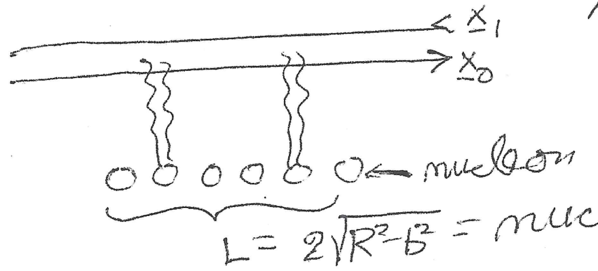
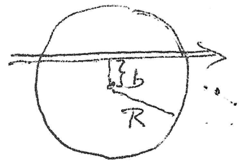
Small-x and Heavy Ion Physics

1. Dipole scattering and evolution on Proton and Nucleus targets

1.1 Evaluation in the MV limit of no evolution. Dipole scattering on a nucleus.



$|\underline{x}_0-\underline{x}_1| = x_{01}$



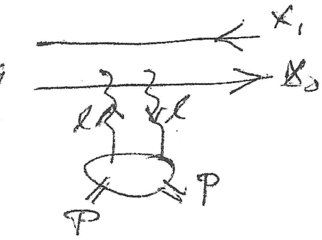
$|S|^2 \simeq S^2$ = probability of no inelastic collision

$S^2 = e^{-L/\lambda}$ $\quad \lambda = \frac{1}{\rho\sigma}$ = mean free path

So

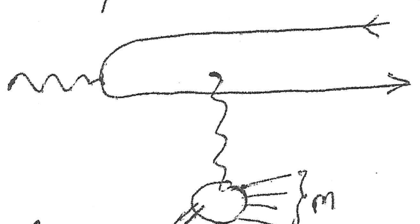
$$S = e^{-2\sqrt{R^2-b^2}\,\rho\sigma/2} \equiv e^{-x_\perp^2 \bar{Q}_s^2/4}$$

$$\sigma = \frac{2\pi^2\alpha C_F}{N_c^2-1}\, xG\, x_\perp^2 = \text{dipole-nucleon scattering}$$



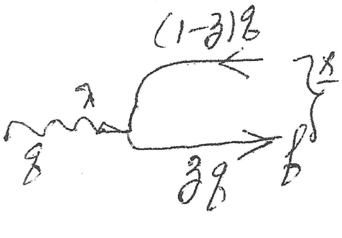
$\bar{Q}_s$ is the "saturation momentum" for quarks. It is the scale at which interactions change from weak, $S \simeq 1$, to strong, $S \simeq 0$. $\bar{Q}_s^2 = \frac{4\pi^2\alpha C_F}{N_c^2-1}\rho L xG = \hat{q}_F L$

1.2 DIS on a proton or large nucleus at small x.

$W_{\mu\nu} \sim \sum_m \Big|$  $\Big|^2$

$q = (q_0, 0, 0, q)$

$p = (m, 0, 0, 0)$



$$\psi^f_{T\lambda}(\underline{x}, z, Q) = \left[\frac{\alpha_{em} N_c\, z(1-z)}{2\pi^2}\left[z^2+(1-z)^2\right]\right]^{1/2} e_f \frac{\underline{\epsilon}\cdot\underline{x}}{|\underline{x}|} K_1\!\left(\sqrt{Q^2 x^2 z(1-z)}\right)$$

$$x q^b_f(x,Q^2) + x\bar{q}^b_f(x,Q^2) = \frac{Q^2}{4\pi\alpha_{em} e_f^2}\sum_\lambda \int d^2b \int \frac{d^2x}{(2\pi)^2}\int_0^1 dz\, |\psi^f_{T\lambda}(x,z,Q)|^2 (1-S(x,b))$$

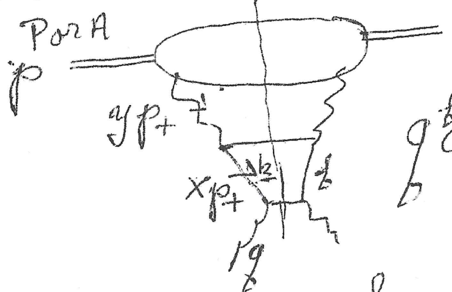
$S \simeq 0 \Leftrightarrow f_q \sim 1 \quad f_g \sim \frac{1}{\alpha N_c}$

Need small-x to view the process as a scattering. All dynamics in S. Unitarity limits the size of the F's. $\left(x = \frac{Q^2}{2p\cdot q} \simeq \frac{Q^2}{s}\right)$

When interaction is weak $S \simeq 1 - x_\perp^2 Q_s^2/4$ and F^b becomes linear in A. When S is small $F \sim A^{2/3}$. When $S \neq 1$ one says that shadowing occurs. (An interaction of the dipole with a nucleon at the front of the nucleus means that later interactions don't count.)

