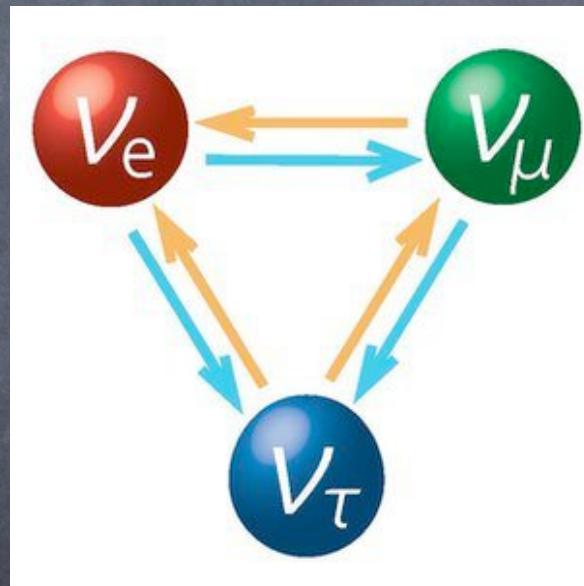


Neutrino oscillations



Neutrino mixing

- Mixing is described by the Maki-Nakagawa-Sakata (MNS) matrix:

$$\nu_{\alpha L} = \sum_k U_{\alpha i} \nu_{kL} \quad U = U_l^\dagger U_\nu$$

- leptonic weak charged current:

$$j_\rho^{\text{CC}\dagger} = 2 \sum_{\alpha=e,\mu,\tau} \overline{\alpha_L} \gamma_\rho \nu_{\alpha L} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^3 \overline{\alpha_L} \gamma_\rho U_{\alpha k} \nu_{kL}$$

- NxN unitary matrix: NxN mixing parameters

→ N(N-1)/2 mixing angles + N(N+1)/2 phases

- Lagrangian invariant under global phase transformations of Dirac fields:

$$\alpha \rightarrow e^{i\theta_\alpha} \alpha, \nu_k \rightarrow e^{i\phi_k} \nu_k$$

$$j_\rho^{\text{CC}\dagger} \rightarrow 2 \sum_{\alpha,k} \overline{\alpha_L} e^{-i(\theta_e - \phi_1)} e^{-i(\theta_\alpha - \theta_e)} \gamma_\rho U_{\alpha k} e^{i(\phi_k - \phi_1)} \nu_{kL}$$

→ 2N-1 phases can be eliminated: (N-1)(N-2)/2 physical phases

Neutrino mixing

- For Majorana neutrinos, the lagrangian is NOT invariant under global phase transformations of the Majorana fields:

$$\nu_k \rightarrow e^{i\phi_k} \nu_k \quad \nu_{kL}^T \mathcal{C}^\dagger \nu_{kL} \rightarrow e^{2i\phi_k} \nu_{kL}^T \mathcal{C}^\dagger \nu_{kL}$$

- only N phases can be eliminated by rephasing charged lepton fields:

$$j_\rho^{\text{CC}\dagger} \rightarrow 2 \sum_{\alpha, k} \overline{\alpha_L} e^{-i\theta_\alpha} \gamma_\rho U_{\alpha k} \nu_{kL}$$

- $N(N-1)/2$ physical phases: $(N-1)(N-2)/2$ Dirac phases → effect in ν oscil.
 $(N-1)$ Majorana phases → relevant for $0\nu\beta\beta$

Exercises: 1) Check number angles/phases NxN matrix
2) how many angles/phases $N=2,3$?

Neutrino mixing

- 2-neutrino mixing depends on 1 angle only (+1 Majorana phase)

$$\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

- 3-neutrino mixing is described by 3 angles and 1 Dirac (+2 Majorana) CP violating phases.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric + LBL
measurements

reactor disapp + LBL
appearance searches

solar + KamLAND
measurements

Neutrino oscillations

- flavour states are admixtures of flavor eigenstates:

$$\nu_{\alpha L} = \sum_k U_{\alpha i} \nu_{kL}$$

- Neutrino evolution equation:

$$-i \frac{d}{dt} |\nu\rangle = H |\nu\rangle$$

in the 2-neutrino mass eigenstates basis ν_j :

$$H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$



$$|\nu_j\rangle \rightarrow e^{-iE_j t} |\nu_j\rangle$$

since neutrinos are relativistic: $t = L$ and

$$E_j \simeq p + \frac{m_j^2}{2p} \simeq p + \frac{m_j^2}{2E}$$

- Hamiltonian in the flavour basis:

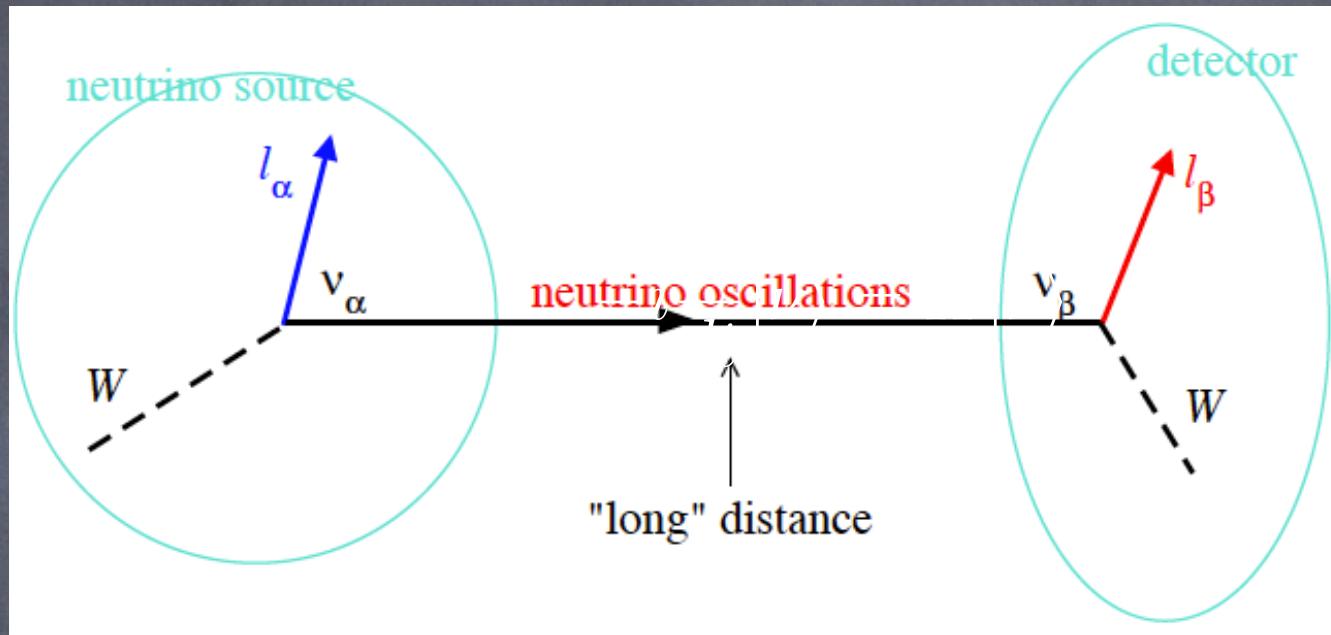
(equal momentum approach)

$$H_{\text{flavour}} = U H_{\text{mass}} U^\dagger = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

with $\Delta m^2 = m_2^2 - m_1^2$

Exercise: derive expression for H_{flavour}

Neutrino oscillations picture



Production

$$|\nu_\alpha\rangle = \sum_j U_{\alpha j}^* |\nu_j\rangle$$

coherent superposition
of massive states

Propagation

$$\nu_j : e^{-iE_j t}$$

different propagation
phases change ν_j
composition

Detection

$$\langle\nu_\beta| = \sum_j \langle\nu_j| U_{\beta j}$$

projection over flavour
eigenstates

Neutrino oscillations

Neutrino oscillation amplitude:

$$\begin{aligned}\mathcal{A}_{\nu_\alpha \rightarrow \nu_\beta} &= \langle \nu_\beta(t) | \nu_\alpha(0) \rangle = \sum_j \langle \nu_\beta | \nu_j(t) \rangle \langle \nu_j(t) | \nu_j(0) \rangle \langle \nu_j(0) | \nu_\alpha \rangle \\ &= \sum_j U_{\beta j} e^{-i \frac{m_j^2}{2E}} U_{\alpha j}^*\end{aligned}$$

Neutrino oscillation probability:

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \left| \sum_j U_{\alpha j}^* U_{\beta j} e^{-i \frac{m_j^2}{2E}} \right|^2$$

$$\begin{aligned}P_{\alpha\beta} &= \delta_{\alpha\beta} - 4 \sum_{i>j} \operatorname{Re}(U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^*) \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) + \\ &\quad + 2 \sum_{i>j} \operatorname{Im}(U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^*) \sin \left(\frac{\Delta m_{ij}^2 L}{2E} \right)\end{aligned}$$

Exercise: derive general formula for oscillation probability

General properties of neutrino oscillations

- Conservation of probability:
$$\sum_{\beta} P(\nu_{\alpha} \rightarrow \nu_{\beta}) = 1$$
- For antineutrinos: $U \rightarrow U^*$
- Neutrino oscillations violate flavour lepton number conservation (expected from mixing) but conserve total lepton number
- Complex phases in the mixing matrix induce CP violation:
$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) \neq P(\overline{\nu}_{\alpha} \rightarrow \overline{\nu}_{\beta})$$
- Neutrino oscillations do not depend on the absolute neutrino mass scale and Majorana phases.
- Neutrino oscillations are sensitive only to mass squared differences:

$$\Delta m_{kj}^2 = m_k^2 - m_j^2$$

2-neutrino oscillations

- 2-neutrino mixing matrix:

$$\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

- 2-neutrino oscillation probability ($\alpha \neq \beta$):

$$P(\nu_\alpha \rightarrow \nu_\beta) = \left| U_{\alpha 1} U_{\beta 1}^* + U_{\alpha 2} U_{\beta 2}^* e^{-i \frac{\Delta m_{21}^2 L}{2E}} \right|^2$$
$$= \sin^2(2\theta) \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$

Exercise

- The oscillation phase:

$$\phi = \frac{\Delta m_{21}^2 L}{4E} = 1.27 \frac{\Delta m_{21}^2 [eV^2] L [km]}{E [GeV]}$$

Exercise

→ short distances, $\phi \ll 1$: oscillations do not develop, $P_{\alpha\beta} = 0$

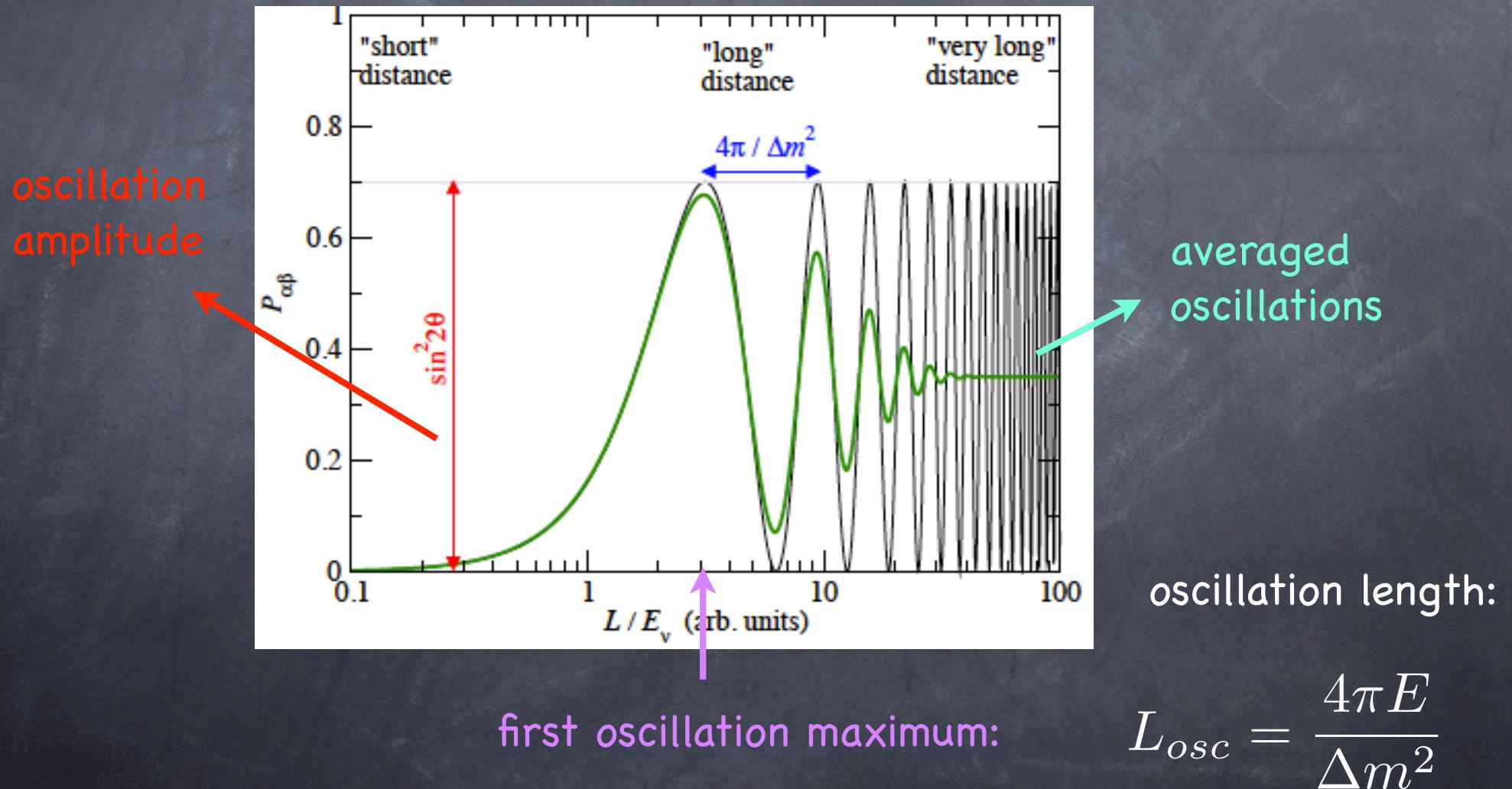
→ long distance, $\phi \sim 1$: oscillations are observable

