Coupling Studies with Simulation and Measurement

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Introduction

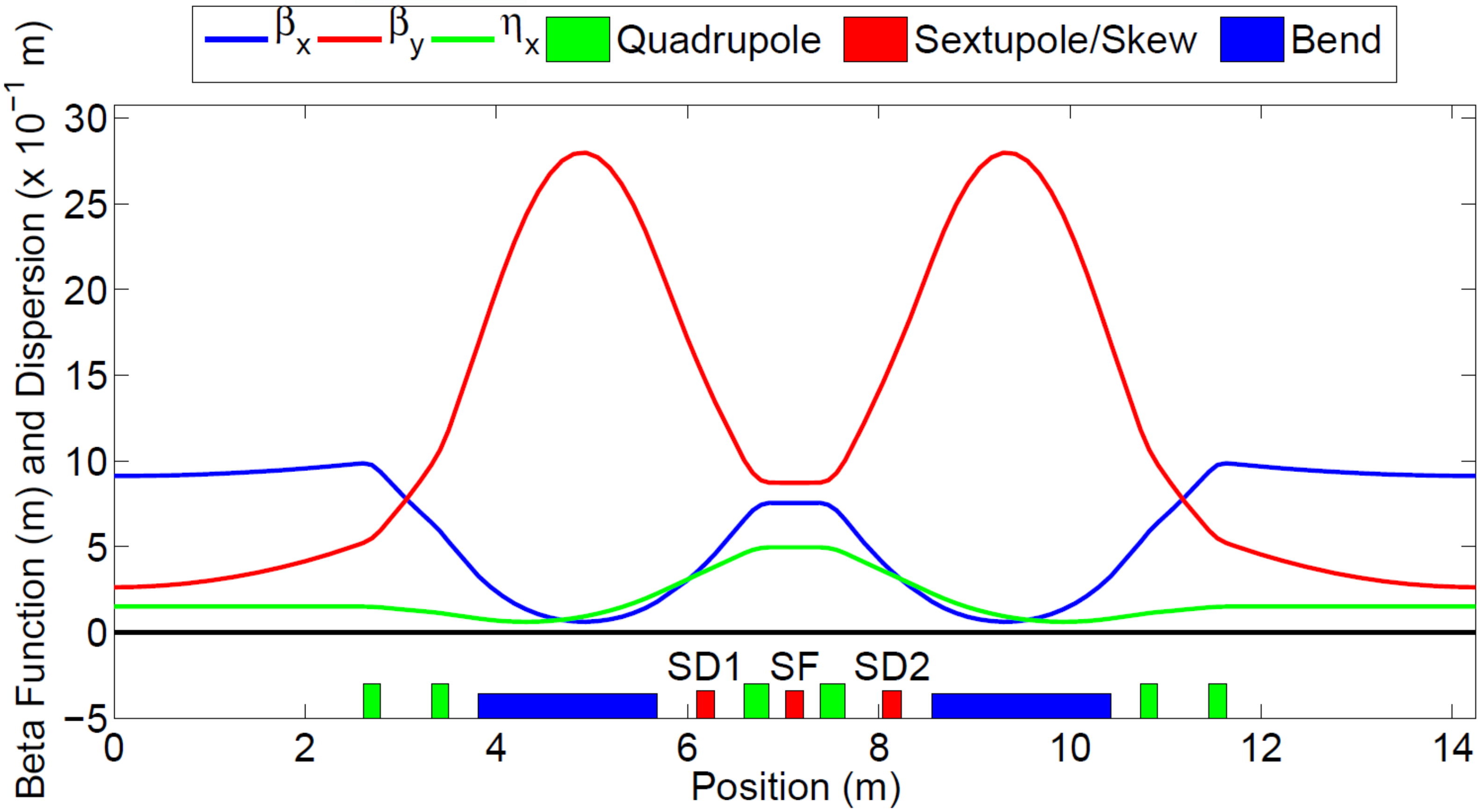
- We used a combination of measurement and simulation to study coupling and its control using skew quadrupoles at the Canadian Light Source
  - 12-cell DBA machine
  - 2.9 GeV
  - 170.88 m circumference
- Used AT as the optics code due to its integration with MATLAB and LOCO

Canadian Light Source Centre canadien de rayonnement synchrotron

The Process

- Measured response matrix and dispersion
- Created an effective model using LOCO
  - Used a legacy version of AT
- Used multi-objective particle swarm optimization (MOPSO) to calculate skew quadrupole configurations
  - Used SourceForge version of AT to evaluate objective functions (downloaded 2014-12-11)
- Tested skew quadrupole configurations on the real machine and made further measurements

CLS Lattice



Sextupoles have skew quadrupole windings

The 36 skew quadrupoles are individually powered

Note: No geometric sextupoles, so no skew quadrupoles in the straights

LOCO

- LOCO takes measurements as inputs
  - Response Matrix
  - Dispersion
  - BPM Noise
- LOCO adjusts selected elements in the lattice until the simulation quantities agree with the measurements
  - Adjust BPM gains and coupling
  - Adjust orbit correction magnet kicks and coupling
  - Adjust lattice quadrupole strengths
  - Adjust lattice quadrupole tilts
- The result is an effective model that is good enough for our purposes

