Higher order chromaticity correction for the future low emittance collider FCC-ee

Bastian Haerer (CERN, Geneva; KIT, Karlsruhe) for the FCC-ee lattice design team
Future Circular Collider Study

- European Strategy Group for Particle Physics 2013:
  “…to propose an ambitious post-LHC accelerator project……, CERN should undertake design studies for accelerator projects in a global context,…with emphasis on proton-proton and electron-positron high-energy frontier machines…..”
CERN Circular Colliders

Conceptual Design Report will be delivered at the next meeting of the European Strategy Group for High Energy Physics in 2018

Michael Benedikt
FCC-ee

One part of the Future Circular Collider Study

- 100 km e+/e- storage ring collider
- Precision studies of Z, W, H, t
  → Beam energies up to 175 GeV
- Beamstrahlung: mom. acceptance required: $\delta = \pm 2\%$
- Design luminosity: $L = O(10^{35} \text{ cm}^{-2}\text{s}^{-1})$
  → Strong focusing in final doublet quadrupoles ($\beta^*_y = 1 \text{ mm}$)
  → Very high chromaticity! ($Q'_y < -2000$)
12-fold layout

Circumference: 100 km
Arc length: 6.8 km
Straight section length: 1.5 km

4 interaction regions (IR) with mini-beta insertions

B = bending magnet, Q = quadrupole, S = sextupole
FODO & low emittance

\[ \varepsilon_x = \frac{C_g \gamma^2 \theta^3 F}{J_x} \]  

\( L = 50 \text{ m}, \mu_{x/y} = 90^\circ/60^\circ \)

- \( \gamma = 342466 \ (175 \text{ GeV}) \)
- \( \theta = 1.96 \text{ mrad} \)
- \( F = 3.125 \)

MADX Emit:
- \( \varepsilon_x = 1.00 \text{ nm rad} \)
- \( U_0 = 8.05 \text{ GeV/turn} \)

(*) L.C.Teng: Minimizing the Emittance in Designing the Lattice of an Electron Storage Ring
Mini-beta insertion

No local chromaticity correction scheme!

\[ L^* = 2 \, \text{m} \]
\[ \beta_x^* = 1 \, \text{m} \]
\[ \beta_y^* = 0.001 \, \text{m} \]
Chromaticity

- Change of the tune with energy deviation

Textbook:

\[ \Delta Q = \xi \cdot \frac{\Delta p}{p} \]

In our case not precise enough:

\[ Q(\delta) = Q_0 + \frac{\partial Q}{\partial \delta} \delta + \frac{1}{2} \frac{\partial^2 Q}{\partial \delta^2} \delta^2 + \frac{1}{6} \frac{\partial^3 Q}{\partial \delta^3} \delta^3 + \ldots \]
FCC-ee: Natural Chromaticities

<table>
<thead>
<tr>
<th>4 IRs</th>
<th>ΔQ (δ=1.5 %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_x$</td>
<td>506.16</td>
</tr>
<tr>
<td>$Q_x'$</td>
<td>$-6.22 \times 10^2$</td>
</tr>
<tr>
<td>$Q_x''$</td>
<td>$-1.37 \times 10^4$</td>
</tr>
<tr>
<td>$Q_x^{(3)}$</td>
<td>$-2.25 \times 10^8$</td>
</tr>
<tr>
<td>$Q_x^{(4)}$</td>
<td>$-1.86 \times 10^{13}$</td>
</tr>
<tr>
<td>$Q_y$</td>
<td>334.28</td>
</tr>
<tr>
<td>$Q_y'$</td>
<td>$-2.06 \times 10^3$</td>
</tr>
<tr>
<td>$Q_y''$</td>
<td>$-8.50 \times 10^6$</td>
</tr>
<tr>
<td>$Q_y^{(3)}$</td>
<td>$-1.94 \times 10^{11}$</td>
</tr>
<tr>
<td>$Q_y^{(4)}$</td>
<td>$-6.40 \times 10^{15}$</td>
</tr>
</tbody>
</table>

- **1st order correction**  
  → Straight forward...

- **Higher orders**  
  → First approach: Montague formalism
Montague functions

- Chromatic variables

\[ B = \frac{1}{\beta} \frac{\partial \beta}{\partial \delta} \quad A = \frac{\partial \alpha}{\partial \delta} - \frac{\alpha}{\beta} \frac{\partial \beta}{\partial \delta} \]

- W-vector

\[ \vec{W} = \frac{1}{2} (B + iA) = \frac{1}{2} \sqrt{A^2 + B^2} e^{i2\mu} \]

Final Focus Quad:
\[ \Delta A \approx -\beta_0 k_0 l_q \]

Sextupole:
\[ \Delta A \approx -\beta_0 k'_0 D_x l_s \]

Rotates with twice the phase advance!
Phase advance FD – 1st Sext.

