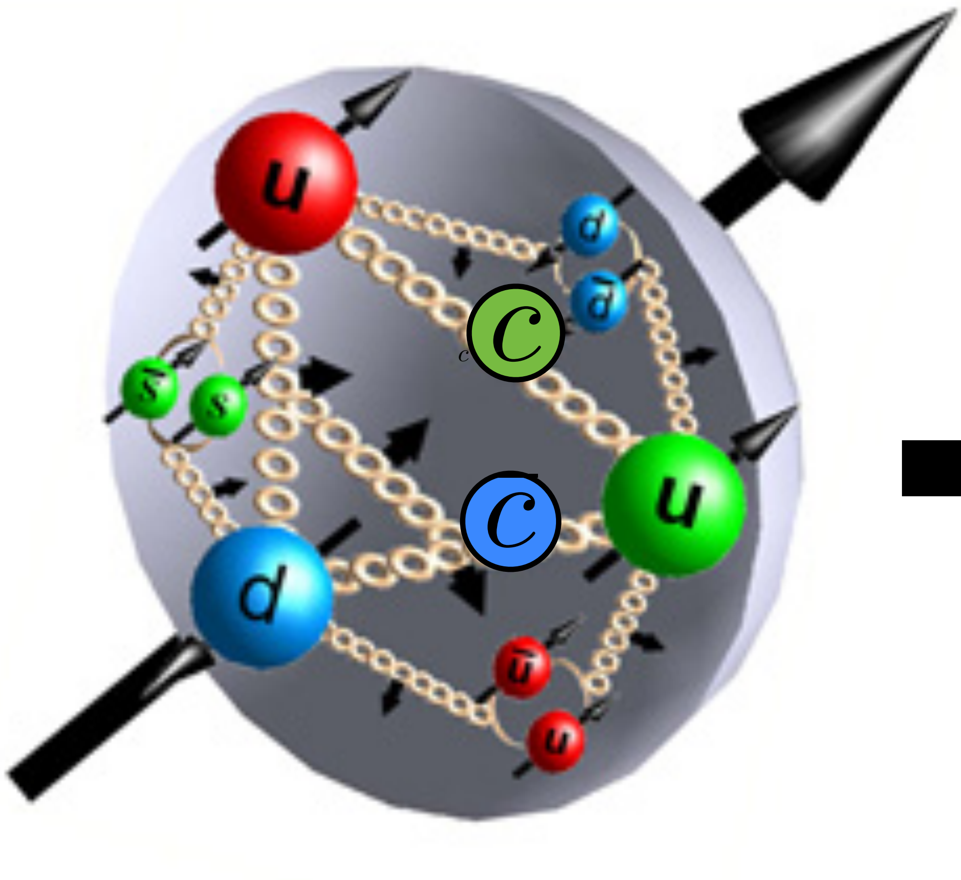
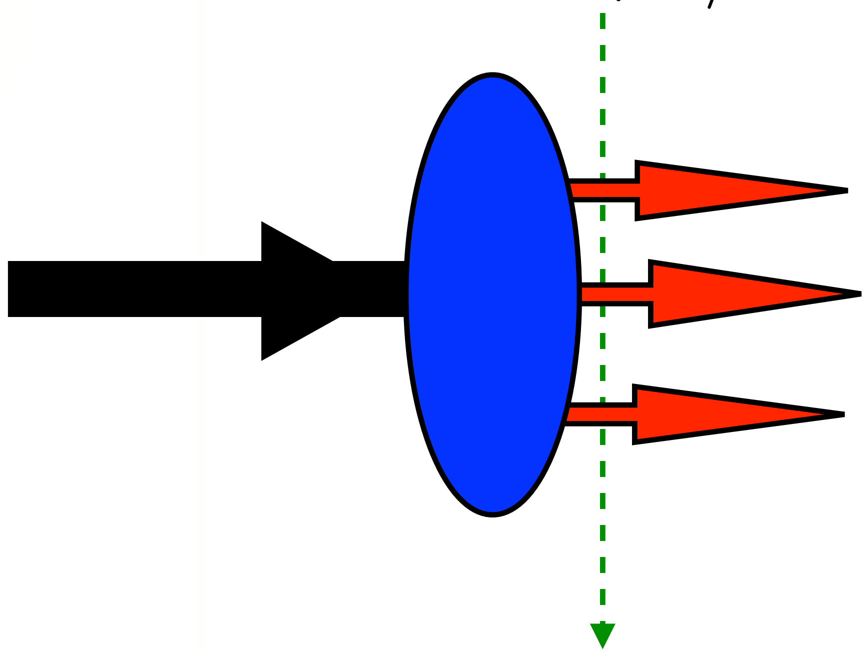
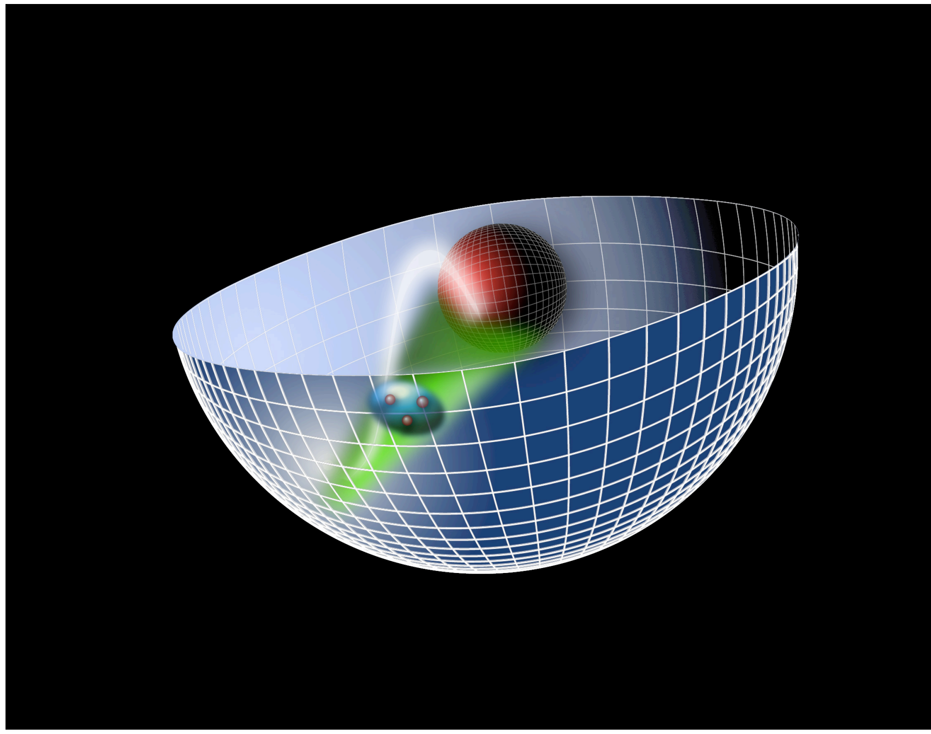
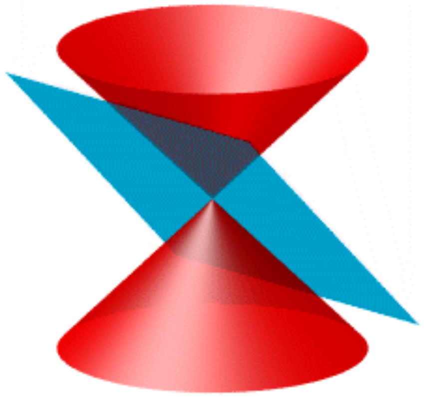
Light-Front QCD









4th Chilean School of High Energy Physics

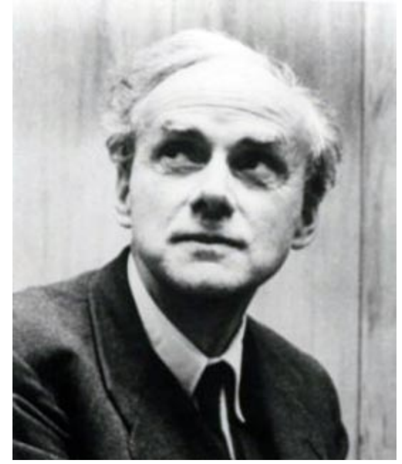
Universidad Técnica Federico Santa María

Student Lecture
January 13,2016

Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY

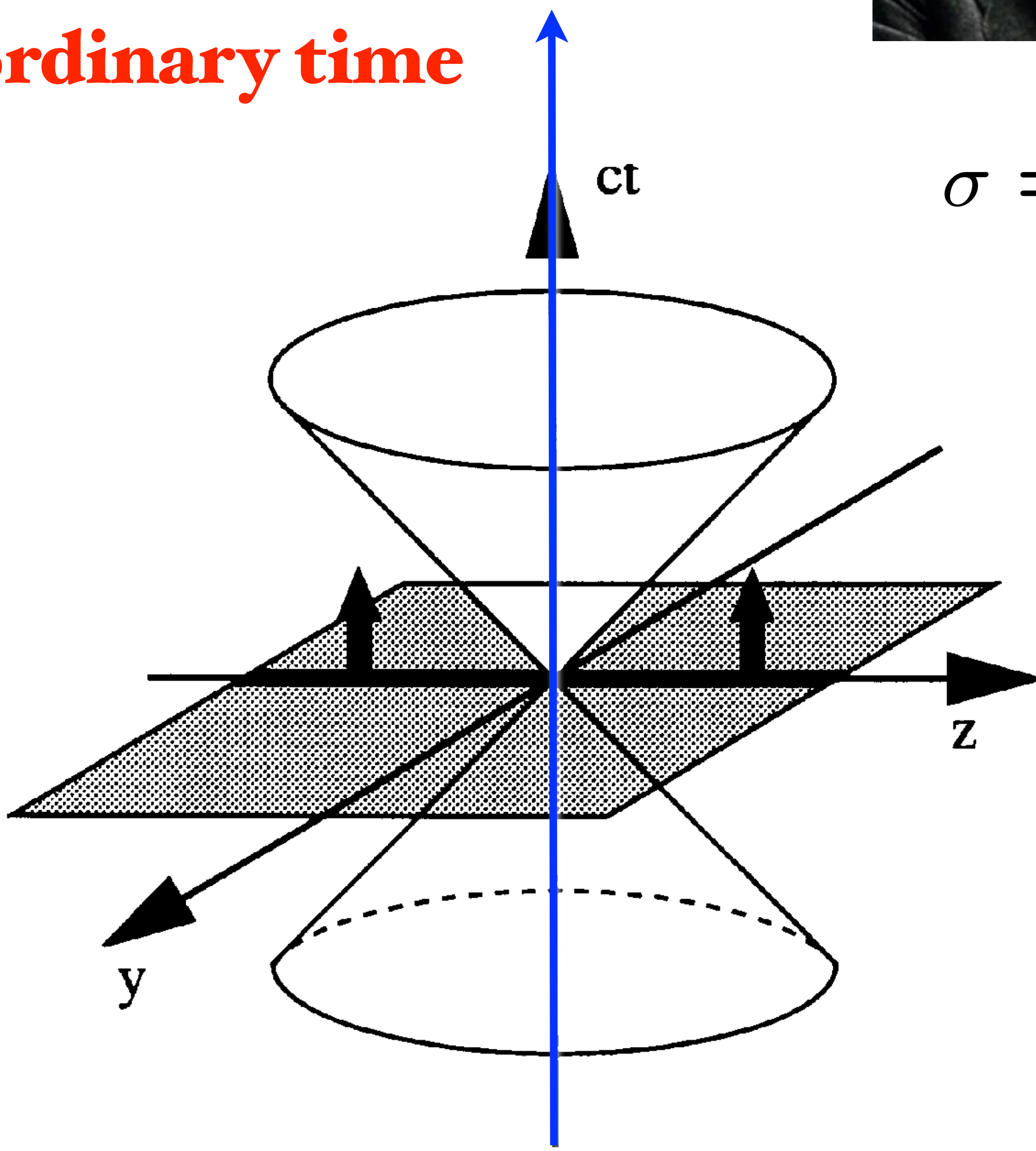
Dirac's Amazing Idea: The "Front Form"



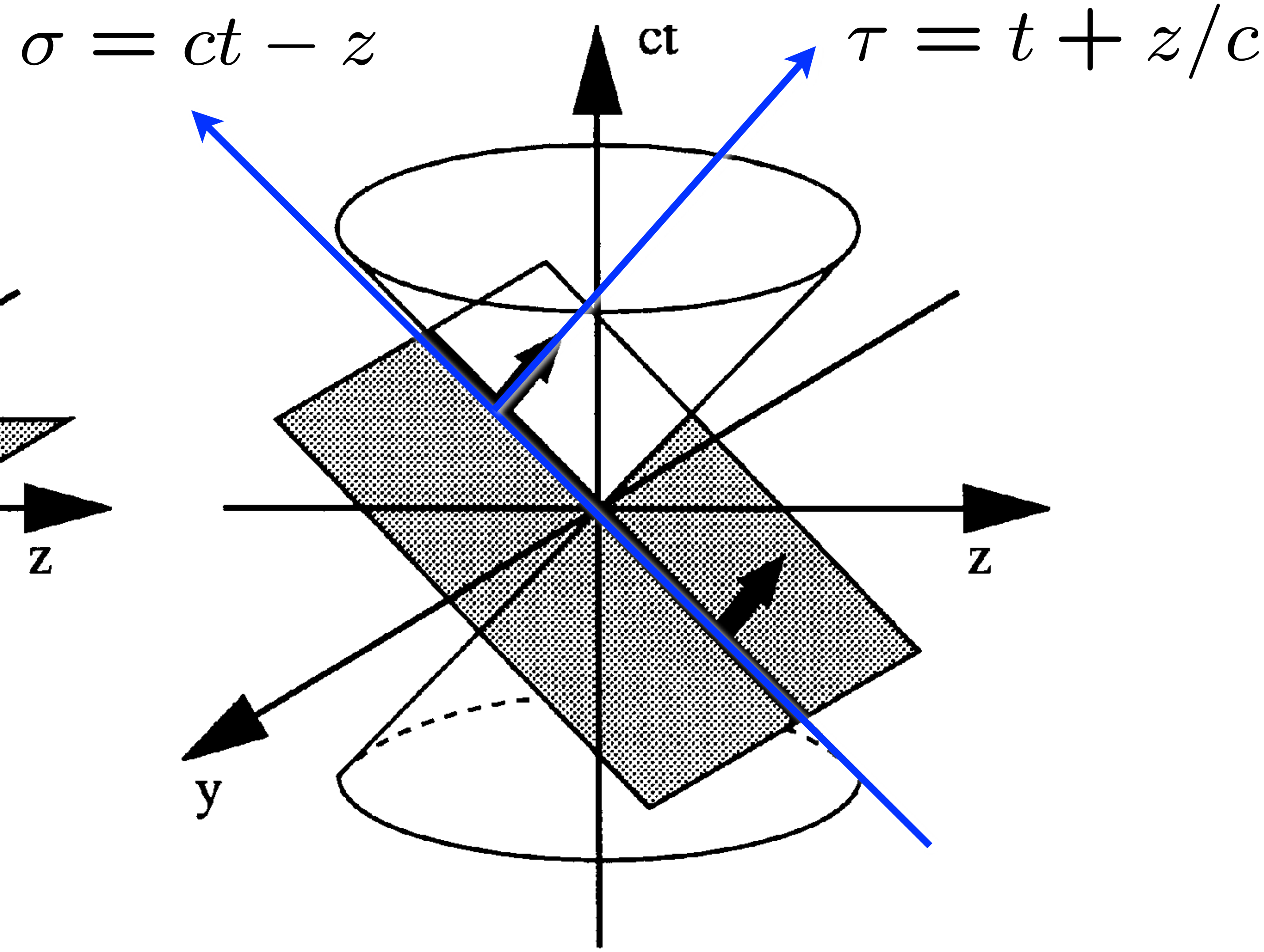
P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Evolve in ordinary time

Evolve in light-front time!

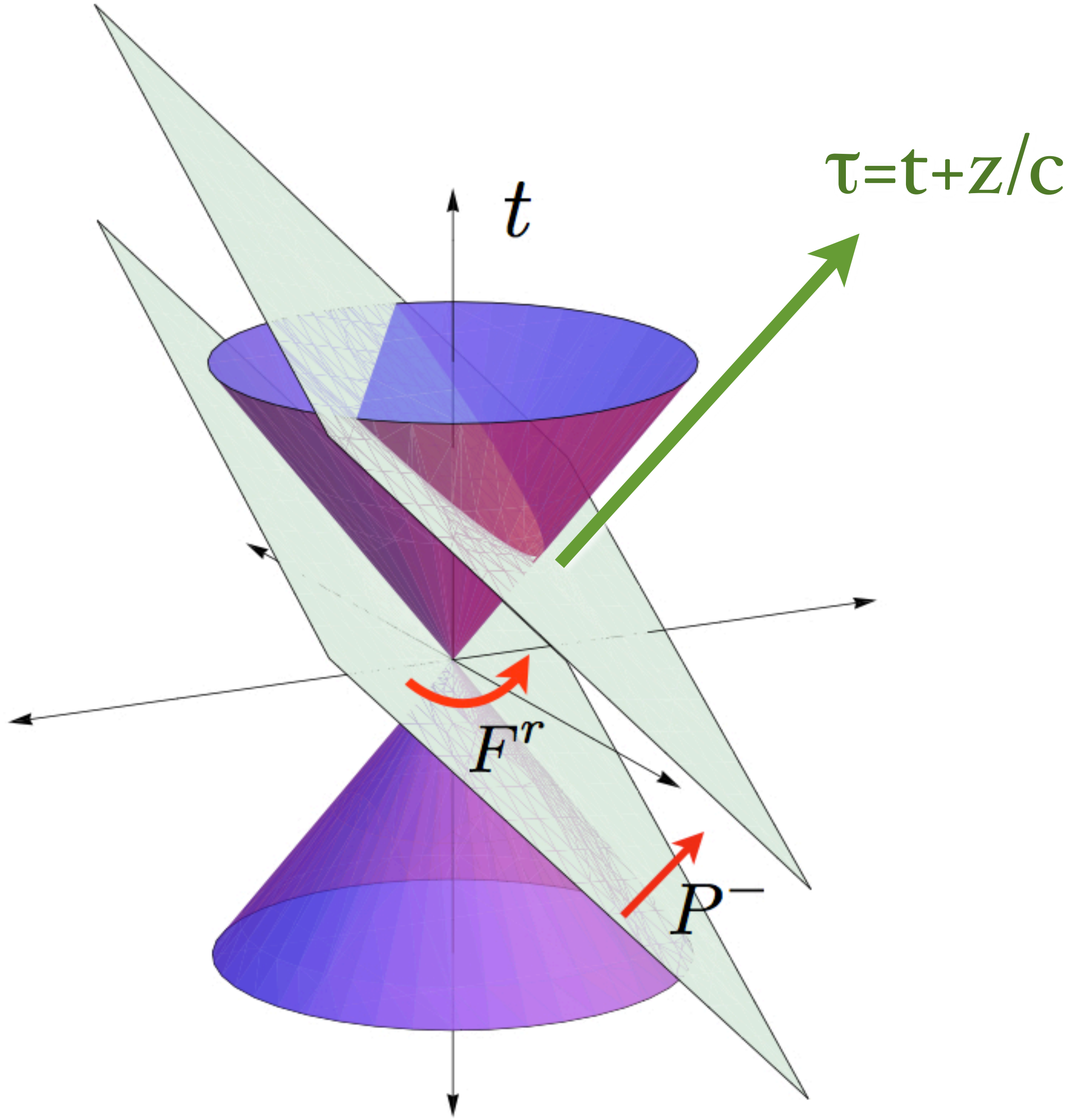


Instant Form



Front Form

Boost Invariant!



Silas R. Beane

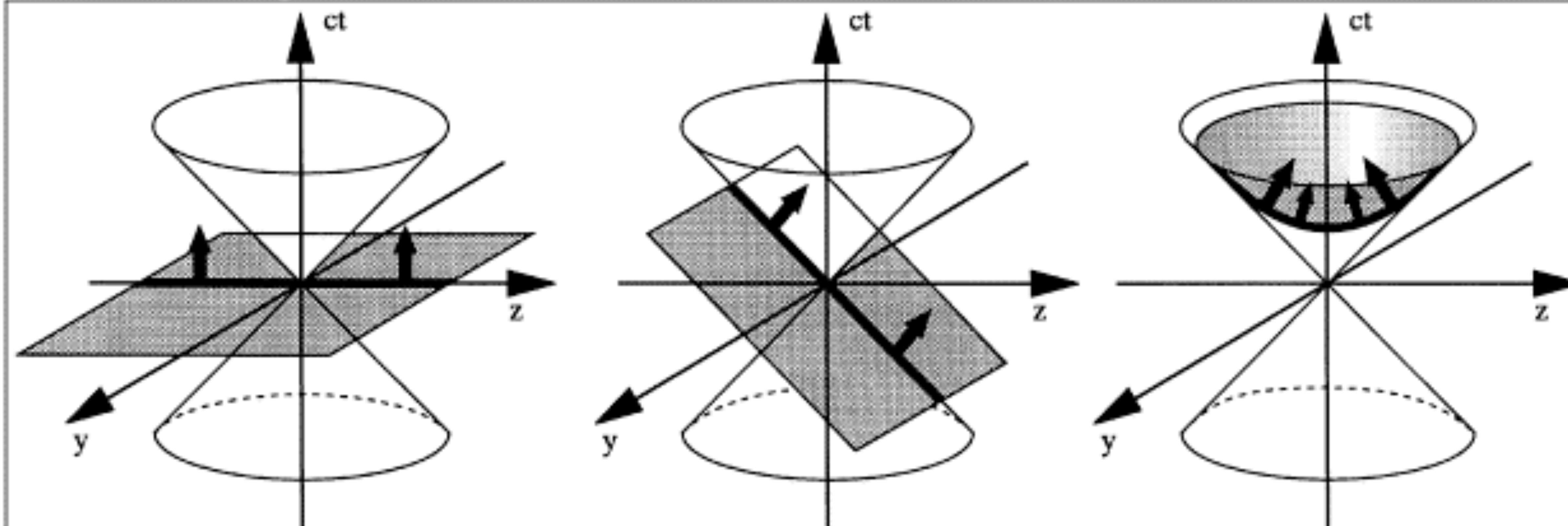
Null plane: a surface tangent to the light cone.

The null-plane Hamiltonians map the initial light-like surface onto some other surface, and therefore describe the dynamical evolution of the system.

The energy P^- translates the system in the null-plane time coordinate x^+, whereas the spin Hamiltonians F_r rotate the initial surface about the surface of the light cone.

P.A.M Dirac,
Rev. Mod. Phys. 21, 392 (1949)

Evolve in ordinary time	Evolve in light-front time	Evolve in point-form time



The instant form

$$\tilde{x}^0 = ct,\quad \tilde{x}^1 = x,\quad \tilde{x}^2 = y,\quad \tilde{x}^3 = z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The front form

$$\tilde{x}^0 = ct+z,\quad \tilde{x}^1 = x,\quad \tilde{x}^2 = y,\quad \tilde{x}^3 = ct-z$$

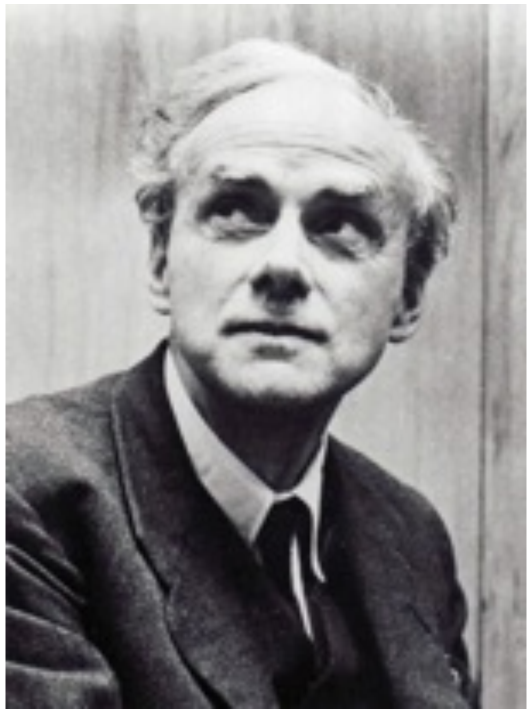
$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

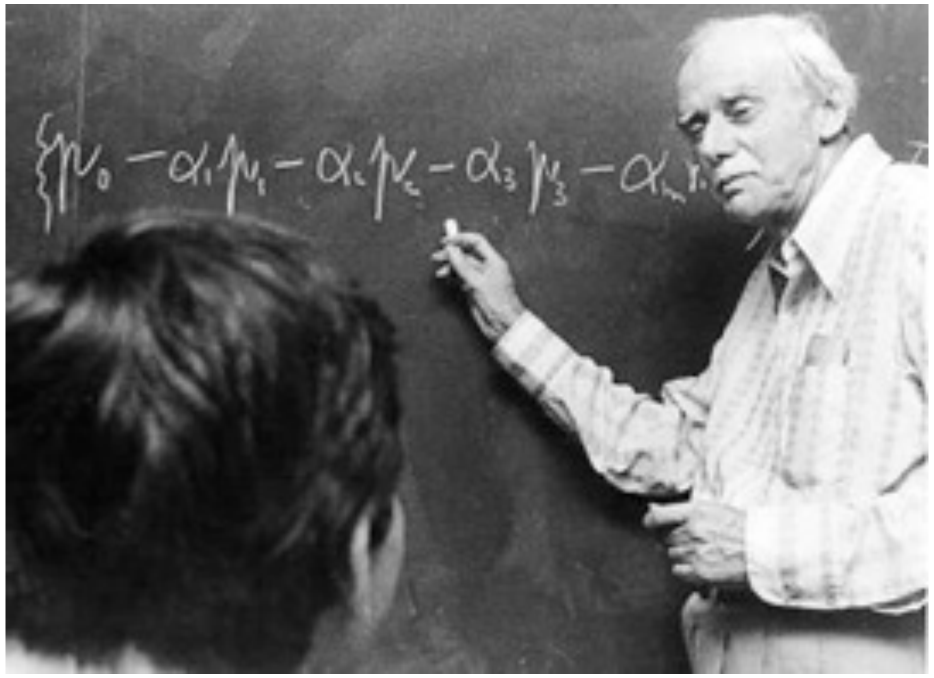
The point form

$$\tilde{x}^0 = \tau \;,\; ct = \tau\cosh\omega$$
$$\tilde{x}^1 = \omega \;,\; x = \tau\sinh\omega\,\sin\theta\,\cos\phi$$
$$\tilde{x}^2 = \theta \;,\; y = \tau\sinh\omega\,\sin\theta\,\sin\phi$$
$$\tilde{x}^3 = \phi \;,\; x = \tau\sinh\omega\,\cos\theta$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\tau^2 & 0 & 0 \\ 0 & 0 & -\tau^2\sinh^2\omega & 0 \\ 0 & 0 & 0 & -\tau^2\sinh^2\omega\,\sin^2\theta \end{pmatrix}$$

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)





"Working with a front is a process that is unfamiliar to physicists.

But still I feel that the mathematical simplification that it introduces is all-important.

I consider the method to be promising and have recently been making an extensive study of it.

It offers new opportunities, while the familiar instant form seems to be played out " - P.A.M. Dirac (1977)

Each element of flash photograph illuminated along the light front

at a fixed

$$\tau = t + z/c$$

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Eigenvalue

$$P^- = \frac{\mathcal{M}^2 + \vec{P}_\perp^2}{P^+}$$

$$H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>$$

- LF coordinates

$x^+ = x^0 + x^3$	light-front time	$P^+ = P^0 + P^3$	longitudinal momentum
$x^- = x^0 - x^3$	longitudinal space variable	$P^- = P^0 - P^3$	light-front Hamiltonian
$\mathbf{x}_\perp = (x^1, x^2)$	transverse space variable	$\mathbf{P}_\perp = (P^1, P^2)$	transverse momentum

- On shell relation $P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2 = \mathcal{M}^2$ leads to dispersion relation for LF Hamilnotian P^-

$$P^- = \frac{\mathbf{P}_\perp^2 + M^2}{P^+}, \quad P^+ > 0$$

- Hamiltonian equation for the relativistic bound state

$$i \frac{\partial}{\partial x^+} |\psi(P)\rangle = P^- |\psi(P)\rangle = \frac{M^2 + \mathbf{P}_\perp^2}{P^+} |\psi(P)\rangle$$

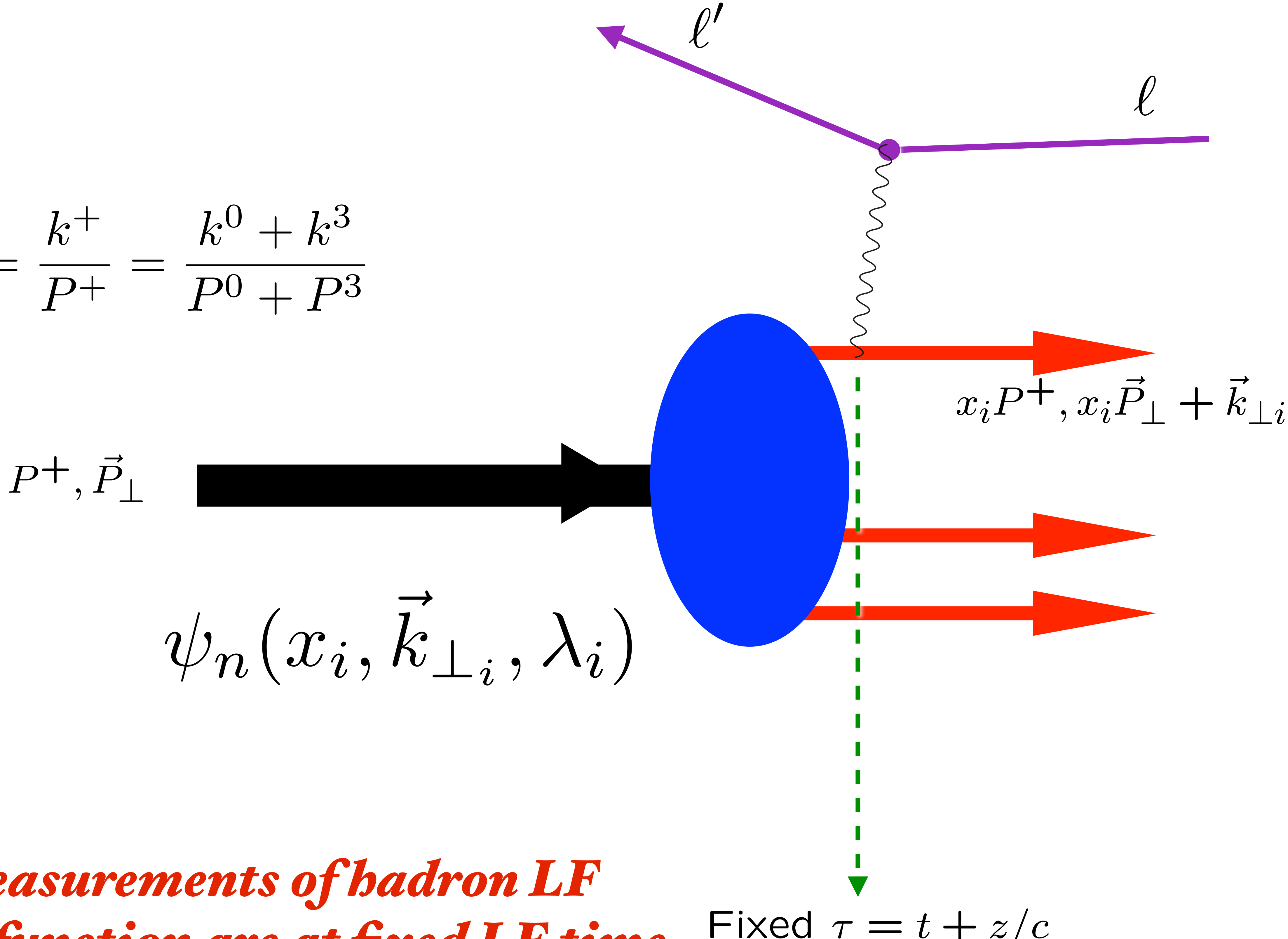
where P^- is derived from the QCD Lagrangian: kinetic energy of partons plus confining interaction

- Construct LF Lorentz invariant Hamiltonian $P^2 = P^- P^+ - \mathbf{P}_\perp^2$

$$P_\mu P^\mu |\psi(P)\rangle = M^2 |\psi(P)\rangle$$

- LF quantization is the ideal framework to describe hadronic structure in terms of constituents: simple vacuum structure allows unambiguous definition of partonic content of a hadron

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

Measurements of hadron LF wavefunction are at fixed LF time

Like a flash photograph

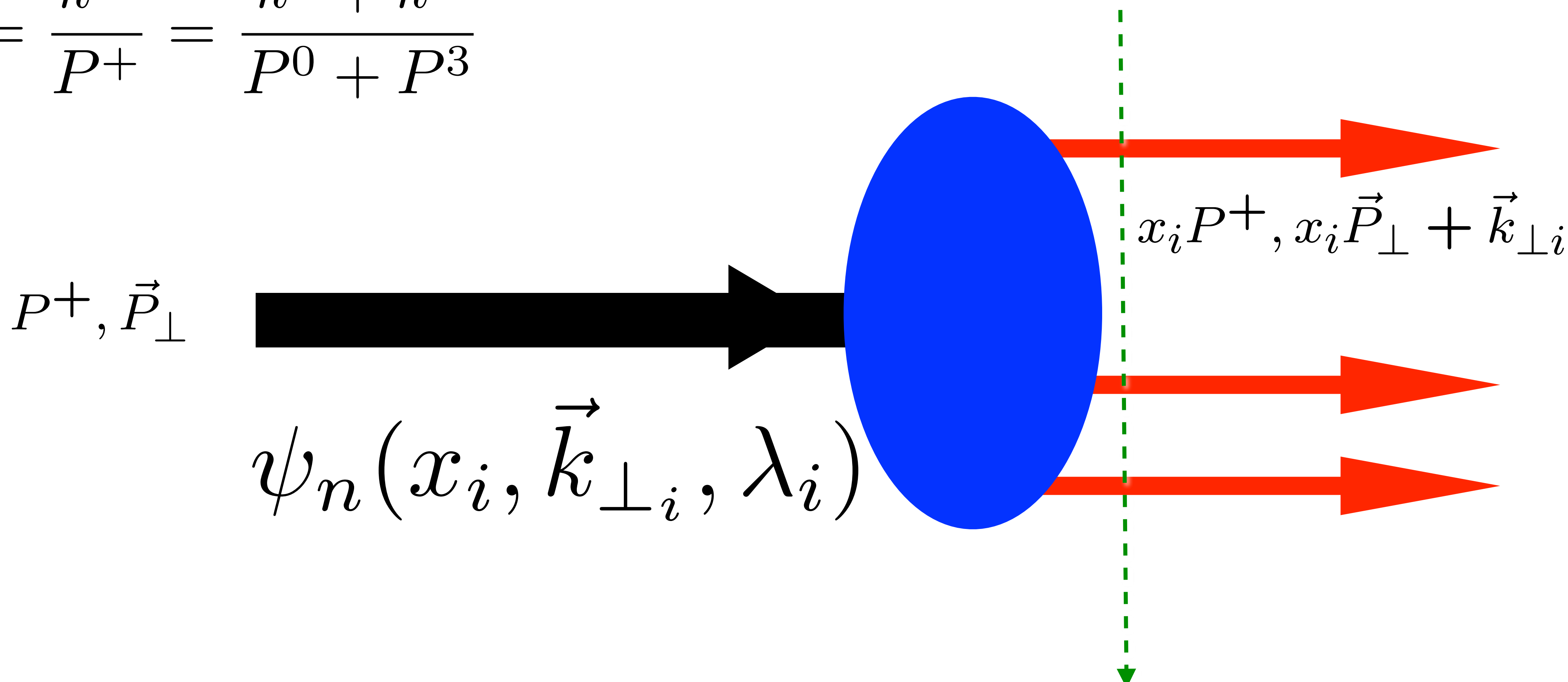
$$x_{bj} = x = \frac{k^+}{P^+}$$

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$



$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) | n; x_i, \vec{k}_{\perp i}, \lambda_i >$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock-State by Fock-State
Every Vertex

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment <--> Nonzero orbital angular momentum

Drell, sjb, Schmidt

Parke-Taylor Amplitudes **Santiago-Cruz, Stasto**

Advantages of the Dirac's Front Form for Hadron Physics

- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent: no boosts, no pancakes!
- Same structure function in e p collider and p rest frame
- No dependence on observer's frame
- LF Holography: Dual to AdS space
- LF Vacuum trivial -- no condensates!
- Profound implications for Cosmological Constant

Roberts, Shrock, Tandy, sjb

QCD Lagrangian

Fundamental Theory of Hadron and Nuclear Physics

gluon dynamics — quark kinetic energy + quark-gluon dynamics — quark mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i \bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classically Conformal if m_q=0

Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

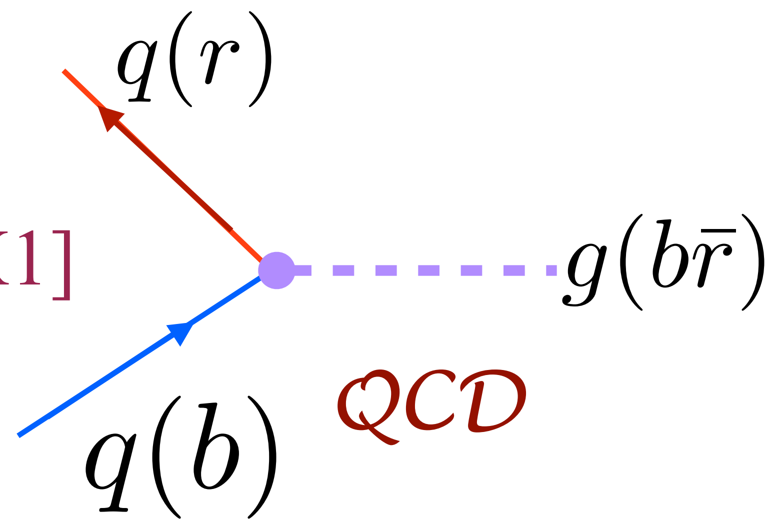
Scale-Invariant Coupling
Renormalizable
Asymptotic Freedom
Color Confinement

QCD Mass Scale from Confinement not Explicit

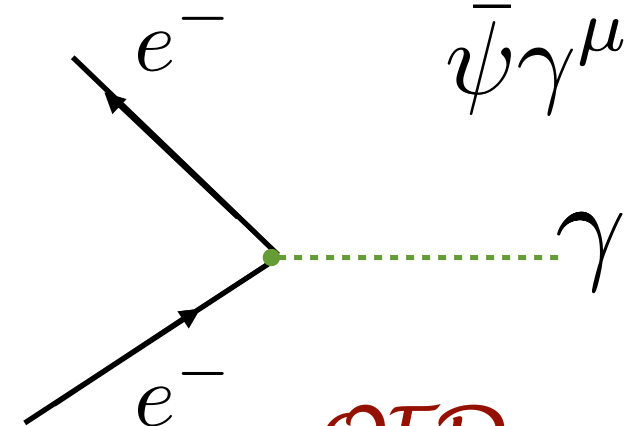
Fundamental Couplings of QCD and QED

$$\bar{\psi}\gamma^{\mu}A^{\mu}\bar{\psi}$$

[1X3] [3X3] [3X1]



QCD



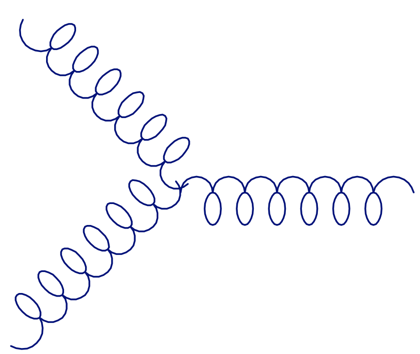
$$\bar{\psi}\gamma^{\mu}A^{\mu}\bar{\psi}$$

QED

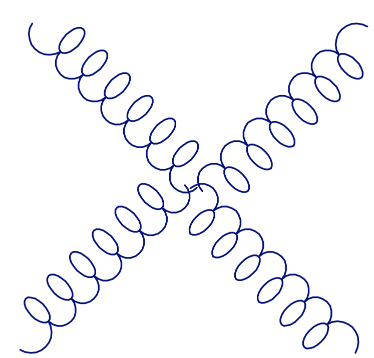
$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_{\mu}\gamma^{\mu}\Psi_f + \sum_{f=1}^{n_f} m_f\bar{\Psi}_f\Psi_f$$

$$G^{\mu\nu} = \partial^{\mu}A^{\mu} - \partial^{\nu}A^{\mu} - g[A^{\mu}, A^{\nu}]$$

Gluon vertices



QCD



$$G^{\mu\nu}G_{\mu\nu}$$

gluon self couplings

QCD Lagrangian

Fundamental Theory of Hadron and Nuclear Physics

gluon dynamics — quark kinetic energy + quark-gluon dynamics — quark mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i \bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

$$lim_{N_C \to 0} : \text{Abelian Theory}$$

$$C_F \alpha_s \to \alpha \qquad C_F = \frac{N_C^2 - 1}{2N_C}$$

P. Huet and SJB

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

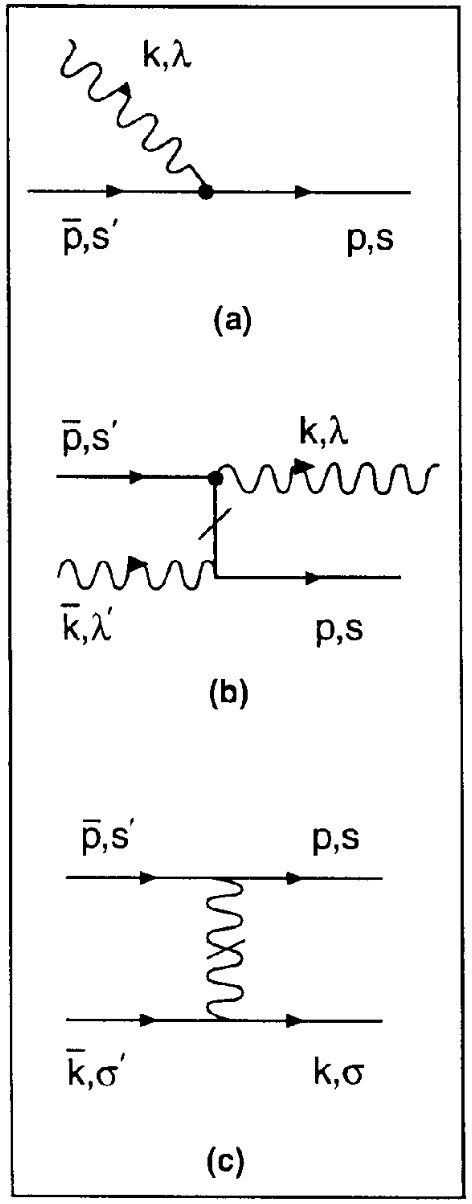
$$L^{QCD} \to H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

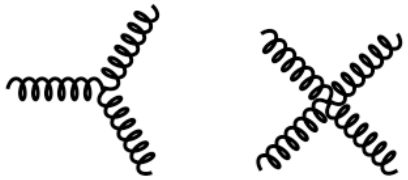
H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$



Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

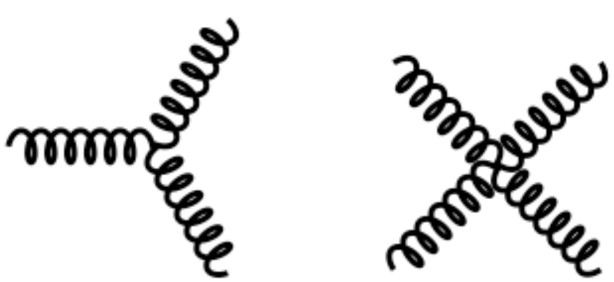


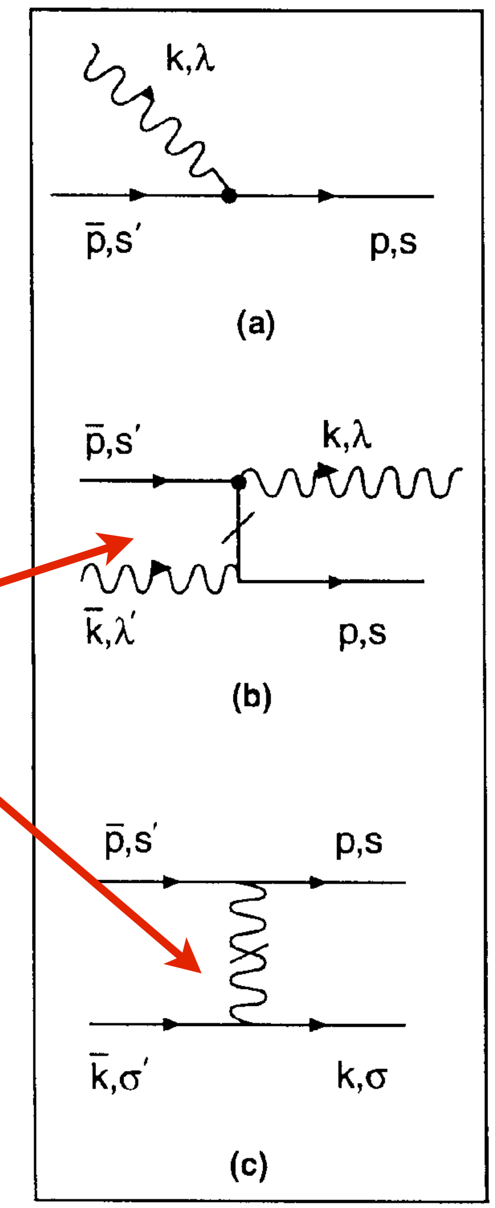
H^{int}_{LF}

LFWFs: Off-shell in P- and invariant mass

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$\begin{aligned}
H^{LF}_{QCD} &= \frac{1}{2}\int d^3x \overline{\tilde{\psi}}\gamma^+ \frac{(\mathrm{i}\partial^\perp)^2 + m^2}{\mathrm{i}\partial^+}\tilde{\psi} - A^i_a(\mathrm{i}\partial^\perp)^2 A_{ia} \\
&- \frac{1}{2}g^2 \int d^3x \,\mathbf{Tr}\left[\tilde{A}^\mu, \tilde{A}^\nu\right]\left[\tilde{A}_\mu, \tilde{A}_\nu\right] \\
&+ \frac{1}{2}g^2 \int d^3x \overline{\tilde{\psi}}\gamma^+ T^a \tilde{\psi}\frac{1}{(\mathrm{i}\partial^+)^2}\overline{\tilde{\psi}}\gamma^+ T^a \tilde{\psi} \\
&- g^2 \int d^3x \overline{\tilde{\psi}}\gamma^+ \left(\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\right)\tilde{\psi} \\
&+ g^2 \int d^3x \,\mathbf{Tr}\left(\left[\mathrm{i}\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\right) \\
&+ \frac{1}{2}g^2 \int d^3x \overline{\tilde{\psi}}\tilde{A\!\!\!/}\frac{\gamma^+}{\mathrm{i}\partial^+}\tilde{A\!\!\!/}\tilde{\psi} \\
&+ g\int d^3x \overline{\tilde{\psi}}\tilde{A\!\!\!/}\tilde{\psi} \\
&+ 2g\int d^3x \,\mathbf{Tr}\left(\mathrm{i}\partial^\mu \tilde{A}^\nu\left[\tilde{A}_\mu, \tilde{A}_\nu\right]\right)
\end{aligned}$$





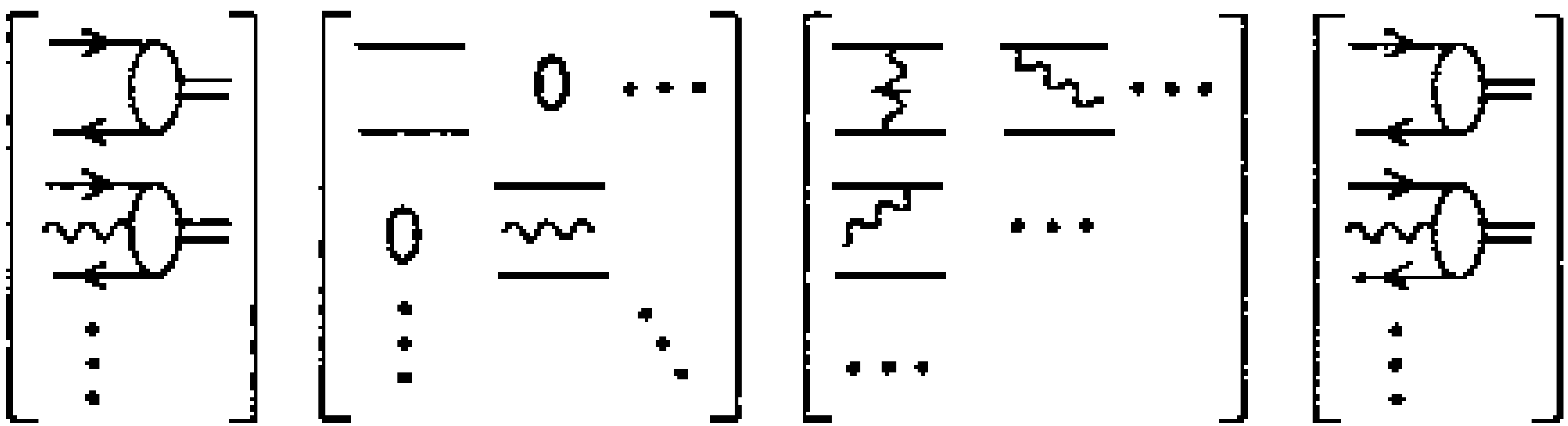
Physical gauge: $A^+ = 0$

LIGHT-FRONT MATRIX EQUATION

Rigorous Method for Solving Non-Perturbative QCD!