J. Safranek NIMA **388**, 27 (1997).

Coupling Confusion

- There are lots of "uncoupled plus" models in the literature with varying conventions
- For our calculations, we primarily use the full six-dimensional sigma matrix with numerical optimization methods
- The other models provide guidance

$$\epsilon_{yq} = C_q \frac{\left\langle \beta_y / \left|\rho\right|^3 \right\rangle}{J_y \left\langle 1/\rho^2 \right\rangle}$$

$$\epsilon_{y0} = C_q \frac{\gamma^2 \left\langle \mathcal{H}_y / \left|\rho\right|^3 \right\rangle}{J_y \left\langle 1/\rho^2 \right\rangle}$$

$$\kappa = \frac{1}{2\pi} \oint k_s \sqrt{\beta_x \beta_y} e^{i\left[\psi_x - \psi_y - \frac{2\pi s}{L}(\nu_x - \nu_y + \ell)\right]} ds$$

$$\Delta \equiv \nu_x - \nu_y + \ell \qquad \Omega = \frac{1}{2}\sqrt{\kappa^2 + \Delta^2}$$

$$\epsilon_y(s) = \frac{\epsilon_{x0}}{4\Omega^2} \kappa^2 \sin^2\left(\frac{2\pi\Omega s}{L}\right)$$

$$\epsilon_x(s) + \epsilon_y(s) = \epsilon_{x0}$$

$$r \equiv \frac{\epsilon_{y,max}}{\epsilon_{x,max}} = \frac{\kappa^2}{\kappa^2 + \Delta^2}$$

$$\frac{\langle \epsilon_y \rangle}{\langle \epsilon_x \rangle} = \frac{\kappa^2/2}{\kappa^2/2 + \Delta^2}$$

xy-Beam Tilt?

Coupling Calculations with AT

- AT has the ohmienvelope() function which computes the six-dimensional sigma matrix using the formalism of Ohmi, Hirata and Oide
- From this matrix, we can calculate the xy-beam tilt and eigenemittances (complex eigenvalues of ΣS)

$$\mathbf{\Sigma} \equiv \begin{pmatrix} \langle x^2 \rangle & \langle xp_x \rangle & \langle xy \rangle & \langle xp_y \rangle & \langle xz \rangle & \langle x\delta \rangle \\ \langle xp_x \rangle & \langle p_x^2 \rangle & \langle yp_x \rangle & \langle p_x p_y \rangle & \langle zp_x \rangle & \langle p_x\delta \rangle \\ \langle xy \rangle & \langle yp_x \rangle & \langle y^2 \rangle & \langle yp_y \rangle & \langle yz \rangle & \langle y\delta \rangle \\ \langle xp_y \rangle & \langle p_x p_y \rangle & \langle yp_y \rangle & \langle p_y^2 \rangle & \langle zp_y \rangle & \langle p_y\delta \rangle \\ \langle xz \rangle & \langle zp_x \rangle & \langle yz \rangle & \langle zp_y \rangle & \langle z^2 \rangle & \langle z\delta \rangle \\ \langle x\delta \rangle & \langle p_x\delta \rangle & \langle y\delta \rangle & \langle p_y\delta \rangle & \langle z\delta \rangle & \langle \delta^2 \rangle \end{pmatrix}$$

$$\tan(2\theta) = \frac{2\langle xy \rangle}{\langle x^2 \rangle - \langle y^2 \rangle}$$

