.C_{-}$=+14

Complete set of charges

CY three- AND four-fold charges with smooth rational sections are constrained to be as follows:

$I_5^{(01)}, I_5^{(0|1)}, I_5^{(0||1)}$:



$$I_5^{(01)} : \quad \{-3, -2, -1, 0, +1, +2, +3\}$$

$\bar{\mathbf{5}}$ $U(1)$ charges:

$$I_5^{(0|1)} : \quad \{-14, -9, -4, +1, +6, +11\}$$

$$I_5^{(0||1)} : \quad \{-13, -8, -3, +2, +7, +12\} \ .$$

Similar analysis for I_1^* yields all possible charges for **10** matter.

$$I_5^{(01)} : \quad \{\mp 3, \mp 2, \mp 1, 0, \pm 1, \pm 2, \pm 3\}$$

10 $U(1)$ charges:

$$I_5^{(0|1)} : \quad \{\mp 13, \mp 8, \mp 3, \pm 2, \pm 7, \pm 12\}$$

$$I_5^{(0||1)} : \quad \{\mp 11, \mp 6, \mp 1, \pm 4, \pm 9\} \ .$$

$U(1)$ charges of GUT-singlets

Similar analysis for $U(1)$-charged GUT singlets: key to break to discrete symmetries $\Gamma \subset U(1)$.

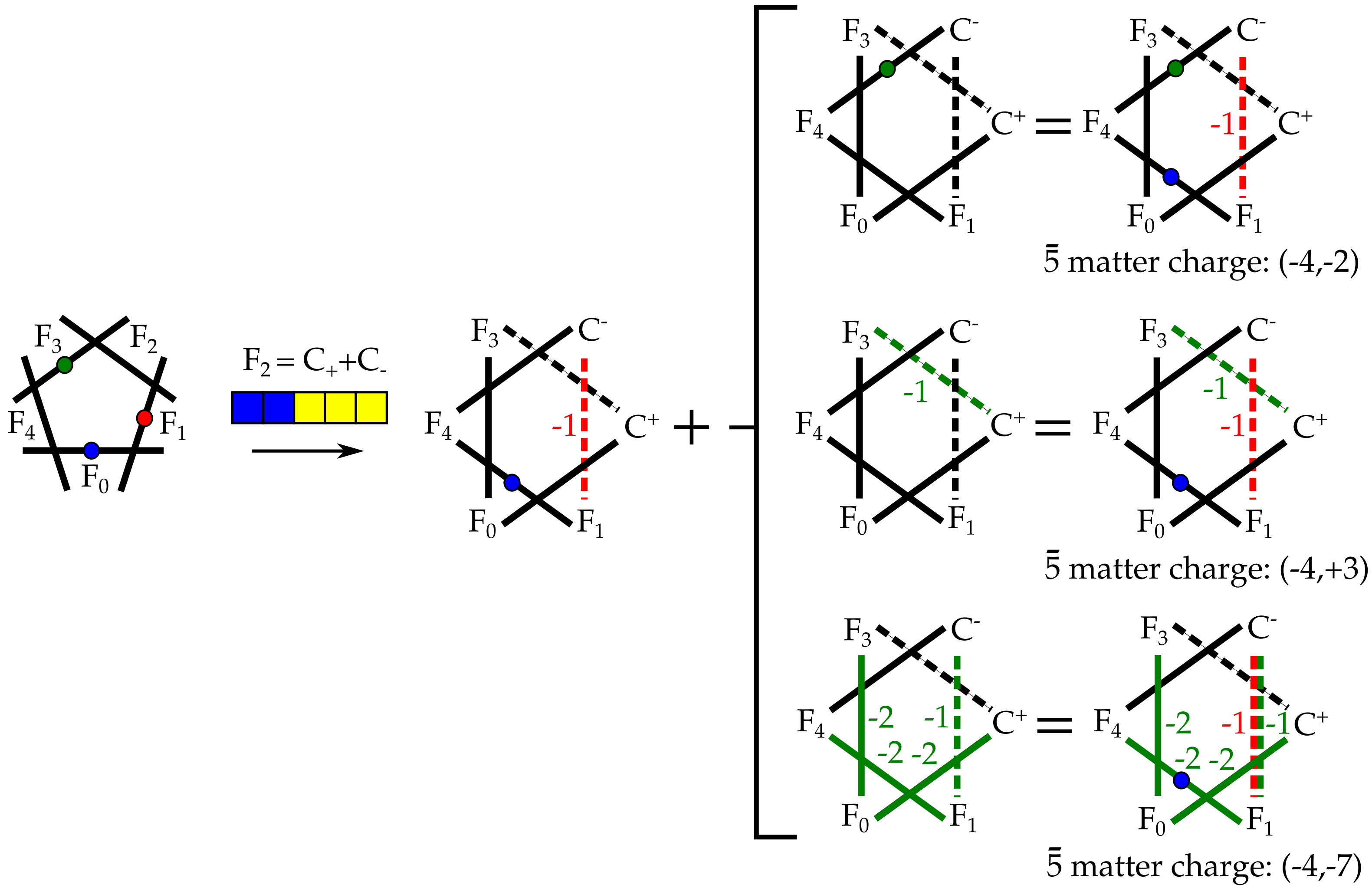
Realizes all the KK-charges $\sigma_0 \cdot C$ for these singlets as well $\Rightarrow$ all elements of Tate-Shafarevich, see also [Mayrhofer, Palti, Till, Weigand], [Cvetic, Donagi, Klever, Piragua, Poretschkin] for charge 2 and 3.