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\, V\, |q\bar{q}\rangle & \langle q\bar{q}|\, V\, |q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\, V\, |q\bar{q}\rangle & \langle q\bar{q}g|\, V\, |q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$

$A^+ = 0$



Minkowski space; frame-independent; no fermion doubling; no ghosts

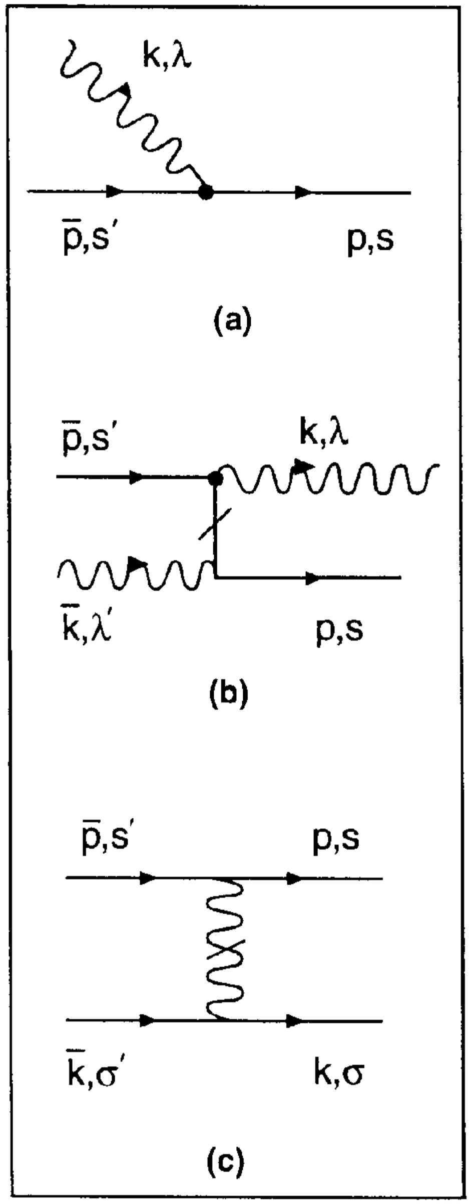
- Light-Front Vacuum = vacuum of free Hamiltonian!

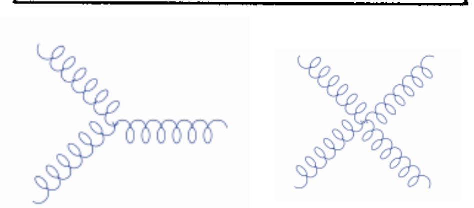
Light-Front QCD Heisenberg Equation

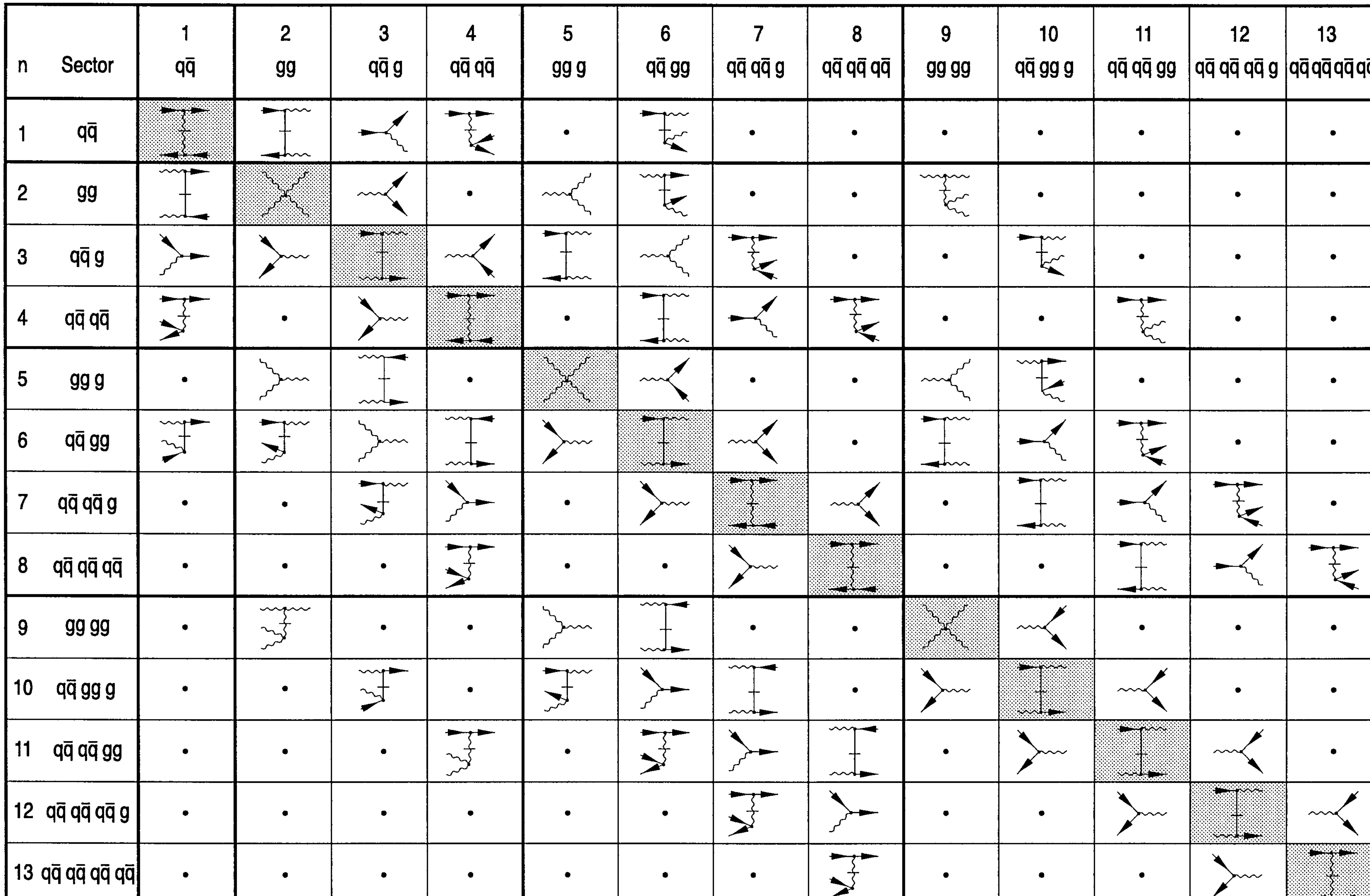
$$H_{LC}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 \, |\Psi_h\rangle$$

DLCQ: Solve QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb

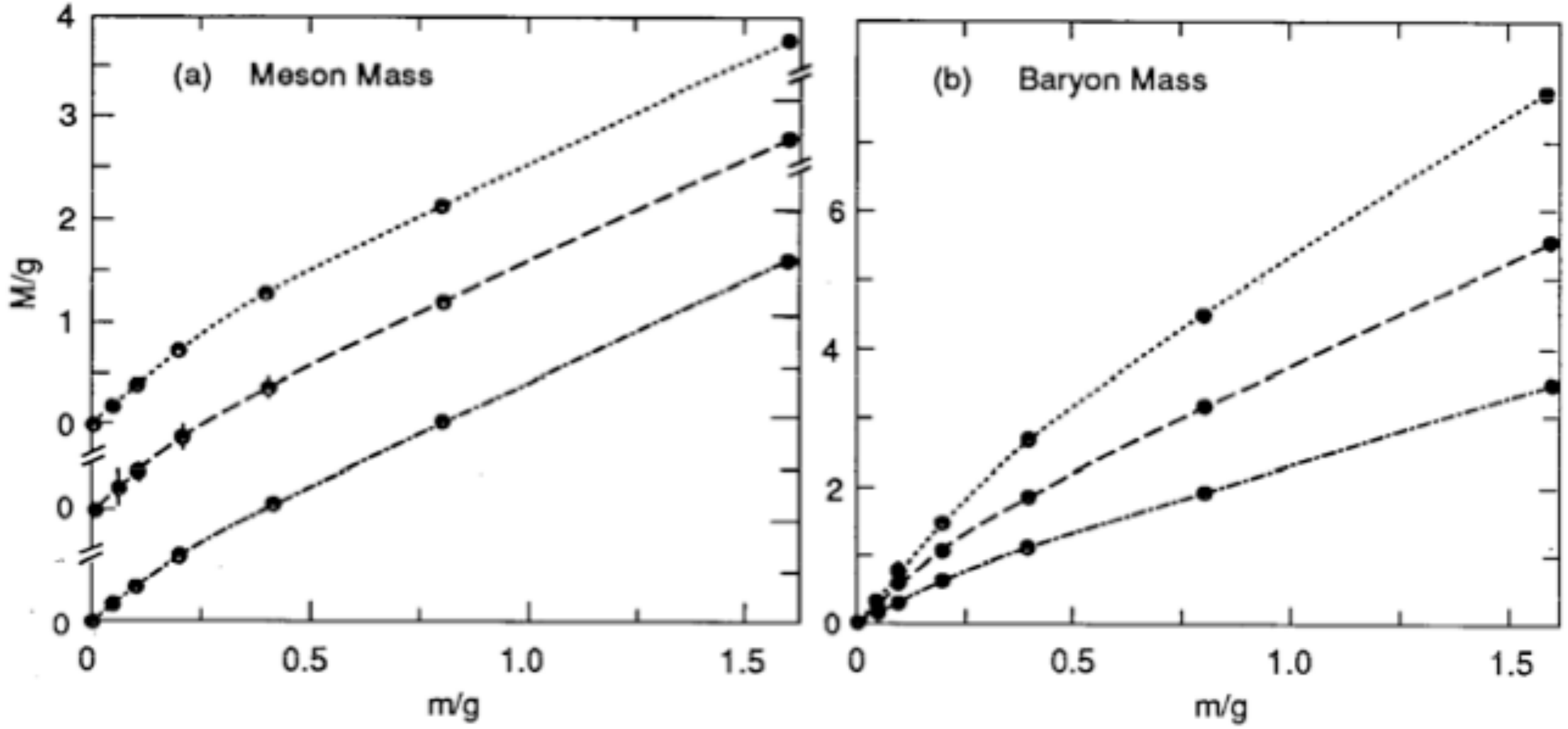




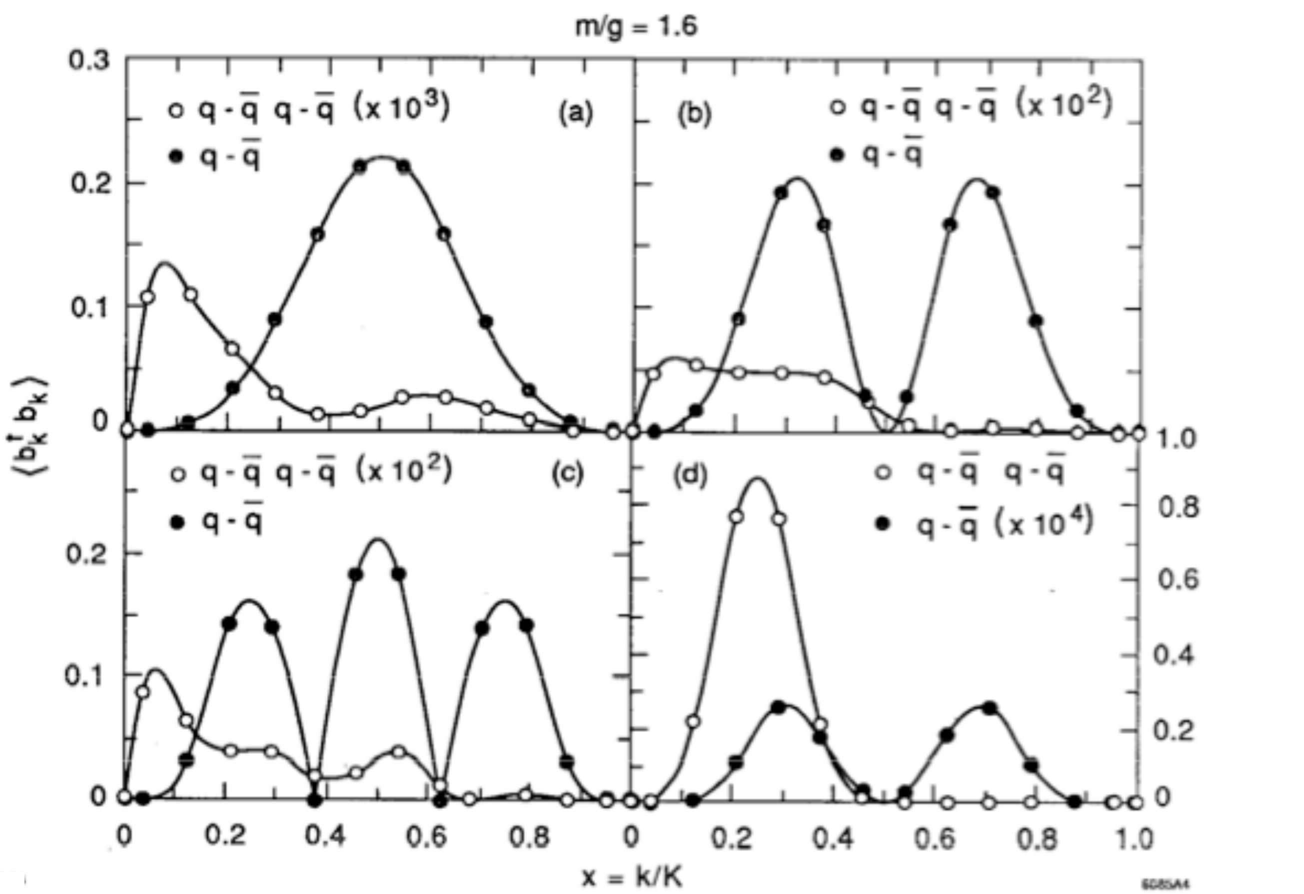


Minkowski space; frame-independent; no fermion doubling; no ghosts trivial vacuum

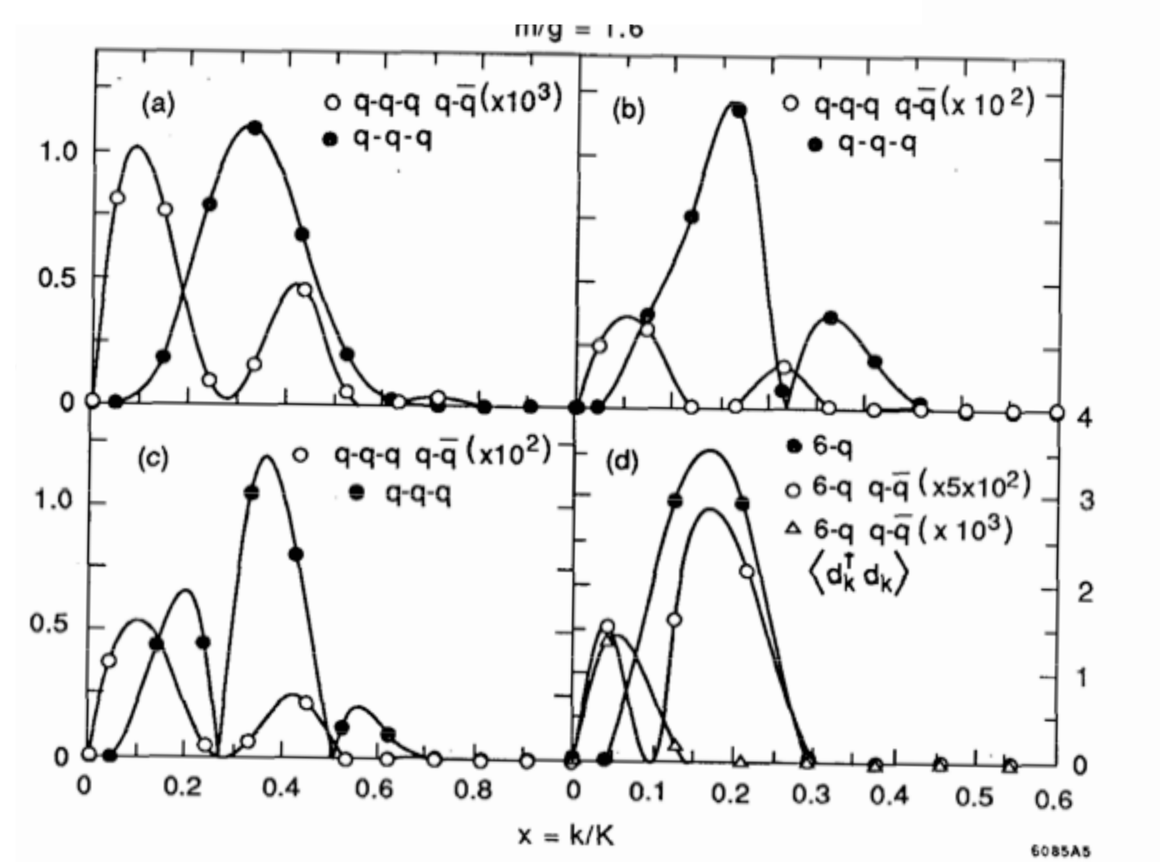
DLCQ: Solve QCD(1+1) for any quark mass and flavors



Extrapolated masses for $N = 2, 3$ and 4 meson and baryon.



a-c) First three states in $N = 3$ meson spectrum for $m/g = 1.6$, 2K=24. d) Eleventh state.

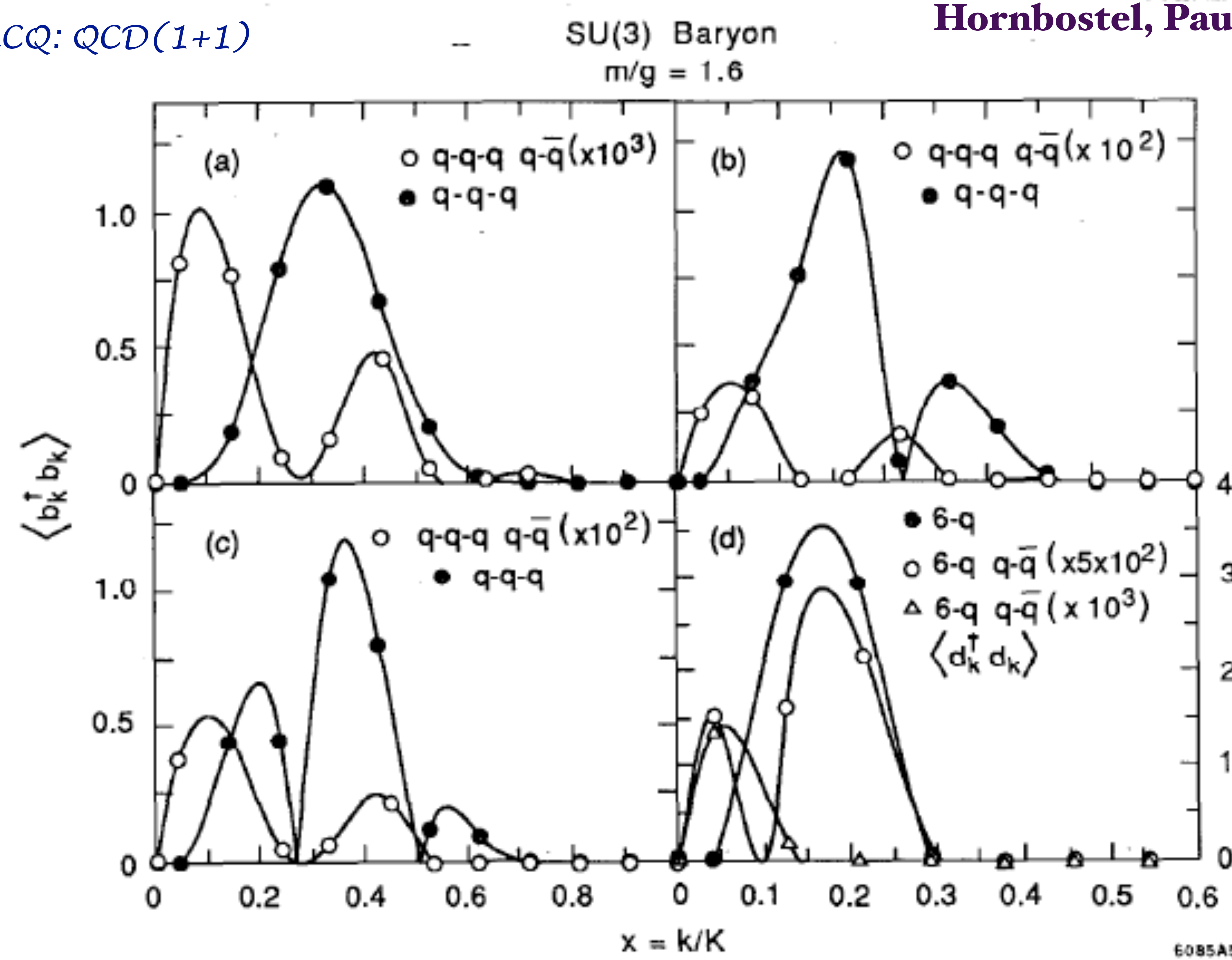


a-c) First three states in $N = 3$ baryon spectrum, 2K=21. d) First $B = 2$ state.

Hornbostel, Pauli, sjb

DLCQ: QCD(1+1)

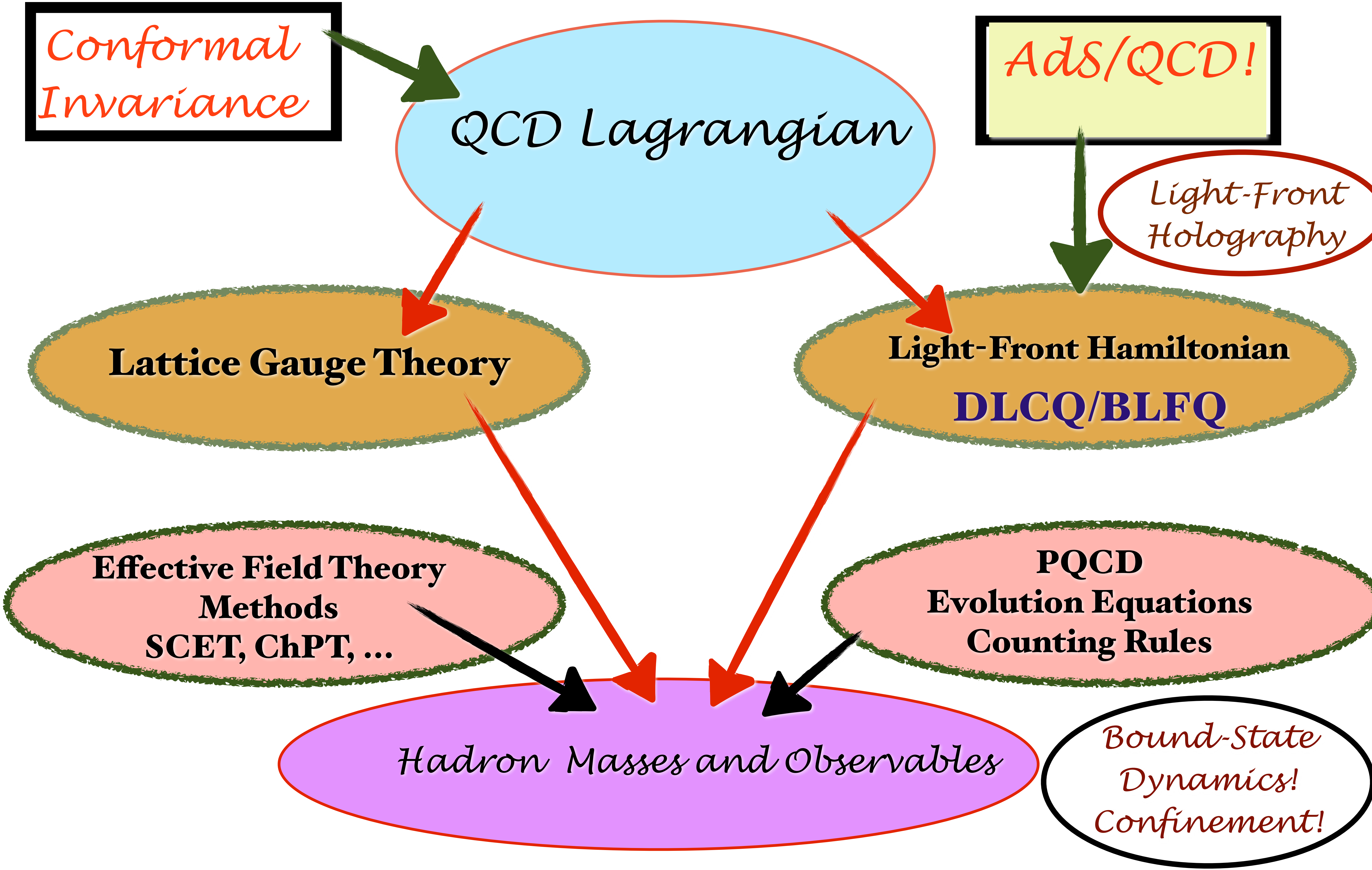
Hornbostel, Pauli, sjb



a-c) First three states in $N = 3$ baryon spectrum, 2K=21. d) First $B = 2$ state.

Predict Hadron Properties from First Principles!

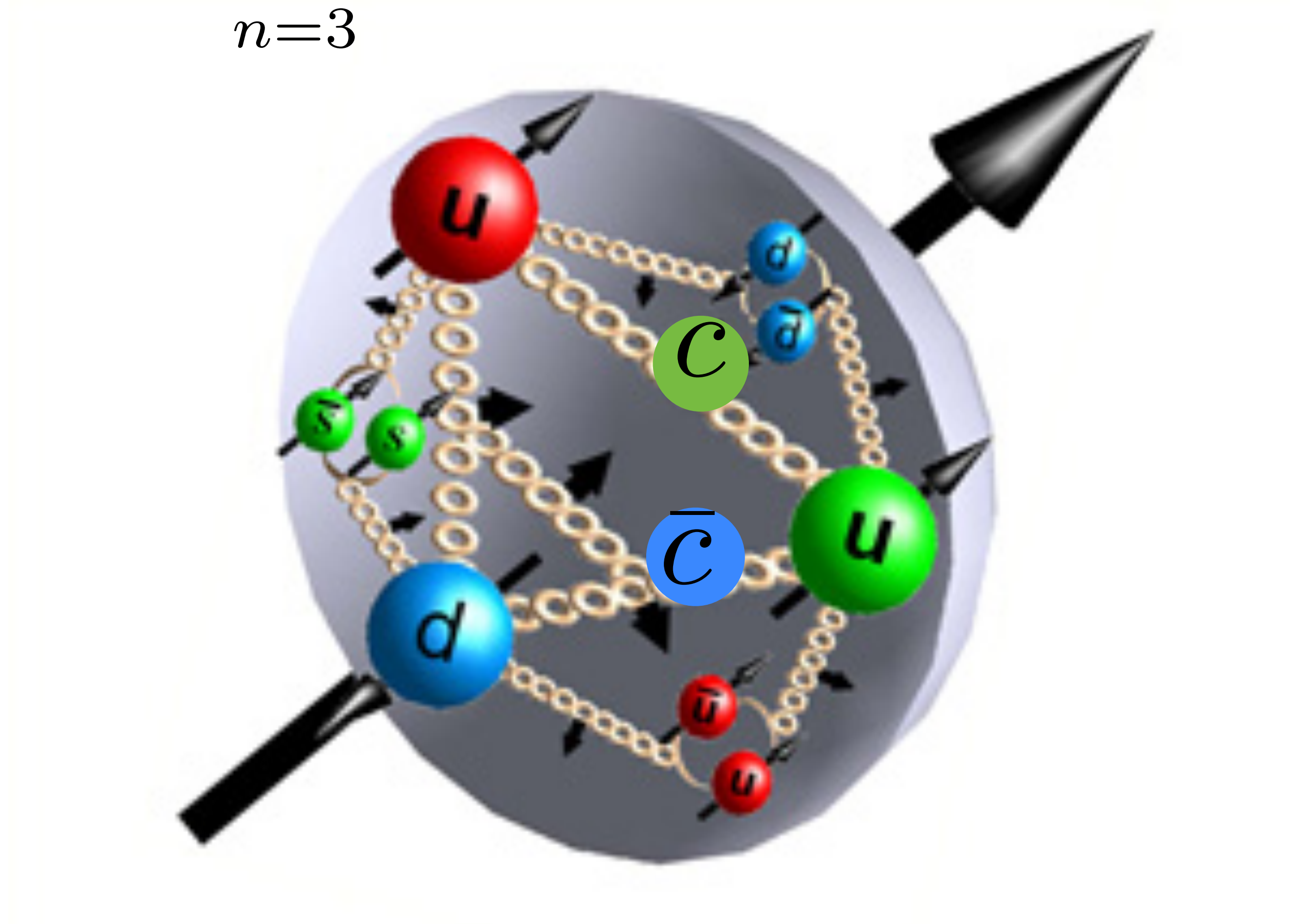
Dynamics and Spectroscopy



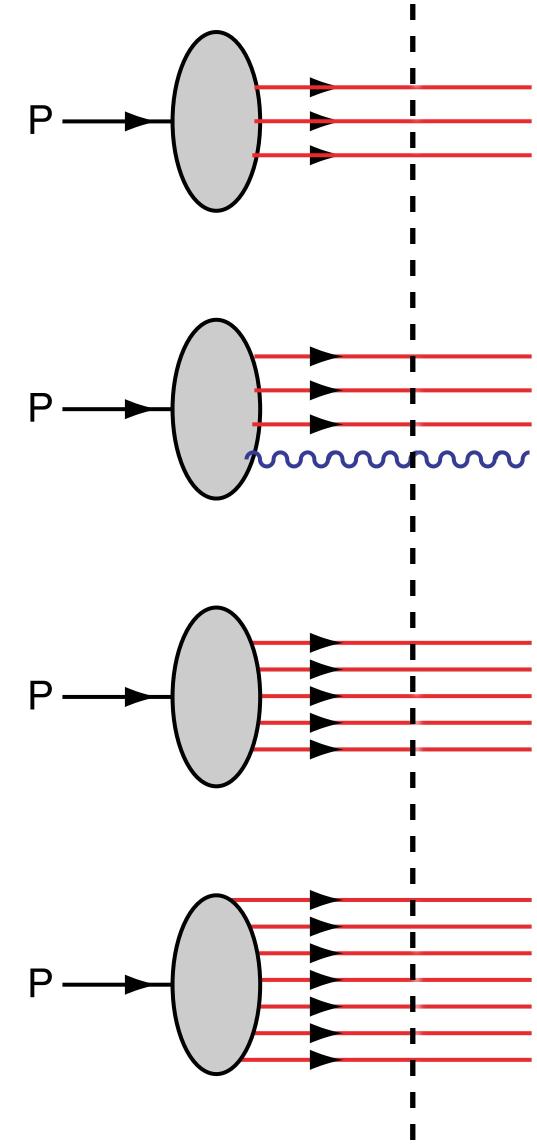
Wavefunction at fixed LF time: Arbitrarily Off-Shell in Invariant Mass

Eigenstate of LF Hamiltonian : all Fock states contribute

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$



Higher Fock States of the Proton



Fixed LF time

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

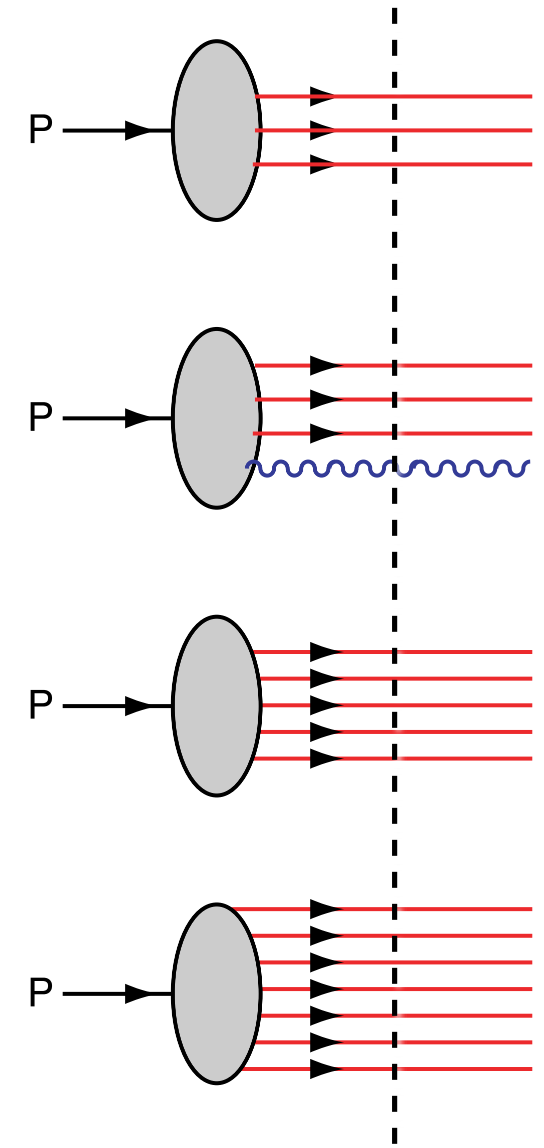
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks
s(x), c(x), b(x) at high x !

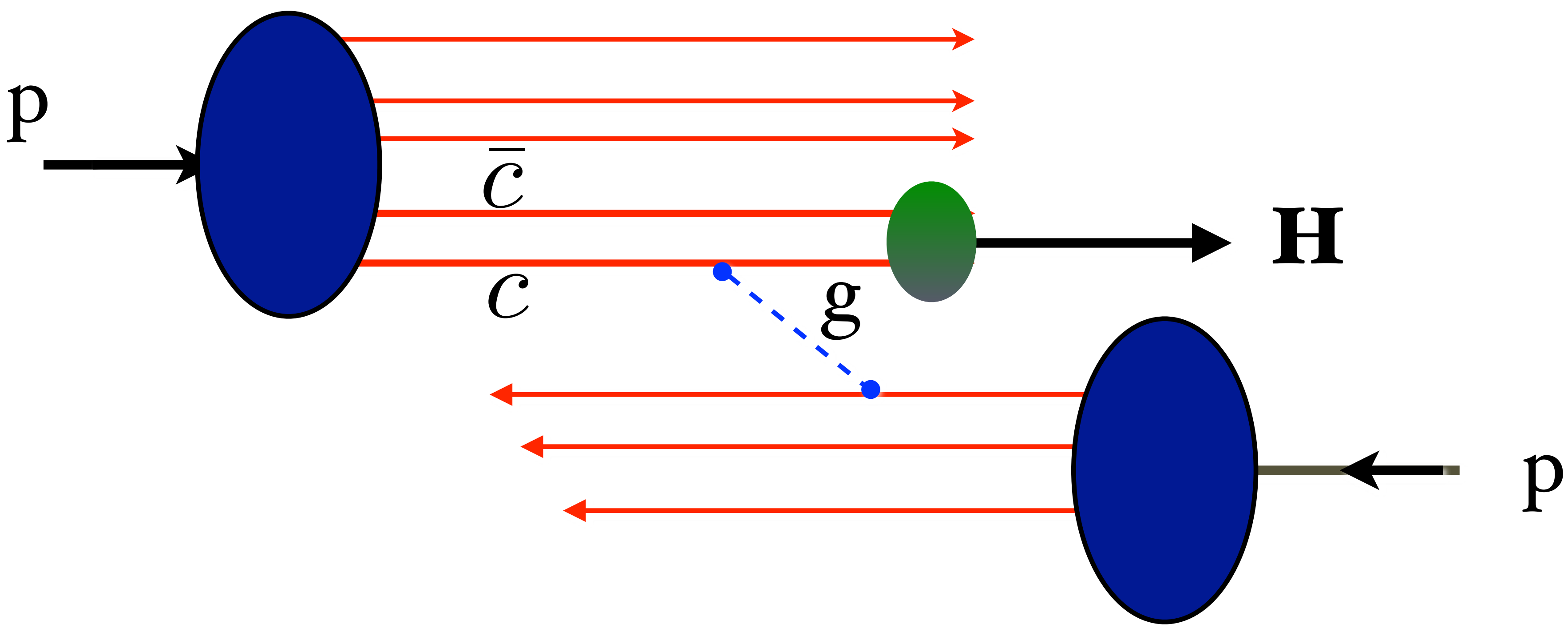
$\bar{s}(x) \neq s(x)$
$\bar{u}(x) \neq \bar{d}(x)$

Mueller: gluon Fock states **BFKL Pomeron**

Hidden Color

Goldhaber, Kopeliovich, Schmidt, Soffer sjb

Intrinsic Charm Mechanism for Inclusive High-X_F Higgs Production



$$pp \to H X$$

Also: intrinsic strangeness, bottom, top

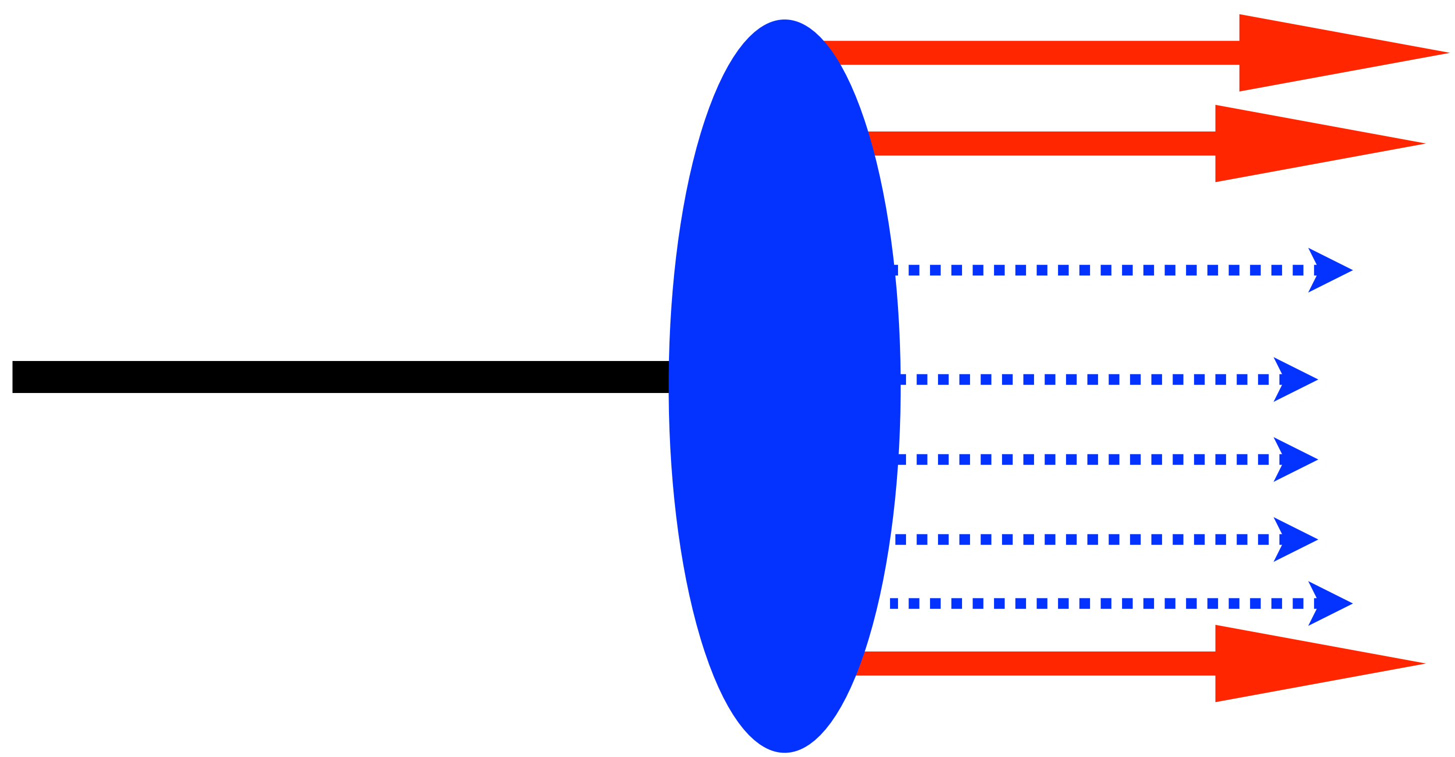
Higgs can have > 80% of Proton Momentum!

New production mechanism for Higgs

Soft gluons in the infinite momentum wave function and the BFKL **pomeron.**
Alfred H. Mueller (SLAC & Columbia U.) . SLAC-PUB-10047, CU-TP-609, Aug 1993. 12pp.
Published in **Nucl.Phys.B415:373-385,1994**.

Light cone wave functions at small x.
F. Antonuccio (Heidelberg, Max Planck Inst. & Heidelberg U.) , S.J. Brodsky (SLAC) , S. Dalley (CERN) .
Phys.Lett.B412:104-110,1997.
e-Print: **hep-ph/9705413**

Mueller: BFKL derived from multi-gluon Fock State



Antonuccio, Dalley, sjb: Ladder Relations

Universidad Técnica Federico Santa María

Light-Front QCD

Stan Brodsky
SLAC NATIONAL ACCELERATOR LABORATORY

Hidden Color in QCD

Lepage, Ji, sjb

- Deuteron six quark wavefunction:
- 5 color-singlet combinations of 6 color-triplets -- one state is |n p>
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- Predict

$$\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn) \text{ at high } Q^2$$

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Light-cone representation of the spin and orbital angular momentum of relativistic composite systems ☆

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[c] *CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China*
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PHYSICAL REVIEW D VOLUME 22, NUMBER 9 1 NOVEMBER 1980

Lepage
sjb

Exclusive processes in perturbative quantum chromodynamics

TABLE II. Dirac matrix elements for the helicity spinors of Appendix A.