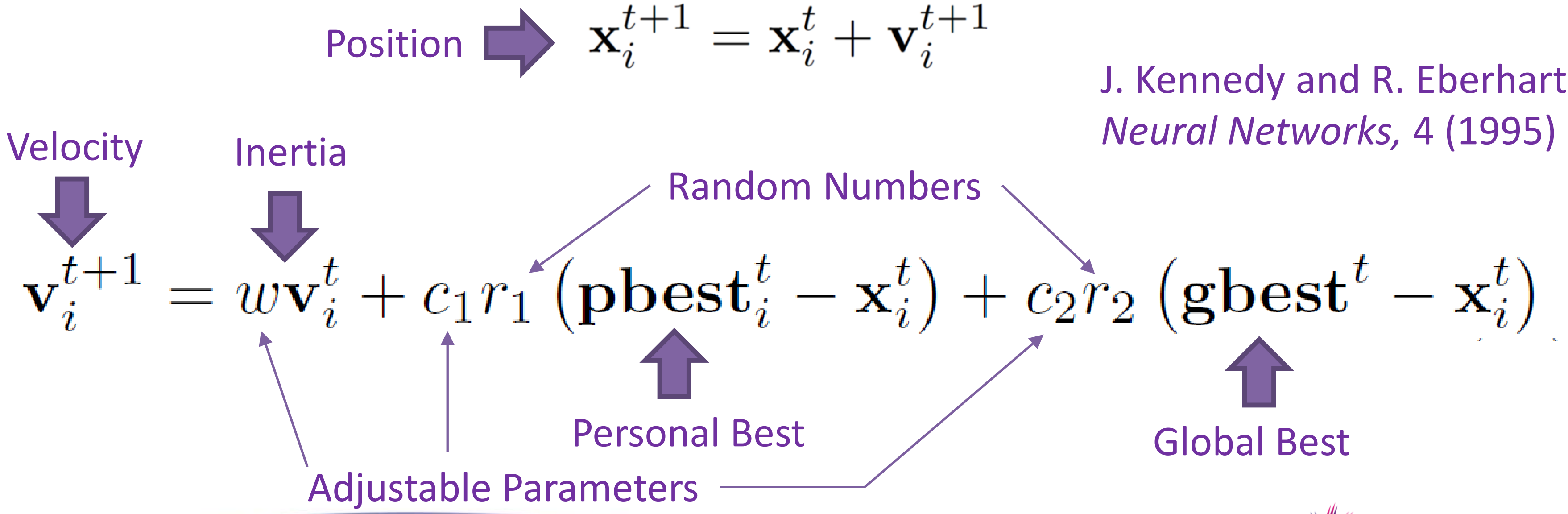
$$\mathbf{S} \equiv \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\epsilon_I,\ \epsilon_{II} \text{ and } \epsilon_{III}$$

K. Ohmi, K. Hirata, and K. Oide, Phys. Rev. E **49**, 751 (1994).

Particle Swarm Optimization (PSO)

- Particles are vectors in a multidimensional vector space
  - 36-dimensional vector space of skew quadrupole settings
- Each particle has a position and a velocity
- The velocity is calculated based on the particle’s previous best and the previous best of the entire population



Position

$$\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \mathbf{v}_i^{t+1}$$

J. Kennedy and R. Eberhart, *Neural Networks,* 4 (1995) 8.

Velocity

Inertia

Random Numbers

$$\mathbf{v}_i^{t+1} = w\mathbf{v}_i^t + c_1 r_1 \left(\mathbf{pbest}_i^t - \mathbf{x}_i^t\right) + c_2 r_2 \left(\mathbf{gbest}^t - \mathbf{x}_i^t\right)$$

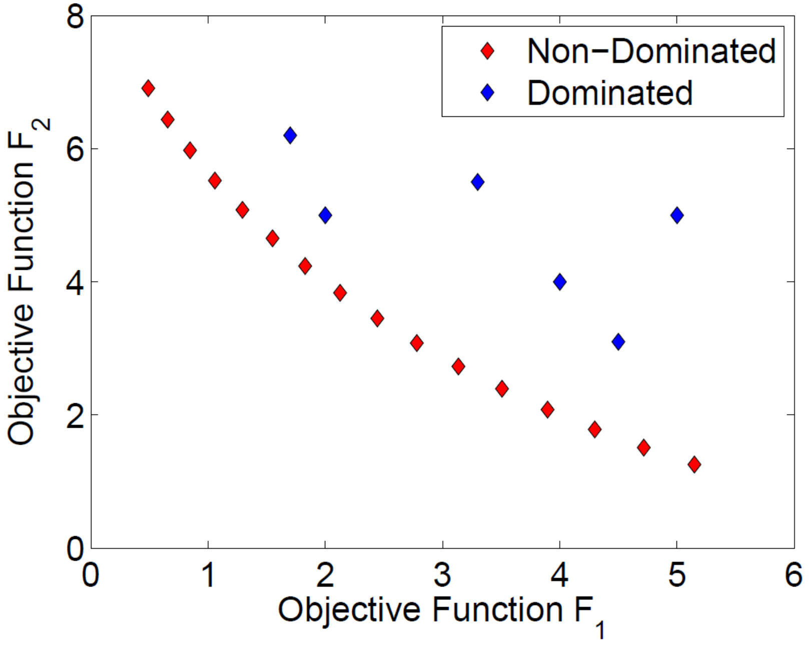
Personal Best

Global Best

Adjustable Parameters

Multiple Objective Particle Swarm Optimization (MOPSO)

- We can handle multiple objectives with PSO
- The global best becomes an archive of non-dominated configurations
- We randomly select a configuration from the archive to use in place of the global best
- This random selection can be weighted to provide an evenly distributed set of non-dominated configurations



Pareto Optimality: optimizing two functions that cannot be simultaneously optimized

A configuration is dominated if there is a another configuration that is better in all objective functions

MOPSO

AT Used Here

1. Load the effective model from LOCO.
2. Initialize a population of 100 particles, $\mathbf{x}_i^0$.
3. Initialize the personal bests, $\mathbf{pbest}_i^0 = \mathbf{x}_i^0$.
4. Initialize the archive from the non-dominated $\mathbf{x}_i^0$.
5. Sort the archive by crowding distance.
6. Calculate $\mathbf{x}_i^{t+1}$ using Eq. (24) selecting $\mathbf{gbest}^t$ from the top 10% of the archive.
7. Perform the mutation operation on $\mathbf{x}_i^{t+1}$.
8. Enforce limits on $\mathbf{x}_i^{t+1}$.