1.3 DGLAP and BFKL evolution

DGLAP evolution is an evolution of parton distributions toward harder scales. For example in



$$q^b(x,Q^2) = \int_x^1 \frac{dy}{y}\int_{\mu^2}^{Q^2} G(y,k^2)\,\frac{\alpha(k^2)}{2\pi}\,2N_f\,P_{G\to q}(x/y)\,\frac{dk^2}{k^2} + \cdots$$

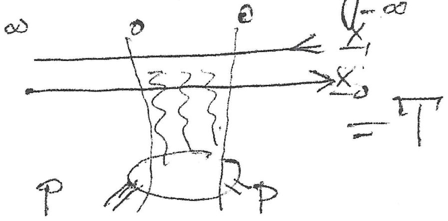
So $$Q^2\frac{\partial}{\partial Q^2} q^b(x,Q^2) = \frac{\alpha(Q)}{2\pi}\int_x^1 \frac{dy}{y}\, G(y,Q^2)\, 2N_f\, P_{g\to q}(x/y)$$

As one looks at smaller transverse sizes, harder transverse momenta, one sees a repeated structure.

DGLAP evolution very important for precise evaluation of hard processes at very hard scales. Not so interesting for high density physics.

<u>BFKL-Dipole</u> small-x evolution

Suppose we scatter a dipole on a proton or large nucleus



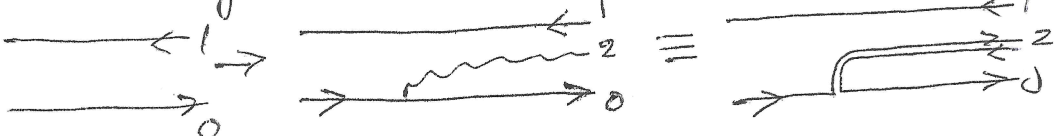
$= T$

$p = (p_+, 0, 0, 0)$

$P = (0, P_-, 0, 0)$

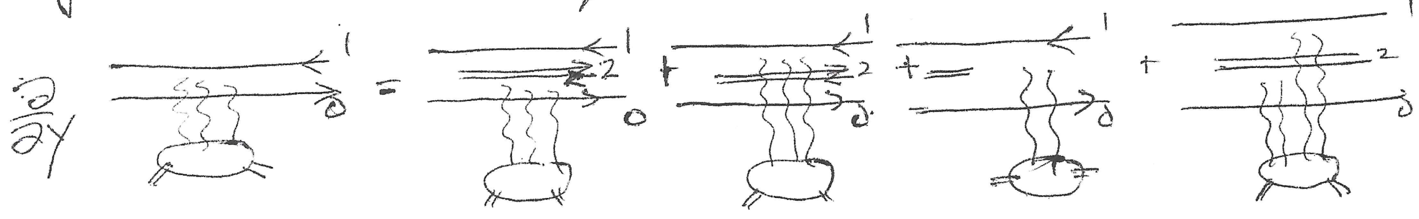
$Y = \ln[\overbrace{(2p_+P_-)x_{01}^2}^{1/x}]$ choose p_+x_{01} not much greater than, ie $\alpha \ln(p_+x_{01}) \ll 1$

At large Y we have put most of the momentum and hence the evolution, into the target. Now increase Y by increasing p_+. Evolution comes about by the dipole creating extra gluons in its wavefunction. In the large N_c limit



dipole $x_{01} \to$ dipoles x_{02}, x_{21} which interact independently on the target.

In pictures the evolution for T is



$$\frac{\partial}{\partial Y} T(x_{01},Y) = \frac{\alpha N_c}{2\pi^2}\int \frac{d^2x_2\, x_{01}^2}{x_{02}^2\, x_{12}^2}\overbrace{\Big[\underbrace{T(x_{02},Y)+T(x_{12},Y)-T(x_{01},Y)}_{\text{BFKL}} - T(x_{02},Y)\,T(x_{12},Y)\Big]}^{\text{BK}}$$

factorization justified for large nuclear target

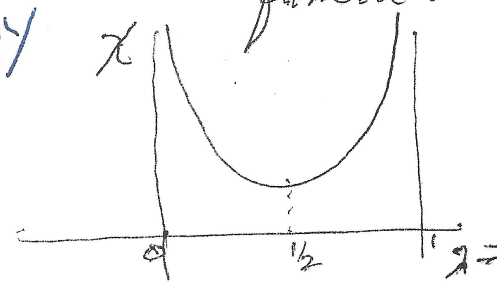
BFKL equation easy to solve

Consider

$$\frac{1}{4\pi}\int \frac{d^2x_2\, x_{01}^2}{x_{02}^2\, x_{12}^2}\left(x_{02}^{2\lambda}+x_{12}^{2\lambda}-x_{01}^{2\lambda}\right) = \chi(\lambda)\,x_{01}^{2\lambda}$$ by dimensions