Matrix element $\bar{u}_{\lambda'}\cdots u_{\lambda}$	Helicity $(\lambda\to\lambda')$ $\uparrow\to\uparrow$ $\downarrow\to\downarrow$	$\uparrow\to\downarrow$ $\downarrow\to\uparrow$
$\frac{\bar{u}(p)}{(p^+)^{1/2}}\gamma^+\frac{u(q)}{(q^+)^{1/2}}$	2	0
$\frac{\bar{u}(p)}{(p^+)^{1/2}}\gamma^-\frac{u(q)}{(q^+)^{1/2}}$	$\frac{2}{p^+q^+}(p_\perp\cdot q_\perp \pm i p_\perp\times q_\perp + m^2)$	$\mp\frac{2m}{p^+q^+}[(p^1\pm ip^2)-(q^1\pm iq^2)]$
$\frac{\bar{u}(p)}{(p^+)^{1/2}}\gamma_\perp^i\frac{u(q)}{(q^+)^{1/2}}$	$\frac{p_\perp^i \mp i\epsilon^{ij}p_\perp^j}{p^+}+\frac{q_\perp^i \pm i\epsilon^{ij}q_\perp^j}{q^+}$	$\mp m\left(\frac{p^+-q^+}{p^+q^+}\right)(\delta^{i1}\pm i\delta^{i2})$
$\frac{\bar{u}(p)}{(p^+)^{1/2}}\frac{u(q)}{(q^+)^{1/2}}$	$m\left(\frac{p^++q^+}{p^+q^+}\right)$	$\mp\left(\frac{p^1\pm ip^2}{p^+}-\frac{q^1\pm iq^2}{q^+}\right)$
$\frac{\bar{u}(p)}{(p^+)^{1/2}}\gamma^-\gamma^+\gamma^-\frac{u(q)}{(q^+)^{1/2}}$	$\frac{8}{p^+q^+}(p_\perp\cdot q_\perp \pm i p_\perp\times q_\perp + m^2)$	$\mp\frac{8m}{p^+q^+}[(p^1\pm ip^2)-(q^1\pm iq^2)]$
$\frac{\bar{u}(p)}{(p^+)^{1/2}}\gamma^-\gamma^+\gamma_\perp^i\frac{u(q)}{(q^+)^{1/2}}$	$4\left(\frac{p_\perp^i \mp i\epsilon^{ij}p_\perp^j}{p^+}\right)$	$\pm\frac{4m}{p^+}(\delta^{i1}\pm i\delta^{i2})$
$\frac{\bar{u}(p)}{(p^+)^{1/2}}\gamma_\perp^i\gamma^+\gamma^-\frac{u(q)}{(q^+)^{1/2}}$	$4\left(\frac{q_\perp^i \pm i\epsilon^{ij}q_\perp^j}{q^+}\right)$	$\mp\frac{4m}{q^+}(\delta^{i1}\pm i\delta^{i2})$
$\frac{\bar{u}(p)}{(p^+)^{1/2}}\gamma_\perp^i\gamma^+\gamma_\perp^j\frac{u(q)}{(q^+)^{1/2}}$	$2(\delta^{ij}\pm i\epsilon^{ij})$	0

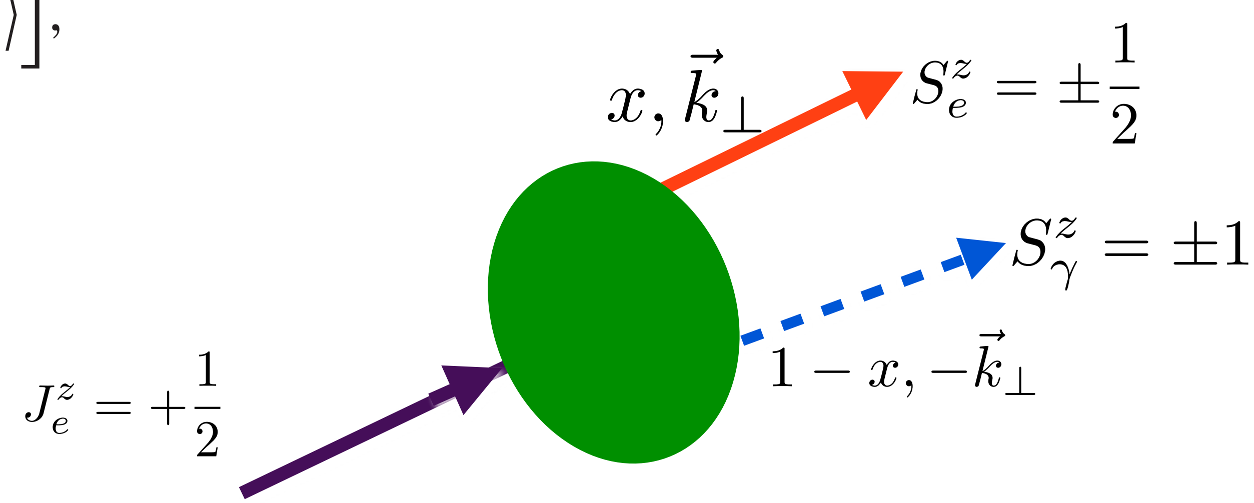
$$\bar{v}_\mu(p)\gamma^\alpha v_\nu(q)=\bar{u}_\nu(q)\gamma^\alpha u_\mu(p) \qquad \bar{v}_\mu(p)v_\nu(q)=-\bar{u}_\nu(q)u_\mu(p)$$

$$\bar{v}_\mu(p)\gamma^\alpha\gamma^\beta\gamma^\delta v_\nu(q)=\bar{u}_\nu(q)\gamma^\delta\gamma^\beta\gamma^\alpha u_\mu(p)$$

MATRIX ELEMENT $\bar{u}_{\lambda'} \ldots u_{\lambda}$	HELICITY $(\lambda \to \lambda')$	
	$\uparrow \to \uparrow$ $\downarrow \to \downarrow$	$\uparrow \to \downarrow$ $\downarrow \to \uparrow$
$\frac{\bar{u}(p)}{\sqrt{p^+}} \gamma^+ \frac{u(q)}{\sqrt{q^+}}$	2	0
$\frac{\bar{u}(p)}{\sqrt{p^+}} \gamma^- \frac{u(q)}{\sqrt{q^+}}$	$\frac{2}{p^+q^+}\left\{p_\perp \cdot q_\perp \pm i p_\perp \times q_\perp + m^2\right\}$	$\mp \frac{2m}{p^+q^+}\left\{(p^1 \pm ip^2)-(q^1 \pm iq^2)\right\}$
$\frac{\bar{u}(p)}{\sqrt{p^+}} \gamma_\perp^i \frac{u(q)}{\sqrt{q^+}}$	$\frac{p_\perp^i \mp i\epsilon^{ij}p_\perp^j}{p^+} + \frac{q_\perp^i \pm i\epsilon^{ij}q_\perp^j}{q^+}$	$\mp m\left\{\frac{p^+ - q^+}{p^+q^+}\right\}(\delta^{i1} \pm i\delta^{i2})$
$\frac{\bar{u}(p)}{\sqrt{p^+}} \frac{u(q)}{\sqrt{q^+}}$	$m\left\{\frac{p^+ + q^+}{p^+q^+}\right\}$	$\mp\left\{\frac{p^1 \pm ip^2}{p^+} - \frac{q^1 \pm iq^2}{q^+}\right\}$
$\frac{\bar{u}(p)}{\sqrt{p^+}} \gamma^-\gamma^+\gamma^- \frac{u(q)}{\sqrt{q^+}}$	$\frac{8}{p^+q^+}\left\{p_\perp \cdot q_\perp \pm i p_\perp \times q_\perp + m^2\right\}$	$\mp \frac{8m}{p^+q^+}\left\{(p^1 \pm ip^2)-(q^1 \pm iq^2)\right\}$
$\frac{\bar{u}(p)}{\sqrt{p^+}} \gamma^-\gamma^+\gamma_\perp^i \frac{u(q)}{\sqrt{q^+}}$	$4\left\{\frac{p_\perp^i \mp i\epsilon^{ij}p_\perp^j}{p^+}\right\}$	$\pm \frac{4m}{p^+}(\delta^{i1} \pm i\delta^{i2})$
$\frac{\bar{u}(p)}{\sqrt{p^+}} \gamma_\perp^i\gamma^+\gamma^- \frac{u(q)}{\sqrt{q^+}}$	$4\left\{\frac{q_\perp^i \pm i\epsilon^{ij}q_\perp^j}{q^+}\right\}$	$\mp \frac{4m}{q^+}(\delta^{i1} \pm i\delta^{i2})$
$\frac{\bar{u}(p)}{\sqrt{p^+}} \gamma_\perp^i\gamma^+\gamma_\perp^j \frac{u(q)}{\sqrt{q^+}}$	$2\left\{\delta^{ij} \pm i\epsilon^{ij}\right\}$	0

The two-particle Fock state for an electron with $J^z = +\frac{1}{2}$ has four possible spin combinations:

$$\left|\Psi^{\uparrow}_{\text{two particle}}(P^+, \vec{P}_\perp = \vec{0}_\perp)\right\rangle$$
$$= \int \frac{\mathrm{d}^2\vec{k}_\perp \, \mathrm{d}x}{\sqrt{x(1-x)}\,16\pi^3}\Big[\psi^{\uparrow}_{+\frac{1}{2}\,+1}(x,\vec{k}_\perp)\left|+\tfrac{1}{2}\,+1;\, xP^+,\, \vec{k}_\perp\right\rangle$$
$$+\,\psi^{\uparrow}_{+\frac{1}{2}\,-1}(x,\vec{k}_\perp)\left|+\tfrac{1}{2}-1;\, xP^+,\, \vec{k}_\perp\right\rangle + \psi^{\uparrow}_{-\frac{1}{2}+1}(x,\vec{k}_\perp)\left|-\tfrac{1}{2}+1;\, xP^+,\, \vec{k}_\perp\right\rangle$$
$$+\,\psi^{\uparrow}_{-\frac{1}{2}-1}(x,\vec{k}_\perp)\left|-\tfrac{1}{2}-1;\, xP^+,\, \vec{k}_\perp\right\rangle\Big],$$



$$\begin{cases}
\psi^{\uparrow}_{+\frac{1}{2}+1}(x,\vec{k}_\perp) = -\sqrt{2}\,\dfrac{(-k^1 + \mathrm{i}k^2)}{x(1-x)}\,\varphi, \\
\psi^{\uparrow}_{+\frac{1}{2}-1}(x,\vec{k}_\perp) = -\sqrt{2}\,\dfrac{(+k^1 + \mathrm{i}k^2)}{1-x}\,\varphi, \\
\psi^{\uparrow}_{-\frac{1}{2}+1}(x,\vec{k}_\perp) = -\sqrt{2}\left(M - \dfrac{m}{x}\right)\varphi, \\
\psi^{\uparrow}_{-\frac{1}{2}-1}(x,\vec{k}_\perp) = 0,
\end{cases}$$

$$\varphi = \varphi(x,\vec{k}_\perp) = \frac{e/\sqrt{1-x}}{M^2 - (\vec{k}^2_\perp + m^2)/x - (\vec{k}^2_\perp + \lambda^2)/(1-x)}.$$

Hwang, Schmidt, Ma, sjb

Angular Momentum on the Light-Front

LC gauge $A^+=0$

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

Conserved
LF Fock state by Fock State

Gluon orbital angular momentum defined in physical lc gauge

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

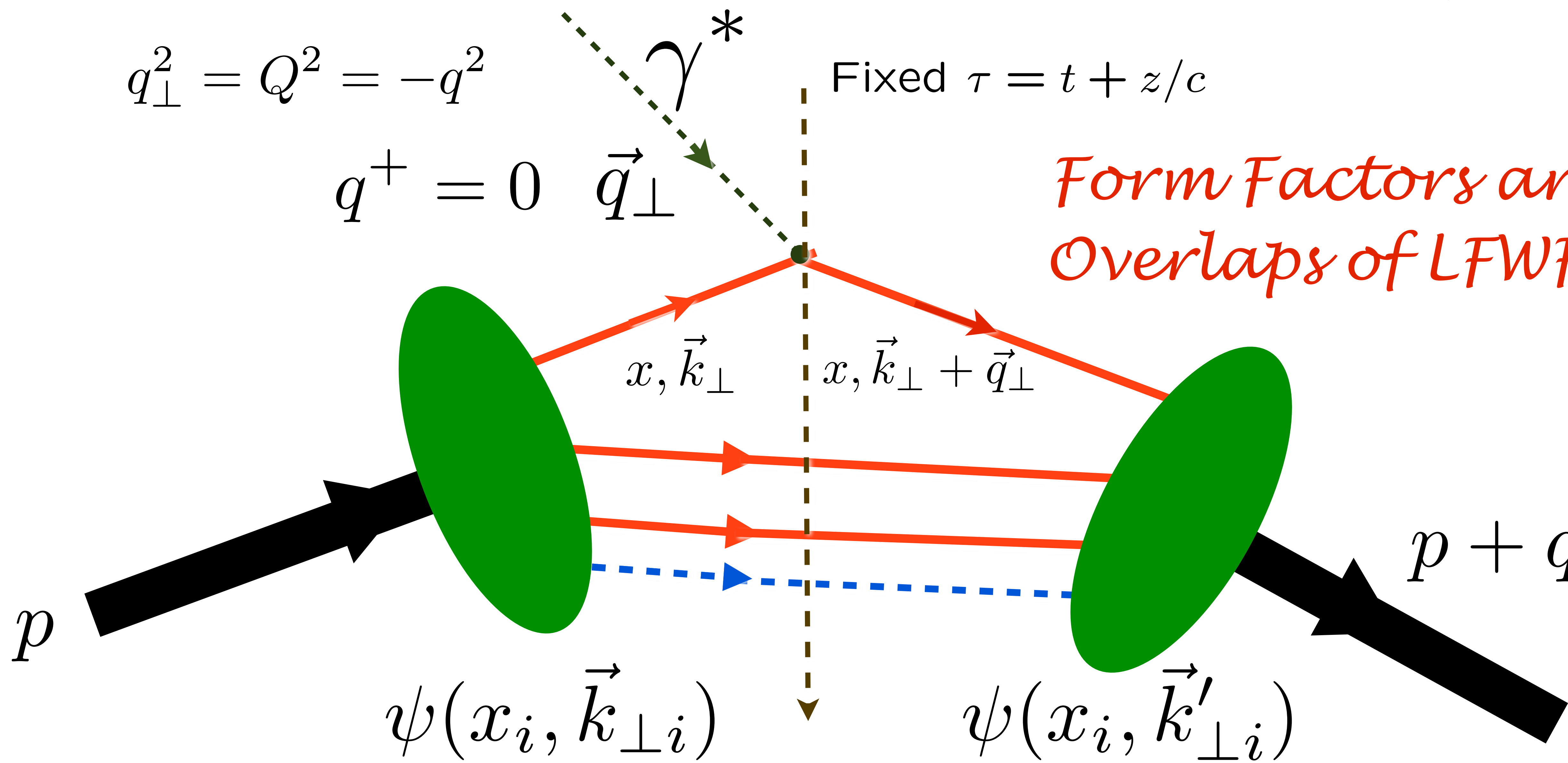
n-1 orbital angular momenta

Orbital Angular Momentum is a property of LFWFS

Nonzero Anomalous Moment -->
Nonzero quark orbital angular momentum!

$$< p+q|j^+(0)|p> = 2p^+ F(q^2)$$

Interaction picture



Form Factors are Overlaps of LFWFs

struck $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i \vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Exact LF Formula for Pauli Form Factor

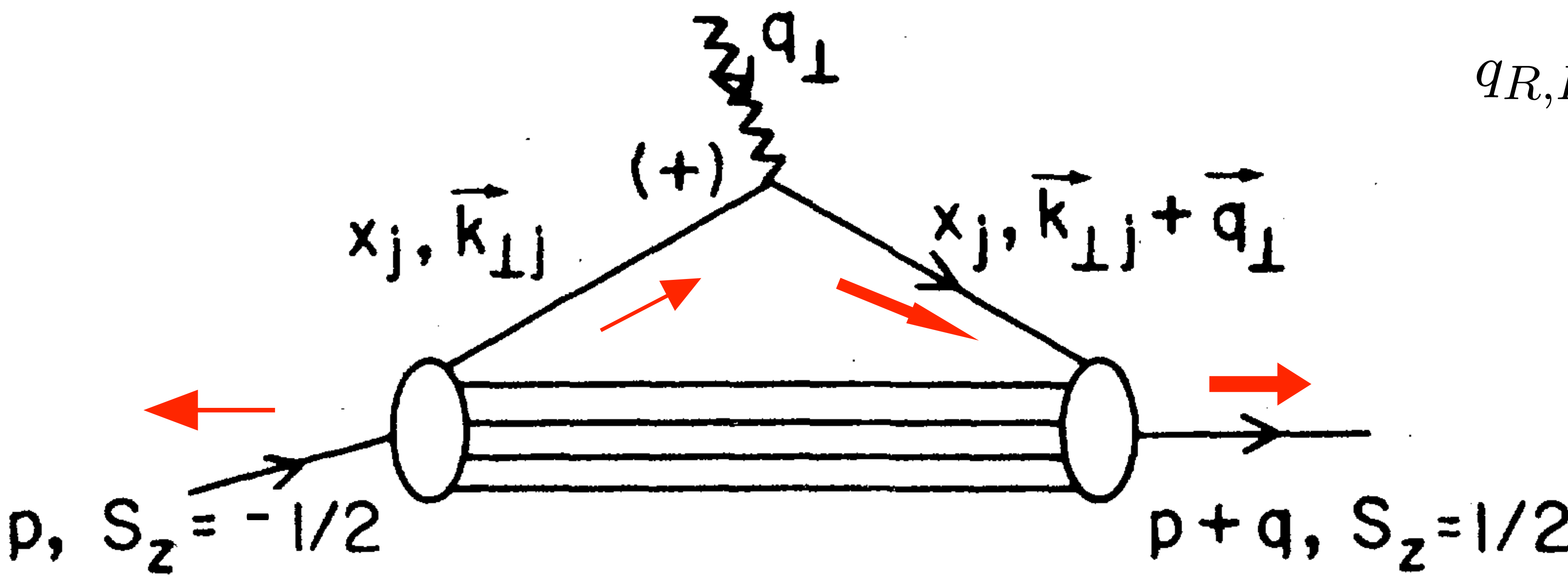
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$



Must have $\Delta\ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

$$\left\langle P+q,\uparrow\left|\frac{J^+(0)}{2P^+}\right|P,\uparrow\right\rangle = F_1(q^2), \tag{5}$$

$$\left\langle P+q,\uparrow\left|\frac{J^+(0)}{2P^+}\right|P,\downarrow\right\rangle = -(q^1 - \mathrm{i}q^2)\frac{F_2(q^2)}{2M}. \tag{6}$$

The magnetic moment of a composite system is one of its most basic properties. The magnetic moment is defined at the $q^2 \to 0$ limit,

$$\mu = \frac{e}{2M}\left[F_1(0) + F_2(0)\right], \tag{7}$$

where e is the charge and M is the mass of the composite system. We use the standard light-cone frame ($q^\pm = q^0 \pm q^3$):

$$\begin{aligned} q &= (q^+, q^-, \vec{q}_\perp) = \left(0, \frac{-q^2}{P^+}, \vec{q}_\perp\right), \\ P &= (P^+, P^-, \vec{P}_\perp) = \left(P^+, \frac{M^2}{P^+}, \vec{0}_\perp\right), \end{aligned} \tag{8}$$

where $q^2 = -2P\cdot q = -\vec{q}_\perp^{\,2}$ is 4-momentum square transferred by the photon.

The Pauli form factor and the anomalous magnetic moment $\kappa = \frac{e}{2M}F_2(0)$ can then be calculated from the expression

$$-(q^1 - \mathrm{i}q^2)\frac{F_2(q^2)}{2M} = \sum_a \int \frac{\mathrm{d}^2\vec{k}_\perp\,\mathrm{d}x}{16\pi^3} \sum_j e_j\, \psi_a^{\uparrow *}(x_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \vec{k}_{\perp i}, \lambda_i), \tag{9}$$

where the summation is over all contributing Fock states a and struck constituent charges e_j. The arguments of the final-state light-cone wavefunction are [1,2]

$$\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp \tag{10}$$

for the struck constituent and

$$\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp \tag{11}$$

$$
\begin{aligned}
F_2(q^2) &= \frac{-2M}{(q^1 - \mathrm{i}q^2)} \langle \Psi^{\uparrow}(P^+, \vec{P}_\perp = \vec{q}_\perp) | \Psi^{\downarrow}(P^+, \vec{P}_\perp = \vec{0}_\perp) \rangle \\
&= \frac{-2M}{(q^1 - \mathrm{i}q^2)} \int \frac{\mathrm{d}^2\vec{k}_\perp \, \mathrm{d}x}{16\pi^3} \Big[\psi^{\uparrow\,*}_{+\frac{1}{2}-1}(x, \vec{k}'_\perp) \psi^{\downarrow}_{+\frac{1}{2}-1}(x, \vec{k}_\perp) \\
&\qquad\qquad + \psi^{\uparrow\,*}_{-\frac{1}{2}+1}(x, \vec{k}'_\perp) \psi^{\downarrow}_{-\frac{1}{2}+1}(x, \vec{k}_\perp) \Big] \\
&= 4M \int \frac{\mathrm{d}^2\vec{k}_\perp \, \mathrm{d}x}{16\pi^3} \frac{(m - Mx)}{x} \varphi(x, \vec{k}'_\perp)^* \varphi(x, \vec{k}_\perp) \\
&= 4Me^2 \int \frac{\mathrm{d}^2\vec{k}_\perp \, \mathrm{d}x}{16\pi^3} \frac{(m - xM)}{x(1-x)} \\
&\quad \times \frac{1}{M^2 - ((\vec{k}_\perp + (1-x)\vec{q}_\perp)^2 + m^2)/x - ((\vec{k}_\perp + (1-x)\vec{q}_\perp)^2 + \lambda^2)/(1-x)} \\
&\quad \times \frac{1}{M^2 - (\vec{k}^2_\perp + m^2)/x - (\vec{k}^2_\perp + \lambda^2)/(1-x)}. \qquad (30)
\end{aligned}
$$

$$
F_2(q^2) = \frac{Me^2}{4\pi^2} \int_0^1 \mathrm{d}\alpha \int_0^1 \mathrm{d}x \, \frac{m - xM}{\alpha(1-\alpha)\frac{1-x}{x}\vec{q}^{\,2}_\perp - M^2 + \frac{m^2}{x} + \frac{\lambda^2}{1-x}}.
$$

The anomalous moment is obtained in the limit of zero momentum transfer:

$$
\begin{aligned}
F_2(0) &= 4Me^2 \int \frac{\mathrm{d}^2\vec{k}_\perp \, \mathrm{d}x}{16\pi^3} \frac{(m - xM)}{x(1-x)} \frac{1}{[M^2 - (\vec{k}_\perp^{\,2} + m^2)/x - (\vec{k}_\perp^{\,2} + \lambda^2)/(1-x)]^2} \\
&= \frac{Me^2}{4\pi^2} \int_0^1 \mathrm{d}x \, \frac{m - xM}{-M^2 + \frac{m^2}{x} + \frac{\lambda^2}{1-x}},
\end{aligned} \tag{32}
$$

which is the result of Ref. [8]. For zero photon mass and $M = m$, it gives the correct order α Schwinger value $a_e = F_2(0) = \alpha/2\pi$ for the electron anomalous magnetic moment for QED.

The form factors of the energy–momentum tensor for a spin-$\frac{1}{2}$ composite are defined by

$$\langle P'|T^{\mu\nu}(0)|P\rangle = \bar{u}(P')\Big[A(q^2)\gamma^{(\mu}\overline{P}^{\nu)} + B(q^2)\frac{i}{2M}\overline{P}^{(\mu}\sigma^{\nu)\alpha}q_\alpha + C(q^2)\frac{1}{M}(q^\mu q^\nu - g^{\mu\nu}q^2)\Big]u(P), \quad (12)$$

where $q^\mu = (P' - P)^\mu$, $\overline{P}^\mu = \frac{1}{2}(P' + P)^\mu$, $a^{(\mu}b^{\nu)} = \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$, and $u(P)$ is the spinor of the system.

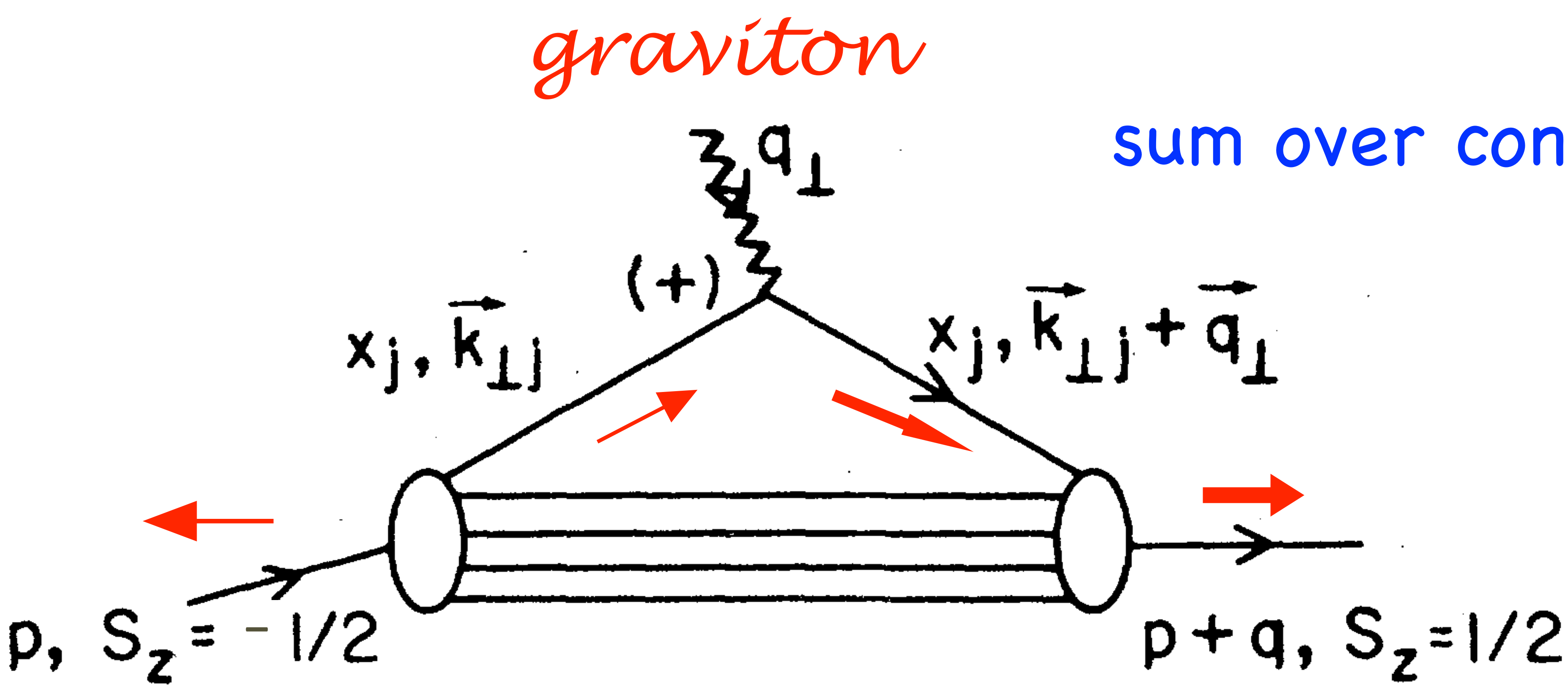
$$\Big\langle P+q,\uparrow\Big|\frac{T^{++}(0)}{2(P^+)^2}\Big|P,\uparrow\Big\rangle = A(q^2),$$

$$\Big\langle P+q,\uparrow\Big|\frac{T^{++}(0)}{2(P^+)^2}\Big|P,\downarrow\Big\rangle = -(q^1 - \mathrm{i}q^2)\frac{B(q^2)}{2M}.$$

$$\langle J^z\rangle = \Big\langle\frac{1}{2}\sigma^z\Big\rangle[A(0) + B(0)].$$

Vanishing Anomalous gravitomagnetic moment B(0)

Terayev, Okun, et al: *B(0) Must vanish because of Equivalence Theorem*



sum over constituents

Hwang, Schmidt, sjb; Holstein et al

B(0) = 0

Each Fock State

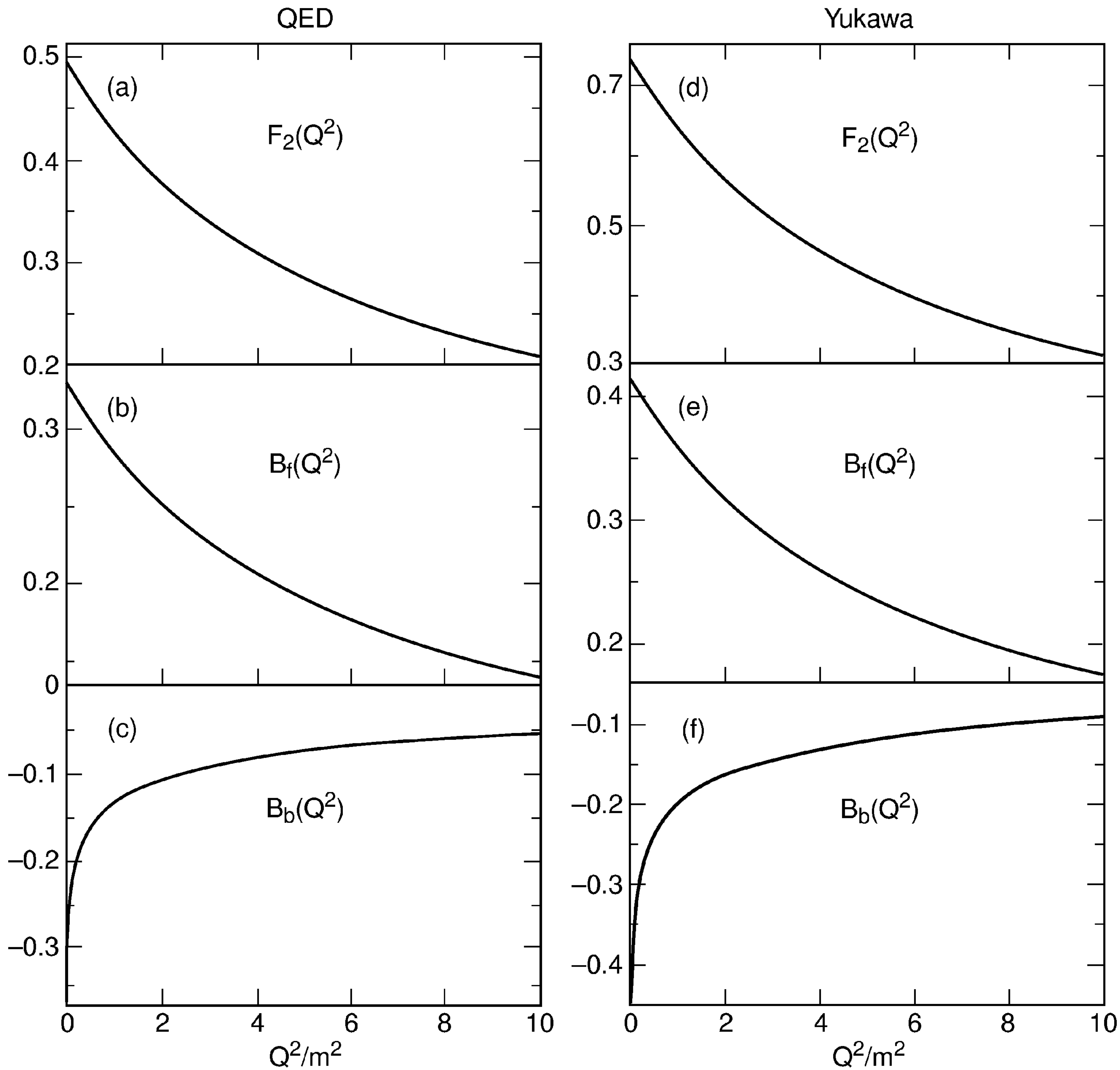
The total contribution for general momentum transfer is

$$
\begin{aligned}
B(q^2) &= B_{\mathrm{f}}(q^2) + B_{\mathrm{b}}(q^2) \\
&= 4Me^2 \int \frac{\mathrm{d}^2\vec{k}_\perp\,\mathrm{d}x}{16\pi^3} \frac{(m - xM)}{(1-x)} \\
&\quad \times \Bigg\{ \frac{1}{M^2 - ((\vec{k}_\perp + (1-x)\vec{q}_\perp)^2 + m^2)/x - ((\vec{k}_\perp + (1-x)\vec{q}_\perp)^2 + \lambda^2)/(1-x)} \\
&\qquad - \frac{1}{M^2 - ((\vec{k}_\perp - x\vec{q}_\perp)^2 + m^2)/x - ((\vec{k}_\perp - x\vec{q}_\perp)^2 + \lambda^2)/(1-x)} \Bigg\} \\
&\quad \times \frac{1}{M^2 - (\vec{k}_\perp^2 + m^2)/x - (\vec{k}_\perp^2 + \lambda^2)/(1-x)} \\
&= \frac{Me^2}{4\pi^2} \int_0^1 \mathrm{d}\alpha \int_0^1 \mathrm{d}x\, x(m - xM) \Bigg(\frac{1}{\alpha(1-\alpha)\frac{1-x}{x}\vec{q}_\perp^{\,2} - M^2 + \frac{m^2}{x} + \frac{\lambda^2}{1-x}} \\
&\qquad - \frac{1}{\alpha(1-\alpha)\frac{x}{1-x}\vec{q}_\perp^{\,2} - M^2 + \frac{m^2}{x} + \frac{\lambda^2}{1-x}} \Bigg).
\end{aligned}
\tag{39}
$$

This is the analog of the Pauli form factor for a physical electron scattering in a gravitational field and in general is not zero. However at zero momentum transfer

$$
B(0) = B_{\mathrm{f}}(0) + B_{\mathrm{b}}(0) = 0, \tag{40}
$$

in agreement with classical arguments based on the equivalence principle and conservation of the energy–momentum tensor [9,18–20].



QED
Yukawa
(a)
$F_2(Q^2)$
(b)
$B_f(Q^2)$
(c)
$B_b(Q^2)$
(d)
$F_2(Q^2)$
(e)
$B_f(Q^2)$
(f)
$B_b(Q^2)$
0.5
0.4
0.3
0.2
0.3
0.2
0
−0.1
−0.2
−0.3
0.7
0.5
0.3
0.4
0.3
0.2
−0.1
−0.2
−0.3
−0.4
0
2
4
6
8
10
Q^2/m^2
0
2
4
6
8
10
Q^2/m^2

Remarkable Advantages of the Front Form

- **Light-Front Time-Ordered Perturbation Theory: Elegant, Physical**

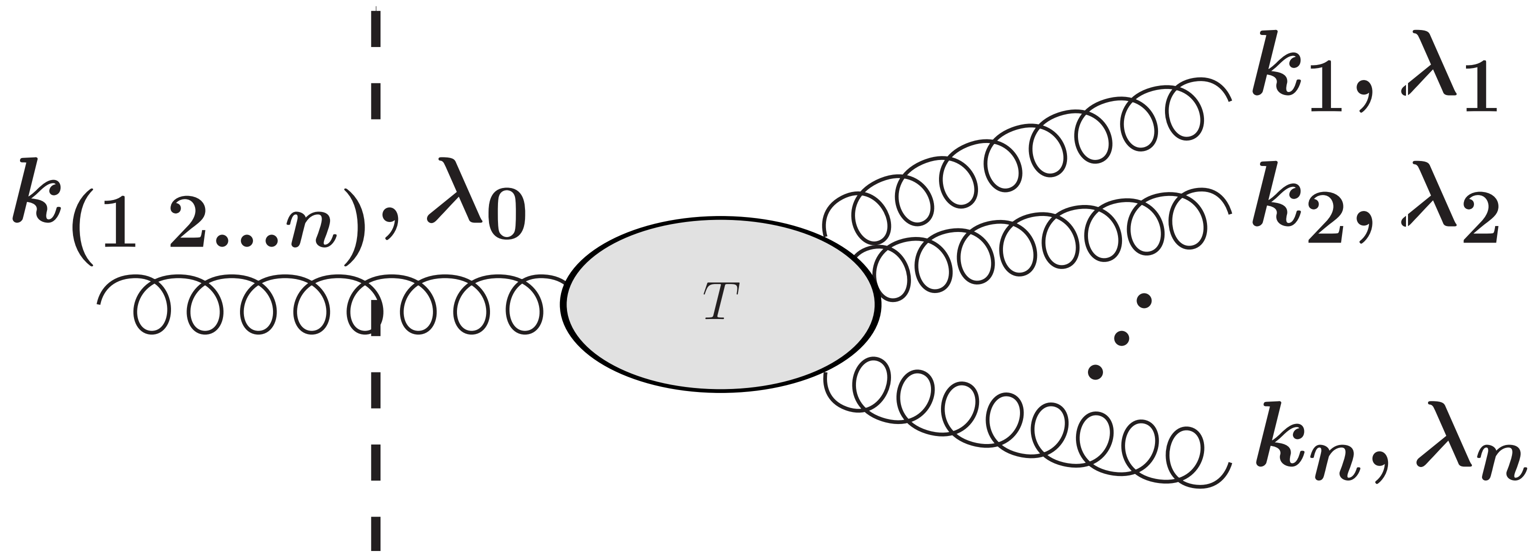
History

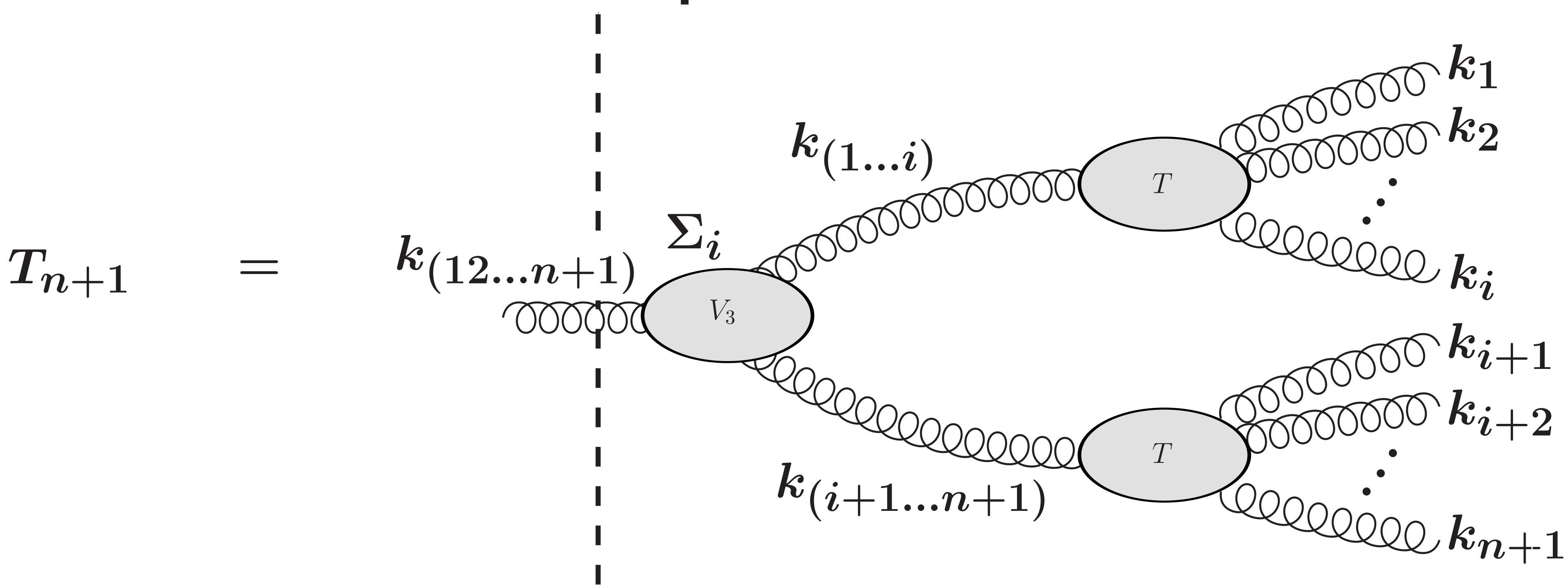
- **Frame-Independent**
- **Few LF Time-Ordered Diagrams (not n!) -- all k^+ must be positive**
- **$J^z = L^z + S^z$ conserved at each vertex**
- **Automatically normal-ordered; LF Vacuum trivial up to zero modes**
- **Renormalization: Alternate Denominator Subtractions: Tested to three loops in QED**
- **Reproduces Parke-Taylor Rules and Amplitudes (Stasto)**
- **Hadronization at the Amplitude Level with Confinement**

Recursion Relations and Scattering Amplitudes in the Light-Front Formalism

Cruz-Santiago & Stasto

Cluster Decomposition Theorem for relativistic systems: **C. Ji & sjb**





Parke-Taylor amplitudes reflect LF angular momentum conservation

$$\langle ij \rangle = \sqrt{z_i z_j}\, \underline{\epsilon}^{(-)} \cdot \left(\frac{\underline{k}_i}{z_i} - \frac{\underline{k}_j}{z_j} \right) =$$

Recursion relations and scattering amplitudes in the light-front formalism

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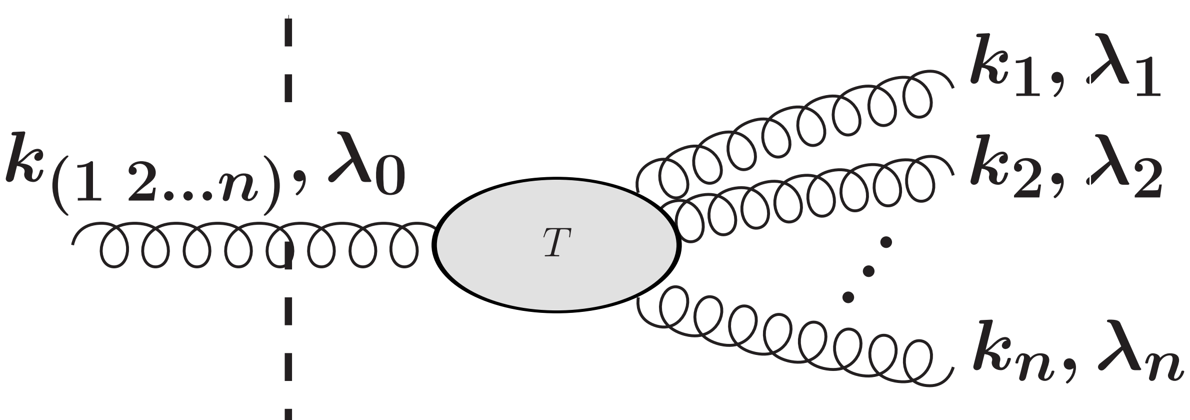
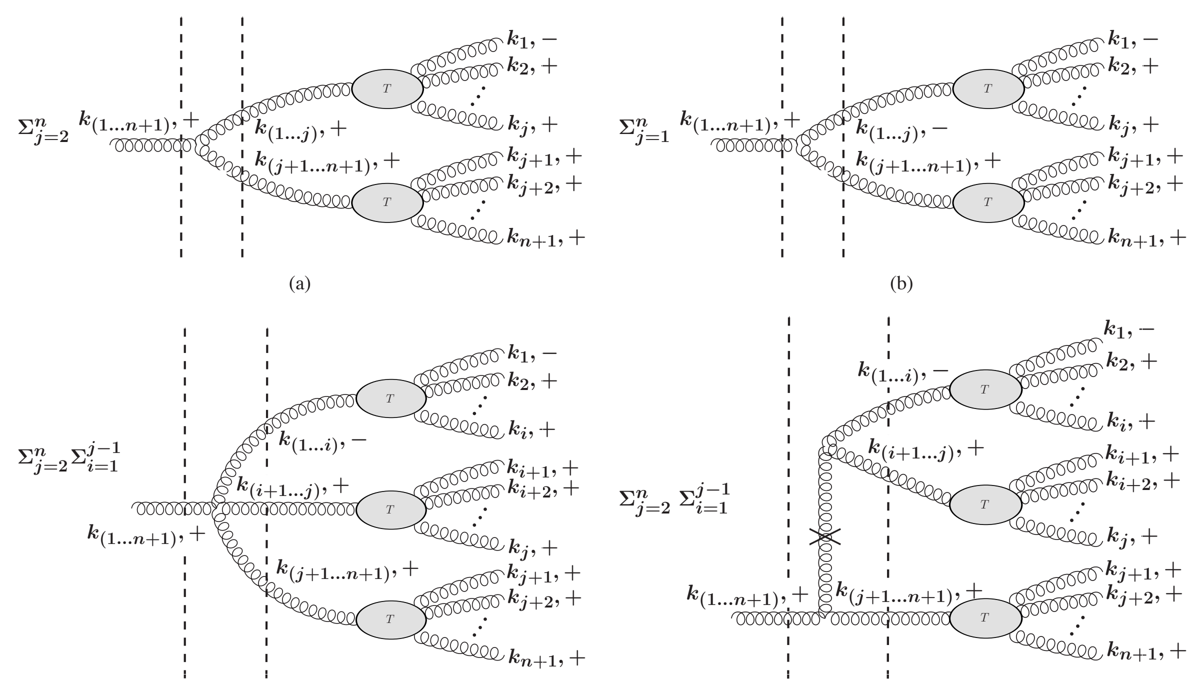
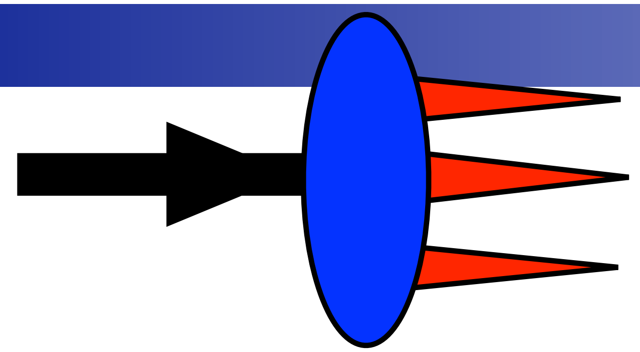
Abstract

The fragmentation functions and scattering amplitudes are investigated in the framework of light-front perturbation theory. It is demonstrated that, the factorization property of the fragmentation functions implies the recursion relations for the off-shell scattering amplitudes which are light-front analogs of the Berends-Giele relations. These recursion relations on the light-front can be solved exactly by induction and it is shown that the expressions for the off-shell light-front amplitudes are represented as a linear combinations of the on-shell amplitudes. By putting external particles on-shell we recover the scattering amplitudes previously derived in the literature.