AT Used Here

9. Evaluate the objective functions $F_1\left(\mathbf{x}_i^{t+1}\right)$ and $F_2\left(\mathbf{x}_i^{t+1}\right)$.
10. Insert any non-dominated $\mathbf{x}_i^{t+1}$ into the archive and remove any archived configurations that are now dominated.
11. Sort the archive by crowding distance and trim it to 100 configurations, if necessary.
12. Update $\mathbf{pbest}_i^{t+1}$.
13. Increment t and go to Step 6 until convergence is satisfactory.
14. Once convergence is satisfactory, return the archive as the set of optimal configurations.

We implemented this algorithm in MATLAB

Example Objective Functions

- Two objective functions:
  - RMS Beam Tilt
  - $(\epsilon_{y0} - 200\ \mathrm{pm})^2$
- The motivation was to generate vertical emittance through synchrotron radiation while trying to minimize beam tilt
  - Seemed like a good idea at the time
- 200 Skew Quadrupole Configurations

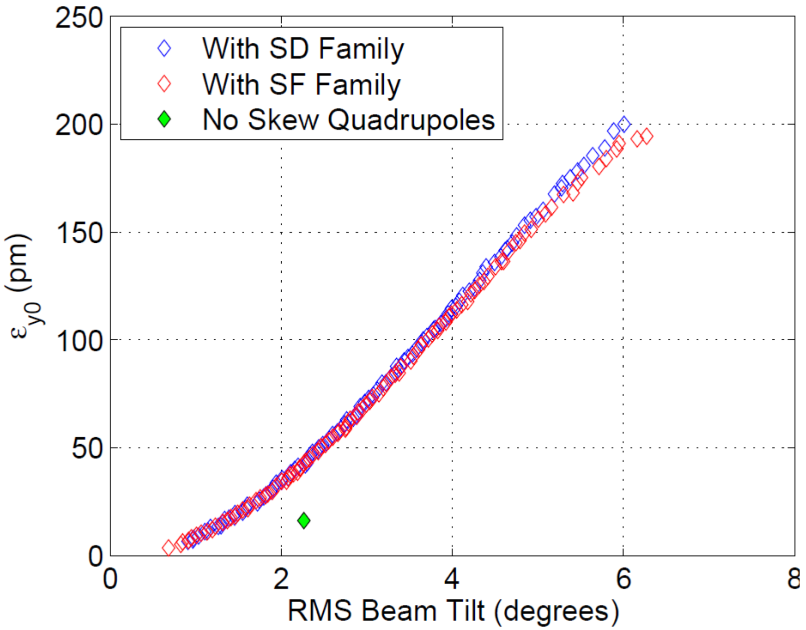


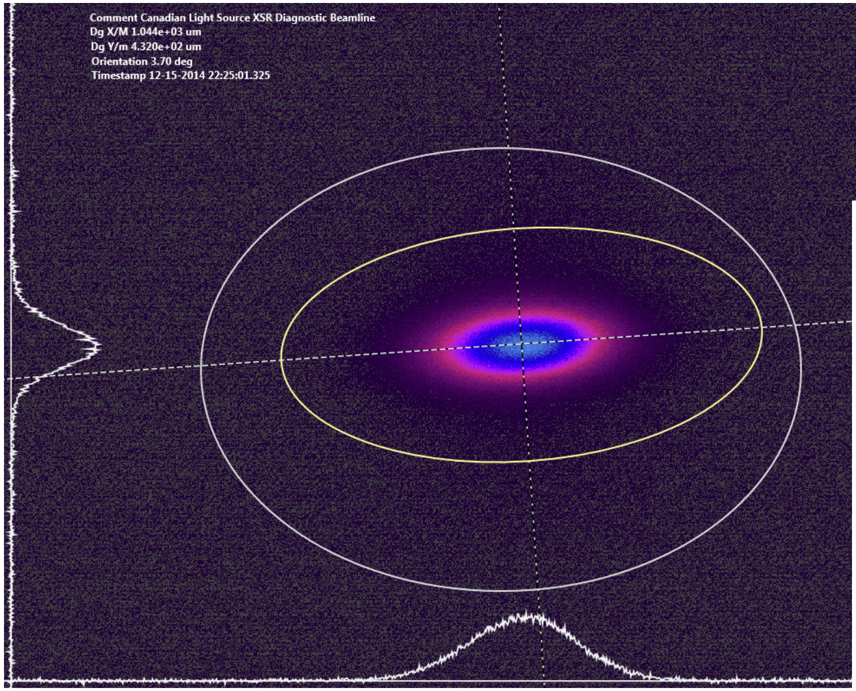
FIG. 3. Results of the MOPSO calculation for the first set of objective functions. The first objective function was the RMS beam tilt. In this plot, we have replaced the second objective function with ϵ_{y0} which is more physically intuitive.

