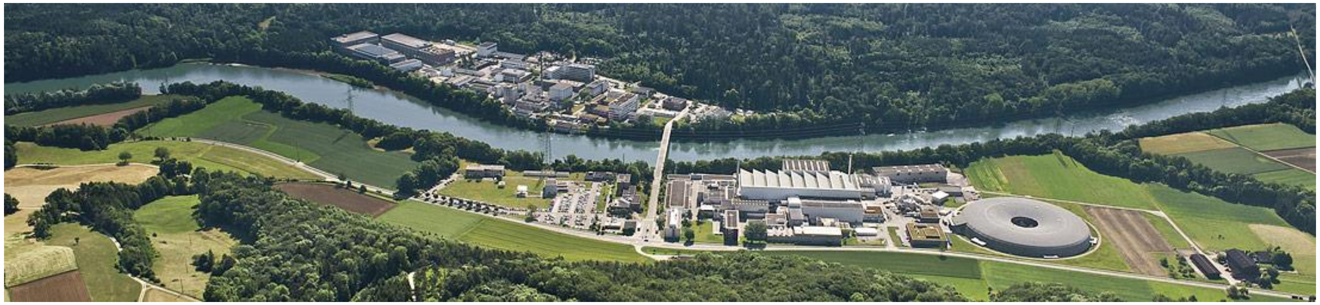
Modelling Longitudinal Gradient Bends in a Storage Ring

Michael Ehrlichman
Paul Scherrer Institut
Noah Bittermann (Summer Student)
University of Minnesota

In collaboration with David Sagan, Cornell University

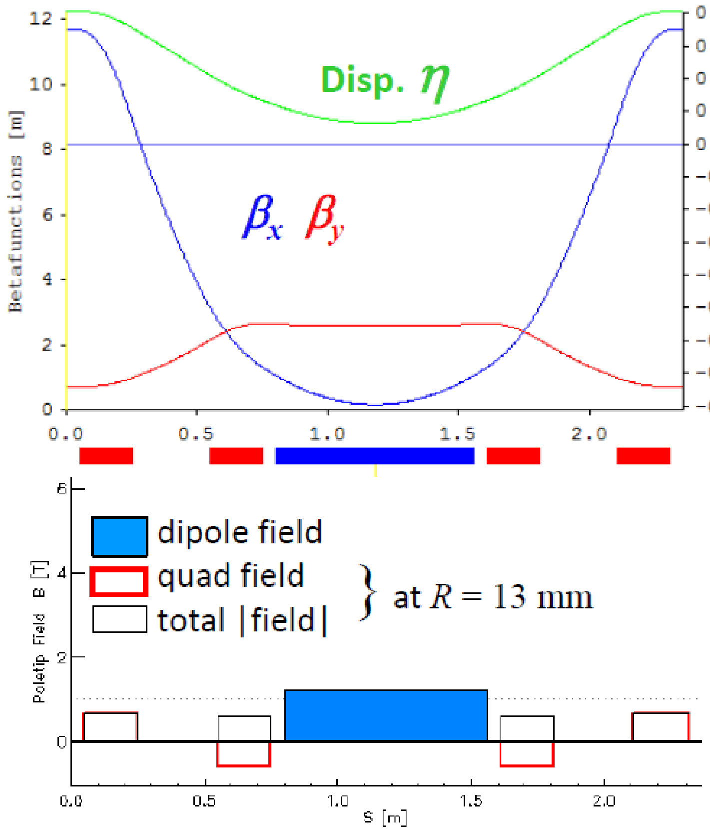


Overview

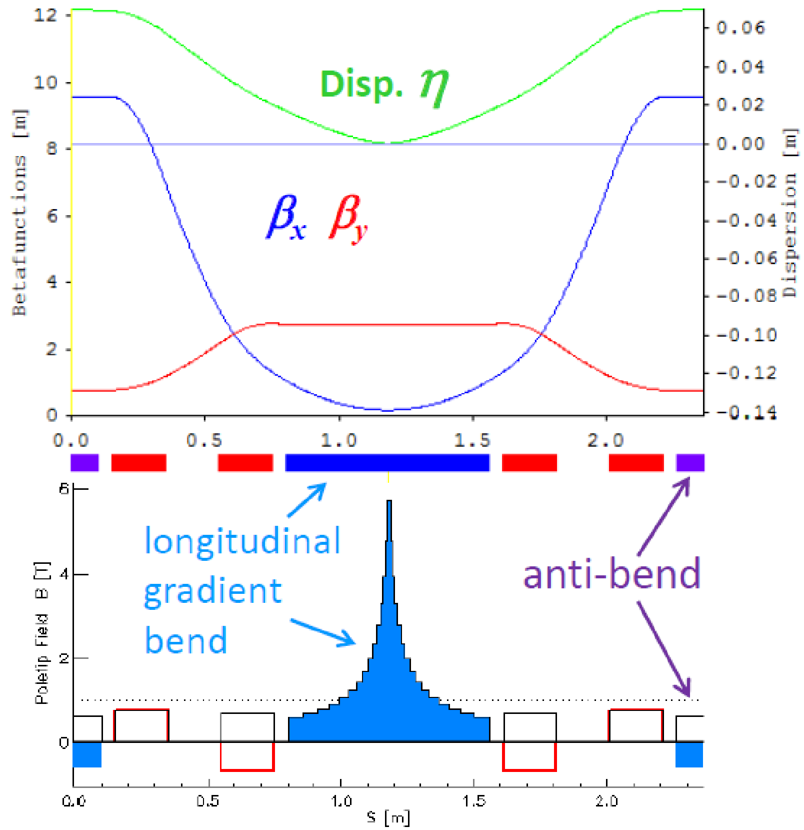
- PSI is planning an SLS upgrade, tentatively called SLS2.
- SLS2 will replace the storage ring with a design that achieves lower emittance by exploiting small NEG-coated vacuum chambers and LGB+AB optics.
  - Small chambers allow for tighter focusing.
  - Longitudinal Gradient Bends (LGB) minimize the I_5 integral.
  - Anti-Bends (AB) allow greater freedom to focus the dispersion function into the bends.
- LGBs are a non-conventional lattice element, whose modelling is not trivial.
- We consider two types of LGB: C-shaped Dipole & Canted Cosine-Theta Solenoid.
- And two modelling techniques: "Stack of sbends" & Fitted Maps

LGB Theory

conventional: $\varepsilon = 990\ \text{pm}\ (F = 3.4)$



LGB/AB: $\varepsilon = 200\ \text{pm}\ (F = 0.69)$



Emittance generated in a bend scales with:

$$I_{5a} = \oint ds\, g^3\, \mathcal{H}_a$$

This bend field profile minimizes I_5.

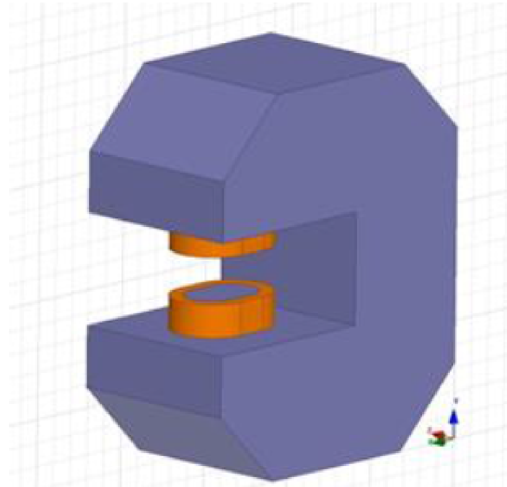
1. C-shaped dipole

ALS-style magnet

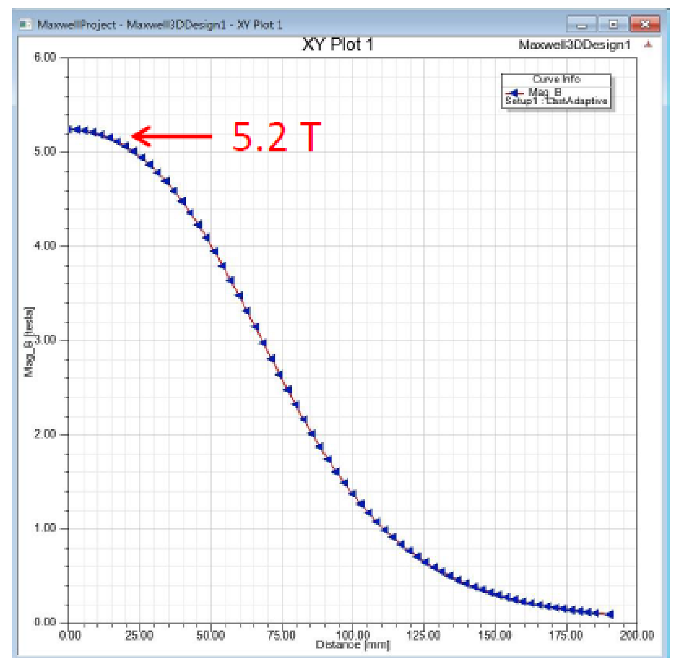
- C-shape yoke
  - open to ring outside (radiation)
  - retractable from vacuum chamber
- Racetrack coils
  - no torsion of conductor
  - any geometry: tape, wire...
- Warm bore
  - removable vacuum chamber
  - decoupling vacuum ↔ cryogenics
- Cold iron
  - avoid air gap between coil and iron to enhance field
  - one cryostat for yoke, upper and lower coils