- *Light Front Wavefunctions:*



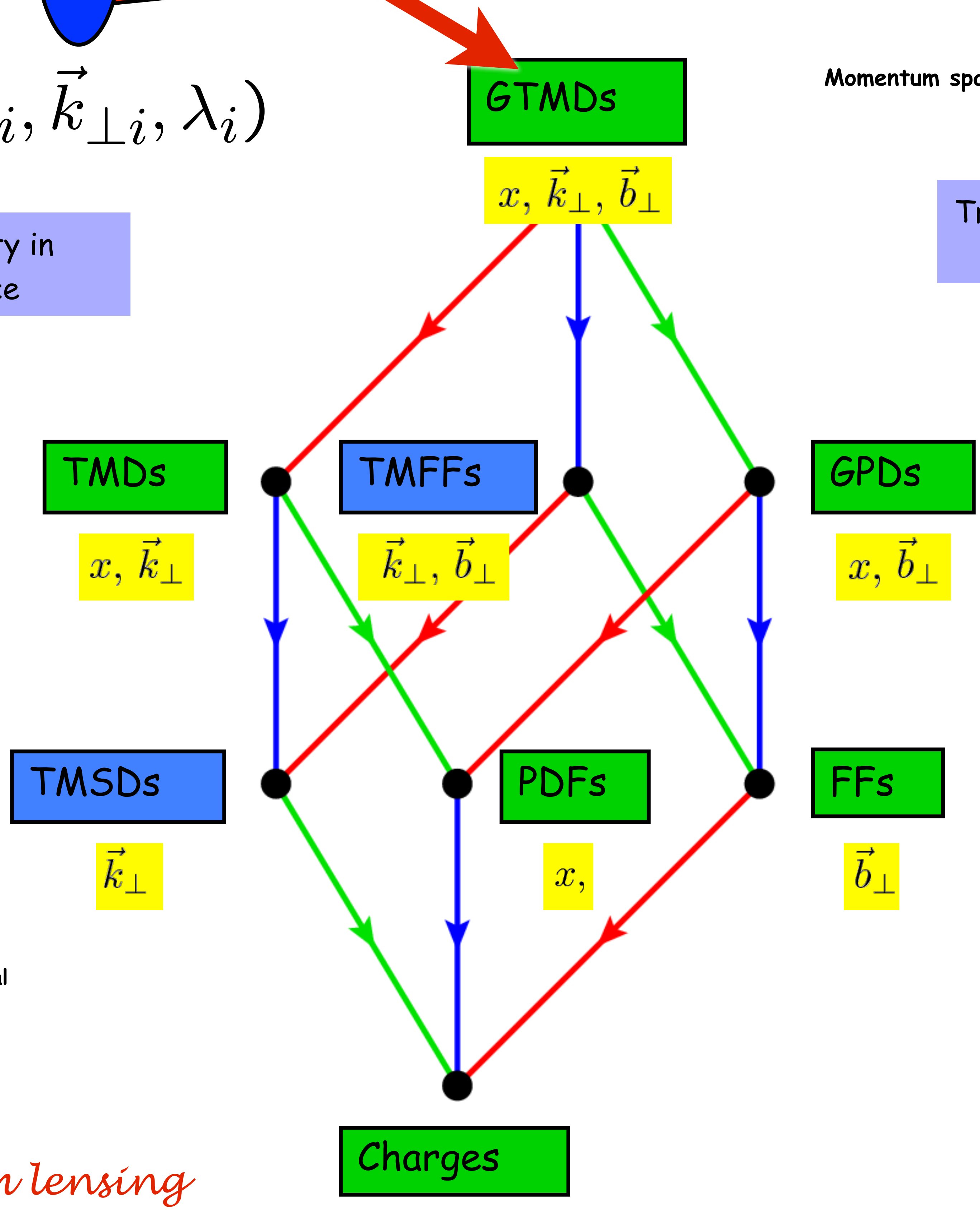
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

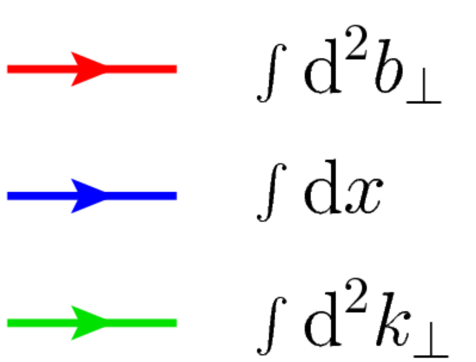
Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

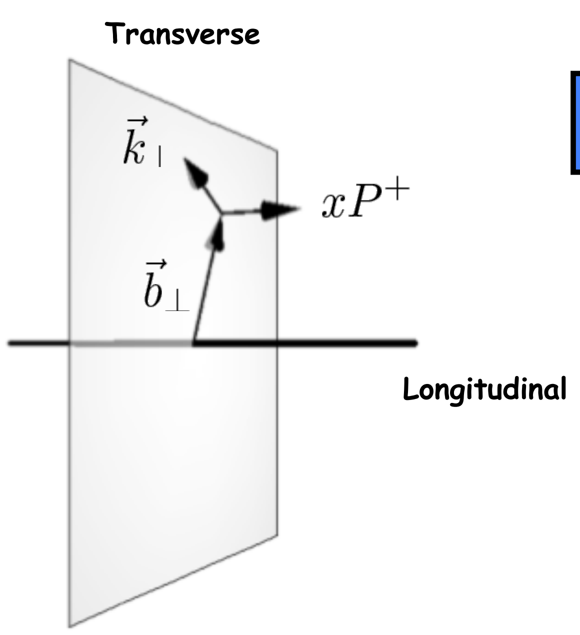
Transverse density in momentum space

Transverse density in position space





Lorce, Pasquini

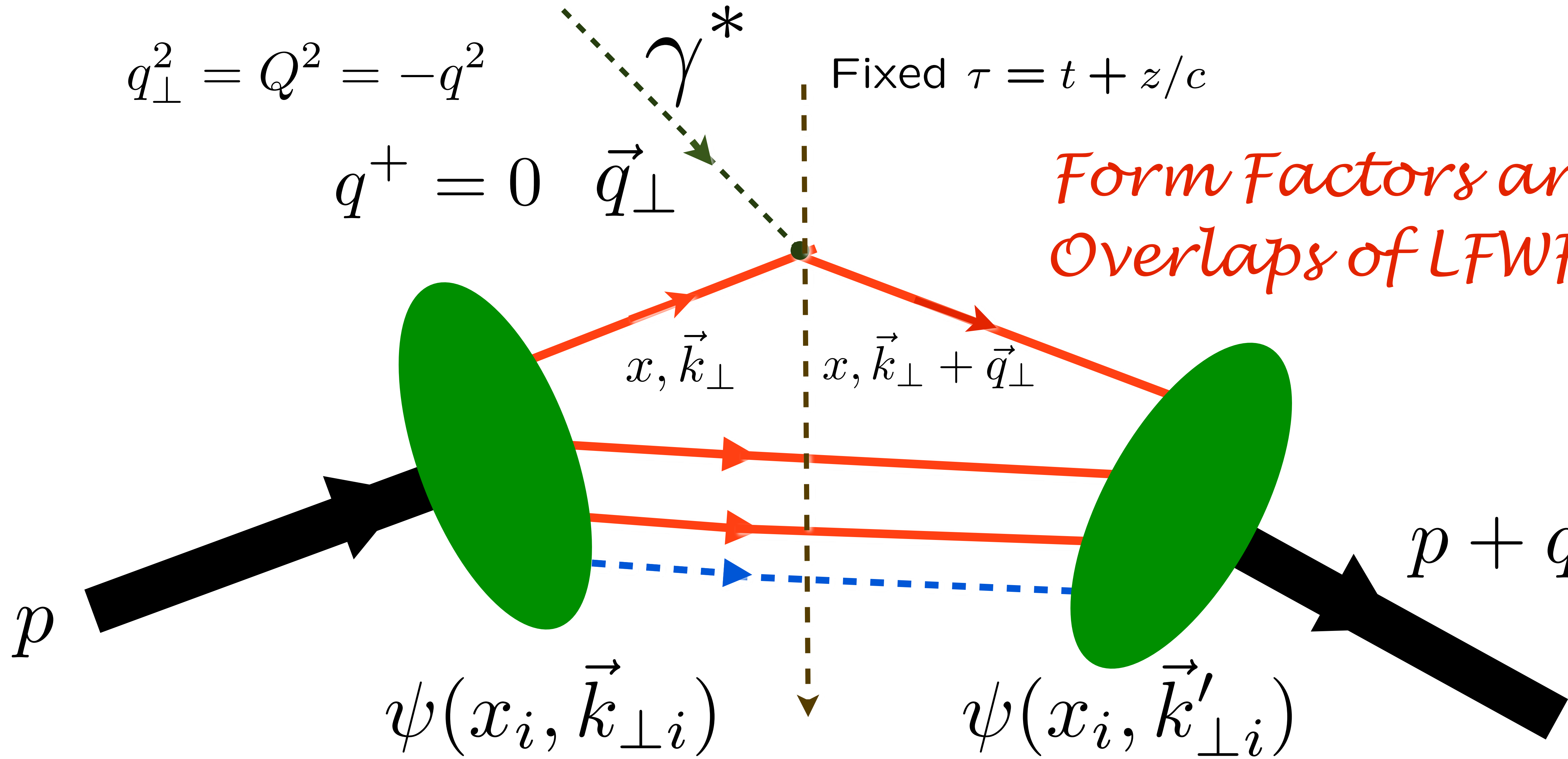


Sivers, T-odd from lensing

$$< p+q|j^+(0)|p > = 2p^+F(q^2)$$

Interaction picture

Form Factors are Overlaps of LFWFs



struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Exact LF Formula for Pauli Form Factor

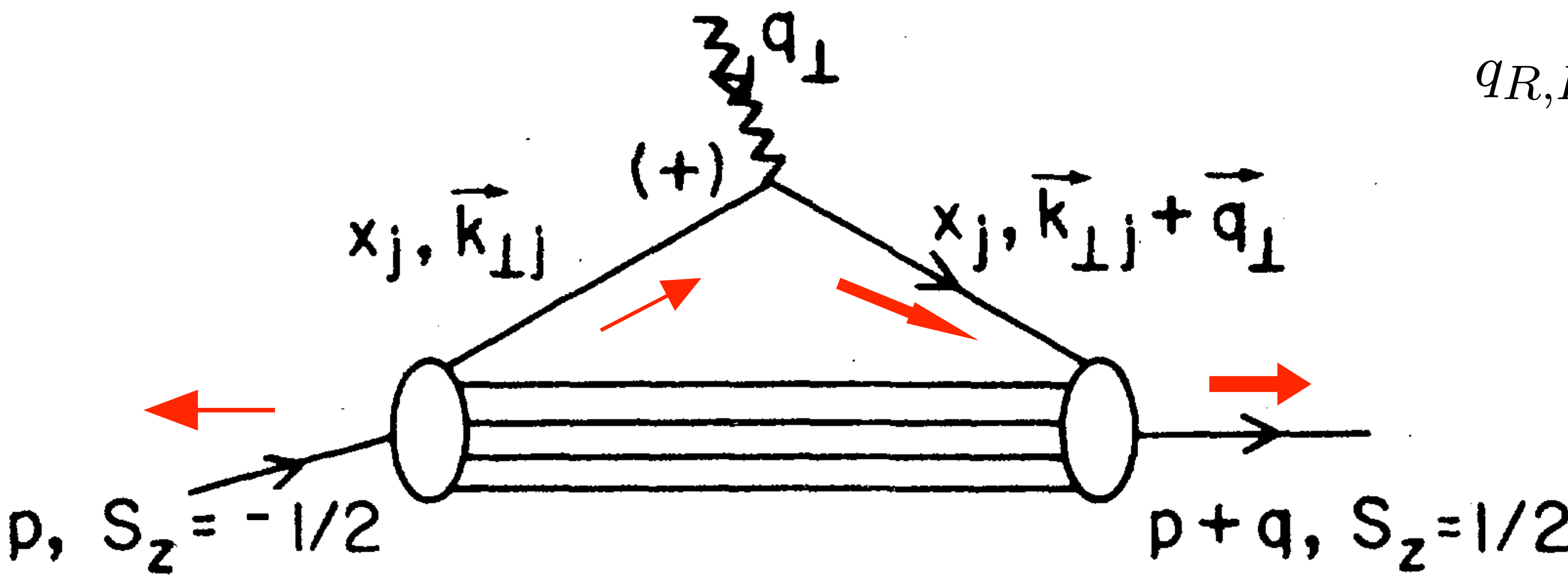
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$



Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Gravitational Form Factors

$$\langle P'|T^{\mu\nu}(0)|P\rangle \;=\; \overline{u}(P')\Big[\,A(q^2)\gamma^{(\mu}\overline{P}^{\nu)} + B(q^2)\frac{i}{2M}\overline{P}^{(\mu}\sigma^{\nu)\alpha}q_\alpha + C(q^2)\frac{1}{M}(q^\mu q^\nu - g^{\mu\nu}q^2)\Big]u(P)\;,$$

where $q^\mu = (P' - P)^\mu,\ \overline{P}^\mu = \frac{1}{2}(P' + P)^\mu,\ a^{(\mu}b^{\nu)} = \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$

$$\left\langle P+q,\uparrow\left|\frac{T^{++}(0)}{2(P^+)^2}\right|P,\uparrow\right\rangle = A(q^2)\;,$$

$$\left\langle P+q,\uparrow\left|\frac{T^{++}(0)}{2(P^+)^2}\right|P,\downarrow\right\rangle = -(q^1 - \mathrm{i}q^2)\frac{B(q^2)}{2M}\;.$$

$$\left|\psi_p(P^+, \vec{P}_\perp)\right\rangle = \sum_n \prod_{i=1}^{n} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{\sqrt{x_i}\,16\pi^3} 16\pi^3 \delta\left(1 - \sum_{i=1}^{n} x_i\right) \delta^{(2)}\left(\sum_{i=1}^{n} \vec{k}_{\perp i}\right)$$

$$\times\, \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)\left|n;\, x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\right\rangle.$$

$$q_{\lambda_q/\Lambda_p}(x, \Lambda) = \sum_{n, q_a} \int \prod_{j=1}^{n} \mathrm{d}x_j\, \mathrm{d}^2\vec{k}_{\perp j} \sum_{\lambda_i} \left|\psi^{(\Lambda)}_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i)\right|^2$$

$$\times\, \delta\left(1 - \sum_{i}^{n} x_i\right) \delta^{(2)}\left(\sum_{i}^{n} \vec{k}_{\perp i}\right) \delta(x - x_q)\delta_{\lambda_a \lambda_q} \Theta\left(\Lambda^2 - \mathcal{M}_n^2\right),$$

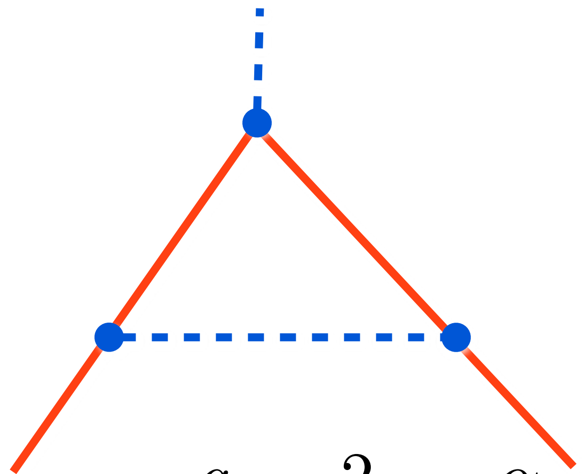
Obeys DGLAP Evolution *Defines quark distributions*

Connection to Bethe-Salpeter:

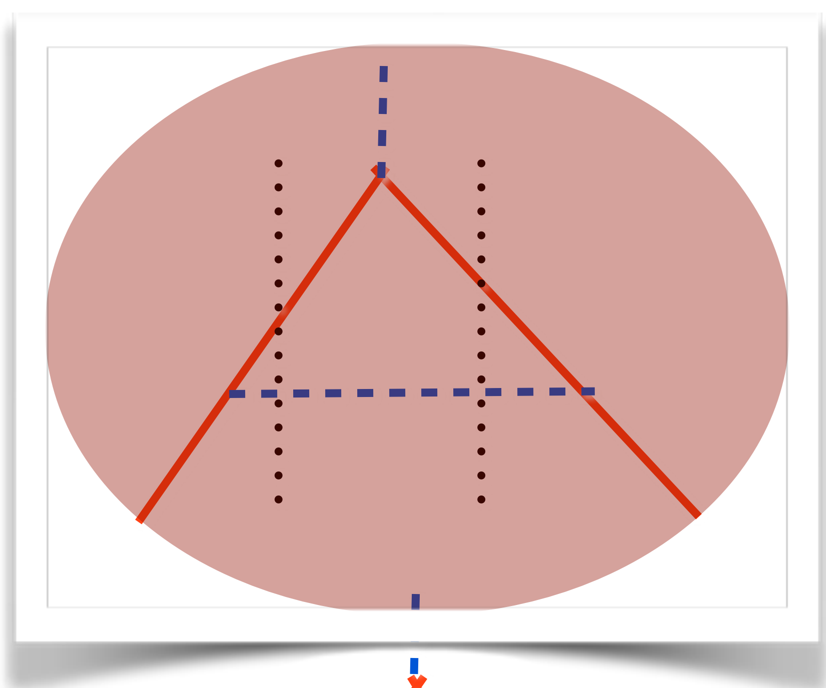
$$\int dk^- \Psi_{BS}(k, P) \rightarrow \psi_{LF}(x, \vec{k}_\perp) \qquad \Psi_{BS}(x, P)|_{x^+=0}$$

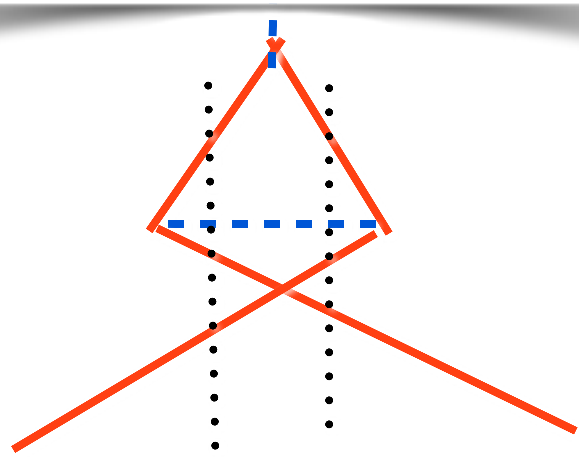
Wick Theorem

Feynman diagram =
single front-form time-ordered diagram!

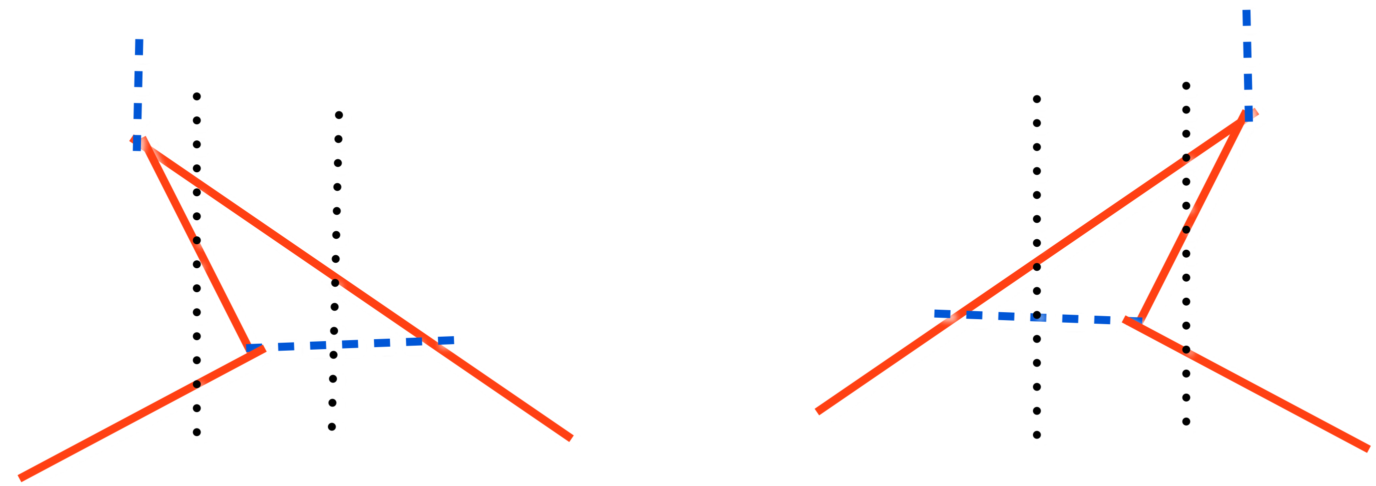


$$a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi}$$

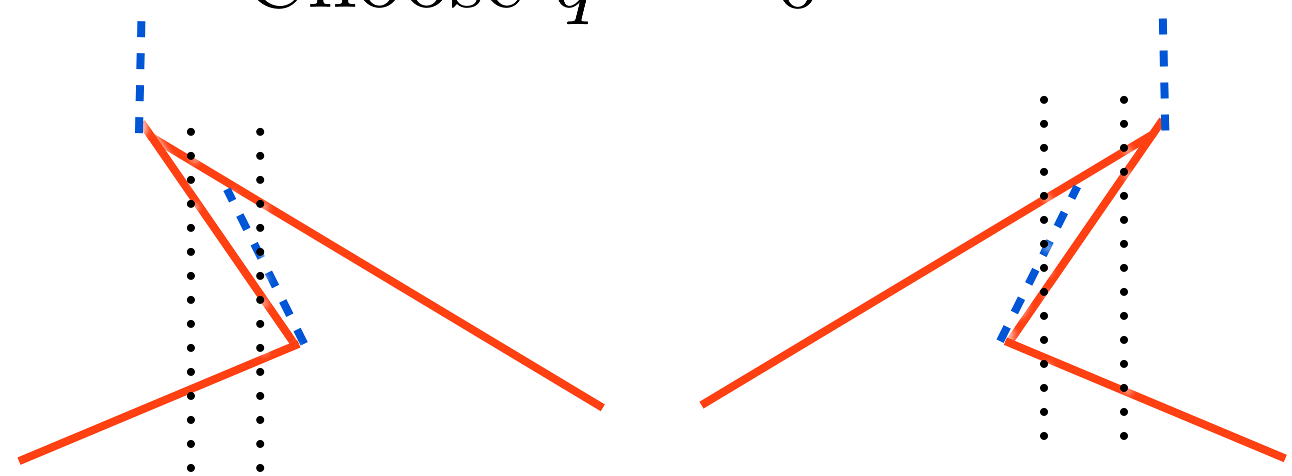




Also $P \to \infty$ observer frame (Weinberg)



Choose $q^+ = 0$



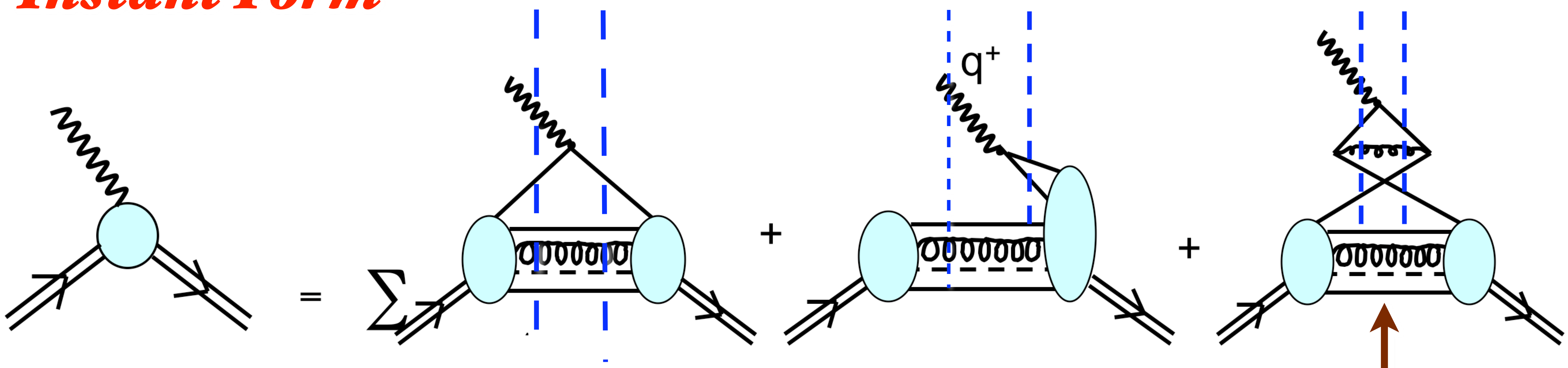
Light-Front QCD

Universidad Técnica
Federico Santa María

Stan Brodsky
SLAC
NATIONAL ACCELERATOR LABORATORY

Calculation of Form Factors in Equal-Time Theory

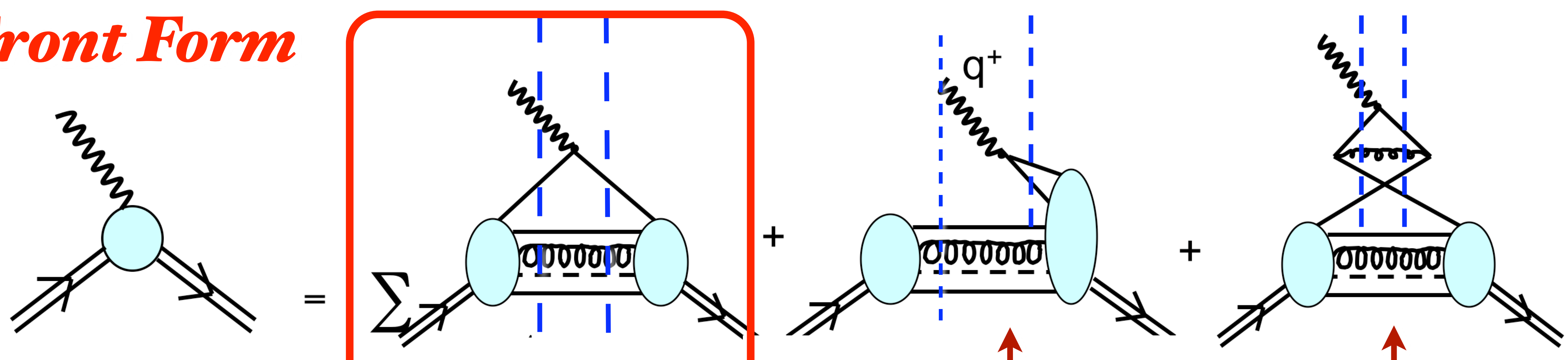
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

Front Form



Exact Answer!

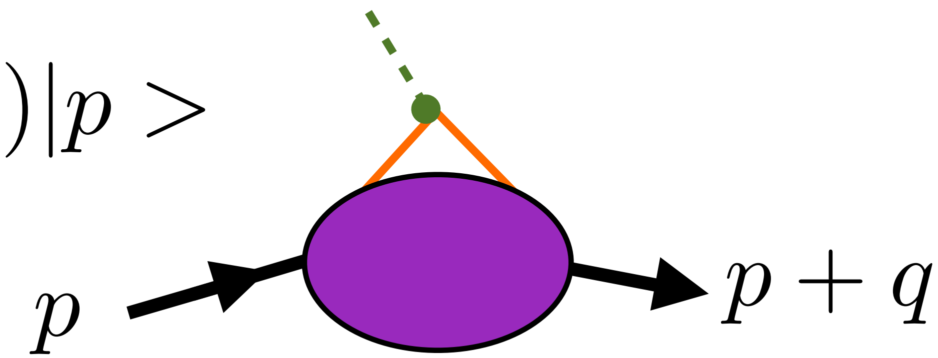
Absent for $q^+ = 0$

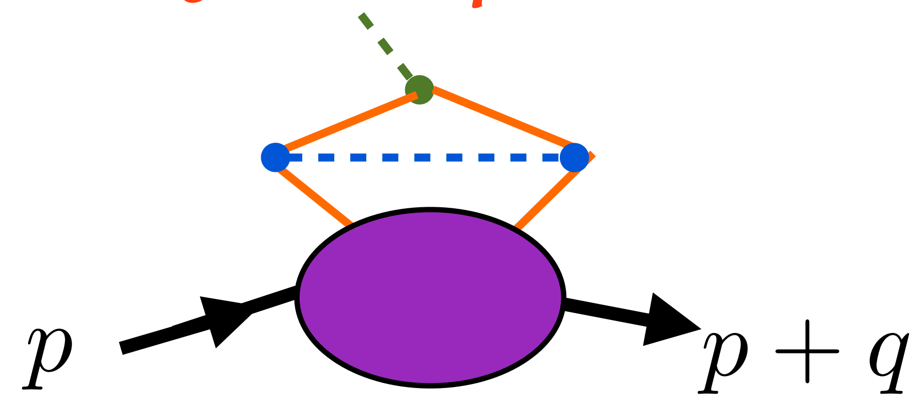
zero !!

No vacuum graphs

Calculation of proton form factor in Instant Form

$< p+q|J^{\mu}(0)|p >$





- **Need to boost proton wavefunction from p to p +q: Extremely complicated dynamical problem; even the particle number changes**
- **Need to couple to all currents arising from vacuum!! Remains even after normal-ordering**
- **Each time-ordered contribution is frame-dependent**
- **Divide by disconnected vacuum diagrams**
- **Instant form: acausal boundary conditions**

PHYSICAL REVIEW VOLUME 174, NUMBER 5 25 OCTOBER 1968

Electromagnetic Interactions of Loosely-Bound Composite Systems*

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Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 13 June 1968)

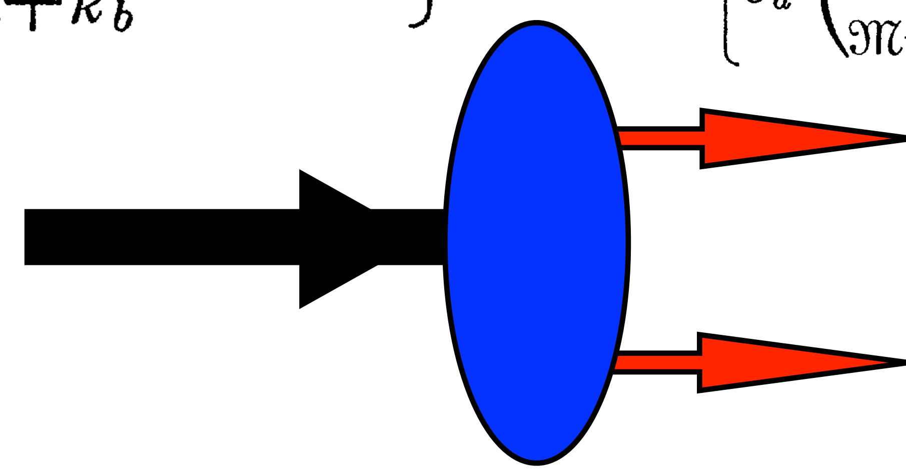
Contrary to popular assumption, the interaction of a composite system with an external electromagnetic field is not equal to the sum of the individual Foldy-Wouthyusen interactions of the constituents if the constituents have spin. We give the correct interaction, and note that it is consistent with the Drell-Hearn-Gerasimov sum rule and the low-energy theorem for Compton scattering. We also discuss the validity of additivity of the individual Dirac interactions, and the corrections to this approximation, with particular reference to the atomic Zeeman effect, which is of importance in the fine-structure and Lamb-shift measurements.

Dynamical boost contribution

$$\vec{P} \neq 0$$

$$\begin{Bmatrix} 1 \\ \dfrac{1}{2m_a+k_a}\boldsymbol{\sigma}_a\cdot\mathbf{p} \end{Bmatrix} \otimes \begin{Bmatrix} 1 \\ \dfrac{1}{2m_b+k_b}\boldsymbol{\sigma}_b\cdot(-\mathbf{p}) \end{Bmatrix} \Rightarrow \begin{Bmatrix} 1+\dfrac{\boldsymbol{\sigma}_a\cdot\mathbf{P}}{\mathfrak{M}+E}\dfrac{\boldsymbol{\sigma}_a\cdot\mathbf{p}}{2m_a+k_a} \\ \boldsymbol{\sigma}_a\cdot\left(\dfrac{\mathbf{P}}{\mathfrak{M}+E}+\dfrac{\mathbf{p}}{2m_a+k_a}\right) \end{Bmatrix} \otimes \begin{Bmatrix} 1-\dfrac{\boldsymbol{\sigma}_b\cdot\mathbf{P}}{\mathfrak{M}+E}\dfrac{\boldsymbol{\sigma}_b\cdot\mathbf{p}}{2m_b+k_b} \\ \boldsymbol{\sigma}_b\cdot\left(\dfrac{\mathbf{P}}{\mathfrak{M}+E}-\dfrac{\mathbf{p}}{2m_b+k_b}\right) \end{Bmatrix}$$

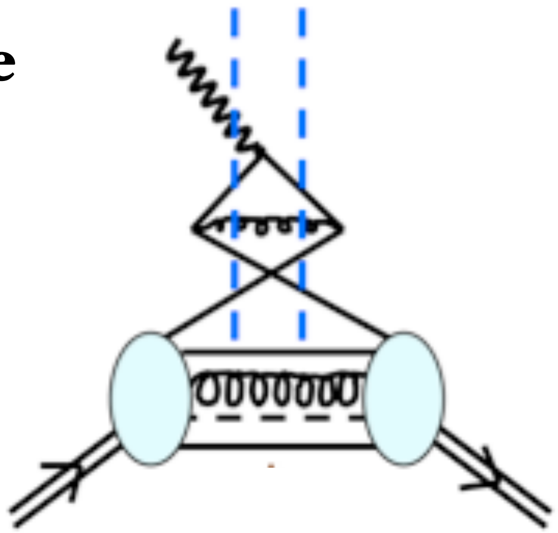
Instant Form WF



Also: Hugh Osborne

Disadvantages of the Instant Form

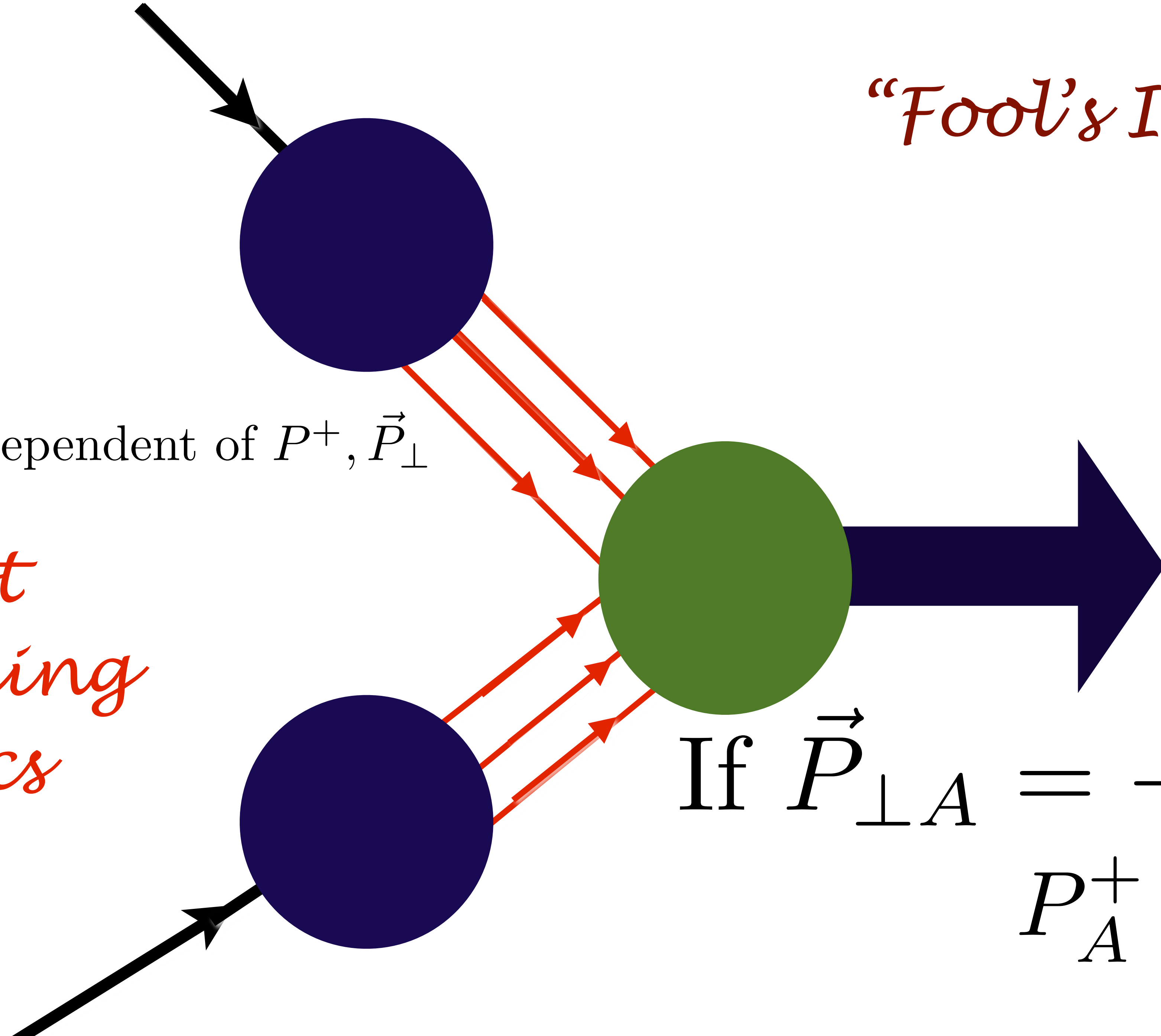
- **Boosts are dynamical, change particle number: not Melosh!**
- **Famous wrong proof showing violation of LET and DHG sum rule**
- **Each Amplitude is Frame-Dependent**
- **States defined at one instant of time over all space - acausal!**
- **Current matrix elements involve connected vacuum currents -- eigensolutions insufficient!**
- **N! time-ordered graphs, each frame-dependent**
- **Vacuum is complex: apparently gives huge vacuum energy density**
- **Normal-ordering required to compute observables**
- **Cluster decomposition theorem fails in relativistic systems**
- **Virtually no valid calculations of dynamics of relativistic composite systems use the instant form**
- **Why Feynman invented Feynman diagrams!**



$$P_A^\mu = (P_A^+, P_A^-, \vec{P}_{\perp A})$$

$$P^- = \frac{P_\perp^2 + M^2}{P^+}$$

"Fool's ISR Frame"



$\psi(x, k_\perp)$ independent of $P^+, \vec{P}_\perp$

Not colliding discs

If $\vec{P}_{\perp A} = -\vec{P}_{\perp B} = \vec{P}_\perp$

$$P_A^+ = P_B^+$$

$$s \simeq 4P_\perp^2$$

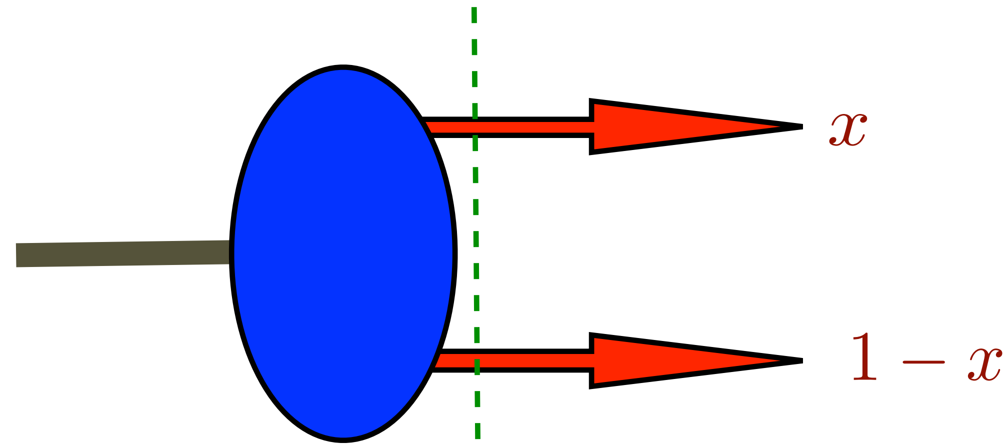
$$P_B^\mu = (P_B^+, P_B^-, \vec{P}_{\perp B})$$

$$s = (P_A + P_B)^2 = M_A^2 + M_B^2 + \frac{P_{\perp A}^2 + M_A^2}{P_A^+} P_B^+ + \frac{P_{\perp B}^2 + M_B^2}{P_B^+} P_A^+ - 2\vec{P}_{\perp A} \cdot \vec{P}_{\perp B}$$

Hadron Distribution Amplitudes

$A^+ = 0$

$$\phi_M(x,Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x,\vec{k}_\perp)$$



$k_\perp^2 < Q^2$

$$\sum_i x_i = 1$$

Fixed $\tau = t + z/c$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

ERBL

- Evolution Equations from PQCD, OPE
- Conformal Expansions
- Compute from valence light-front wavefunction in light-cone gauge