Did the calculation twice:

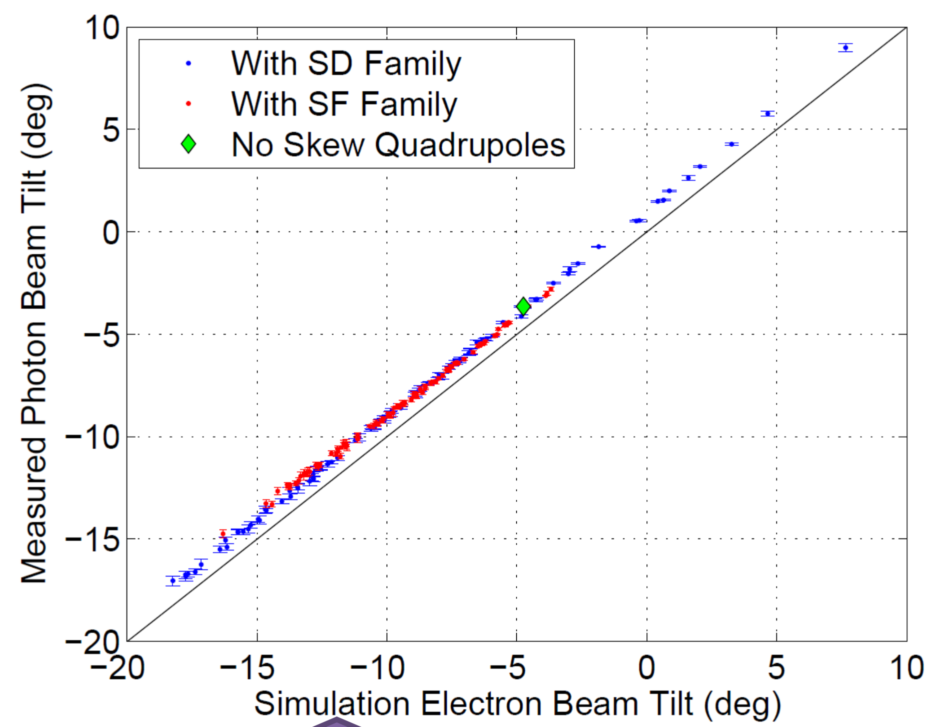
SD family of skew quadrupoles (24)

SF family of skew quadrupoles (12)

Beam Tilt Measurements Using X-ray Pinhole Camera



Other than the constant offset, the agreement between AT and the measurement is good



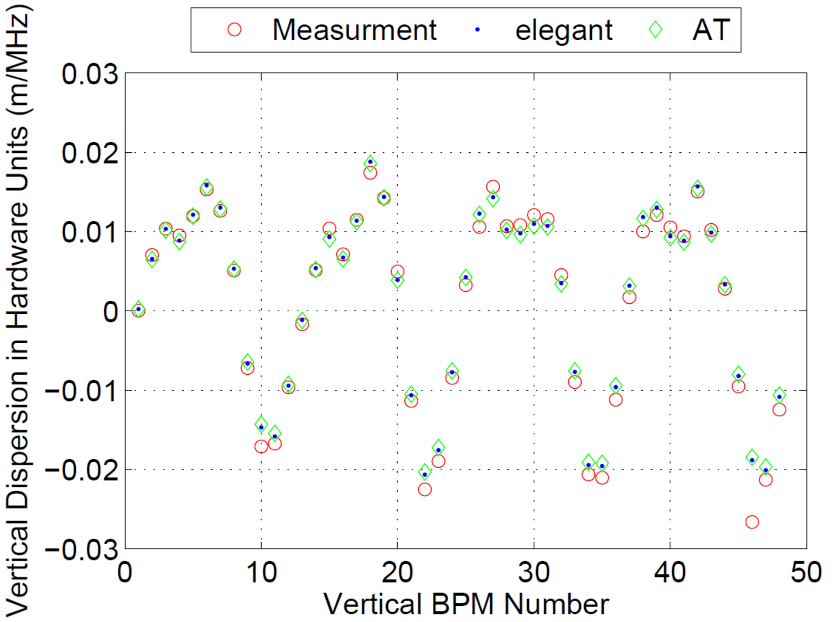
AT – from sigma matrix

Constant offset is not understood:

- Aligned camera using plumb-bob
- Possibly due to the effective model calculated by LOCO

Vertical Dispersion Measurement

- Measured vertical dispersion for 22 of the skew quadrupole configurations
- We see satisfactory agreement between the measurements and the simulations
- Simulations are corrected for BPM coupling, obtained from LOCO