Final doublet

MQDIR1  MQFIR1

First arc FODO cell

SD1  SD1  SF1  SF1

\[ \mu_x = m^*\pi \]

\[ \mu_y = n^*\pi \]

(m,n integer)
FCC-ee sextupole scheme

$\mu_x = 90^\circ = \pi/2$

$\mu_y = 60^\circ = \pi/3$

$\mu_x = 180^\circ = \pi$ (→ -I transformation)

Even number of sextupoles per family!
-l transformation

• Sextupoles of each family are in phase

→ W-vector rotates by $2\pi$
W functions in the 1st quarter

$W_x, W_y$

Arc 1 | Arc 2 | Arc 3

MAD-X 5.02.03 13/01/15 16.32.19
Momentum acceptance

\([-0.22\%, 0.06\%]\)

\([-0.08\%, 0.06\%]\)
**Corrected Chromaticity**

<table>
<thead>
<tr>
<th></th>
<th>Natural Chromaticities</th>
<th>Corrected Chromaticities</th>
<th>ΔQ (δ=0.05 %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_x$</td>
<td>506.16</td>
<td>506.16</td>
<td></td>
</tr>
<tr>
<td>$Q_x'$</td>
<td>$-6.22 \times 10^2$</td>
<td>$-3.47 \times 10^{-6}$</td>
<td>0.000</td>
</tr>
<tr>
<td>$Q_x''$</td>
<td>$-1.37 \times 10^4$</td>
<td>$-2.70 \times 10^3$</td>
<td>0.000</td>
</tr>
<tr>
<td>$Q_x^{(3)}$</td>
<td>$-2.25 \times 10^8$</td>
<td>$-8.24 \times 10^6$</td>
<td>0.000</td>
</tr>
<tr>
<td>$Q_x^{(4)}$</td>
<td>$-1.86 \times 10^{13}$</td>
<td>$4.55 \times 10^{12}$</td>
<td>0.012</td>
</tr>
<tr>
<td>$Q_y$</td>
<td>334.28</td>
<td>334.28</td>
<td></td>
</tr>
<tr>
<td>$Q_y'$</td>
<td>$-2.06 \times 10^3$</td>
<td>$-1.22 \times 10^{-5}$</td>
<td>0.000</td>
</tr>
<tr>
<td>$Q_y''$</td>
<td>$-8.50 \times 10^6$</td>
<td>$9.07 \times 10^3$</td>
<td>0.001</td>
</tr>
<tr>
<td>$Q_y^{(3)}$</td>
<td>$-1.94 \times 10^{11}$</td>
<td>$-2.17 \times 10^9$</td>
<td><strong>-0.045</strong></td>
</tr>
<tr>
<td>$Q_y^{(4)}$</td>
<td>$-6.40 \times 10^{15}$</td>
<td>$4.87 \times 10^{13}$</td>
<td><strong>0.126</strong></td>
</tr>
</tbody>
</table>
3\textsuperscript{rd} order chromaticity

\[ b_2 = \frac{1}{\beta} \frac{\partial^2 \beta}{\partial \delta^2} \]

Put Sextupole where \( b_2 \) is large

The non-linear dispersion (D') is less than 1 m and therefore very small, so the correction of the 3rd order chromaticity via the 3rd term of equation (3.15) does not work (the sextupole strength would be too high). Therefore he uses the 4th term. The dispersion at the end of the final focus telescope is very high and the derivative of \( a_1 + b_1 \) as well (compare figure 3.1).
IR with local chromaticity correction

Anton Bogomyagkov
Advantage of local CCS

![Graph showing local CCS advantage]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>ΔQ(2%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_x$</td>
<td>124.54</td>
<td></td>
</tr>
<tr>
<td>$Q'_x$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Q''_x$</td>
<td>170</td>
<td>0.034</td>
</tr>
<tr>
<td>$Q'''_x$</td>
<td>$-4.5 \cdot 10^4$</td>
<td>$-0.059$</td>
</tr>
<tr>
<td>$Q_x''''$</td>
<td>$-5.3 \cdot 10^6$</td>
<td>$-0.035$</td>
</tr>
<tr>
<td>$Q_y$</td>
<td>84.57</td>
<td></td>
</tr>
<tr>
<td>$Q'_y$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Q''_y$</td>
<td>387</td>
<td>0.077</td>
</tr>
<tr>
<td>$Q'''_y$</td>
<td>$-5.3 \cdot 10^5$</td>
<td>$-0.7$</td>
</tr>
<tr>
<td>$Q''''_y$</td>
<td>$-4.3 \cdot 10^6$</td>
<td>$-0.029$</td>
</tr>
</tbody>
</table>

3 orders of magn. smaller!!!

Anton Bogomyagkov
Next steps

- Get 3\textsuperscript{rd} order chromaticity under control
  \(\rightarrow\) Analyse higher order derivatives of \(\beta\) functions

- Optimise bandwidth and dynamic aperture using more sextupole families

- Try non-interleaved sextupole scheme

- Combine optimised arc and local chromaticity correction scheme for best performance
Discussions

• Which experience do you have with higher order chromaticity control?

• Which method and software do you use to increase energy acceptance and DA?
Thank you for your attention!