Explicit evaluation gives $\chi(\lambda) = 2\psi(1) - \frac{1}{2}2\psi(\lambda) - \frac{1}{2}2\psi(1-\lambda)$ = BFKL characteristic function

Then $$T(x_{01},Y) = \int_{1/2-i\infty}^{1/2+i\infty} \frac{d\lambda}{2\pi}\, T_\lambda \cdot x_{01}^{2\lambda}\, e^{\frac{2\alpha N_c}{\pi}\chi(\lambda) Y}$$



Saddle point at $\lambda = \frac{1}{2}$ gives

$$T(x_{01},Y) = T^{(0)}_{1/2}\,\frac{x_{01}\, e^{(\alpha_P-1)Y}}{\sqrt{56\,\alpha N_c\,\zeta(3)\,Y}}$$

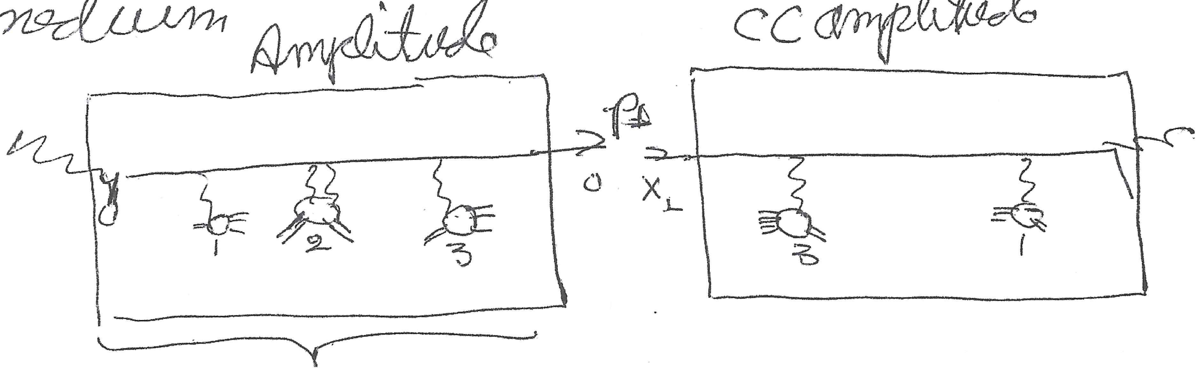
$$\alpha_P - 1 = \frac{2\alpha N_c}{\pi}\chi(\tfrac{1}{2}) = \frac{4\alpha N_c \ln 2}{\pi}$$

$$\chi''(\lambda) = 14\zeta(3)$$

2. $p_\perp$-broadening and energy loss in hot and cold matter

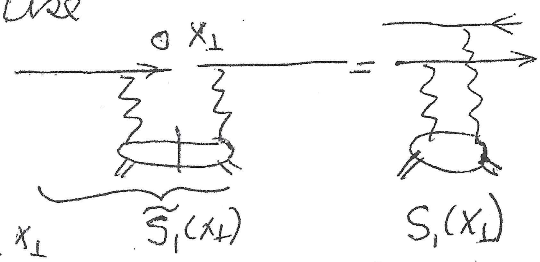
2.1 An almost theorem

Suppose a jet is produced in a nucleus at a well defined position and with zero transverse momentum, for example in large-x DIS. Let's try to calculate the transverse momentum it gets as it passes through the medium



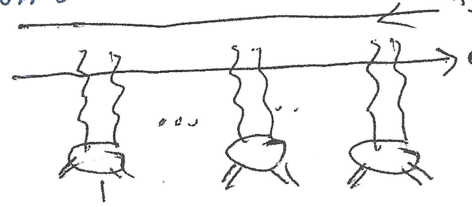
$$\frac{d^2N}{d^2p_\perp} = \int \frac{d^2x_\perp}{4\pi^2}\, e^{-i p_\perp \cdot x_\perp} \underbrace{\tilde{S}(x_\perp)}_{-\tilde{S}_1(x_\perp)}$$