→ very long distances, $\phi \gg 1$: oscillations are averaged out:

$$P_{\alpha\beta} \simeq \frac{1}{2} \sin^2(2\theta)$$

2-neutrino oscillation probability

$$P_{\alpha\beta} = \sin^2(2\theta) \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$



Appearance vs disappearance experiments

► appearance experiments:

$$\alpha \neq \beta$$

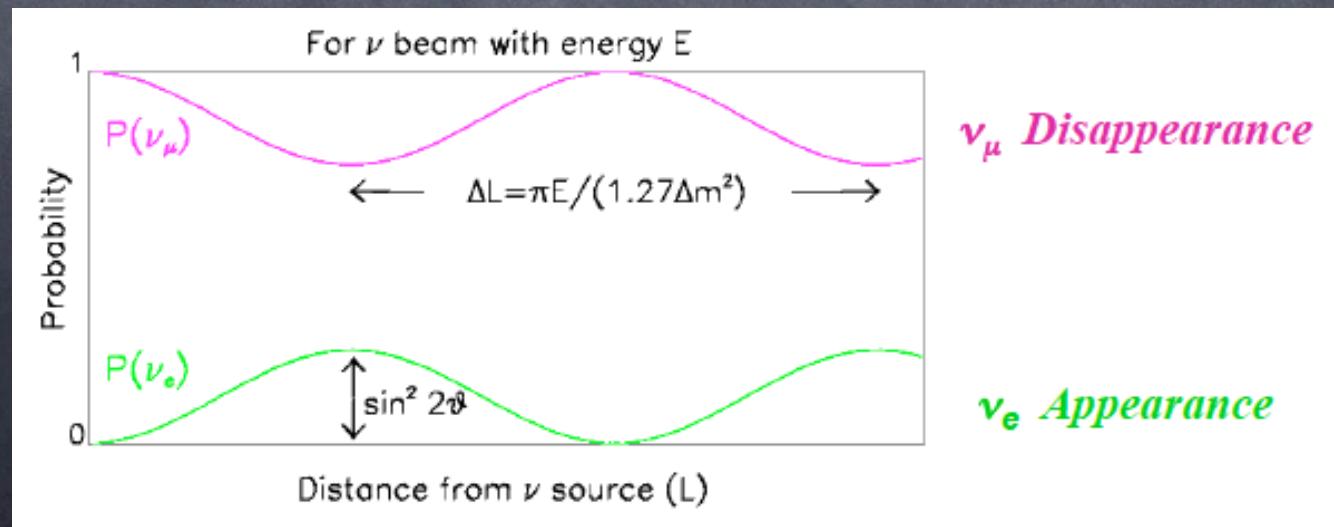
$$P_{\alpha\beta} = \sin^2(2\theta) \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$

→ appearance of a neutrino of a new flavour β in a beam of ν_α

► disappearance experiments:

$$P_{\alpha\alpha} = 1 - \sin^2(2\theta) \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$

→ measurement of the survival probability of a neutrino of given flavour



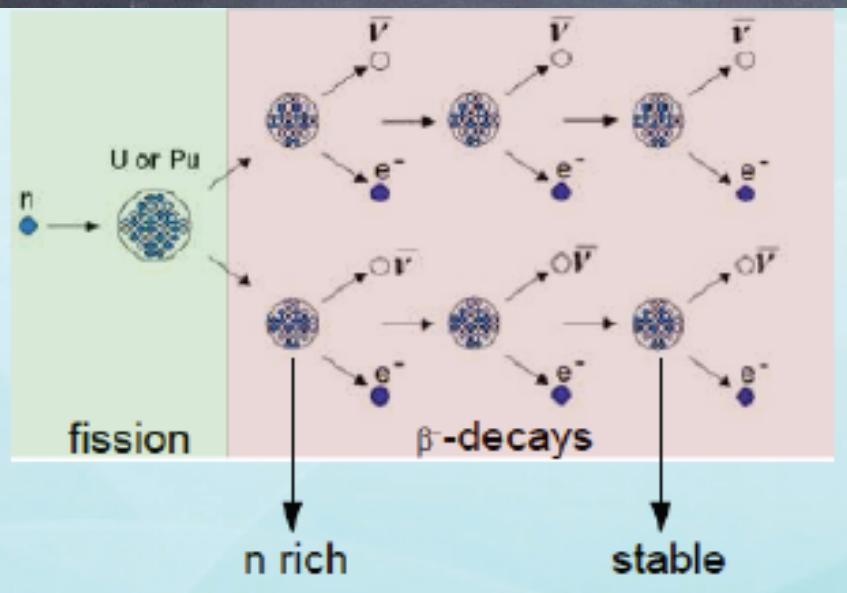
Example for 2-neutrino oscillations in vacuum: reactor experiment



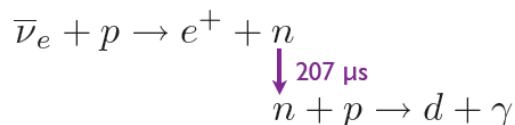
Production and detection of reactor neutrinos

Emission: β -decay of fission products

- $\sim 6 \bar{\nu}_e$ / fission
- $\sim 10^{21} \bar{\nu}_e$ / s for a 1 GW_{el} reactor



Inverse beta decay



Scintillator is both target and detector

- Distinct two step process:

- prompt event: positron

$$E_{\bar{\nu}_e} \simeq E_{prompt} + 0.8 \text{ MeV}$$

- delayed event: neutron capture after $\sim 207 \mu\text{s}$

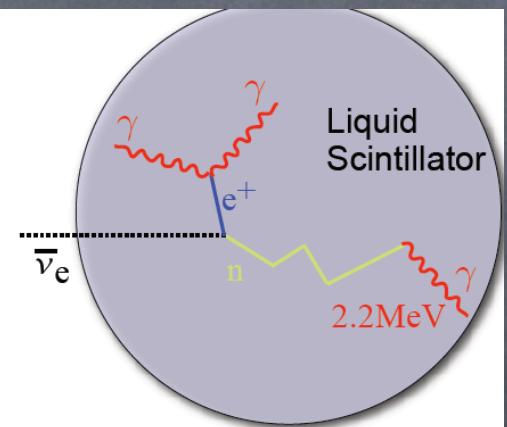
- 2.2 MeV gamma

Delayed coincidence: good background rejection

Detection reaction cross section $\sim 10^{-43} \text{ cm}^2$

→ typical ν detector masses: many 10 tons - some ktons

**1ton target @ 25m of 1GW_{el} reactor gives
~4600 int./day → 1% stat within 2 days**



The CHOOZ reactor experiment

- ▶ $2\nu \approx (\Delta m^2, \theta)$:

$$P_{ee} = 1 - \sin^2(2\theta) \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$

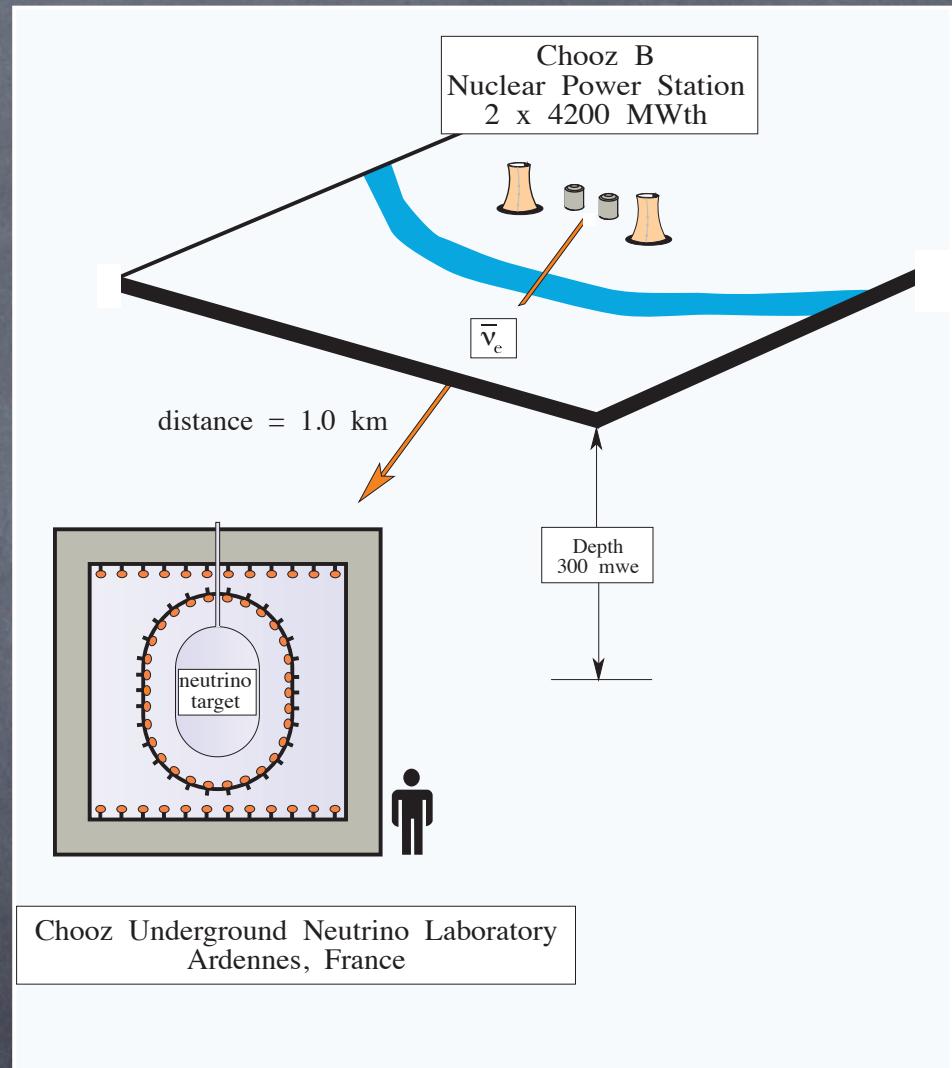
- ▶ $L = 1 \text{ km}$, $E \sim \text{MeV}$

→ sensitive to $\Delta m^2 \sim 10^{-3} \text{ eV}^2$

- ▶ non-observation of ν_e disappearance:

R = $1.01 \pm 2.8\%(\text{stat}) \pm 2.7\%(\text{syst})$

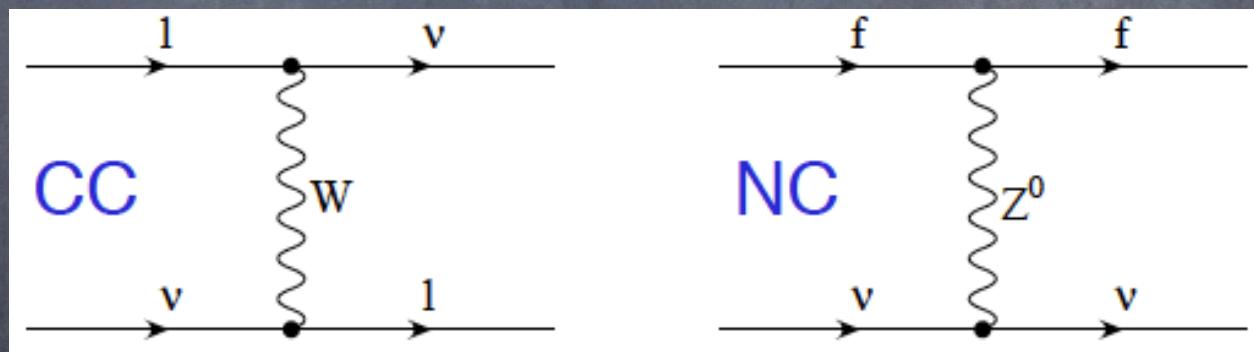
ratio between measured and expected number of events



Matter effects on neutrino oscillations

► When neutrinos pass through matter, the interactions with the particles in the medium induce an **effective potential** for the neutrinos.

[→ the coherent forward scattering amplitude leads to an index of refraction neutrinos. L. Wolfenstein, 1978]



→ modifies the mixing between flavor states and propagation states as well as the eigenvalues of the Hamiltonian, leading to a different oscillation probability wrt vacuum.

Effective matter potential

- Effective four-fermion interaction Hamiltonian (CC+NC)

$$H_{\text{int}}^{\nu_\alpha} = \frac{G_F}{\sqrt{2}} \overline{\nu_\alpha} \gamma_\mu (1 - \gamma_5) \nu_\alpha \sum_j \overline{f} \gamma^\mu (g_V^{\alpha,f} - g_A^{\alpha,f} \gamma_5) f$$

in ordinary matter: $f = e^-, p, n$

$$J_{\text{matt}}^{\mu\alpha}$$

To obtain the matter-induced potential we integrate over f-variables:

for a	non-relativistic	medium:	$\langle \overline{f} \gamma^\mu f \rangle = \frac{1}{2} N_f \delta_{\mu,0}$
	unpolarised		$\langle f \gamma_5 \gamma^\mu f \rangle = 0$
	neutral		$N_e = N_p$

$$J_{\text{matt}}^{\mu\alpha} = \frac{1}{2} [N_e (g_V^{\alpha,e} + g_V^{\alpha,p}) + N_n g_V^{\alpha,n}]$$

Effective matter potential

$$J_{\text{matt}}^{\mu\alpha} = \frac{1}{2} [N_e(g_V^{\alpha,e} + g_V^{\alpha,p}) + N_n g_V^{\alpha,n}]$$

g_V	e^-	p	n
ν_e	$2 \sin^2 \Theta_W + \frac{1}{2}$	$-2 \sin^2 \Theta_W + \frac{1}{2}$	$-\frac{1}{2}$
$\nu_{\mu,\tau}$	$2 \sin^2 \Theta_W - \frac{1}{2}$	$-2 \sin^2 \Theta_W + \frac{1}{2}$	$-\frac{1}{2}$

$$\rightarrow J_{\text{matt}}^{\mu\alpha} = (N_e - \frac{1}{2}N_n, -\frac{1}{2}N_n, -\frac{1}{2}N_n)$$
$$V_{\text{matt}} = \sqrt{2}G_F \text{ diag}(N_e - \frac{1}{2}N_n, -\frac{1}{2}N_n, -\frac{1}{2}N_n)$$

- ▶ only νe are sensitive to CC (no μ, τ in ordinary matter)
- ▶ NC has the same effect for all flavours → it has no effect on evolution (however it can be important in presence of sterile neutrinos)
- ▶ for antineutrinos the potential has opposite sign