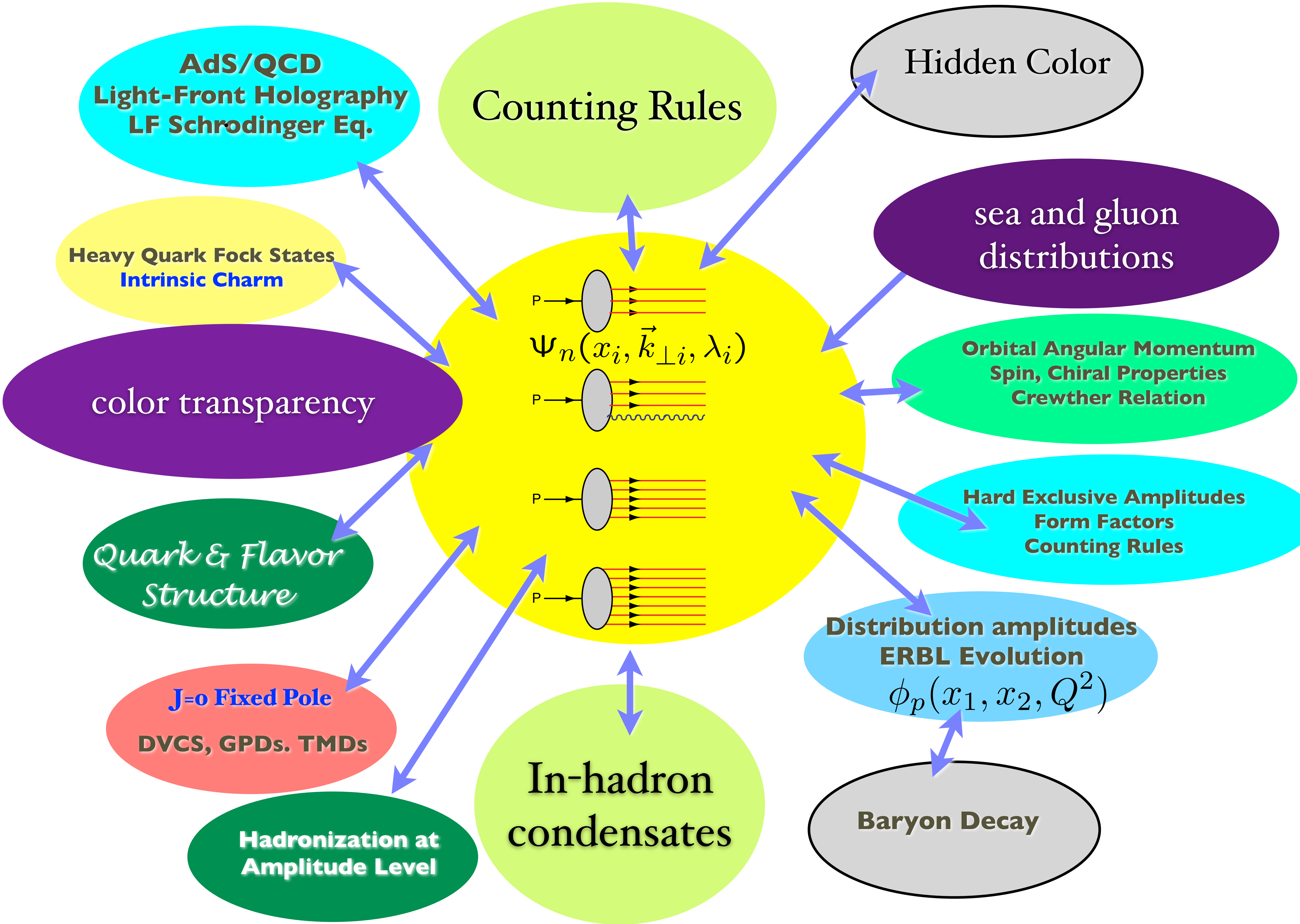
Lepage, sjb

Efremov, Radyushkin

Sachrajda, Frishman Lepage, sjb

Braun, Gardi

QCD and the LF Hadron Wavefunctions

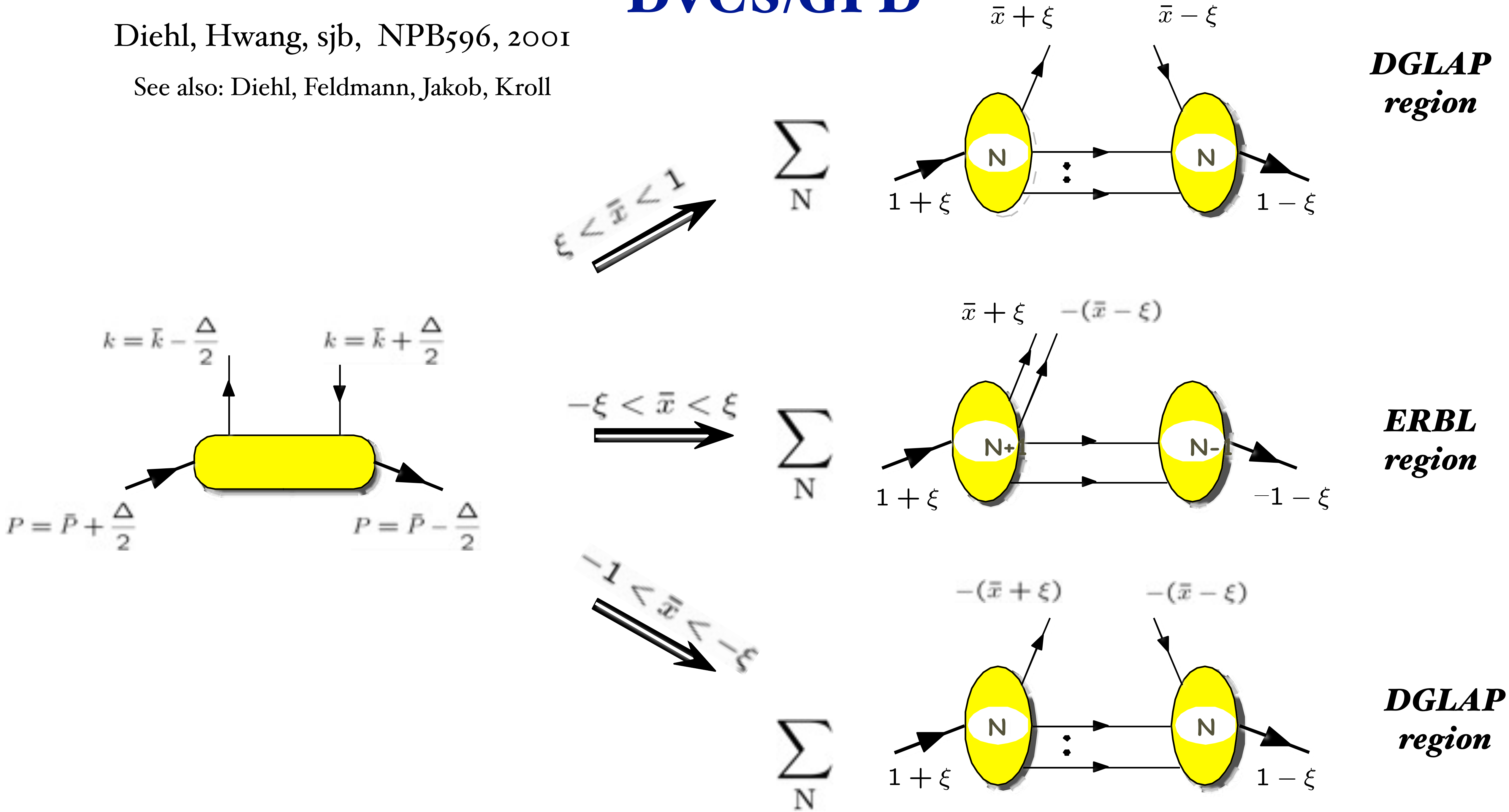


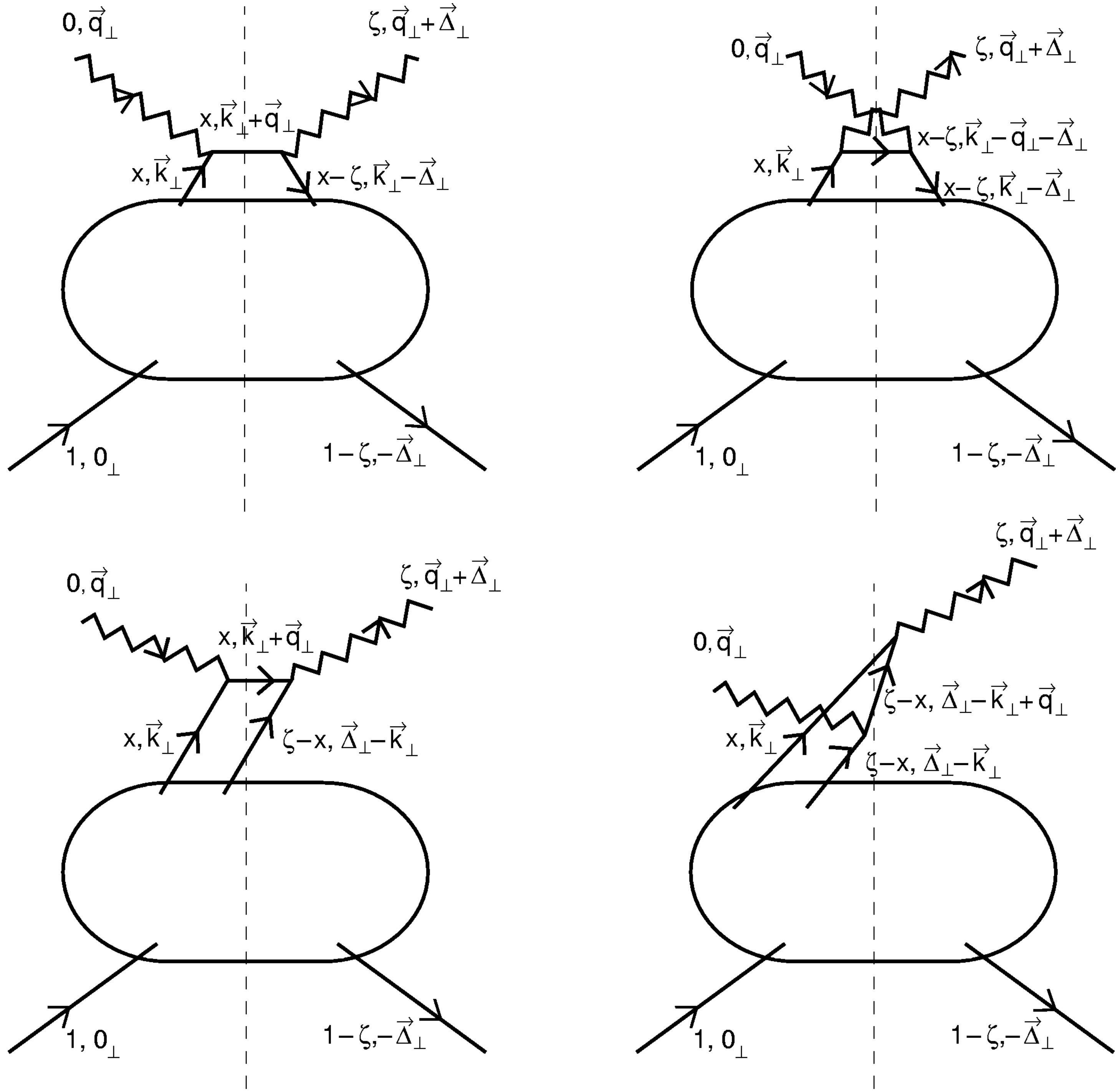
Light-Front Wave Function Overlap Representation

DVCS/GPD

Diehl, Hwang, sjb, NPB596, 2001

See also: Diehl, Feldmann, Jakob, Kroll





Light-front wavefunctions representation of deeply virtual Compton scattering

Stanley J. Brodsky [a], Markus Diehl [a,1], Dae Sung Hwang [b]

Universidad Técnica
Federico Santa María

Light-Front QCD

Stan Brodsky
SLAC
NATIONAL ACCELERATOR LABORATORY

Example of LFWF representation of GPDs (n => n)

Diehl, Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}} \frac{\Delta^1 - i\,\Delta^2}{2M} E_{(n\to n)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{2-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\, 16\pi^3 \delta\left(1-\sum_{j=1}^{n} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n} \vec{k}_{\perp j}\right)$$

$$\times\, \delta(x-x_1)\, \psi^{\uparrow *}_{(n)}(x'_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi^{\downarrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i),$$

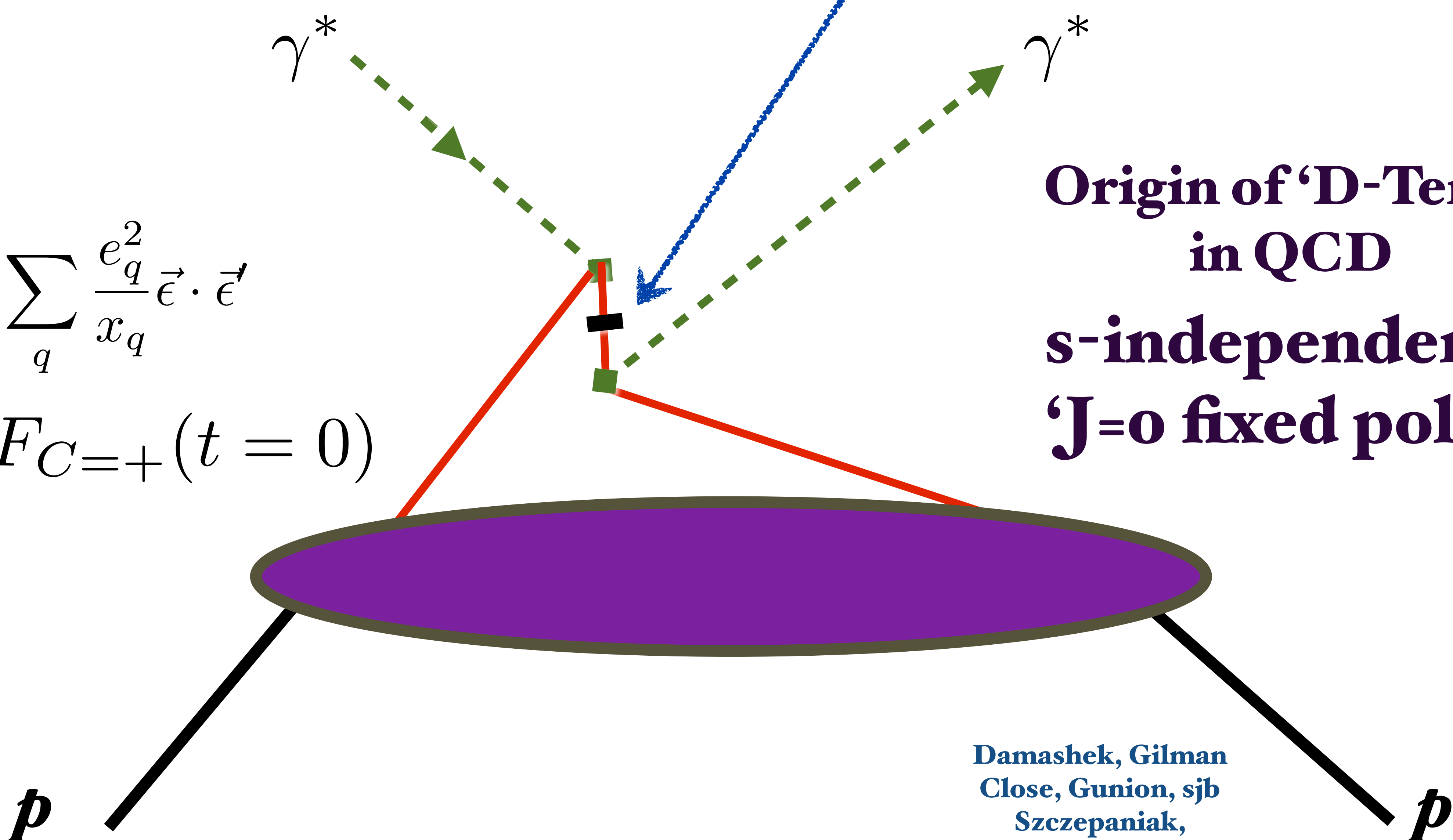
where the arguments of the final-state wavefunction are given by

$$x'_1 = \frac{x_1-\zeta}{1-\zeta}, \qquad \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{1-x_1}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the struck quark,}$$

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the spectators } i = 2, \ldots, n.$$

Leading-Twist Contribution to Real Part of DVCS

LF Instantaneous interaction



$$T = -2\sum_q \frac{e_q^2}{x_q}\vec{\epsilon}\cdot\vec{\epsilon}'$$

$$T \propto s^0 F_{C=+}(t=0)$$

Origin of 'D-Term' in QCD

s-independent 'J=0 fixed pole'

Damashek, Gilman
Close, Gunion, sjb
Szczepaniak,
Llanes Estrada, sjb

Single-spin asymmetries

Leading Twist Sivers Effect

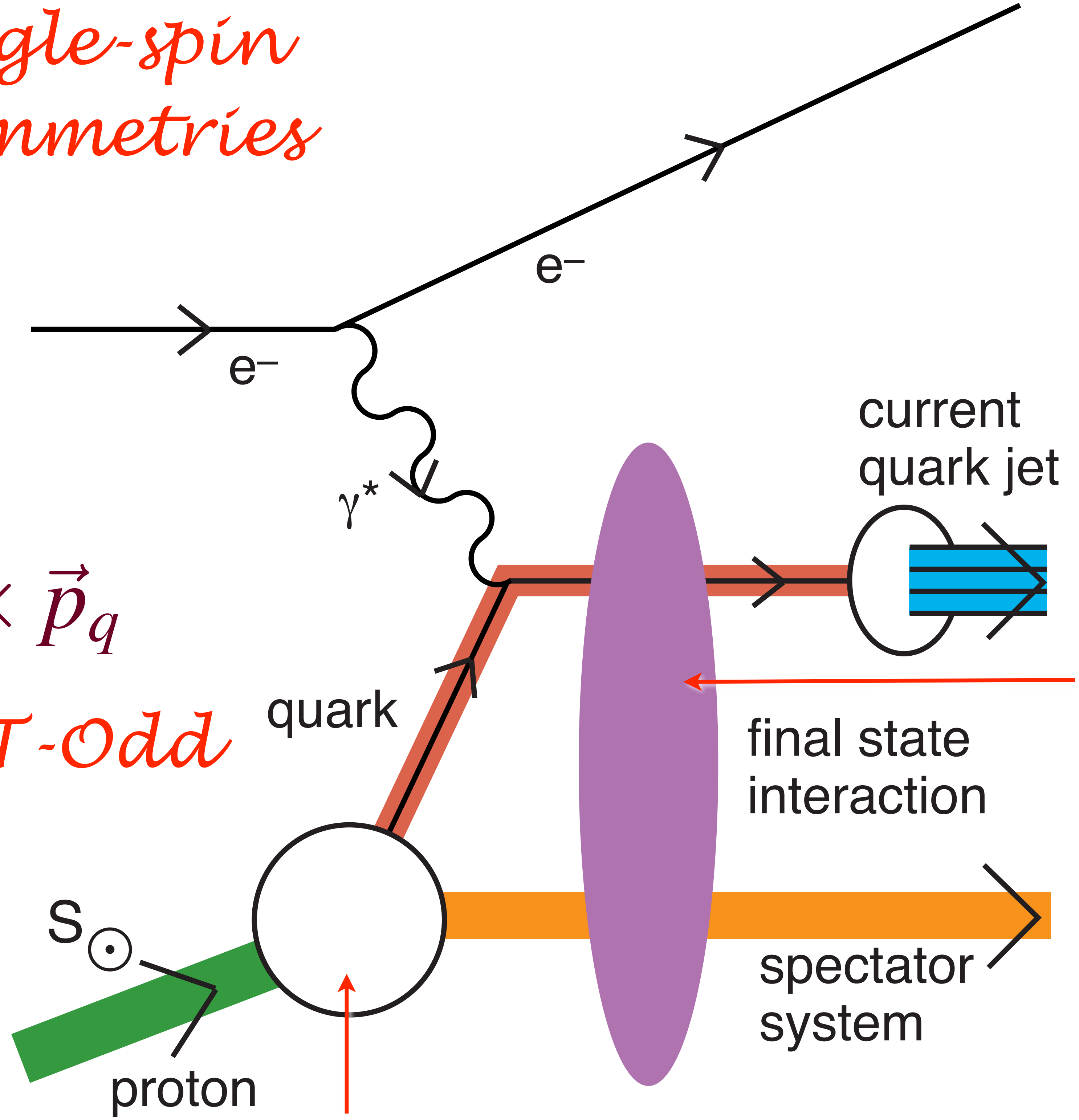
Hwang, Schmidt, sjb

Collins, Burkardt, Ji, Yuan. Pasquini, ...

$$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo- T-Odd

QED: Lensing involves soft scales



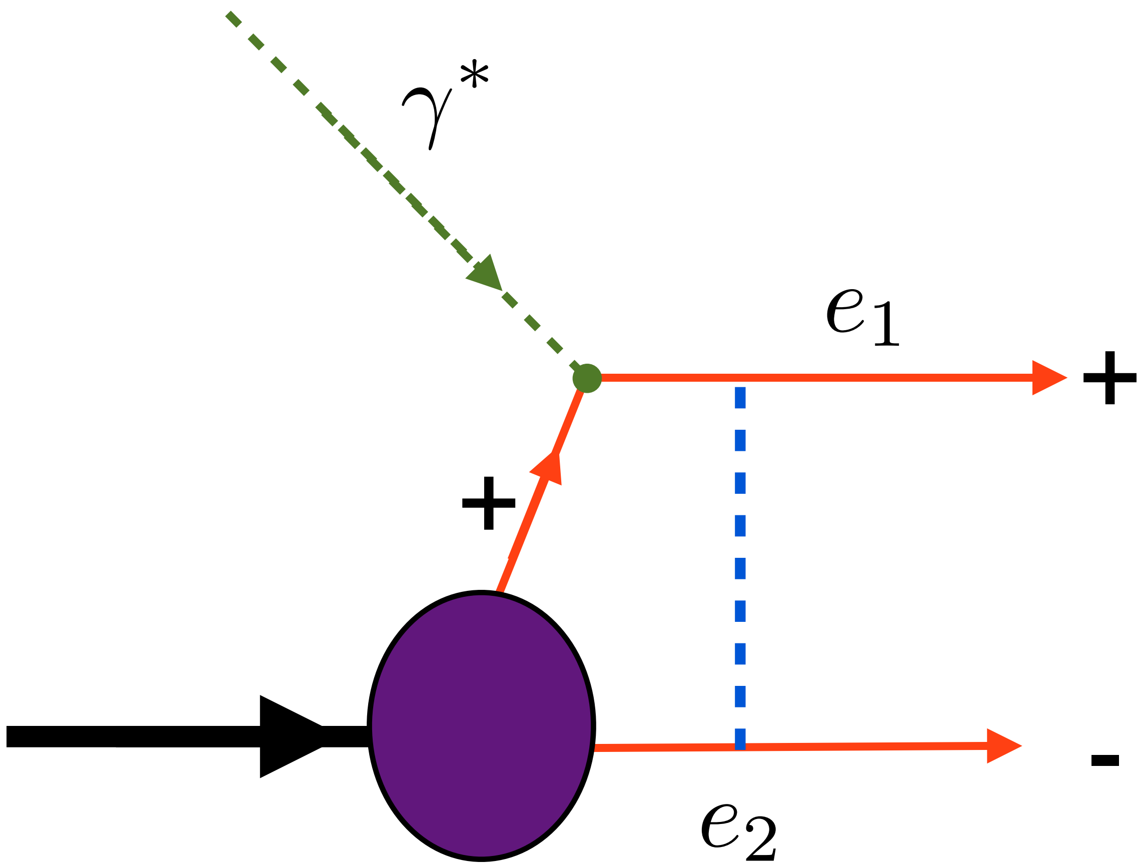
Light-Front Wavefunction S and P- Waves!

QCD S- and P- Coulomb Phases --Wilson Line

"Lensing Effect"

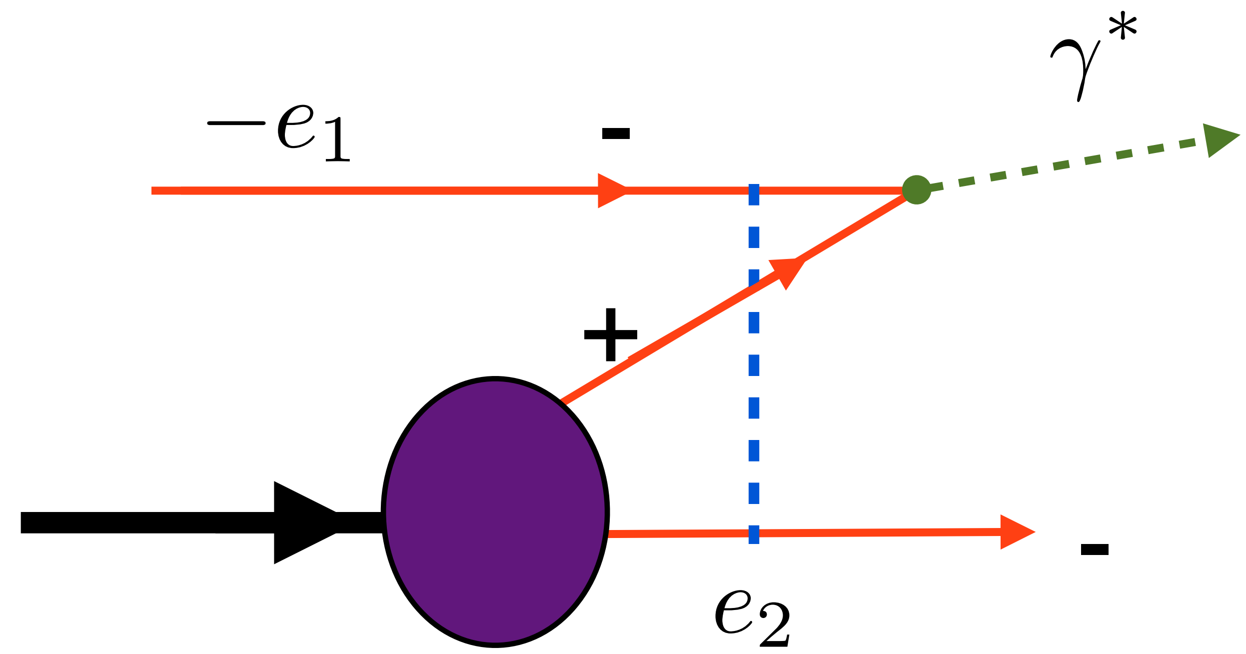
Leading-Twist Rescattering Violates pQCD Factorization!

Sign reversal in DY!



DIS

Attractive, opposite-sign rescattering potential



DY

Repulsive, same-sign scattering potential

Dae Sung Hwang, Yuri V. Kovchegov, Ivan Schmidt, Matthew D. Sievert, sjb

Static	Dynamic
Square of Target LFWFs	Modified by Rescattering: ISI & FSI
No Wilson Line	Contains Wilson Line, Phases
Probability Distributions	No Probabilistic Interpretation
Process-Independent	Process-Dependent - From Collision
T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
Sum Rules: Momentum and J^z	Sum Rules Not Proven
DGLAP Evolution; mod. at large x	DGLAP Evolution
No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

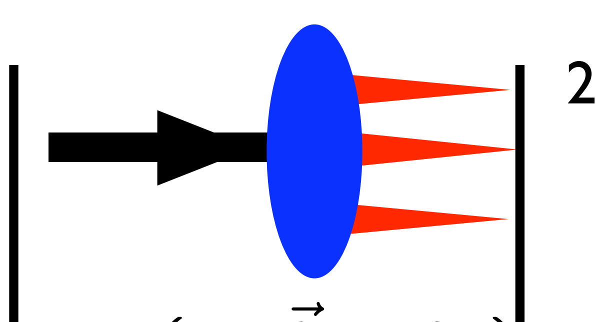
Hwang, Schmidt, sjb,

Mulders, Boer

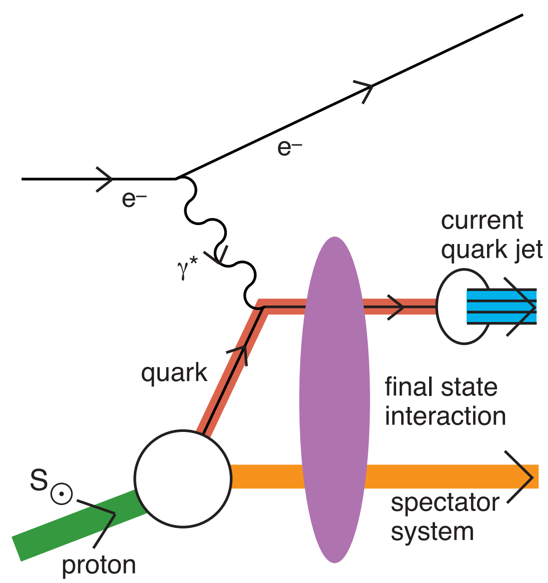
Qiu, Sterman

Collins, Qiu

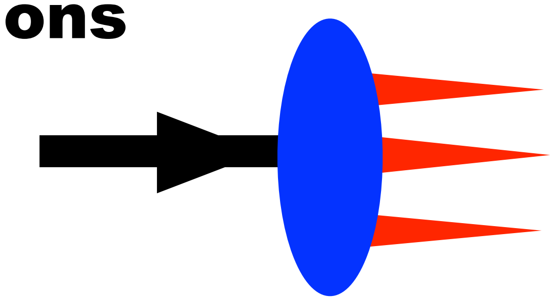
Pasquini, Xiao, Yuan, sjb



$$\left| \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \right|^2$$



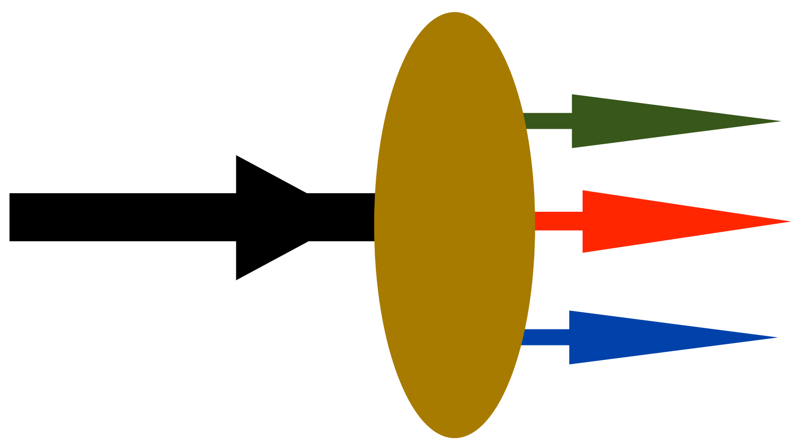
- LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

- LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian
- Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors
- Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, ... modulo `lensing' from ISIs, FSIs
- Cannot compute current matrix elements using instant form from eigensolutions alone -- need to include vacuum currents!
- Hadron Physics without LFWFs is like Biology without DNA!

- *Hadron Physics without LFWFs is like Biology without DNA!*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

No length contraction — no pancakes!

Penrose Terrell Weiskopf

Goal: an analytic first approximation to QCD

- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- Confinement in QCD -- What sets the QCD mass scale?
- QCD Coupling at all scales
- Hadron Spectroscopy
- Light-Front Wavefunctions
- Form Factors, Structure Functions,Hadronic Observables
- Constituent Counting Rules
- Hadronization at the Amplitude Level
- Insights into QCD Condensates
- Chiral Symmetry
- Systematically improvable

Light-Front: Universal Tool for atoms, nuclei, hadrons

- LFWFs are Frame Independent
- No colliding pancakes
- One-dimensional *Light-Front Schrödinger Equation*
- Precision QED; Atoms in flight
- Avoid dynamical boosts
- Avoid vacuum currents!
- Angular momentum conservation
- Goal: Hadronization at amplitude level

Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

Origin of hadronic mass scale if $m_q=0$

Atomic Physics from First Principles

$$\mathcal{L}_{QED} \rightarrow H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi> = E\ |\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S}, \vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r, S, \ell)]\ \psi(r) = E\ \psi(r)$$

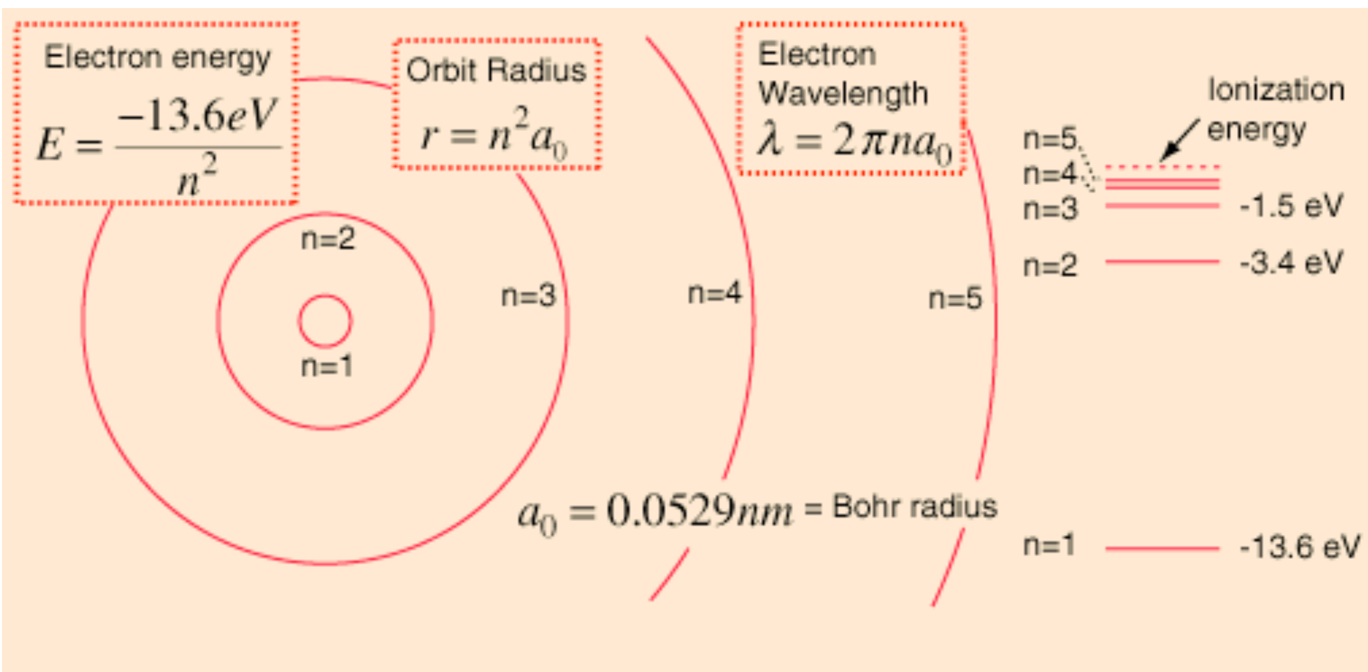
Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

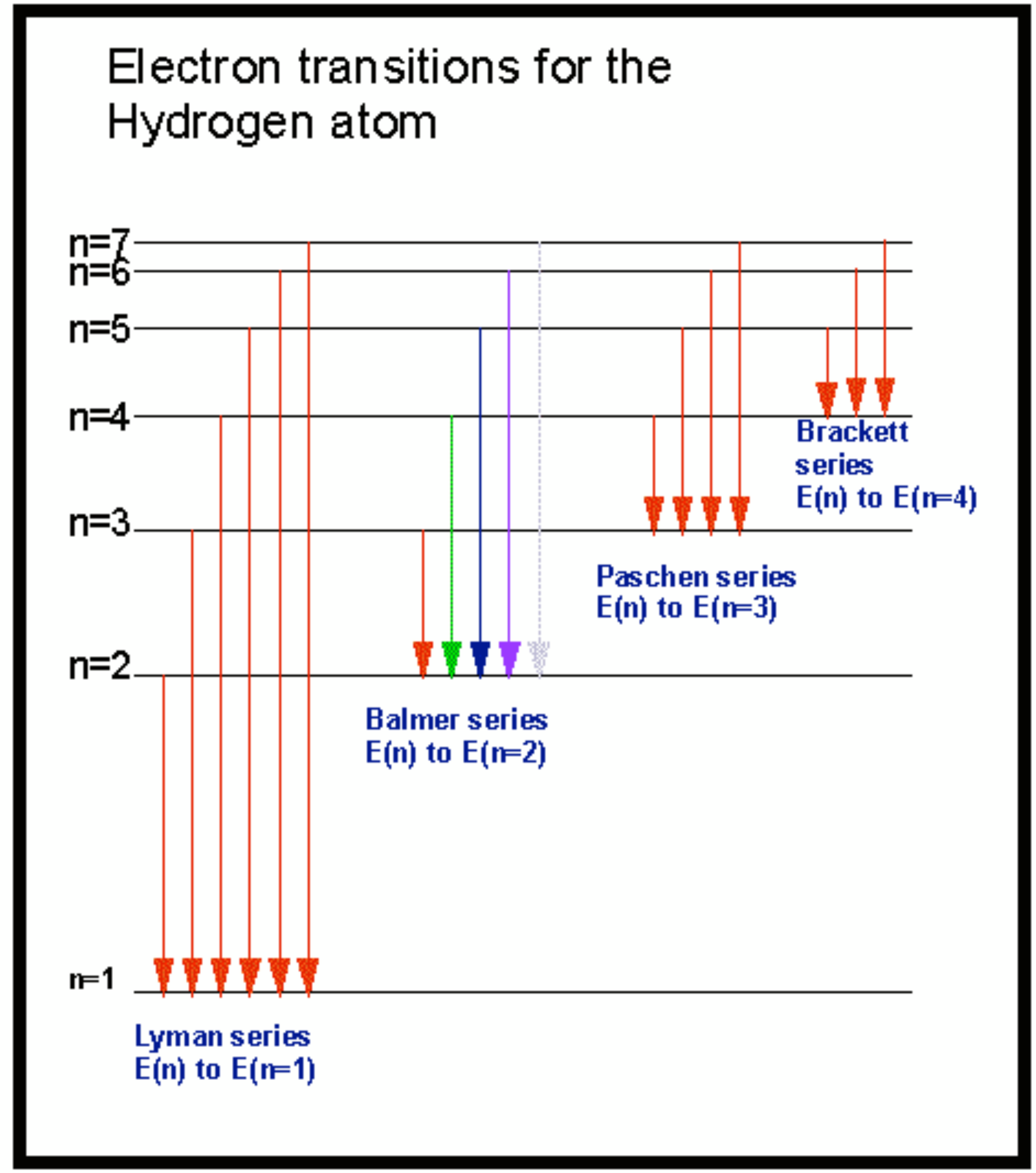
Coulomb potential

Semiclassical first approximation to QED --> **Bohr Spectrum**

Bohr Atom



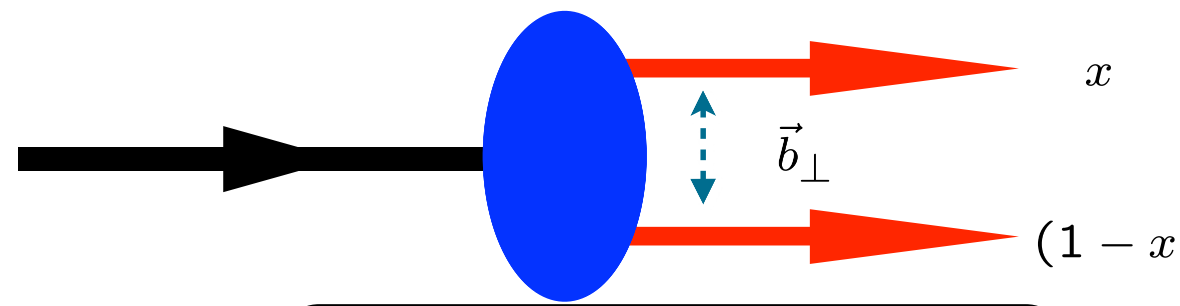
Electron energy
$E = \frac{-13.6eV}{n^2}$
Orbit Radius
$r = n^2 a_0$
Electron Wavelength
$\lambda = 2\pi n a_0$
Ionization energy
n=5
n=4
n=3
-1.5 eV
n=2
-3.4 eV
n=1
-13.6 eV
n=2
n=1
n=3
n=4
n=5
$a_0 = 0.0529nm$ = Bohr radius



Electron transitions for the Hydrogen atom
n=7
n=6
n=5
n=4
n=3
n=2
n=1
Brackett series E(n) to E(n=4)
Paschen series E(n) to E(n=3)
Balmer series E(n) to E(n=2)
Lyman series E(n) to E(n=1)

Light-Front QCD

Fixed $\tau = t + z/c$



$$\zeta^2 = x(1-x)b_\perp^2$$

$$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$\left[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \; \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)\right] \psi_{LF}(\zeta) = M^2 \; \psi_{LF}(\zeta)$$

Azimuthal Basis

ζ, ϕ

AdS/QCD:

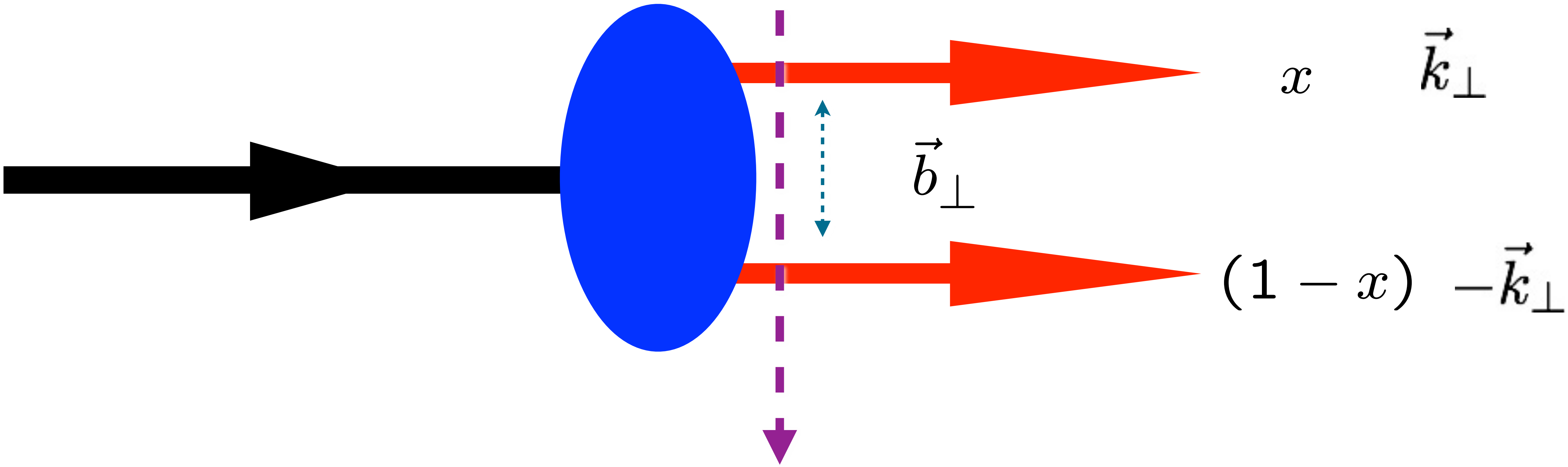
$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

Confining AdS/QCD potential!

Sums an infinite # diagrams

Semiclassical first approximation to QCD

Fixed $\tau = t + z/c$



$$\zeta^2 \equiv b_\perp^2 x(1-x)$$

Invariant transverse separation

$$\zeta^2 \text{ conjugate to } \frac{k_\perp^2}{x(1-x)} = (p_q + p_{\bar{q}})^2 = \mathcal{M}_{q+\bar{q}}^2$$

$$\int dk^- \Psi_{BS}(P,k) \to \psi_{LF}(x, \vec{k}_\perp)$$

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions}$$
$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp\, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions.}$$

Change variables $(\vec{\zeta},\varphi),\ \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp:\quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$

$$\mathcal{M}^2 = \int d\zeta\, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta\, \phi^*(\zeta)U(\zeta)\phi(\zeta)$$
$$= \int d\zeta\, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)$$

- de Teramond, sjb

Effective QCD LF Bound-State Equation

- Factor out the longitudinal $X(x)$ and orbital kinematical dependence from LFWF ψ

$$\psi(x,\zeta,\varphi) = e^{iL\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

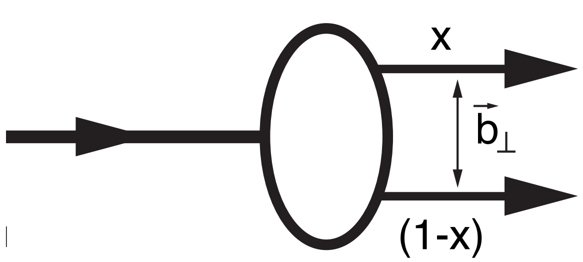
- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple and LF Hamiltonian equation $P_\mu P^\mu|\psi\rangle = M^2|\psi\rangle$ is a LF wave equation for ϕ

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

- Invariant transverse variable in impact space

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2$$

conjugate to invariant mass $\mathcal{M}^2 = \mathbf{k}_\perp^2/x(1-x)$



- Critical value $L=0$ corresponds to lowest possible stable solution: ground state of the LF Hamiltonian
- Relativistic and frame-independent LF Schrödinger equation: U is instantaneous in LF time and comprises all interactions, including those with higher Fock states.

Light-Front Schrödinger Equation

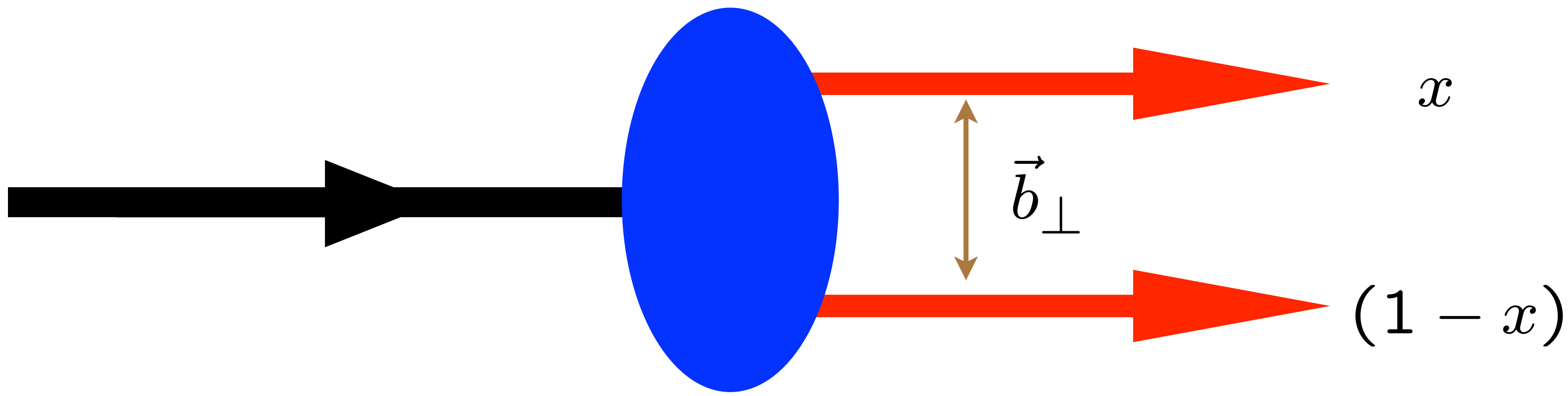
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

$$\left[-\frac{d^2}{d\zeta^2}+\frac{4L^2-1}{4\zeta^2}+U(\zeta^2,J,L,M^2)\right]\Psi_{J,L}(\zeta^2)=M^2\Psi_{J,L}(\zeta^2)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2 .$$



where the potential $U(\zeta^2, J, L, M^2)$ represents the contributions from higher Fock states. It is also the kernel for the forward scattering amplitude $q\bar{q} \to q\bar{q}$ at $s = M^2$. It has only "proper" contributions; i.e. it has no $q\bar{q}$ intermediate state. The potential can be constructed systematically using LF time-ordered perturbation theory. Thus the exact QCD theory has the identical form as the AdS theory, but with the quantum field-theoretic corrections due to the higher Fock states giving a general form for the potential. This provides a novel way to solve nonperturbative QCD. Complex eigenvalues for excited states n>0

- Invariant mass $\mathcal{M}^2$ in terms of LF mode ϕ

$$\mathcal{M}^2 = \int d\zeta\, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta\, \phi^*(\zeta)U(\zeta)\phi(\zeta)$$

$$= \int d\zeta\, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2}\right)\phi(\zeta) + \int d\zeta\, \phi^*(\zeta)U(\zeta)\phi(\zeta)$$

where the interaction terms are summed up in the effective potential $U(\zeta)$ and the orbital angular momentum in ∇^2 has the $SO(2)$ Casimir representation $SO(N) \sim S^{N-1}:\ L(L+N-2)$

$$-\frac{\partial^2}{\partial\varphi^2}|\phi\rangle = L^2|\phi\rangle$$

- LF eigenvalue equation $H_{LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle$ is a LF wave equation for ϕ

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = \mathcal{M}^2\phi(\zeta)} \qquad m_q = 0$$

- Effective light-front Schrödinger equation: relativistic, covariant and analytically tractable.