For singlets for CY3: exists criterion for contractibility of rational curves [Reid, Laufer] $N_{C/Y}$ has degree $(0,-2), (1,1), (-3,1)$ (For CY4, we determine all possibilities, but don't impose contractability)

[Lawrie, SSN, Wong]

$U(1)$ charges of GUT singlets in
$$\begin{cases} I_5^{(01)} \in \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6\} \\ I_5^{(0|1)} \in \{0, \pm 5, \pm 10, \pm 15, \pm 20, \pm 25\} \\ I_5^{(0||1)} \in \{0, \pm 5, \pm 10, \pm 15, \pm 20, \pm 25\} \,. \end{cases}$$

Multiple $U(1)$s



III. Phenomenology

1. Uses of Symmetries

Input: $U(1)$s and 4d spectra from F-theory.

Uses of $U(1)$s:

- Suppress unwanted couplings: Proton decay
- Forbid tree-level μ-term
- Flavor: $U(1)$s for Froggatt-Nielsen

Rapid Proton Decay

Protect model from Proton Decay: half-life $> 10^{36}$ years.

- Dim 4: B/L-violating operators (R-parity violating)

$$W_{\text{dim }4} = \lambda^{(4)}_{ija}\bar{\mathbf{5}}_i\bar{\mathbf{5}}_j\mathbf{10}_a \supset \lambda^0_{ija}L_iL_j\bar{e}_a + \lambda^1_{ija}\bar{d}_iL_jQ_a + \lambda^2_{ija}\bar{d}_i\bar{d}_j\bar{u}_a$$

$$\sqrt{\lambda^1\lambda^2} \leq \left(\frac{M_{SUSY}}{\text{TeV}}\right)10^{-12}$$

- Dim 5:

$$\begin{aligned} W_{\text{dim5}} &= \delta^{(5)}_{abci}\mathbf{10}_a\mathbf{10}_b\mathbf{10}_c\bar{\mathbf{5}}_i \\ &\supset \delta^1_{abci}Q_aQ_bQ_cL_i + \delta^2_{abci}\bar{u}_a\bar{u}_b\bar{e}_c\bar{d}_i + \delta^3_{abci}Q_a\bar{u}_b\bar{e}_cL_i \end{aligned}$$

$$\delta^1_{112i} \leq 16\pi^2\left(\frac{M_{SUSY}}{M^2_{GUT}}\right) \qquad i = 1,2$$

$\Rightarrow$ $U(1)$s or discrete symmetries Γ to control spectrum

(C1.) μ-term:

$$\mu \mathbf{5}_{H_u} \bar{\mathbf{5}}_{H_d}$$

(C2.) Dimension five proton decay:

$$\delta^{(5)}_{abci} \mathbf{10}_a \mathbf{10}_b \mathbf{10}_c \bar{\mathbf{5}}_i \,, \qquad a, b, c, i = \text{matter}$$

(C3.) Bilinear lepton number violating superpotential coupling:

$$\beta_i \bar{\mathbf{5}}_i \mathbf{5}_{H_u} \supset \beta_I L_I H_u \,, \qquad i = \text{matter}, I = 1, 2, 3$$

(C4.) Dimension four proton decay:

$$\lambda^{(4)}_{ija} \bar{\mathbf{5}}_i \bar{\mathbf{5}}_j \mathbf{10}_a \,, \qquad i, j, a = \text{matter}$$