2-neutrino oscillations in matter

- 2-neutrino Hamiltonian in vacuum (mass basis): $H^{\text{vac}} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$
- In the **flavour basis**, where effective matter potential is diagonal:

$$H_f^{\text{matt}} = H_f^{\text{vac}} + V_{\text{eff}} = \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta + V_{CC} & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix}$$
$$V_{CC} = \sqrt{2} G_F N_e$$

Diagonalizing the Hamiltonian, we identify the mixing angle and mass splitting in matter:

$$H_f^{\text{matt}} = \frac{\Delta M^2}{4E} \begin{pmatrix} -\cos 2\theta_M & \sin 2\theta_M \\ \sin 2\theta_M & \cos 2\theta_M \end{pmatrix}$$

Exercise: calculate θ_M and ΔM^2

In general: $N_e = N_e(x)$, so θ_M and ΔM^2 will be function of x as well

→ however, in some cases analytical solutions can be obtained

2-V oscillations in constant matter

- If N_e is constant (good approximation for oscillations in the Earth crust):
 - θ_M and ΔM^2 are constant as well
 - we can use vacuum expression for oscillation probability, replacing “vacuum” parameters by “matter” parameters:

$$P_{\alpha\beta} = \sin^2(2\theta_M) \sin^2 \left(\frac{\Delta M^2 L}{4E} \right)$$

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2} \quad A = \frac{2EV}{\Delta m^2}$$

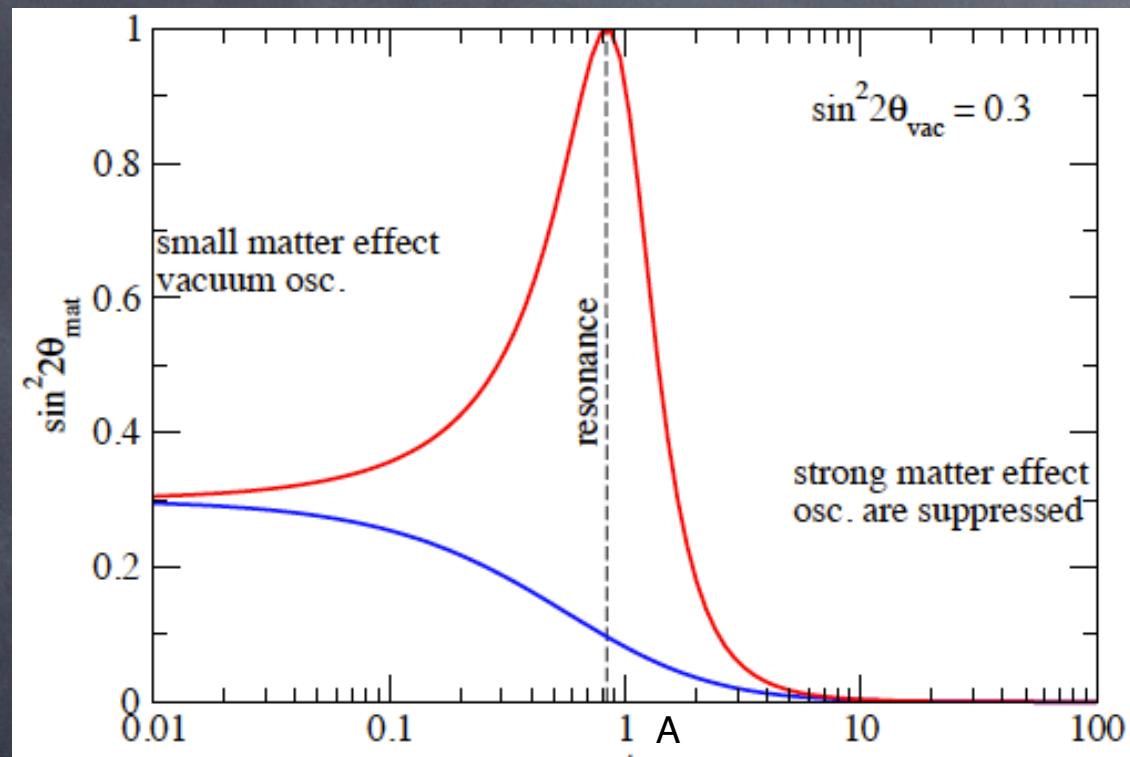
$$\Delta M^2 = \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

There is a resonance effect for $A = \cos 2\theta \rightarrow$ MSW effect

Wolfenstein, 1978

Mikheyev & Smirnov, 1986

2-V oscillations in constant matter



mixing angle in matter:

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

$$A = \frac{2EV}{\Delta m^2}$$

- $A \ll \cos 2\theta$, small matter effect \rightarrow vacuum oscillations: $\theta_M = \theta$
- $A \gg \cos 2\theta$, matter effects dominate \rightarrow oscillations are suppressed: $\theta_M \approx 0$
- $A = \cos 2\theta$, resonance takes place \rightarrow maximal mixing $\theta_M \approx \pi/4$

→ resonance condition is satisfied for neutrinos for $\Delta m^2 > 0$

for antineutrinos for $\Delta m^2 < 0$

Solar neutrinos: the MSW effect

- neutrino oscillations in matter were first discussed by Wolfenstein, Mikheyev and Smirnov (MSW effect)

- electron neutrino is born at the center of the Sun as:

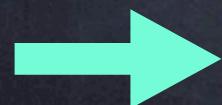
$$|\nu_e\rangle = \cos\theta_M |\nu_1^m\rangle + \sin\theta_M |\nu_2^m\rangle$$

→ ν_1^m and ν_2^m evolve adiabatically until the solar surface and propagate in vacuum from the Sun to the Earth:

$$P(\nu_e \rightarrow \nu_e) = P_{e1}^{\text{prod}} P_{1e}^{\text{det}} + P_{e2}^{\text{prod}} P_{2e}^{\text{det}}$$

$$P_{e1}^{\text{prod}} = \cos^2 \theta_M, \quad P_{1e}^{\text{det}} = \cos^2 \theta$$

$$P_{e2}^{\text{prod}} = \sin^2 \theta_M, \quad P_{2e}^{\text{det}} = \sin^2 \theta$$



$$P_{ee} = \cos^2 \theta_M \cos^2 \theta + \sin^2 \theta_M \sin^2 \theta$$

Solar neutrinos: the MSW effect

$$P_{ee} = \cos^2 \theta_M \cos^2 \theta + \sin^2 \theta_M \sin^2 \theta$$

► In the center of the Sun:

$$A = \frac{2EV}{\Delta m^2} \simeq 0.2 \left(\frac{E_\nu}{\text{MeV}} \right) \left(\frac{8 \times 10^{-5} \text{eV}^2}{\Delta m^2} \right)$$

Exercise: check this formula: $n_e \sim 100 \text{ mol/cm}^3$

and resonance occurs for $A = \cos(2\theta) = 0.4$

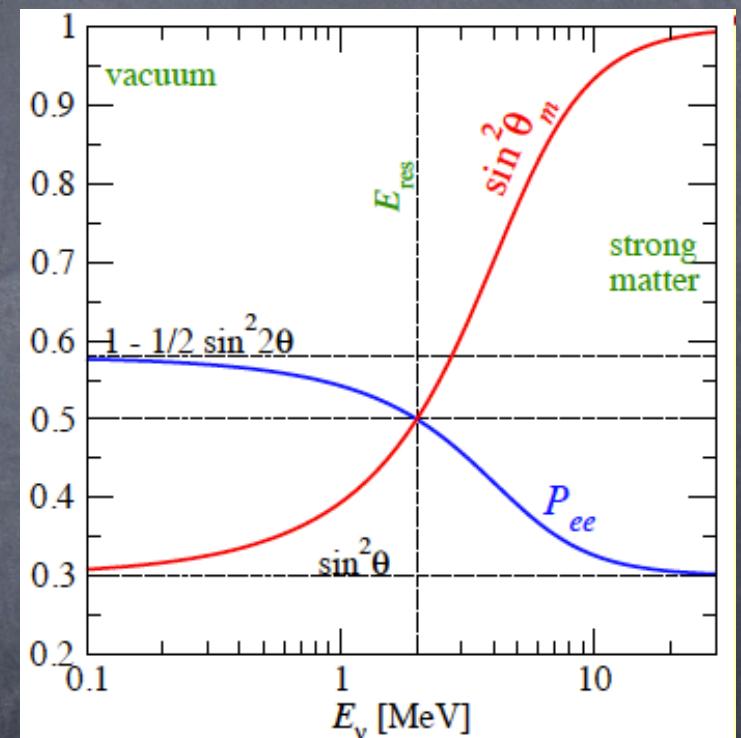
$$\rightarrow E_{\text{res}} \approx 2 \text{ MeV}$$

► For $E < 2 \text{ MeV} \rightarrow \text{vacuum osc: } \theta_M = \theta$

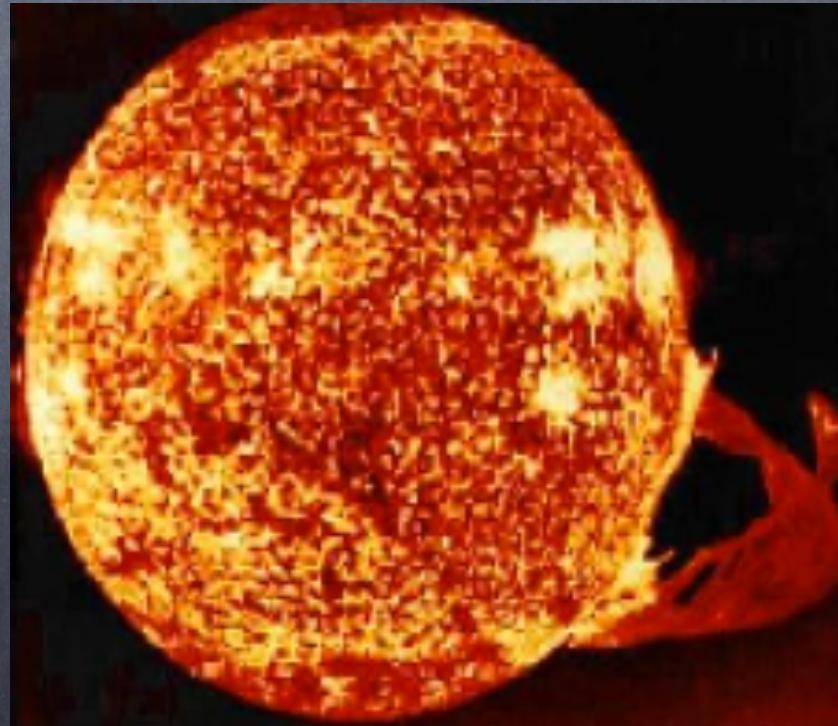
$$P_{ee} = 1 - \frac{1}{2} \sin^2 2\theta$$

► For $E > 2 \text{ MeV} \rightarrow \text{strong matter effect: } \theta_M = \pi/2 \quad P_{ee} = \sin^2 \theta$

→ $P_{ee}(E)$ will be crucial to understand solar neutrino data

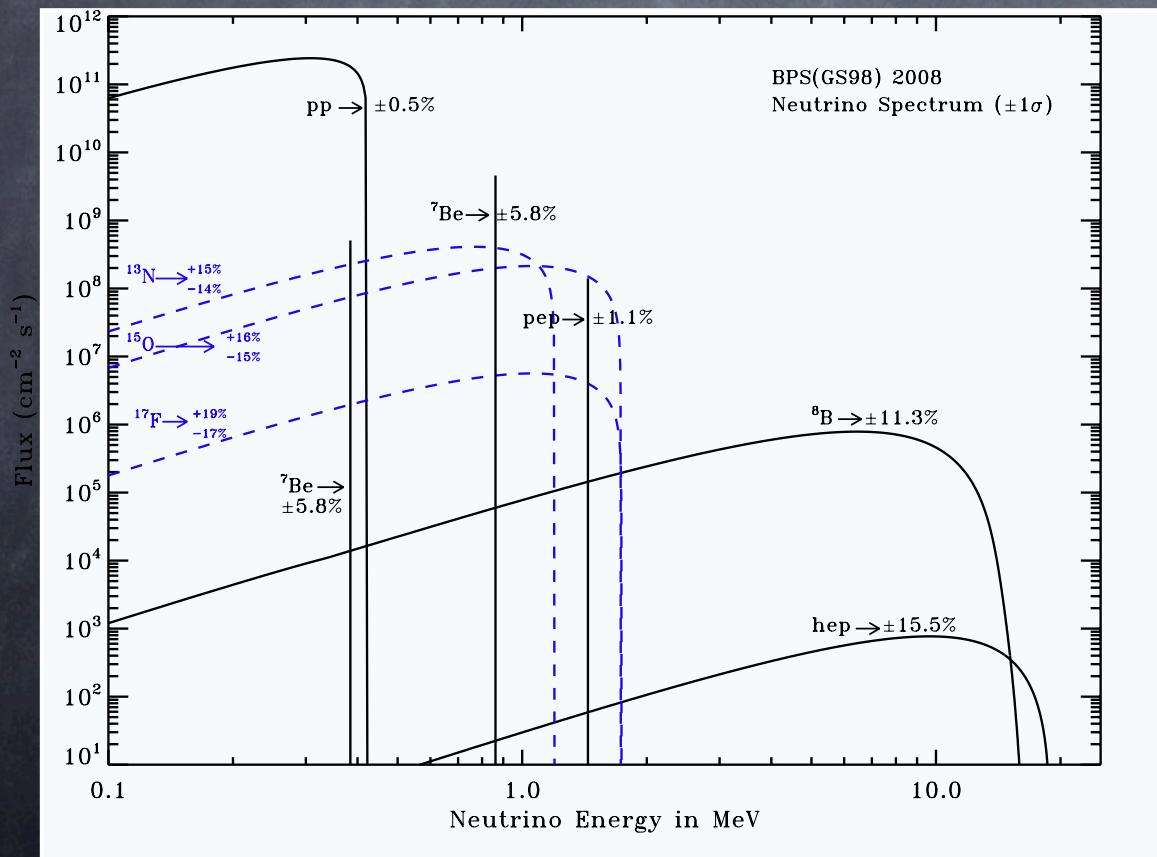


Example for 2-neutrino
oscillations in matter:
solar neutrinos

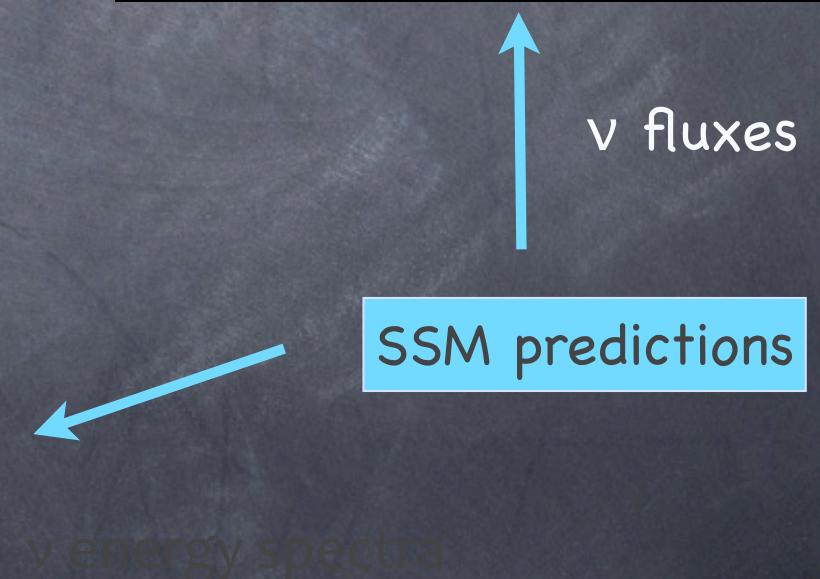


Solar neutrinos

* produced in nuclear reactions in the core of the Sun:



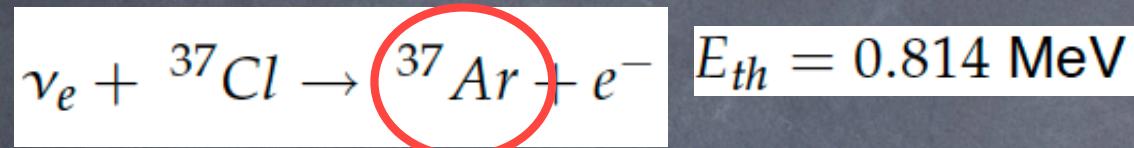
Reaction	source	Flux ($\text{cm}^{-2} \text{s}^{-1}$)
$p p \rightarrow d e^+ \nu$	pp	$5.97(1 \pm 0.006) \times 10^{10}$
$p e^- p \rightarrow d \nu$	pep	$1.41(1 \pm 0.011) \times 10^8$
${}^3\text{He} p \rightarrow {}^4\text{He} e^+ \nu$	hep	$7.90(1 \pm 0.15) \times 10^3$
${}^7\text{Be} e^- \rightarrow {}^7\text{Li} \nu \gamma$	${}^7\text{Be}$	$5.07(1 \pm 0.06) \times 10^9$
${}^8\text{B} \rightarrow {}^8\text{Be}^* e^+ \nu$	${}^8\text{B}$	$5.94(1 \pm 0.11) \times 10^6$
${}^{13}\text{N} \rightarrow {}^{13}\text{C} e^+ \nu$	${}^{13}\text{N}$	$2.88(1 \pm 0.15) \times 10^8$
${}^{15}\text{O} \rightarrow {}^{15}\text{N} e^+ \nu$	${}^{15}\text{O}$	$2.15(1 \pm 0.17) \times 10^8$
${}^{17}\text{F} \rightarrow {}^{17}\text{O} e^+ \nu$	${}^{17}\text{F}$	$5.82(1 \pm 0.19) \times 10^6$



Radiochemical solar experiments

Homestake (Cl) experiment: 1967-2002

- ▶ gold mine in Homestake (South Dakota)
- ▶ 615 tons of perchloro-ethylene (C_2Cl_4)
- ▶ detection process (radiochemical)



- ▶ only 1/3 of SSM prediction detected:

$$R_{Cl}^{\text{SSM}} = 8.12 \pm 1.25 \text{ SNU}$$

$$R_{Cl} = 2.56 \pm 0.16 \text{ (stat.)} \pm 0.16 \text{ (syst.) SNU}$$

Gallium radiochemical experiments:

$$R_{SAGE} = 66.9 \pm 3.9 \text{ (stat.)} \pm 3.6 \text{ (syst.) SNU}$$

$$R_{GALLEX/GNO} = 69.3 \pm 4.1 \text{ (stat.)} \pm 3.6 \text{ (syst.) SNU}$$

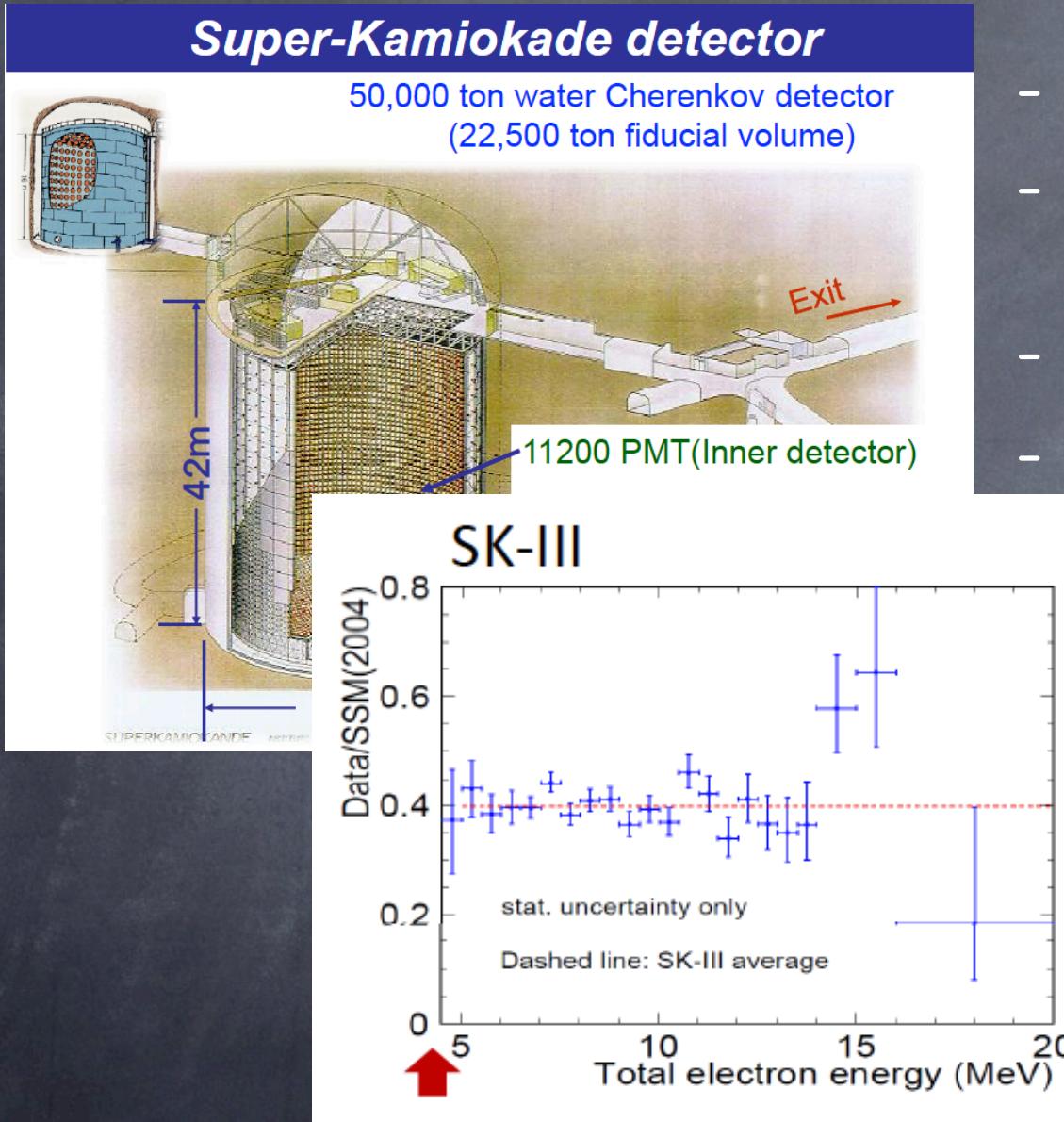
$$R_{Ga}^{\text{SSM}} = 126.2 \pm 8.5 \text{ SNU}$$

→ 50% deficit

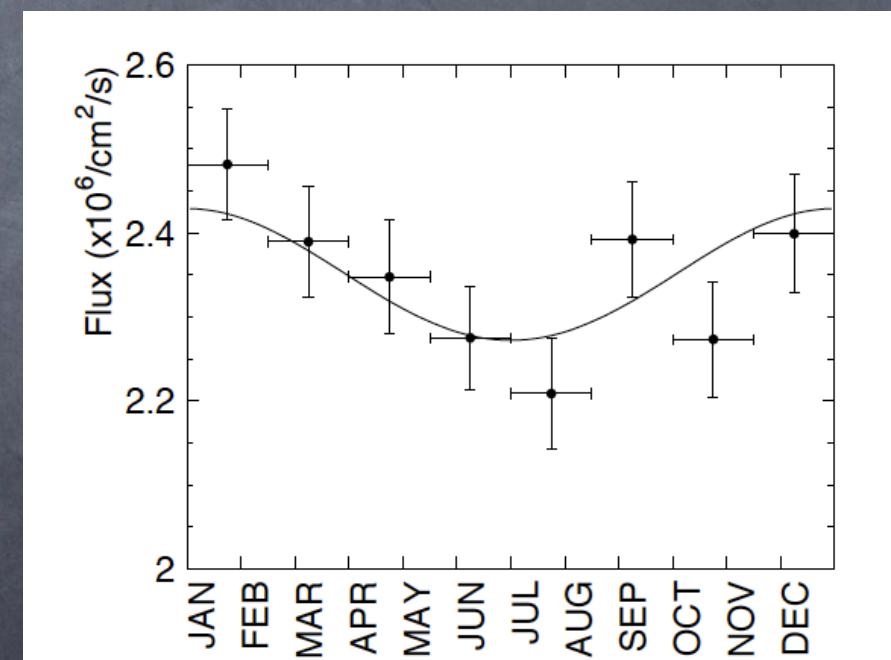


Solar neutrinos in Super-Kamiokande

Super-Kamiokande detector

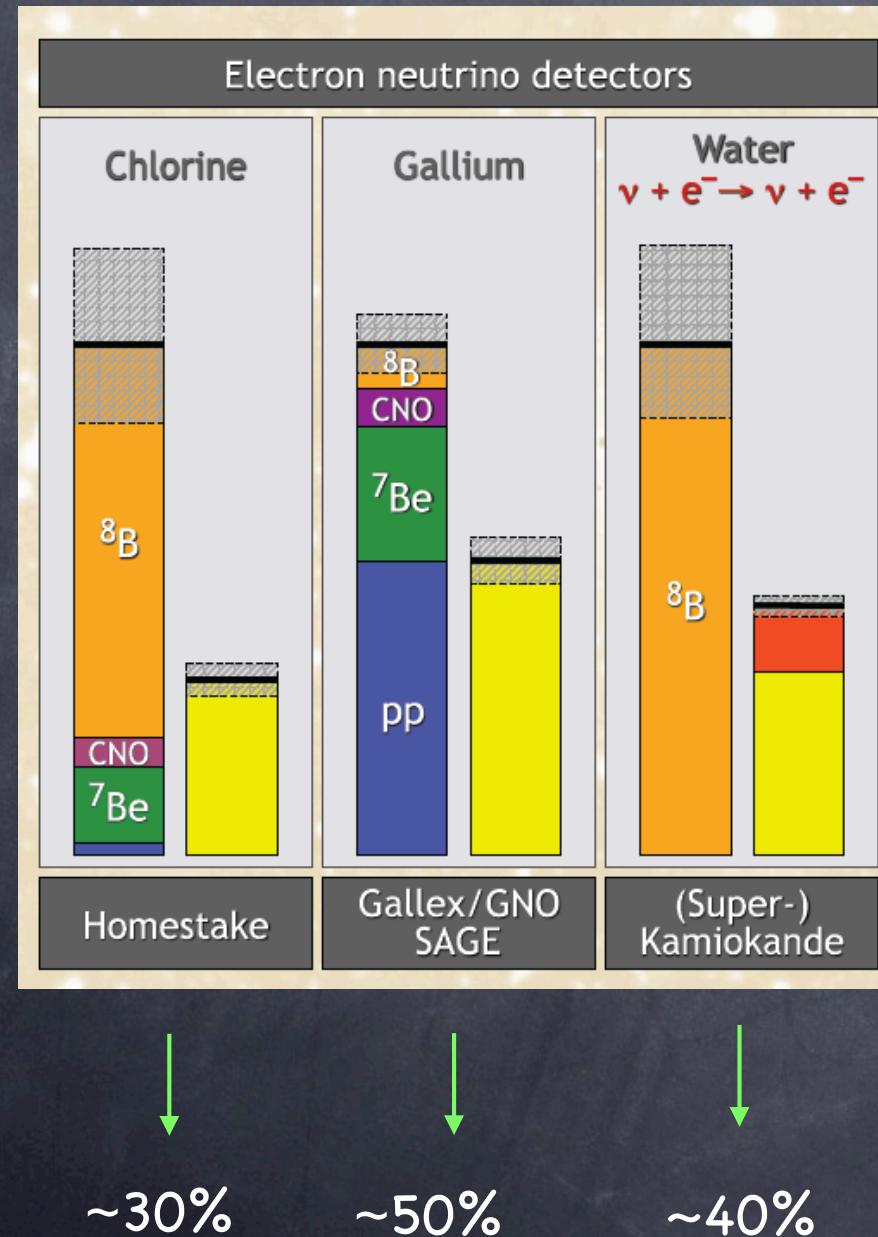


- water cherenkov detector
- sensitive to all neutrino flavors:
 $\nu_x e^- \rightarrow \nu_x e^-$
- threshold energy $\sim 4\text{-}5 \text{ MeV}$
- real-time detector: (E, t)



→ Super-Kamiokande detects less neutrinos than expected according to the SSM (40%)