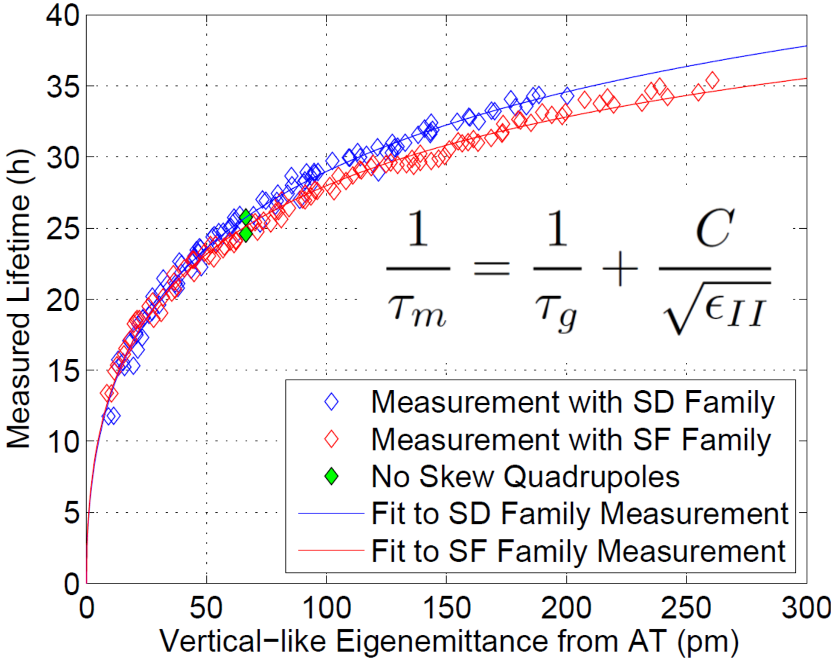


Simulation dispersion converted to hardware units

$$\frac{\Delta y}{\Delta f_{rf}} = -\frac{\eta_y}{\alpha_c f_{rf}}$$

Lifetime Measurements

- Measured lifetime
- Assumed Touschek lifetime is proportional to the root of the second eigenemittance (vertical-like eigenemittance)
- Fit gas lifetime and C to the data (2 parameters)
- Discrepancy between families likely due to AT model of skew quads not quite matching reality



Optics Code	Family	τ_g (h)	C ($\mathrm{pm}^{\frac{1}{2}}\mathrm{h}^{-1}$)
AT	SD	64.8	0.190
AT	SF	56.3	0.180
elegant	SD	65.1	0.190
elegant	SF	56.4	0.179

$$\tau_T \sim \sqrt{\epsilon_{II}}$$

From AT Sigma matrix

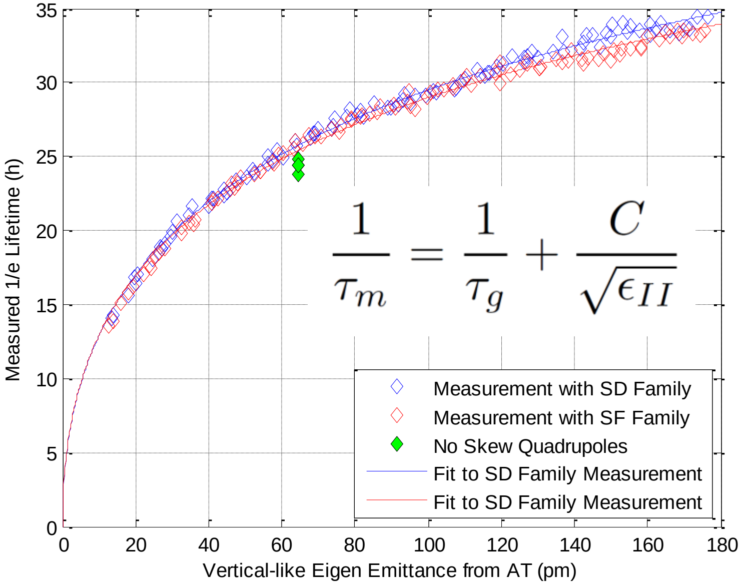
Review of this Measurement

- Our original goal was to minimize beam tilt while generating emittance from synchrotron radiation, rather than coupling to the horizontal phase space
- Generated 200 skew quadrupole configurations
- Found that it is not useful for CLS as we cannot simultaneously optimize the two objective functions
- Found that the simulation and measurement agree
  - Beam tilt from pinhole camera
  - Vertical dispersion
- Found that the second eigenemittance (vertical-like eigenemittance) can be used as a proxy for the Touschek lifetime
  - Can do fine tuning of the lifetime
  - Provides measurement of gas scattering lifetime

A Second Set of Objective Functions

- Can we generate vertical emittance by coupling to the horizontal phase space and does the expected lifetime relationship still hold?
- New objective functions:
  - Minimize emittance due to the emission of synchrotron radiation
  - Set vertical-like eigen-emittance to a specified value