(C5.) Tri-linear lepton number violating Kähler potential couplings:

$$\kappa_{abi} \mathbf{10}_a \mathbf{10}_b \bar{\mathbf{5}}^\dagger_i \supset \kappa_{ABI} Q_A \bar{u}_B L^\dagger_I \,, \qquad a, b, i = \text{matter}, A, B, I = 1, 2, 3$$

(C6.) Dimension five lepton violating superpotential coupling:

$$\gamma_i \bar{\mathbf{5}}_i \bar{\mathbf{5}}_{H_d} \mathbf{5}_{H_u} \mathbf{5}_{H_u} \supset \gamma_I L_I H_d H_u H_u \,, \qquad i = \text{matter}, I = \text{generation index}$$

(C7.) Dimension five lepton violating Kähler potential coupling:

$$\rho_a \bar{\mathbf{5}}_{H_d} \mathbf{5}^\dagger_{H_u} \mathbf{10}_a \supset \rho_A \bar{\mathbf{5}}_{H_d} \mathbf{5}^\dagger_{H_u} \bar{e}_A \,, \qquad a = \text{matter}, A = 1, 2, 3$$

2. Anomalies

F_Y GUT breaking generates chiral spectrum
$\Rightarrow$ In presence of $U(1)$s: Require $G^2_{MSSM} \times U(1)$ and $U(1)_Y \times U(1) \times U(1)'$ anomaly cancellation

[Dudas Palti], [Marsano, Saulina, SS-N], [Marsano], [Palti]

$\Rightarrow$ Compatibility constraints between charges and F_Y restriction N:

$$\mathbf{10}_a : \quad \begin{cases} (\mathbf{3},\mathbf{2})_{1/6} : & M_a \\ (\bar{\mathbf{3}},\mathbf{1})_{-2/3} : & M_a - N_a \\ (\mathbf{1},\mathbf{1})_1 : & M_a + N_a \end{cases} \qquad \bar{\mathbf{5}}_i : \quad \begin{cases} (\bar{\mathbf{3}},\mathbf{1})_{1/3} : & M_i \\ (\bar{\mathbf{1}},\mathbf{2})_{-1/2} : & M_i + N_i \end{cases}$$

$q = U(1)$-charges:

$$\sum_a q_a^\alpha N_a + \sum_i q_i^\alpha N_i = 0 \qquad (A1.)$$

$$3\sum_a q_a^\alpha q_a^\beta N_a + \sum_i q_i^\alpha q_i^\beta N_i = 0 \qquad (A2.)$$

$\Rightarrow$ Constraints on M, N and $U(1)$ charges.

3. Flavor and Froggatt-Nielsen

Long History of Flavor in F-theory: [Font, Ibañez, Heckman, Vafa, Dudas, Palti, Marchesano, Aparicio, Uranga, Regalado, Zoccarato, King, Leontaris, Ross, Hayashi, Kawano, Tsuchiya, Watari, ...]

$U(1)$s to generate flavor textures, Froggatt-Nielsen (FN) type. Tree-level Yukawas + subleading terms from $U(1)$-charged singlets $\epsilon = \frac{\langle S \rangle}{\Lambda}$.
Consistent with $SU(5)$ GUT e.g. [Dreiner, Thormeier]

$$Y_u \sim \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix} , \quad Y_d \sim \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^4 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ 1 & 1 & 1 \end{pmatrix} .$$

For local F-theory GUTs: no realistic FN models from E_8 [Dudas, Palti].
Why reconsider now?
New insights and general understanding of $U(1)$s in F-theory.

New insights from Geometry

Idea of this Program:

1. Phenomenological constraints on Symmetries, 2. Anomalies, and 3. Realistic Flavor combined with global, geometric consistencies imply constraints on resulting 4d EFT.

F-theory/String theory input:

Constraints on F-theory compactification geometries for GUTs with extra $U(1)$s. Use classification of $U(1)$s in part II.

Search Strategy

- Strict minimal spectrum (no exotics) solving anomaly eqs (A1.), (A2.)
- Only F-theoretic U(1) charges
- Absence at leading order of all operators (C1.)-(C7.) at leading order.
- Generating of at least one charge neutral coupling

$$Y^t_{ab}: \qquad \lambda^t_{ab}\mathbf{10}_a\mathbf{10}_b\mathbf{5}_{H_u} \supset Y^u_{AB}Q_A\bar{u}_BH_u\,,\ a,b = \text{matter}$$

- Leading order Y_b can be absent, but regeneration with singlet vevs should violate bounds on (C1.)-(C7.)

$$Y^b_{ai}: \qquad \lambda^b_{ai}\mathbf{10}_a\bar{\mathbf{5}}_i\bar{\mathbf{5}}_{H_d} \supset Y^d_{AI}Q_A\bar{d}_IH_d + Y^L_{AI}L_I\bar{e}_AH_d\,,\ i,a = \text{matter}$$

$\Rightarrow$ Search by #$U(1)$s, #$\bar{\mathbf{5}}$ and **10** matter curves.