The solar neutrino problem

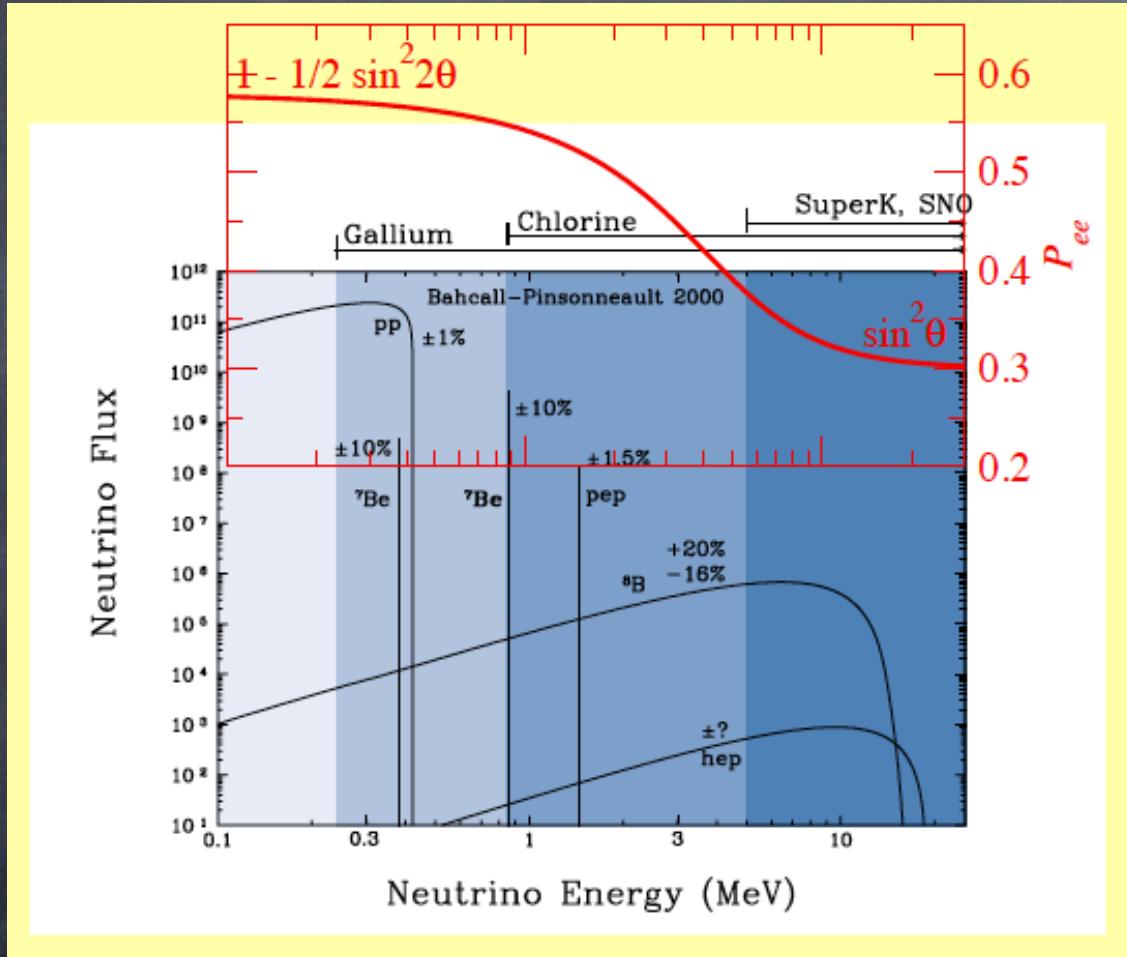


→ All the experiments detect less neutrinos than expected (30-50%)

Why the deficit observed is different?

- ▶ different type of neutrinos observed
 - radiochemical: ν_e while Super-K: ν_α
- ▶ different E-range sensitivity:
 - Cl: $E > 0.814$ MeV
 - Ga: $E > 0.233$ MeV
 - Super-K: $E > 5$ MeV

Different energy suppression of solar fluxes



- Ga experiments: pp neutrinos

$$P_{ee} = 1 - \frac{1}{2} \sin^2 2\theta$$

with $\sin^2 2\theta \simeq 0.84$ $P_{ee} > 0.5$

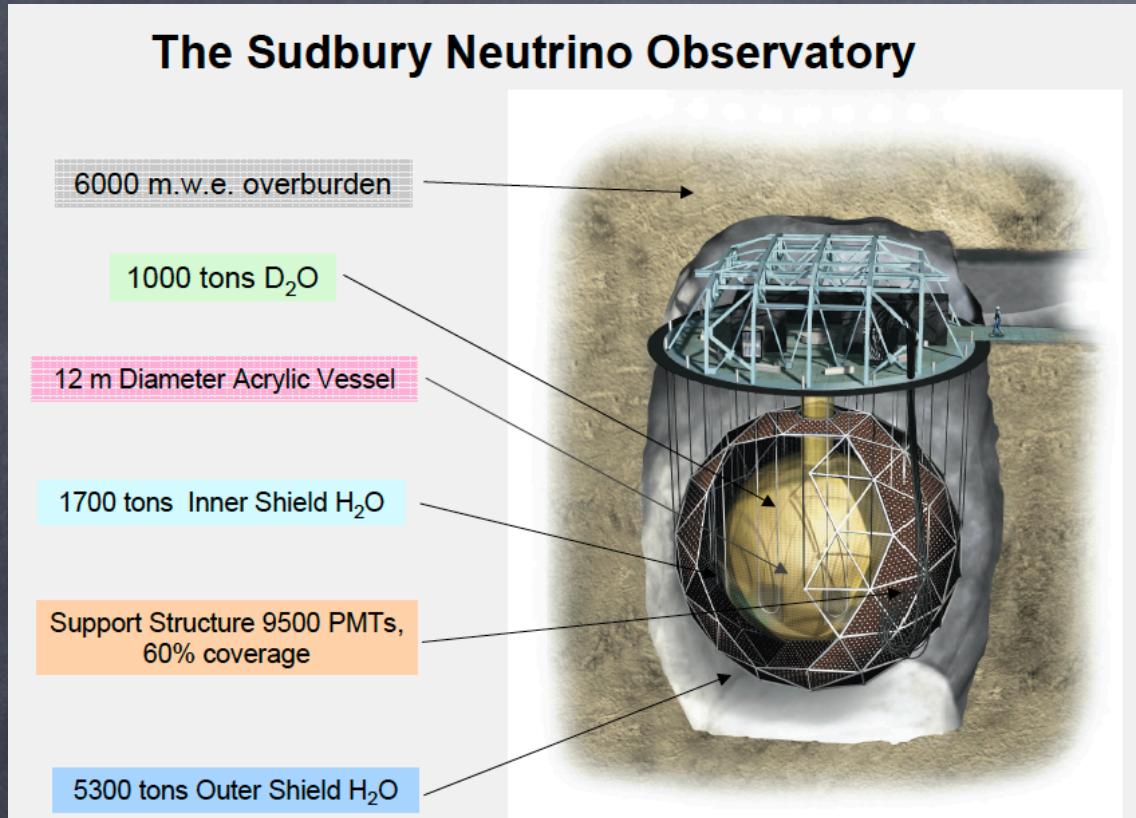
- Cl + Super-K: ^8B neutrinos

$$P_{ee} = \sin^2 \theta$$

→ $P_{ee} \sim 0.3$

→ stronger neutrino deficit is expected

The Sudbury Neutrino Observatory, SNO



SNO is sensitive to all ν flavors:

SNO interactions

Elastic-scattering (ES):

$$\nu_x + e^- \rightarrow \nu_x + e^-$$

ν_e mainly strong directional sensitivity

Charged-currents (CC):

$$\nu_e + d \rightarrow p + p + e^-$$

ν_e only E_e well correlated with E_ν

Neutral-currents (NC):

$$\nu_x + d \rightarrow p + n + \nu_x$$

All flavors equally Total neutrino flux

3 phases:

D₂O phase:

$$n + d \rightarrow t + \gamma + 6.25 \text{ MeV}$$

Salt phase (D₂O + 2 tons of NaCl):

$$n + ^{35}\text{Cl} \rightarrow ^{36}\text{Cl} + \gamma\text{'s} + 8.6 \text{ MeV}$$

NCD phase (³He proportional counters):

$$n + ^3\text{He} \rightarrow p + t + 0.76 \text{ MeV}$$

ν_e flux (CC): $\frac{\phi_{\text{CC}}^{\text{SNO}}}{\phi_{\text{NC}}^{\text{SNO}}} = 0.301 \pm 0.033 \quad 30\%$

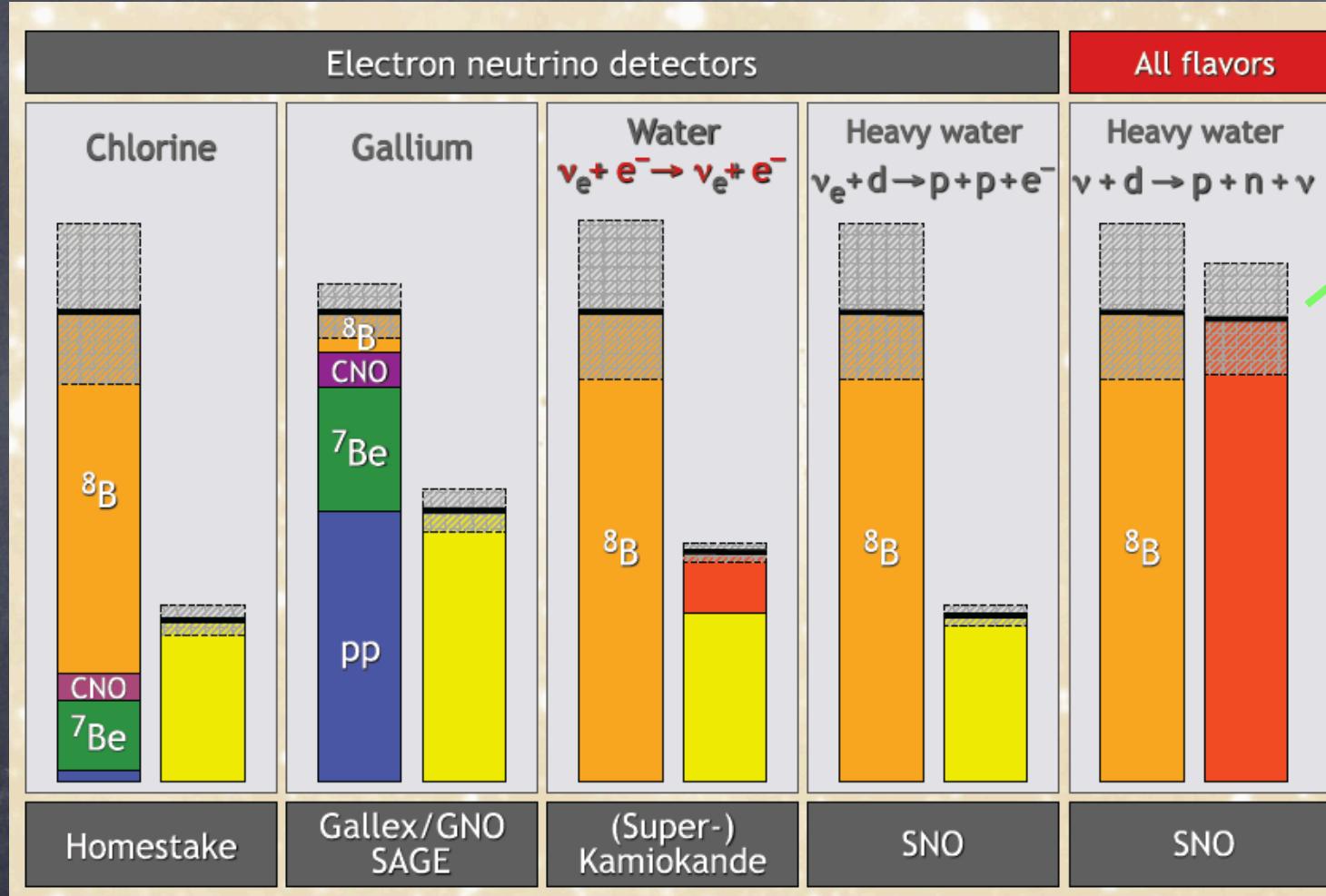
total ν flux (NC): $\phi_{\text{NC}}^{\text{SNO}} = 5.54^{+0.33}_{-0.31}(\text{stat})^{+0.36}_{-0.34}(\text{syst})$



100% !!

The solar neutrino problem

All neutrinos
are there!!



The Sun produces ν_e that arrive to the Earth as $1/3 \nu_e + 1/3 \nu_\mu + 1/3 \nu_\tau$

→ flavor conversion: $\nu_e \rightarrow \nu_x$

Conversion mechanism ?
Neutrino oscillations ??