Light-Front Schrödinger Equation

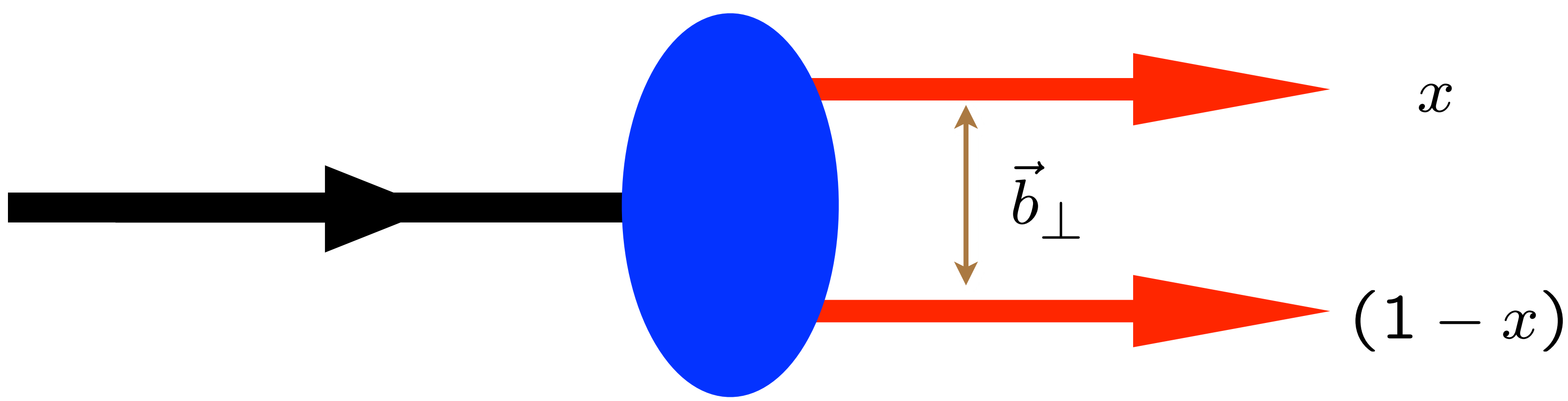
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

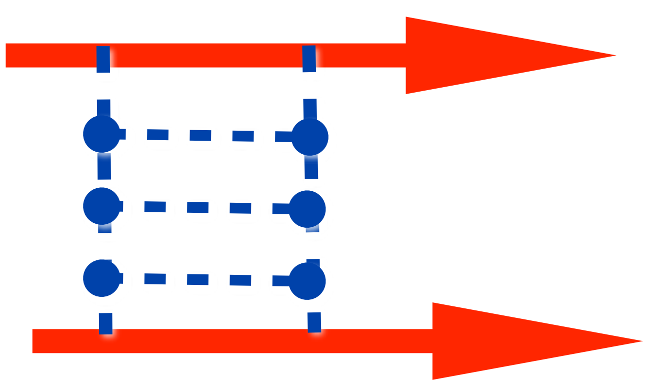
Frame Independent!

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2 .$$



U is the confining QCD potential
Conjecture: 'H'-diagrams generate



$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L+S-1)$$

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Light-Front QCD

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Light-Front Schrödinger Equation

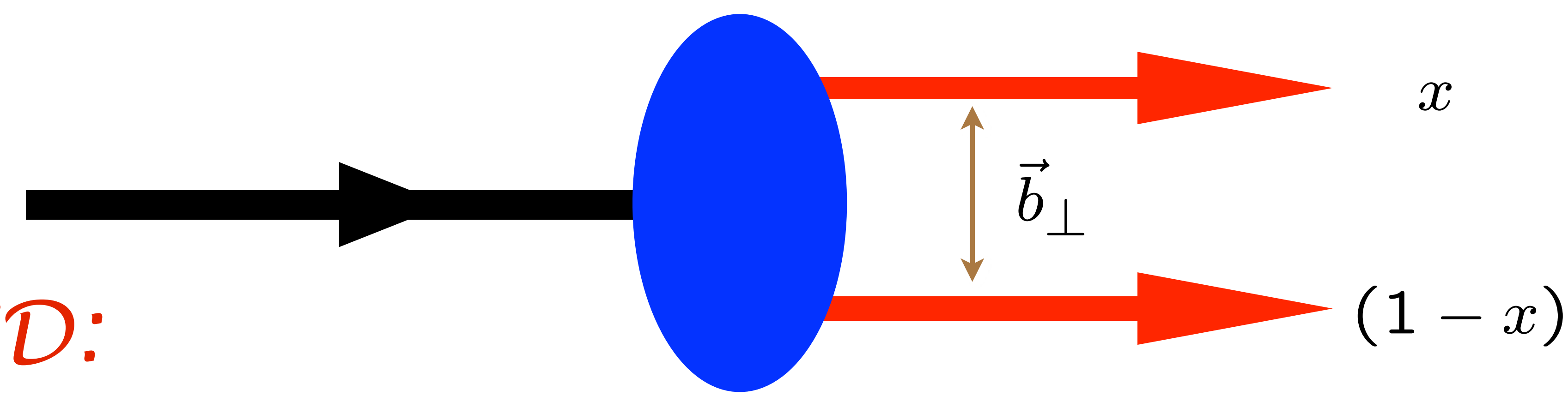
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

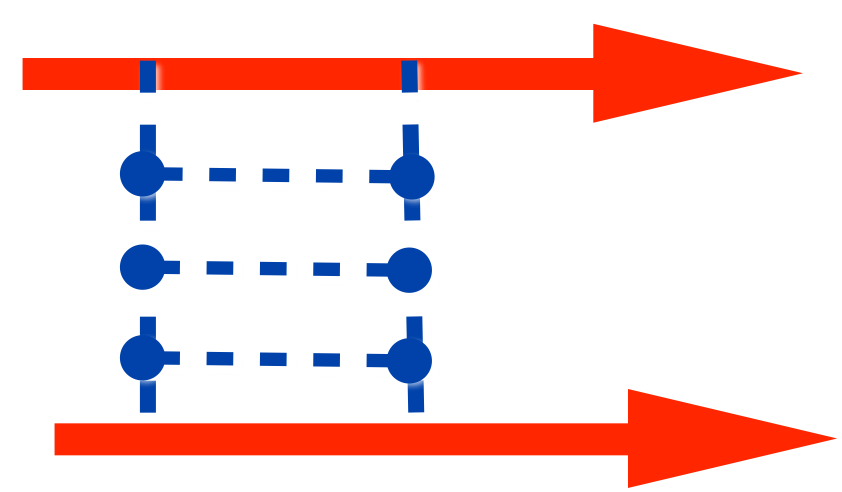
$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)\right] \psi_{LF}(\zeta) = M^2 \, \psi_{LF}(\zeta)$$

$$m_q \sim 0 \qquad \zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



AdS/QCD:

$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

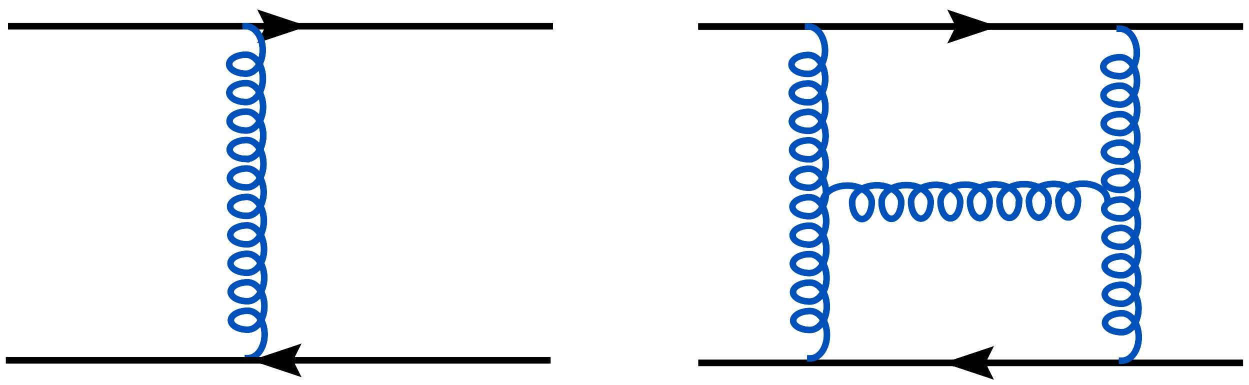


U is the exact QCD potential
Conjecture: 'H'-diagrams generate U?

Heavy Quark Potential is IR Divergent in QCD

$$V(Q^2) = -\frac{(4\pi)^2 C_F}{Q^2} a(Q^2) \Big[1 + (c_{2,0} + c_{2,1} N_f) a(Q^2) + (c_{3,0} + c_{3,1} N_f + c_{3,2} N_f^2) a(Q^2)^2$$
$$+ (c_{4,0} + c_{4,1} N_f + c_{4,2} N_f^2 + c_{4,3} N_f^3) a(Q^2)^3 + 8\pi^2 C_A^3 \ln \frac{\mu_{IR}^2}{Q^2} a(Q^2)^3 \Big]$$

Smirnov, Smirnov, Steinhauser, 2010



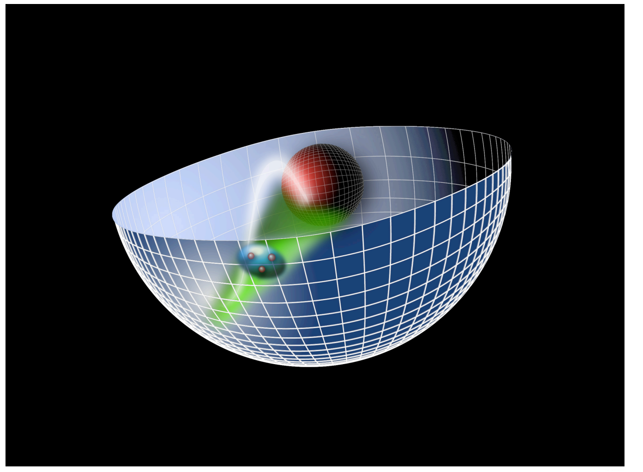
$\log \kappa^2 \zeta^2$

Summation of H graphs: confining potential

Confinement eliminates IR divergences
Self-consistent mass scale κ

de Tèramond, Dosch, sjb

AdS/QCD Soft-Wall Model



Light-Front Holography

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2.$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale:

$$\kappa \simeq 0.6\ GeV$$

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Meson Spectrum in Soft Wall Model

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

G. de Teramond, H. G. Dosch, sjb

Quark separation increases with L

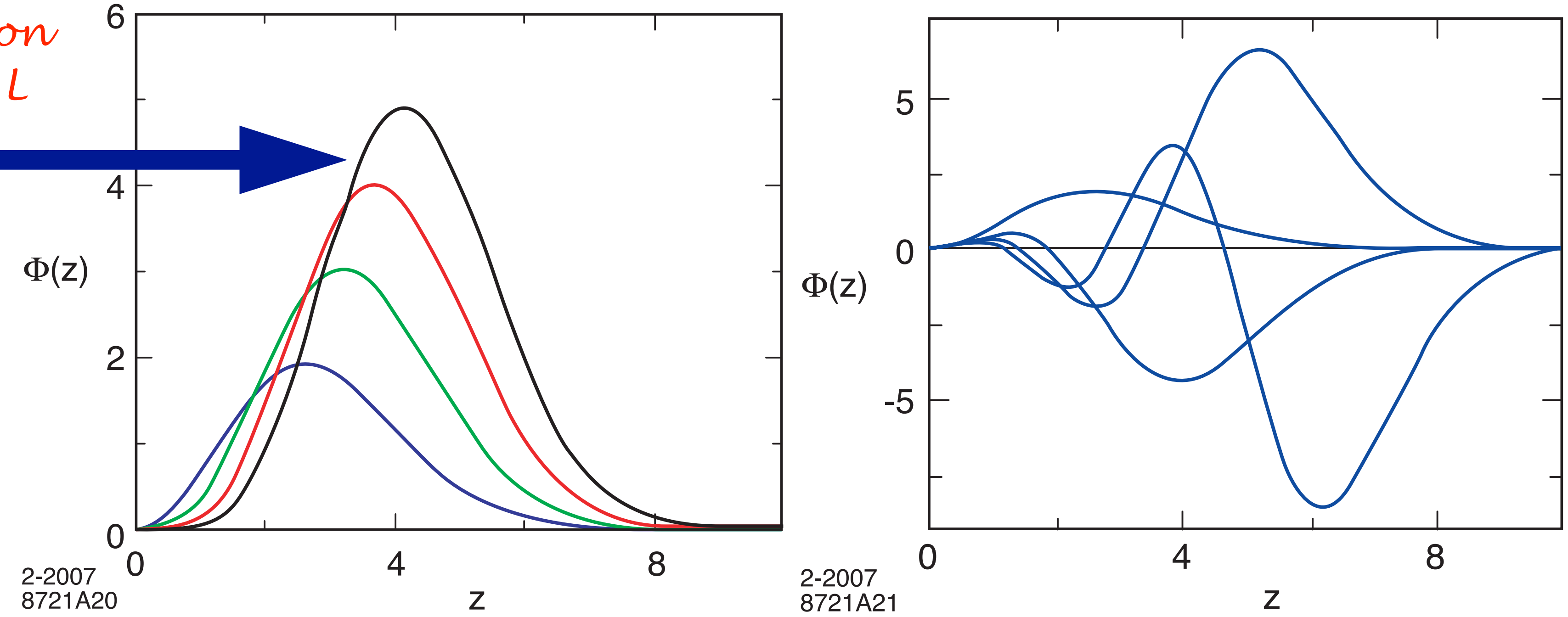
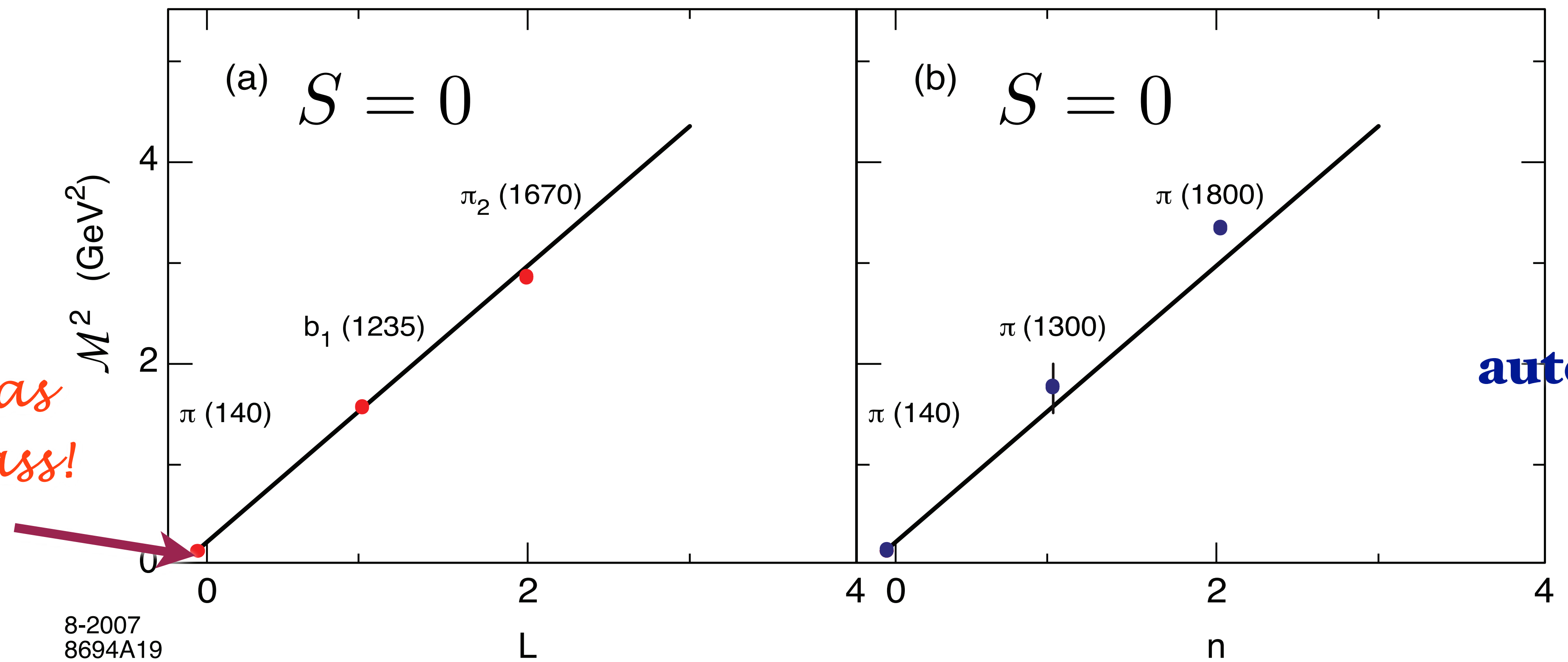


Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV .

Soft Wall Model

Same slope in n and L!

Pion has zero mass!



Pion mass automatically zero!

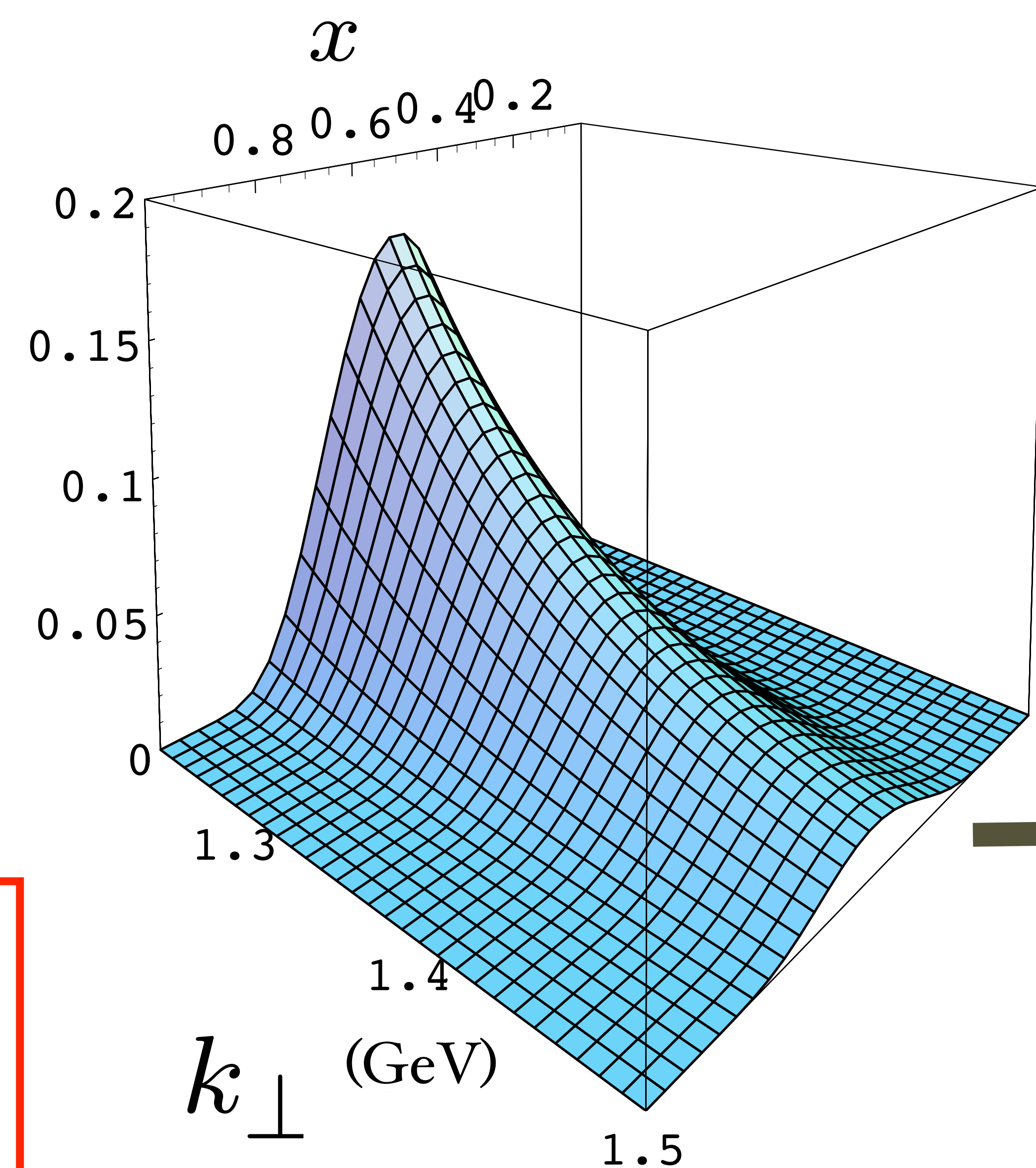
$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

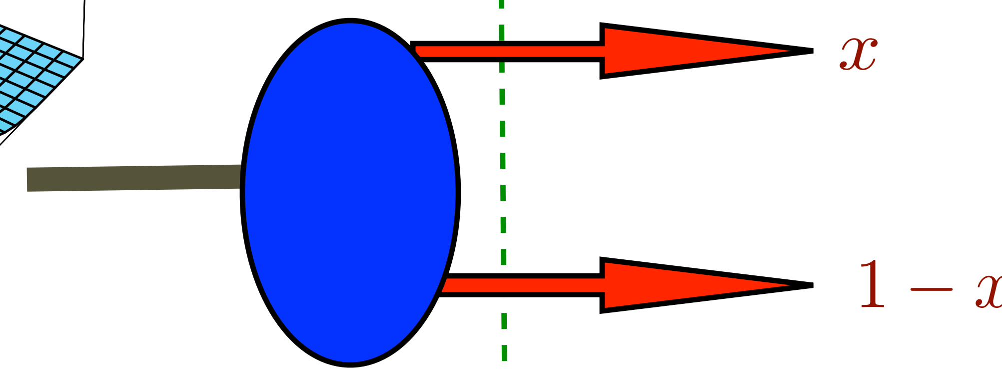
Prediction from AdS/QCD: Meson LFWF

de Teramond, Cao, sjb

$\psi_M(x, k_\perp^2)$



"Soft Wall" model



massless quarks

Note coupling

$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

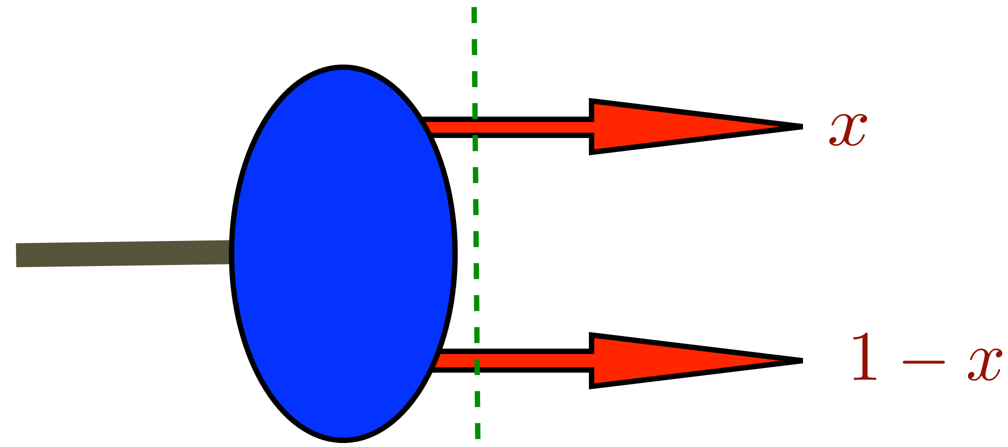
$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

Provides Connection of Confinement to Hadron Structure

Hadron Distribution Amplitudes

$A^+ = 0$

$$\phi_M(x,Q) = \int^{Q} d^2\vec{k}\ \psi_{q\bar{q}}(x,\vec{k}_\perp)$$



$$k_\perp^2 < Q^2$$

$$\sum_i x_i = 1$$

Fixed $\tau = t + z/c$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

  Lepage, sjb

  Efremov, Radyushkin

- Evolution Equations from PQCD, OPE

  Sachrajda, Frishman Lepage, sjb

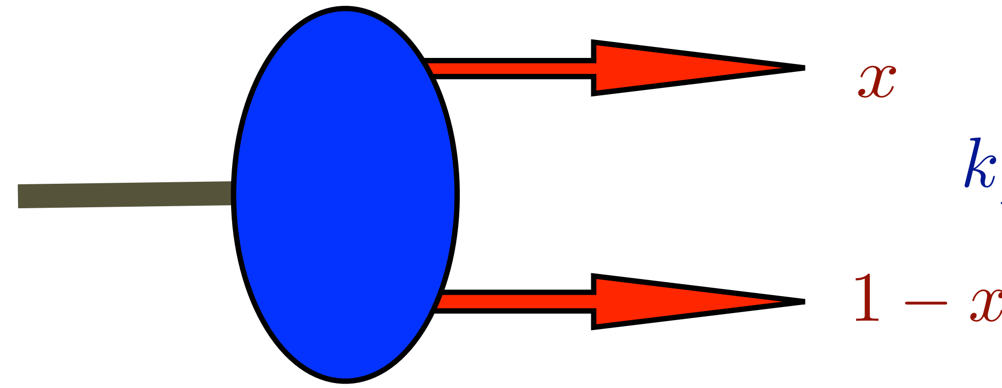
- Conformal Expansions

  Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

Pion Distribution Amplitude in Non-Perturbative Domain

$$\phi_M(x,Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x,\vec{k}_\perp)$$



Factorization Theorem for Hard Exclusive Processes

$\phi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$

and pion decay constant

$f_\pi = \sqrt{P_{\bar{q}q}} \frac{\sqrt{3}}{8}\kappa$

In contrast with the asymptotic DA

$\phi(x) = \sqrt{3} f_\pi x(1-x)$

and decay constant

$f_\pi = \sqrt{P_{\bar{q}q}} \frac{\kappa}{\sqrt{2}\pi}$

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

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Consortium for Fundamental Physics, School of Physics and Astronomy, University of Manchester, Oxford Road, Manchester M13 9PL, United Kingdom

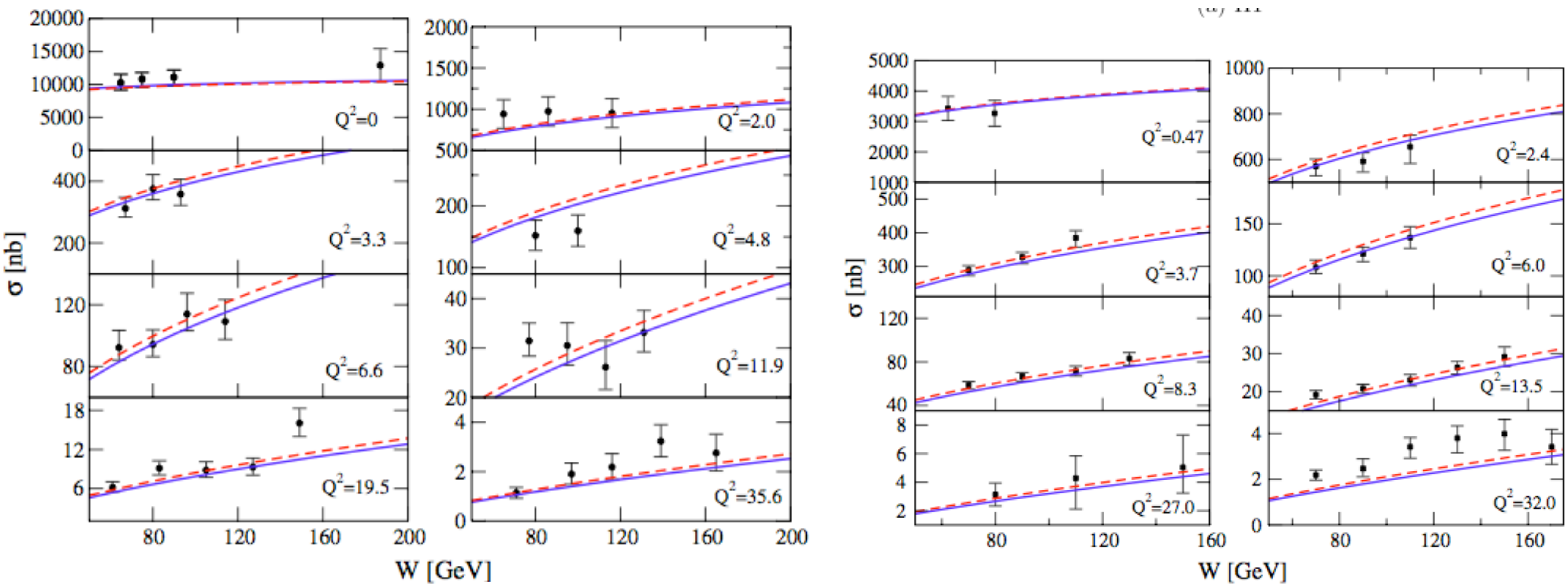
R. Sandapen†
Département de Physique et d'Astronomie, Université de Moncton, Moncton, New Brunswick E1A3E9, Canada
(Received 5 April 2012; published 20 August 2012)

We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ-meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron–proton collider.

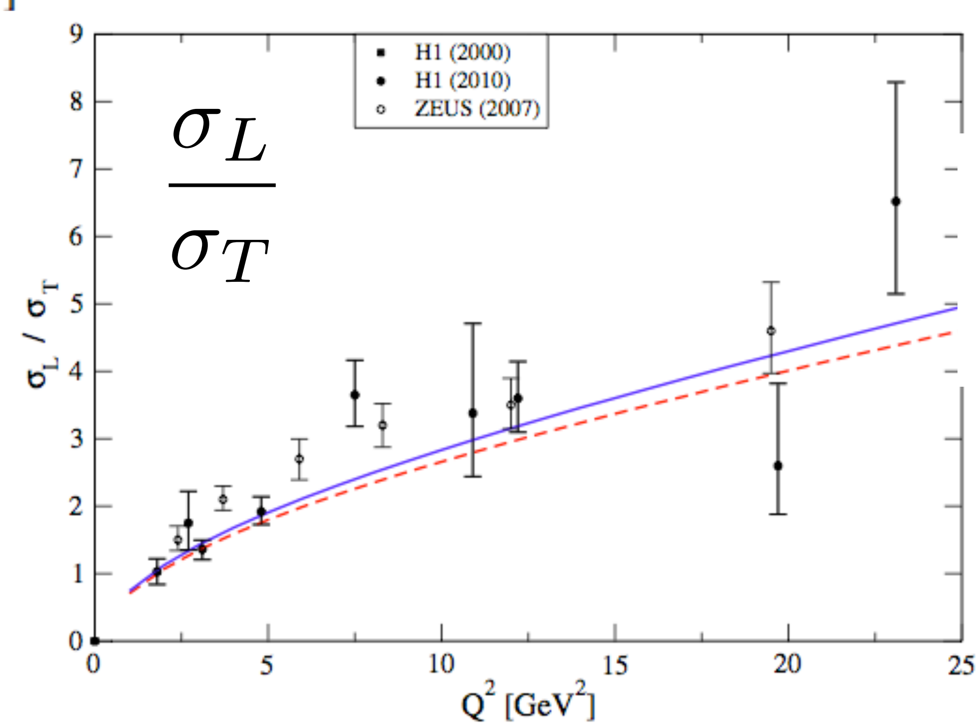
$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

PRL **109,** 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



(a) H (b) ZEUS



$$\frac{\sigma_L}{\sigma_T}$$

J. R. Forshaw, R. Sandapen

$$\gamma^* p \to \rho^0 p'$$

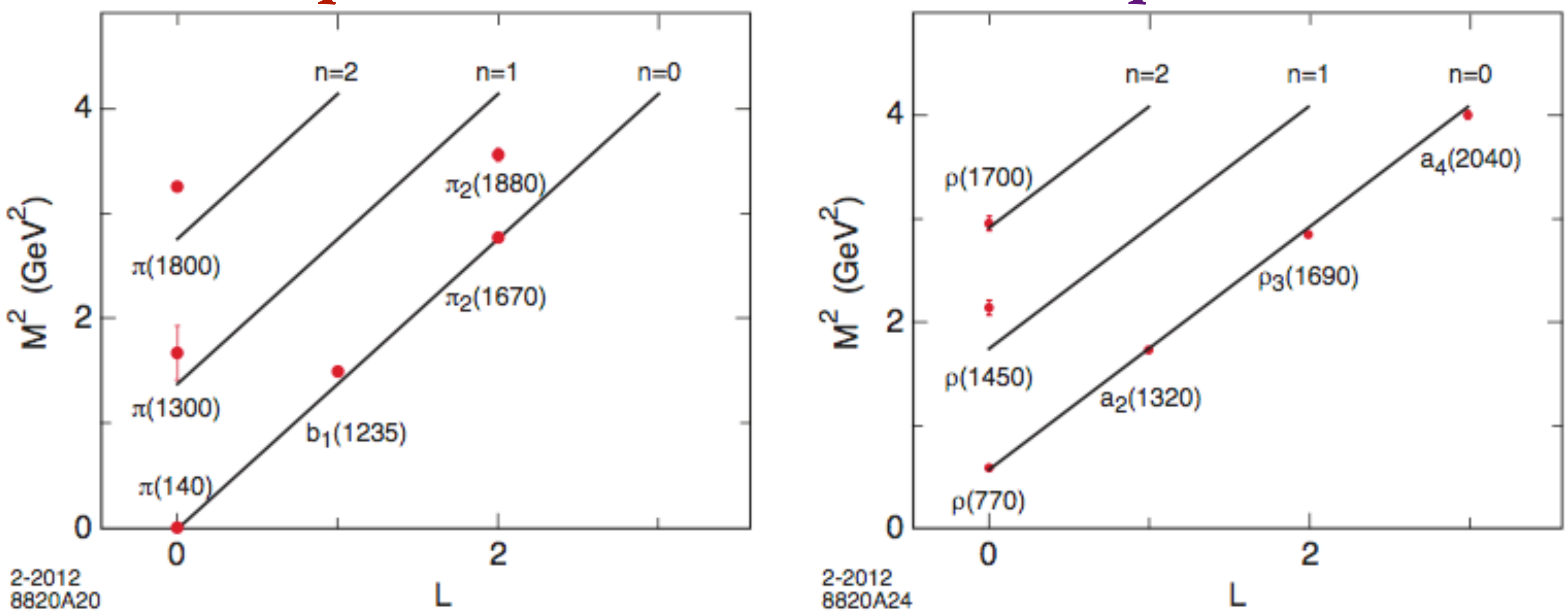
$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

- $J = L + S, I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2 (n + L + S/2)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

$m_q = 0$

Massless pion in Chiral Limit! ***Same slope in n and L!***



I=1 orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ-meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1, L = 1, J = 0, 1, 2$, vector meson a-states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules

G. de Teramond, H. G. Dosch, sjb

De Teramond, Dosch, sjb

- Results easily extended to light quarks masses (Ex: K-mesons)

- First order perturbation in the quark masses

$$\Delta M^2 = \langle\psi|\sum_a m_a^2/x_a|\psi\rangle$$

- Holographic LFWF with quark masses

$$\lambda \equiv \kappa^2$$

$$\psi(x,\zeta) \sim \sqrt{x(1-x)}\, e^{-\frac{1}{2\lambda}\left(\frac{m_q^2}{x}+\frac{m_q^2}{1-x}\right)} e^{-\frac{1}{2}\lambda\zeta^2}$$

- Ex: Description of diffractive vector meson production at HERA
  [J. R. Forshaw and R. Sandapen, PRL **109**, 081601 (2012)]

- For the K^*

$$M_{n,L,S}^2 = M_{K^\pm}^2 + 4\lambda\left(n + \frac{J+L}{2}\right)$$

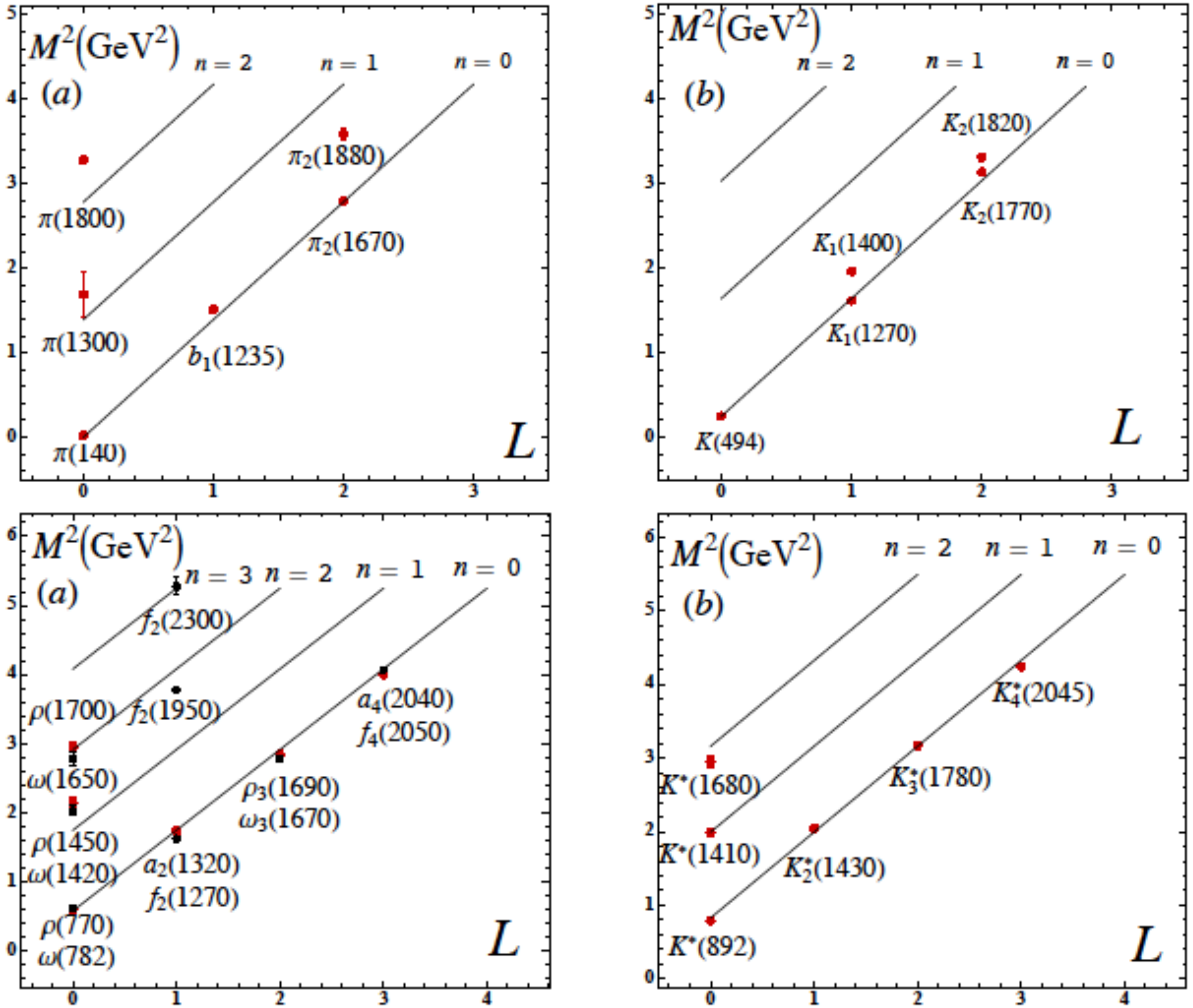
- Effective quark masses from reduction of higher Fock states as functionals of the valence state:

$$m_u = m_d = 46\ \mathrm{MeV}, \quad m_s = 357\ \mathrm{MeV}$$

De Teramond, Dosch, sjb

$m_u = m_d = 46$ MeV, $\quad m_s = 357$ MeV

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar q}^2}{1-x} \right| X \right\rangle$$



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Light-Front QCD

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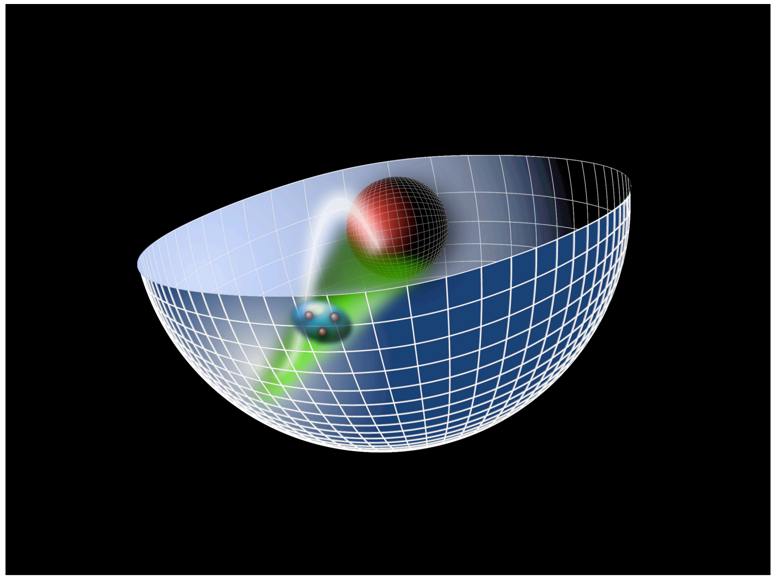
$\phi(z)$

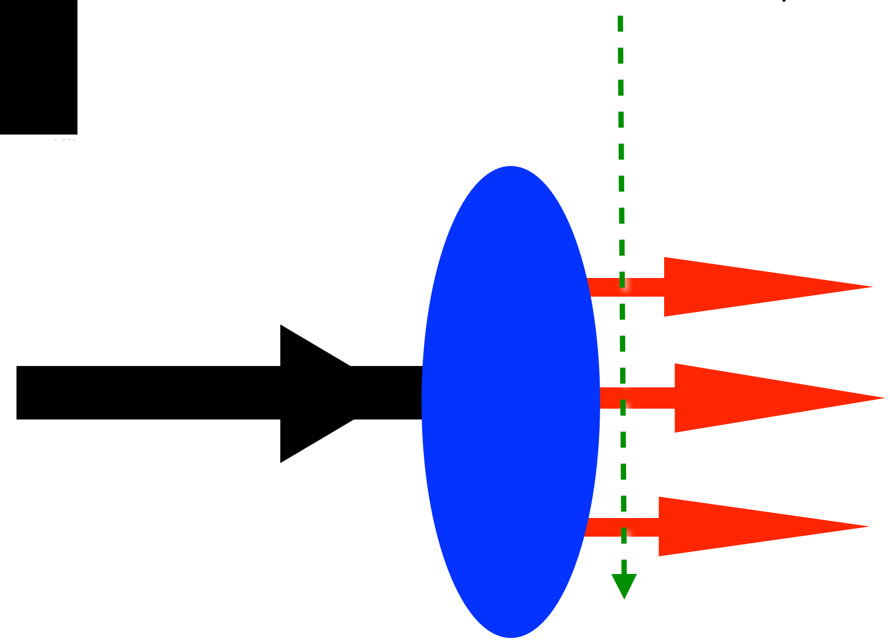
AdS5: Conformal Template for QCD



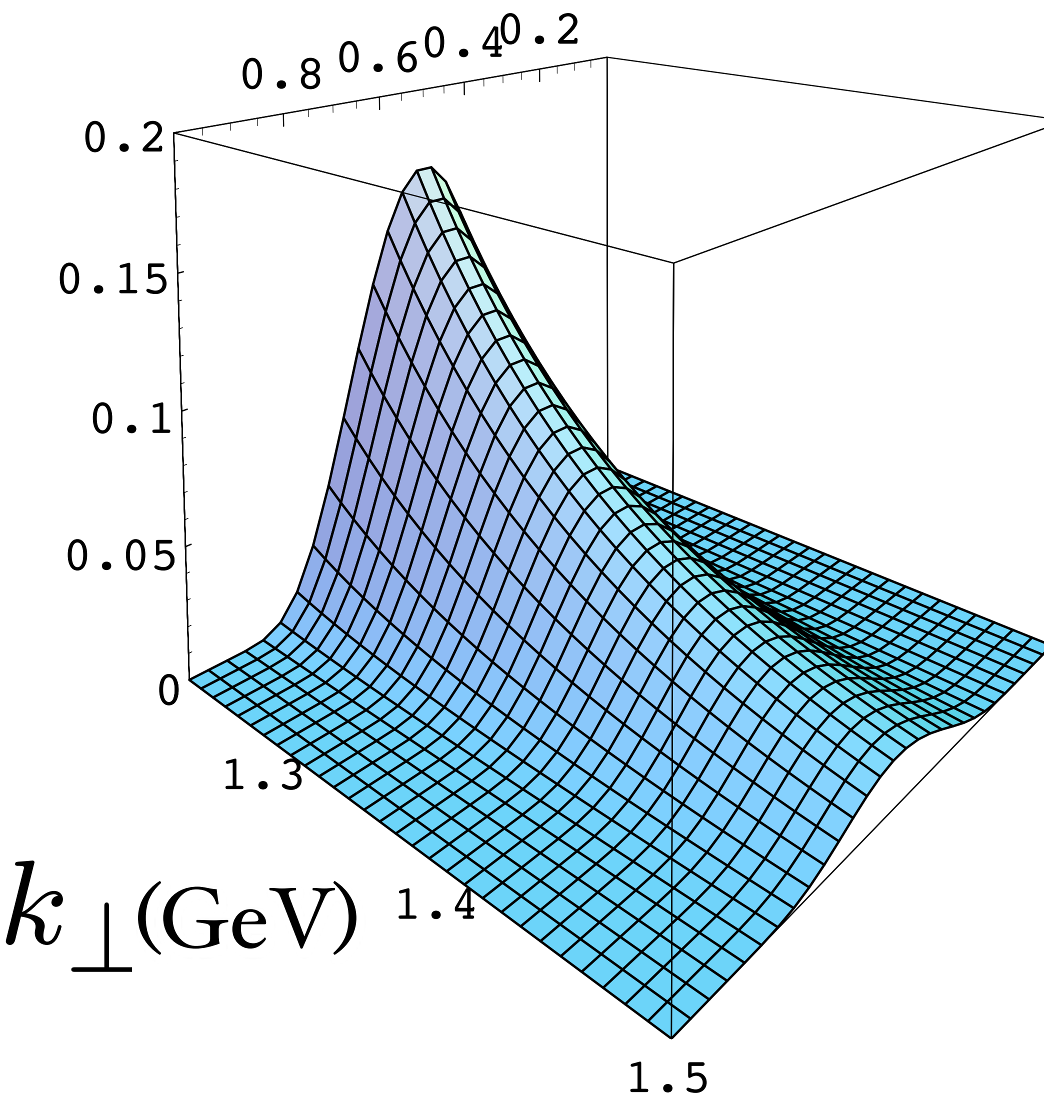
- *Light-Front Holography*

Duality of AdS_5 with LF Hamiltonian Theory

Fixed $\tau = t + z/c$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



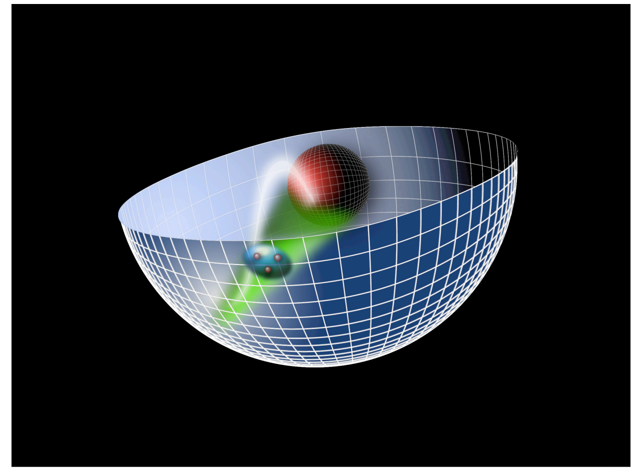
- *Light Front Wavefunctions:*

Light-Front Schrödinger Equation
Spectroscopy and Dynamics

de Tèramond, Dosch, sjb

AdS/QCD Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

Confinement scale:

$$\kappa \simeq 0.6\ GeV$$

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Generalized parton distributions in AdS/QCD