Viable models

Two classes of solutions to all above requirements:

1. Solutions which also generate via Froggatt-Nielsen mechanism realistic Yukawa textures
   $\Rightarrow$ ✓

2. Solutions which have charges within known explicit geometric constructions
   $\Rightarrow$ ✓ (lots of solutions) but need additional structure like fluxes to generate additional flavor hierarchies

FN Benchmark Model

- Require 2 $U(1)$s, 3 $\mathbf{10}$ for good flavor models
- Singlet vevs with charge q: GUT scale mass ratios ($\epsilon \sim .22$)

$$\begin{aligned}
m_t : m_c : m_u &\sim 1 : \epsilon^4 : \epsilon^8 \\
m_b : m_s : m_d &\sim 1 : \epsilon^2 : \epsilon^4 \\
m_\tau : m_\mu : m_e &\sim 1 : \epsilon^2 : \epsilon^{4,5} \\
\frac{m_b}{m_t} = \epsilon^x \tan^{-1}\beta \sim \epsilon^3 & \qquad m_b \sim m_\tau \ , \ m_t \sim \langle H_u \rangle \\
\text{Quark mixing: } \theta_{12} &\sim \epsilon, \quad \theta_{23} \sim \epsilon^2, \quad \theta_{31} \sim \epsilon^3
\end{aligned} \tag{1}$$

Yukawa textures that are compatible with $SU(5)$ GUT:

$$(*) \quad Y_u \sim \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \quad Y_d \sim \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^4 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ 1 & 1 & 1 \end{pmatrix}.$$

Benchmark FN-F-theory model:

GUT Reps	Charges	M	N	MSSM Matter
$\mathbf{10}_1$	$(10,-7)$	1	0	$Q_1, \bar{u}_1, \bar{e}_i,\ i=1,2$
$\mathbf{10}_2$	$(5,-7)$	1	0	$Q_2, \bar{u}_2, \bar{e}_j,\ j\neq i,\ j=1,2$
$\mathbf{10}_3$	$(0,-7)$	1	0	$Q_3, \bar{u}_3, \bar{e}_3$
$\bar{\mathbf{5}}_{H_u}$	$(0,14)$	0	-1	H_u
$\bar{\mathbf{5}}_{H_d}$	$(0,6)$	0	1	H_d
$\bar{\mathbf{5}}_1$	$(0,-9)$	0	2	$L_i, i=1,2$
$\bar{\mathbf{5}}_2$	$(0,1)$	3	-2	$L_3, \bar{d}_i, i=1,2,3$

μ-term charge: $(0,20)$. Yukawa charge matrix:

$$Q_{Y^u} \sim \begin{pmatrix} (20,0) & (15,0) & (10,0) \\ (15,0) & (10,0) & (5,0) \\ (10,0) & (5,0) & (0,0) \end{pmatrix}, \quad Q_{Y^d} \sim \begin{pmatrix} (10,0) & (10,0) & (10,0) \\ (5,0) & (5,0) & (5,0) \\ (0,0) & (0,0) & (0,0) \end{pmatrix}.$$

- Singlet vev: $S = \mathbf{1}_{(-5,0)} \Rightarrow$ with $s = \frac{\langle S \rangle}{M_{GUT}} = \epsilon^2$: generates successful FN: Yukawas (*)
- All (C1.)-(C7). suppressed, NOT regenerated at same order as Yukawas.
- $U(1)_1$ ensures flavor texture, $U(1)_2$ ensures absence of (C1).-(C7).

Discussion

We determined the most general $U(1)^n$ charges for $SU(5)$ models with $\bar{\mathbf{5}}$ and $\mathbf{10}$ matter.

1. Validity: Smooth versus singular sections
   Singular divisors also contribute to $(1,1)$ forms. Normal bundle exact sequence does not hold. What replaces it, and what are constraints?

2. Global Patching:
   What are the global compatibility conditions between the fibers?

3. Geometric Construction

4. Physics: [Krippendorf, SSN, Wong]
   Combine with 1. Phenomenological constraints on Symmetries,
   2. Anomalies and 3. Realistic Flavor from FN
   ⇒ Successful model: there are only very few similarly successful models within the class of F-theory charges!
   ⇒ Construct geometry, fluxes, moduli stabilization.

Thank you