Solar neutrino oscillations

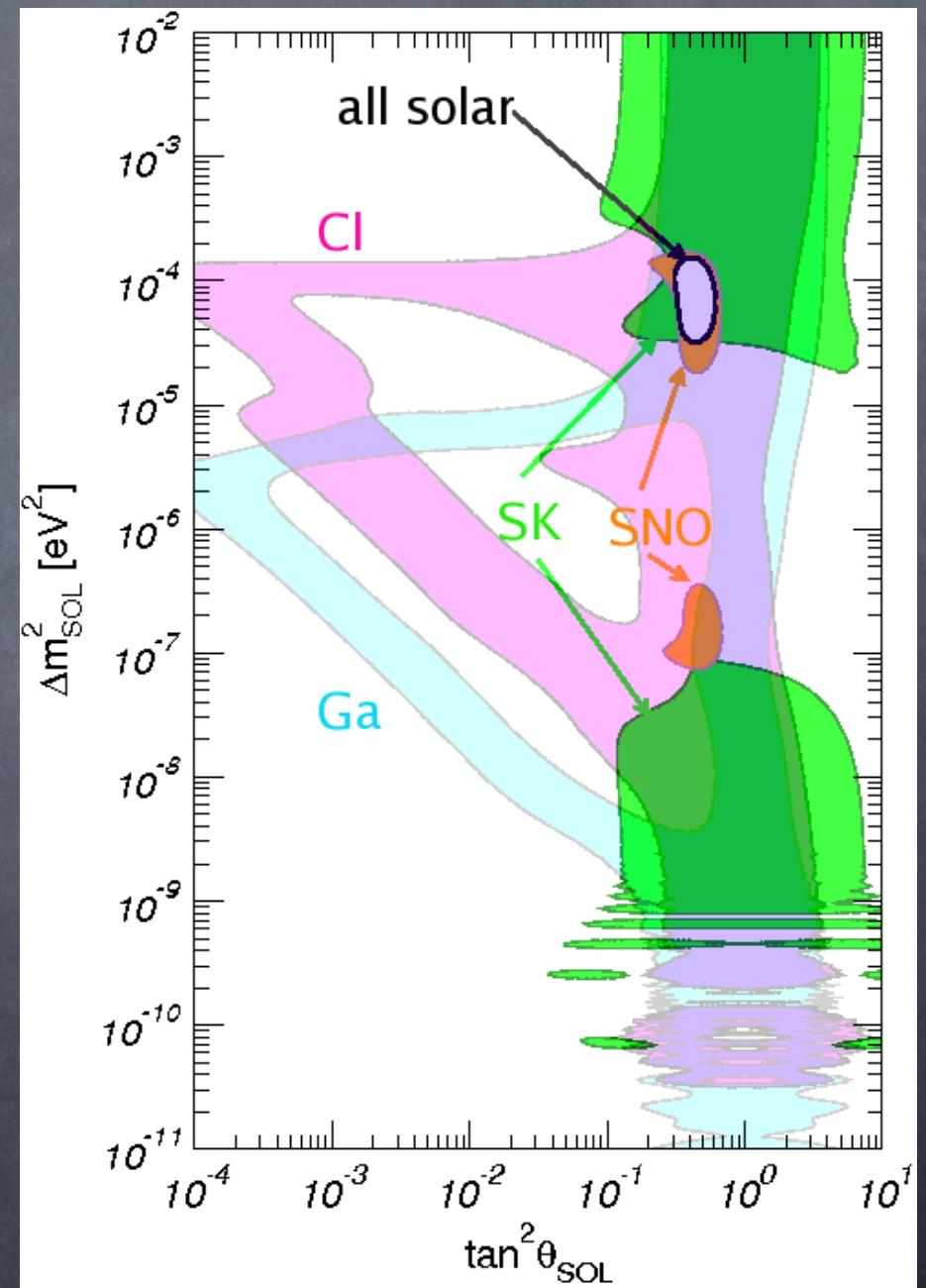
Homestake $(E_\nu > 0.814 \text{ MeV})$
 $\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$

SAGE/GALLEX-GNO $(E_\nu > 0.233 \text{ MeV})$
 $\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-$

Super-Kamiokande $(E_e \gtrsim 5 \text{ MeV})$
 $\nu_x + e^- \rightarrow \nu_x + e^-$

SNO $(E_e \gtrsim 5 \text{ MeV})$
[CC] $\nu_e + d \rightarrow p + p + e^-$
[NC] $\nu_x + d \rightarrow \nu_x + n + p$
[ES] $\nu_x + e^- \rightarrow \nu_x + e^-$

- only LMA allowed at 3σ
- max. mixing excluded at 5σ



The KamLAND reactor experiment

Kamioka Liquid scintillator Anti-Neutrino Detector

* reactor experiment:

$$\bar{\nu}_e + p \rightarrow n + e^+$$

* 55 commercial power reactors

* average distance ~ 180 km

$\rightarrow E_\nu/L$ sensitivity range: $\Delta m^2 \sim 10^{-5} \text{ eV}^2$

\rightarrow correct order of magnitude to test
solar neutrino oscillations in LMA region

* CPT invariance: same oscillation
channel as solar ν_e (Δm^2_{21} , θ_{12})



Results from KamLAND

2002: First evidence $\bar{\nu}_e$ disappearance
→ confirmation of solar LMA ν oscillations

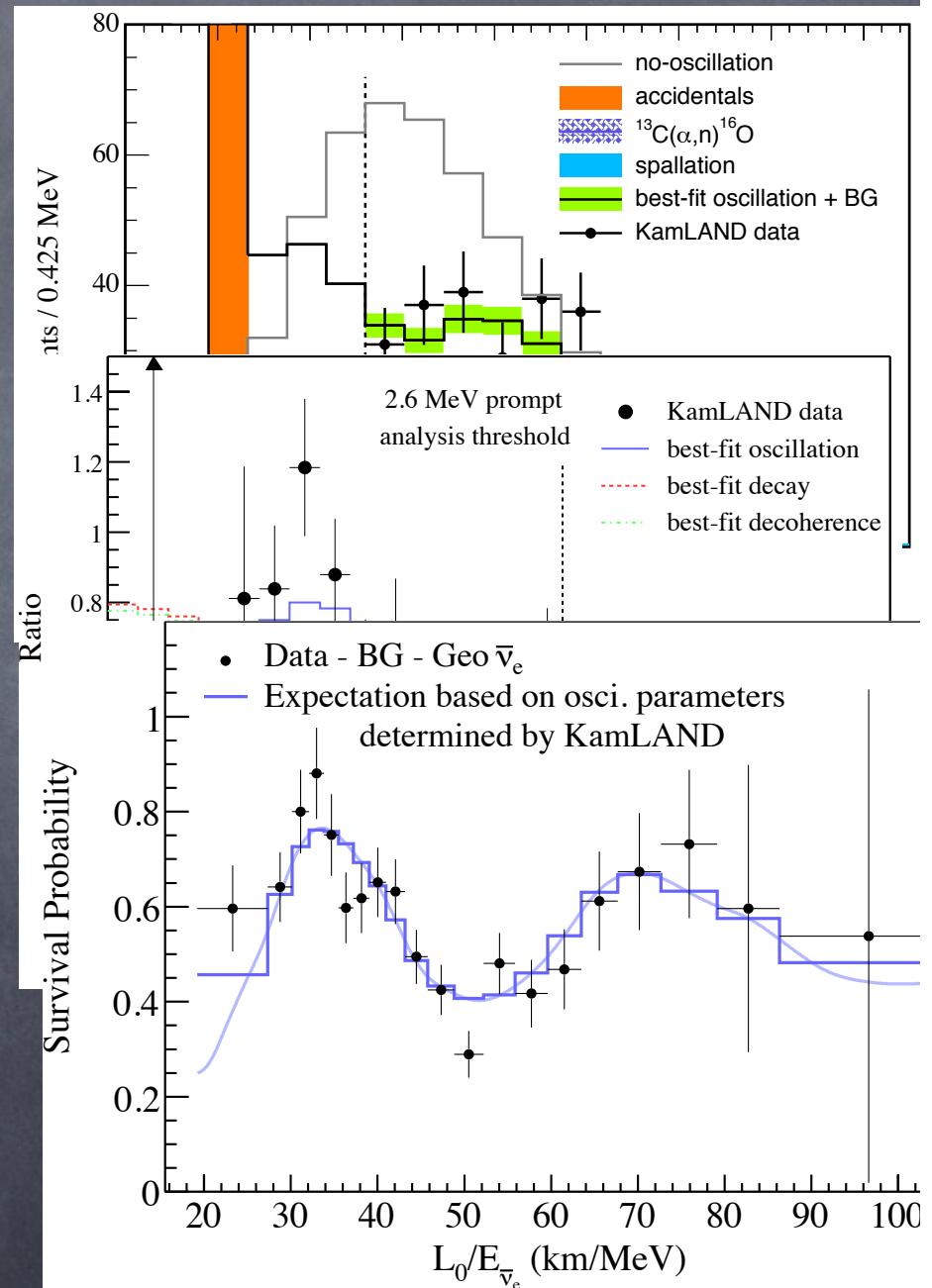
KamLAND Coll, PRL 90 (2003) 021802

2004: spectral distortions (L/E)

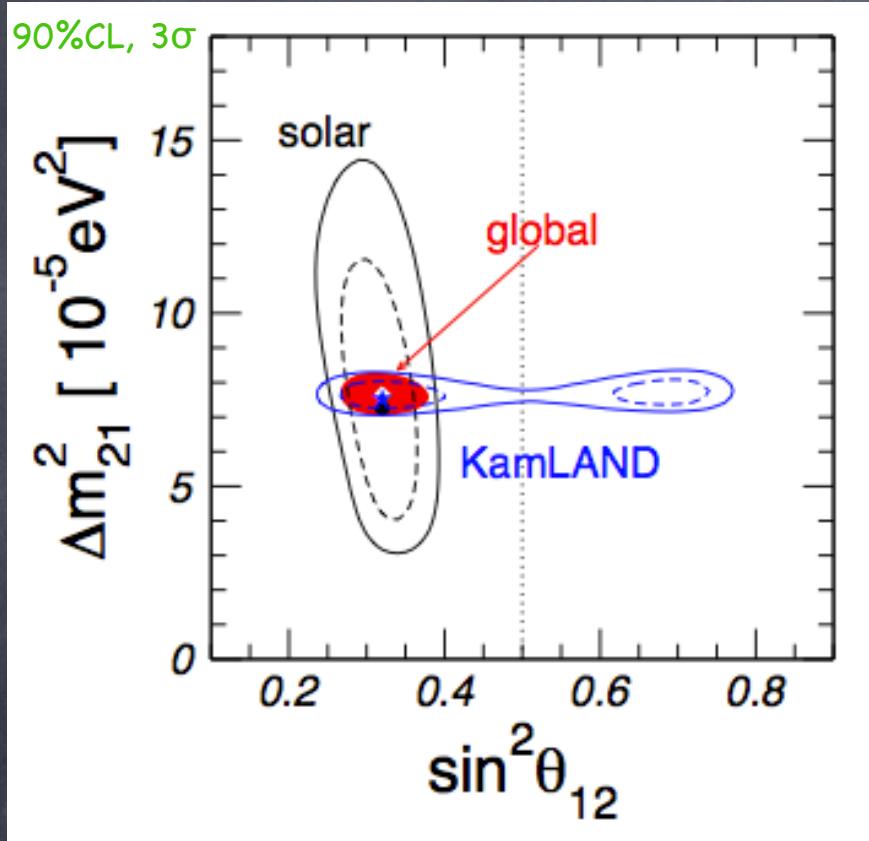
KamLAND Coll, PRL 94 (2005) 081801

2008: 1-period oscillations observed
→ high precision determination Δm^2_{21}

KamLAND Coll, PRL 100 (2008) 221803



Combined analysis solar + KamLAND



- * KamLAND confirms solar neutrino oscillations.
- * Best fit point:
 $\sin^2\theta_{12} = 0.320^{+0.016}_{-0.017}$
 $\Delta m^2_{21} = 7.62 \pm 0.19 \times 10^{-5} \text{ eV}^2$
- * max. mixing excluded at more than 7σ

Forero, M.T., Valle, PRD86, 073012 (2012)
arXiv:1205.4018 [hep-ph]

- Bound on θ_{12} dominated by solar data.
- Bound on Δm^2_{21} dominated by KamLAND.

The atmospheric neutrino sector

The atmospheric neutrino anomaly

Cosmic rays interacting with the Earth atmosphere producing pions and kaons, that decay generating neutrinos:

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

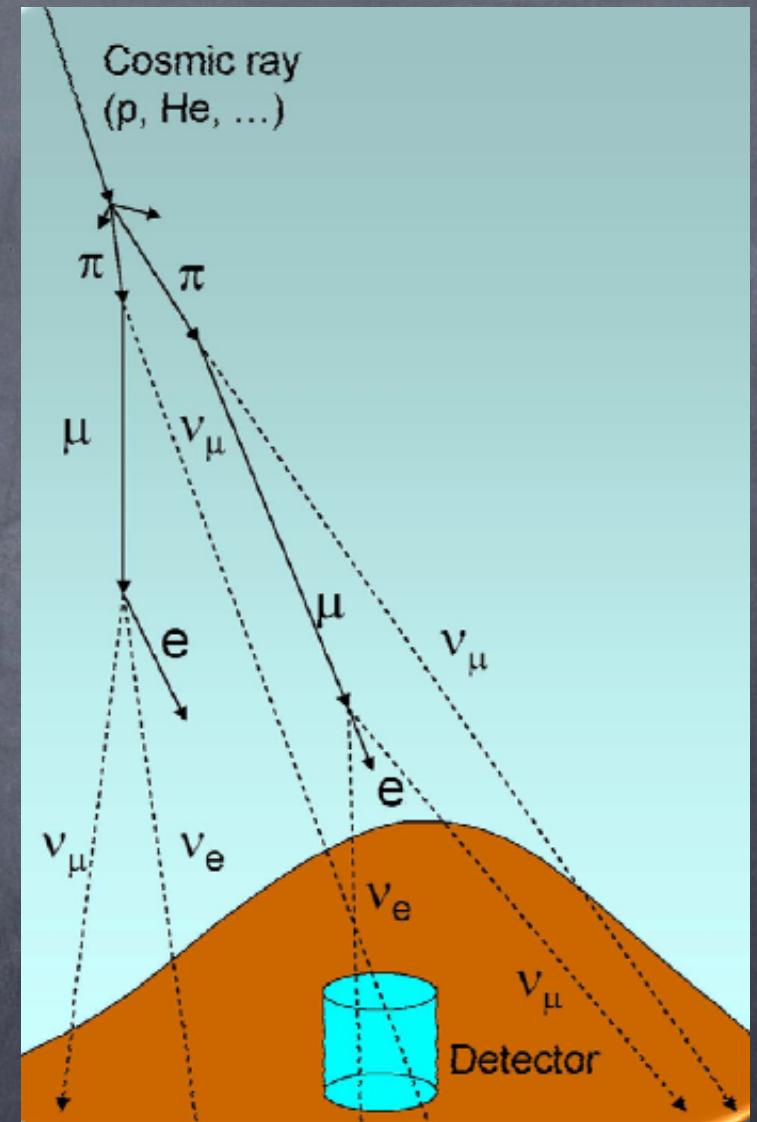
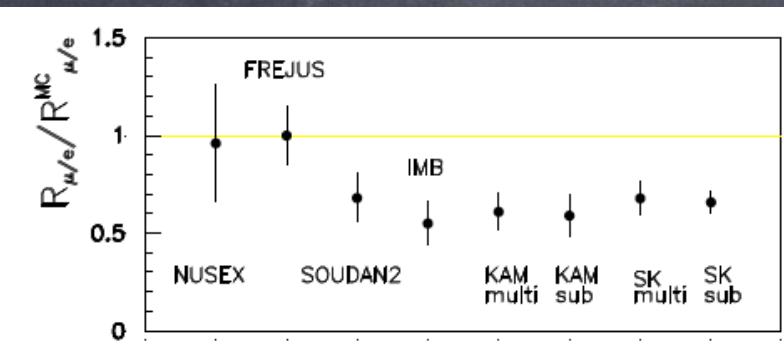
$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

then, one expects:

$$R_{\mu/e} = \frac{N_{\nu_\mu} + N_{\bar{\nu}_\mu}}{N_{\nu_e} + N_{\bar{\nu}_e}} \simeq 2$$