☺ simple geometry

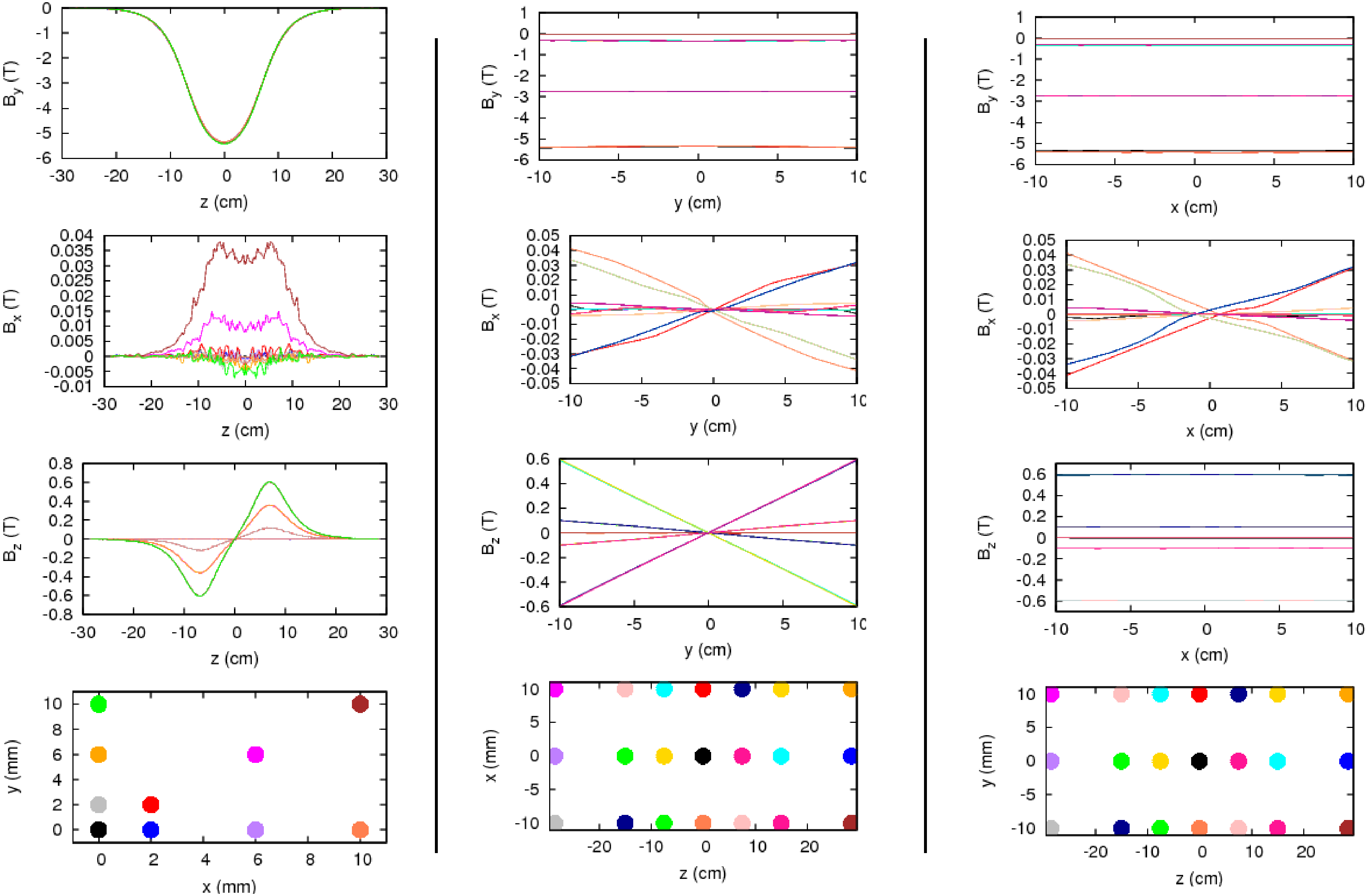
☹ bell shaped rather than hyperbolic field
⇨ +45% in emittance



Courtesy Alexander Anghel PSI



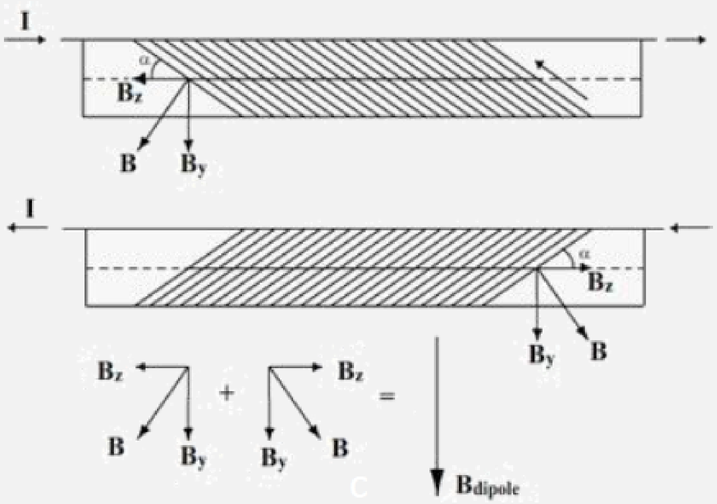
Field of C-Shape Magnet from CAD Model



- **For this C-Shape magnet CAD model, B_x is relatively small and dominated noise, and assumed to be an artifact.**

2. Canted-cosine-theta (CCT) solenoid

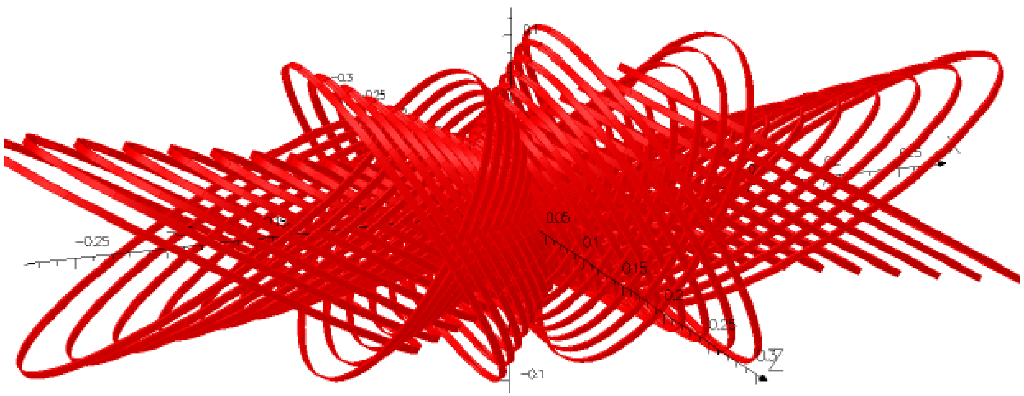
CCT principle



- Pure dipole field ($\cos\vartheta$) from superposition of canted solenoids
- Any field profile by adding layers
- Mechanical support of each winding → low magnetic forces

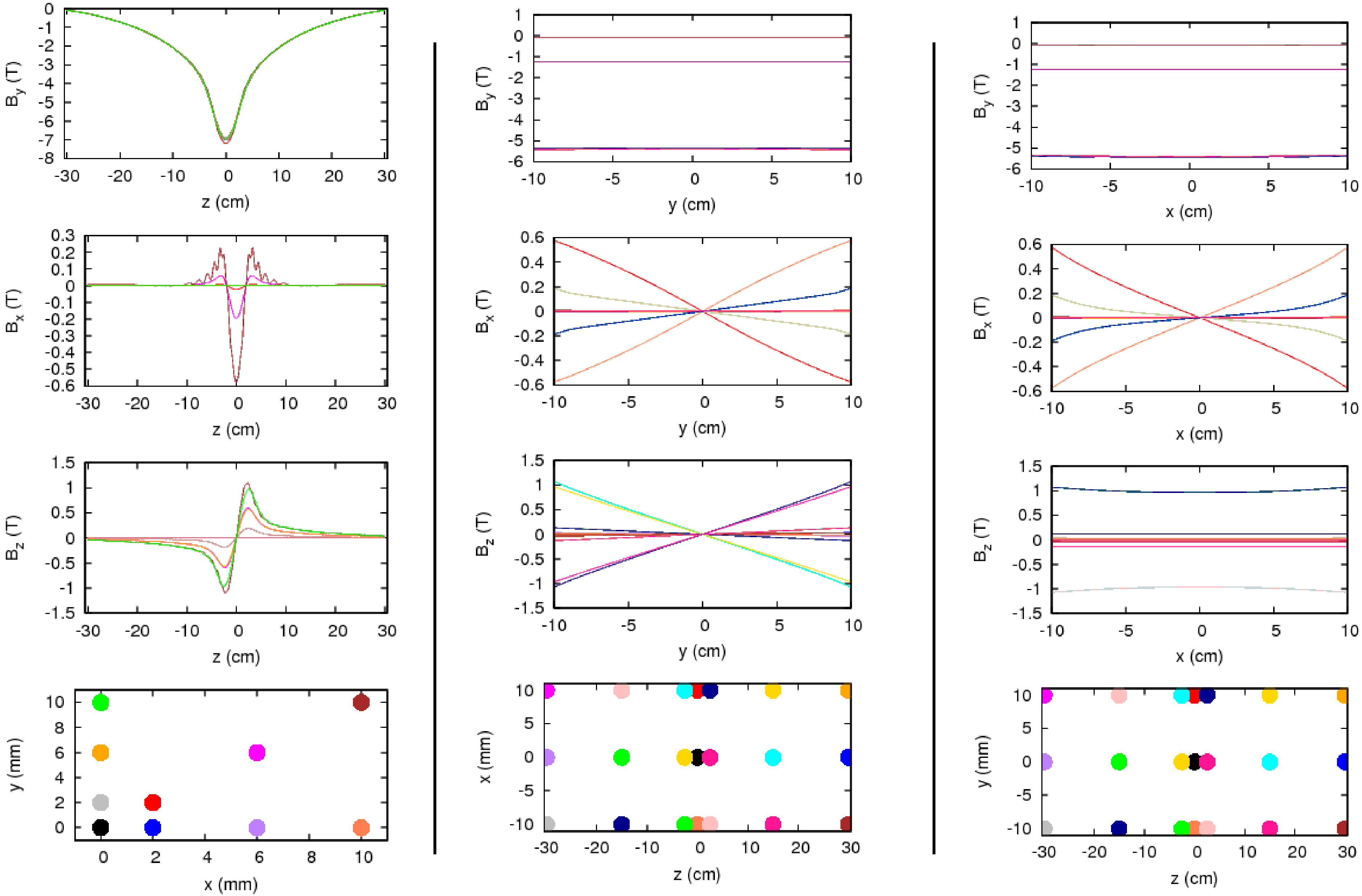
- L. Caspi and L. Brouwer, BeMa2 workshop, Bad Zurzach, Dec. 1-4, 2014
- D. I. Meyer and R. Flasck, NIM 80 (1970) 339

Modification for SLS-2 super-LGB



- hyperbolic mandrel
  - follow radiation fan opening
  - lower field towards edges
- superimposed layers
  - approximate hyperbolic field profile
- ☺ very low magnetic forces
- ☹ complex geometry & engineering

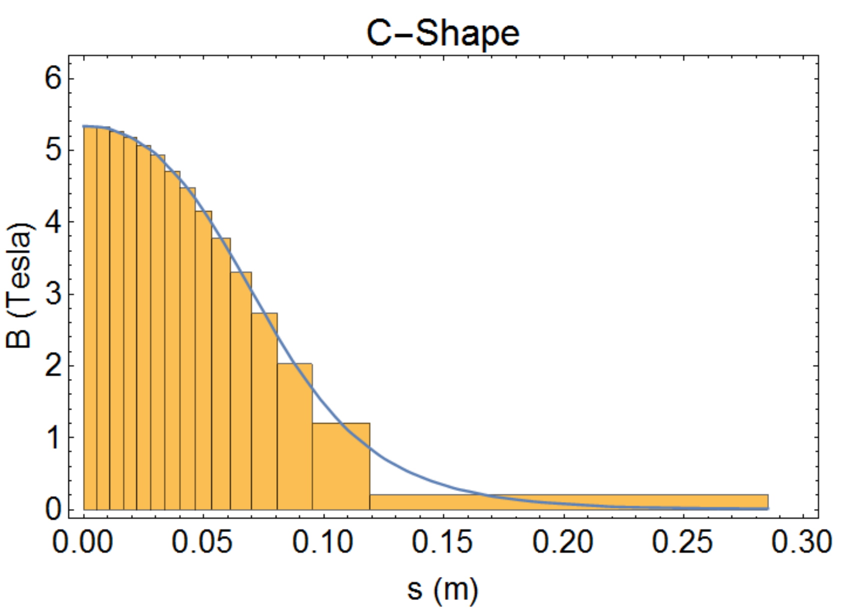
Field of Canted Solenoid Magnet from CAD Model

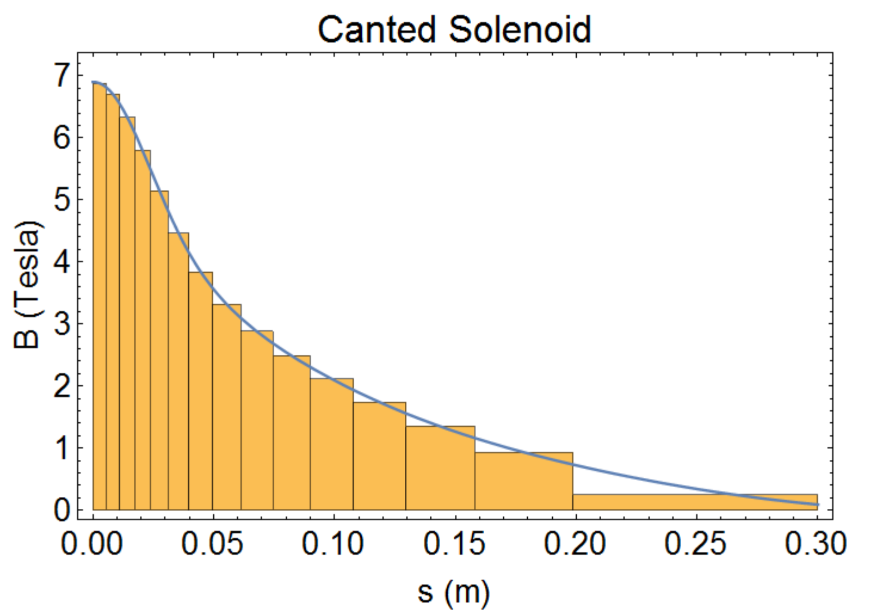


- For canted solenoid magnet, B_x is not negligible.

Stack of Sector Bends Model

- A sequence of standard sbends is fit to the LGB profile.
  - Can be easily implemented in any accelerator code.
- Based on bend field B_y along axis of magnet body.
- B_y is from either a rad-int optimization process or magnet designers (CAD).
- Construction Method:
  1. Divide total bend angle by some number of slices.
  2. Stack slices one-at-a-time, adjusting length to get bend-angle. Field strength for slice set by B_y data.
  3. Face angles are set to yield a stack with parallel faces. Stack is rectangular.
     1. Fringe fields are





Map Model (1/2)

- For CESR and CESR-TA, fitted field maps were used to perform lattice calculations on wiggler-dominated storage rings.
  - Documented in Bmad Manual [1].
- Generalized the wiggler-modeling technique to work with LGB-dominated storage rings.
  - Work by Noah Bittermann (summer student U. Minn).
  - Developed in collaboration with David Sagan at Cornell.
- Wigglers have a net-zero bend field, and so a constant term was not needed.
  - The basis was not complete.
- LGBs have a net bend field, and require a constant offset.
  - Modify series to allow for constant terms.
  - Makes the series complete.
  - Updates are documented in latest edition of Bmad Manual.

[1] “The Bmad Reference Manaul,” David Sagan, available online.

Map Model (2/2)

- Series from solution to source-free Ampere's Law in Coulomb gauge.
- Satisfies Maxwell's equations term-wise.
- Complete, but not orthogonal.
- Using Lie exponential of Hamiltonian, series is easily transformed into symplectic integrator.
- Fit to 3D gridded B-vector data from CAD model of magnet.
- Result: ability to symplectically integrate through arbitrary magnetic fields with arbitrary precision and accuracy.
- **Can treat map-model elements same as any other lattice element.**
  - **Linear optics calculations**
  - **Long-term tracking studies**
  - **Allows for misalignment studies**

$$\mathbf{B}(x,y,z) = \sum_i \mathbf{B}_i(x,y,z;C,k_x,k_y,k_z,x_0,y_0,\phi_z,plane)$$

$$\begin{aligned}
B_x &= -C\,\frac{k_x}{k_y}\,\sin(k_x x+\phi_x)\,\sinh(k_y y+\phi_y)\,\cos(k_z z+\phi_z)\\
B_y &= \quad C\,\cos(k_x x+\phi_x)\,\cosh(k_y y+\phi_y)\,\cos(k_z z+\phi_z)\\
B_s &= -C\,\frac{k_z}{k_y}\,\cos(k_x x+\phi_x)\,\sinh(k_y y+\phi_y)\,\sin(k_z z+\phi_z)
\end{aligned}$$

$$\text{with } k_y^2 = k_x^2 + k_z^2\ .$$

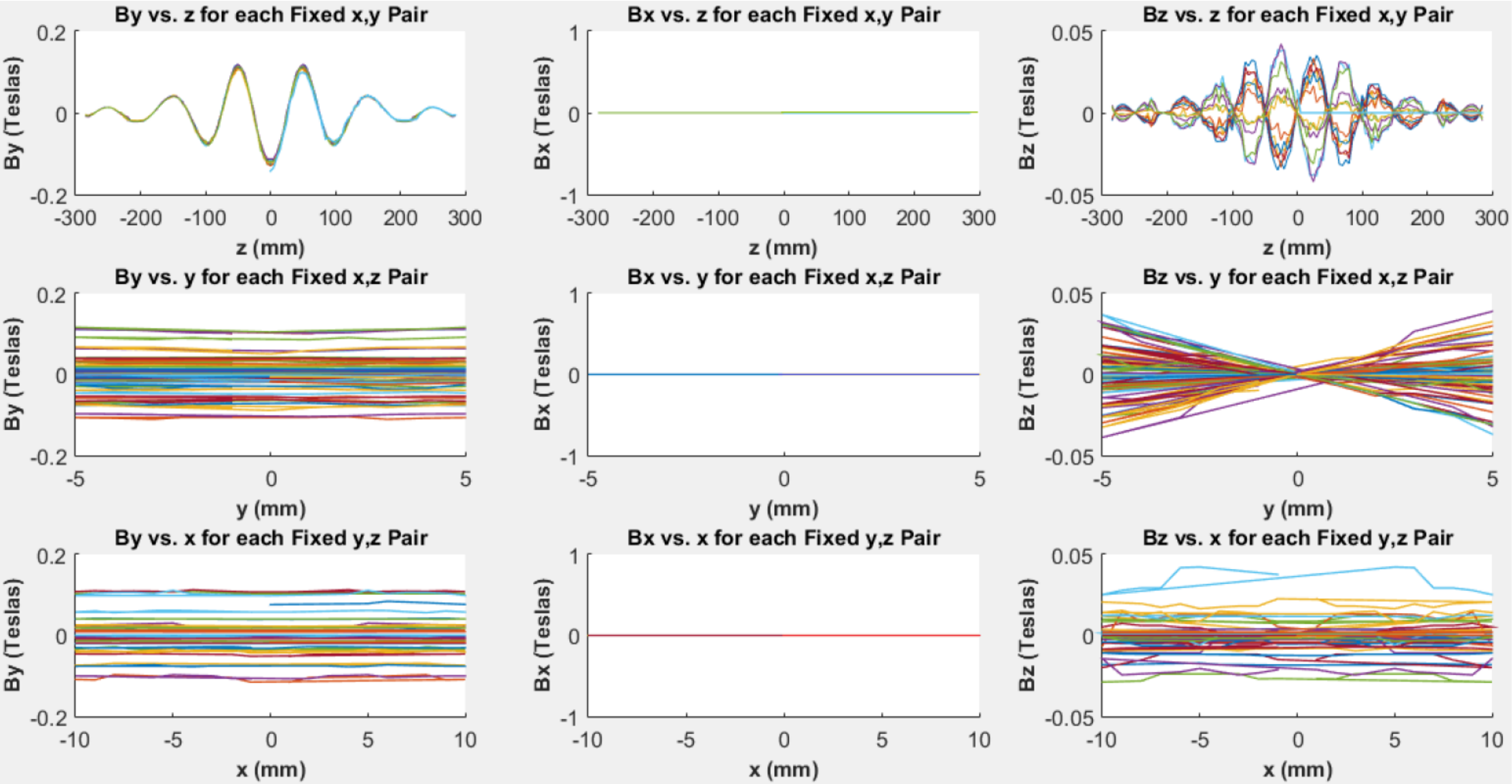
$$\begin{aligned}
B_x &= \quad C\,\frac{k_x}{k_z}\,\sinh(k_x x+\phi_x)\,\sinh(k_y y+\phi_y)\,\cos(k_z z+\phi_z)\\
B_y &= \quad C\,\frac{k_y}{k_z}\cosh(k_x x+\phi_x)\,\cosh(k_y y+\phi_y)\,\cos(k_z z+\phi_z)\\
B_s &= -C\,\cosh(k_x x+\phi_x)\,\sinh(k_y y+\phi_y)\,\sin(k_z z+\phi_z)
\end{aligned}$$

$$\text{with } k_z^2 = k_x^2 + k_y^2\ ,$$

$$\begin{aligned}
B_x &= \quad C\,\sinh(k_x x+\phi_x)\,\sin(k_y y+\phi_y)\,\cos(k_z z+\phi_z)\\
B_y &= \quad C\,\frac{k_y}{k_x}\cosh(k_x x+\phi_x)\,\cos(k_y y+\phi_y)\,\cos(k_z z+\phi_z)\\
B_s &= -C\,\frac{k_z}{k_x}\,\cosh(k_x x+\phi_x)\,\sin(k_y y+\phi_y)\,\sin(k_z z+\phi_z)
\end{aligned}$$

$$\text{with } k_x^2 = k_y^2 + k_z^2\ .$$

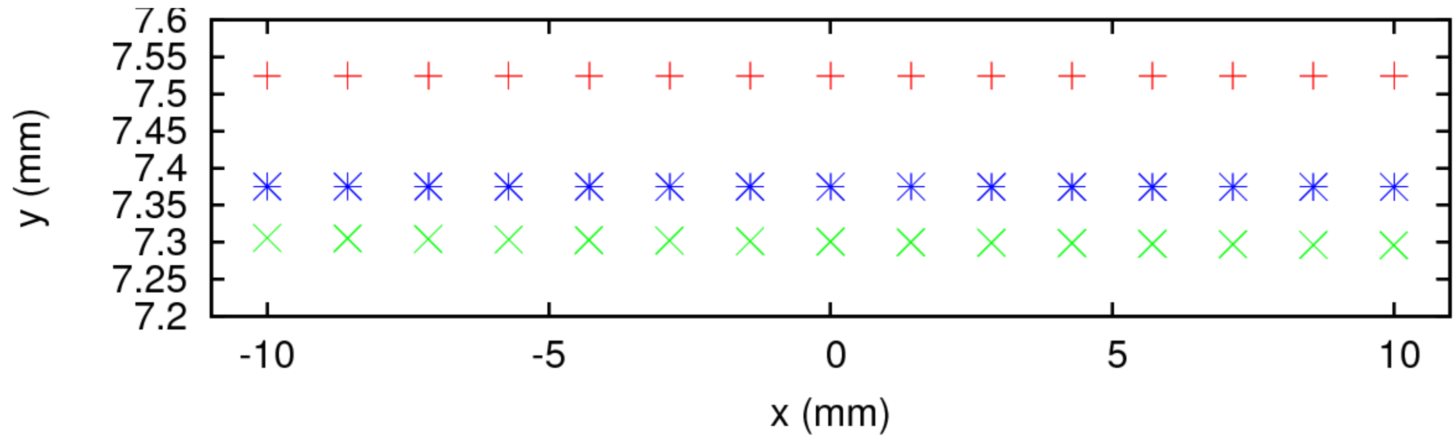
Residuals of 7-Term Fit to C-Shape Magnet

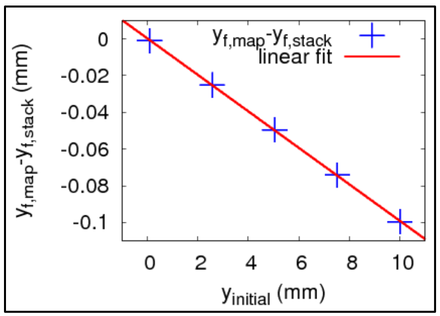


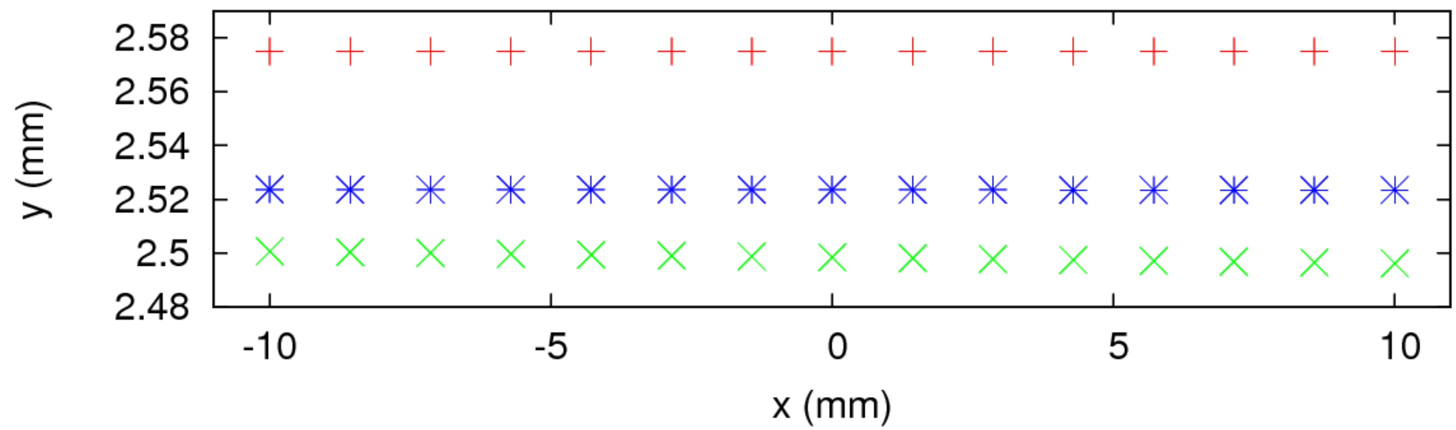
C-Shaped LGB–7 Terms	B_x (T)	B_y (T)	B_z (T)
Maximum Residual Magnitude	0.0	1.4×10^{-1}	4.2×10^{-1}
Sum of Residual Magnitudes	0.0	8.6×10^{2}	1.6×10^{2}
Average Residual (23,776 data points)	0.0	3.6×10^{-2}	6.8×10^{-3}

Note: Lattice with 12 Map LGBs is about 30% slower to track than with stack LGBs.

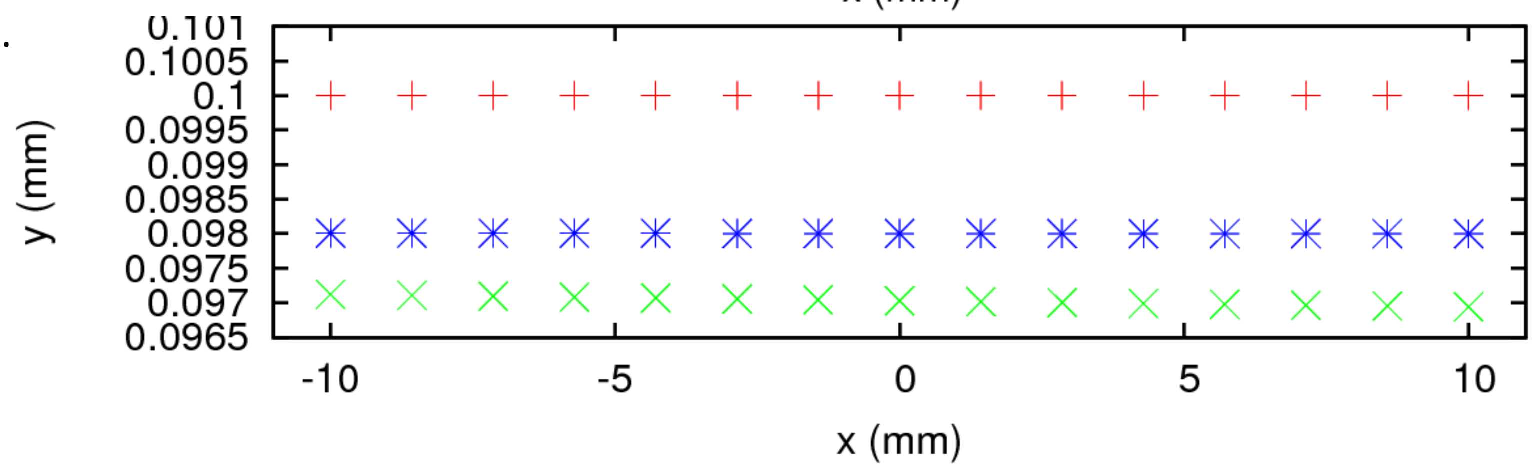
C-Shape Trajectory Comparisons





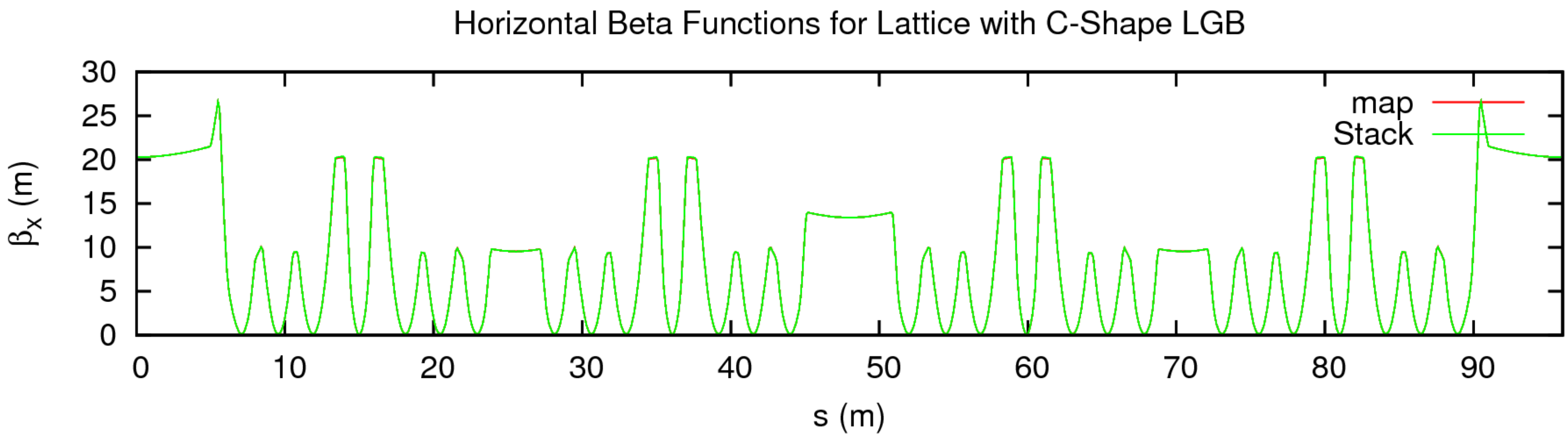


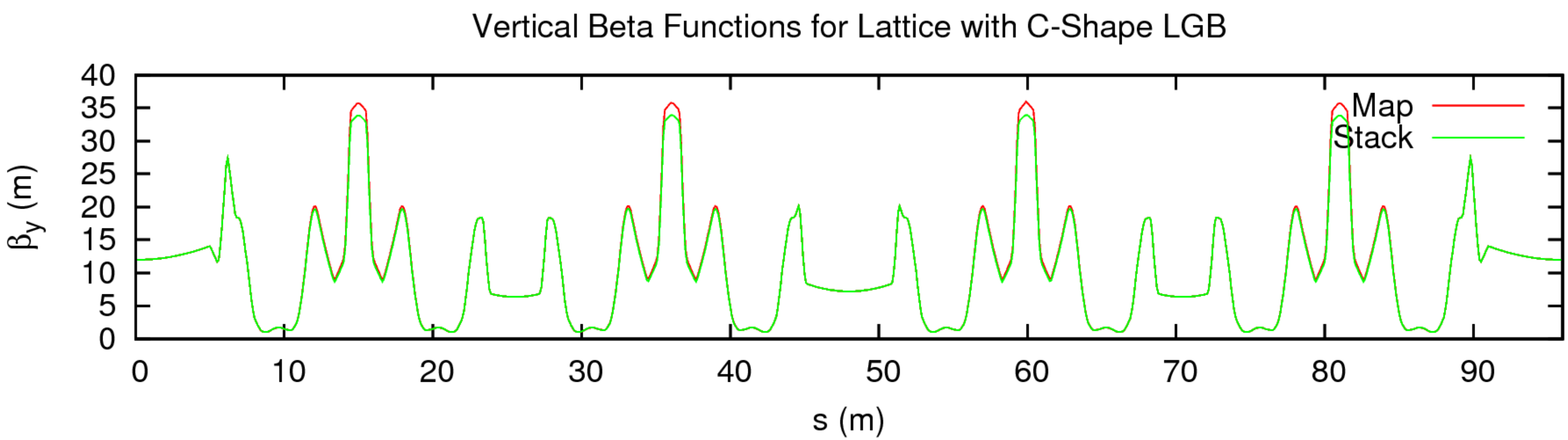
- Vert. residual versus offset is mostly linear, and independent of x.



Linear optics (C-Shape)

- Arc quadrupoles and dispersion-focusing bends matched for Map or Stack model.





Emittance Map: 262 pm

Emittance Stack: 230 pm

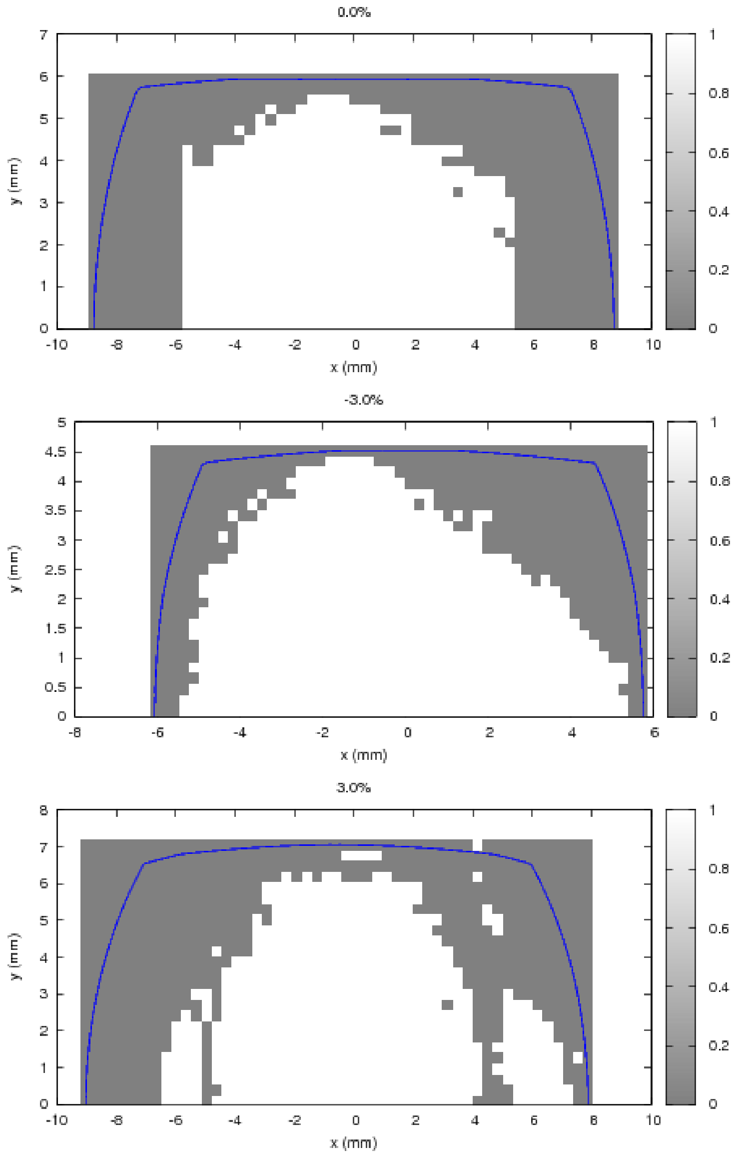
Natural Chromaticity Map: (-162, -80.7)

Natural Chromaticity Stack: (-162, -76.5)

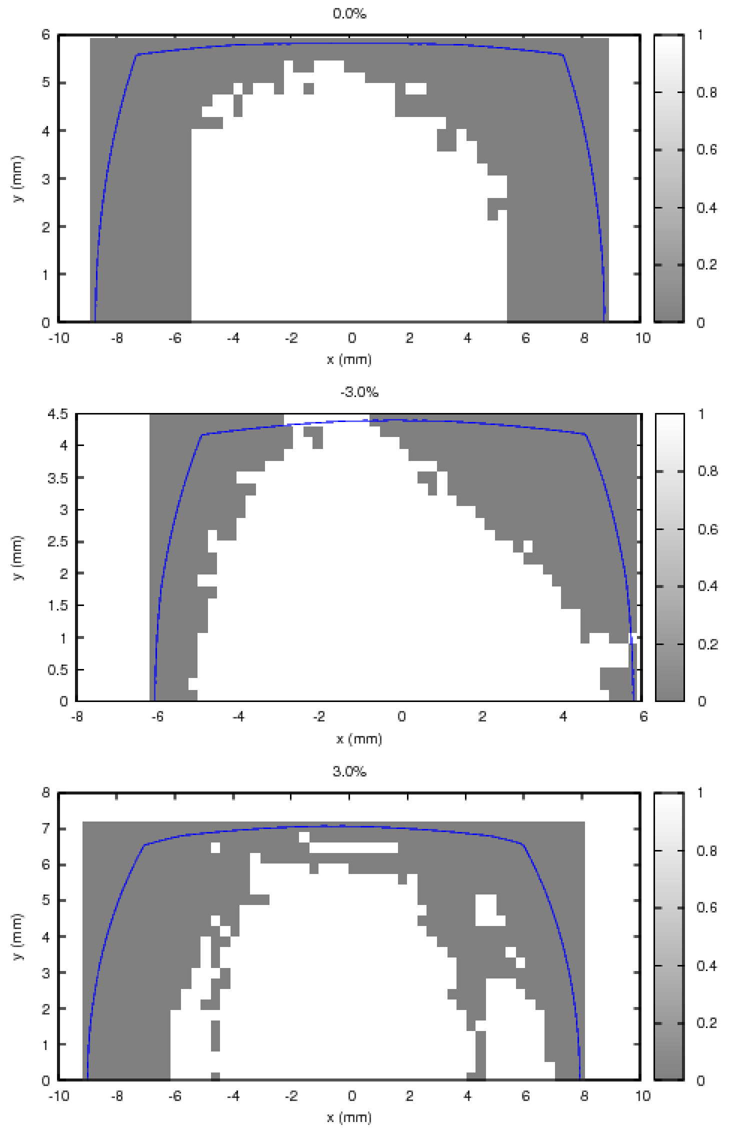
DA Correction (C-Shape)

- Sextupoles & Octupoles set using genetic optimizer.
- Completely random initial populations, no seeding.
- Both arrive at similar results.

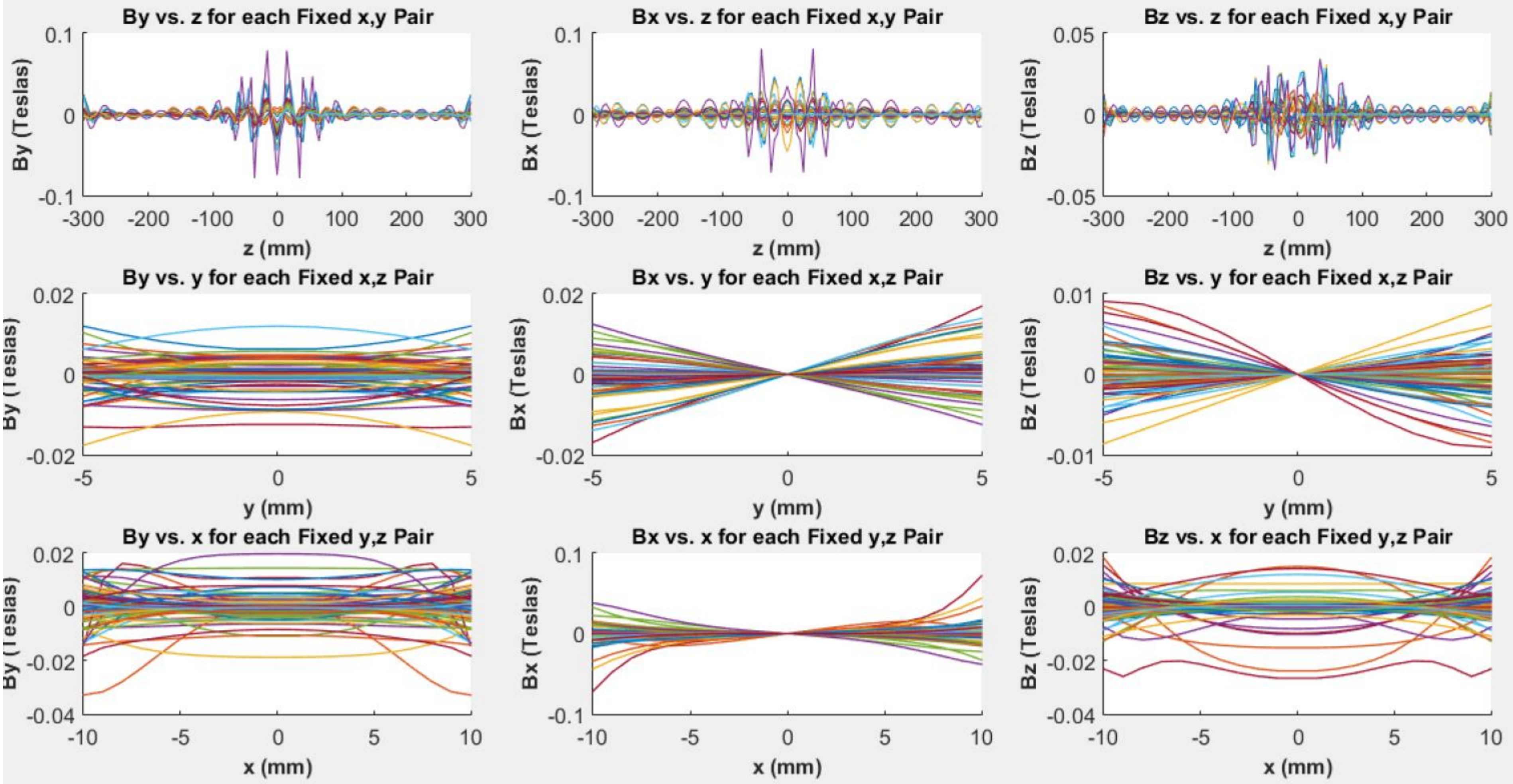
Stack Model



Map Model



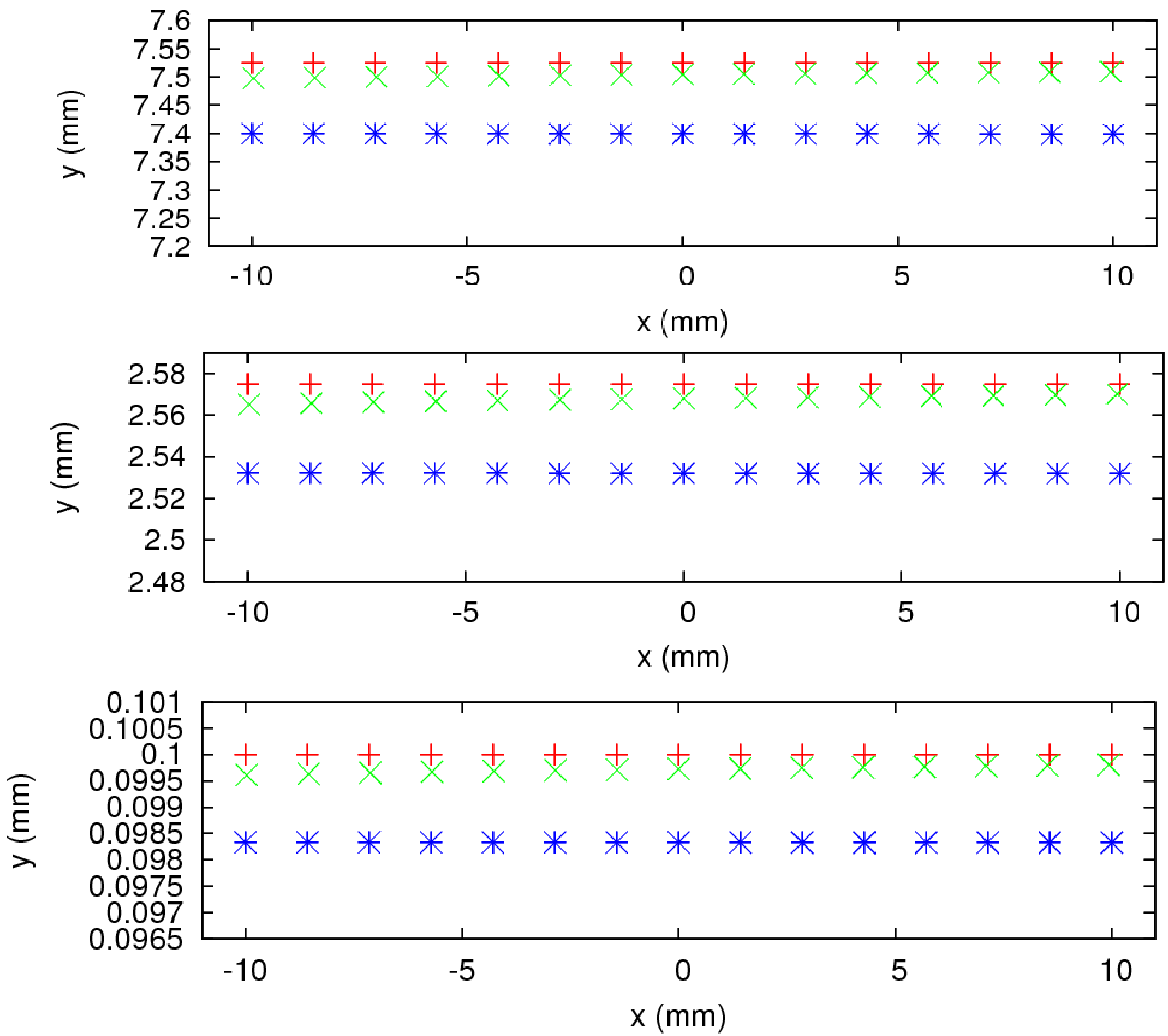
Residuals of 9-Term Fit to CCT Magnet



Canted Solenoid LGB–9 Terms	B_x (T)	B_y (T)	B_z (T)
Maximum Residual Magnitude	8.1×10^{-2}	7.9×10^{-2}	3.4×10^{-2}
Sum of Residual Magnitudes	1.78×10^{2}	2.0×10^{2}	1.3×10^{2}
Average Residual (50,900 data points)	3.0×10^{-3}	3.3×10^{-3}	2.1×10^{-3}

Note: Lattice with 12 Map LGBs is about 30% slower to track than with stack LGBs.

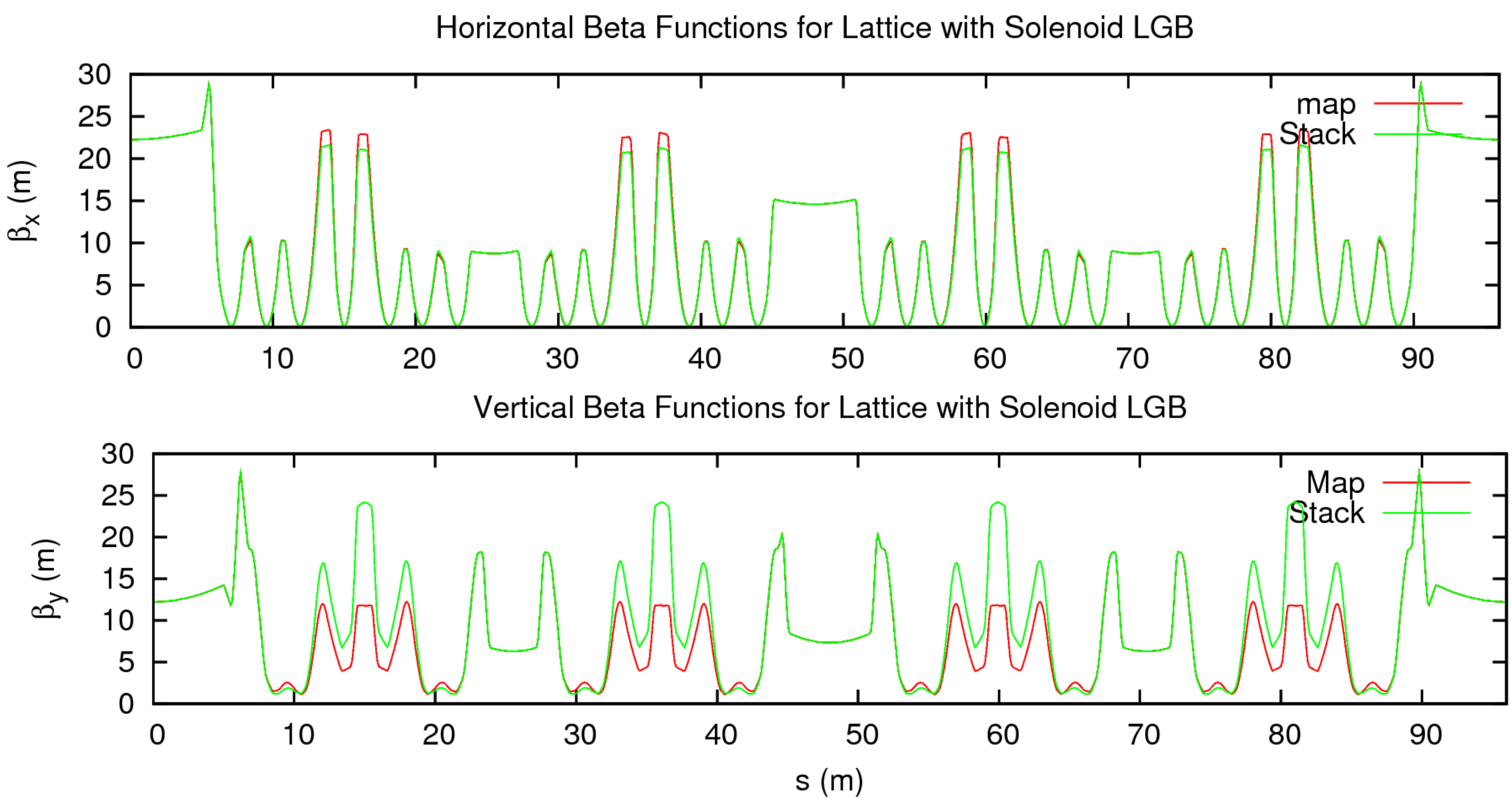
Canted Solenoid Trajectory Comparisons



- B_x field along mid-plane complicates trajectory.
- Vertical residual depends on x.
- A noticeable horizontal residual is present.

Linear optics (Solenoid)

- Arc quadrupoles and dispersion-focusing bends matched for Map or Stack model.



Emittance Map: 198 pm

Emittance Stack: 187 pm

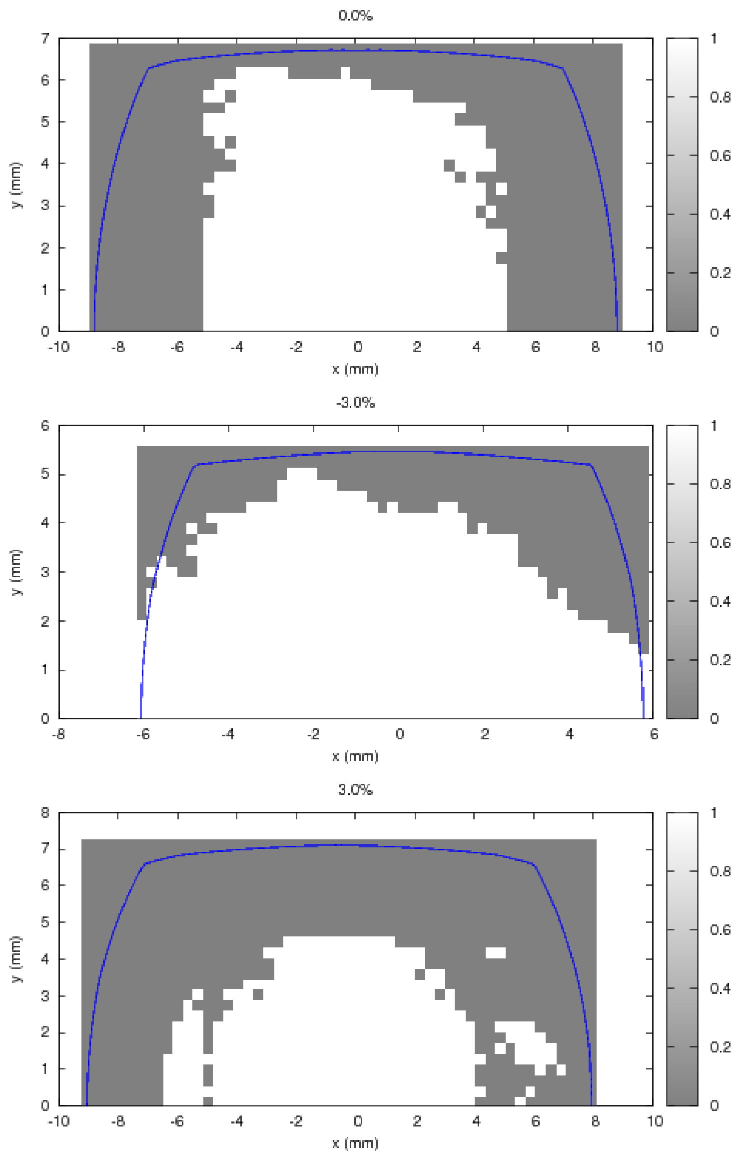
Natural Chromaticity Map: (-171, -53.0)

Natural Chromaticity Stack: (-166, -66.6)

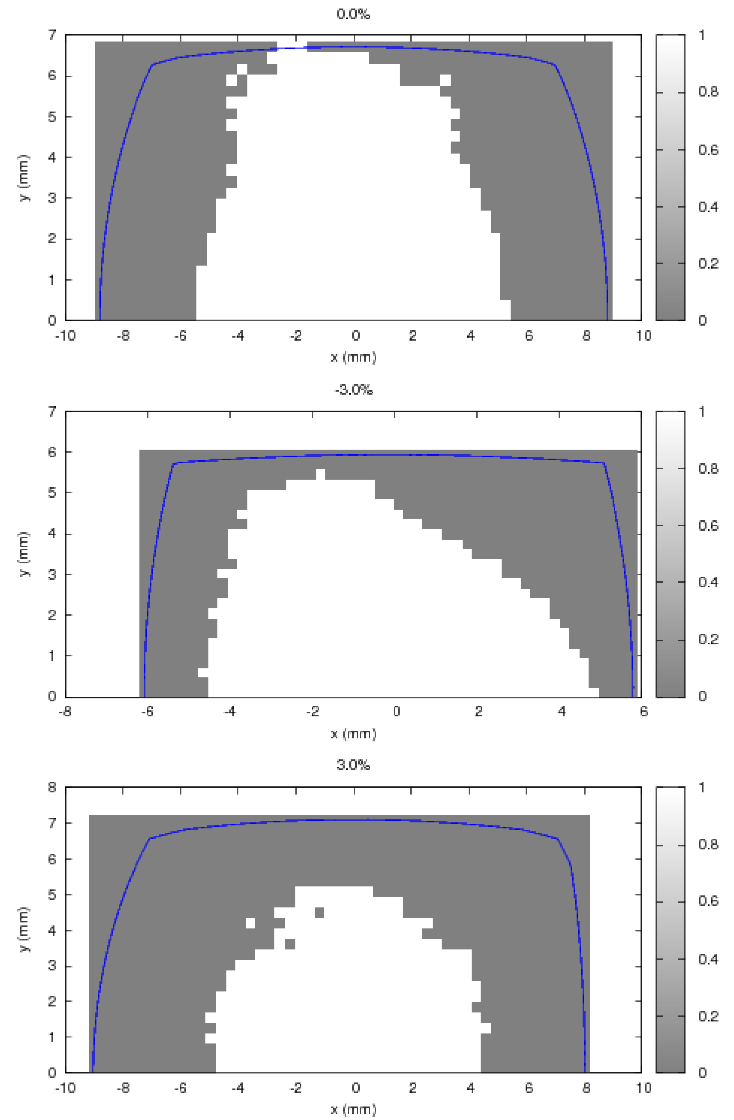
DA Correction (Solenoid)

- Sextupoles & Octupoles set using genetic optimizer.
- Completely random initial populations, no seeding.
- B_x field seems acceptable to large J particles.

Stack Model



Map Model



Further Techniques

- For c-shape magnet, use multipole expansion about closed orbit to find slice-by-slice focusing correction for stack (M. Aiba, PSI). (in progress)
  - Theory promising for C-Shaped magnet, not yet tested.
  - Expansion suggests that higher order components of field should be small (agrees with DA map result).
- Use kick-map (collaboration with Weiming Guo, BNL) as a check or alternative to fitted map method. (in progress)

Conclusion

- Magnet modelling method suitable for both linear optics calculations and long-term particle tracking implemented for SLS2 project.
  - Linear optics and rad. int. calculations depend on modeling method.
  - Stack method seems to get the linear optics wrong for CCT.
- Lattice optics depends on particular design of LGB magnet.
  - Bend field along axis is not a sufficient description of LGB.
- Modelling and theory suggest a slice-by-slice vertical focusing term may be sufficient to make stack method fairly accurate for C-Shaped magnet.