

# Feature Selection Topics

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# Outline



- Motivation
- What is Feature Selection
- Feature Selection Methods
- Recent work and ideas
- Caveats

# Motivation



- Common **Machine Learning (ML)** problems in HEP:
  - **Classification** or class discrimination
    - Higgs event or background?
  - **Regression** or function estimation
    - How to best model particle energy based on detector measurements

# Motivation continued



- While performing data analysis one of the most **crucial decisions** is which features to use
  - Garbage In = Garbage Out
  - Ingredients:
    - **Relevance** to the problem
    - **Level of understanding** of the feature
    - **Power of the feature** and its relationship with others

# Goal



- **How to:**



**Feature set**  
**used to solve the problem**

# Example



- Build a **classifier** to discriminate events of different classes based on event kinematics
- Typical initial feature set:
  - **Functions of object four-vectors** in event
  - **Basic kinematics:** transverse momenta, invariant masses, angular separations
    - More complex features relating objects in the event topology using **physics knowledge** to help discriminate among classes (thrusters, helicity e.t.c.)

# Initial Selection



- Features initially chosen due to their individual **performance**
  - How well does **X** discriminate between signal and background?
    - **Vetos:** Is **X** well-understood?
    - Theoretical and other uncertainties
    - Monte-Carlo and data agreement
  - Arrive at order of 10-30 features (95% use cases)

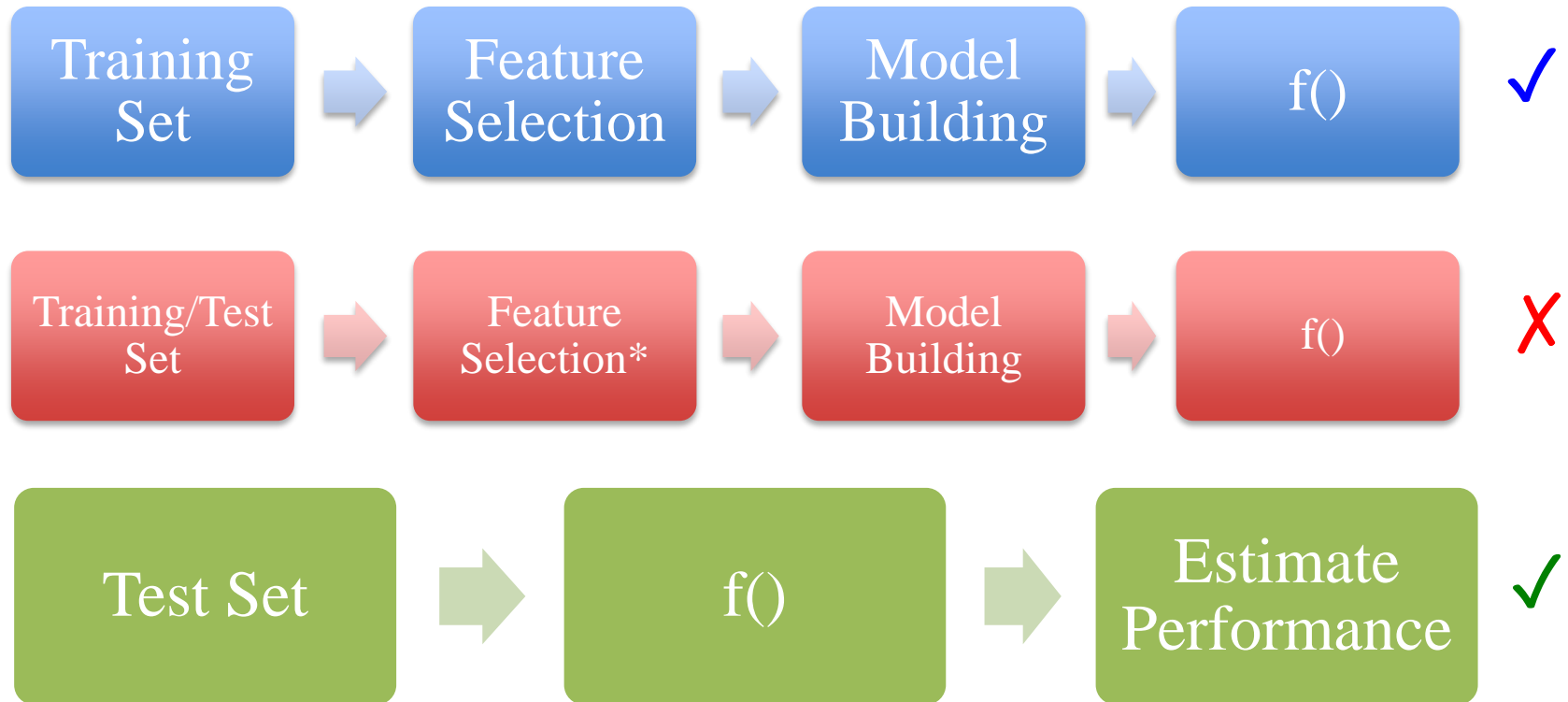
# Feature Engineering



- **By combining features** with each other, **boosting into other frames** of reference\*  
this set can grow quickly from tens to hundreds of features
  - That's ok if you have enough of computational power
    - Still small compared to 100k features of cancer/image recognition datasets
  - Balance between Occam's razor and need for additional performance/power

\* JHEP 1104:069,2011 K. Black, et. al.



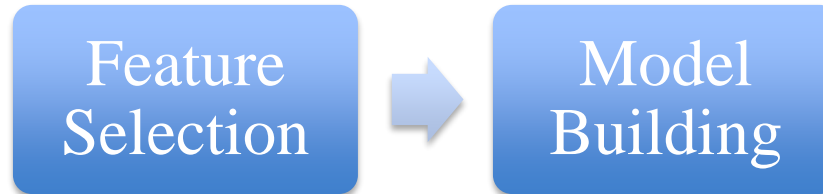


**\*Feature Selection Bias**

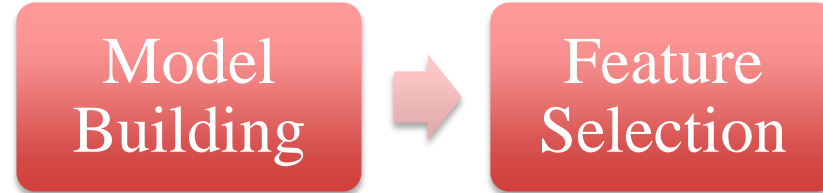
# Methods



## Filters



## Wrappers



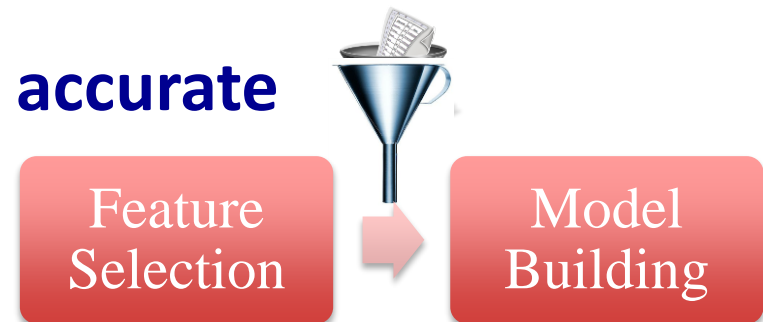
## Embedded-Hybrid



# Filter Methods



- **Filters: usually fast**
    - No **feedback** from the Classifier
    - Use correlations/mutual information gain
- “quick and dirty” and less accurate**
- Useful in pre-processing**



**Example algorithms:** information gain,  
Relief, rank-sum test, e.t.c.

# Wrapper Methods



- **Wrappers: typically slower and relatively more accurate (due to model-building)**
  - **Tied to a chosen model:**
    - Use it to evaluate features
    - Assess feature interactions
    - Search for optimal subset of features
  - **Different types:**
    - Methodical
    - Probabilistic (random hill-climbing)
    - Heuristic (forward backward elimination)

Model  
Building



Feature  
Selection

# Ex: Feature Importance



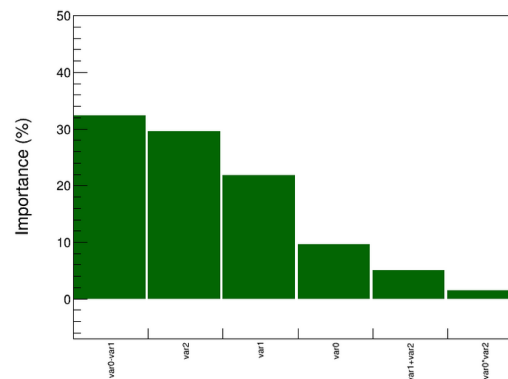
- **Feature Importance**  $\longrightarrow$  **proportional to classifier performance in which feature participates**

$$FI(X_i) = \sum_{S \subseteq V: X_i \in S} F(S) \times W_{X_i}(S)$$

- **Full feature set  $\{V\}$**
- **Feature subsets  $\{S\}$**
- **Classifier performance  $F(S)$**

$$W_{X_i}(S) \equiv 1 - \frac{F(S - \{X_i\})}{F(S)}$$

- **Fast stochastic version uses random subset seeds**



# Example: RuleFit



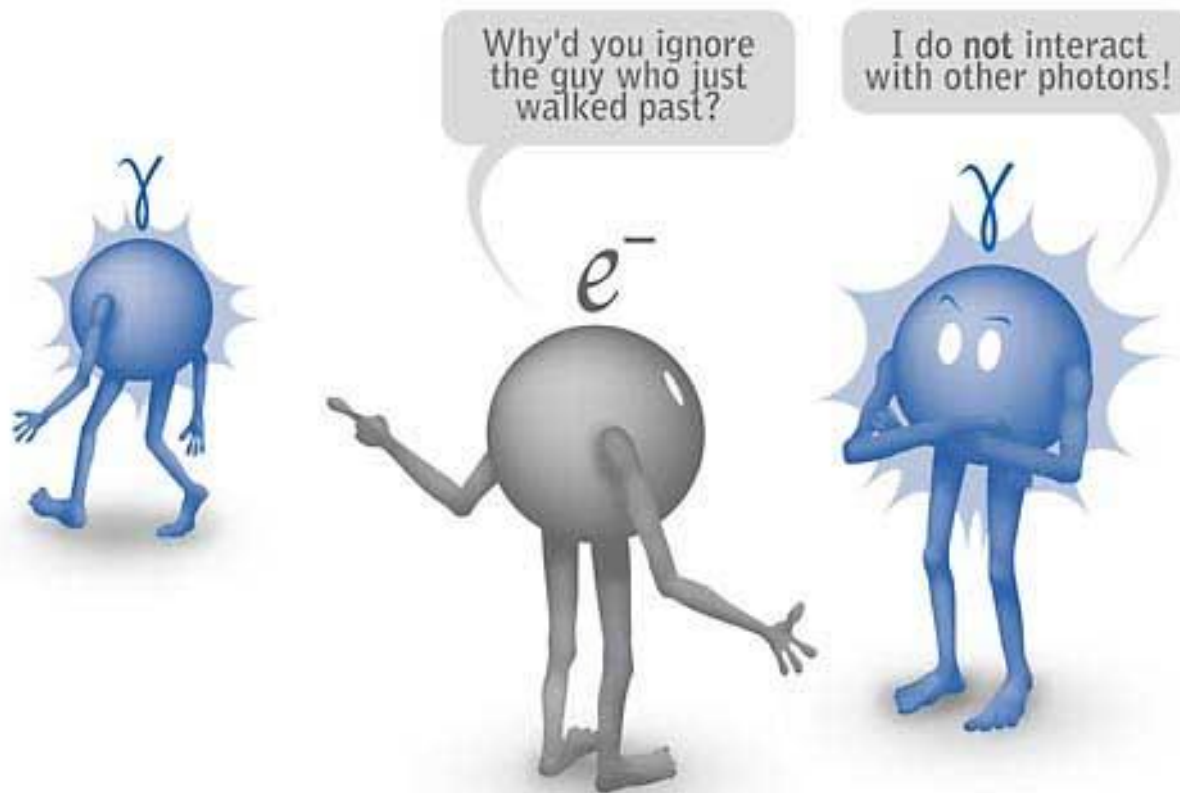
- **Rulefit:** rule-based binary classification and regression (J. Friedman)
  - Transforms decision trees into rule ensembles
  - A powerful classifier even if some rules are poor
- **Feature Importance:**
  - Proportional to performance of classifiers in which features participate (similar)
  - Difference: no  $W_i(S)$ 
    - Individual Classifier Performance evenly divided among participating features

# Selection Caveats



- **Feature Selection Bias**
  - Common mistake leading to **over-optimistic** evaluation of classifiers from “usage” of the testing dataset
  - **Solution:**
    - M-fold cross-validation/Bootstrap
    - Second testing sample for evaluation

# Feature Interactions



**Features Like to Interact Too**



# Feature Interactions



- Features often **interact** strongly in the classification process.
  - Their removal affects the performance of remaining interacting partners
    - Strength of interaction quantified by some wrapper methods
  - In some classifiers features can be overlooked (or shadowed) by their interacting partners



Beware of  
hidden reefs

# Importance Landscape

# Has Changed

## Holds for any criterion that doesn't incorporate interactions

# Global Loss Function



- **GLoss Function** ➡ **Global measure of loss**
  - Selects feature subsets for global removal

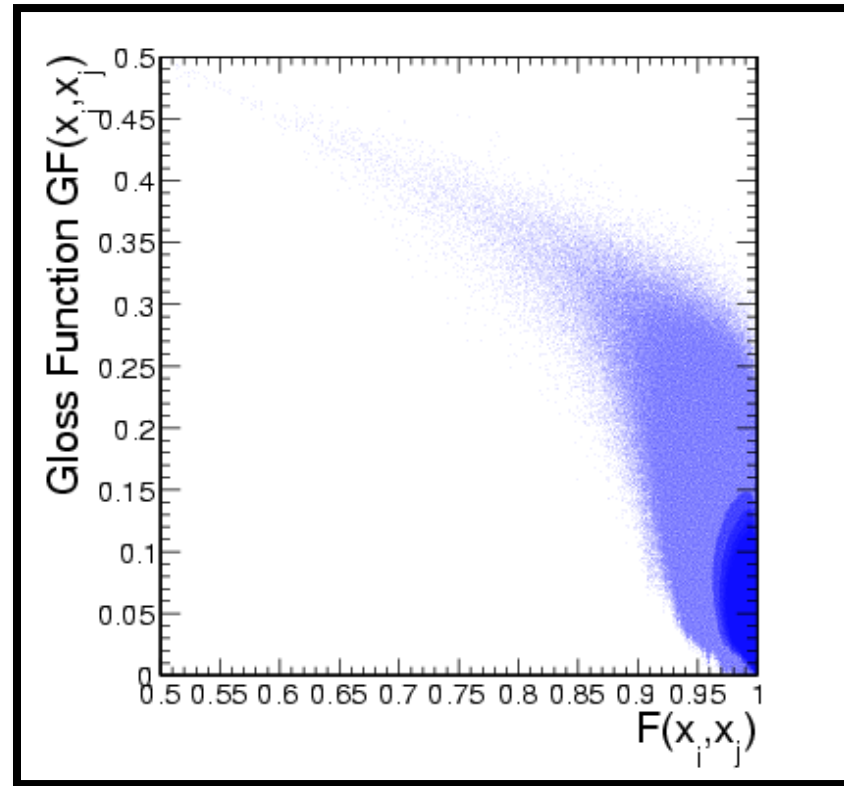
$$GF(S') \equiv 1 - \frac{\sum_{S \subset (V-S')} F(S)}{2^{|V-S'|}}$$

**S' is the subset to be removed**

- Shows the amount of predictive power loss relative to the upper bound of performance of remaining classifiers

$$\sum_{S \subset (V-S')} F(S)_{\max} = 2^{|V-S'|}$$

# Global Loss and Classifier Performance

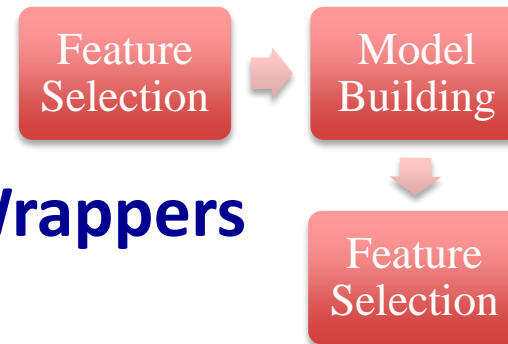


**GLoss Function minimization NOT EQUIVALENT to Maximization of  $F(S)$  – i.e. finding the highest performing classifier and its constituent features**

# Recent Work



- **Probabilistic Wrapper Methods:**
  - Stochastic approach
- **Hybrid Algorithms:**
  - combine Filters and Wrappers
- **Embedded Methods**



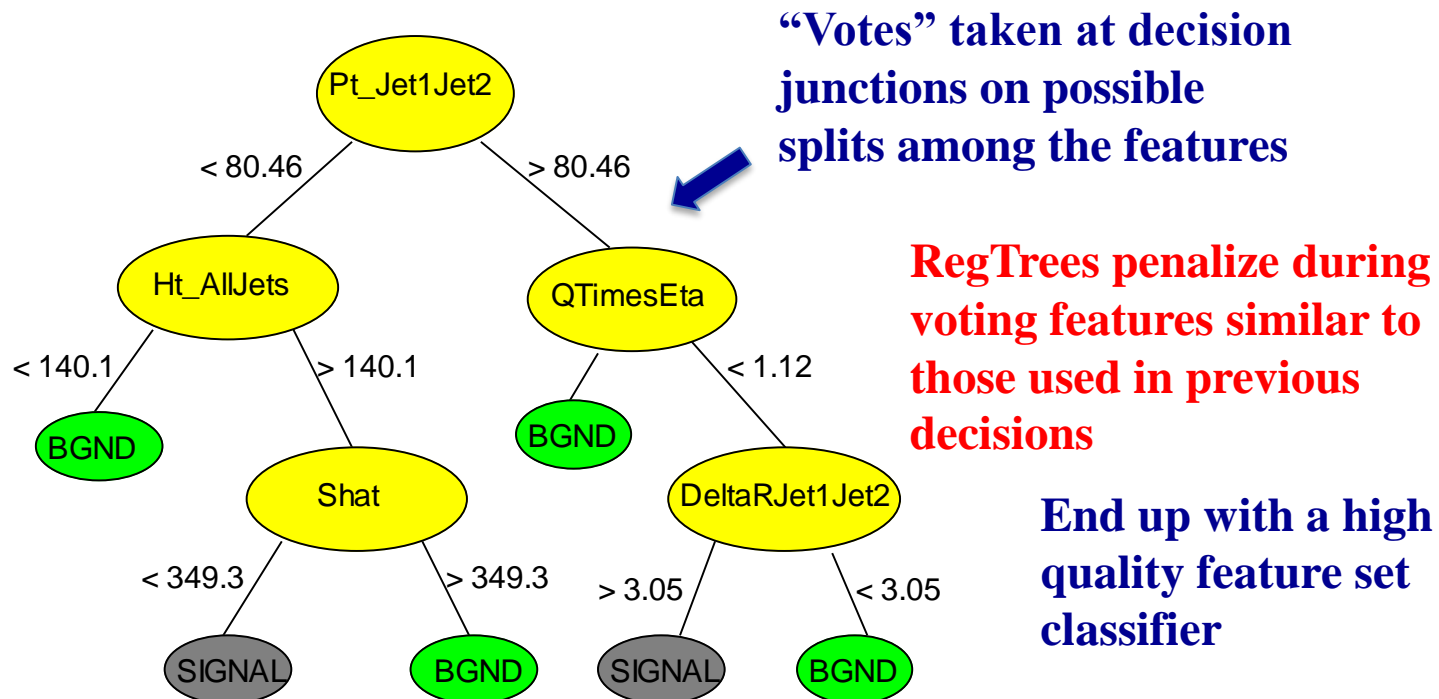
Feature Selection  
during Model  
Building

- At model-building stage assess **feature importance** and incorporate it in the process
  - Way to penalize/remove features in the classification or regression process
    - Regularization
    - Examples: LASSO, RegTrees

# Regularized Trees



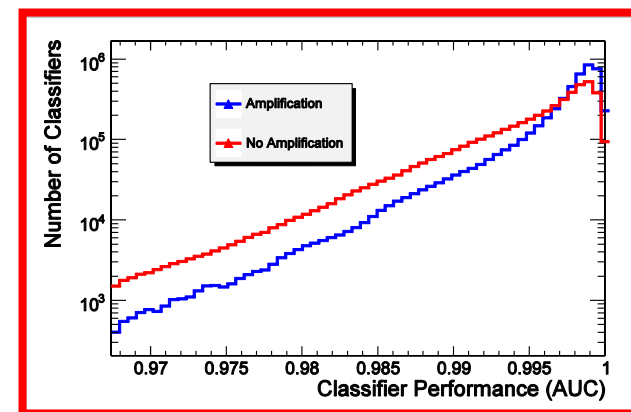
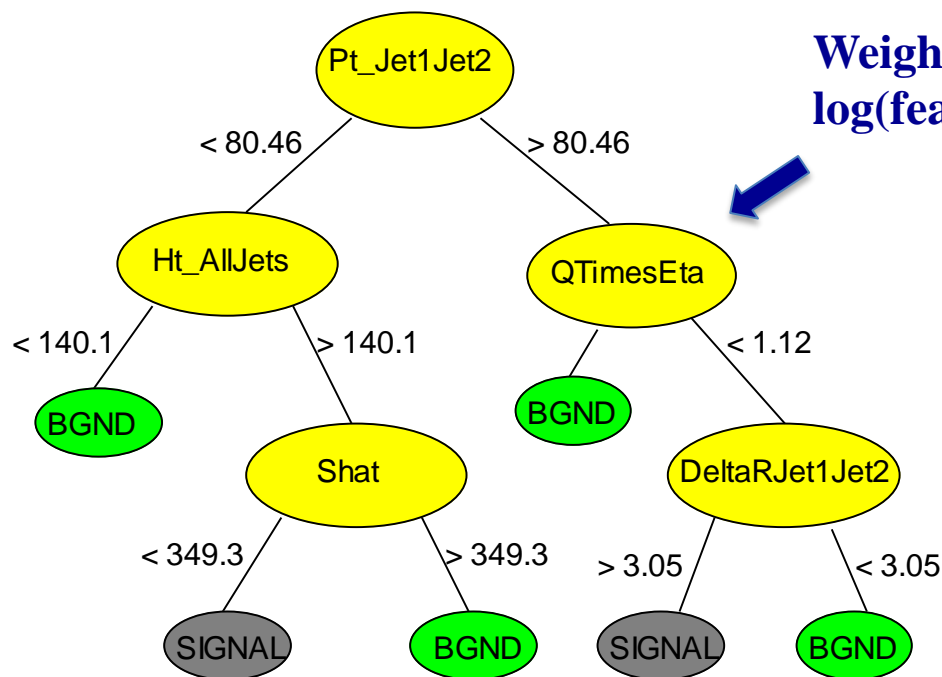
- Inspired by Rules Regularization in Friedman and Popescu 2008
- Decision Tree Reminder:



# Feature Amplification



- **Another example: feedback feature importance into classifier building**

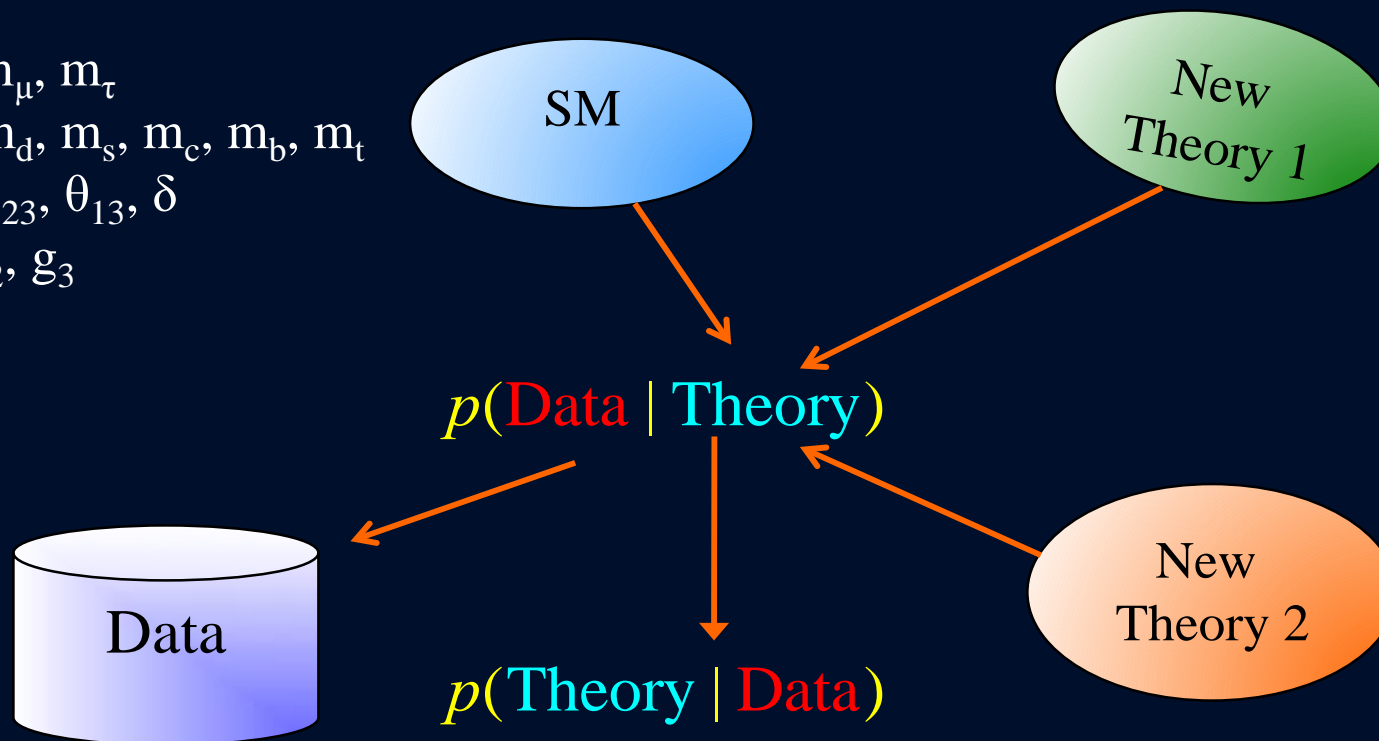




All interesting theories are *multi-parameter* models

H. Prosper

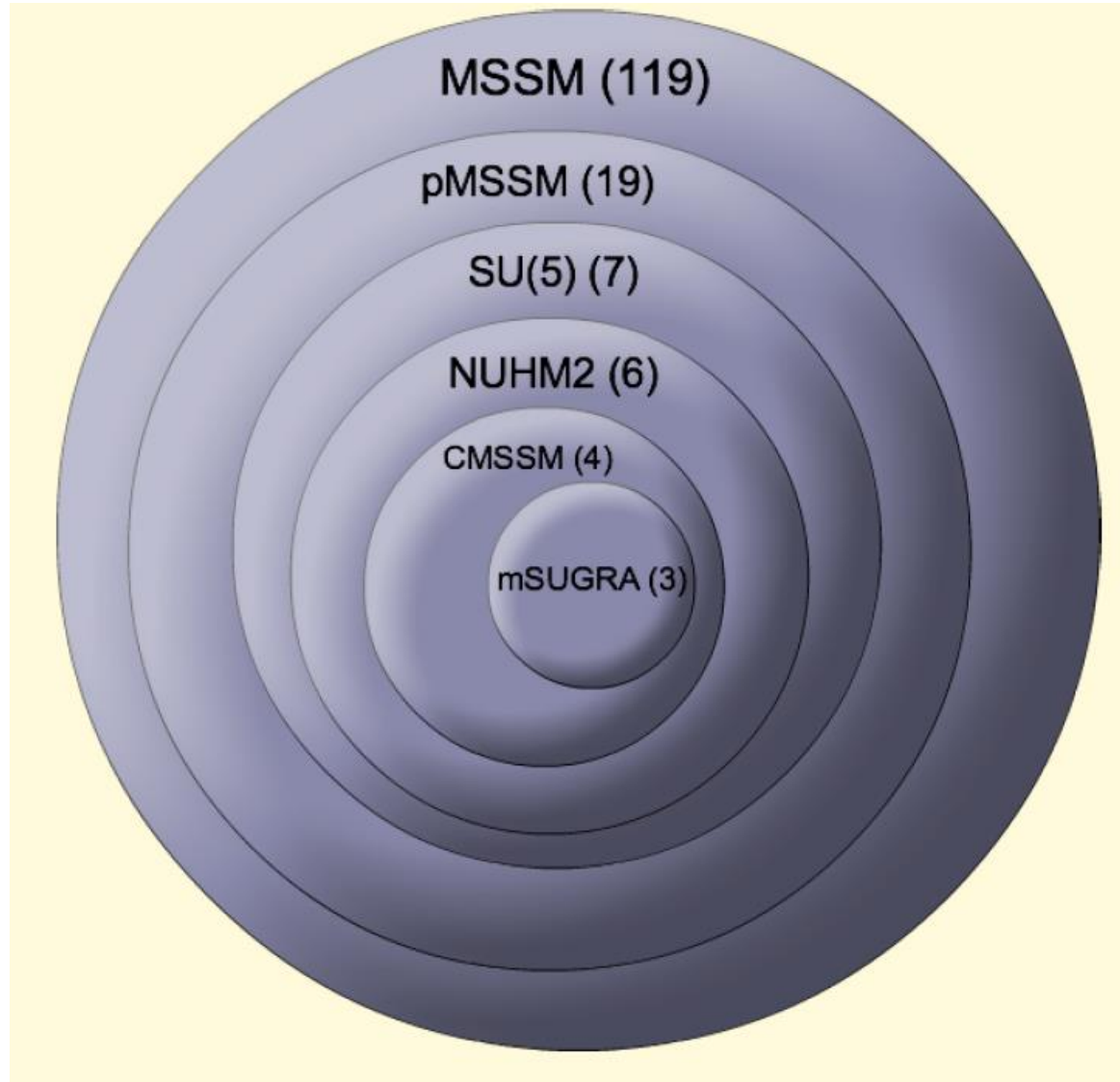
$m_e, m_\mu, m_\tau$   
 $m_u, m_d, m_s, m_c, m_b, m_t$   
 $\theta_{12}, \theta_{23}, \theta_{13}, \delta$   
 $g_1, g_2, g_3$   
 $\theta_{\text{QCD}}$   
 $\mu, \lambda$



Basic *statistical* questions:

1. Which theories are preferred, given the data?
2. And which parameter sub-spaces within these theories?

# Minimal SUSY



- **Often in HEP one searches for new phenomena and applies classifiers trained on MC for at least one of the classes (signal) or sometimes both to real data**
  - **Flexibility is KEY to any search**
  - **It is more beneficial to choose a reduced parameter space that consistently produces strong performing classifiers at actual analysis time**
    - **Useful for general SUSY and other new phenomena searches**

# Feature Selection Tools



- **R (CRAN):** Boruta, RFE, CFS, Fselector, caret
- **TMVA:** FAST algo (stochastic wrapper), Global Loss function
- **Scikit-Learn**
- **Bioconductor**

# Summary



- Feature selection is important part of robust HEP Machine Learning applications
- Many methods available
- Watch out for caveats
- **Happy ANALYZING**