Alfredo Vega[1], Ivan Schmidt[1], Thomas Gutsche[2], Valery E. Lyubovitskij[2*]

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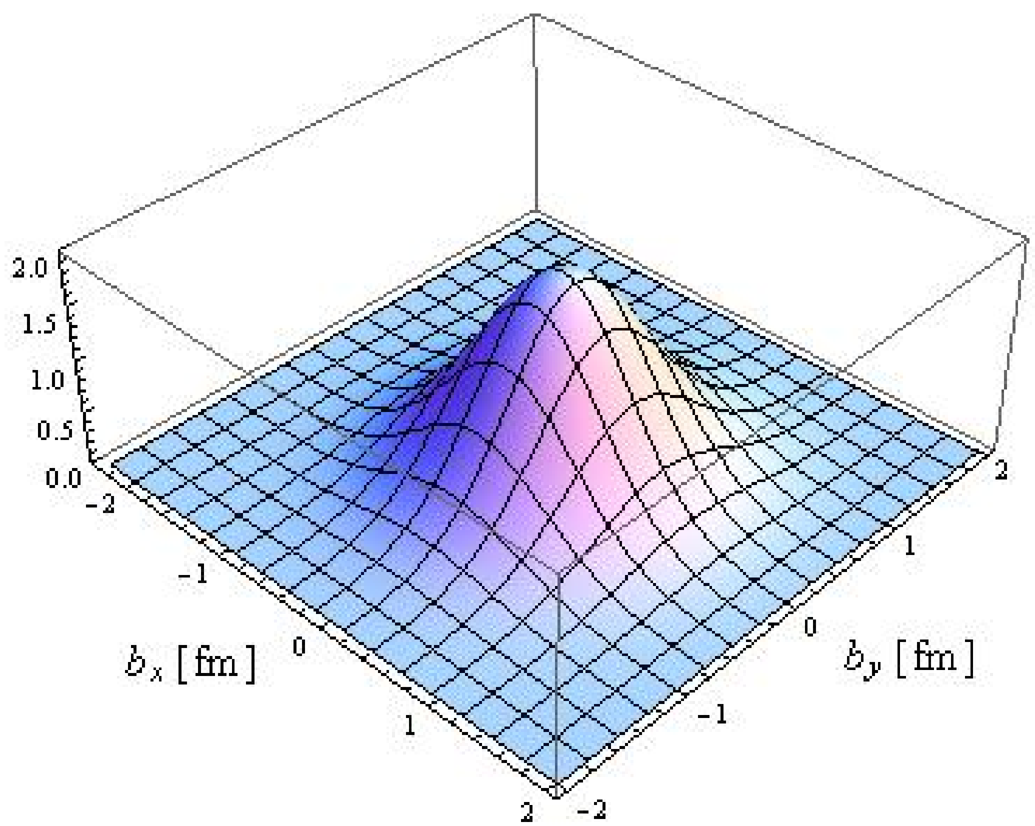
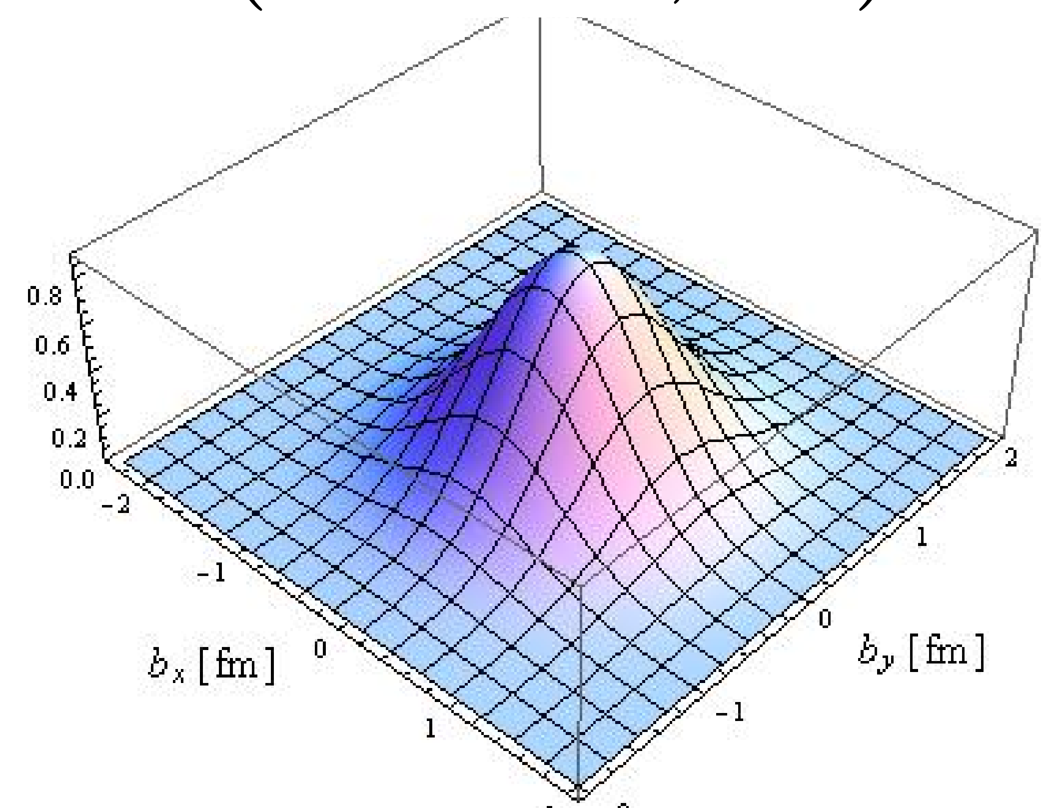
(Dated: January 19, 2011)

$$u(x = 0.1, \vec{b}_\perp)$$



$$d(x = 0.1, \vec{b}_\perp)$$

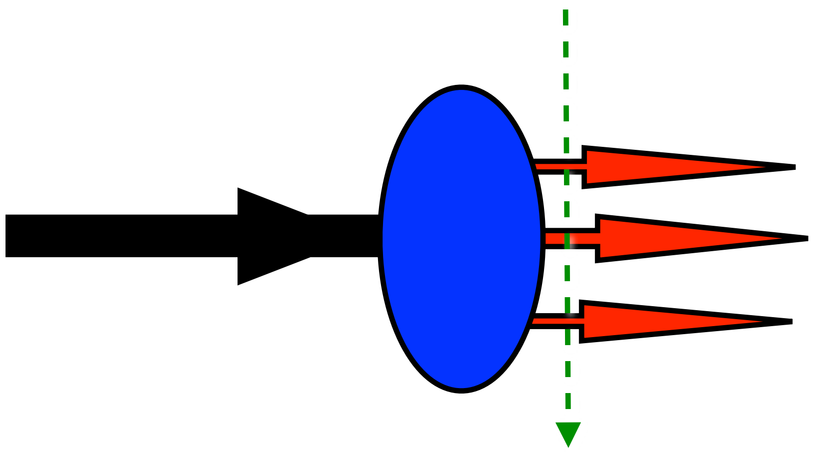


Also: Heavy quark bound states

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

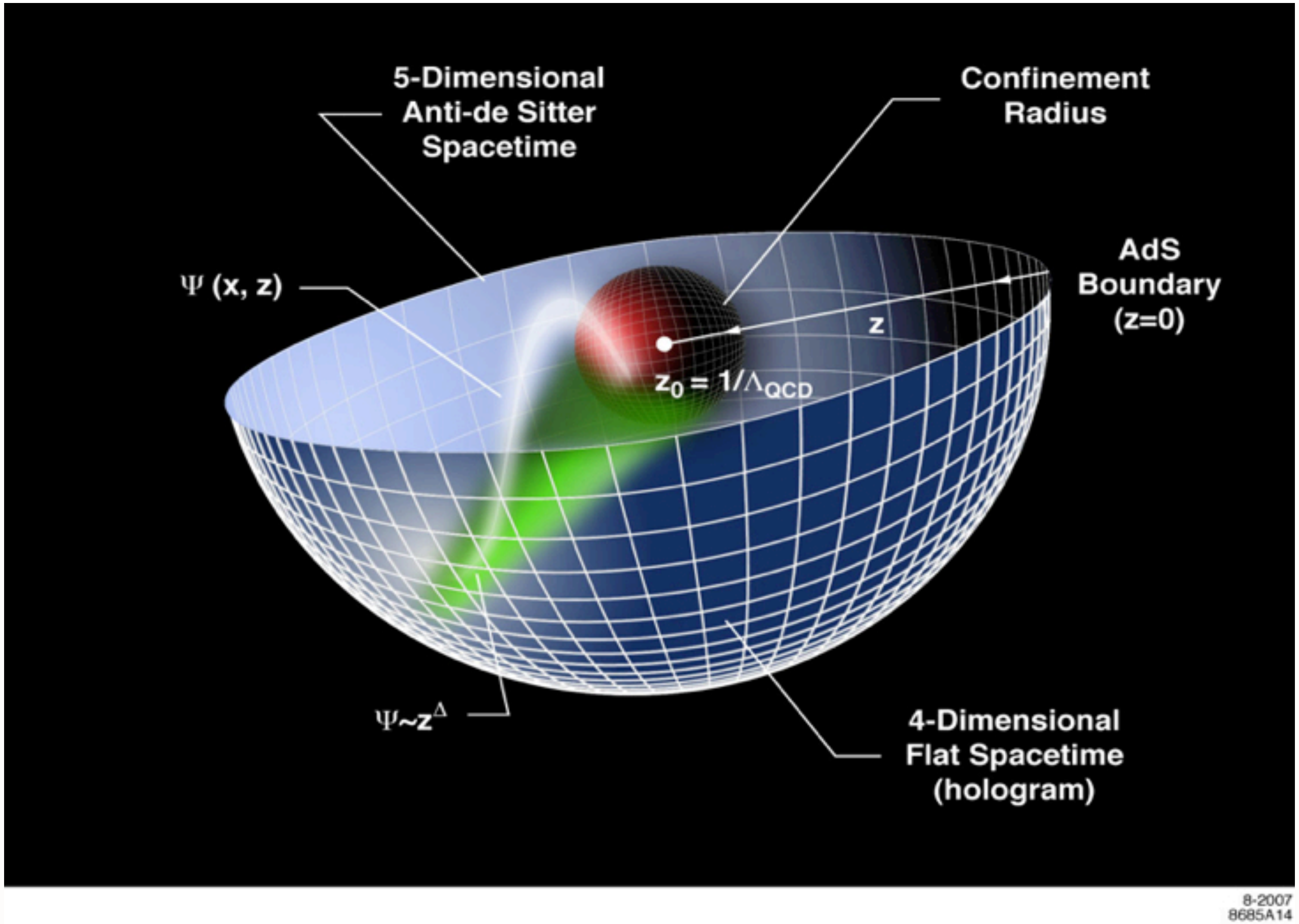
$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD} |\psi> = M^2 |\psi>$$

Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space



Changes in physical length scale mapped to evolution in the 5th dimension z

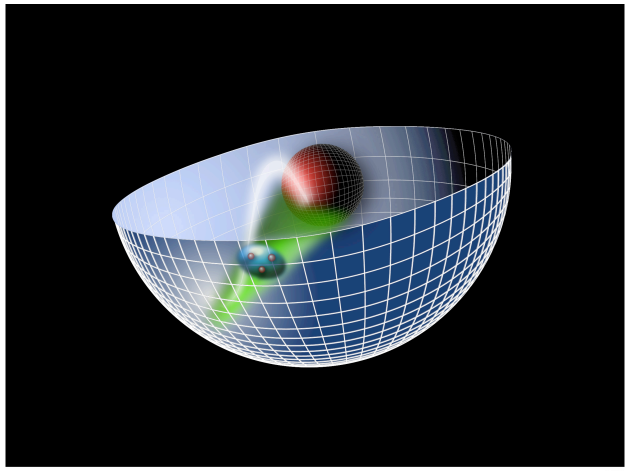
- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

$$\kappa \simeq 0.6\ GeV$$

Confinement scale:

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

AdS/CFT

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

invariant measure

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$.
- Action for massive scalar modes on AdS$_{d+1}$:

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \, \tfrac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g} \, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ-coordinates , $\Phi_P(x,z) = e^{-iP\cdot x} \, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\left[z^2 \partial_z^2 - (d-1) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z\mathcal{M}) \qquad \Delta = \tfrac{1}{2}\left(d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

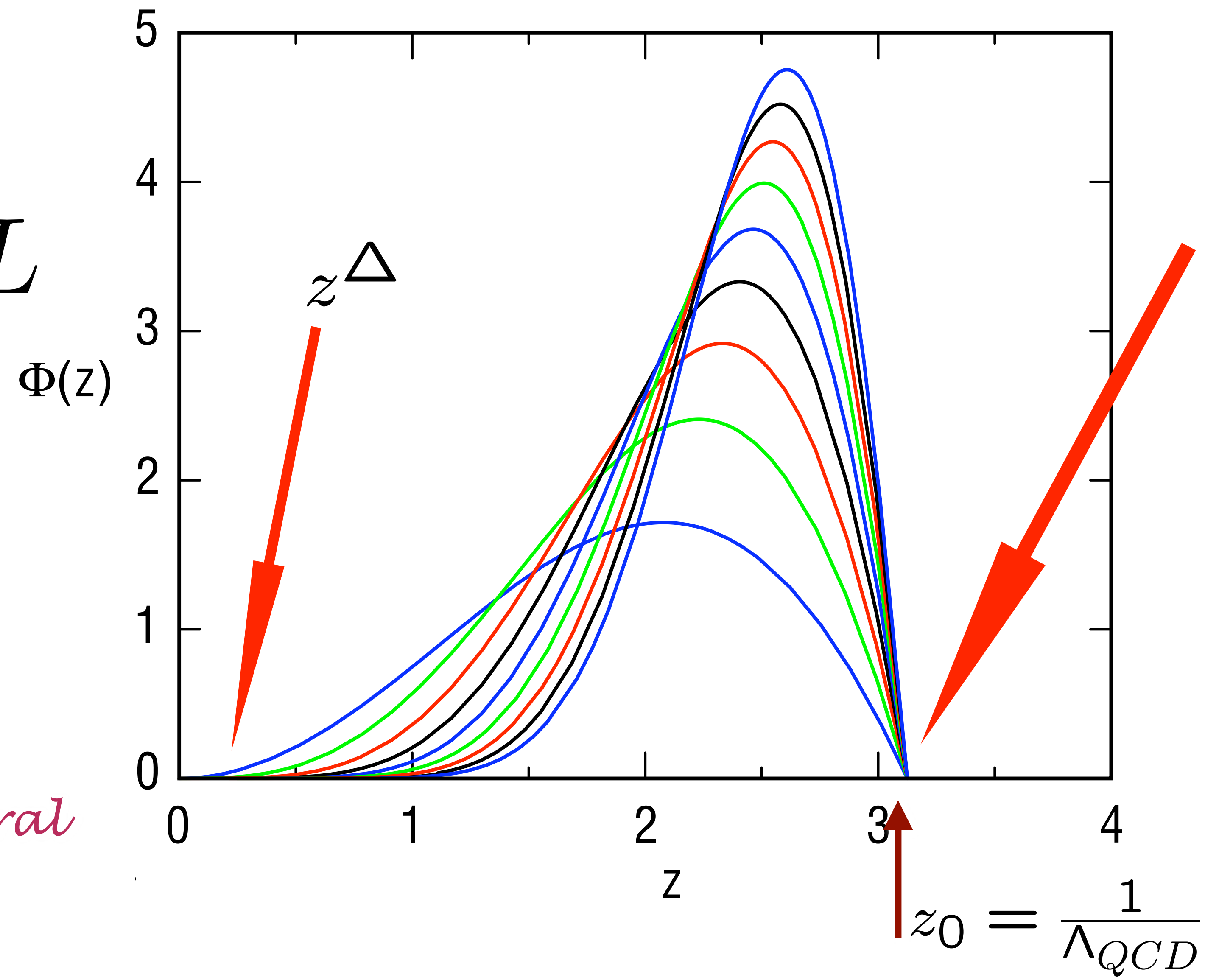
$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

- Physical AdS modes $\Phi_P(x,z) \sim e^{-iP\cdot x}\,\Phi(z)$ are plane waves along the Poincaré coordinates with four-momentum P^μ and hadronic invariant mass states $P_\mu P^\mu = \mathcal{M}^2$.
- For small-z $\Phi(z) \sim z^\Delta$. The scaling dimension Δ of a normalizable string mode, is the same dimension of the interpolating operator $\mathcal{O}$ which creates a hadron out of the vacuum: $\langle P|\mathcal{O}|0\rangle \neq 0$.

$$\Delta = 2 + L$$

Twist dimension of meson

equivalent to dimensions of chiral superfields



Hard Wall

Confinement in the 5th dimension

de Teramond, sjb

Identify hadron by its interpolating operator at z --> o

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$

- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement**
- **Introduces confinement scale** κ
- **Uses AdS$_5$ as template for conformal theory**

Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

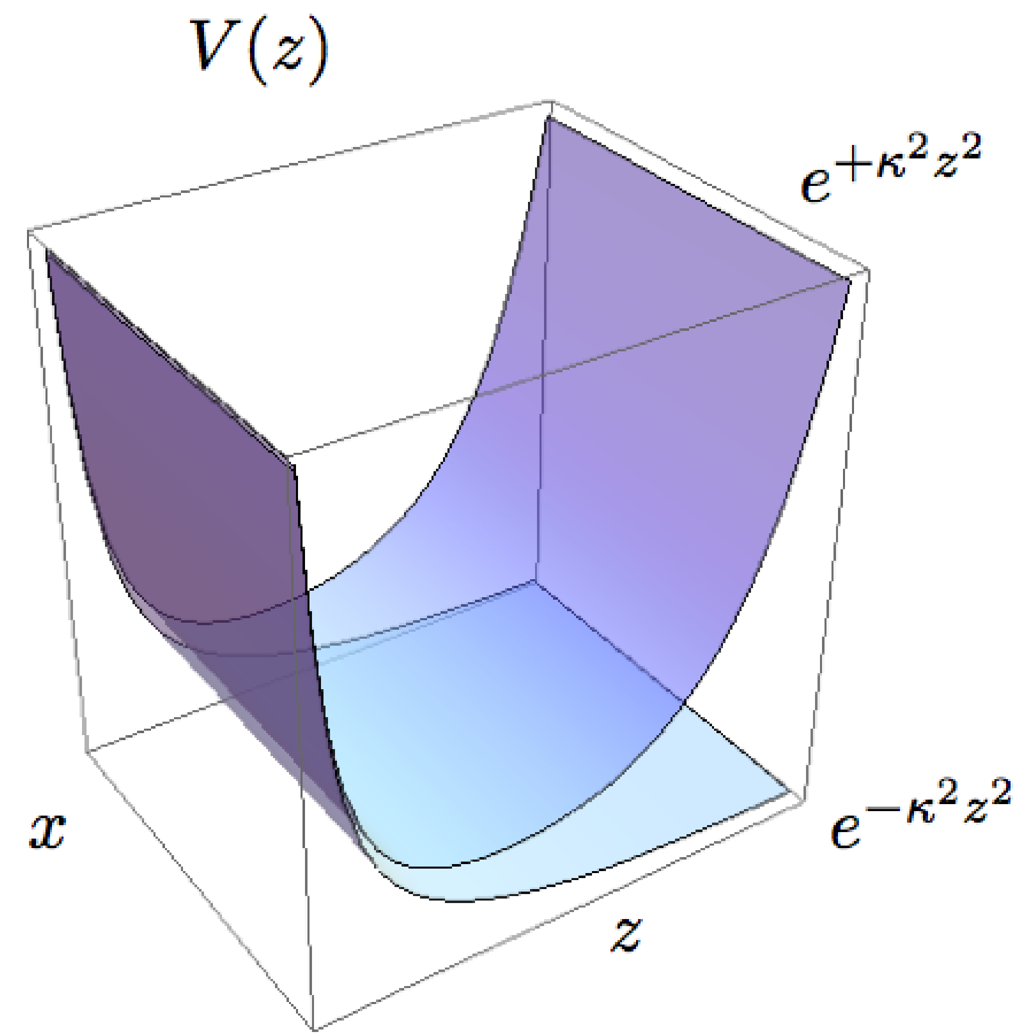
where $\varphi(z) \to 0$ at small z for geometries which are asymptotically AdS_5

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \, \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$

- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Klebanov and Maldacena

$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

Positive-sign dilaton

- de Teramond, sjb

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- Dosch, de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Light-Front Bound State Equation!

$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

de Teramond, Dosch, sjb

General-Spin Hadrons

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

- Upon substitution $z \rightarrow \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2}\, \Phi_J(\zeta)$$

we find the LF wave equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}$$

with $(\mu R)^2 = -(2 - J)^2 + L^2$

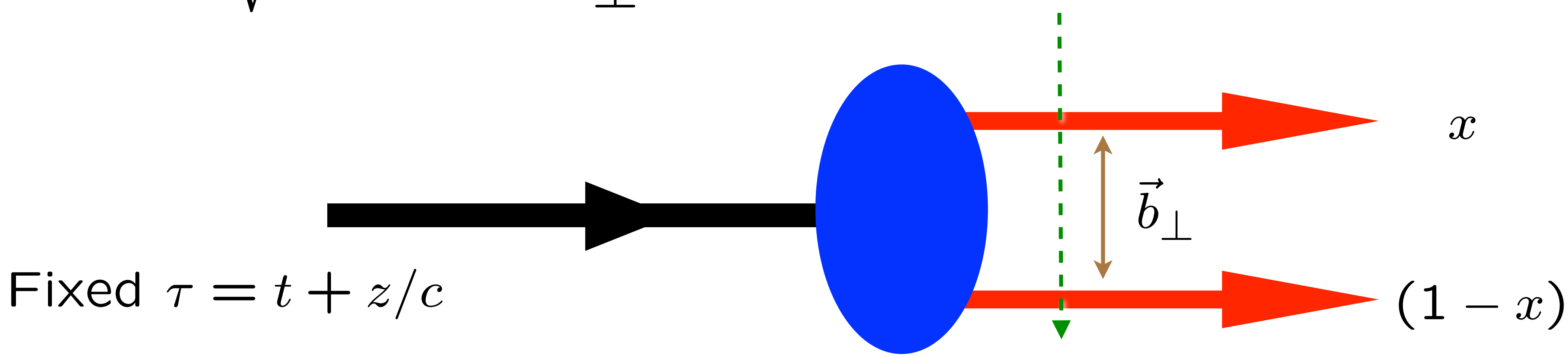
de Teramond, sjb

LF(3+1) ↔ AdS_5

Light-Front Holography

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



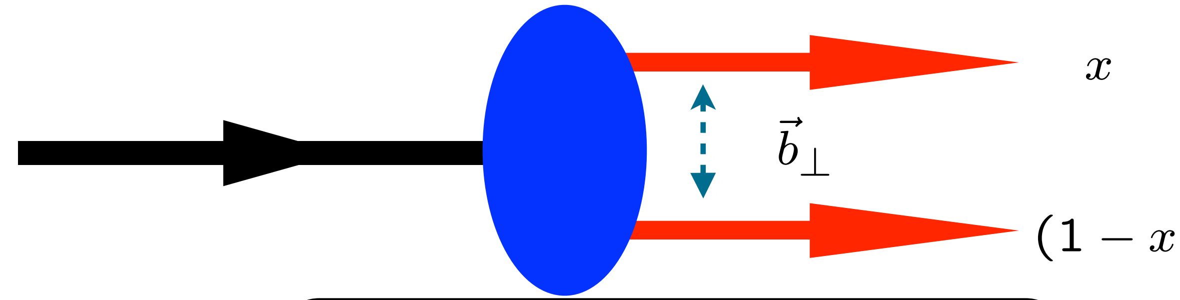
$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

Light-Front QCD

Fixed $\tau = t + z/c$



$$\zeta^2 = x(1-x)b_\perp^2$$

$$H^{LF}_{QCD}$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states (retarded interactions)

$$[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1 + 4L^2}{4\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

Azimuthal Basis

$$\zeta, \phi$$

AdS/QCD:

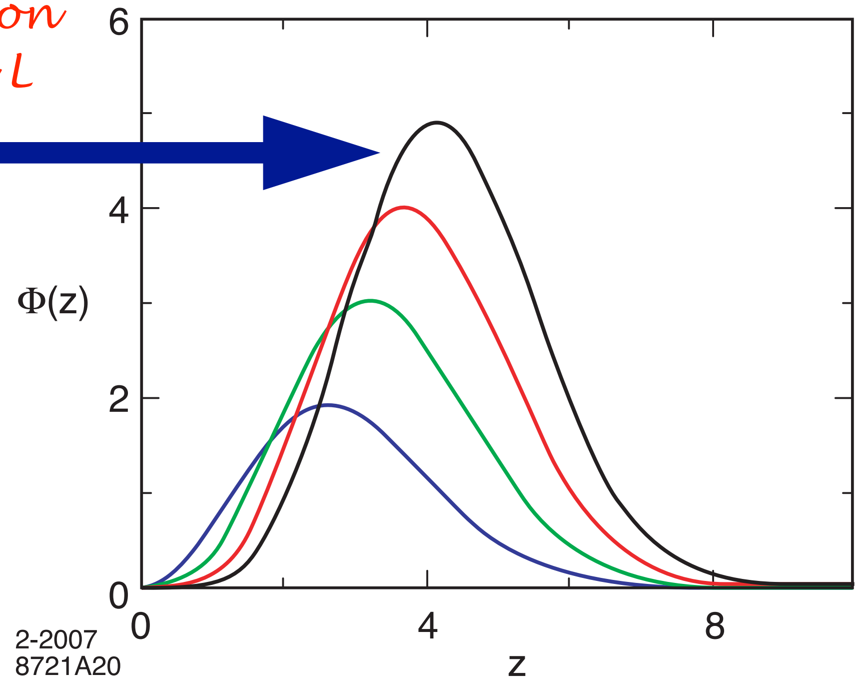
$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

Confining AdS/QCD potential!

Sums an infinite # diagrams

Semiclassical first approximation to QCD

Quark separation increases with L



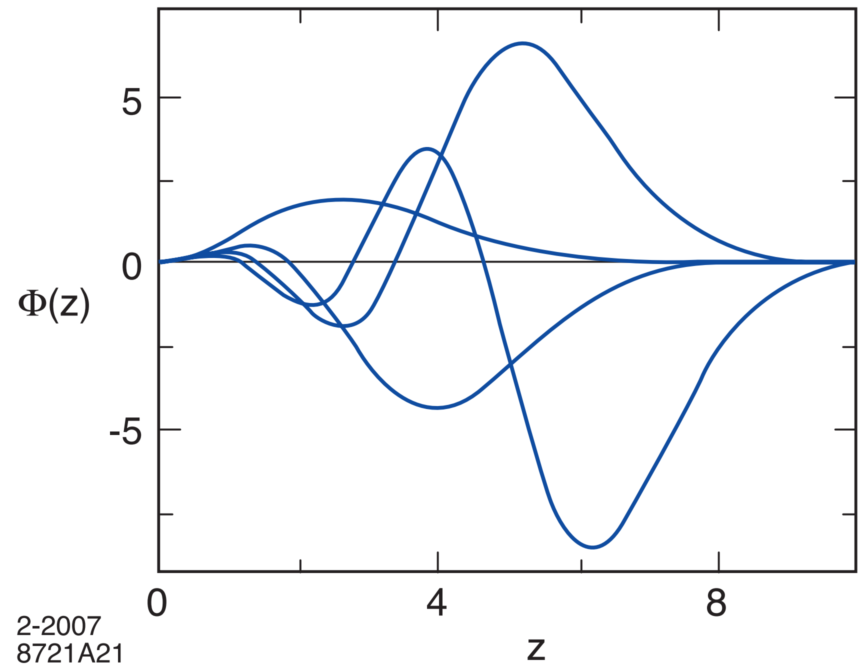
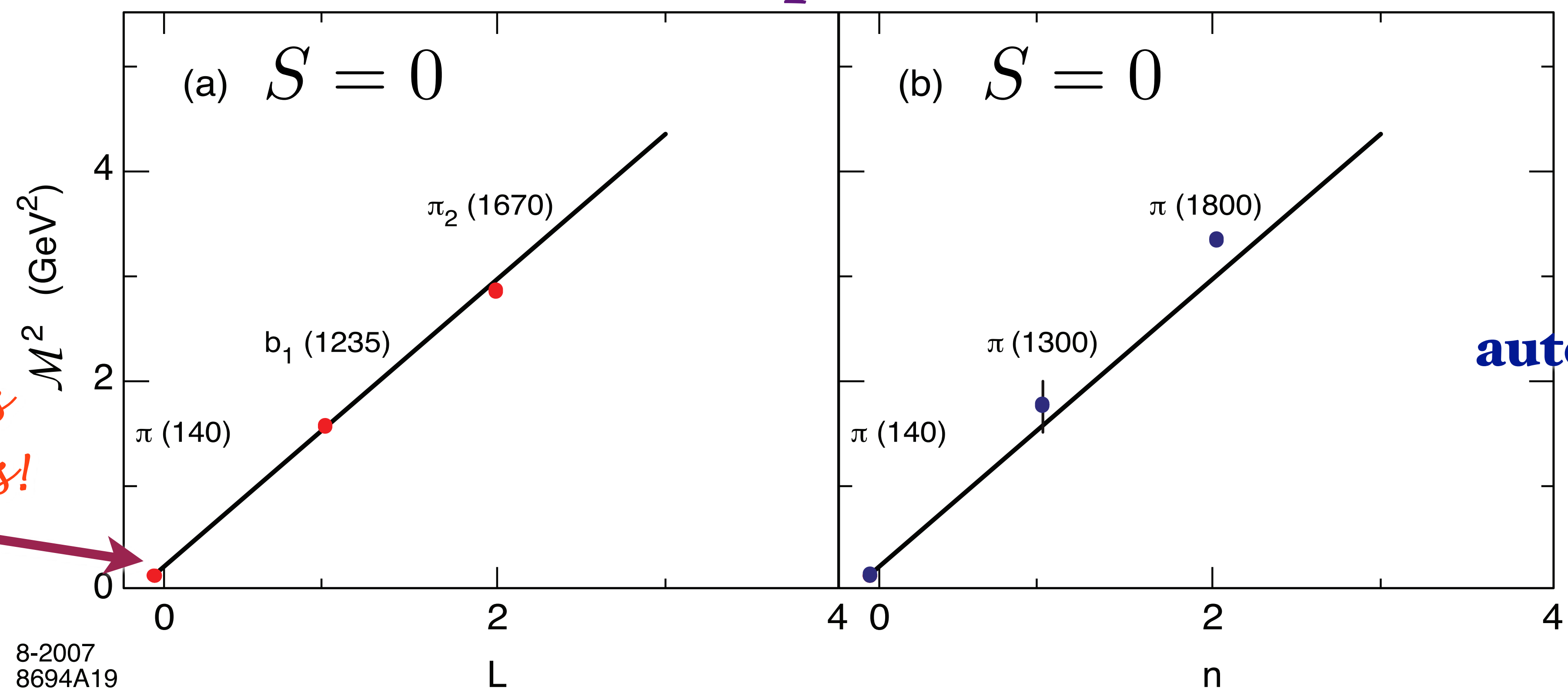


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model

Same slope in n and L!



Pion has zero mass!

Pion mass automatically zero!

$$m_q = 0$$

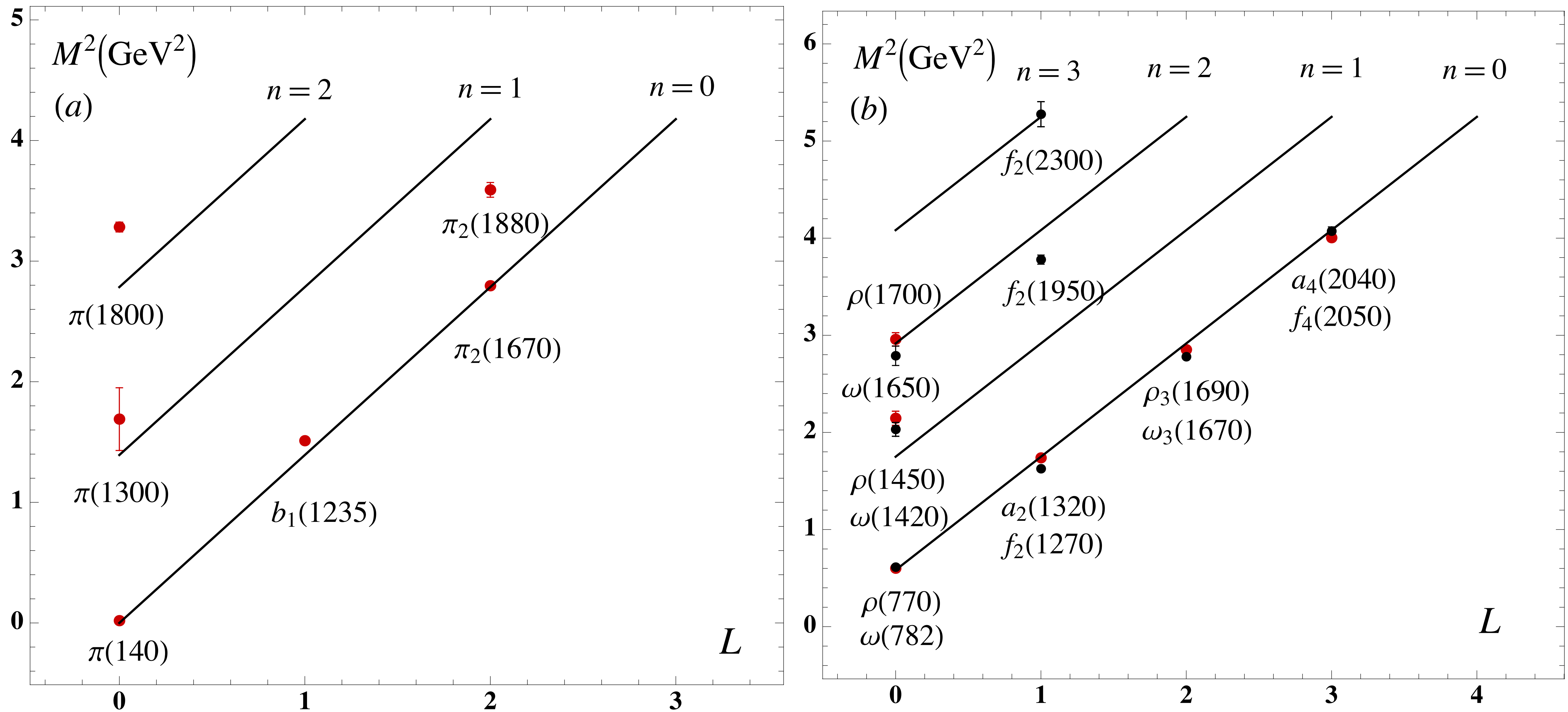
Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Universidad Técnica Federico Santa María

Light-Front QCD

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY



$M^2(\mathrm{GeV}^2)$
(a)
$n=2$
$n=1$
$n=0$
$\pi(1800)$
$\pi_2(1880)$
$\pi_2(1670)$
$\pi(1300)$
$b_1(1235)$
$\pi(140)$
L
$M^2(\mathrm{GeV}^2)$
(b)
$n=3$
$n=2$
$n=1$
$n=0$
$f_2(2300)$
$\rho(1700)$
$f_2(1950)$
$a_4(2040)$
$f_4(2050)$
$\omega(1650)$
$\rho_3(1690)$
$\omega_3(1670)$
$\rho(1450)$
$\omega(1420)$
$a_2(1320)$
$f_2(1270)$
$\rho(770)$
$\omega(782)$
L

G. de Teramond and sjb, PRL 102 081601 (2009)

$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

$$z \Leftrightarrow \zeta, \quad \Phi_P(z) \Leftrightarrow |\psi(P)\rangle$$

General dilaton profile

- Upon substitution $z \to \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2} \, \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \left(\frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$

find LFWE $(d = 4)$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

with

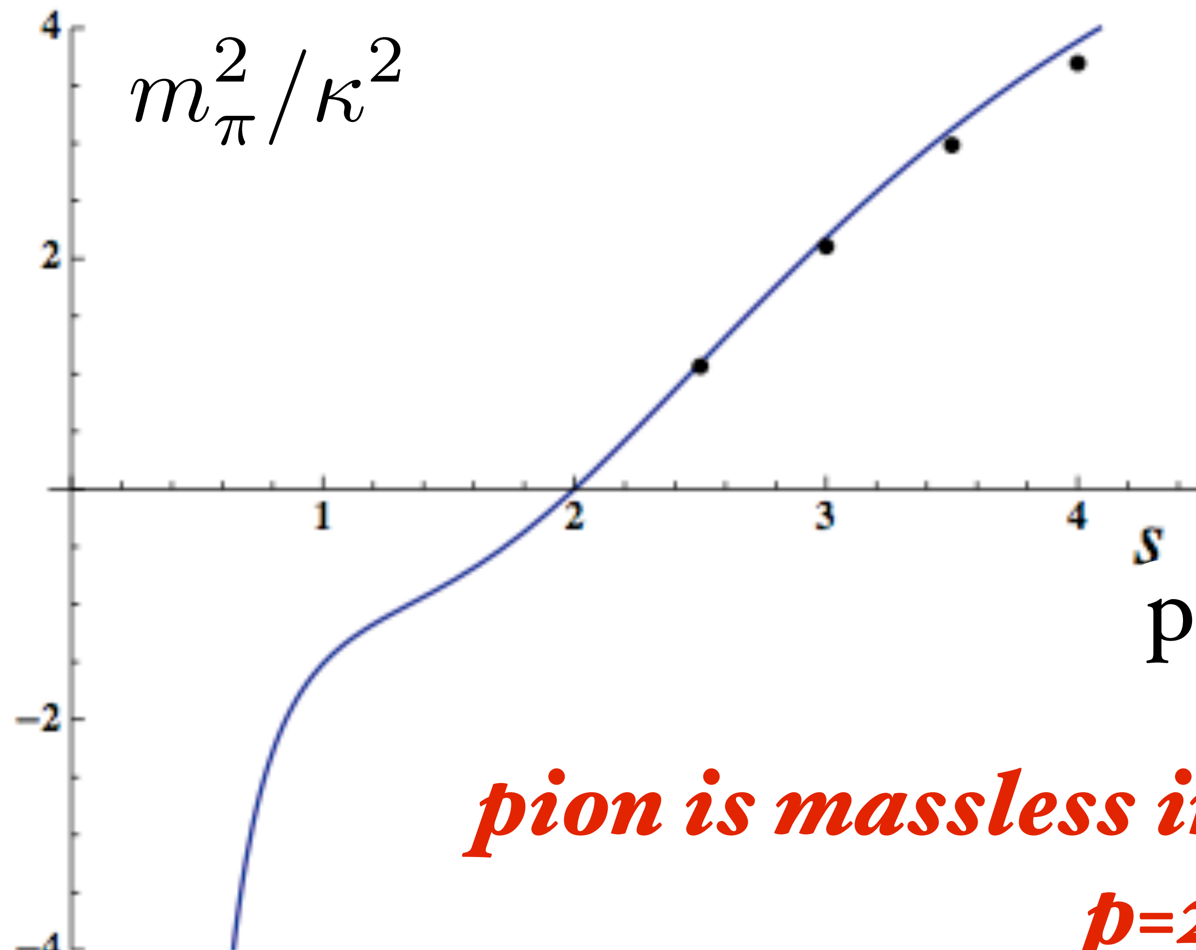
$$U(\zeta) = \frac{1}{2} \phi''(\zeta) + \frac{1}{4} \phi'(\zeta)^2 + \frac{2J - 3}{2\zeta} \phi'(\zeta)$$

and $(\mu R)^2 = -(2 - J)^2 + L^2$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
- Scaling dimension τ of AdS mode $\hat{\Phi}_J$ is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD and determined by QM stability condition

Uniqueness of Dilaton

$$\varphi_p(z) = \kappa^p z^p$$



pion is massless in chiral limit iff p=2!