Lifetime Measurement for Second Set of Objective Functions



$$\frac{1}{\tau_m} = \frac{1}{\tau_g} + \frac{C}{\sqrt{\epsilon_{II}}}$$

Again, the relationship holds

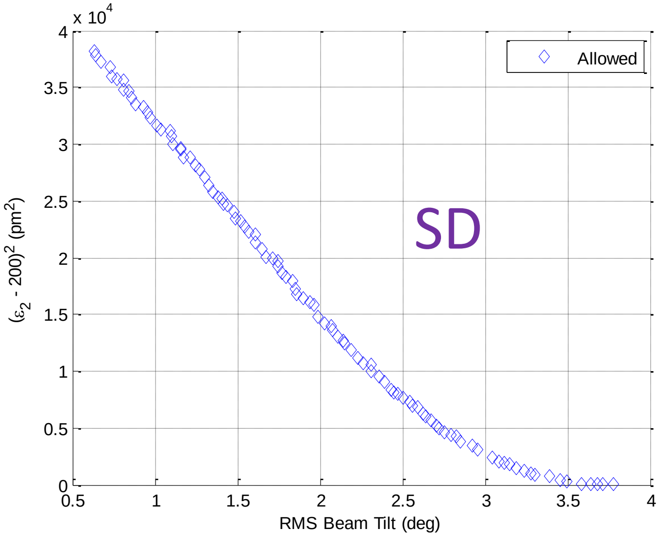
Note: the gas lifetime has gone up between measurements (correlates to improved vacuum in the storage ring after maintenance)
C has gone up (correlates to a less uniform fill pattern)

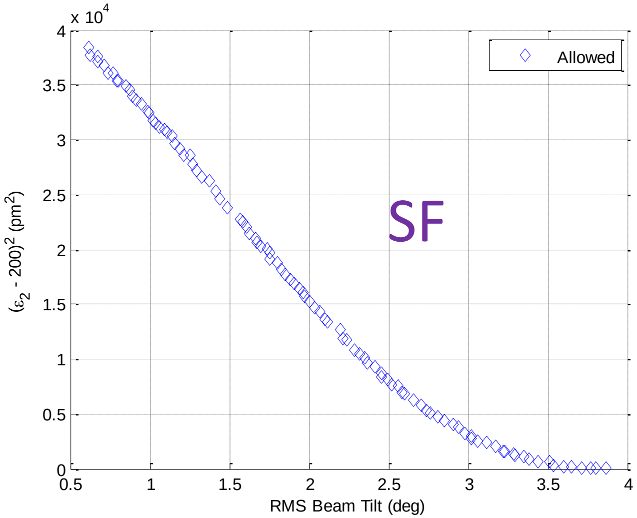
Optics code	Skew Quads	Gas scattering lifetime (h)	C ($pm^{1/2}h^{-1}$)
AT	SD	72.5	0.201
AT	SF	68.0	0.198
elegant	SD	72.6	0.201
elegant	SF	67.9	0.197

Another Set of Objective Functions Studied in Simulation Only

- The objective functions:
  - Minimize RMS Beam Tilt
  - Desired vertical-like eigen-emittance of 200 pm