However, this prediction is in disagreement with the experimental results:

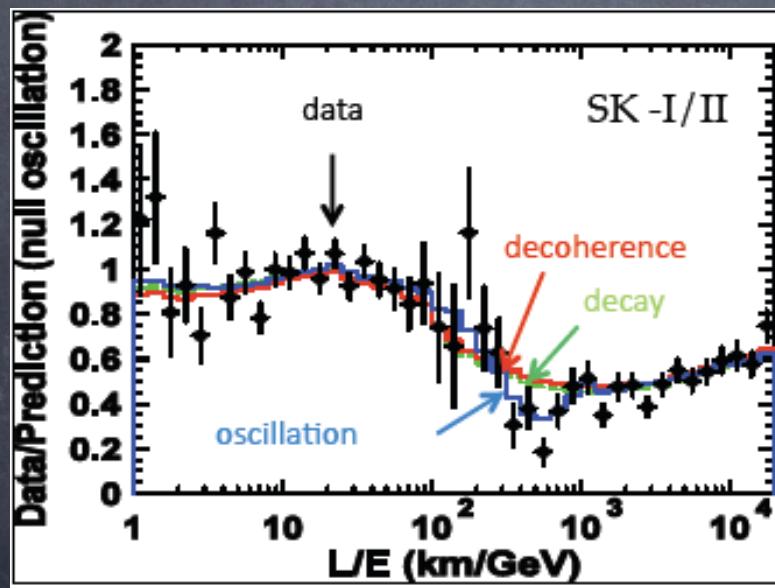


Atmospheric neutrinos

1998: Evidence ν_μ oscillations at Super-K

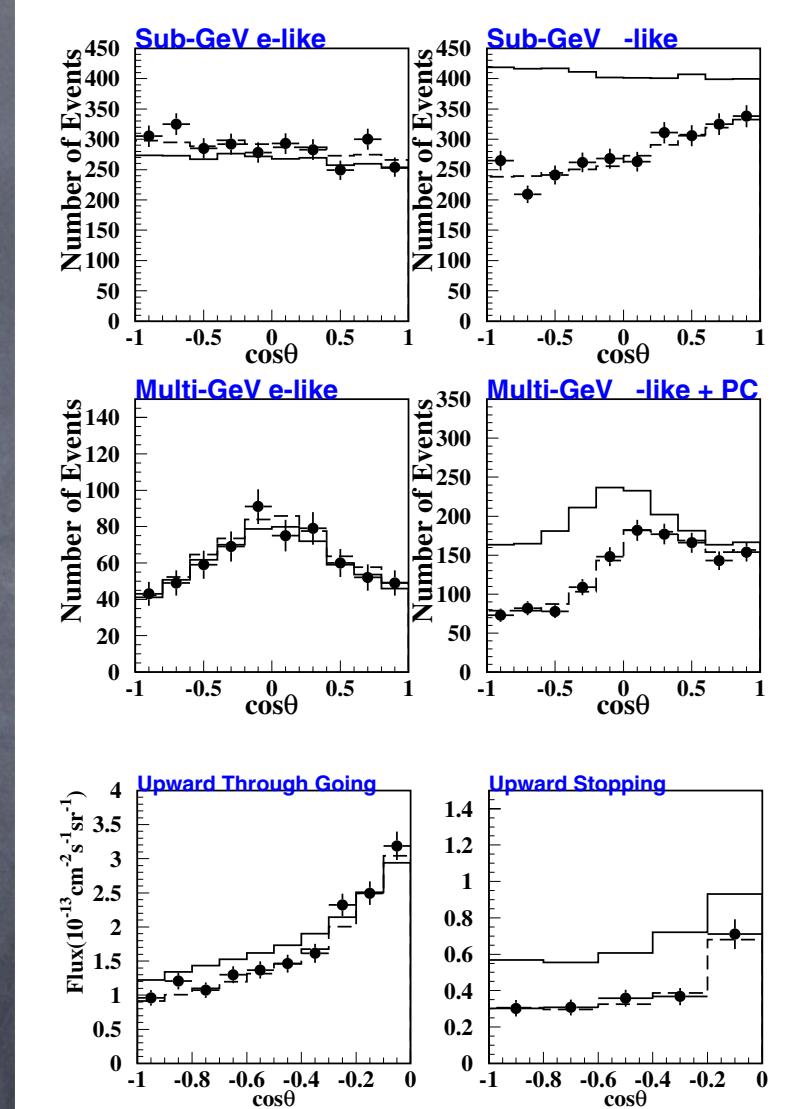
oscillation channel $\nu_\mu \rightarrow \nu_\tau$

2004: oscillatory L/E pattern



Super-K Coll, PRL93, 101801 (2004)

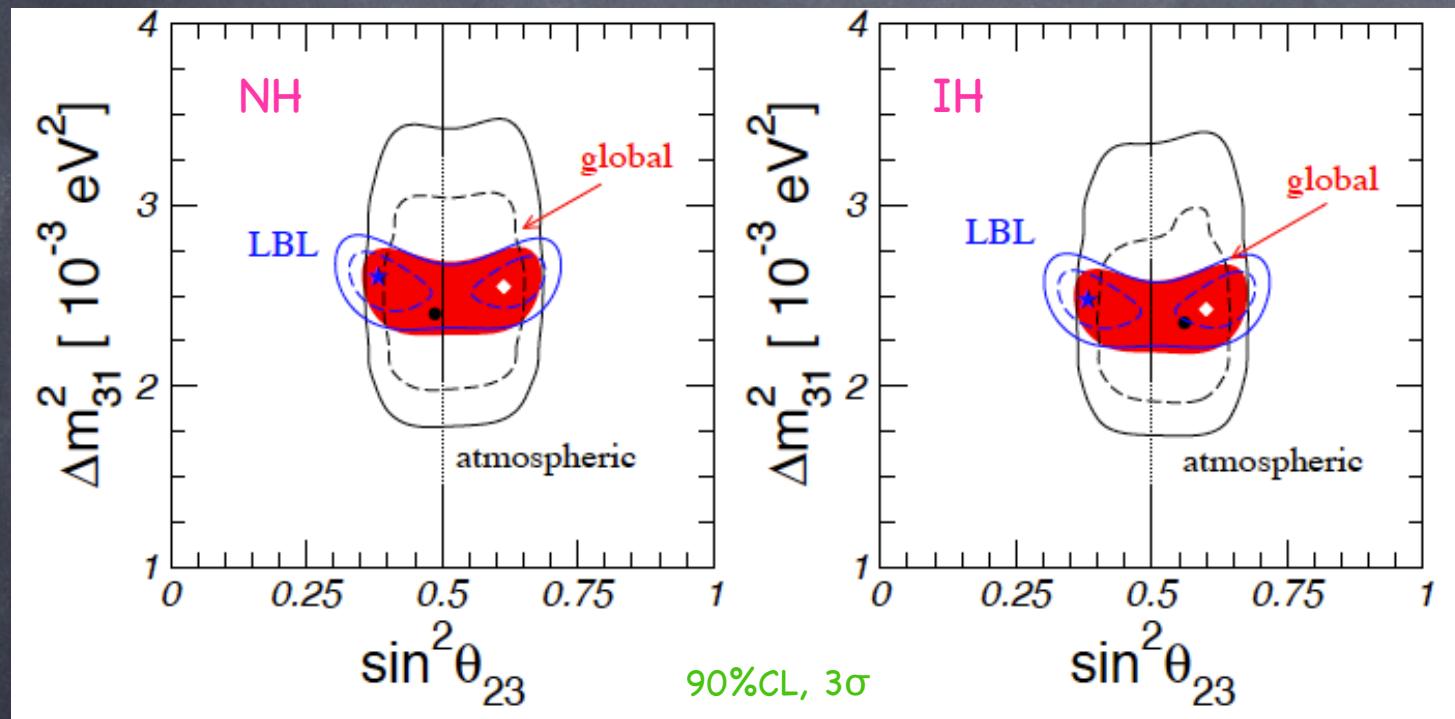
$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2 \left(\frac{\Delta m_{32}^2}{4} \frac{L}{E_\nu} \right)$$



Super-K Coll., PRL 8 (1998) 1562.

Combined analysis atmospheric + LBL data

→ Super-Kamiokande (I + II + III) + K2K + MINOS + T2K LBL data



→ Determination of θ_{23} and Δm^2_{31} is now dominated by LBL data

Forero, M.T., Valle, 2012

* Best fit point:

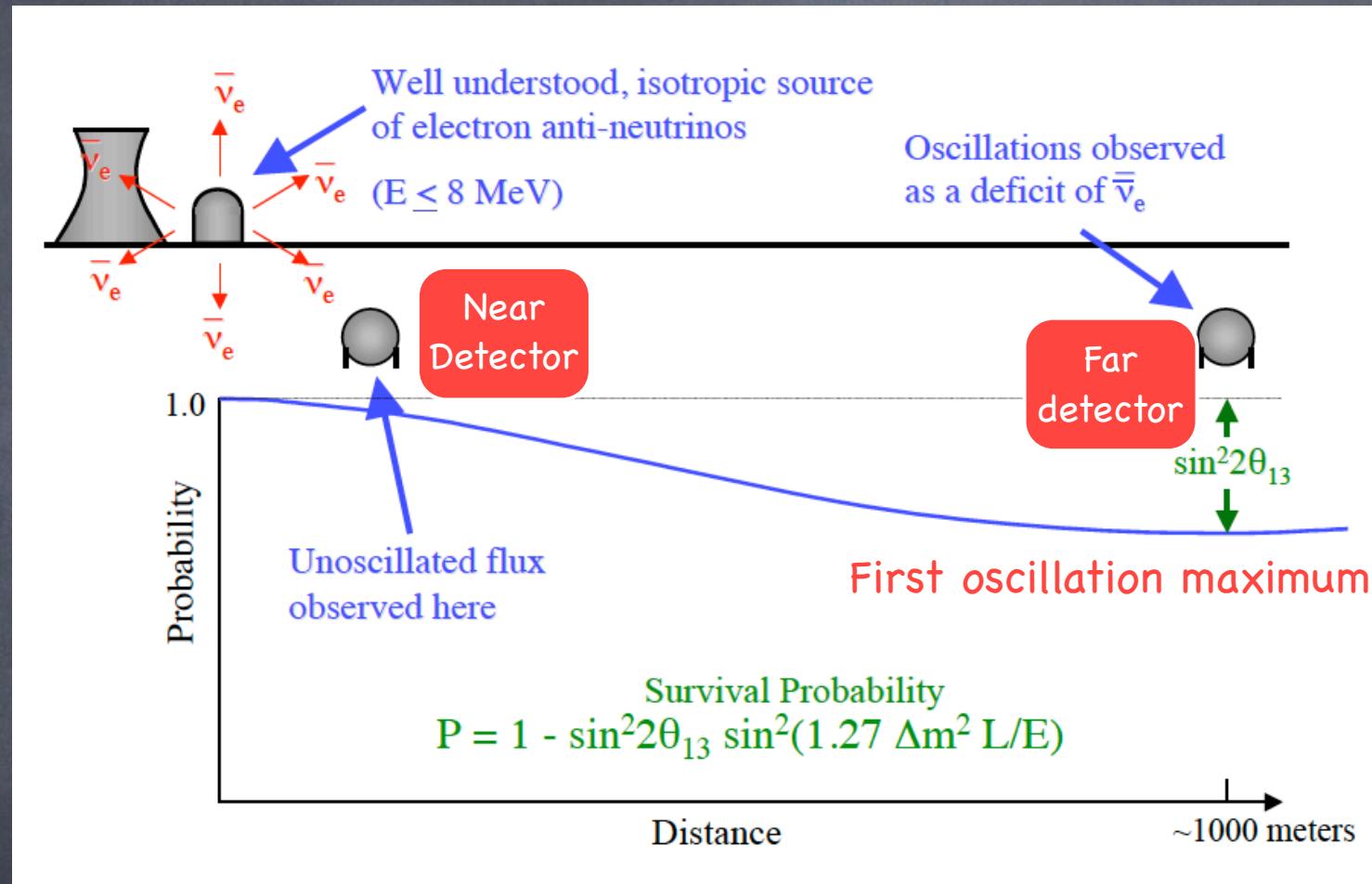
$$\sin^2 \theta_{23} = 0.613^{+0.022}_{-0.040} * \text{local bf in } 0.427 \quad \sin^2 \theta_{23} = 0.600^{+0.026}_{-0.031}$$

$$\Delta m^2_{31} = 2.55^{+0.06}_{-0.08} \times 10^{-3} \text{ eV}^2$$

$$\Delta m^2_{31} = -(2.43^{+0.07}_{-0.06} \times 10^{-3}) \text{ eV}^2$$

Short-baseline reactor experiments

New generation of reactor experiments



- * more powerful reactors (multi-core)
- * larger detector volume
- * 2-6 detectors at 100 m - 1 km.