Use



To get (do one scattering in detail)

$\tilde{S}(x_\perp) \equiv S(x_\perp) =$

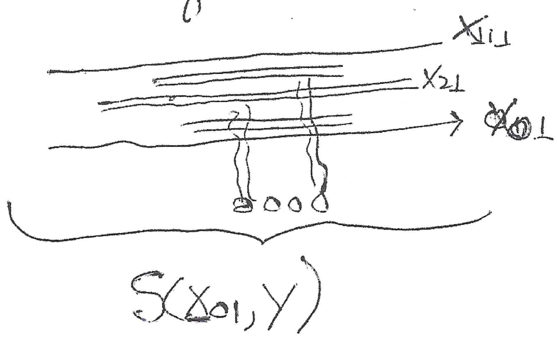


= Same dipole S matrix we had earlier

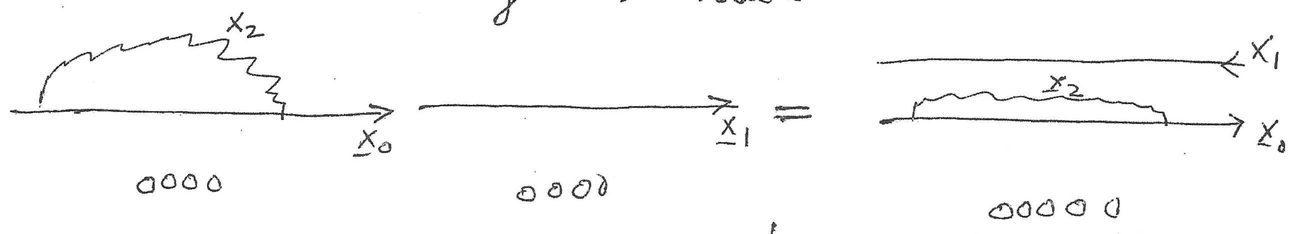
$S = e^{-x_\perp^2 Q_s^2/4}$

This holds in the presence of evolution!

$$\frac{dN}{d^2p_\perp} = \int \frac{d^2x_\perp}{4\pi^2}\, e^{-i p_\perp \cdot x_{01}}$$