We get Pareto fronts with 200 potentially useful configurations

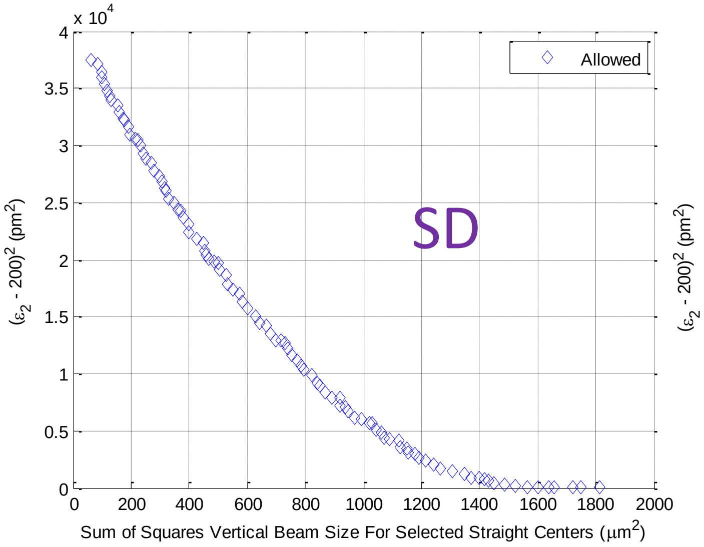


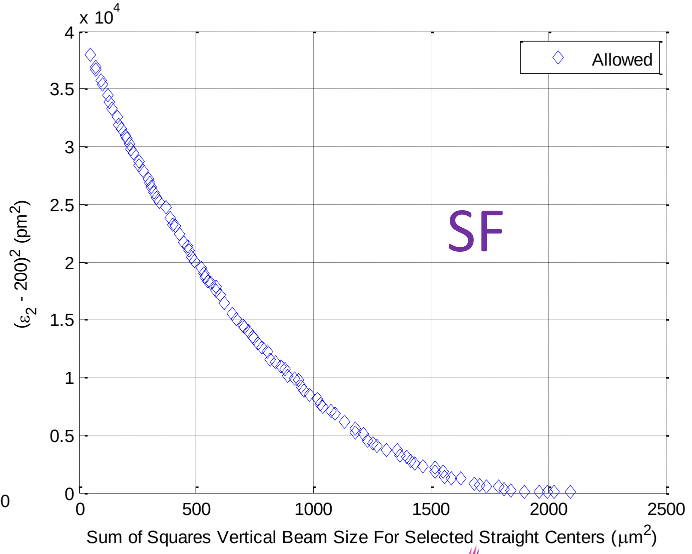


Yet Another Set of Objective Functions Studied in Simulation

- Attempt to reduce the vertical beam size at strategic locations while maintaining high lifetime
- The objective functions:
  - Minimize sum of squared vertical beam sizes at the centers of straights 8, 9, 10
  - Desired vertical-like eigen-emittance of 200 pm

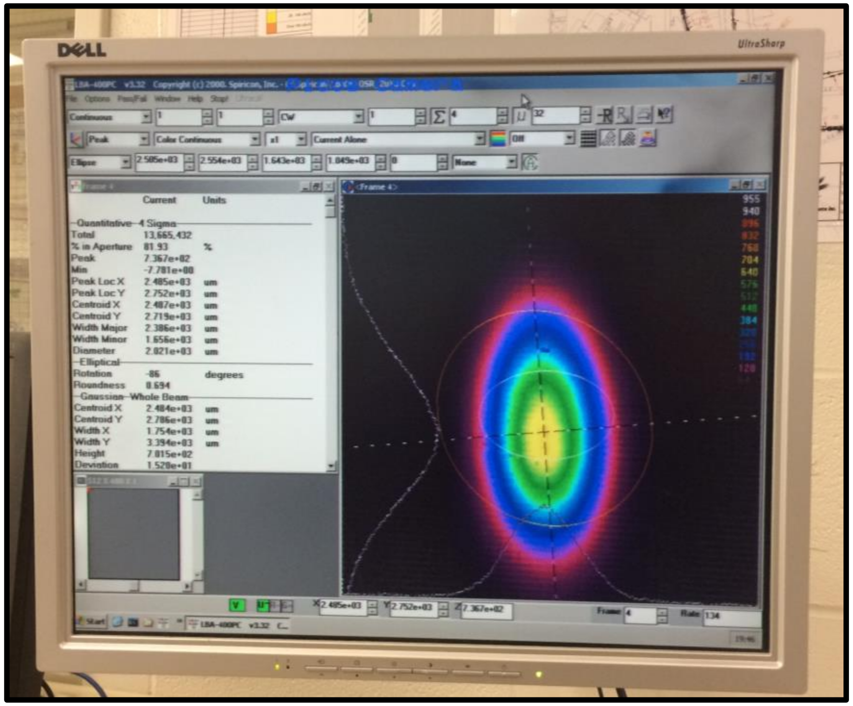
We get Pareto fronts with 200 potentially useful configurations





Yet Another Useful Solution

- Maximize second eigenemittance (vertical-like eigenemittance) to maximize Touschek lifetime
- Useful to increase lifetime during Coherent Synchrotron Radiation (CSR) studies as vertical emittance is less important for THz radiation
- Affectionately known as “Pancake Beam”



	Current	Units
Quantitative 4 Sigma		
Total	13,665,432	
% in Aperture	81.93	%
Peak	7.367e+02	
Min	-7.781e+00	
Peak Loc X	2.485e+03	um
Peak Loc Y	2.752e+03	um
Centroid X	2.487e+03	um
Centroid Y	2.719e+03	um
Width Major	2.386e+03	um
Width Minor	1.656e+03	um
Diameter	2.021e+03	um
Elliptical		
Rotation	-86	degrees
Roundness	0.694	
Gaussian Whole Beam		
Centroid X	2.484e+03	um
Centroid Y	2.786e+03	um
Width X	1.754e+03	um
Width Y	3.394e+03	um
Height	7.015e+02	
Deviation	1.520e+01	

Conclusion

- We have demonstrated agreement between simulation and measurement when controlling coupling with skew quadrupoles
- AT was integral to the calculations
  - Ease of integrating with our MATLAB implementation of MOPSO
  - Integration with LOCO and the MATLAB middle layer

Thoughts about the Future of AT

- I like having AT available through SourceForge
- Ease of programming in MATLAB is the main benefit of AT for me
  - Excellent for small and medium sized problems
  - For complex problems involving a lot of tracking, I have historically used non-parallel elegant on a cluster
- Integration with the MATLAB Middle Layer and LOCO is invaluable

Thank You For Your Atttention