New generation reactor experiments



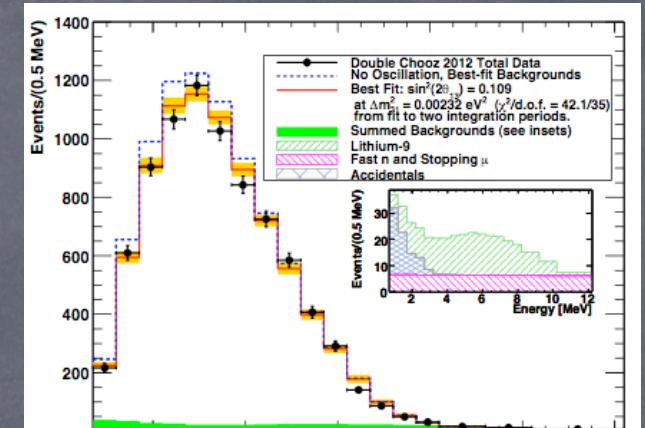
2 reactor cores + 1 FD (ND 2013)

livetime: 227.9 days

$$\sin^2 2\theta_{13} = 0.109 \pm 0.030 \text{ (stat)} \pm 0.025 \text{ (syst)}$$

→ $\sin^2 2\theta_{13} = 0$ excluded at 2.9σ

Double Chooz Coll, PRD 86 (2012) 052008



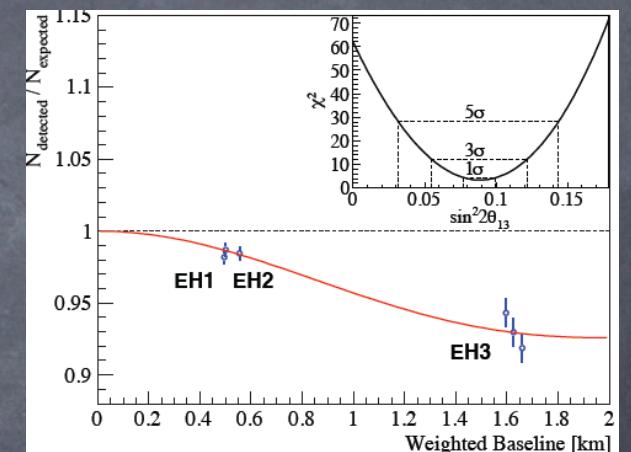
6 reactor cores + 6 detectors (3ND,3FD)

livetime: 139 days

$$\sin^2 2\theta_{13} = 0.089 \pm 0.010 \text{ (stat)} \pm 0.005 \text{ (syst)}$$

→ $\sin^2 2\theta_{13} = 0$ excluded at 7.7σ

Daya Bay Coll., Chin. Phys. C 37 (2013) 011001



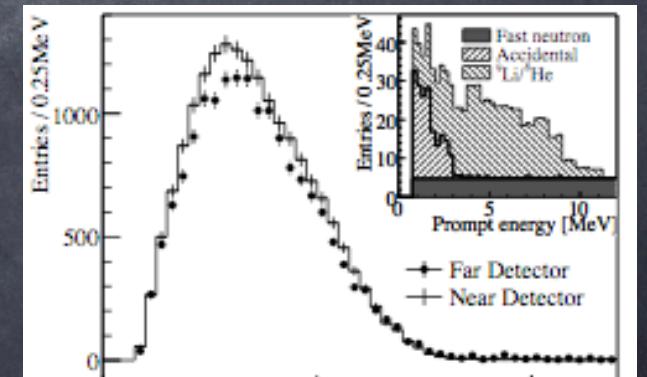
6 reactor cores + 2 detectors (ND,FD)

livetime: 229 days

$$\sin^2 2\theta_{13} = 0.113 \pm 0.013 \text{ (stat)} \pm 0.019 \text{ (syst)}$$

→ $\theta_{13}=0$ excluded at 4.9σ

RENO Coll., PRL 108 (2012) 191802



3-neutrino probabilities in vacuum

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^*) \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) + 2 \sum_{i>j} \text{Im}(U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^*) \sin \left(\frac{\Delta m_{ij}^2 L}{2E} \right)$$

ν_e disappearance channel

$$\begin{aligned} P(\nu_e \rightarrow \nu_e) &= 1 - 4|U_{e3}|^2|U_{e1}|^2 \sin^2 \Delta_{31} \\ &\quad - 4|U_{e3}|^2|U_{e2}|^2 \sin^2 \Delta_{32} \\ &\quad - 4|U_{e2}|^2|U_{e1}|^2 \sin^2 \Delta_{21} \end{aligned}$$

$$\Delta_{ij} = \frac{\Delta m_{ij}^2 L}{4E}$$

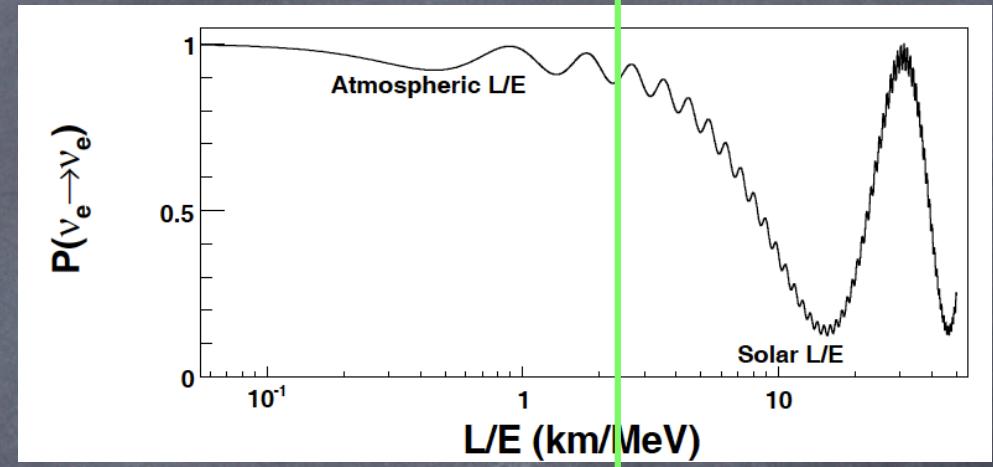
$$P_{ee}^{\text{reactor}} \approx 1 - \sin^2 2\theta_{13} \sin^2 \Delta_{31} - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21}$$

↓

reactor exp with atm L/E
(CHOOZ)

↓

reactor exp with solar L/E
(KamLAND)



$\nu_\mu \rightarrow \nu_e$ appearance channel

$$\begin{aligned} P_{\mu e} &= |2U_{\mu 3}^* U_{e3} \sin \Delta_{31} e^{-i\Delta_{32}} + 2U_{\mu 2}^* U_{e2} \sin \Delta_{21}|^2 = \\ &\approx \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta_{31} + \cos^2 \theta_{23} \cos^2 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21} + \\ &+ \sin 2\theta_{13} \sin 2\theta_{23} \sin 2\theta_{12} \cos \theta_{13} \sin \Delta_{31} \sin \Delta_{21} \cos(\Delta_{32} + \delta) \end{aligned}$$

for antineutrinos: $\delta \rightarrow -\delta$

$$\Delta P = P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = -2 \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \cos \theta_{13} \sin \Delta_{21} \sin \Delta_{31} \sin \Delta_{32} \sin \delta$$

CP violating term: only present for appearance probabilities $\alpha \neq \beta$.
Genuine 3-flavor effect

3-neutrino masses and mixings

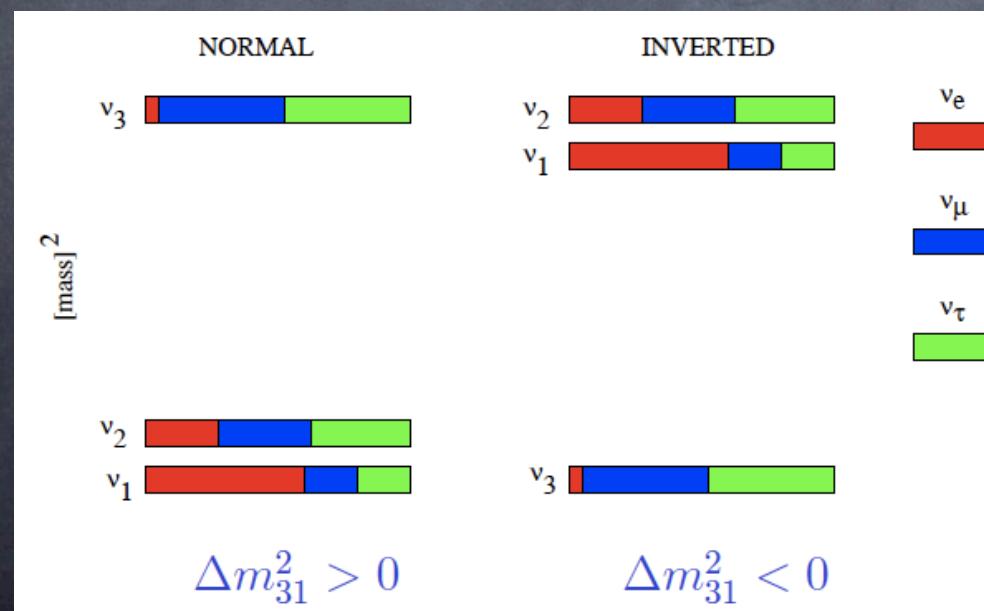
- 3-neutrino mixing is described by 3 angles and 1 Dirac (+2 Majorana) CP violating phases.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric + LBL
measurements

reactor disapp + LBL
appearance searches

solar + KamLAND
measurements



Δm^2_{31} : atmospheric +
long-baseline

Δm^2_{21} : solar + KamLAND

3-neutrino masses and mixings

- 3-neutrino mixing is described by 3 angles and 1 Dirac (+2 Majorana) CP violating phases.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric + LBL
measurements

reactor disapp + LBL
appearance searches

solar + KamLAND
measurements

- From the experimental data we know:

$$\Delta m^2_{31} \gg \Delta m^2_{21} \quad \text{and} \quad \theta_{13} \ll 1$$

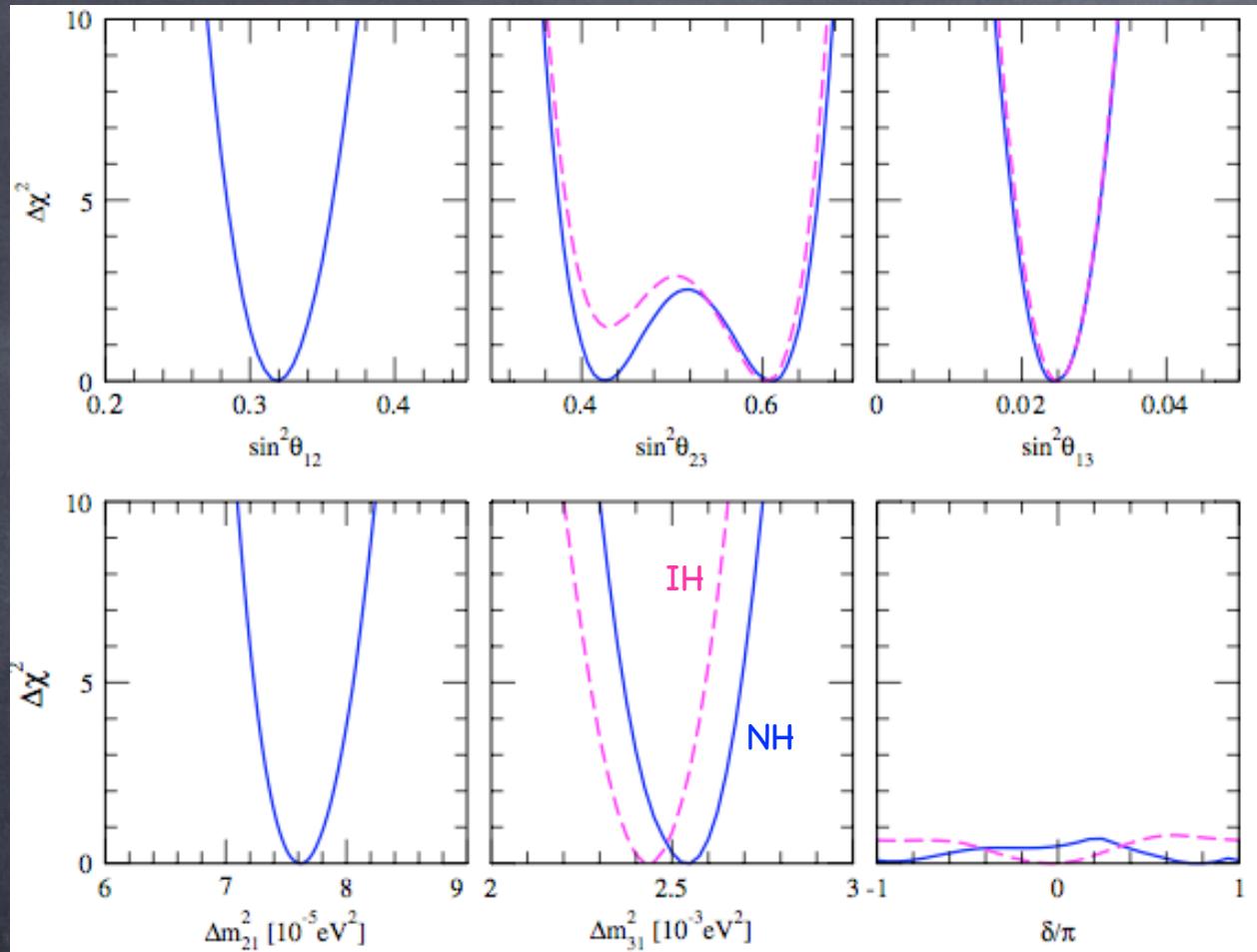
→ 3-flavour effects are suppressed: dominant oscillations are well described by effective 2-flavour oscillations

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$

Three-neutrino effects

- ▶ θ_{13} effects in oscillations with Δm^2_{21} : KamLAND + solar neutrinos
- ▶ θ_{13} effects in oscillations with Δm^2_{31} : atmospheric and accelerator experiments
 - ν_e appearance in ν_μ beam
- ▶ Δm^2_{21} effects in oscillations with Δm^2_{31} : reactor and atmospheric experiments
- ▶ effects of CP violation: atmospheric and accelerator experiments

3-flavour oscillation parameters



parameter	best fit $\pm 1\sigma$	
$\Delta m_{21}^2 [10^{-5} \text{eV}^2]$	7.62 ± 0.19	3%
$\Delta m_{31}^2 [10^{-3} \text{eV}^2]$	$2.55^{+0.06}_{-0.09}$ $-(2.43^{+0.07}_{-0.06})$	3%
$\sin^2 \theta_{12}$	$0.320^{+0.016}_{-0.017}$	5%
$\sin^2 \theta_{23}$	$0.613^{+0.022}_{-0.040}$ $(0.427^{+0.034}_{-0.027})$ $0.600^{+0.026}_{-0.031}$	10%
$\sin^2 \theta_{13}$	$0.0246^{+0.0029}_{-0.0028}$ $0.0250^{+0.0026}_{-0.0027}$	15%
δ	0.80π -0.03π	

- Deviations from 2-3 maximal mixing, $\theta_{23} > 45^\circ$ preferred for IH.
- Poor sensitivity to δ_{CP}
- No indication for correct mass ordering