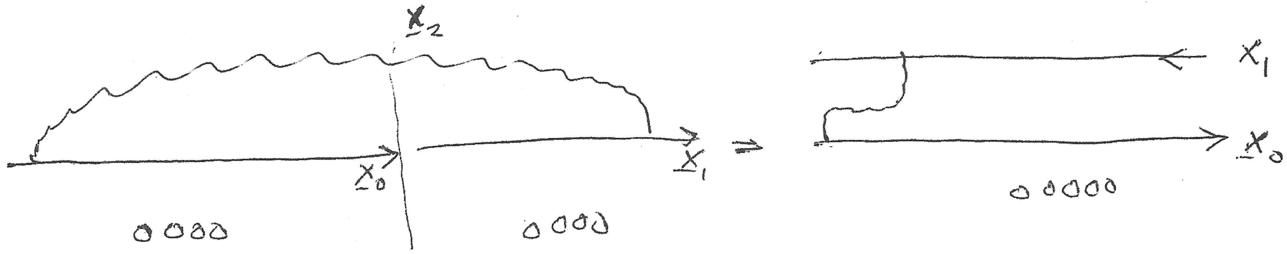


Look a bit closer. One gluon emission



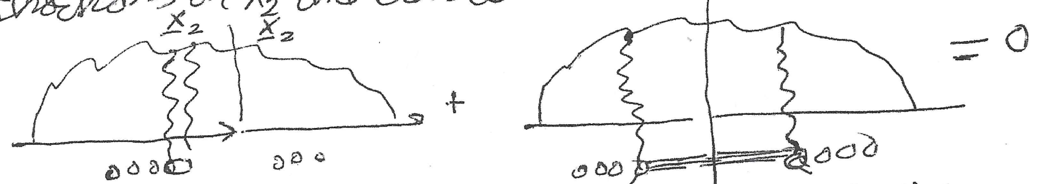
Large N_c

$S(x_{02})\,S(x_{12})$ $\qquad$ $S(x_{02})\,S(x_{12})$ ∴

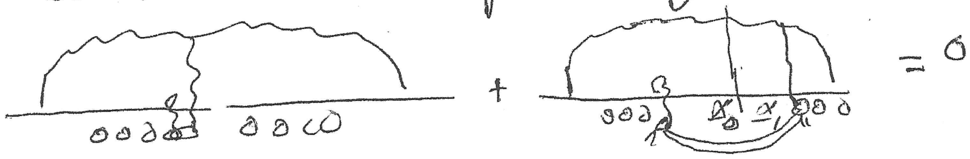


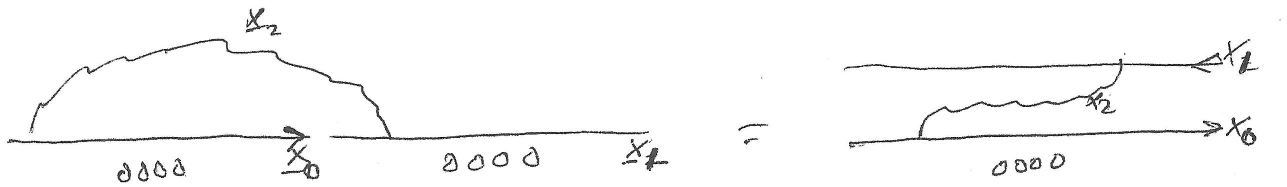
$S(x_{01})$ $\qquad$ $S(x_{01})$

Interactions on x_2 line cancel



Don't measure $\underline{x}_2$ so probability is 1 that it survives





$S(x_{02})S(x_{12})$ $\qquad$ $S(x_{02})\,S(x_{12}$

Equality between $\frac{dN}{d^3p_\perp}$ and $S(x_\perp)$ not completely general. But survives at next to leading logs. Probably breaks down at next-to-next-leading logs. $\underbrace{\text{Schwinger, Keldysh} \longleftrightarrow \text{time ordered}}_{\text{ok to NLO}}$

$$\langle p_\perp^2\rangle = \int d^2p_\perp \int \frac{d^2x_\perp}{4\pi^2} \underbrace{p_\perp^2 e^{-ip_\perp\cdot x_\perp}}_{-\nabla^2_{x_\perp} e^{-ip_\perp\cdot x_\perp}} S(x_\perp) = \int d^2p_\perp \int \frac{d^2x_\perp}{4\pi^2} e^{-ip_\perp\cdot x_\perp} \nabla^2_{x_\perp} S(x_\perp)$$

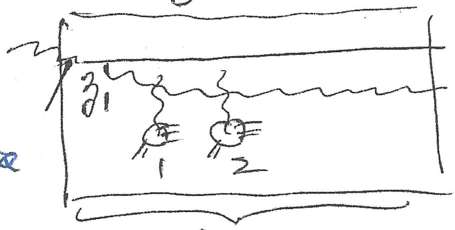
$$\langle p_\perp^2\rangle = \underbrace{-\nabla^2_{x_\perp} S(x_\perp)\big|_{x_\perp=0}}_{\text{single scattering } \times \rho L} = \underbrace{\hat q L = \langle p_\perp^2\rangle}_{\hat q \text{ transport coefficient}}$$

Recall without evolution $\nabla^2_{x_\perp} S|_{x=0} = -Q_s^2$ so $\underline{Q_s^2 = \hat q L}$

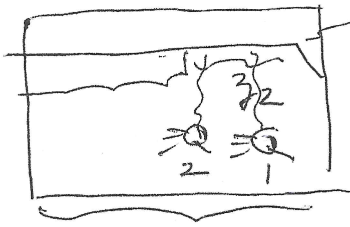
2.2 Energy loss (radiative) related to $p_\perp$-broadening

$$\boxed{-\frac{dE}{dz} = \frac{\alpha}{4}\hat q L^2}$$

do calculation (without '1/4') here



L



L

L^2 comes from $dz_1 dz_2$

Holds to higher order corrections

$$\langle p_\perp^2\rangle = \hat q_0 L\left(1 + \frac{\alpha N_c}{8\pi}\ln^2(L/l_0)^2\right) = \hat q L$$

$\hat q = \hat q_0\left(1 + \frac{\alpha N_c}{8\pi}\ln^2(L/l_0)^2\right)$ also gives $\frac{dE}{dz}$ leading order corrections using above formula for $-\frac{dE}{dz}$.

$\frac{dE}{dz}$ is believed to be main source of jet quenching

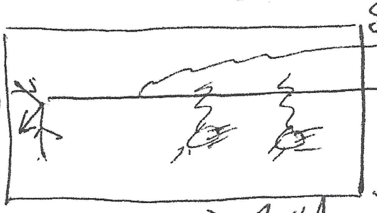
Hard probes collaboration determines $\hat q \simeq 1\,\mathrm{GeV}^2/\mathrm{fm}$ RHIC
$\simeq 2\,\mathrm{GeV}^2/\mathrm{fm}$ LHC

For cold matter $\hat q \simeq 0.075\,\mathrm{GeV}^2/\mathrm{fm}$ at Fermilab fixed target

<u>Roughly</u> $\hat q$ counts # of scatterers in medium

2.3 Important difference; energy loss from jet produced in medium or jet coming into medium

(i) in medium



$\Delta E = \hat{q} L^2$ jet produced in medium

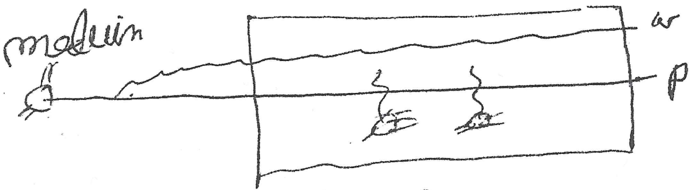
Simple "derivation"

$$\tau_g = \frac{2\omega}{k_\perp^2} = \frac{2\omega}{\hat{q} L} = L \qquad \omega = \frac{1}{2}\hat{q}L^2$$

prob of emission $= \alpha$

$$-\Delta E = \alpha\omega \cancel{\alpha}\ \alpha\hat{q}L^2, \quad -\Delta E = \frac{\alpha}{4}\hat{q}L^2$$

(ii) entering medium



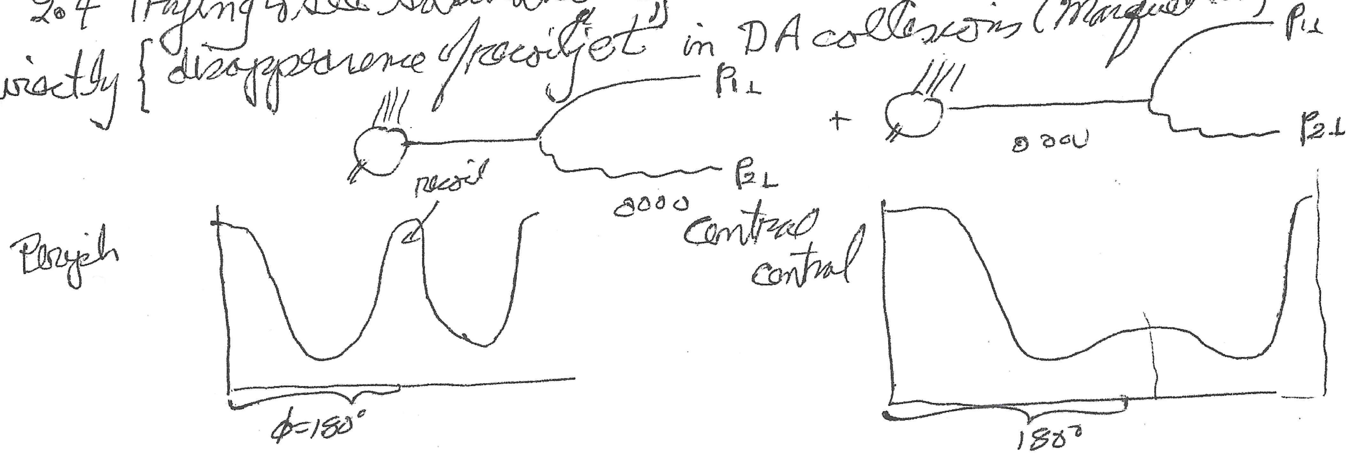
$\omega \sim \alpha p$ J/ψ, forward jet prod in pA collisions

$-\Delta E \sim \alpha p$

Arleo's talk at conference

2.4* Trying to see saturation

directly { "disappearance of recoil jet" in DA collisions (Marquet ...)



indirectly (Stasto, Watanabe, Xiao, Yuan, Zaslavsky) $y = 2\text{–}3$ $y \sim 0$

Fit hadron production in p(D)A collisions at RHIC, LHC. Use BK match onto DGLAP at large $p_\perp$. Cross sections go negative for $p_\perp$ too big in BK. However, BK stops the data from becoming too large at small $p_\perp \lesssim 1$

DIS: New H1-ZEUS joint fits show DGLAP starts to break down for $Q^2 \lesssim 10$ (Albacete) BK seems to fix this.

(Iancu et al ...)

3. Equilibration in Heavy Ion Collisions

Big topic not yet completely settled down
Classical Y-M vs Kinetic theory. Significant overlap

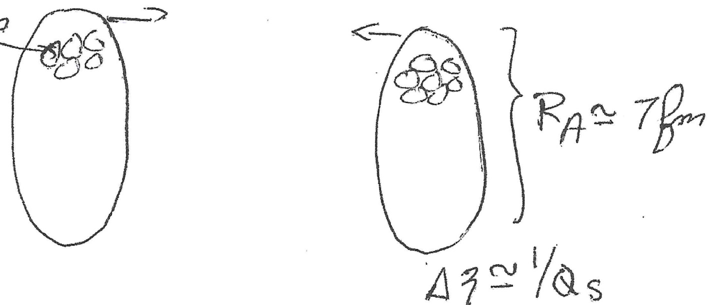
Overall picture of H I Collision BMSS Phys Lett B502 (2001) 51
A. Kurkela & Y. Zhu arXiv: 1506.06647 [hep-ph]
Berges et al (many papers)
Gelis et al

Take CM frame

Just before collision

$\Delta x_\perp \sim 1/Q_s$

$f_g \sim 1/\alpha$



Nucleus has single layer of about $\frac{1}{\alpha} Q_s^2 R^2$ gluons per unit of rapidity having $p_\perp \sim Q_s$. All gluons with $p_\perp \le Q_s$ are freed; all gluons with $p_\perp > Q_s$ are not.

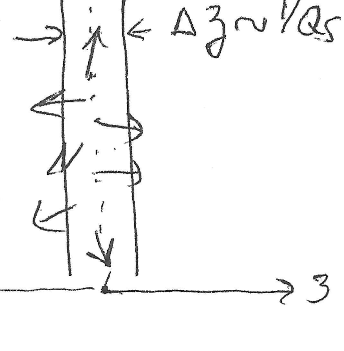
After a time $\Delta t \sim 1/Q_s$ the gluons in the central unit of rapidity $-\frac{1}{2} < y < \frac{1}{2}$ are left by themselves as gluons with higher rapidity move away. This is the time over which these gluons are freed.

Sequence of steps in equilibration:

(a) Very early times $1 < Q_s t < \alpha^{-3/2}$ (Classical f.t.)

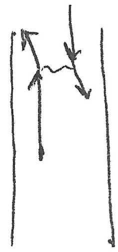
At $Q_s t \sim 1$ $\quad f_g \sim 1/\alpha, \; k \sim Q$

Gluons run away toward larger z leaving gluons having predominately p_z small, $k_\perp \sim Q_s$



$N_h \sim \frac{Q_s^3}{\alpha(Q_s t)}$ reflects linear expansion

However there is also elastic scattering



$$\frac{dN_{col}}{dt} = \underbrace{\sigma}_{\frac{\alpha^2}{m_D^2}} N_h (1+\underbrace{f_h}_{\frac{N_h}{Q_s^2 p_z}}) = N_h^2 \frac{\alpha^2}{m_D^2} \frac{1}{Q_s^2 p_z} = \frac{\alpha N_h}{m_D^2 p_z t}$$

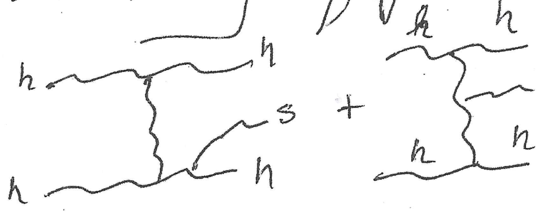
and $p_z^2 \sim \frac{dN_{col}}{dt} \cdot t \, m_D^2 = \frac{\alpha N_h \, m_D^2}{m_D^2 p_z} = \frac{\alpha N_h}{p_z}$

$$\boxed{p_z = (\alpha N_h)^{1/3} = \frac{Q_s}{(Q_s t)^{1/3}} = k_s}$$

So $f_h = \frac{N_h}{Q_s^2 p_z} = \frac{Q_s^3}{\alpha(Q_s t)} \frac{1 (Q_s t)^{1/3}}{Q_s^2 Q_s}$; $\boxed{f_h = \frac{1}{\alpha}\frac{1}{(Q_s t)^{2/3}}}$

f_h large until $Q_s t \sim (\frac{1}{\alpha})^{3/2}$. This is domain of classical field theory.

Now how many soft particles are there?



$+ \cdots$ $\qquad m_D^2 \sim \alpha \frac{N_h}{Q_s} = \frac{Q_s^2}{Q_s \tau}$

$$N_s = t \frac{dN_s}{dt} = t \frac{\alpha^3}{m_D^2} N_h^2 (1+f_h)^2 \sim \frac{Q_s^3}{\alpha} \frac{1}{(Q_s t)^{4/3}}$$

$$\boxed{f_s = \frac{N_s}{k_s^3} = \frac{1}{\alpha}\frac{1}{(Q_s t)^{1/3}}}$$

(b) $\alpha^{-3/2} < Q_s\tau < \alpha^{-5/2}$

Now $m_D^2 \sim \frac{\alpha N_s}{k_s}$ but $N_s \ll N_h$ ∴ $f_h < 1$

$k_s^2 = N_{col} m_D^2 = t\sigma N_h m_D^2$

$\frac{\alpha^2}{m_D^2} \frac{Q^3}{\alpha Q_s t}$

$N_s = t\frac{\alpha^3}{m_D^2} N_h^2 = \frac{\alpha Q_s^4}{m_D^2 t}$

$\boxed{k_s^2 = \alpha Q_s^2}$ ∴

$$\boxed{N_s = \frac{\alpha^{1/4} Q_s^3}{(Q_s\tau)^{1/2}} \qquad m_D^2 \sim \frac{\alpha^{3/4} Q_s^2}{(Q_s\tau)^{1/2}}}$$

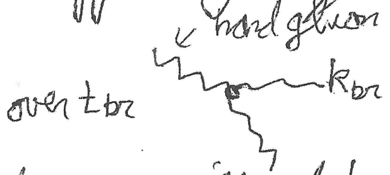
$N_s \sim N_h$ when $\frac{\alpha^{1/4} Q_s^3}{(Q_s t)^{1/2}} \sim \frac{Q_s^3}{\alpha(Q_s t)}$ or $\boxed{Q_s t \sim \alpha^{-5/2}}$

at $Q_s t \sim \alpha^{-5/2}$ $\quad f_s = \frac{\alpha^{1/4} Q_s^3}{(Q_s\tau)^{1/2}} \frac{1}{\alpha^{3/2} Q_s^3} \sim \alpha^{\frac{1}{4} - \frac{3}{2} + \frac{5}{4}} \sim O(1)$

$f_s k_s^3 = \alpha^{3/2} Q_s^3 = N_h = \frac{Q_s^3}{\alpha(Q_s t)} \sim Q_s^3 \alpha^{3/2}$

(c) $Q_s t > \alpha^{-5/2}$

When $N_s > N_h$ $f_s \sim 1$ thermalization of soft gluons occurs. Most scatterings are by soft gluons. Most energy still in hard gluons but they no longer affect scattering. Hard gluons disappear by the following (turbulent) mechanism:



such that k_{br} decays over t_{br} in soft particles

then we will set $t_{br} = t$ = thermalization time!

$t_{br} = \frac{1}{\alpha} t_f = \frac{1}{\alpha}\frac{k_{br}}{k_t^2} = \frac{1}{\alpha}\frac{k_{br}}{\cancel{m_D^2}\,\frac{t_{br}}{\alpha t_{br}}\, N_s \frac{\sigma \alpha^2}{\cancel{m_D^2}}} \Rightarrow \boxed{\frac{1}{t_{br}} = \alpha^2\sqrt{\frac{N_s}{k_{br}}}}$

$N_s = T^3$, $t_{br} = t \Rightarrow \boxed{k_{br} = \alpha^4 T^3 t^2}$

Now flow of energy from hard to soft is

$$k_{br}\frac{dN(k_{br})}{dt} = k_{br}\cdot\frac{N_h}{t_{br}} = k_{br}\frac{Q_s^3}{\alpha Q_s t}\frac{1}{t} = k_{br}\frac{Q_s^2}{\alpha t^2}$$

$$= \alpha^4 T^3 t^2 \frac{Q_s^2}{\alpha t^2} = \alpha^3 Q_s^2 T^3$$

This must be $\frac{dT^4}{dt} = T^3\frac{dT}{dt}$ so $\frac{dT}{dt} = \alpha^3 Q_s^2$

$$\boxed{T = \alpha^3 Q_s^2 t}$$

Now set $k_{br} = Q_s \Rightarrow Q_s = \alpha^4 T^3 t^2 = \alpha^4 \alpha^9 Q_s^6 t^5$ or $T^4 = \frac{Q_s^4}{\alpha (Q_s t)}$

or $\boxed{t = 1/Q_s\left(\frac{1}{\alpha}\right)^{13/5}}$ = parametric thermalization time.

The $k_{br}\frac{dN(k_{br})}{dt}$ = flow of energy is a turbulent flow. Gives good? picture of jet energy loss in thermal medium