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **Dosch, de Teramond, sjb**

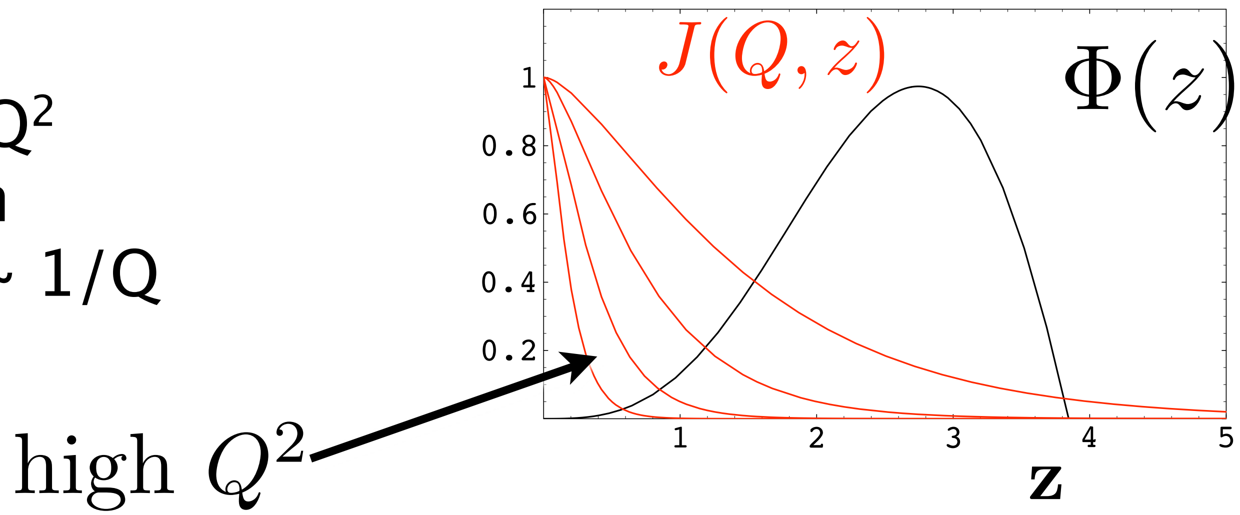
Hadron Form Factors from AdS/QCD

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

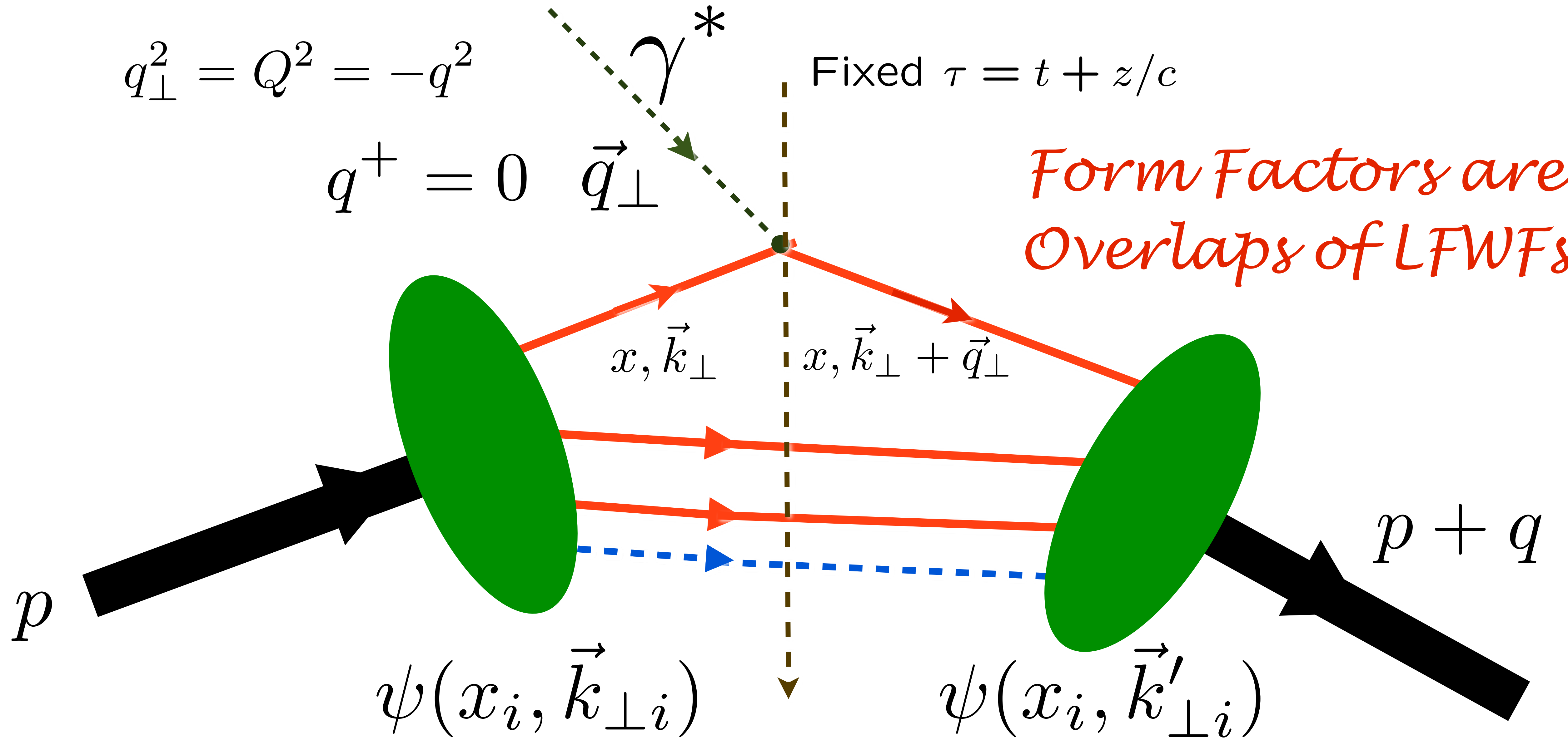
where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n}\sigma_i$.

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

Twist $\tau = n + L$

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture



struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Soper: DYW: Product of LFWFs in transverse space

Holographic Mapping of AdS Modes to QCD LFWFs

Drell-Yan-West: Form Factors are Convolution of LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

de Teramond, sjb

Identical to Polchinski-Strassler Convolution of AdS Amplitudes

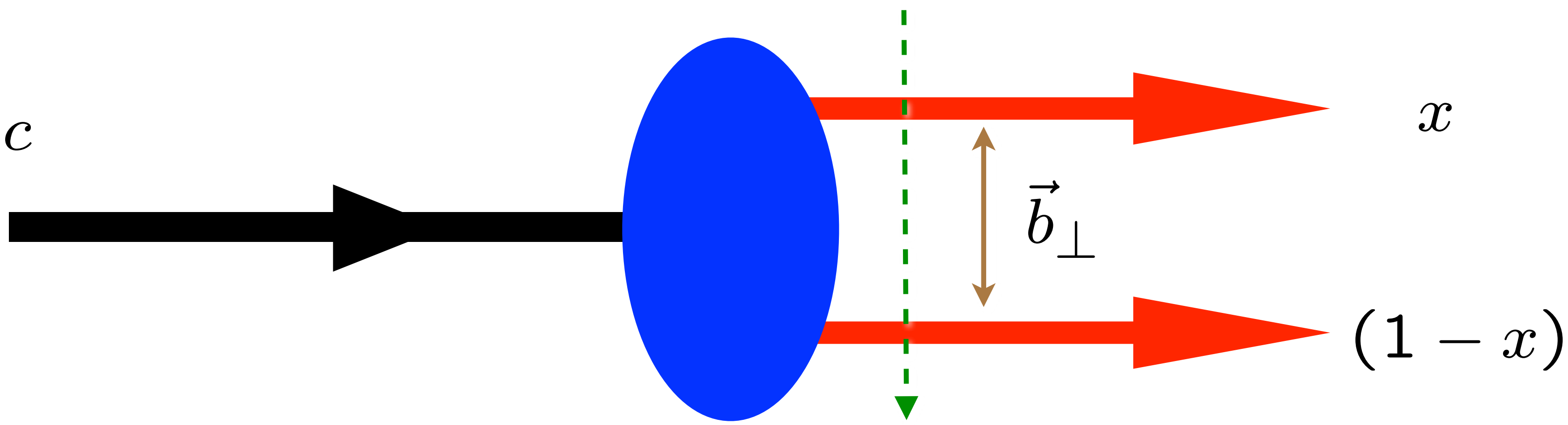
de Teramond, sjb

LF(3+1) ⟷ AdS_5

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$

Fixed $\tau = t + z/c$



$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3}\, H(Q^2, z)\, |\Phi_\pi(z)|^2 \,,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x\, dx\, J_0\!\left(zQ\sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x\, dx \int \frac{dz}{z^3}\, J_0\!\left(zQ\sqrt{\frac{1-x}{x}}\right) |\Phi_\pi(z)|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\left|\tilde\psi_{q\bar q/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi}\, x(1-x)\, \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4},$$

de Teramond & sjb

Identical to LF Holography obtained from electromagnetic current

Light-Front Holography

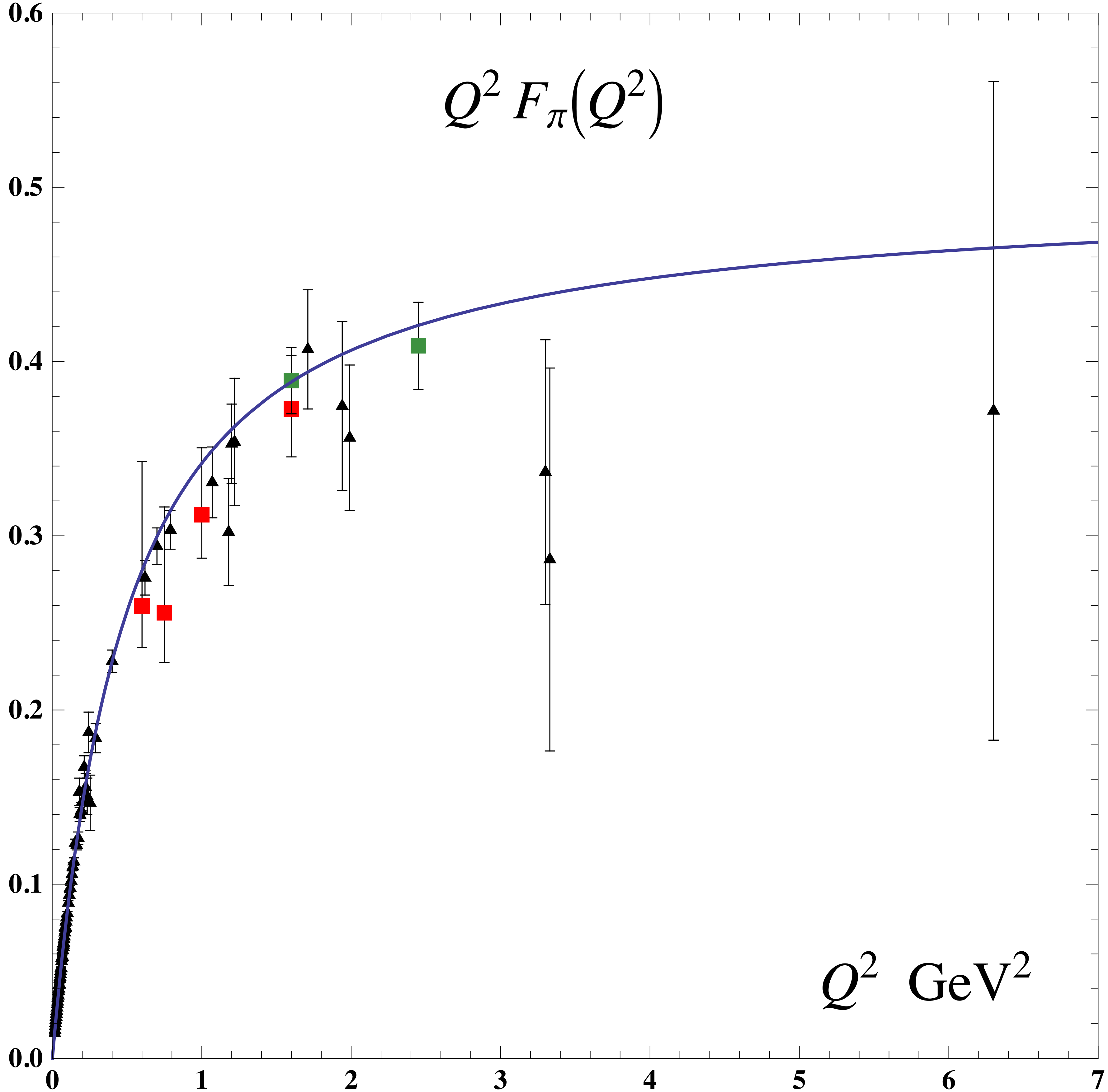
- AdS_5/CFT_4 **Duality between AdS5 and Conformal Gauge Theory in 3+1 at fixed LF time** G. de Téramond, H. G. Dosch, sjb

  Valery E. Lyubovitskij, Tanja Branz, Thomas Gutsche, Ivan Schmidt, Alfredo Vega

- AdS_4/CFT_3 **Construction from Collective Fields"** Robert de Mello Koch, Antal Jevicki, Kewang Jin, João P. Rodrigues

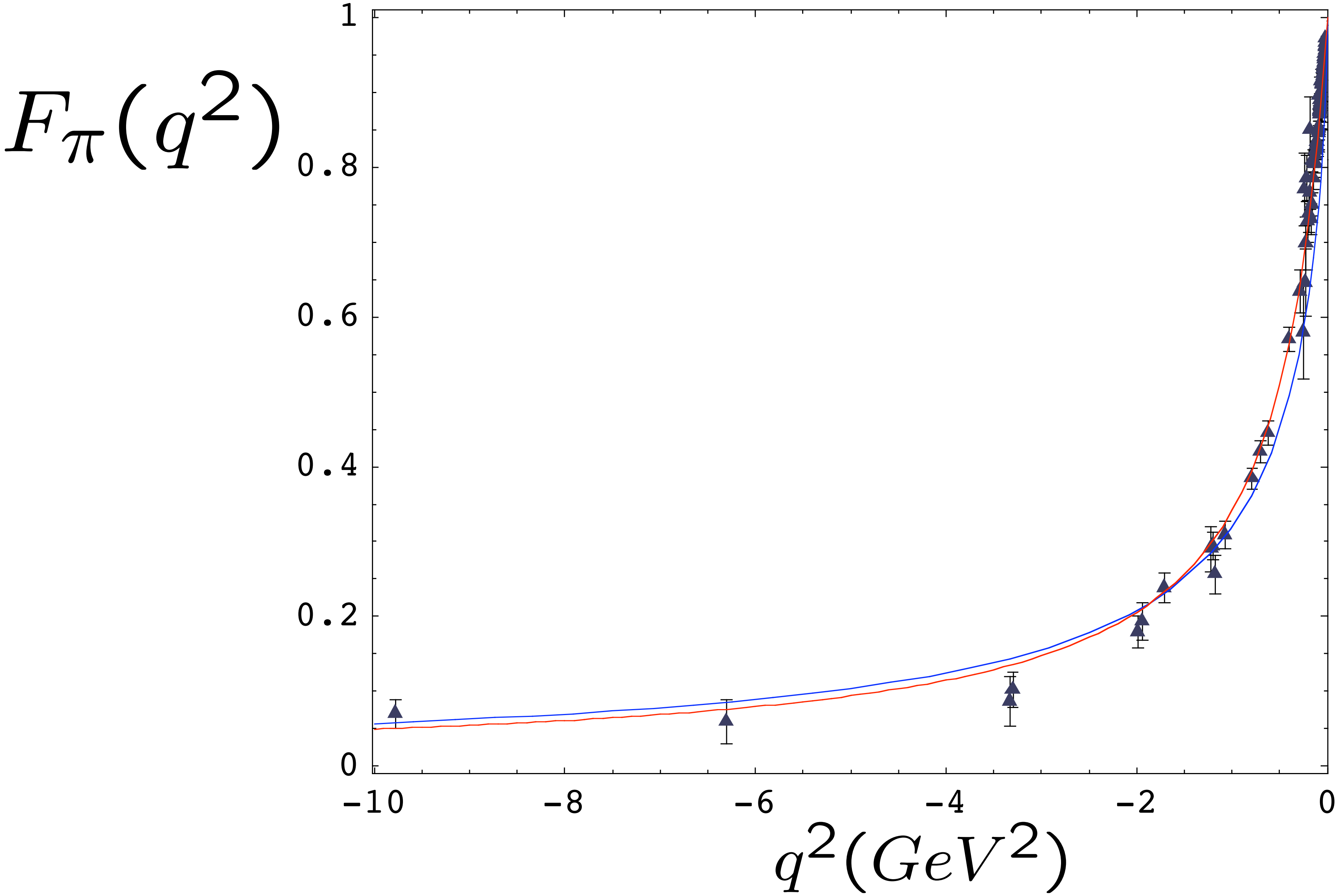
- **"Exact holographic mapping and emergent space-time geometry"** Xiao-Liang Qi

- **Ehrenfest arguments:** Glazek and Trawinski



$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

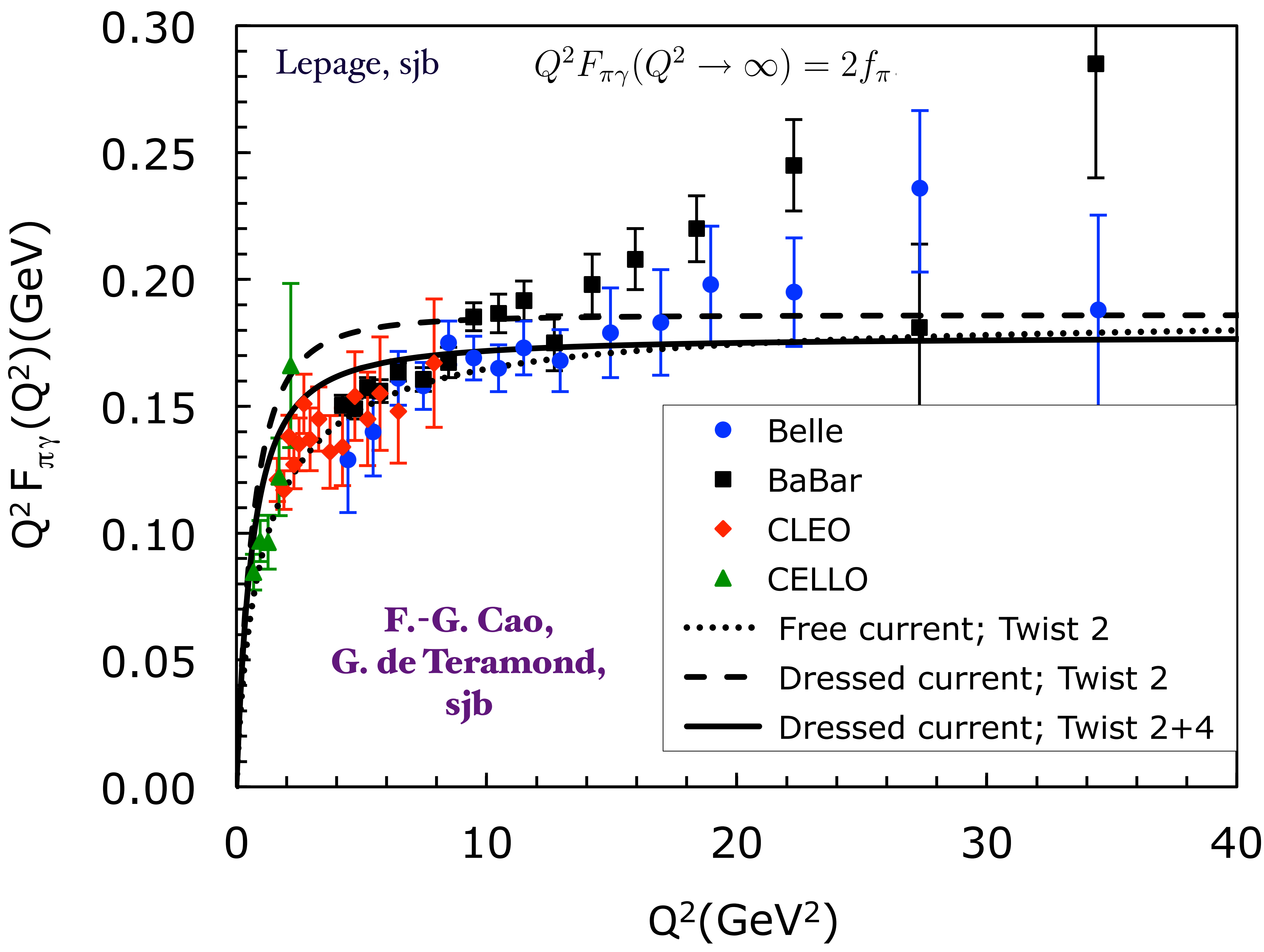
Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

Photon-to-pion transition form factor



Current Matrix Elements in AdS Space (SW)

sjb and GdT
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q,z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a,b,c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

Dressed Current in Soft-Wall Model

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

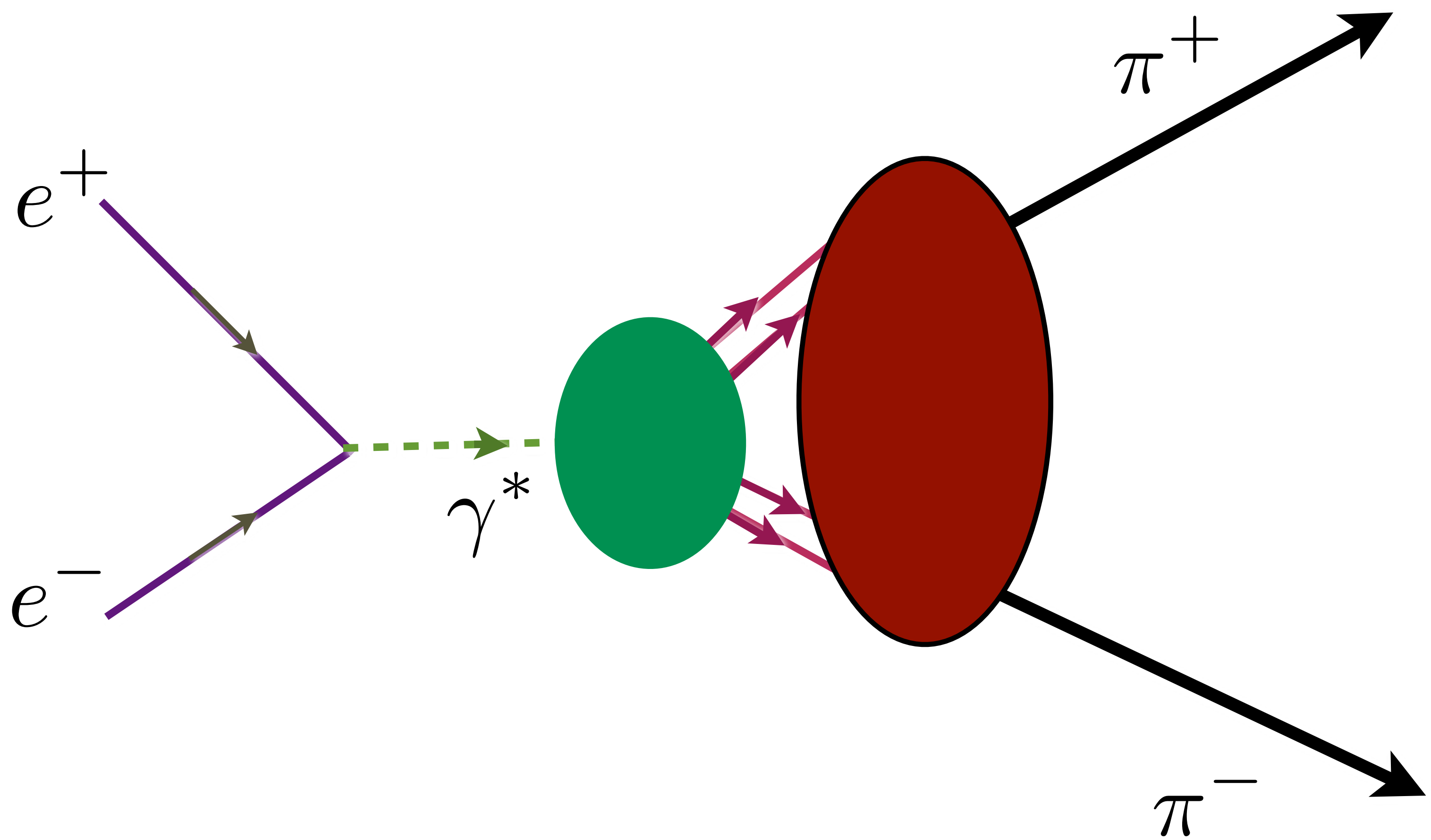
$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q,z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q,z) \to zQK_1(zQ) = J(Q,z),$$

the external current decouples from the dilaton field.

Dressed soft-wall current brings in higher Fock states and more vector meson poles

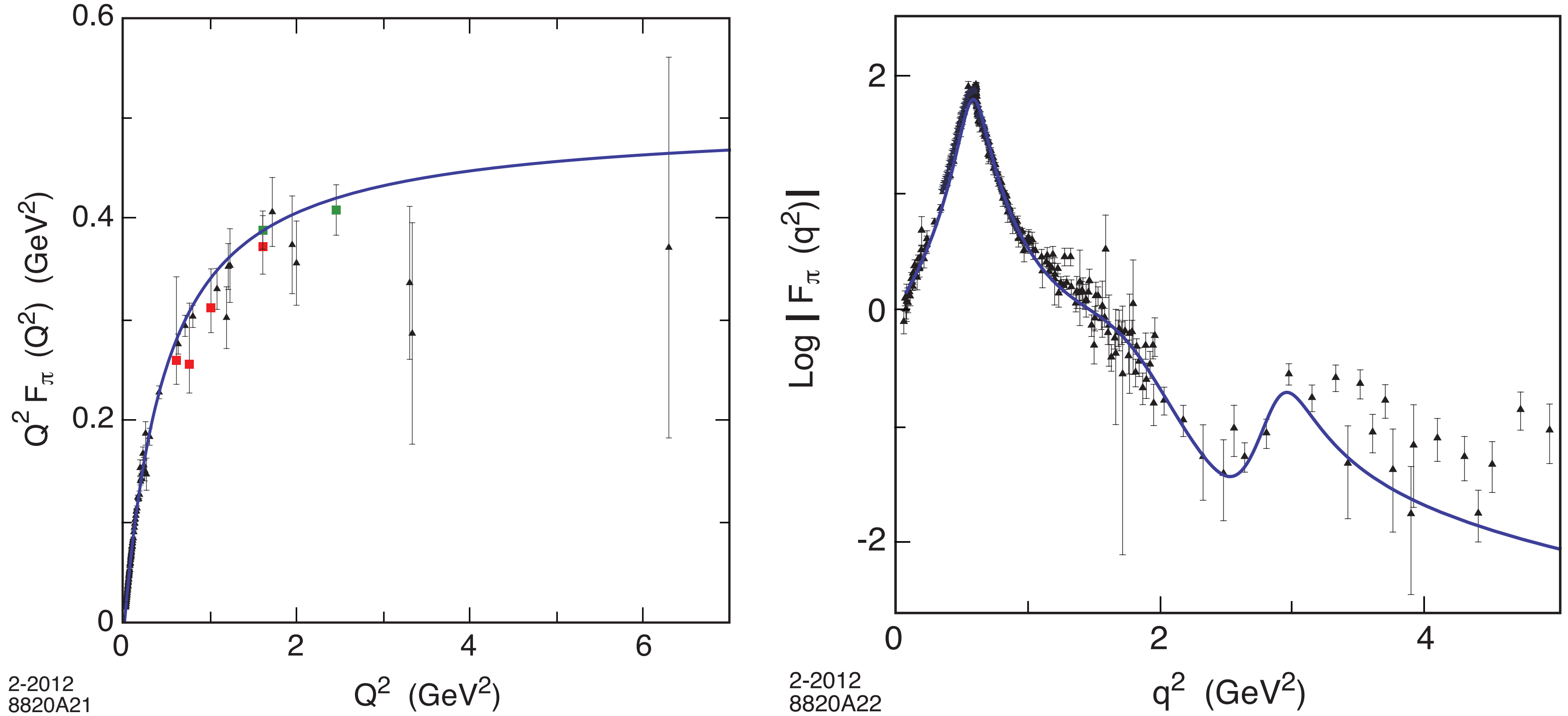


Higher Fock Components in LF Holographic QCD

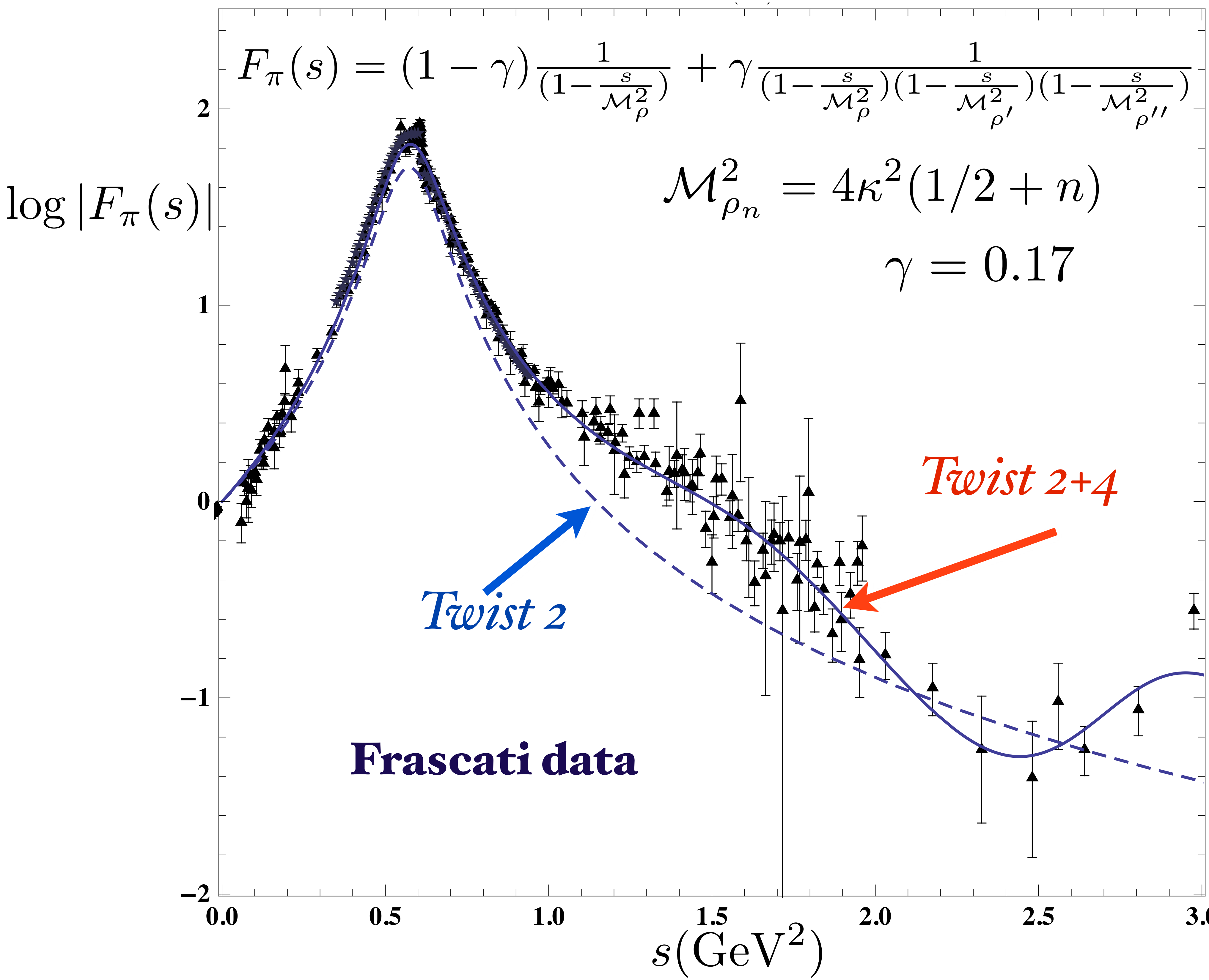
- Effective interaction leads to $qq \to qq$, $q\overline{q} \to q\overline{q}$ but also to $q \to qq\overline{q}$ and $\overline{q} \to \overline{q}q\overline{q}$
- Higher Fock states can have any number of extra $q\overline{q}$ pairs, but surprisingly no dynamical gluons
- Example of relevance of higher Fock states and the absence of dynamical gluons at the hadronic scale

$$|\pi\rangle = \psi_{q\overline{q}/\pi}|q\overline{q}\rangle_{\tau=2} + \psi_{q\overline{q}q\overline{q}}|q\overline{q}q\overline{q}\rangle_{\tau=4} + \cdots$$

- Modify form factor formula introducing finite width: $q^2 \to q^2 + \sqrt{2}i\mathcal{M}\Gamma$ $\quad (P_{q\overline{q}q\overline{q}}$ = 13 %)



Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



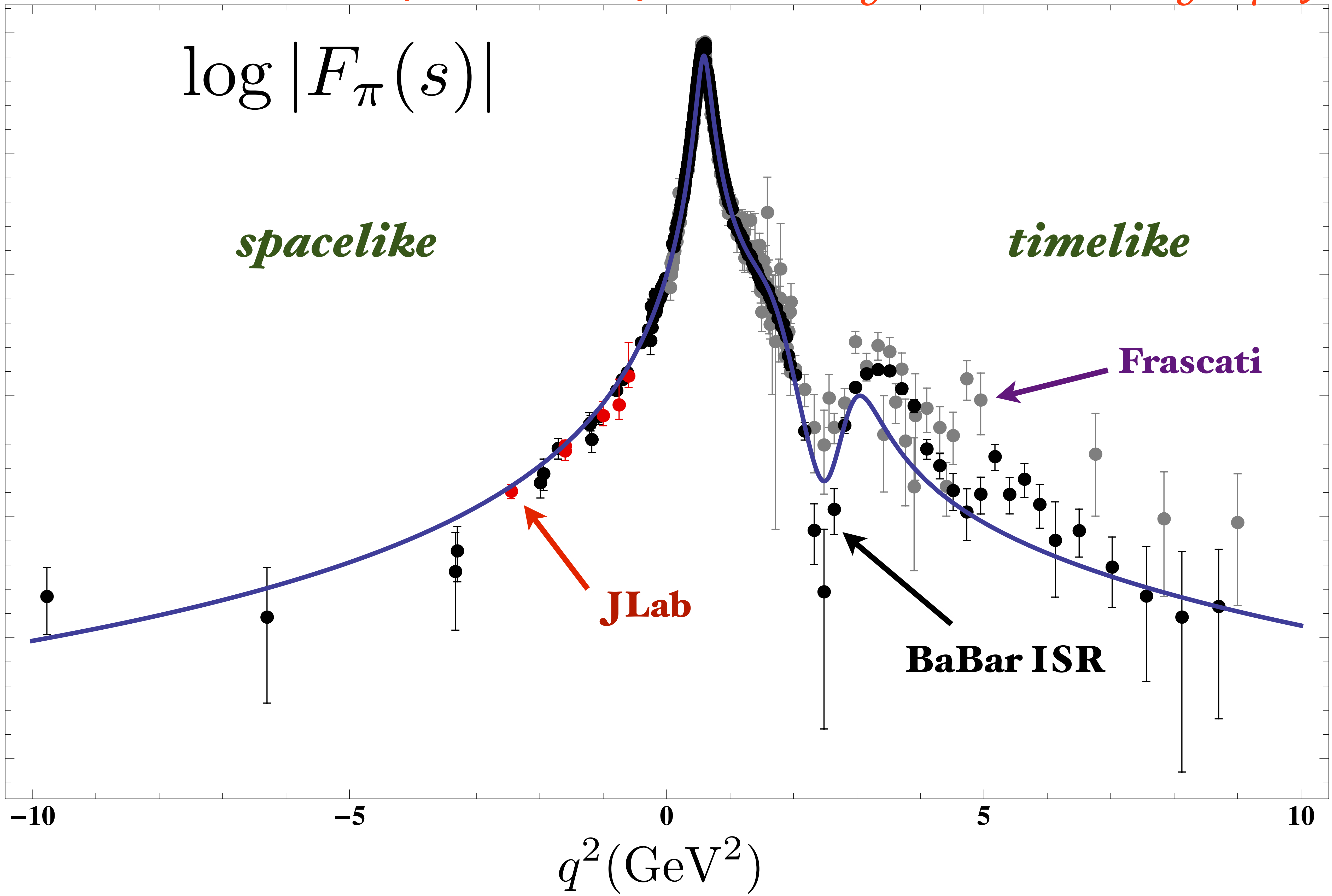
Prescription for Timelike poles :

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

14% four-quark probability

G. de Teramond & sjb

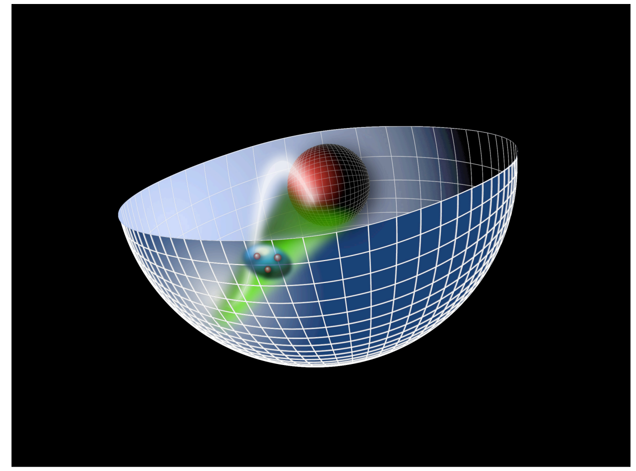
Pion Form Factor from AdS/QCD and Light-Front Holography



de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

Single scheme-independent fundamental mass scale κ



Light-Front Holography

$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp .$$

$$\Big[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\Big]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale:

(m_q=0)

$$\kappa \simeq 0.6 \ GeV$$

$$1/\kappa \simeq 1/3 \ fm$$

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale Λ_{QCD} come from?

How does color confinement arise?

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Unique confinement potential!

Uniqueness

de Teramond, Dosch, sjb

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \qquad e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **ζ^2 confinement potential and dilaton profile unique!**
- **Linear Regge trajectories in n and L: same slope!**
- **Massless pion in chiral limit!   No vacuum condensate!**
- **Conformally invariant action for massless quarks retained despite mass scale**
- **Same principle, equation of motion as de Alfaro, Furlan, Fubini, <u>Conformal Invariance in Quantum Mechanics</u> Nuovo Cim. A34 (1976) 569**

Conformal Invariance in Quantum Mechanics.

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S. FUBINI and G. FURLAN (*)

CERN - Geneva

(ricevuto il 3 Maggio 1976)

Summary. — The properties of a field theory in one over-all time dimension, invariant under the full conformal group, are studied in detail. A compact operator, which is not the Hamiltonian, is diagonalized and used to solve the problem of motion, providing a discrete spectrum and normalizable eigenstates. The role of the physical parameters present in the model is discussed, mainly in connection with a semi-classical approximation.

- **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau)>= i\frac{\partial}{\partial\tau}|\psi(\tau)>$$

New term

$$G = uH + vD + wK$$

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

- **Dosch, de Teramond, sjb**

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

What determines the QCD mass scale Λ_{QCD}?

- Mass scale does not appear in the QCD Lagrangian (massless quarks)
- Dimensional Transmutation? Requires external constraint such as $\alpha_s(M_Z)$
- dAFF: Confinement Scale κ appears spontaneously via the Hamiltonian: $G = uH + vD + wK$ $4uw - v^2 = \kappa^4 = [M]^4$
- The confinement scale regulates infrared divergences, connects Λ_{QCD} to the confinement scale κ
- Only dimensionless mass ratios (and M times R) predicted
- Mass and time units [GeV] and [sec] from physics external to QCD
- New feature: bounded frame-independent relative time between constituents

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

- **Identify with difference of LF time $\Delta x^+/P^+$ between constituents**
- **Finite range**
- **Measure in Double Parton Processes**

Remarkable Features of Light-Front Schrödinger Equation

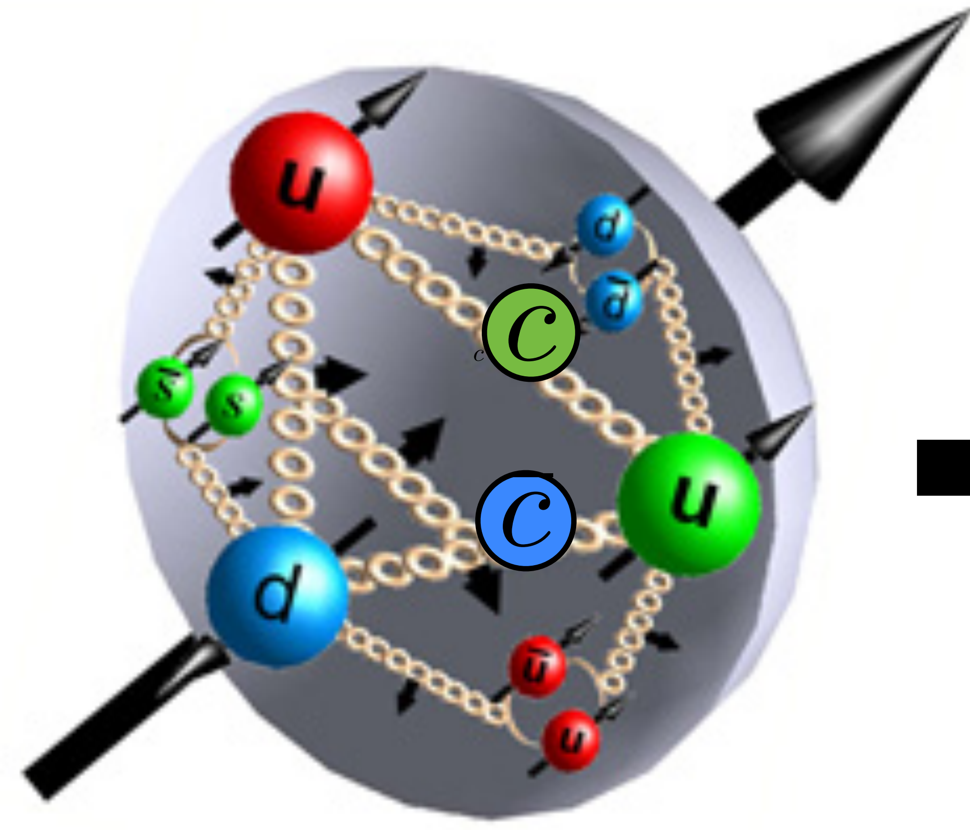
- **Relativistic, frame-independent**
- **QCD scale appears - unique LF potential**
- **Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter**
- **Zero-mass pion for zero mass quarks!**
- **Regge slope same for n and L -- not usual HO**
- **Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry**
- **Phenomenology: LFWFs, Form factors, electroproduction**
- **Extension to heavy quarks**

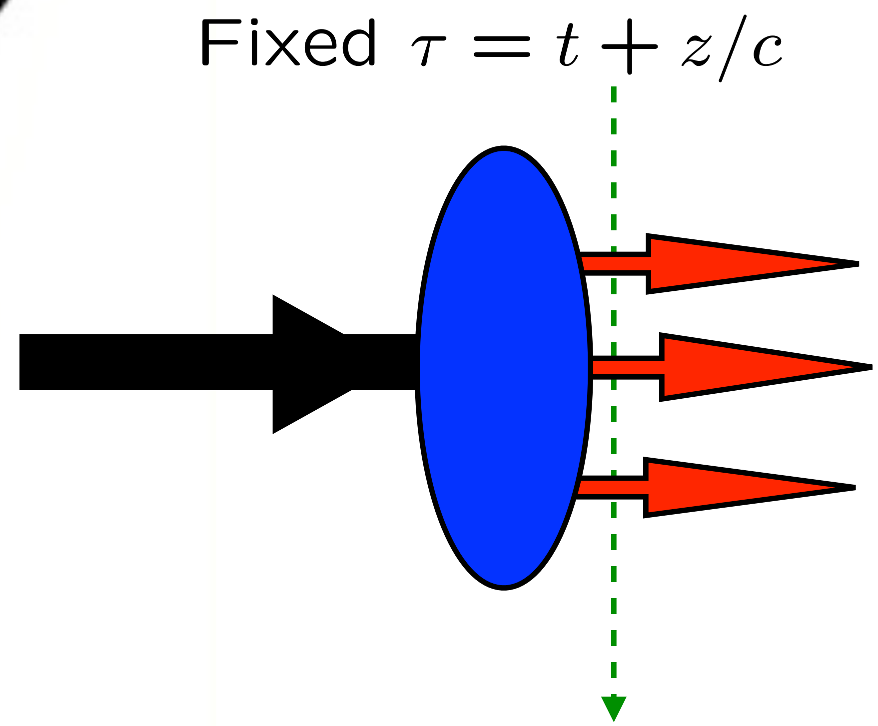
$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

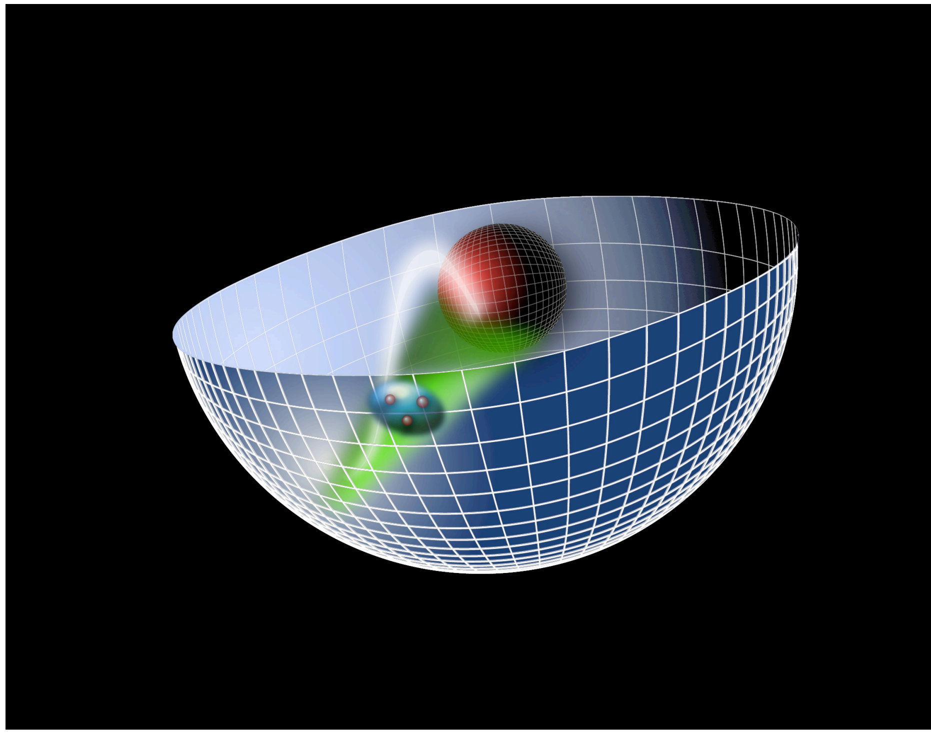
Interpretation of Mass Scale κ

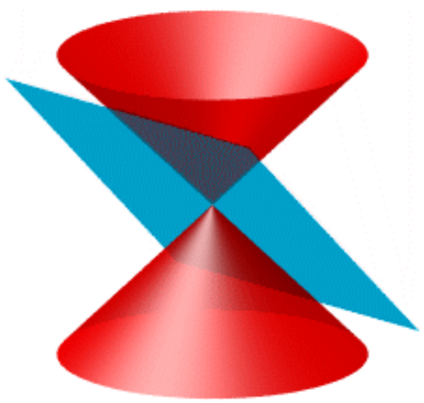
- Does not affect conformal symmetry of QCD action
- Self-consistent regularization of IR divergences
- Determines all mass and length scales for zero quark mass
- Compute scheme-dependent $\Lambda_{\overline{MS}}$ determined in terms of
- Value of κ itself not determined -- place holder
- Need external constraint such as f_π

Light-Front QCD









4th Chilean School of High Energy Physics

Universidad Técnica Federico Santa María

Student Lecture
January 13,2016

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY