# Scaling Gaussian Processes and the search for exoplanets 

Dan Foreman-Mackey<br>Sagan Fellow, University of Washington<br>github.com/dfm // @exoplaneteer // dfm.io



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Sagan Fellow, University of Washington github.com/dfm // @exoplaneteer // dfm.io

## | study astronomy.

Photo credit NASA Ames/SETI Institute/JPL-Caltech
this isn't what my data look like

## I study astronomy.

## I do data science...

## this is not what my data science looks like.



this is what my data science looks like.
$f a r(a), \omega \in \Delta$.
Cox (a). w A


## convince you that <br> exoplanets are cool

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demonstrate some
sick Python code

## Why Astronomy?

# simple but interesting physical models 

precise open-access data
observational only

## Why Astronomy?

# simple but interesting physical models 

## precise open-access data

## observational only

no chance of financial gain ever

## ex•o•plan•et

'eksō,planət/
noun. a planet that orbits a star outside the solar system.

## How do we find \& study exoplanets?

## 1307 transit

644 radial velocity 48 direct imaging
37 microlensing
24 timing
0 astrometry


the transit method


Earth
that's not what most stars look like!


## everything is against us!



# need to look at the right place at the right time 

and measure extremely precise<br>photometry

## Kepler






H N且
为


|  |
| :---: |
|  |  |




## Kepler-32



## Kepler-32






Credit Fabrycky et al. (2012)

that looks pretty good...





The anatomy of a transit observation


The anatomy of a transit observation


The anatomy of a transit observation


The anatomy of a transit observation


The anatomy of a transit observation


The anatomy of a transit observation



## Standard practice: Filtering



Exoplanets are hard to find


Figure credit: Petigura, Howard \& Marcy (2013)

## What about Gaussian Processes?

gaussianprocess.org/gpml

Rasmussen \& Williams

## Modeling a light curve using a Gaussian Processes



## Modeling a light curve using a Gaussian Processes




## Modeling a light curve using a Gaussian Processes



## What is a Gaussian Process?








## the data are drawn from one <br> 

* the dimension is the number of data points.

The mathematical model

$$
\boldsymbol{y} \sim \mathcal{N}\left(\boldsymbol{f}_{\boldsymbol{\theta}}(\boldsymbol{x}), K_{\boldsymbol{\alpha}}(\boldsymbol{x}, \boldsymbol{\sigma})\right)
$$

where

$$
\left[K_{\boldsymbol{\alpha}}(\boldsymbol{x}, \boldsymbol{\sigma})\right]_{i j}={\sigma_{i}}^{2} \delta_{i j}+k_{\boldsymbol{\alpha}}\left(x_{i}, x_{j}\right)
$$

## The mathematical model

$$
\begin{aligned}
\log p(\boldsymbol{y} \mid \boldsymbol{x}, \boldsymbol{\sigma}, \boldsymbol{\theta}, \boldsymbol{\alpha})= & -\frac{1}{2}\left[\boldsymbol{y}-\boldsymbol{f}_{\boldsymbol{\theta}}(\boldsymbol{x})\right]^{\mathrm{T}} K_{\boldsymbol{\alpha}}(\boldsymbol{x}, \boldsymbol{\sigma})^{-1}\left[\boldsymbol{y}-\boldsymbol{f}_{\boldsymbol{\theta}}(\boldsymbol{x})\right] \\
& -\frac{1}{2} \log \operatorname{det} K_{\boldsymbol{\alpha}}(\boldsymbol{x}, \boldsymbol{\sigma})-\frac{N}{2} \log 2 \pi
\end{aligned}
$$

where

$$
\left[K_{\boldsymbol{\alpha}}(\boldsymbol{x}, \boldsymbol{\sigma})\right]_{i j}={\sigma_{i}}^{2} \delta_{i j}+k_{\boldsymbol{\alpha}}\left(x_{i}, x_{j}\right)
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\text { kernel function } \\
\text { (where the magic happens) }
\end{array}}
$$

The choice of kernel

$$
k_{\boldsymbol{\alpha}}\left(x_{i}, x_{j}\right)=\exp \left(-\frac{\left[x_{i}-x_{j}\right]^{2}}{2 \ell^{2}}\right)
$$



The choice of kernel

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$$



The choice of kernel

$$
k_{\boldsymbol{\alpha}}\left(x_{i}, x_{j}\right)=\left[1+\frac{\sqrt{3}\left|x_{i}-x_{j}\right|}{\ell}\right] \exp \left(-\frac{\left|x_{i}-x_{j}\right|}{\ell}\right) \cos \left(\frac{2 \pi\left|x_{i}-x_{j}\right|}{P}\right)
$$



The choice of kernel
$k_{\boldsymbol{\alpha}}\left(x_{i}, x_{j}\right)=\left[1+\frac{\sqrt{3}\left|x_{i}-x_{j}\right|}{\ell}\right] \exp \left(-\frac{\left|x_{i}-x_{j}\right|}{\ell}\right) \cos \left(\frac{2 \pi\left|x_{i}-x_{j}\right|}{P}\right)$


The choice of kernel


The choice of kernel


## Does this matter?






After.


After.


How to use Gaussian Processes?

## The mathematical model

$$
\begin{aligned}
\log p(\boldsymbol{y} \mid \boldsymbol{x}, \boldsymbol{\sigma}, \boldsymbol{\theta}, \boldsymbol{\alpha})= & -\frac{1}{2}\left[\boldsymbol{y}-\boldsymbol{f}_{\boldsymbol{\theta}}(\boldsymbol{x})\right]^{\mathrm{T}} K_{\boldsymbol{\alpha}}(\boldsymbol{x}, \boldsymbol{\sigma})^{-1}\left[\boldsymbol{y}-\boldsymbol{f}_{\boldsymbol{\theta}}(\boldsymbol{x})\right] \\
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\text { kernel function } \\
\text { (where the magic happens) }
\end{array}}
$$

A simple \& efficient Python implementation

```
import numpy as np
from scipy.linalg import cho_factor, cho_solve
def kernel(x1, x2):
    # ...
def gp_lnlike(x, y, yerr):
    C = kernel(x[:, None], x[None, :])
    C[np.diag_indicies_from(C)] += yerr ** 2
    factor, flag = cho_factor(C)
    logdet = 2*np.sum(np.log(np.diag(factor)))
    return -0.5 * (np.dot(y, cho_solve((factor, flag), y))
    + logdet + len(x)*np.log(2*np.pi))
```


## Using George

```
import george
import numpy as np
# kernel = george.kernels...
def george_lnlike(x, y, yerr):
    gp = george.GP(kernel)
    gp.compute(x, yerr)
    return gp.lnlikelihood(y)
```

What's the catch?

## What's the catch?

# My Problem <br> Big Data 

(by some definition)

Note: I hate myself for this slide too...

## Computational complexity.

$$
\begin{aligned}
\log p(\boldsymbol{y} \mid \boldsymbol{x}, \boldsymbol{\sigma}, \boldsymbol{\theta}, \boldsymbol{\alpha})= & -\frac{1}{2}\left[\boldsymbol{y}-\boldsymbol{f}_{\boldsymbol{\theta}}(\boldsymbol{x})\right]^{\mathrm{T}} K_{\boldsymbol{\alpha}}(\boldsymbol{x}, \boldsymbol{\sigma})^{-1}\left[\boldsymbol{y}-\boldsymbol{f}_{\boldsymbol{\theta}}(\boldsymbol{x})\right] \\
& -\frac{1}{2} \log \operatorname{det} K_{\boldsymbol{\alpha}}(\boldsymbol{x}, \boldsymbol{\sigma})-\frac{N}{2} \log 2 \pi
\end{aligned}
$$

compute factorization // evaluate log-det // apply inverse
naïvely: $\mathcal{O}\left(N^{3}\right)$

## Using George

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```


github.com/dfm/george

github.com/dfm/george

How can we scale?

$$
\begin{aligned}
\log p(\boldsymbol{y} \mid \boldsymbol{x}, \boldsymbol{\sigma}, \boldsymbol{\theta}, \boldsymbol{\alpha})= & -\frac{1}{2}\left[\boldsymbol{y}-\boldsymbol{f}_{\boldsymbol{\theta}}(\boldsymbol{x})\right]^{\mathrm{T}} K_{\boldsymbol{\alpha}}(\boldsymbol{x}, \boldsymbol{\sigma})^{-1}\left[\boldsymbol{y}-\boldsymbol{f}_{\boldsymbol{\theta}}(\boldsymbol{x})\right] \\
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\end{aligned}
$$

Aren't kernel matrices Hierarchical Off-Diagonal Low-Rank?

- not me


Ambikasaran, DFM, et al. (arXiv:1403.6015)


Ambikasaran, DFM, et al. (arXiv:1403.6015)
github.com/sivaramambikasaran/HODLR

## 2. dfm@moka | tmux ne...:header (tmux)

```
164 \
```

```
164 \
```




```
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```

temp.block(nRank[0], 0, nRank[1] , n)\ =\ Vinverse[0]*matrix.block(start, 0 , chil
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166 \ \ \ //\ Computes tempSolve\ =\ Kinverse\temp-
167 ?
167 ?
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169 ᄀ
170 ( ) //| Computes matrix\ = matrix-Uinverse*tempSolve` 170 ( ) //| Computes matrix\ = matrix-Uinverse*tempSolve`
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173 \ \ matrix.block(start + child[0]->nSize, 0, child[1]->nSize, n)\ => matrix.block(sta
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1 7 8 Computes the determinant of the matrix.7
1 7 9 ~ * / \urcorner ~
1 7 9 ~ * / \urcorner ~
180 void compute_Determinant() {` 180 void compute_Determinant() {`
181 if (Kinverse.rows()>0) { // Check needed when the matrix is predomin
181 if (Kinverse.rows()>0) { // Check needed when the matrix is predomin
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183 determinant = log(fabs(LU(0,0)));7
183 determinant = log(fabs(LU(0,0)));7
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1 8 4 ~ f o r ~ ( i n t ~ k = 1 ; ~ k < K i n v e r s e . r o w s ( ) ; ~ + + k ) ~ \{ ~ \ , ~
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190 };` 190 };`
HODLR_Node.hpp [cpp]
HODLR_Node.hpp [cpp]

## The HODLR solver from George

```
import george
import numpy as np
# kernel = george.kernels...
def george_lnlike(x, y, yerr):
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    return gp.lnlikelihood(y)
```


## The HODLR solver from George

```
import george
import numpy as np
# kernel = george.kernels...
def george_lnlike(x, y, yerr):
    gp = george.GP(kernel, solver=george.HODLRSolver)
    gp.compute(x, yerr)
    return gp.lnlikelihood(y)
```


github.com/dfm/george

github.com/dfm/george

Does this work?

Yes.

## K2 Campaign 1 exoplanet discoveries

# 21,703 stars <br> 80 days of data 36 planet candidates 18 confirmed planets 

Published:
Foreman-Mackey, Montet, Hogg, et al. (arXiv:1502.04715)
Montet, Morton, Foreman-Mackey, et al. (arXiv:1503.07866)
Schölkopf, Hogg, Wang, Foreman-Mackey, et al. (arXiv:1505.03036)




# Probabilistic modeling-combining physical and data-driven models-enables the discovery of new planets using open data and open source software 

gaussianprocess.org/gpml<br>github.com/dfm/george<br>dfm.io/george

## extra



